New essentials of economic theory II. Economic transactions, expectations and asset pricing

Olkhov, Victor

TVEL

20 April 2019

New Essentials of Economic Theory II.

Economic Transactions, Expectations and Asset Pricing

Victor Olkhov

Moscow, Russia

victor.olkhov@gmail.com

Abstract

This paper presents further development of our economic model. We describe economic and financial transactions between agents as factors that define evolution of economic variables. We show that change of risk ratings of agents as their coordinates on economic space due to economic activity or due to other reasons induce flows of economic transactions that contribute significantly to macroeconomic evolution. Transactions are made under numerous expectations of agents and agents establish their expectations on base of economic variables, transactions, other factors that impact economic evolution. We argue that economic value of expectations should be regarded proportionally to economic value of transactions made under these expectations. We describe transition from modeling transactions and expectations of separate agents to description of density functions of transactions and expectations on economic space. We derive systems of equations that describe density functions of transactions, expectations and their flows. We explain how transactions and expectations determine asset pricing and derive price equations. We use our model equations on economic variables, transactions, expectations and their flows for description of particular economic problems in Part III.

Keywords: Economic Theory, Risk Ratings, Economic Space, Economic Transactions, Expectations, Asset Pricing

JEL: C00, C02, C10, E00

This research did not receive any assistance, specific grant or financial support from TVEL or funding agencies in the public, commercial, or not-for-profit sectors.

1. Introduction

Preliminary notions, definitions and general treatment of our economic model are given in Part I. (Olkhov, 2019c). Here we explain only basic points of our approach to economic modeling. We use risk ratings of economic agents as their coordinates on economic space. That helps approximate economic and financial variables, transactions and expectations of numerous separate agents by description of variables, transactions and expectations as density functions on economic space. We take into account flows of economic variables, transactions and expectations induced by motion of separate agents on economic space due to change of agents risk ratings and describe macroeconomic impact of such economic flows. We introduce notions of economic transactions and expectations and their flows on economic space and study transactions, expectations and asset pricing (Olkhov, 2016a-2017b; 2018-2019b). In Sec. 2 we describe transactions between agents at points $\boldsymbol{x}$ and $\boldsymbol{y}$ on economic space and derive equations that model evolution of these transactions. In Sec.3 we argue description of expectations as density functions on economic space and derive equations on expectations. In Sec.4 we describe impact of transactions on asset pricing and derive equations on price evolution. Sec. 5 – Conclusion. In Appendix A we give derivation of assets pricing equations. In Part III of our paper we apply our economic model equations to description of several economic problems. We model business cycles, describe wave propagation for disturbances of economic variables and transactions, model asset price fluctuations and argue hidden complexities of classical Black-Scholes-Merton option pricing. We refer as (I.7) equation (7) in Part I and use bold letters to denote vectors and roman for scalars.

2. Economic transactions on economic space

In this Section we argue description of economic and financial transactions between agents. In Part I. (Olkhov, 2019c) we show that risk assessments of economic agents permit distribute them by their risk rating as coordinates on economic space. Here let's model economic transactions alike to description of economic variables (Part I. Sec.3). Let's study additive economic and financial variables that are subject of transactions between agents. Such additive variables are changed by transactions between agents. For example let's propose that agent i sell some amount of variable E to agent j. Let's take as E any economic or financial variable like goods, capital, service or commodities as Oil, Steel, Energy and etc. For example let's assume that agent i provide credits C to agent j. Such transactions between agents i and j change amount of credits C provided by i and change amount of loans L

received by *j*. Each transactions take certain time *dt* and we consider transactions as rate or speed of change of corresponding variable *E* of agents involved into transaction. For example all transactions of agent *i* at moment *t* during time [0, *t*] define change of variable *E* (Steel, Energy, Shares, Credits, Assets and etc.) owned by agent *i* during period [0, *t*].

To avoid excess specification of transactions between numerous separate agents let's replace description of transactions between separate agents by rougher description of transactions between points of economic space and average it during time Δ alike to (I.3; 4). Let's neglect granularity of separate agents and transactions between them and replace it by density functions of transactions on economic space. Below we study macroeconomic evolution under action on *n* risks and hence (I. 1.1) define economic domain on *n*-dimensional economic space R^n (Olkhov, 2019c). Similar to Part I. Sec.3 let's take that agents on economic space R^n at moment *t* have coordinates $\boldsymbol{x}=(x_1,...x_n)$ and risk velocities $\boldsymbol{\upsilon}=(\upsilon_1,...\upsilon_n)$. Risk velocities describe change of agents risk coordinates during time *dt*. Let's remind that all agents have coordinates inside *n*-dimensional economic domain (I.1.1). Hence for economy under action on *n* risks transactions between agents with coordinates $\boldsymbol{x}$ and agents with coordinates $\boldsymbol{y}$ are determined on *2n*-dimensional economic domain, $\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y})$:

$$\boldsymbol{z} = (\boldsymbol{x}, \boldsymbol{y}) \;\; ; \;\; \boldsymbol{x} = (x_1 \dots x_n) \;\; ; \; \boldsymbol{y} = (y_1 \dots y_n) \tag{1.1}$$

$$0 \le x_i \le 1 \,, i = 1, \dots n \;\; ; \;\; 0 \le y_j \le 1 \,, j = 1, \dots n \tag{1.2}$$

Relations (1.1; 1.2) define economic domain that is filled by pairs of agents with coordinates $\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y})$ on *2n*-dimensional economic space R^{2n}. Let's rougher description of transactions between agents and replace it by description of transactions between all agents at points $\boldsymbol{x}$ and $\boldsymbol{y}$. Let's take a unit volume $dV(z)$

$$dV(\boldsymbol{z}) = dV(\boldsymbol{x})dV(\boldsymbol{y}) \;\; ; \;\; \boldsymbol{z} = (\boldsymbol{x}, \boldsymbol{y}) \tag{1.3}$$

and assume that $dV(\boldsymbol{x})$ and $dV(\boldsymbol{y})$ follow relations (I.2) and their scales are small to compare with scales of economic domain (I.1.1) for $\boldsymbol{x}$ and $\boldsymbol{y}$. Let's propose:

$$dV_i \ll 1 \,, i = 1, \dots n \, ; \; dV(\boldsymbol{x}) = \prod_{i=1,..n} dV_i \tag{1.4}$$

$$dV_j \ll 1 \,, j = 1, \dots n \, ; \; dV(\boldsymbol{y}) = \prod_{j=1,..n} dV_j \tag{1.5}$$

Let's assume that each unit volume $dV(\boldsymbol{x})$ and $dV(\boldsymbol{y})$ contain a lot of agents with risk coordinates inside $dV(\boldsymbol{x})$ and $dV(\boldsymbol{y})$. Let's take time Δ small to compare with time scales of macroeconomic problem under consideration but assume that during time Δ agents inside $dV(\boldsymbol{x})$ and $dV(\boldsymbol{y})$ perform a lot of transactions between them. Let's rougher space description of transactions on (1.1; 1.2) by scales dV_i and rougher time description by scale Δ. As we keep space scales dV_i small to compare with scales of economic domain (1.1; 1.2) and time scale Δ small to compare with time scales of the macroeconomic problem hence we still use continuous approximation, but with rougher scales.

Let's denote $bs_{1,2}(t,\boldsymbol{x},\boldsymbol{y})$ as buy-sell transactions by variable *E* from agent *1* at point $\boldsymbol{x}$ to agent *2* at point $\boldsymbol{y}$. Economic variable *E* may be Oil, Steel, Shares, Credits, Assets and etc. that are supplied from agent *1* as seller at point $\boldsymbol{x}$ to agent 2 as buyer at point $\boldsymbol{y}$ at moment *t*. Let's aggregate all transactions with variable *E* performed by all agents inside $dV(\boldsymbol{x})$ and agents inside $dV(\boldsymbol{y})$. Similar to (I. 3;4) let's define transaction $BS(t,z)$ at point $z=(\boldsymbol{x},\boldsymbol{y})$ as sum of transactions $bs_{i,j}(t,\boldsymbol{x},\boldsymbol{y})$ between all agents *i* in a unit volume $dV(\boldsymbol{x})$ at point $\boldsymbol{x}$ and agents *j* in a unit volume $dV(\boldsymbol{y})$ at point $\boldsymbol{y}$ and average this sum during time Δ:

$$BS(t,\boldsymbol{x},\boldsymbol{y}) = \sum_{i\in dV(x);\, j\in dV(y);\, \Delta} bs_{i,j}(t,\boldsymbol{x},\boldsymbol{y}) \tag{2.1}$$

$$\sum_{i\in dV(x);\, \Delta} bs_{i,j}(t,\boldsymbol{x},\boldsymbol{y}) = \frac{1}{\Delta}\int_t^{t+\Delta} d\tau \sum_{i\in dV(x);\, j\in dV(y)} bs_{i,j}(t,\boldsymbol{x},\boldsymbol{y}) \tag{2.2}$$

Integral of $BS(t,z)$ by variable $d\boldsymbol{y}$ over economic domain (I.1.1) defines all sells $BS(t,\boldsymbol{x})$ of variable *E* performed by agents inside a unit volume $dV(\boldsymbol{x})$ at point $\boldsymbol{x}$

$$BS(t,\boldsymbol{x}) = \int d\boldsymbol{y}\; BS(t,\boldsymbol{x},\boldsymbol{y}) \tag{3}$$

and integral of $BS(t,\boldsymbol{x})$ by variable $d\boldsymbol{x}$ over economic domain define all sells $BS(t)$ of variable *E* performed by all agents in macroeconomics at moment *t*.

$$BS(t) = \int d\boldsymbol{x}\; BS(t,\boldsymbol{x}) = \int d\boldsymbol{x}d\boldsymbol{y}\; BS(t,\boldsymbol{x},\boldsymbol{y}) \tag{4.1}$$

For example, if $CI(t)$ (4.2) defines cumulative investment made in entire economy during term [0,t] and $BS(t,\boldsymbol{x},\boldsymbol{y})$ – investment transactions made from $\boldsymbol{x}$ to $\boldsymbol{y}$ during time term dt then

$$\frac{d}{dt}CI(t) = BS(t) = \int d\boldsymbol{x}d\boldsymbol{y}\; BS(t,\boldsymbol{x},\boldsymbol{y}) \tag{4.2}$$

Hence transactions define time derivative of cumulative macroeconomic and financial variables like investment made during time term, credits provided during time term, year GDP, and etc. Macros transactions $BS(t,z)$ on economic domain (1.1; 1.2) describe evolution of macroeconomic and financial variables. Relations (4.1-4.2) define macroeconomic variables as integrals of transactions $BS(t,z)$ over economic domain. Let's call $BS(t,z)$ as transactions density functions on *2n*-dimensional economic domain similar to economic density function $A(t,\boldsymbol{x})$ (I.3).

Let's remind that transactions densities $BS(t,z)$, $z=(\boldsymbol{x},\boldsymbol{y})$ are determined as aggregates of transactions between agents at points $\boldsymbol{x}$ and $\boldsymbol{y}$. As we argue in Part I each agent *i* on economic domain is described by its risk coordinates $\boldsymbol{x}_i$ and its velocity $\boldsymbol{v}_i$. Thus similar to (I.6) let's define flows $\boldsymbol{p}_{ij}(t,z)$ (5.1;5.2) of transactions $bs_{ij}(t,z)$ between agents:

$$\boldsymbol{p}_{ij}(t,\boldsymbol{z}) = \left(\boldsymbol{p}_{xij}(t,\boldsymbol{z}), \boldsymbol{p}_{yij}(t,\boldsymbol{z})\right) \tag{5.1}$$

$$\boldsymbol{p}_{xij}(t,\boldsymbol{z}) = bs_{i,j}(t,\boldsymbol{z})\boldsymbol{v}_i(t,\boldsymbol{x}) \quad ; \quad \boldsymbol{p}_{yij}(t,\boldsymbol{z}) = bs_{i,j}(t,\boldsymbol{z})\boldsymbol{v}_j(t,\boldsymbol{y}) \tag{5.2}$$

Flows $\boldsymbol{p}_{ij}(t,\boldsymbol{x},\boldsymbol{y})$ describe amounts of transactions $bs_{ij}(t,\boldsymbol{x},\boldsymbol{y})$ carried by agents i as sellers and carried by agents j as buyers of variable E. Flows $\boldsymbol{p}_{xij}(t,\boldsymbol{x},\boldsymbol{y})$ describe motion of sellers along axis $\boldsymbol{X}$ and flows $\boldsymbol{p}_{yij}(t,\boldsymbol{x},\boldsymbol{y})$ describe motion of buyers along axis $\boldsymbol{Y}$. Aggregates of flows $\boldsymbol{p}_{ij}(t,\boldsymbol{x},\boldsymbol{y})$ over all agents i with coordinates inside $dV(\boldsymbol{x})$ at point $\boldsymbol{x}$ and all agents j with coordinates inside $dV(\boldsymbol{y})$ at point $\boldsymbol{y}$ define transactions flows $\boldsymbol{P}(t,\boldsymbol{x},\boldsymbol{y})$ between points $\boldsymbol{x}$ and $\boldsymbol{y}$ similar to (I.7) and (2.1; 2.2) as:

$$\boldsymbol{P}(t,\boldsymbol{z}) = \left(\boldsymbol{P}_{\boldsymbol{x}}(t,\boldsymbol{z}), \boldsymbol{P}_{\boldsymbol{y}}(t,\boldsymbol{z})\right) \;;\; \boldsymbol{z} = (\boldsymbol{x},\boldsymbol{y}) \tag{5.3}$$

$$\boldsymbol{P}_{\boldsymbol{x}}(t,\boldsymbol{z}) = \sum_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y})\,\Delta} \boldsymbol{p}_{xij}(t,\boldsymbol{z}) = \sum_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y})\,\Delta} bs_{i,j}(t,\boldsymbol{z})\boldsymbol{v}_i(t,\boldsymbol{x}) \tag{5.4}$$

$$\boldsymbol{P}_{\boldsymbol{x}}(t,\boldsymbol{z}) = \sum_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y})\,\Delta} \boldsymbol{p}_{yij}(t,\boldsymbol{z}) = \sum_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y})\,\Delta} bs_{i,j}(t,\boldsymbol{z})\boldsymbol{v}_j(t,\boldsymbol{y}) \tag{5.5}$$

Transactions flows $\boldsymbol{P}(t,z)$ (5.3-5.5) between points $\boldsymbol{x}$ and $\boldsymbol{y}$ describe amounts of transactions $BS(t,z)$ carried by transactions velocities $\boldsymbol{v}(t,z)$ through $2n$-dimensional economic domain (1.1;1.2). Similar to (I.9) let's define transactions velocities $\boldsymbol{v}(t,z)$ as:

$$\boldsymbol{P}(t,\boldsymbol{z}) = BS(t,\boldsymbol{z})\boldsymbol{v}(t,\boldsymbol{z}) \;;\; \boldsymbol{v}(t,\boldsymbol{z}) = (\boldsymbol{v}_{\boldsymbol{x}}(t,\boldsymbol{z}); \boldsymbol{v}_{\boldsymbol{y}}(t,\boldsymbol{z})) \tag{5.6}$$

$$\boldsymbol{P}_{\boldsymbol{x}}(t,\boldsymbol{z}) = BS(t,\boldsymbol{z})\,\boldsymbol{v}_{\boldsymbol{x}}(t,\boldsymbol{z}) \;;\; \boldsymbol{P}_{\boldsymbol{y}}(t,\boldsymbol{z}) = BS(t,\boldsymbol{z})\boldsymbol{v}_{\boldsymbol{y}}(t,\boldsymbol{z}) \tag{5.7}$$

Similar to (I.8;9) integrals over economic domain (1.1;1.2) by $d\boldsymbol{x}$ and $d\boldsymbol{y}$ define macroeconomic flows of transactions $BS(t)$ (4.1) with velocity $\boldsymbol{v}(t)$ as:

$$\boldsymbol{P}(t) = \int d\boldsymbol{x}d\boldsymbol{y}\,\boldsymbol{P}(t,\boldsymbol{x},\boldsymbol{y}) = BS(t)\boldsymbol{v}(t) \quad ; \quad \boldsymbol{v}(t) = (\boldsymbol{v}_x(t); \boldsymbol{v}_y(t)) \tag{5.8}$$

For example let's take $BS(t)$ as investments made in macroeconomics during time dt. Then relations (5.8) describe flow of investment transactions with velocity $\boldsymbol{v}(t)$ on economic space. Components $\boldsymbol{v}_x(t)$ and $\boldsymbol{v}_y(t)$ describe motion of aggregated investors and aggregated recipients of investments. Positive or negative values of components of velocity $\boldsymbol{v}_{xi}(t)$ along axis x_i of economic space describe motion of investors in risky of safer directions. Positive values of components of velocity $\boldsymbol{v}_{yj}(t)$ along axis y_j of economic space describe risk growth of recipients of investments and negative $\boldsymbol{v}_{yj}(t)$ describes decline of risks of recipients of investments along axis y_j. Aggregated investors and recipients of investments have coordinates inside economic domain (1.1;1.2). Thus velocities (5.8) can't be constant and must change signature and fluctuate as borders of economic domain (1.1; 1.2) reduce motion along each risk axes. Fluctuations of macroeconomic velocities (5.8) of investment transactions describe motion of investors and recipients of investments from safer to risky areas of economic domain (1.1; 1.2) and back from risky to safer areas. Such fluctuations of investors and recipients of investments due to oscillations of velocity $\boldsymbol{v}(t)$ (5.8) describe Investment business cycles. Credit transactions, buy-sell transactions and etc., induce similar macroeconomic transactions flows (5.8) and describe corresponding credit cycles, buy-sell cycles and etc., (Olkhov, 2017d; 2018).

Relations (2.1-2.2; 5.3-5.5) allow derive equations on transactions density *BS(t,z)* and transactions flows ***P****(t,z), z=(****x,y****)* on *2n*-dimensional economic domain similar to equations (I.14; 17). To derive equations on transactions density *BS(t,z)* (2.1; 2.2) and flows ***P****(t,z)* (5.6) let's describe their change in a small unit volume *dV(z)* (1.4; 1.5). Two factors change *BS(t,z)* in a unit volume *dV(z)*. The first change *BS(t,z)* in time as:

$$\int d\boldsymbol{z}\ \frac{\partial}{\partial t} BS(t,\boldsymbol{z}) \tag{5.9}$$

The second factor describe change of *BS(t,z)* due to flows ***P****(t,z)*: amount of *BS(t,z)* in a unit volume *dV(z)* (1.4; 1.5) can grow up or decrease due to in- or out- flows ***P****(t,z)*. If in-flows ***P****(t,z)* are more then out-flows then *BS(t,z)* will increase in a volume *dV(z)*. To calculate balance of in- and out-flows let's take integral of flow ***P****(t,z)* over the surface of *dV(z)*:

$$\oint ds\, \boldsymbol{P}(t,\boldsymbol{z}) = \oint ds\ \ BS(t,\boldsymbol{z})\boldsymbol{v}(t,\boldsymbol{z}) \tag{5.10}$$

Due to divergence theorem (Strauss 2008, p.179) surface integral (5.10) of the flow ***P****(t,z)=BS(t,z)****v****(t,z)* through surface equals its volume integral by divergence of the flow:

$$\oint ds\, BS(t,\boldsymbol{z})\boldsymbol{v}(t,\boldsymbol{z}) = \int d\boldsymbol{x}\ \nabla\cdot\left(BS(t,\boldsymbol{z})\boldsymbol{v}(t,\boldsymbol{z})\right) \tag{5.11}$$

Relations (5.9; 5.11) give total change of variable *BS(t,z)* in a unit volume *dV(z)*:

$$\int d\boldsymbol{z}\ \left[\frac{\partial}{\partial t} BS(t,\boldsymbol{z}) + \nabla\cdot\big(BS(t,\boldsymbol{z})\ \boldsymbol{v}(t,\boldsymbol{z})\big)\right]$$

As a unit volume *dV(z)* is arbitrary one can take equations on economic density *BS(t,z)* as

$$\frac{\partial}{\partial t} BS(t,\boldsymbol{z}) + \nabla\cdot\big(BS(t,\boldsymbol{z})\ \boldsymbol{v}(t,\boldsymbol{z})\big) = F(t,\boldsymbol{z}) \tag{5.12}$$

$$\frac{\partial}{\partial t} \boldsymbol{P}(t,\boldsymbol{z}) + \nabla\cdot\big(\boldsymbol{P}(t,\boldsymbol{z})\ \boldsymbol{v}(t,\boldsymbol{z})\big) = \boldsymbol{G}(t,\boldsymbol{z}) \tag{5.13}$$

Similar to (I.18.1; 18.2) integrals of (5.12; 5.13) by *dz*=(***dx,dy***) over economic domain (1.1; 1.2) give for (4.1) ordinary time derivation equations:

$$\int d\boldsymbol{z}\left[\frac{\partial}{\partial t} BS(t,\boldsymbol{z}) + \nabla\cdot\big(BS(t,\boldsymbol{z})\ \boldsymbol{v}(t,\boldsymbol{z})\big)\right] = \frac{d}{dt} BS(t) = F(t) = \int d\boldsymbol{z}\, F(t,\boldsymbol{z}) \tag{6.1}$$

$$\int d\boldsymbol{z}\left[\frac{\partial}{\partial t} \boldsymbol{P}(t,\boldsymbol{z}) + \nabla\cdot\big(\boldsymbol{P}(t,\boldsymbol{z})\ \boldsymbol{v}(t,\boldsymbol{z})\big)\right] = \frac{d}{dt} \boldsymbol{P}(t) = \boldsymbol{G}(t) = \int d\boldsymbol{z}\, \boldsymbol{G}(t,\boldsymbol{z}) \tag{6.2}$$

Relations (6.1; 6.2) illustrate that operators in the left hand of (5.12; 5.13) for *BS(t,z)* and flows ***P****(t,z), z=(****x,y****)* on *2n*-dimensional economic space play role alike to ordinary time derivative for macro transactions *BS(t)* (4.1) and flows ***P****(t)* (5.8). Different transactions have different densities, flows and velocities and thus are described by different operators (5.12; 5.13) with different functions *F(t,z)* and ***G****(t,z)*. It is assumed that agents are engaged into transactions *BS(t,z)* with other agents under various expectations. Thus we propose that functions *F(t,z)* in (5.12) may describe action of expectations of agents involved into transactions $BS(t,z)$ between points $\boldsymbol{x}$ and $\boldsymbol{y}$. In the next section we introduce definitions of expectations between points $\boldsymbol{x}$ and $\boldsymbol{y}$. Functions $\boldsymbol{G}(t,z)$ in (5.13) describe action of factors that impact evolution of transactions flows $\boldsymbol{P}(t,z)$. Thus functions $F(t,z)$ and $\boldsymbol{G}(t,z)$ in (5.12; 5.13) define particular evolution model of transactions $BS(t,z)$ and flows $\boldsymbol{P}(t,z)$. Economic reasons that define dependence of functions $F(t,z)$ and $\boldsymbol{G}(t,z)$ on other transactions, economic variables or expectations permit study different models of evolution of transactions $BS(t,z)$ and flows $\boldsymbol{P}(t,z)$. The simplest case describes mutual dependence between two transactions $BS_E(t,z)$ and $BS_Q(t,z)$ that describe exchange by economic variables E and Q in the assumption that functions $F_E(t,z)$ and $\boldsymbol{G}_E(t,z)$ depend on transactions $BS_Q(t,z)$ and its flows $\boldsymbol{P}_Q(t,z)$ and functions $F_Q(t,z)$ and $\boldsymbol{G}_Q(t,z)$ depend on transactions $BS_E(t,z)$ and flows $\boldsymbol{P}_E(t,z)$. This approximation models self-consistent dynamics of two transactions and their flows and describes evolution of corresponding variables E and Q. One can study equations (5.12; 5.13) with functions $F(t,z)$ and $\boldsymbol{G}(t,z)$ that depend on several transactions, expectations or economic variables. Such models describe approximations of economic evolution of transactions and macro variables for different functions $F(t,z)$ and $\boldsymbol{G}(t,z)$.

Due to definition (2.1; 2.2) of $BS(t,z)$ it aggregates transactions $bs_{ij}(t,z)$ performed by agents i and agents j. It is assumed that agents take decisions and perform transactions under different expectations. To describe impact of expectations on functions $F(t,z)$ and $\boldsymbol{G}(t,z)$ for equations (5.12; 5.13) let's introduce definitions of expectations densities similar to above models of economic variables and economic transactions.

3. Expectations on economic space

Expectations are the most "etheric" economic substance. In Sec 3, Part I., we argue macroeconomic variables as main properties of economic state and as a ground for economic and financial policy decisions that govern and manage countries' economic development. In the above Sec. 2 we argue that transactions between agents should be treated as only economic tool that change macroeconomic variables and determine their growth or decline dynamics. In this Section we consider expectation as economic substance that determine performance of transactions and thus have substantial impact on evolution of macroeconomic variables.

Expectations are treated as factors that govern economic and financial transactions, price and return at least by Keynes (1936) and actively studied since Muth (1961) and Lucas (1972) and in numerous further publications (Sargent and Wallace, 1976; Hansen and Sargent, 1979; Kydland and Prescott, 1980; Blume and Easley, 1984; Brock and Hommes, 1998; Manski, 2004; Brunnermeier and Parker, 2005; Dominitz and Manski, 2005; Klaauw et al, 2008; Janžek and Ziherl, 2013; Caporin, Corazzini and Costola, 2014; Greenwood and Shleifer, 2014; Lof, 2014; Manski, 2017; Thaler, 2018).

Expectations concern all macroeconomic and financial variables like inflation and demand, currency exchange rates, market demand, bank rates, price trends and etc. Economic agents may establish their expectations on base of market transactions, dynamics of macroeconomic and financial variables, on base of expectations of other agents, policy decisions, technology forecasts and etc.

There are a lot of studies on expectations measurements (Manski, 2004; Dominitz and Manski, 2005; Klaauw et al, 2008; Stangl, 2009; Bachmann and Elstner, 2013; Janžek and Ziherl, 2013; Manski, 2017; Tanaka et al, 2018). Manski (2004) indicate that "It would be better to measure expectations as - subjective probabilities". Dominitz and Manski (2005 "analyze probabilistic expectations of equity returns". Stangl (2009) suggest that "Visual Analog Scale (VAS) enables scores between categories, and the respondent can express not only the direction of his attitude but also its magnitude on a 1-to-100 point scale, which comes close to an interval scale measurement". Measurement of such "etheric" economic substance as expectations is a really tough problem. Our approach to expectations modeling as important factor that impact macroeconomic evolution requires that all expectations under consideration should have similar measure. Let's omit here discussion on expectations measure and assume that all expectations are measured as index. It is clear, that scale of index is not important. Expectations can take any values between 0 and 100 or 0 and 1. The only requirement – all expectations under consideration are measured by same measure with same scale. For certainty let's take that measure of expectations is an interval [0,1].

Each economic agent can have a lot of different expectations and different expectations force agents accomplish transactions. Let's assume that in economy there are $j=1,..K$ expectations those may impact transactions between agents. Let's transfer description of expectations that define transactions between separate agents to aggregate expectations that describe transactions between points on economic space. To aggregate value and economic importance of agents expectations let's state that economic value of particular expectation of agent should be proportional to value of transactions made under this particular expectation. Indeed, if particular transactions amount 90% of all deals and are made under expectation 1 then this particular expectation 1 is ninety times more important then expectation 2 that is responsible for only 1% of same deals. Thus aggregation of expectations and description of most valuable expectations should be done for expectations weighted by value of transactions made under these expectations.

Let's study buy-sell transactions $bs_{ij}(t,\boldsymbol{x},\boldsymbol{y})$ that describe transfer of economic variable E - assets, shares, commodities, service, credits and etc., from agent i as seller at point $\boldsymbol{x}$ to agent j as buyer at point $\boldsymbol{y}$. Let's denote $ex_i(k;t,\boldsymbol{x})$ as expectations of type $k=1,..K$ of agent i as seller at point $\boldsymbol{x}$. Let's assume that expectations $ex_i(k;t,\boldsymbol{x})$ approve $bs_{ij}(k;t,\boldsymbol{x},\boldsymbol{y})$ - part of transactions $bs_{ij}(t,\boldsymbol{x},\boldsymbol{y})$ with economic variable E made under sellers expectations of type k from agent i as seller at point $\boldsymbol{x}$ to agent j as buyer at point $\boldsymbol{y}$. Further let's denote expectations of buyer $ex_j(t,\boldsymbol{y};l)$ of type $l=1,..K$ that approve part $bs_{ij}(t,\boldsymbol{x},\boldsymbol{y};l)$ of transactions $bs_{ij}(t,\boldsymbol{x},\boldsymbol{y})$ made under buyers expectations of type l by the agent j as buyer at point $\boldsymbol{y}$.

Economic value of sellers expectations $ex_i(k;t,\boldsymbol{x})$ is proportional to amount of transactions $bs_{ij}(k;t,\boldsymbol{x},\boldsymbol{y})$ with variable E made under this type of expectations. For k, $l=1,..K$ let's introduce expected transactions $et_{ij}(k;t,\boldsymbol{x},\boldsymbol{y};l)$ as follows:

$$\boldsymbol{et}_{ij}(k;t,\boldsymbol{z};l) = \left(et_{ij}(k;t,\boldsymbol{x},\boldsymbol{y})\,;\; et_{ij}(t,\boldsymbol{x},\boldsymbol{y};l)\right) \;;\; \boldsymbol{z} = (\boldsymbol{x},\boldsymbol{y}) \tag{7.1}$$

$$et_{ij}(k;t,\boldsymbol{z}) = ex_i(k;t,\boldsymbol{x})bs_{ij}(k;t,\boldsymbol{z}) \;;\; et_{ij}(t,\boldsymbol{z};l) = ex_j(t,\boldsymbol{y};l)bs_{ij}(t,\boldsymbol{z};l)$$

Expected transactions $et_{ij}(k;t,z)$ (7.1) describe sellers expectations $ex_i(k;t,\boldsymbol{x})$ at point $\boldsymbol{x}$ weighted by transactions $bs_{ij}(k;t,z)$ performed between agents i as sellers at $\boldsymbol{x}$ and agents j as buyers at $\boldsymbol{y}$ under expectations of type k. Expected transactions $et_{ij}(t,z;l)$ (7.1) describe buyers expectations $ex_j(t,\boldsymbol{y};l)$ at $\boldsymbol{y}$ weighted by transactions $bs_{ij}(t,z;l)$ performed under buyers expectations $ex_j(t,\boldsymbol{y};l)$ between agents i as sellers at $\boldsymbol{x}$ and agents j as buyers at $\boldsymbol{y}$. Transactions $bs_{ij}(k;t,z)$ between agents i and j are made with variable E under sellers expectations k and transactions $bs_{ij}(t,z;l)$ are made under buyers expectations l and are additive functions.

Let's rougher description of transactions $bs_{ij}(k;t,z)$ and $bs_{ij}(t,z;l)$ and define transactions $BS(k;t,z)$ and transactions $BS(t,z;l)$ with variable E performed by sellers at $\boldsymbol{x}$ under expectations of type k and by buyers at $\boldsymbol{y}$ under expectations of type l as:

$$BS(k;t,\boldsymbol{z}) = \textstyle\sum_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y});\Delta} bs_{i,j}(k;t,\boldsymbol{z}) \;;\; \boldsymbol{z} = (\boldsymbol{x},\boldsymbol{y}) \tag{7.2}$$

$$BS(t,\boldsymbol{z};l) = \textstyle\sum_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y});\Delta} bs_{i,j}(k;t,\boldsymbol{z}) \tag{7.3}$$

Functions $BS(k;t,\boldsymbol{x},\boldsymbol{y})$ (7.2) describe part of transactions $BS(t,\boldsymbol{x},\boldsymbol{y})$ (4.2) performed by sellers at $\boldsymbol{x}$ under expectations of type k of with agents at $\boldsymbol{y}$ and all types of buyers expectations. Functions $BS(t,\boldsymbol{x},\boldsymbol{y};l)$ (7.3) describe part of transactions $BS(t,\boldsymbol{x},\boldsymbol{y})$ (4.2) performed by buyers at $\boldsymbol{y}$ under expectations of type l with agents at $\boldsymbol{x}$ and all types of sellers expectations.

$$BS(t,\boldsymbol{z}) = \textstyle\sum_k BS(k;t,\boldsymbol{z}) = \sum_l BS(t,\boldsymbol{z};l) \tag{7.4}$$

Sum by k of transactions $BS(k;t,z)$ (7.2) equals sum by l of transactions $BS(t,z;l)$ (7.3) and that equals transactions $BS(t,z)$ (2.1;2.2) performed under all expectations a z=($\boldsymbol{x}$, $\boldsymbol{y}$).

Now let's define expected transactions $\boldsymbol{Et}(k;t,\boldsymbol{x},\boldsymbol{y};l)$ between points $\boldsymbol{x}$ and $\boldsymbol{y}$ made under sellers expectations of type k and buyers expectations of type l. Let's aggregate (7.1) in unit volumes (1.3) and average alike to (2.1;2.2) as:

$$\boldsymbol{Et}(k;t,\boldsymbol{z};l) = \big(Et(k;t,\boldsymbol{z})\ ;\ Et(t,\boldsymbol{z};l)\big)\ ;\ \boldsymbol{z} = (\boldsymbol{x},\boldsymbol{y}) \tag{7.5}$$

$$Et(k;t,\boldsymbol{z}) = \textstyle\sum_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y});\Delta} ex_i(k;t,\boldsymbol{x})bs_{ij}(k;t,\boldsymbol{z}) \tag{7.6}$$

$$Et(t,\boldsymbol{z};l) = \textstyle\sum_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y});\Delta} ex_j(t,\boldsymbol{y};l)bs_{ij}(t,\boldsymbol{z};l) \tag{7.7}$$

Definitions of $BS(k;t,z)$ (7.2) and $BS(t,z;l)$ (7.3) permit use expected transactions $Et(k;t,z)$ and $Et(t,z;l)$ (7.5-7.7) and introduce expectations densities $Ex(k;t,z)$, $z=(\boldsymbol{x},\boldsymbol{y})$ of type k of sellers at $\boldsymbol{x}$ and expectations densities $Ex(t,z;l)$ of type l of buyers at $\boldsymbol{y}$ as:

$$Et(k;t,\boldsymbol{z}) = Ex(k;t,\boldsymbol{z})BS(k;t,\boldsymbol{z}) \tag{7.8}$$

$$Et(t,\boldsymbol{z};l) = Ex(t,\boldsymbol{z};l)BS(t,\boldsymbol{z};l) \tag{7.9}$$

Index E for expected transactions $Et(k;t,z)$ and expectations $Ex(t,z;l)$ underlines that these functions are determined with respect to transactions with selected economic variable E. Transactions with different variables E – commodities, service, assets and etc., - define different functions of expected transactions and expectations densities. Functions $Ex(k;t,\boldsymbol{x},\boldsymbol{y})$ (7.8) $z=(\boldsymbol{x},\ \boldsymbol{y})$, describe sellers expectations of type k at point $\boldsymbol{x}$ for transactions $BS(k;t,\boldsymbol{x},\boldsymbol{y})$ (7.2) made under sellers expectations of type k and for all expectations of buyers at $\boldsymbol{y}$. Functions $Ex(t,\boldsymbol{x},\boldsymbol{y};l)$ (7.9) describe buyers expectations of type l at point $\boldsymbol{y}$ for transactions $BS(t,\boldsymbol{x},\boldsymbol{y};l)$ (7.3) performed under all expectations of Sellers at $\boldsymbol{x}$. To define expectations $Ex(k;t)$ of sellers and expectations of buyers $Ex(t;l)$ let's take integrals over economic domain (1.1; 1.2):

$$BS(k;t,\boldsymbol{x}) = \int d\boldsymbol{y}\, BS(k;t,\boldsymbol{x};\boldsymbol{y})\ ;\ BS(t,\boldsymbol{y};l) = \int d\boldsymbol{x}\, BS(t,\boldsymbol{x};\boldsymbol{y};l) \tag{8.1}$$

$$Et(k;t,\boldsymbol{x}) = \int d\boldsymbol{y}\, Et(k;t,\boldsymbol{x},\boldsymbol{y}) = Ex(k;t,\boldsymbol{x})BS(k;t,\boldsymbol{x}) \tag{8.2}$$

$$Et(t,\boldsymbol{y};l) = \int d\boldsymbol{x}\, Et(t,\boldsymbol{x},\boldsymbol{y};l) = Ex(t,\boldsymbol{y};l)BS(t,\boldsymbol{y};l) \tag{8.3}$$

$$BS(k;t) = \int d\boldsymbol{x}d\boldsymbol{y}\, BS(k;t,\boldsymbol{x};\boldsymbol{y})\ ;\ BS(t;l) = \int d\boldsymbol{x}d\boldsymbol{y}\, BS(t,\boldsymbol{x};\boldsymbol{y};l) \tag{8.4}$$

$$Et(k;t) = \int d\boldsymbol{x}d\boldsymbol{y}\, Et(k;t,\boldsymbol{x},\boldsymbol{y}) = Ex(k;t)BS(k;t) \tag{8.5}$$

$$Et(t;l) = \int d\boldsymbol{x}d\boldsymbol{y}\, Et(t,\boldsymbol{x},\boldsymbol{y};l) = Ex(k;t)BS(t;l) \tag{8.6}$$

Relations (8.1) define transactions $BS(k;t,\boldsymbol{x})$ with economic variable E performed by sellers at $\boldsymbol{x}$ under their expectations of type k with all buyers of entire economics. Transactions $BS(t,\boldsymbol{y};l)$ (8.1) are performed by buyers at $\boldsymbol{y}$ under their expectations of type l with all sellers of entire economics. Relations (8.2) define expected transactions $Et(k;t,\boldsymbol{x})$ made by sellers at $\boldsymbol{x}$ under sellers expectations $Ex(k;t,\boldsymbol{x})$ of type k with all buyers of entire economics. Relations (8.3) define expected transactions $Et(t,\boldsymbol{y};l)$ made by buyers at $\boldsymbol{y}$ under buyers expectations

$Ex(t,\boldsymbol{y};l)$ of type l with all sellers of entire economics. Relations (8.4) define all transactions $BS(k;t)$ with economic variable E made in the entire economics under sellers expectations of type k. Functions $BS(t;l)$ (8.4) define all transactions with economic variable E made in the entire economics under buyers expectations of type l. Relations (8.5) define macroeconomic sellers expectations $Ex(k;t)$ of type k for the transactions $BS(k;t)$ with economic variable E. Relations (8.6) define macroeconomic buyers expectations $Ex(t;l)$ of type l for the transactions $BS(t;l)$ with economic variable E. Thus starting with definitions of expected transactions (7.1) and definitions of partial transactions $BS(k;t,\boldsymbol{x},\boldsymbol{y})$ (7.2) and $BS(t,\boldsymbol{x},\boldsymbol{y};l)$ (7.3) we deliver reasonable definitions of macroeconomic expectations of sellers (8.5) and buyers (8.6) for transactions with economic variable E. Let's outline that expectations of type k play different role for transactions with different economic variables E and that makes observations and measurements of expectations a really complex problem.

Now let's describe how expected transactions and expectations can flow on economic space alike to flows of economic variables (I.6-10) and transactions flows (5.1-5.5). Motion of agents i and j at points $\boldsymbol{x}$ and $\boldsymbol{y}$ with velocities $\boldsymbol{v}_i(t,\boldsymbol{x})$ and $\boldsymbol{v}_j(t,\boldsymbol{y})$ on e-space induce flows $\boldsymbol{p}_{ij}(k;t,z)$ and $\boldsymbol{p}_{ij}(t,z;l)$ of expected transactions $et_{ij}(k;t,z)$ and $et_{ij}(k;t,z)$ (7.1) of agents i at point $\boldsymbol{x}$ similar to flows $\boldsymbol{p}_{xij}(t,z)$ of transactions $bs_{ij}(t,z)$, $z=(\boldsymbol{x},\boldsymbol{y})$, as:

$$\boldsymbol{p}_{ij}(k;t,\boldsymbol{z};l) = \left(\boldsymbol{p}_{ij}(k;t,\boldsymbol{z}),\boldsymbol{p}_{ij}(t,\boldsymbol{z};l)\right) \quad ; \quad \boldsymbol{z} = (\boldsymbol{x},\boldsymbol{y}) \tag{9.1}$$

$$\boldsymbol{p}_{ij}(k;t,\boldsymbol{z}) = et_{ij}(k;t,\boldsymbol{z})\boldsymbol{v}_i(\boldsymbol{x}) = ex_i(k;t,\boldsymbol{x})bs_{i,j}(k;t,\boldsymbol{z})\boldsymbol{v}_i(\boldsymbol{x}) \tag{9.2}$$

$$\boldsymbol{p}_{ij}(t,\boldsymbol{z};l) = et_{ij}(t,\boldsymbol{z};l)\boldsymbol{v}_j(\boldsymbol{y}) = ex_j(t,\boldsymbol{y};l)bs_{i,j}(t,\boldsymbol{z};l)\boldsymbol{v}_j(\boldsymbol{y}) \tag{9.3}$$

Flows $\boldsymbol{p}_{ij}(k;t,z)$ describe amount of expected transactions $et_{ij}(k;t,z)$ of type k carried by agent i in the direction of velocity $\boldsymbol{v}_i$. To define aggregate flows of expected transactions at points $\boldsymbol{x}$ and $\boldsymbol{y}$ let's collect flows $\boldsymbol{p}_{ij}(k;t,z)$ of expected transactions $et_{ij}(k;t,z)$ (9.2) of agents i in a unit $dV(t,\boldsymbol{x})$ (1.3-1.5) and flows $\boldsymbol{p}_{ij}(t,z;l)$ of expected transactions $et_{ij}(t,z;l)$ (9.3) of agents j in a unit volume $dV(t,\boldsymbol{y})$ and then average the sum during time Δ similar to (2.1;2.2; 5.4; 5.5) as:

$$\boldsymbol{P}(k;t,\boldsymbol{z};l) = \left(\boldsymbol{P}_{\boldsymbol{x}}(k;t,\boldsymbol{z}),\boldsymbol{P}_{\boldsymbol{y}}(t,\boldsymbol{z};l)\right) \quad ; \quad \boldsymbol{z} = (\boldsymbol{x},\boldsymbol{y}) \tag{9.4}$$

$$\boldsymbol{P}_x(k;t,\boldsymbol{z}) = \textstyle\sum_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y})\,\Delta} et_{ij}(k;t,\boldsymbol{z})\boldsymbol{v}_i(\boldsymbol{x}) \tag{9.5}$$

$$\boldsymbol{P}_y(t,\boldsymbol{z};l) = \textstyle\sum_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y})\,\Delta} et_{ij}(t,\boldsymbol{z};l)\boldsymbol{v}_j(\boldsymbol{y}) \tag{9.6}$$

$$\boldsymbol{P}_x(k;t,\boldsymbol{z}) = Et_E(k;t,\boldsymbol{z})\,\boldsymbol{v}_x(k;t,\boldsymbol{z}) = Ex(k;t,\boldsymbol{z})BS(k;t,\boldsymbol{z})\boldsymbol{v}_x(k;t,\boldsymbol{z}) \tag{9.7}$$

$$\boldsymbol{P}_y(t,\boldsymbol{z};l) = Et_E(t,\boldsymbol{z};l)\boldsymbol{v}_{\boldsymbol{y}}(t,\boldsymbol{z};l) = Ex(t,\boldsymbol{z};l)BS(t,\boldsymbol{z};l)\boldsymbol{v}_y(t,\boldsymbol{z};l) \tag{9.8}$$

$$\boldsymbol{v}(k;t,\boldsymbol{z};l) = \left(\boldsymbol{v}_x(k;t,\boldsymbol{z});\boldsymbol{v}_y(t,\boldsymbol{z};l)\right) \tag{9.9}$$

For transactions $BS(t,\boldsymbol{x},\boldsymbol{y})$ that describe deals with economic variable E (shares, commodities, service and etc.) from sellers at point $\boldsymbol{x}$ to buyers at point $\boldsymbol{y}$ relations (9.5) define aggregated flows $\boldsymbol{P}_x(k;t,z)$ of expected transactions of type k of sellers at point $\boldsymbol{x}$. Relations (9.6) define aggregated flows $\boldsymbol{P}_y(t,z;l)$ of expected transactions of type l of buyers at point $\boldsymbol{y}$. Relations (9.7-9.9) and expected transactions $Et(k;t,z)$ and $Et(t,z;l)$ (7.5-7.9) define velocities $\boldsymbol{v}_x(k;t,z)$ (9.7) of sellers at point $\boldsymbol{x}$ of expected transaction of type k and velocities $\boldsymbol{v}_y(t,z;l)$ (9.8) of buyers at point $\boldsymbol{y}$ of expected transaction of type l as function of $z=(\boldsymbol{x},\boldsymbol{y})$. Similar to definitions of macroeconomic flows of variables (I. 6-9) and macro flows of transactions (5.3-5.8) integrals by $dz=d\boldsymbol{x}d\boldsymbol{y}$ over economic domain (1.1; 1.2) of relations (9.4-9.9) define macroeconomic flows $\boldsymbol{P}_x(k;t)$, $\boldsymbol{P}_y(t;l)$ and macroeconomic velocities $\boldsymbol{v}_x(k;t)$ and $\boldsymbol{v}_y(t;l)$ of expected transactions $Et_x(k;t)$, $Et_y(t;l)$ and macroeconomic expectations $Ex_x(k;t)$, $Ex_y(t;l)$ as:

$$\boldsymbol{P}_x(k;t) = \int d\boldsymbol{x}d\boldsymbol{y}\, Et_x(k;t,\boldsymbol{x},\boldsymbol{y})\boldsymbol{v}_x(k;t,\boldsymbol{x},\boldsymbol{y}) \tag{10.1}$$

$$\boldsymbol{P}_x(k;t) = Et_x(k;t)\,\boldsymbol{v}_x(k;t) = Ex_x(k;t)BS(k;t)\boldsymbol{v}_x(k;t) \tag{10.2}$$

$$\boldsymbol{P}_y(\boldsymbol{t};l) = \int \boldsymbol{dx}d\boldsymbol{y}\, Et_y(t,\boldsymbol{x},\boldsymbol{y};l)\boldsymbol{v}_y(\boldsymbol{t},\boldsymbol{x},\boldsymbol{y};l) \tag{10.3}$$

$$\boldsymbol{P}_y(t;l) = Et_y(t;l)\,\boldsymbol{v}_y(t;l) = Ex_y(t;l)BS(t;l)\boldsymbol{v}_y(t;l) \tag{10.4}$$

Relations (10.1) define macroeconomic flows of $\boldsymbol{P}_x(k;t)$ and relations (10.2) define macroeconomic velocities $\boldsymbol{v}_x(k;t)$ of sellers of expected transaction of type k. Flows $\boldsymbol{P}_x(k;t)$ describe motion of sellers macroeconomic expectations $Ex_x(k;t)$ of type k and transactions of sellers $BS(k;t)$ with velocities $\boldsymbol{v}_x(k;t)$ on economic domain (1.1; 1.2). Relations (10.3) define macroeconomic flows $\boldsymbol{P}_y(t;l)$ and relations (10.4) define velocities $\boldsymbol{v}_y(t;l)$ of buyers expected transaction of type l as function of time t. Flows $\boldsymbol{P}_y(t;l)$ describe motion of buyers macroeconomic expectations $Ex_y(t;l)$ of type l and transactions of buyers $BS(t;l)$ with velocities $\boldsymbol{v}_y(t;l)$ on economic domain (1.1;1.2). In other words, sellers expectations $Ex_x(k;t)$ of type k change in time due to motion on economic domain with velocity $\boldsymbol{v}_x(k;t)$. Borders of economic domain (1.1;1.2) reduce motion along risk axes and hence values and direction of sellers flows $\boldsymbol{P}_x(k;t)$ and velocities $\boldsymbol{v}_x(k;t)$ should fluctuate. That induce time oscillations of macroeconomic expectations $Ex_x(k;t)$ and transactions $BS(k;t)$ and correlates with the business cycles induced by oscillations of flows $\boldsymbol{P}(t)$ and velocities $\boldsymbol{v}(t)$ (5.8).

Let's underline that velocities of $\boldsymbol{v}_x(t)$ of sellers and velocities $\boldsymbol{v}_y(t)$ of buyers (5.8) differs from velocities $\boldsymbol{v}_x(k;t)$ of sellers expectations $Ex_x(k;t)$ of type k and velocities $\boldsymbol{v}_y(t;l)$ of buyers expectations $Ex_y(t;l)$ of type l. Flows of different variables, transactions and expectations have different velocities and their mutual interaction on economic domain reflect high complexity of real economic processes.

Definitions (7.5-7.7) of expected transactions $Et(k;t,z)$ and $Et(t,z;l)$ and definitions (9.4-9.6) of their flows $\boldsymbol{P}_x(k;t,z)$ and $\boldsymbol{P}_y(t,z;l)$ and definitions (9.7; 9.8) of their velocities $\boldsymbol{v}_x(k;t,z)$ and $\boldsymbol{v}_y(t,z;l)$ allow take equations on expected transactions and their flows similar to equations on transactions and their flows (5.12; 5.13) as:

$$\frac{\partial}{\partial t}Et(k;t,\boldsymbol{z}) + \nabla\cdot\left(Et(k;t,\boldsymbol{z})\,\boldsymbol{v}_x(k;t,\boldsymbol{z})\right) = W_x(k;t,\boldsymbol{z}) \tag{10.5}$$

$$\frac{\partial}{\partial t}Et(t,\boldsymbol{z};l) + \nabla\cdot\left(Et(t,\boldsymbol{z};l)\,\boldsymbol{v}_{yl}(t,\boldsymbol{z};l)\right) = W_y(t,\boldsymbol{z};l) \tag{10.6}$$

$$\frac{\partial}{\partial t}\boldsymbol{P}_x(k;t,\boldsymbol{z}) + \nabla\cdot\left(\boldsymbol{P}_x(k;t,\boldsymbol{z})\,\boldsymbol{v}_x(k;t,\boldsymbol{z})\right) = \boldsymbol{R}_x(k;t,\boldsymbol{z}) \tag{10.7}$$

$$\frac{\partial}{\partial t}\boldsymbol{P}_y(t,\boldsymbol{z};l) + \nabla\cdot\left(\boldsymbol{P}_y(t,\boldsymbol{z};l)\,\boldsymbol{v}_y(t,\boldsymbol{z};l)\right) = \boldsymbol{R}_y(t,\boldsymbol{z};l) \tag{10.8}$$

Functions W_x, W_y and $\boldsymbol{R}_x$, $\boldsymbol{R}_y$ in equations (10.5-10.8) describe action of economic and financial variables, transactions and different expectations, technology, political and other factors that may impact change of expectations $Ex(k;t,z)$ and $Ex(t,z;l)$ and hence change of expected transactions $Et(k;t,z)$ and $Et(t,z;l)$ and their flows $\boldsymbol{P}_x(k;t,z)$ and $\boldsymbol{P}_y(t,z;l)$. That makes economic modeling a really exciting problem.

Equations (I.14; 17) on macroeconomic and financial variables $A(t,\boldsymbol{x})$ and their flows $\boldsymbol{P}_A(t,\boldsymbol{x})$, equations (5.12; 5.13) on transactions $BS(t,z)$ and transactions flows $\boldsymbol{P}(t,z)$ and equations (10.5-10.8) on expected transaction $Et(k;t,z)$ and $Et(t,z;l)$ and their flows $\boldsymbol{P}_x(k;t,z)$ and $\boldsymbol{P}_y(t,z;l)$ complete our approximation of macroeconomic evolution based on description of relations between macroeconomic and financial variables, transactions and expectations on economic space. It is obvious that description of any particular macroeconomic problem requires definition of right hand side of equations (I.14; 17), (5.12; 5.13), (10.5-10.8). All specifics and details of macroeconomic processes are hidden in and are determined by function $F_A(t,\boldsymbol{x})$ and $\boldsymbol{G}_A(t,\boldsymbol{x})$, $F(t,z)$ and $\boldsymbol{G}(t,z)$, W_x and W_x, W_y and $\boldsymbol{R}_x$, $\boldsymbol{R}_y$. We describe some particular economic problems in Part III. "Economic Applications" of our paper.

4. Transactions, expectations and asset pricing

Asset pricing is one of the most important problems of economics and finance. We refer (Cochrane and Hansen, 1992; Cochrane and Culp, 2003; Hansen, 2013; Campbell, 2014; Fama, 2014; Cochrane, 2017) as only small part of asset pricing studies.

Let's mention that in this paper we don't argue ***why*** asset prices should take certain values, but study ***how*** economic equations on variables, transactions expectations and their flows determine equations on asset prices. Below we show that expectations and economic flows induce equations on asset pricing and argue different definitions of transactions prices.

Above in Sec.3 and 4 we derive equations (5.12; 5.13) and (10.5-10.8) on transactions $BS(t,z)$ with economic variable E and expected transactions $Et(k;t,z)$ and $Et(t,z;l)$. As variable E one can take assets, investment, credits, commodities and etc. Meanwhile any economic transactions from agent i to agent j with particular asset or commodities implies payments for assets or commodities from agent j to agent i. Thus transactions with variable E between agents i and j should describe trading volume Q_{ij} from i to j and trading value or cost C_{ij} from j to i. For example let's assume that agent i sell Q_{ij} *=100 bbl.* of Brent crude oil to agent j for C_{ij}*=6000 \$.* Thus Brent oil price p_{ij} of this particular transaction equals $p_{ij}= C_{ij}/Q_{ij}$ *=60 \$/bbl.* Let's treat transactions as two component functions and describe prices of separate deals between two agents. That helps describe prices of aggregate transactions between points $\boldsymbol{x}$ and $\boldsymbol{y}$ and prices aggregated over entire economics.

In Appendix A we give notion (A.1) of transaction as two component function. Transactions $\boldsymbol{BS}$ with variable E as two components function define trading volume Q and cost C of variable E:

$$\boldsymbol{BS}(\boldsymbol{k};t,\boldsymbol{z}) = \big(Q(k_1;t,\boldsymbol{z});C(k_2;t,\boldsymbol{z})\big) \;\; ; \quad \boldsymbol{k} = (k_1,k_2) \tag{11.1}$$

Relations (11.1) double the number of equations that describe transactions and expectations. Indeed, each transaction should be approved by sellers expectations of type k_1 that approve trading volume $Q(k_1;t,z)$ and sellers expectations of type k_2 that approve trading value or cost $C(k_2;t,z)$ of transaction. Thus sellers expectations $\boldsymbol{k}=(k_1,k_2)$ approve price $p(\boldsymbol{k};t,z)$ (A.12.7) or (11.2) of variable E for the transaction $\boldsymbol{BS}$

$$C(k_2;t,\boldsymbol{z}) = p(\boldsymbol{k};t,\boldsymbol{z})Q(k_1;t,\boldsymbol{z}) \;\; ; \quad \boldsymbol{k} = (k_1,k_2) \tag{11.2}$$

All transactions transaction $\boldsymbol{BS}$ with variable E performed in the entire economics at moment t define (A.12.14) price $p(t)$ as:

$$C(t) = p(t)Q(t) \tag{11.3}$$

In Appendix we derive equations that describe sellers transactions $\boldsymbol{BS}(\boldsymbol{k};t,z)$ (A.12.1) of type $\boldsymbol{k}=(k_1;k_2)$ made under sellers expectations $Ex_Q(k_1;t,z)$ (A.13.7) on trading volume $Q(k_1;t,z)$ (A.12.2) of type k_1 and sellers expectations $Ex_C(k_2;t,z)$ (A.13.8) on cost $C(k_2;t,z)$ of transaction (A.12.3) of type k_2. In other words – sellers expectations $Ex_Q(k_1;t,z)$ of type k_1 approve trading volumes $Q(k_1;t,z)$ (A.12.2) of variable E for transactions $\boldsymbol{BS}(\boldsymbol{k};t,z)$ (A.12.1). Sellers expectations $Ex_C(k_2;t,z)$ of type k_2 approve trading values or costs $C(k_2;t,z)$ (A.12.3) of transactions with variable E. We derive similar equations on buyers transactions of type $\boldsymbol{l}=(l_1;l_2)$ $\boldsymbol{BS}(t,z;\boldsymbol{l})$ (A.12.4) that are made under buyers expectations $Ex_Q(t,z;l_1)$ (A.13.9) of type l_1 on trading volumes $Q(t,z;l_1)$ (A.12.5) of variable E and buyers expectations $Ex_C(t,z;l_2)$ (A.13.10) on costs $C(t,z;l_2)$ (A.12.6) of type k_2.

Let's state that notion of price should always be treated in regard to definite transactions only. For example, sellers price $p(\boldsymbol{k};t,z)$ (A.12.7) or (11.2) correspond to all transactions made under sellers expectations of type $k=(k_1,k_2)$ at moment t between points $\boldsymbol{x}$ and $\boldsymbol{y}$; $z=(\boldsymbol{x},\boldsymbol{y})$. Definition of price $p(t,z)$ (A.12.9) corresponds to all transactions performed between points $\boldsymbol{x}$ and $\boldsymbol{y}$; $z=(\boldsymbol{x},\boldsymbol{y})$ under all expectations of sellers and buyers. Price $p(t)$ (A.12.14) or (11.3) corresponds to all transactions in economy made at moment t with variable E. Different definitions of price describe different states of prices due to different aggregations of transactions and cause different equations.

Economic equations on transactions $\boldsymbol{BS}(\boldsymbol{k};t,z)$ (A.18.1-4) made under sellers expectations and equations on transactions made under buyers expectations (A.19.1-4) describe evolution of transactions as two component functions and their flows. Further we derive equations on sellers expected transactions and their flows (A.20.1-4) and buyers expected transactions and their flows (A.21.1-4). Equations (A.18.1–21.4) complete system of equations on transactions and expected transactions and their flows made under expectations of type $\boldsymbol{k}=(k_1;k_2)$ and $\boldsymbol{l}=(l_1;l_2)$.

Equations on transactions and their flows define equations on prices (A.12.7-16). For example, (A.22.3-4) define equations on Sellers price $p(k_1,k_2;t)$ (A.12.7) for transactions (A.12.15) follows equations made in the entire economics under expectations of type $\boldsymbol{k}=(k_1;k_2)$. Relations (A.23.1-6) define equations on price $p(t)$ (A.12.14) of all transactions made in economy at moment t with variable E.

$$\frac{d}{dt}Q(t) = F_Q(t) \quad ; \quad Q(t)\frac{d}{dt}p(t) + p(t)F_Q(t) = F_C(t) \qquad (11.4)$$

$$Q(t)\frac{d}{dt}\boldsymbol{v}_Q(t) + F_Q(t)\boldsymbol{v}_Q(t) = \boldsymbol{G}_Q(t) \;\; ; \;\; Q(t)p(t)\frac{d}{dt}\boldsymbol{v}_C(t) + \boldsymbol{v}_C(t)F_C(t) = \boldsymbol{G}_C(t) \quad (11.5)$$

We apply above equations on asset pricing to model price fluctuations in Part III.

5. Conclusion

Economic theory is an endless problem. We present only beginnings, essentials of economic theory and argue some outcomes. We model economy by three elements – economic variables, transactions and expectations of economic agents. Starting with these properties of economic agents we model macroeconomic variables, transactions and expectations. We show that change of risk ratings of agents due to their economic activity or other factors induce economic flows of variables, transactions and expectations and these flows make significant contribution to macroeconomic evolution. Economic flows of variables, transactions and expectations double number of properties that determine state and evolution of macroeconomy.

Our approach permits study different approximations of real economic processes and arises new problems. Let's mention some of them. First, one should chose set of economic or financial risks that determine representation of economic space. In the assumption, that this set of risks doesn't change one should provide risk assessments of economic agents. Economic model is determined by selection of k types of economic and financial transactions between agents. These transactions define evolution of $2k$ additive macroeconomic variables that are change by k types of transactions. This set of $2k$ additive variables involved into k transactions defines approximation of real economy. Transactions are made under certain expectations and model should select W expectations that approve k types of transactions. These expectations can be formed by variables determined by k transactions. If so such case describes self-consistent approximation of macroeconomics. Otherwise some expectations may be formed by exogenous variables, transactions, other expectations or any factors that are not described by initial set of k transactions and $2k$ additive variables. This case describes macroeconomic model in the presence of exogenous environment.

There exist different simplifications of general approach. For example, let's study dynamics of several economic variables and their flows in the assumptions that factors $\boldsymbol{F}_A(t,\boldsymbol{x})$ and $\boldsymbol{G}_A(t,\boldsymbol{x})$ that define economic equations on variables and their flows like (I.14; 17) depend on other economic variables and their flows only. Such approximation may describe mutual dependence between several macroeconomic variables and their flows. The simplest approximation of this kind describes mutual dependence between two economic variables and their flows in a self-consistent manner and as we show in Part III permits model wave propagation of economic disturbances and simplest model of business cycles.

Similar approximation describes dynamics of several transactions in the approximation that functions $\boldsymbol{F}(t,\boldsymbol{x})$ and $\boldsymbol{G}(t,\boldsymbol{x})$ for equations on transactions and their flows (5.9-10) depend on other transactions and their flows only. Simplest form of such approximation describes mutual dependence between two transactions and their flows in a self-consistent manner. We use this self-consistent approximation in Part III to model business cycles and wave propagation of economic disturbances.

We hope that different approximations of economic processes can help describe complex economic evolution in a unified frame and improve our understanding of economic phenomena.

Appendix A

Transactions and expectations as two component functions and assets pricing equations

To describe trading volume Q_{ij} and cost C_{ij} of transaction $\boldsymbol{bs}_{i,j}(t,z)$ with economic variable E let's define transaction as two component function:

$$\boldsymbol{bs}_{i,j}(t,\boldsymbol{z}) = \left(Q_{ij}(t,\boldsymbol{z}); C_{ij}(t,\boldsymbol{z})\right) \quad ; \quad \boldsymbol{z} = (\boldsymbol{x},\boldsymbol{y}) \tag{A.1}$$

Each component Q_{ij} and C_{ij} (A.1) of transaction $\boldsymbol{bs}_{i,j}(t,z)$ should be approved by expectations of agent i as seller and expectations of agent j as buyer. Let's define transaction $\boldsymbol{bs}_{i,j}(k;t,z;l)$ performed under sellers expectations of type $\boldsymbol{k}=(k_1;k_2)$ and buyers expectations of type $\boldsymbol{l}=(l_1;l_2)$, $k1,k2,\ l_1,l_2=1,\dots K$ as:

$$\boldsymbol{bs}_{i,j}(\boldsymbol{k};t,\boldsymbol{z};\boldsymbol{l}) = \left(Q_{ij}(k_1;t,\boldsymbol{z};l_1); C_{ij}(k_2;t,\boldsymbol{z};l_2)\right) \tag{A.2}$$

$$\boldsymbol{k} = (k_1,k_2) \quad ; \quad \boldsymbol{l} = (l_1,l_2)$$

Relation (A.2) define transactions $\boldsymbol{bs}_{i,j}(\boldsymbol{k};t,z;\boldsymbol{l})$ determined by trading volume Q_{ij} and cost C_{ij}. Relations (A.2) define price $p_{i,j}(\boldsymbol{k};t,z;\boldsymbol{l})$ of variable E for transaction $\boldsymbol{bs}_{i,j}(\boldsymbol{k};t,z;\boldsymbol{l})$ between agents i and j as:

$$C_{ij}(k_2;t,\boldsymbol{z};l_2) = p_{ij}(\boldsymbol{k};t,\boldsymbol{z};\boldsymbol{l})Q_{ij}(k_1;t,\boldsymbol{z};l_1) \tag{A.2.1}$$

Sum over all buyers expectations of $\boldsymbol{l}=(l_1;l_2)$ define sellers price $p_{i,j}(\boldsymbol{k};t,z)$

$$C_{ij}(k_2;t,\boldsymbol{z}) = p_{ij}(\boldsymbol{k};t,\boldsymbol{z})Q_{ij}(k_1;t,\boldsymbol{z}) \tag{A.2.2}$$

$Q_{i,j}(k_1;t,z)$ and $C_{i,j}(k_2;t,z)$ are defined by (A.7). Sum over all sellers expectations of $\boldsymbol{k}=(k_1;k_2)$ define buyers price $p_{i,j}(t,z;\boldsymbol{l})$

$$C_{ij}(t,\boldsymbol{z};l_2) = p_{ij}(t,\boldsymbol{z};\boldsymbol{l})Q_{ij}(t,\boldsymbol{z};l_1) \tag{A2.3}$$

$Q_{i,j}(t,z;l_1)$ and $C_{i,j}(t,z;l_2)$ are defined by (A.11). And sum over sellers and buyers expectations define price $p_{i,j}(t,z)$ of transactions between agents i and j at $\boldsymbol{x}$ and $\boldsymbol{y}$, $z=(\boldsymbol{x},\boldsymbol{y})$ as:

$$C_{ij}(t,\boldsymbol{z}) = p_{ij}(t,\boldsymbol{z})Q_{ij}(t,\boldsymbol{z}) \tag{A.2.4}$$

$$Q_{ij}(t,\boldsymbol{z}) = \sum\nolimits_{k_1;l_1} Q_{ij}(k_1;t,\boldsymbol{z};l_1) \ ; C_{ij}(t,\boldsymbol{z}) = \sum\nolimits_{k_2;l_2} C_{ij}(k_2;t,\boldsymbol{z};l_2)$$

Trading volumes Q_{ij} are approved by sellers expectations of type k_1 and buyers expectations of type l_1. The trading values or costs C_{ij} of transaction are approved by sellers expectations of type k_2 and buyers expectations of type l_2. Let's introduce seller's expectations $\boldsymbol{ex}_i(\boldsymbol{k};t,\boldsymbol{x})$ of type $\boldsymbol{k}=(k_1;k_2)$ of agent i at $\boldsymbol{x}$ as

$$\boldsymbol{ex}_i(\boldsymbol{k};t,\boldsymbol{x}) = \left(ex_{iQ}(k_1;t,\boldsymbol{x});\ ex_{iC}(k_2;t,\boldsymbol{x})\right) \tag{A.3}$$

and buyer's expectations $\boldsymbol{ex}_j(t,\boldsymbol{y};\ \boldsymbol{l})$ of type $\boldsymbol{l}=(l_1;l_2)$ of agent j at $\boldsymbol{y}$ as

$$\boldsymbol{ex}_j(t,\boldsymbol{y};\boldsymbol{l}) = \left(ex_{jQ}(t,\boldsymbol{y};l_1);\ ex_{jC}(t,\boldsymbol{y};l_2)\right) \tag{A.4}$$

that approve Q_{ij} and C_{ij} (A.2) of transaction $\boldsymbol{bs}_{i,j}(k;t,z;\boldsymbol{l})$ respectively. Similar to (7.1) let's define sellers and buyers expected transactions of as:

$$\boldsymbol{et}_{ij}(\boldsymbol{k};t,\boldsymbol{z}) = \left(et_{ijQ}(k_1;t,\boldsymbol{z})\ ;\ et_{ijC}(k_2;t,\boldsymbol{z})\right) \tag{A.4}$$

$$et_{ijQ}(k_1;t,\boldsymbol{z}) = ex_{iQ}(k_1;t,\boldsymbol{x})Q_{ij}(k_1;t,\boldsymbol{z}) \tag{A.5}$$

$$et_{ijC}(k_2;t,\boldsymbol{z}) = ex_{iC}(k_2;t,\boldsymbol{x})C_{ij}(k_2;t,\boldsymbol{z}) \tag{A.6}$$

$$Q_{ij}(k_1;t,\boldsymbol{z}) = \sum\nolimits_{l_1} Q_{ij}(k_1;t,\boldsymbol{z};l_1)\ \ ;\ \ C_{ij}(k_2;t,\boldsymbol{z}) = \sum\nolimits_{l_2} C_{ij}(k_2;t,\boldsymbol{z};l_2) \tag{A.7}$$

$$\boldsymbol{et}_{ij}(t,\boldsymbol{z};\boldsymbol{l}) = \left(et_{ijQ}(t,\boldsymbol{z};l_1)\ ;\ et_{ijC}(t,\boldsymbol{z};l_2)\right) \tag{A.8}$$

$$et_{ijQ}(t,\boldsymbol{z};l_1) = ex_{jQ}(t,\boldsymbol{y};l_1)Q_{ij}(t,\boldsymbol{z};l_1) \tag{A.9}$$

$$et_{ijC}(t,\boldsymbol{z};l_2) = ex_{jC}(t,\boldsymbol{y};l_2)C_{ij}(t,\boldsymbol{z};l_2) \tag{A.10}$$

$$Q_{ij}(t,\boldsymbol{z};l_1) = \sum\nolimits_{k_1} Q_{ij}(k_1;t,\boldsymbol{z};l_1)\ \ ;\ \ C_{ij}(t,\boldsymbol{z};l_2) = \sum\nolimits_{k_2} C_{ij}(k_2;t,\boldsymbol{z};l_2) \tag{A.11}$$

Relations (A.4) define sellers expected transactions of type $\boldsymbol{k}=(k_1,k_2)$. Relations (A.5) define sellers expected transactions for trading volume Q_{ij} and (A.6) define sellers expected transactions for cost C_{ij} of the transaction. Relations (A.7-A.9) define expected transactions for buyers of type $\boldsymbol{l}=(l_1,l_2)$. Relations (11.2) for transaction $\boldsymbol{bs}_{i,j}(\boldsymbol{k};t,z;\boldsymbol{l})$ and (A.4-A.11) for expected transactions $\boldsymbol{et}_{i,j}(\boldsymbol{k};t,z)$ and $\boldsymbol{et}_{i,j}(t,z;\boldsymbol{l})$ derive sellers aggregated transactions $\boldsymbol{BS}(\boldsymbol{k};t,z)$ and buyers aggregated transactions $\boldsymbol{BS}(t,z;\boldsymbol{l})$ and expected transactions $\boldsymbol{Et}(\boldsymbol{k};t,z)$ and $\boldsymbol{Et}(t,z;\boldsymbol{l})$ similar to (2.1; 2.2) as:

$$\boldsymbol{BS}(\boldsymbol{k};t,\boldsymbol{z}) = \left(Q(k_1;t,\boldsymbol{z});C(k_2;t,\boldsymbol{z})\right)\ ;\ \boldsymbol{z} = (\boldsymbol{x},\boldsymbol{y}) \tag{A.12.1}$$

$$Q(k_1;t,\boldsymbol{z}) = \sum\nolimits_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y});\Delta} \sum\nolimits_{l_1} Q_{ij}(k_1;t,\boldsymbol{z};l_1) \tag{A.12.2}$$

$$C(k_2;t,\boldsymbol{z}) = \sum\nolimits_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y});\Delta} \sum\nolimits_{l_2} C_{ij}(k_2;t,\boldsymbol{z};l_2) \tag{A.12.3}$$

$$\boldsymbol{BS}(t,\boldsymbol{z};\boldsymbol{l}) = \left(Q(t,\boldsymbol{z};l_1);C(t,\boldsymbol{z};l_2)\right) \tag{A.12.4}$$

$$Q(t,\boldsymbol{z};l_1) = \sum\nolimits_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y});\Delta} \sum\nolimits_{k_1} Q_{ij}(k_1;t,\boldsymbol{z};l_1) \tag{A.12.5}$$

$$C(t,\boldsymbol{z};l_2) = \sum\nolimits_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y});\Delta} \sum\nolimits_{k_2} C_{ij}(k_2;t,\boldsymbol{z};l_2) \tag{A.12.6}$$

Relations (A.12.2; 12.3) define sellers aggregated price $p(\boldsymbol{k};t,z)$ of variable E for the transaction $\boldsymbol{BS}(\boldsymbol{k};t,z)$ (A.12.1) under expectations of type $\boldsymbol{k}=(k_1;k_2)$ as:

$$C(k_2;t,\boldsymbol{z}) = p(\boldsymbol{k};t,\boldsymbol{z})Q(k_1;t,\boldsymbol{z}) \tag{A.12.7}$$

Relations (A.12.5; 12.6) define buyers aggregated price $p(t,z;\boldsymbol{l})$ for expectations of type $\boldsymbol{l}=(l_1;l_2)$ as:

$$C(t,\boldsymbol{z};l_2) = p(t,\boldsymbol{z};\boldsymbol{l})Q(t,\boldsymbol{z};l_1) \tag{A.12.8}$$

Sum by all sellers expectations (A.12.10) or all buyers expectations (A.12.11) define aggregate price $p(t,z)$ of transactions between agents at $z=(\boldsymbol{x}, \boldsymbol{y})$:

$$C(t,\boldsymbol{z}) = p(t,\boldsymbol{z})Q(t,\boldsymbol{z}) \tag{A.12.9}$$

$$Q(t,\boldsymbol{z}) = \sum_{k_1} Q_{ij}(k_1;t,\boldsymbol{z}) = \sum_{l_1} Q_{ij}(t,\boldsymbol{z};l_1) \tag{A.12.10}$$

$$C(t,\boldsymbol{z}) = \sum_{k_2} C_{ij}(k_2;t,\boldsymbol{z}) = \sum_{l_2} C_{ij}(t,\boldsymbol{z};l_2) \tag{A.12.11}$$

Integral of $C(t,\boldsymbol{x},\boldsymbol{y})$ and $Q(t,\boldsymbol{x},\boldsymbol{y})$ by $d\boldsymbol{y}$ over economic domain (1.1; 1.2) defines mean price $p_S(t,\boldsymbol{x})$ of sellers for transactions with variable E from point $\boldsymbol{x}$:

$$C_S(t,\boldsymbol{x}) = \int d\boldsymbol{y}\, C(t,\boldsymbol{x},\boldsymbol{y}) = p_S(t,\boldsymbol{x})Q_S(t,\boldsymbol{x}) \;\; ; \;\; Q_S(t,\boldsymbol{x}) = \int d\boldsymbol{y}\, Q(t,\boldsymbol{x},\boldsymbol{y}) \tag{A.12.12}$$

Relations (A.12.12) define sellers trading volume $Q_S(t,\boldsymbol{x})$ and cost $C_S(t,\boldsymbol{x})$ of all transactions from $\boldsymbol{x}$ and thus define sellers price $p_S(t,\boldsymbol{x})$ from point $\boldsymbol{x}$. Integral of $C(t,\boldsymbol{x},\boldsymbol{y})$ and $Q(t,\boldsymbol{x},\boldsymbol{y})$ by $d\boldsymbol{x}$ over economic domain (1.1; 1.2) defines mean price $p_B(t,\boldsymbol{y})$ of buyers at $\boldsymbol{y}$:

$$C_B(t,\boldsymbol{y}) = \int d\boldsymbol{x}\, C(t,\boldsymbol{x},\boldsymbol{y}) = p_B(t,\boldsymbol{y})Q_B(t,\boldsymbol{y}) \;\; ; \;\; Q_B(t,\boldsymbol{y}) = \int d\boldsymbol{x}\, Q(t,\boldsymbol{x},\boldsymbol{y}) \tag{A.12.13}$$

Relations (A.12.13) define buyers trading volume $Q_B(t,\boldsymbol{y})$ and cost $C_B(t,\boldsymbol{y})$ of all transactions to $\boldsymbol{y}$ and thus define buyers price $p_B(t,\boldsymbol{y})$ at point $\boldsymbol{y}$.

$$C(t) = \int d\boldsymbol{x}d\boldsymbol{y}\, C(t,\boldsymbol{x},\boldsymbol{y}) = p(t)Q(t) \;\; ; \;\; Q(t) = \int d\boldsymbol{x}d\boldsymbol{y}\, Q(t,\boldsymbol{x},\boldsymbol{y}) \tag{A.12.14}$$

Relations (A.12.14) define trading volume $Q(t)$ and cost $C(t)$ of all transactions with variable E in economy thus define price $p(t)$ of variable E in macroeconomics at time t. Relations (A.12.15) define sellers price $p(\boldsymbol{k};t)=p(k_1,k_2;t)$

$$C(k_2;t) = \int d\boldsymbol{z}\, C(k_2;t,\boldsymbol{z}) = p(\boldsymbol{k};t)Q(k_1;t)\; ; \; Q(k_1;t) = \int d\boldsymbol{z}\, Q(k_1;t,\boldsymbol{z}) \tag{A.12.15}$$

for transactions with trading volume $Q(k_1;t)$ and cost $C(k_2;t)$ of economic variable E under sellers expectations of type $\boldsymbol{k}=(k_1,k_2)$.

$$C(t;l_2) = \int d\boldsymbol{z}\, C(t,\boldsymbol{z};l_2) = p(t;\boldsymbol{l})Q(t;l_1) \;\; ; \;\; Q(t;l_1) = \int d\boldsymbol{z}\, Q(t,\boldsymbol{z};l_1) \tag{A.12.16}$$

Relations (A.12.16) define buyers price $p(t;\boldsymbol{l})= p(t;l_1,l_2)$ of variable E for transactions with trading volume $Q(t;l_1)$ and cost $C(t;l_2)$ under buyers expectations of type $\boldsymbol{l}=(l_1,l_2)$. Definitions (A.2.1-2.4) and (A.12.7-12.16) define different sellers and buyers states of price p of economic variable E under transactions and different expectations. We show below that relations (A.12.7-12.16) define equations on price evolution of economic variable E.

Relations (A.12.1-12.6) define transactions $\boldsymbol{BS}(\boldsymbol{k};t,z)$ made under sellers expectations of type $\boldsymbol{k}=(k_1;k_2)$ and transactions $\boldsymbol{BS}(t,z;\boldsymbol{l})$ made under buyers expectations of type $\boldsymbol{l}=(l_1;l_2)$.

$$\boldsymbol{Et}(\boldsymbol{k};t,\boldsymbol{z}) = \left(Et_Q(k_1;t,\boldsymbol{z})\; ; \; Et_C(k_2;t,\boldsymbol{z})\right) \;\; ; \; \boldsymbol{z} = (\boldsymbol{x},\boldsymbol{y}) \tag{A.13.1}$$

$$Et_Q(k_1;t,\boldsymbol{z}) = \sum_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y});\,\Delta} \sum_{l_1} ex_{iQ}(k_1;t,\boldsymbol{x})\, Q_{ij}(k_1;t,\boldsymbol{z};l_1) \tag{A.13.2}$$

$$Et_C(k_2;t,\boldsymbol{z}) = \sum_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y});\,\Delta} \sum_{l_2} ex_{iC}(k_2;t,\boldsymbol{x})\, C_{ij}(k_2;t,\boldsymbol{z};l_2) \tag{A.13.3}$$

$$\boldsymbol{Et}(t,\boldsymbol{z};\boldsymbol{l}) = \left(Et_Q(t,\boldsymbol{z};l_1)\; ; \; Et_C(t,\boldsymbol{z};l_2)\right) \tag{A.13.4}$$

$$Et_Q(t,\boldsymbol{z};l_1) = \sum_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y});\Delta}\sum_{k_1} ex_{jQ}\,(t,\boldsymbol{y};l_1)Q_{ij}(k_1;t,\boldsymbol{z};l_1) \quad (A.13.5)$$

$$Et_C(t,\boldsymbol{z};l_2) = \sum_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y});\Delta}\sum_{k_2} ex_{jC}\,(t,\boldsymbol{y};l_2)C_{ij}(k_2;t,\boldsymbol{z};l_2) \quad (A.13.6)$$

$$Et_Q(k_1;t,\boldsymbol{z}) = Ex_Q(k_1;t,\boldsymbol{z})Q(k_1;t,\boldsymbol{z}) \quad (A.13.7)$$

$$Et_C(k_2;t,\boldsymbol{z}) = Ex_C(k_2;t,\boldsymbol{z})C(k_2;t,\boldsymbol{z}) \quad (A.13.8)$$

$$Et_Q(t,\boldsymbol{z};l_1) = Ex_Q(t,\boldsymbol{y};l_1)Q(t,\boldsymbol{z};l_1) \quad (A.13.9)$$

$$Et_C(t,\boldsymbol{z};l_2) = Ex_C(t,\boldsymbol{y};l_2)C(t,\boldsymbol{z};l_2) \quad (A.13.10)$$

Relations (A.13.1-13.6) define expected transactions $\boldsymbol{Et(k;t,z)}$ of sellers made under expectations of type $\boldsymbol{k}=(k_1;k_2)$ and expected transactions $\boldsymbol{Et(t,z;l)}$ of buyers made under buyers expectations of type $\boldsymbol{l}=(l_1;l_2)$. Relations (A.13.7) for variable E define sellers aggregate expectations $Ex_Q(k_1;t,z)$ of type k_1 on trading volume $Q(k_1;t,z)$ (A.12.2) and (A.13.8) sellers aggregate expectations $Ex_C(k_2;t,z)$ of type k_2 on cost $C(k_2;t,z)$ of transaction (A.12.3) with variable E. Relations (A.13.9) define buyers aggregate expectations $Ex_Q(t,z;l_1)$ of type l_1 on trading volume $Q(t,z;l_1)$ (A.12.5) and (A.13.10) define buyers expectations $Ex_C(t,z;l_2)$ of type k_2 on cost $C(t,z;l_2)$ of transaction (A.12.6) with variable E. Now similar to (2.1; 2.2; 5.1; 5.2) and (7.1) let's introduce flows $\boldsymbol{p}_{ij}(\boldsymbol{k};t,z)$ and $\boldsymbol{p}_{ij}(t,z;\boldsymbol{l})$, $z=(\boldsymbol{x},\boldsymbol{y})$ of transactions (A.2; A.4; A.8):

$$\boldsymbol{p}_{i,j}(\boldsymbol{k};t,\boldsymbol{z}) = \left(\boldsymbol{p}_{Qij}(k_1;t,\boldsymbol{z});\boldsymbol{p}_{Cij}(k_2;t,\boldsymbol{z})\right)\ ;\ \boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y}) \quad (A.14.1)$$

$$\boldsymbol{p}_{Qij}(k_1;t,\boldsymbol{z}) = Q_{ij}(k_1;t,\boldsymbol{z})\boldsymbol{v_i}(t,\boldsymbol{x}) \quad (A.14.2)$$

$$\boldsymbol{p}_{Cij}(k_2;t,\boldsymbol{z}) = C_{ij}(k_2;t,\boldsymbol{z})\boldsymbol{v_i}(t,\boldsymbol{x}) \quad (A.14.3)$$

$$\boldsymbol{p}_{i,j}(t,\boldsymbol{z};\boldsymbol{l}) = \left(\boldsymbol{p}_{Qij}(t,\boldsymbol{z};l_1);\boldsymbol{p}_{Cij}(t,\boldsymbol{x},\boldsymbol{y};l_2)\right) \quad (A.14.4)$$

$$\boldsymbol{p}_{Qij}(t,\boldsymbol{z};l_1) = Q_{ij}(t,\boldsymbol{z};l_1)\boldsymbol{v}_j(t,\boldsymbol{y}) \quad (A.14.5)$$

$$\boldsymbol{p}_{Cij}(t,\boldsymbol{z};l_2) = C_{ij}(t,\boldsymbol{z};l_2)\boldsymbol{v}_j(t,\boldsymbol{y}) \quad (A.14.6)$$

Flows $\boldsymbol{pe}_{ij}(\boldsymbol{k};t,z)$ and $\boldsymbol{pe}_{ij}(t,z;\boldsymbol{l})$ of expected transactions $\boldsymbol{et}_{i,j}(\boldsymbol{k};t,z)$ (A.4-6) and $\boldsymbol{et}_{i,j}(t,z;\boldsymbol{l})$ (A.8-10) take form:

$$\boldsymbol{pe}_{i,j}(\boldsymbol{k};t,\boldsymbol{z}) = \left(\boldsymbol{pe}_{Qij}(k_1;t,\boldsymbol{z});\boldsymbol{pe}_{Cij}(k_2;t,\boldsymbol{z})\right)\ ;\ \boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y}) \quad (A.15.1)$$

$$\boldsymbol{pe}_{Qij}(k_1;t,\boldsymbol{z}) = ex_i(k_1;t,\boldsymbol{x})Q_{ij}(k_1;t,\boldsymbol{z})\boldsymbol{v_i}(t,\boldsymbol{x}) \quad (A.15.2)$$

$$\boldsymbol{pe}_{Cij}(k_2;t,\boldsymbol{z}) = ex_i(k_2;t,\boldsymbol{x})C_{ij}(k_2;t,\boldsymbol{z})\boldsymbol{v_i}(t,\boldsymbol{x}) \quad (A.15.3)$$

$$\boldsymbol{pe}_{i,j}(t,\boldsymbol{z};\boldsymbol{l}) = \left(\boldsymbol{pe}_{Qij}(t,\boldsymbol{z};l_1);\boldsymbol{pe}_{Cij}(t,\boldsymbol{z};l_2)\right) \quad (A.15.4)$$

$$\boldsymbol{pe}_{Qij}(t,\boldsymbol{z};l_1) = ex_j(t,\boldsymbol{y};l_1)Q_{ij}(t,\boldsymbol{z};l_1)\boldsymbol{v}_j(t,\boldsymbol{y}) \quad (A.15.5)$$

$$\boldsymbol{pe}_{Cij}(t,\boldsymbol{z};l_2) = ex_j(t,\boldsymbol{y};l_2)C_{ij}(t,\boldsymbol{z};l_2)\boldsymbol{v}_j(t,\boldsymbol{y}) \quad (A.15.6)$$

Relations (A.14.1-6) are required to define flows $\boldsymbol{P(k;t,z)}$ and $\boldsymbol{P(t,z;l)}$ and velocities $\boldsymbol{v(k;t,z)}$

and $\boldsymbol{v}(t,z;\boldsymbol{l})$ of transactions $\boldsymbol{BS}(\boldsymbol{k};t,z)$ and $\boldsymbol{BS}(t,z;\boldsymbol{l})$ (A.12.1-6). Relations (A.15.1-6) allow define flows $\boldsymbol{Pe}(\boldsymbol{k};t,z)$ and $\boldsymbol{Pe}(t,z;\boldsymbol{l})$ and velocities $\boldsymbol{v}_{et}(\boldsymbol{k};t,z)$ and $\boldsymbol{v}_{et}(t,z;\boldsymbol{l})$ of expected transactions $\boldsymbol{Et}(\boldsymbol{k};t,z)$ and $\boldsymbol{Et}(t,z;\boldsymbol{l})$ (A.13.1-6). Let's define flows $\boldsymbol{P}(\boldsymbol{k};t,z)$ and $\boldsymbol{P}(t,z;\boldsymbol{l})$, $z=(\boldsymbol{x},\boldsymbol{y})$ similar to (9.4-9.9) as:

$$\boldsymbol{P}(\boldsymbol{k};t,\boldsymbol{z}) = \left(\boldsymbol{P}_Q(k_1;t,\boldsymbol{z});\boldsymbol{P}_C(k_2;t,\boldsymbol{z})\right) \quad ; \quad \boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y}) \tag{A.16.1}$$

$$\boldsymbol{P}_Q(k_1;t,\boldsymbol{z}) = \sum_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y})\,\Delta} Q_{ij}(k_1;t,\boldsymbol{z})\boldsymbol{v_i}(t,\boldsymbol{x}) \tag{A.16.2}$$

$$\boldsymbol{P}_C(k_2;t,\boldsymbol{z}) = \sum_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y})\,\Delta} C_{ij}(k_2;t,\boldsymbol{z})\boldsymbol{v_i}(t,\boldsymbol{x}) \tag{A.16.3}$$

$$\boldsymbol{P}(t,\boldsymbol{z};\boldsymbol{l}) = \left(\boldsymbol{P}_Q(t,\boldsymbol{z};l_1);\boldsymbol{P}_C(t,\boldsymbol{x},\boldsymbol{y};l_2)\right) \tag{A.16.4}$$

$$\boldsymbol{P}_Q(t,\boldsymbol{z};l_1) = \sum_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y})\,\Delta} Q_{ij}(t,\boldsymbol{z};l_1)\boldsymbol{v}_j(t,\boldsymbol{y}) \tag{A.16.5}$$

$$\boldsymbol{P}_C(t,\boldsymbol{z};l_2) = \sum_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y})\,\Delta} C_{ij}(t,\boldsymbol{z};l_2)\boldsymbol{v}_j(t,\boldsymbol{y}) \tag{A.16.6}$$

$$\boldsymbol{P}_Q(k_1;t,\boldsymbol{z}) = Q(k_1;t,\boldsymbol{z})\boldsymbol{v}_Q(k_1;t,\boldsymbol{z}) \;;\; \boldsymbol{P}_C(k_1;t,\boldsymbol{z}) = C(k_2;t,\boldsymbol{z})\boldsymbol{v}_C(k_2;t,\boldsymbol{z}) \tag{A.16.7}$$

$$\boldsymbol{P}_Q(t,\boldsymbol{z};l_1) = Q(t,\boldsymbol{z};l_1)\boldsymbol{v}_Q(t,\boldsymbol{z};l_1) \;;\; \boldsymbol{P}_C(t,\boldsymbol{z};l_2) = C(t,\boldsymbol{z};l_2)\boldsymbol{v}_C(t,\boldsymbol{z};l_2) \tag{A.16.8}$$

$$\boldsymbol{v}(\boldsymbol{k};t,\boldsymbol{z}) = \left(\boldsymbol{v}_Q(k_1;t,\boldsymbol{z});\boldsymbol{v}_C(k_2;t,\boldsymbol{z})\right) \;;\; \boldsymbol{k}=(k_1,k_2) \tag{A.16.9}$$

$$\boldsymbol{v}(t,\boldsymbol{z};\boldsymbol{l}) = \left(\boldsymbol{v}_Q(t,\boldsymbol{z};l_1);\boldsymbol{v}_C(t,\boldsymbol{z};l_2)\right) \;;\; \boldsymbol{l}=(l_1,l_2) \tag{A.16.10}$$

Relations (A.16.7-16.8) define velocities $\boldsymbol{v}(\boldsymbol{k};t,z)$ (16.9) and $\boldsymbol{v}(t,z;\boldsymbol{l})$ (16.10). These velocities determine equations on transactions $\boldsymbol{BS}(\boldsymbol{k};t,z)$ (A.12.1-12.3) made under sellers expectations of type $\boldsymbol{k}=(k_1;k_2)$ and transactions $\boldsymbol{BS}(t,z;\boldsymbol{l})$ (A.12.4-12.6) made under buyers expectations of type $\boldsymbol{l}=(l_1;l_2)$. Flows $\boldsymbol{Pe}(\boldsymbol{k};t,z)$ and $\boldsymbol{Pe}(t,z;\boldsymbol{l})$, $z=(\boldsymbol{x},\boldsymbol{y})$ of expected transactions $\boldsymbol{Et}(\boldsymbol{k};t,z)$ and $\boldsymbol{Et}(t,z;\boldsymbol{l})$ (A.13.1-10) take form:

$$\boldsymbol{Pe}(\boldsymbol{k};t,\boldsymbol{z}) = \left(\boldsymbol{Pe}_Q(\boldsymbol{k};t,\boldsymbol{z});\boldsymbol{Pe}_C(k_2;t,\boldsymbol{z})\right) \quad ; \quad \boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y}) \tag{A.17.1}$$

$$\boldsymbol{Pe}_Q(k_1;t,\boldsymbol{z}) = \sum_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y})\,\Delta} ex_{iQ}(k_1;t,\boldsymbol{x})Q_{ij}(k_1;t,\boldsymbol{z})\boldsymbol{v_i}(t,\boldsymbol{x}) \tag{A.17.2}$$

$$\boldsymbol{Pe}_C(k_2;t,\boldsymbol{z}) = \sum_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y})\,\Delta} ex_{iC}(k_2;t,\boldsymbol{x})C_{ij}(k_2;t,\boldsymbol{z})\boldsymbol{v_i}(t,\boldsymbol{x}) \tag{A.17.3}$$

$$\boldsymbol{Pe}(t,\boldsymbol{z};\boldsymbol{l}) = \left(\boldsymbol{Pe}_Q(t,\boldsymbol{z};l_1);\boldsymbol{Pe}_C(t,\boldsymbol{z};l_2)\right) \tag{A.17.4}$$

$$\boldsymbol{Pe}_Q(t,\boldsymbol{z};l_1) = \sum_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y})\,\Delta} ex_{jQ}(t,\boldsymbol{y};l_1)Q_{ij}(t,\boldsymbol{z};l_1)\boldsymbol{v}_j(t,\boldsymbol{y}) \tag{A.17.5}$$

$$\boldsymbol{Pe}_C(t,\boldsymbol{z};l_2) = \sum_{i\in dV(\boldsymbol{x});j\in dV(\boldsymbol{y})\,\Delta} ex_{jC}(t,\boldsymbol{y};l_2)C_{ij}(t,\boldsymbol{z};l_2)\boldsymbol{v}_j(t,\boldsymbol{y}) \tag{A.17.6}$$

$$\boldsymbol{Pe}_Q(k_1;t,\boldsymbol{z}) = Et_Q(k_1;t,\boldsymbol{z})\boldsymbol{v}_{eQ}(k_1;t,\boldsymbol{z}) = Ex_Q(k_1;t,\boldsymbol{z})Q(k_1;t,\boldsymbol{z})\boldsymbol{v}_{eQ}(k_1;t,\boldsymbol{z}) \tag{A.17.7}$$

$$\boldsymbol{Pe}_C(k_2;t,\boldsymbol{z}) = Et_C(k_2;t,\boldsymbol{z})\boldsymbol{v}_{eC}(k_2;t,\boldsymbol{z}) = Ex_C(k_2;t,\boldsymbol{z})C(k_2;t,\boldsymbol{z})\boldsymbol{v}_{eC}(k_2;t,\boldsymbol{z}) \tag{A.17.8}$$

$$\boldsymbol{Pe}_Q(t,\boldsymbol{z};l_1) = Ex_Q(t,\boldsymbol{z};l_1)Q(t,\boldsymbol{z};l_1)\boldsymbol{v}_{eQ}(t,\boldsymbol{z};l_1) \tag{A.17.9}$$

$$\boldsymbol{Pe}_C(t,\boldsymbol{z};l_2) = Ex_C(t,\boldsymbol{z};l_2)C(t,\boldsymbol{z};l_2)\boldsymbol{v}_{eC}(t,\boldsymbol{z};l_2) \tag{A.17.10}$$

$$\boldsymbol{v}_e(\boldsymbol{k};t,\boldsymbol{z}) = \left(\boldsymbol{v}_{eQ}(k_1;t,\boldsymbol{z});\ \boldsymbol{v}_{eC}(k_2;t,\boldsymbol{z})\right) \tag{A.17.11}$$

$$\boldsymbol{v}_e(t,\boldsymbol{z};\boldsymbol{l}) = \left(\boldsymbol{v}_{eQ}(t,\boldsymbol{z};l_1);\ \boldsymbol{v}_{eC}(t,\boldsymbol{z};l_2)\right) \tag{A.17.12}$$

Relations (A.17.1-17.3) and (A.17.7-17.8) for $z=(\boldsymbol{x},\boldsymbol{y})$ define expectations $Ex_Q(k_1;t,z)$ and $Ex_C(k_2;t,z)$ of sellers that approve transactions with trading volume $Q(k_1;t,z)$ (A.12.2) and cost $C(k_2;t,z)$ (A.12.3) as well as velocities $\boldsymbol{v}_{eQ}(k_1;t,z)$ and $\boldsymbol{v}_{eC}(k_2;t,z)$ (A.17.11) that describe motion of sellers expectations. Relations (A.17.4-17.6) and (A.17.9-17.10) define expectations $Ex_Q(t,z;l_1)$, $z=(\boldsymbol{x},\boldsymbol{y})$ of buyers that approve transactions with trading volume $Q(t,z;l_1)$ (A.12.5) and expectations $Ex_C(t,z;l_2)$ that approve transactions with trading cost $C(t,z;l_2)$ (A.12.6) as well as velocities $\boldsymbol{v}_{eQ}(t,z;l_1)$ and $\boldsymbol{v}_{eC}(t,z;l_2)$ (A.17.12) that describe motion of buyers expectations.

Equations (A.18.1-18.4) describe transactions $\boldsymbol{BS}(\boldsymbol{k};t,z)$ (A.12.1-12.3) and flows $\boldsymbol{P}(\boldsymbol{k};t,z)$ (A.16.1-16.3) made under sellers expectations of type $\boldsymbol{k}=(k_1;k_2)$

$$\frac{\partial}{\partial t}Q(k_1;t,\boldsymbol{z}) + \nabla\cdot\left(Q(k_1;t,\boldsymbol{z})\boldsymbol{v}_Q(k_1;t,\boldsymbol{z})\right) = F_Q(k_1;t,\boldsymbol{z}) \tag{A.18.1}$$

$$\frac{\partial}{\partial t}\boldsymbol{P}_Q(k_1;t,\boldsymbol{z}) + \nabla\cdot\left(\boldsymbol{P}_Q(k_1;t,\boldsymbol{z})\boldsymbol{v}_Q(k_1;t,\boldsymbol{z})\right) = \boldsymbol{G}_Q(k_1;t,\boldsymbol{z}) \tag{A.18.2}$$

$$\frac{\partial}{\partial t}C(k_2;t,\boldsymbol{z}) + \nabla\cdot\left(C(k_2;t,\boldsymbol{z})\boldsymbol{v}_C(k_2;t,\boldsymbol{z})\right) = F_C(k_2;t,\boldsymbol{z}) \tag{A.18.3}$$

$$\frac{\partial}{\partial t}\boldsymbol{P}_C(k_2;t,\boldsymbol{z}) + \nabla\cdot\left(\boldsymbol{P}_C(k_2;t,\boldsymbol{z})\boldsymbol{v}_C(k_2;t,\boldsymbol{z})\right) = \boldsymbol{G}_C(k_2;t,\boldsymbol{z}) \tag{A.18.4}$$

Equations (A.19.1-19.4) describe transactions $\boldsymbol{BS}(t,z;\boldsymbol{l})$ (A.12.4-12.6) and flows $\boldsymbol{P}(t,z;\boldsymbol{l})$ (A.16.4-16.7) made under Buyers expectations of type $\boldsymbol{l}=(l_1;l_2)$ are similar to (6.1; 6.2):

$$\frac{\partial}{\partial t}Q(t,\boldsymbol{z};l_1) + \nabla\cdot\left(Q(t,\boldsymbol{z};l_1)\boldsymbol{v}_Q(t,\boldsymbol{z};l_1)\right) = F_Q(t,\boldsymbol{z};l_1) \tag{A.19.1}$$

$$\frac{\partial}{\partial t}\boldsymbol{P}_Q(t,\boldsymbol{z};l_1) + \nabla\cdot\left(\boldsymbol{P}_Q(t,\boldsymbol{z};l_1)\boldsymbol{v}_Q(t,\boldsymbol{z};l_1)\right) = \boldsymbol{G}_Q(t,\boldsymbol{z};l_1) \tag{A.19.2}$$

$$\frac{\partial}{\partial t}C(t,\boldsymbol{z};l_2) + \nabla\cdot\left(C(t,\boldsymbol{z};l_2)\boldsymbol{v}_C(t,\boldsymbol{z};l_2)\right) = F_C(t,\boldsymbol{z};l_2) \tag{A.19.3}$$

$$\frac{\partial}{\partial t}\boldsymbol{P}_C(t,\boldsymbol{z};l_2) + \nabla\cdot\left(\boldsymbol{P}_C(t,\boldsymbol{z};l_2)\boldsymbol{v}_C(t,\boldsymbol{z};l_2)\right) = \boldsymbol{G}_C(t,\boldsymbol{z};l_2) \tag{A.19.4}$$

Velocities $\boldsymbol{v}_e(\boldsymbol{k};t,z)$ (A.17.11) and $\boldsymbol{v}_e(t,z;\boldsymbol{l})$ (A.17.12) define equations (A.20.1-20.4) on expected transactions $\boldsymbol{Et}(\boldsymbol{k};t,z)$ (13.6-13.8) and their flows $\boldsymbol{Pe}(\boldsymbol{k};t,z)$ (A.17.1-17.3):

$$\frac{\partial}{\partial t}Et_Q(k_1;t,\boldsymbol{z}) + \nabla\cdot\left(Et_Q(k_1;t,\boldsymbol{z})\boldsymbol{v}_{eQ}(k_1;t,\boldsymbol{z})\right) = F_{eQ}(k_1;t,\boldsymbol{z}) \tag{A.20.1}$$

$$\frac{\partial}{\partial t}\boldsymbol{Pe}_Q(k_1;t,\boldsymbol{z}) + \nabla\cdot\left(\boldsymbol{Pe}_Q(k_1;t,\boldsymbol{z})\boldsymbol{v}_{eQ}(k_1;t,\boldsymbol{z})\right) = \boldsymbol{G}_{eQ}(k_1;t,\boldsymbol{z}) \tag{A.20.2}$$

$$\frac{\partial}{\partial t}Et_C(k_2;t,\boldsymbol{z}) + \nabla\cdot\left(Et_C(k_2;t,\boldsymbol{z})\boldsymbol{v}_{eC}(k_2;t,\boldsymbol{z})\right) = F_{eC}(k_2;t,\boldsymbol{z}) \tag{A.20.3}$$

$$\frac{\partial}{\partial t}\boldsymbol{Pe}_C(k_2;t,\boldsymbol{z}) + \nabla\cdot\left(\boldsymbol{Pe}_C(k_2;t,\boldsymbol{z})\boldsymbol{v}_{eC}(k_2;t,\boldsymbol{z})\right) = \boldsymbol{G}_{eC}(k_2;t,\boldsymbol{z}) \tag{A.20.4}$$

Equations (A.21.1-21.4) on expected transactions $\boldsymbol{Et}(t,z;\boldsymbol{l})$ (A.13.1-6) and their flows $\boldsymbol{Pe}(t,z;\boldsymbol{l})$ (A.17.4-17.6):

$$\frac{\partial}{\partial t}Et_Q(t,\boldsymbol{z};l_1)+\nabla\cdot\left(Et_Q(t,\boldsymbol{z};l_1)\boldsymbol{v}_{eQ}(t,\boldsymbol{z};l_1)\right)=F_{eQ}(t,\boldsymbol{z};l_1) \tag{A.21.1}$$

$$\frac{\partial}{\partial t}\boldsymbol{Pe}_Q(t,\boldsymbol{z};l_1)+\nabla\cdot\left(\boldsymbol{Pe}_Q(t,\boldsymbol{z};l_1)\boldsymbol{v}_{eQ}(t,\boldsymbol{z};l_1)\right)=\boldsymbol{G}_{eQ}(t,\boldsymbol{z};l_1) \tag{A.21.2}$$

$$\frac{\partial}{\partial t}Et_C(t,\boldsymbol{z};l_2)+\nabla\cdot\left(Et_C(t,\boldsymbol{z};l_2)\boldsymbol{v}_{eC}(t,\boldsymbol{z};l_2)\right)=F_{eC}(t,\boldsymbol{z};l_2) \tag{A.21.3}$$

$$\frac{\partial}{\partial t}\boldsymbol{Pe}_C(t,\boldsymbol{z};l_2)+\nabla\cdot\left(\boldsymbol{Pe}_C(t,\boldsymbol{z};l_2)\boldsymbol{v}_{eC}(t,\boldsymbol{z};l_2)\right)=\boldsymbol{G}_{eC}(t,\boldsymbol{z};l_2) \tag{A.21.4}$$

Equations (A.18.1 – 21.4) complete system of equations on transactions and expected transactions and their flows made under expectations of type $\boldsymbol{k}=(k_1;k_2)$ and $\boldsymbol{l}=(l_1;l_2)$. Equations (A.18.1 – 21.4) and definitions of price *p* (A.12.7-12.16) permit derive equations on price of economic variable *E* due to transactions ***BS*** (A.12.1-6). To derive equations on price $p(k_1,k_2;t)$ (A.12.7) for transactions (A.12.15) made under sellers expectations k_1 and k_2 let's take integrals of (A.18.1-18.4) by $d\boldsymbol{z}=d\boldsymbol{x}d\boldsymbol{y}$ over economic domain:

$$C(k_2;t)=p(k_1,k_2;t)Q(k_1;t)$$

$$\frac{d}{dt}Q(k_1;t)=F_Q(k_1;t)\ \ ;\ \ \frac{d}{dt}C(k_2;t)=F_C(k_2;t) \tag{A.22.1}$$

$$Q(k_1;t)\frac{d}{dt}p(k_1,k_2;t)+p(k_1,k_2;t)F_Q(k_1;t)=F_C(k_2;t) \tag{A.22.2}$$

Transactions made in economy at moment *t* with variable *E* under all expectations of sellers and buyers define equations on price $p(t)$ (A.12.14):

$$C(t)=p(t)Q(t)$$

$$\frac{d}{dt}Q(t)=F_Q(t)\ \ ;\ \ \frac{d}{dt}C(t)=F_C(t) \tag{A.23.1}$$

$$Q(t)\frac{d}{dt}p(t)+p(t)F_Q(t)=F_C(t) \tag{A.23.2}$$

Let's underline two issues on equations (A.23.2). First – price $p(t)$ (A.23.2) depends on functions $F_Q(t)$ that determine evolution of quantity $Q(t)$ (A.23.1) and $F_C(t)$ that determine cost $C(t)$ (A.12.14) of transactions. Second - complexity of price $p(t)$ definition by equation (A.23.2) is hidden by direct form of functions $F_Q(t)$, $F_C(t)$ that define dependence of transactions (A.18.1) and (A.18.3) on $F_Q(k_1;t,z)$, $F_C(k_2;t,z)$ under sellers expectations of type $\boldsymbol{k}=(k_1;k_2)$ or (A.19.1) and (A.19.3) on $F_Q(t,\boldsymbol{z};l_1)$, $F_C(t,\boldsymbol{z};l_2)$ under buyers expectations of type $\boldsymbol{l}=(l_1;l_2)$. These functions describe dependence of transactions on expectations and their flows. Expectations may depend on economic variables, transactions, other expectations and their flows. Thus expectations play core role for transmitting impact of different economic variables, transactions and their flows on price $p(t)$ (A.23.2) of variable *E*. That makes description of price $p(t)$ a really tough problem. Let's repeat that dependence of expectations on flows of variables, transactions and other expectations may cause dependence of price $p(t)$ on flows and velocities $\boldsymbol{v}_Q(t)$ and $\boldsymbol{v}_C(t)$ or velocities of transactions and etc. Analysis of price evolution and fluctuations requires development of econometrics data that can verify model dependence of expectations on economic variables, transactions and their flows.

Equations (A.22.1-4) describe sellers price $p(k_1,k_2;t)$ (A.12.15) that reflect price in entire economics due to sellers expectations of type $\boldsymbol{k}=(k_1, k_2)$. Let's mention that sellers price $p(k_1,k_2;t)$ (A.12.15) can differs from buyers price $p(t;l_1,l_2)$ (A.12.16) but nevertheless they both define same price $p(t)$ (A.12.14) determined by all transactions with variable E in the entire economics. Fluctuations of sellers $p(k_1,k_2;t)$ (A.12.15) can differs from statistics of buyers price $p(t;l_1,l_2)$ (A.12.16). This and many other problems concern modeling price dynamics and fluctuations should be studied further.

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