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Effective Leadership Selection in Complementary Teams*

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Abstract

This paper considers effective leadership selection in a simple two-person team production model with heterogeneous agents. We demonstrate *leadership success through synergy* by showing that the existence of synergy makes effort complementary, implying that the leader devote more effort than the follower and that a team with a leader yields greater production than a team without a leader. We also show that, to elicit greater team production, a principal should appoint the agent with higher (lower) opportunity cost as the leader (follower). Even if the agents' opportunity costs are unobservable to the principal, the principal can select a better leader by proposing a larger position allowance for the leader. The results may explain why many organizations indeed favor leadership styles and why real-world leaders receive higher compensation than followers.

JEL Classification: H41; D21; M54

Keywords: Team production; Leadership selection; Synergy effect; Complementary team.

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1 Introduction

The issue of leadership in teams is important for many organizations (e.g., firms, sports teams, and universities). Most of these organizations favor leadership styles and pay higher compensation to leaders in their departments or business units. Various explanations exist for the essence and benefits of leadership. In general, an effective team leader is one who succeeds in providing guidance, instruction, and direction to the team and motivates the team members to use their knowledge, skills, and influence. Hermalin (2012) notes that “the essences of leadership is the ability to induce others to follow absent the power to compel or to provide formal contractual incentives.”

In economic theory, especially game theory, there is a famous concept of a leader called a *Stackelberg leader*. Although it may be thought that the concept of a Stackelberg leader means simply a “first mover” in some sequential-move games, a Stackelberg leader in a model of team production (or of private provision of public goods) can capture the following important features of a real-world leader. First, the Stackelberg leader has the power and authority to commit his/her workload. Second, by committing his/her workload, the Stackelberg leader can indirectly control, influence, and even motivate the followers. Finally, the Stackelberg leader must have an informational advantage against the followers because s/he is required to expect the followers’ reactions in advance.

However, in the conventional economic theory of *team production*, the Stackelberg leader’s equilibrium behavior is not consistent with the general image of leaders’ behavior. For example, considering a leader-follower version of the private provision of public goods, Varian (1994) shows that a Stackelberg leader can free ride on the follower’s effort and devote smaller effort than the follower. In his model, a first-mover advantage exists due to the strategic substitutability between the agents’ efforts. This result implies that agents want to be a Stackelberg leader, *even if lower payments are received*. Varian (1994) also shows that a team with a leader (sequential-move game) results in smaller team achievements than a team without a leader (simultaneous-move game), indicating that the upper organizations of the team do not favor leadership style in teams. These results contradict the above explanations for the benefits of leadership.

Constructing a simple team production model with heterogeneous agents, we reconsider the benefits of leadership in teams and provide a theoretical rationale for why and when leadership style is good for teams. An important feature of our model is the incorporation of a “synergy effect” into a conventional team production model. In our study, synergy is defined as the interaction of two or more agents to produce an effect greater than the sum of their individual efforts. In addition, we assume that synergy is more likely to be exerted effectively when agents make similar efforts in their team project.¹ We show that the existence of synergy effect is crucial for leadership style to be effective.

Within the above framework of the model, we first consider the question of which team structure, a team with a leader (which we call a Stackelberg team) or a team without a leader (which we call a Nash team), is good for teams and for a principal (or upper management). If

¹It can be justified by, for example, positive motivational effects in teams that result from peer pressure.

the leadership style is shown to be better for the principal, we next consider which agent, an agent with high opportunity costs or an agent with low opportunity costs, should be a leader. Then, we consider a leadership selection problem: how a principal who has imperfect knowledge of the agents' type can construct an effective team structure. Finally, we ask how synergies affect the efficiency of the team and the equilibrium compensation of agents.

We find that the existence of synergy makes agents' efforts strategic complements, implying that a leader devotes more than a follower. Without any compensation for the leader, each agent wants to be a follower in order to enjoy the second-mover advantages. The principal, as well as the agents, favors a Stackelberg team because such a team yields greater team production than a Nash team does. Therefore, the principal has to provide an additional incentive to the agents, depending on their position (leader or follower), to build an appropriate team structure. In addition, the agent with higher opportunity costs should be the leader in order to produce greater team achievements.

We also find that the principal, even if s/he does not exactly know each agent's type, can select a desirable leader with higher opportunity costs by offering the agents an additional payment scheme contract in which some amount of the position allowance is redistributed from the potential follower to the potential leader. As a result, the leader's compensation is greater than the follower's, as is often the case in the real world. Furthermore, we find that the stronger the synergy is, the more effort the leader devotes than the follower, and a greater position bonus is required to compensate the leader's greater effort. Thus, the wage gap between the leader and the follower increases as the synergies become more prominent. These results reconcile the property and behavior of a Stackelberg leader in team production theory with those of a leader in reality.

We extend the basic model by incorporating some important aspects: endogenous labor share, voluntary leadership, larger team, larger project, and heterogeneous skills. First, we show that the optimal labor share for maximizing the principal's payoff is increasing in synergies, implying that the principal (manager) should set a higher labor share of the profits for their agents (employees) when the synergies are large. Second, we consider a situation of endogenous leadership in which agents voluntarily decide who should be the leader to investigate what happens in the absence of a principal. We show that in a mixed strategy Nash equilibrium of the endogenous leadership game, a desirable Stackelberg team is the least likely to be voluntarily organized. This result suggests the importance of the principal's role in selecting the leader. Third, we consider a team that consists of more than two agents. We find that the optimal team size and the equilibrium position allowance for the leader crucially depend on the degree of scale synergy. Fourth, incorporating various profitabilities of a team project, we investigate the effect of a larger project on the leader's compensation. We find that the larger the project is, the greater the position allowance the principal should provide for the leader. Finally, incorporating heterogeneous skill in engaging team production, we confirm the robustness of our main result that effective leadership comes from leaders with higher opportunity costs.

This study contributes to the literature in three important ways. First, it reconciles the model prediction of Stackelberg leaders with real-world observations of leaders without imposing

additional assumptions, such as asymmetric information and psychological factors in teams. Our simple model incorporating the synergy effect into the traditional model of team production shows that (i) a leader exerts greater effort than the follower(s) to leverage the synergy effect, (ii) leadership style is beneficial to the team and upper management, (iii) a more successful leader is one with high opportunity costs, and (iv) the leader's compensation should be greater than that of the follower(s). Second, this study briefly points out that the degree of synergy is crucial for leadership style to be effective. Finally, by building a model in which the team leader is appointed by a principal outside the team, who may not have information about the agents' type, it provides a theoretical foundation on how a leader's high rewards are rationalized and how important it is for the leader to be selected by the principal.

This paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the basic structure of the model. Section 4 considers the benefits of leadership by comparing the equilibria of Nash and Stackelberg games. Section 5 considers leadership selection by a principal and characterizes the subgame-perfect Nash equilibrium of the full game. Section 6 considers various extensions of the analysis, and Section 7 concludes the paper.

2 Related literature

The conventional team production model has qualitatively similar features with the standard model of private provision of public goods, such as Bergstrom et al. (1986). As mentioned in the Introduction, Varian (1994) considers a leader-follower structure and shows that a Stackelberg leader free rides on a follower's contribution; therefore, the leader-follower style produces less than the Nash style. His results come from the strategic substitutability of agents' effort: the total is equal to the sum of the agents' efforts, that is, there is no synergy in teams.² Romano and Yildirim (2001) and Kempf and Rota-Graziosi (2010) consider voluntary leadership in a two-player public-good model by applying an endogenous timing game developed by Hamilton and Slutsky (1990). They find that a leader-follower style can endogenously emerge when the agents' contributions exhibit strategic complementarity. Incorporating 'conformity preferences', in which agents dislike effort differentials, into a two-player team production model, Huck and Rey-Biel (2006) confirm that beneficial leadership can endogenously arise.³ In contrast to these studies, our study explicitly attributes the complementary team to the existence of synergy in effort.⁴ In addition, our study focuses on leadership selection by an outside coordinator (i.e., a

²Andreoni (1998) considers increasing returns at low levels of provision of public good and shows that an announcement of 'leadership giving' may be effective in eliciting greater contributions from the followers. The property of increasing returns comes from a minimum threshold for the production of the public good. In our study, team production exhibits increasing returns stemming from the existence of synergy in effort.

³For more theoretical and experimental studies on leadership that consider various psychological factors, see Potters et al. (2005), Potters et al. (2007), Gächter et al. (2010), Arbak and Villeval (2013), and Amegashie (2016).

⁴Vislie (1994) considers the optimal sharing rule in an extreme case in which efforts are strict complements in team production. In contrast, our study considers a team production function in which the degree of complementarity varies depending on the degree of synergy.

principal) who can design a compensation system for the agents rather than leadership selection by team members (i.e., agents).

Our study considers heterogeneous agents in terms of their opportunity costs for team production. Some studies on the simultaneous-move game of the private provision of public goods explicitly incorporate heterogeneous agents and provide interesting result on the *strategic advantages of being less skilled*, in which agents may have an incentive to remain unskilled (e.g., Buchholz and Konrad (1994, 1995), Ihori (1996), Buchholz et al. (1998), and Hattori (2005)). However, recent studies, such as Hattori and Yamada (2018) and Buchholz and Eichenseer (2019), show that a leadership style may enhance team performance in the long run because a leader never has the strategic advantage of being less skilled. Compared to these studies, our study considers a complementary team in which no agent has such strategic advantages; therefore, leadership style is beneficial even in the short run.

In the paper perhaps most closely related to ours, Bose et al. (2010) considers leader-follower style in a team production model with (two) symmetric agents. They show that when contributions are strategic complements, a principal (a manager) can enjoy the benefits of leadership by paying less for the leader than the follower. As a result of lower compensation, the leader is worse off than the follower, which implies that neither agent wants to be the leader. This result comes from the assumption that the leadership role is determined by a principal who has perfect information of the agents' type. By contrast, in our study, the leadership role is determined by a principal who does not know the agents' type to satisfy the incentive constraint for each agent. Because the complementarity in effort generates the first-mover disadvantage, higher payment is required to encourage the agents to voluntarily step forward as a leader.

In the literature of organizational economics, path-breaking studies of Hermalin (1998) consider a Stackelberg leader in team production who has privileged information about what a team should do. He shows that the Stackelberg leader will make greater contributions to signal the high value of the team project, i.e., the leader *leads by example*. Since then, a variety of studies have focused on the informational aspect of the leadership role. For example, Vesterlund (2003) and Andreoni (2006) provide a signaling explanation for leadership giving in charitable fundraising. They find that the announcement of a contribution to a charity by a first giver reveals the charity to be high quality to subsequent givers. Komai et al. (2007) consider a signaling model of team production and show that restricting (or centralizing) information to a leader can induce followers to work harder and improve efficiency.⁵ The common feature of these studies is asymmetric information between agents (i.e., leader(s) and follower(s)) within a team. By contrast, our model explores a situation in which there is information asymmetry between agents and a principal outside the team but no information asymmetry within a team. This assumption simplifies the strategic relationship between agents and allows us to explicitly analyze the process in which a superior (principal) selects leaders of subordinate teams.

⁵For more studies focusing on the informational aspect of the leadership role, see Kobayashi and Suehiro (2005), Zhou (2016), and Cato and Ishihara (2017).

3 The model

Consider two agents, 1 and 2, contributing to a team project. The achievement of the team project, G , is defined by

$$G \equiv \sum_{i \in \{1,2\}} g_i + \alpha \prod_{i \in \{1,2\}} g_i, \quad (1)$$

where g_i represents agent i 's ($i \in \{1,2\}$) effort on the team project and $\alpha > 0$ represents the degree of synergy. If $\alpha = 0$, team achievements are given by the sum of the efforts, as in most of the literature. In our setting of $\alpha > 0$, assuming the sum of each agent's effort to be fixed, the more similar the efforts, the greater the total production that can be achieved. For example, if $g_1 = 5$ and $g_2 = 5$, then $G = 10 + 25\alpha$, which is strictly greater than $G = 10 + 16\alpha$ if $g_1 = 2$ and $g_2 = 8$. This simple specification of the team production function captures the important features of synergies and makes the analysis tractable. We also assume that the individual efforts are not verifiable for the principal and, therefore, contracts cannot be written contingent on them.⁶

Each agent i 's payoff is defined by

$$U_i = \frac{\beta}{2} G - c_i g_i^2 + b_k, \quad (2)$$

where $\beta \in [0, 1]$ represents the labor (or wage) share, i.e., the fraction of joint profits from the team project allocated equally to the agents (thus, each agent receives a $\beta/2$ fraction of the joint profits), and b_k represents a position allowance based on the agent's role $k = \{L, F\}$ (L stands for the position of a leader and F for the position of a follower). The position allowance b_k depends only on the agent's role and is independent of their effort. The position allowances, b_L and b_F , may take positive or negative values.⁷

In Eq. (2), c_i represents agent i 's "opportunity cost" parameter; thus, the term $c_i g_i^2$ represents agent i 's total opportunity cost or sacrificed value of engaging in the team project, e.g., the subjective value of leisure and the profits from engaging in an alternative project. Therefore, a higher value of c_i implies that agent i has higher status (because leisure is more important) or holds many other profitable jobs. Agents may have a different opportunity cost parameter c_i , which is perfectly known for both agents. However, the principal (she) does not know each agent's opportunity cost parameter.

The principal does not directly engage in the team project and serves only as a coordinator in choosing the team structure (Nash horizontal team or Stackelberg vertical team) and, if necessary, appointing one agent as the leader. The principal allocates the $\beta \in (0, 1)$ fraction of

⁶There are two ways to interpret the nature of the synergy in our specification of the team production function. One interpretation is that synergy captures the degree to which the whole is greater than the sum of the efforts exerted by agents who perform the *same* task. Another interpretation is that synergy broadly captures the degree of complementarity in efforts exerted by agents who share *different* tasks. For example, synergy includes a team in sports where players play different roles, such as forward and defense players in soccer, a team of salespersons and administrative staffs, and workers assembling different parts on a production line.

⁷Note that there is no need for the principal to pay this additional compensation to the agents when she decides to organize a Nash (horizontal) team.

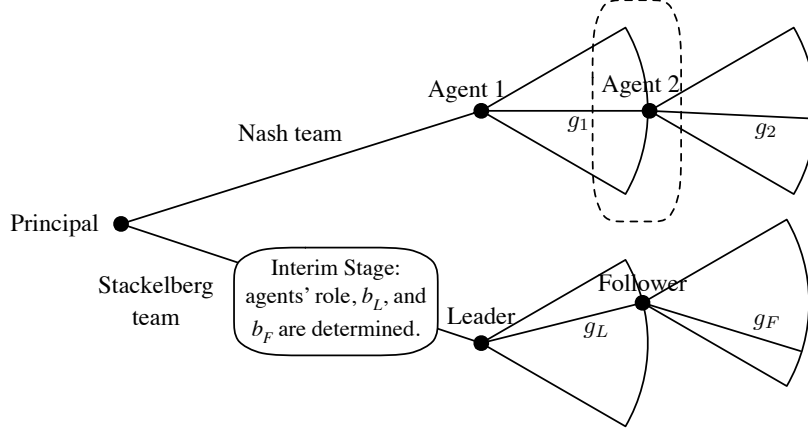


Figure 1: Game tree of leadership selection and team production

the team achievements equally to agents and the $(1 - \beta)$ fraction to the principal herself. In addition, she can pay the position allowances for the leader and the follower, b_L and b_F , when she decides to organize the agents into a Stackelberg team. Therefore, the principal's problem is to coordinate the team and design the compensation scheme to maximize her payoff:

$$U_P = (1 - \beta)G - b_L - b_F.$$

The game structure is illustrated in Figure 1. In the first stage, the principal chooses whether to employ a leadership style. When she decides to organize a Nash team, the agents engage in a simultaneous-move team production game in the subsequent stage without any position allowances offered by the principal.⁸ When she decides to organize a Stackelberg team, she appoints one of the two agents as the leader of the team via an auction-like procedure and proposes a position allowance scheme to the designated leader and the follower in the interim stage. Then, given the agents' roles and the determined position allowances, the agents engage in a sequential-move team production game in the subsequent stage.

4 The benefits of leadership

In this section, we consider the benefits of leadership by comparing the second-stage equilibria for Nash and (two types of) Stackelberg teams.

4.1 A team without a leader (Nash team)

In a team without a leader, each agent i simultaneously chooses her/his contribution, g_i , to maximize her/his payoff, U_i , taking the other agent's choice of g_j ($j \neq i$) as given. Solving the payoff maximization problem, we obtain the following reaction function from the first-order

⁸In the figure, the dotted loop represents the information set of agent 2, indicating that agents 1 and 2 play a simultaneous-move team production game on a Nash team.

condition:

$$g_i(g_j) = \frac{\beta}{4c_i} (1 + \alpha g_j), \quad (3)$$

where the second-order condition is satisfied ($-2c_i < 0$). The reaction function shows that the agents' efforts are strategic complements ($dg_i/dg_j > 0$) for $\alpha > 0$. Note that if α is zero, there are no strategic interdependencies between agents.

The following assumption ensures the uniqueness and stability of the equilibrium:

Assumption 1 $8c_1c_2 > \alpha^2\beta^2$.

Solving the above reaction functions, we derive the equilibrium efforts as

$$g_{iN} = \frac{\beta(\alpha\beta + 4c_j)}{16c_ic_j - \alpha^2\beta^2}.$$

We denote the equilibrium value of this Nash team-production game by a subscript N . We find that $g_{1N} < g_{2N}$ holds for $c_1 > c_2$, indicating that an agent with a greater opportunity cost parameter devotes less effort than an agent with a smaller one.

The equilibrium team achievement, G_N , and equilibrium payoffs for agent i , U_{iN} , can be obtained by

$$G_N = \frac{\beta [16c_1c_2 (4c_1 + 4c_2 + 3\alpha\beta) - \alpha^3\beta^3]}{(16c_1c_2 - \alpha^2\beta^2)^2}, \quad (4)$$

$$U_{iN} = \frac{\beta^2 [32c_1c_2 (c_i + 2c_j + \alpha\beta) - \alpha^2\beta^2 (2c_j + \alpha\beta)]}{2(16c_1c_2 - \alpha^2\beta^2)^2}. \quad (5)$$

4.2 A team with a leader (Stackelberg team)

In a team with a leader, one agent acts as the first mover and chooses her/his contribution to the team project first. The other agent acts as a second mover: the second mover observes the decision of the first mover and then chooses her/his contribution. That is, we consider the equilibrium in the Stackelberg team-production game. We denote the Stackelberg team-production model by S . Each agent is indexed by ik ($i \in \{1, 2\}$, $i \neq j$, and $k \in \{L, F\}$), where L denotes a leader and F denotes a follower. By backward induction, we first solve agent jF 's problem given agent iL 's contribution, g_{iL} . Agent jF 's reaction function is the same as Eq.(3), so agent iL chooses g_{iL} to maximize U_{iL} in anticipation of agent jF 's reaction. Solving the problem, we derive the leader's and follower's contribution in the equilibrium as

$$g_{iL} = \frac{\beta(2c_j + \alpha\beta)}{8c_1c_2 - \alpha^2\beta^2} \quad \text{and} \quad g_{jF} = \frac{\beta[2c_i + (1/2)\alpha\beta]}{8c_1c_2 - \alpha^2\beta^2}.$$

Result 1 For $\alpha > 0$, (i) $g_{iL} > g_{iF} > g_{iN}$, (ii) $g_{iL} > g_{jF}$ if $c_1 = c_2$, (iii) $\frac{\partial g_{iL}}{\partial \alpha} > \frac{\partial g_{jF}}{\partial \alpha}$.

The first assertion indicates that an agent will exert more effort if he is in the leader position, which is obvious from the strategic complementarity of agents' effort. Assertion (ii) simply shows that the leader devotes more effort than the follower if both agents have the same opportunity

cost parameter. Assertion (iii) states that the effect of synergy on the leader's effort is greater than the effect on the follower's effort.

The equilibrium team achievement, G_S , can be obtained as

$$G_S = \frac{\beta [16c_1c_2(c_1 + c_2 + \alpha\beta) - \alpha^2\beta^2(c_j + \alpha\beta)]}{(8c_1c_2 - \alpha^2\beta^2)^2}. \quad (6)$$

Then, we obtain the equilibrium payoffs as

$$\begin{aligned} U_{iL} &= \frac{\beta^2(2c_i + c_j + \alpha\beta)}{16c_1c_2 - 2\alpha^2\beta^2} + b_L, \\ U_{jF} &= \frac{\beta^2[8c_1c_2(2c_i + 4c_j + 3\alpha\beta) - \alpha^2\beta^2(3c_j + 2\alpha\beta)]}{4(8c_1c_2 - \alpha^2\beta^2)^2} + b_F. \end{aligned}$$

Result 2 For $\alpha > 0$, $U_{iL} < U_{iF}$ if $b_k = 0$.

This result indicates that in the absence of position allowances, each agent wants to be a follower (rather than a leader) in our complementary team model.

4.3 Nash versus Stackelberg teams

We consider the benefits of leadership for both the principal and the team as a whole. Because the agents can have heterogeneous opportunity cost parameters, there are two types of Stackelberg team. In the following, we describe a team in which agent 1 is a leader (follower) and agent 2 is a follower (leader) as a team LF (FL). Comparing the equilibrium team achievements under the Nash and two Stackelberg teams (LF and FL), we obtain the following result:

Result 3 For $\alpha > 0$, (i) $G_{LF} > G_{FL} > G_N$ if $c_1 > c_2$, (ii) $G_{LF} = G_{FL} > G_N$ if $c_1 = c_2$.

The results imply that the two Stackelberg teams produce more than the Nash team; therefore, the principal favors the leadership style. Moreover, the principal should, if she can, select the agent with higher opportunity cost as the leader because assigning the agent with a lower opportunity cost as a follower increases the leader's marginal benefits of effort, reduces the effort gap between the leader and the follower compared to the opposite case, and enhances the synergy effect.

Result 4 For $\alpha > 0$ and $b_K = 0$, (i) $U_{iF} > U_{iL} > U_{iN} \forall i$, (ii) $U_P(G_{LF}) > U_P(G_{FL}) > U_P(G_N)$ if $c_1 > c_2$.

This result confirms that all the players (the principal and both agents) strictly prefer a Stackelberg team to a Nash team in the absence of any position allowances. The next section considers the optimal choices of team structure and leader made by the principal.

For $\alpha > 0$, it holds that

$$G_{LF} - G_{FL} = \frac{\alpha^2\beta^3(c_1 - c_2)}{(8c_1c_2 - \alpha^2\beta^2)^2} > 0 \quad \text{if} \quad c_1 > c_2.$$

Result 5 For $\alpha > 0$, $\frac{d(G_{LF} - G_{FL})}{d(c_1 - c_2)} > 0$ holds if $c_1 > c_2$.

This result implies that the difference in team achievement between effective and ineffective leadership styles is an increasing function of the difference in opportunity costs between agents. Therefore, the principal's choice of a leader is more important when the agents are more diverse.

Finally, when there is no synergy in a team project (i.e., $\alpha = 0$), the equilibrium team achievements and payoffs are both independent of the team structure because there is no strategic interdependence between agents. In addition, the advantages of the leadership style increases as the synergy effect increases. These results indicate that synergy is critical to effective leadership.

5 Effective leadership selection by a principal

This section considers the leadership selection by a principal. As shown in the last section, a Stackelberg team with a leader who has higher opportunity cost is better for the principal, but the problem is how she can select the appropriate leader by choosing b_L and b_F , the position allowances for the leader and the follower, without knowing the types of the agents.

We consider a simple procedure for selecting a leader: (i) The principal announces that b amount of wages will be redistributed from a potential follower to a potential leader, i.e., $b_L = b > 0$ and $b_F = -b < 0$.⁹ (ii) The principal requests each agent i to simultaneously report $\phi_i \in [0, \infty)$, which is the amount of compensation (that should be transferred from a follower) required for agent i to become the leader on the promise that the agent who reports lower ϕ will be selected as the leader. That is, if $\phi_1 < \phi_2$, then agent 1 is selected as the leader and receives ϕ_1 from agent 2.¹⁰ This procedure is similar to open competitive bidding in procurement.

In the following, we derive the Nash equilibrium of the above auction-like procedure for selecting a leader. First, agent i 's minimum position allowance (transfer from the follower) that is required for himself to be a leader, \bar{b}_i , should satisfy

$$U_{iL}|_{b_L=\bar{b}_i} = U_{iF}|_{b_F=-\bar{b}_i},$$

which yields the following:

$$\bar{b}_i = \frac{\alpha\beta^3 \{8c_1c_2 + (2c_j + c_i)\alpha\beta\}}{8(8c_1c_2 - \alpha^2\beta^2)^2}.$$

Then, we have $\bar{b}_1 \leq \bar{b}_2$ if $c_1 \geq c_2$, which implies that an agent with higher opportunity cost is willing to be a leader with a lower position allowance.

Now we have the following result regarding the equilibrium position allowance (transfer from the follower to the leader) to select effective leadership. The proof is given in the Appendix.

Result 6 In our auction-like procedure for selecting a leader (and follower), $b_L^* = \max[\bar{b}_i, \bar{b}_j]$ and $b_F^* = -b_L^*$ constitute the equilibrium position allowances for selecting effective leadership.

⁹This simplification implies a payoff-neutral position allowance for the principal, which enables us to focus on the effect of the additional monetary incentives for agents' roles.

¹⁰If their reported valuations are the same ($\phi_1 = \phi_2$), the principal should randomly select one agent as the leader.

If $c_1 > c_2$, in equilibrium, agent 1 reports a slightly lower value of \bar{b}_2 (i.e., $\phi_1 = \max[\bar{b}_1, \bar{b}_2] - \epsilon = \bar{b}_2 - \epsilon$, with $\epsilon > 0$ small enough) and agent 2 reports any value equal to or greater than \bar{b}_2 (i.e., $\phi_2 \in [\bar{b}_2, \infty)$). Thus, the principal selects agent 1 as the leader and pays position allowances $b_L^* = \bar{b}_2$ (ignoring ϵ) and $b_F^* = -\bar{b}_2$.¹¹ In the following, we denote the subgame-perfect Nash equilibrium values of the full game with an asterisk over the variable.

Two remarks concerning the procedure of effective leadership selection may be of interest. First, we have assumed the position allowances to be simply a transfer from the follower to the leader (i.e., neutral for the principal). The reason for this assumption is that if the principal pays a reward only to the leader in order to select effective leadership, the principal cannot be better off than under random leadership selection without any position allowances. Notably, effective leadership selection is possible even when position allowances are not necessarily neutral for the principal, as long as the gains for the principal from having effective leadership outweigh the net payments to the agents (i.e., as long as $(1 - \beta)G_{LF} - (1 - \beta)(G_{FL} + G_{LF})/2 \geq b_L + b_F$ for $c_1 > c_2$).¹²

Second, under the equilibrium levels of b_L^* and b_F^* , an agent who has lower opportunity cost is indifferent to being a leader or a follower while an agent who has higher opportunity cost strictly prefers to be a leader. As a result, comparing the equilibrium payoffs for the leader with those for the follower, we have

$$U_{1L}^* - U_{2F}^* = \frac{(c_1 - c_2)\beta^2}{16c_1c_2 - 2\alpha^2\beta^2} > 0 \quad \text{for } c_1 > c_2,$$

which implies that the position allowance overcomes the first-mover disadvantage for the leader shown in Result 2 and even allows the leader to be better off than the follower.

Because the rewards from the team achievement are the same for the leader and the follower, the equilibrium wage gap between them is simply $2b_L^*$. Then, we have

$$\begin{aligned} \frac{db_L^*}{d\alpha} &= \frac{\beta^3 [(4c_1c_2 + \alpha\beta(2c_1 + c_2))(8c_1c_2 + \alpha^2\beta^2) + 8c_1c_2\alpha^2\beta^2]}{4(8c_1c_2 - \alpha^2\beta^2)^3} > 0, \\ \frac{d^2b_L^*}{d\alpha^2} &= \frac{\beta^4 [(2c_1^2c_2 + 2c_1\alpha^2\beta^2)(32c_2^2 + 24c_2\alpha\beta + 3\alpha^2\beta^2) + 2c_1^2c_2(168c_2\alpha\beta + 61\alpha^2\beta^2 + 64c_1c_2) + 3c_2\alpha^4\beta^4]}{4(8c_1c_2 - \alpha^2\beta^2)^4} > 0. \end{aligned}$$

Result 7 (i) $\frac{db_L^*}{d\alpha} > 0$, (ii) $\frac{d^2b_L^*}{d\alpha^2} > 0$.

This result shows that the equilibrium wage premium for the leader is increasing and convex in synergy. The result is intuitive. The stronger the synergy is, the greater the effort made by the leader than the follower, as shown in Result 1-(iii). To give agents an incentive to be the leader,

¹¹This procedure for leadership selection is obviously strategy proof, meaning that the dominant strategy of each agent i is to report his true value of ϕ_i because there is no private information between agents.

¹²In the two-agent Stackelberg game of the private provision of public goods, Buchholz et al. (1997) show that the follower may have an incentive to make monetary transfers to the leader to eliminate the leader's free-riding incentives. They also show that the monetary transfer generates a Pareto superior outcome that benefits both agents. Although the mechanism is quite different, a redistribution of compensation from the follower to the leader (i.e., a position allowance) enables a principal to organize a Pareto-superior Stackelberg team in our model of team production with complementary inputs.

the principal should compensate the leader's effort by increasing the position allowance for the leader.

Finally, we investigate the effect of a marginal decrease in the follower's opportunity cost on the equilibrium wage gap between the leader and the follower. A decrease in the follower's opportunity cost may represent a situation in which the follower becomes more specialized in the team project. We have already shown that when $c_1 > c_2$, agent 2 will be the follower. Thus, we have

$$-\frac{db_L^*}{dc_2} = \frac{\alpha\beta^3 [\alpha^3\beta^3 + 8c_1\alpha\beta(c_2 + \alpha\beta) + 32c_1^2(2c_2 + \alpha\beta)]}{8(8c_1c_2 - \alpha^2\beta^2)^3} > 0,$$

which indicates that a marginal decrease in the follower's opportunity cost increases the equilibrium wage gap between the leader and the follower.

Result 8 For $\alpha > 0$, $-\frac{db_L^*}{dc_2} > 0$.

This result shows that the more specialized the follower is in the team project, the greater the compensation of the leader and the larger the wage gap. Although the improvement of the follower's expertise increases the team output, it also increases the position allowance required for the follower to being the leader. Therefore, the benefits are exploited more by the leader of the team than by the follower.

6 Extensions

In this section, we consider some extensions of our basic model: endogenous labor share, voluntary leadership, larger team, larger project, and heterogeneous skills.

6.1 Optimal labor share for management

This subsection investigates the relationship between the optimal labor share for the principal and the degree of synergy. The equilibrium payoff for the principal obtained in the previous section is $U_P^*(\alpha, \beta) = (1 - \beta)G_{LF}(\alpha, \beta)$ if $c_1 > c_2$. Thus, the optimal labor share (β_{LF}^*) that maximizes the principal's payoff is given by the first-order condition:

$$\frac{\partial U_P^*(\alpha, \beta)}{\partial \beta} = -G_{LF} + (1 - \beta_{LF}^*) \frac{\partial G_{LF}}{\partial \beta} = 0, \quad (7)$$

where the first term is the negative direct effect of decreased dividend and the second term is the positive indirect effect of the agents' improved motivations for the team project. The optimal labor share β_{LF}^* is necessarily interior ($\beta_{LF}^* \in (0, 1)$) because

$$\left. \frac{\partial U_P^*(\alpha, \beta)}{\partial \beta} \right|_{\beta=0} = \frac{\partial G_{LF}}{\partial \beta} > 0 \quad \text{and} \quad \left. \frac{\partial U_P^*(\alpha, \beta)}{\partial \beta} \right|_{\beta=1} = -G_{LF} < 0.$$

Furthermore, if $\alpha = 0$, then team achievement is a linear function of β (as in Eq. (6)), and it holds that $\beta(\partial G_{LF}/\partial \beta) = G_{LF}$. Then, the above first-order condition can be written as

$$\left. \frac{\partial U_P^*(\alpha, \beta)}{\partial \beta} \right|_{\alpha=0} = (\beta_{LF}^*)^{-1} G_{LF} (1 - 2\beta_{LF}^*) = 0,$$

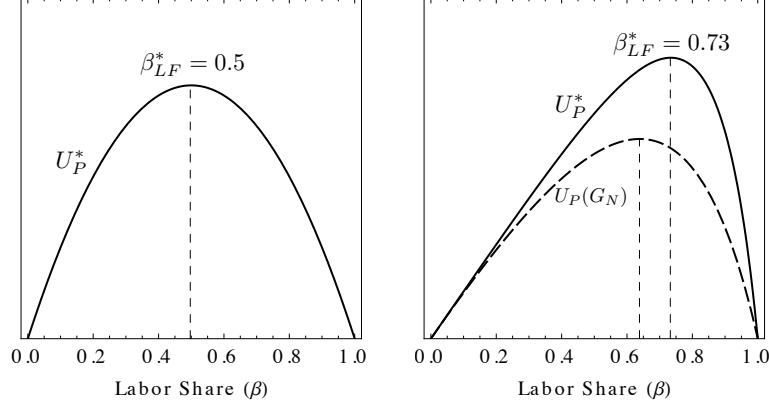


Figure 2: The labor share and the principal's payoff under team LF

No synergies (left): $c_1 = 3, c_2 = 2, \alpha = 0$; Large synergies (right): $c_1 = 3, c_2 = 2, \alpha = 5$

which reduces to the maximum $\beta_{LF}^* = 1/2$. We also find that β_{LF}^* becomes greater than $1/2$ for $\alpha > 0$ because

$$\beta \frac{\partial G_{LF}}{\partial \beta} - G_{LF} = \frac{\alpha \beta^2 (2c_2 + \alpha \beta) [64c_1^2 c_2 + \alpha \beta (24c_1 c_2 - \alpha^2 \beta^2)]}{(8c_1 c_2 - \alpha^2 \beta^2)^2} > 0$$

holds for positive synergy. Now, we have the following result.

Result 9 $\beta_{LF}^* \geq 1/2$ for $\alpha \geq 0$.

Fig. 2 illustrates the relationship between the principal's payoff with effective leadership, U_P^* , and the labor share, β . As shown in the left panel of the figure, the labor share that maximizes the principal's payoffs is $1/2$ when there is no synergy. From the right panel of the figure, we see that the stronger the synergy is, the larger the labor share the principal should choose. Note that the principal cannot set the labor share at exactly the optimal level because, in our model setting, she does not know the agents' cost parameters.

The following result compares the optimal labor share under the desirable leadership style (β_{LF}^*) with that under a Nash team (β_N).

Result 10 For $\alpha > 0$, $\beta_{LF}^* > \beta_N$.

The proof is in the Appendix. The results show that the optimal labor share for a Stackelberg team is greater than that for a Nash team.¹³ In the right panel of Fig. 2, the dotted curve represents the principal's payoff when a Nash team is organized. The optimal labor share in the case of a Nash team β_N is lower than that in the case of a Stackelberg team. Therefore, leadership style is beneficial for agents not only through the direct effects of increased compensation from increased team achievement but also through the indirect effects of an increase in the organization's labor share.

¹³Intuitively, the result comes from the fact that the elasticity of team output with respect to labor incentives (share) $E_K \equiv \frac{\partial G_K}{\partial \beta} \cdot \frac{\beta}{G_K}$ is greater under a team with leadership than under a team without leadership (i.e., $E_{LF} > E_N$).

| | | |
|----------------|--|--|
| A1\A2 | <i>Leads</i> | <i>Follows</i> |
| <i>Leads</i> | U_1^N, U_2^N | $\underline{U_{1L}}, \underline{U_{2F}}$ |
| <i>Follows</i> | $\underline{U_{1F}}, \underline{U_{2L}}$ | U_1^N, U_2^N |

Table 1: Payoff matrix of endogenous timing in team production

6.2 Voluntary leadership

Here, we consider the case of voluntary leadership in which agents autonomously coordinate their role (leader or follower) in the absence of an outside coordinator (principal). We apply the endogenous timing game developed by Hamilton and Slutsky (1990) in which two agents non-cooperatively choose their preference between moving early or late in team production stage. If both agents choose to move early (strategy *Leads*) or to move late (strategy *Follows*), a Nash (simultaneous-move) team production will take place. If one agent chooses *Leads* and the other chooses *Follows*, a Stackelberg (sequential-move) team production will take place. Table 1 represents the normal form representation of the timing game.

From Result 4, each agent's best responses are underlined in the payoff matrix. We immediately find that two sequential-move situations (leadership styles) are Nash equilibria of the voluntary leadership game.

Now, we derive the mixed strategy Nash equilibrium. Let P_{iL} and $P_{iF} = 1 - P_{iL}$ be the probability that agent i chooses *Leads* and *Follows*, respectively. The equilibrium probability can be obtained as

$$P_{iL}^* = \frac{U_{jL} - U_{jN}}{U_{jF} + U_{jL} - 2U_{jN}}.$$

After some manipulations, we have

$$P_{iL}^* - P_{jL}^* = -(c_i - c_j) \left\{ \frac{\alpha^3 \beta^7 [128c_1^2 c_2^2 + \alpha\beta (c_1 + c_2) (32c_1 c_2 - \alpha^2 \beta^2)]}{8(8c_1 c_2 - \alpha^2 \beta^2)^3 (16c_1 c_2 - \alpha^2 \beta^2)^2} \right\}.$$

From Assumption 1, we obtain the following result:

Result 11 *In the mixed-strategy Nash equilibrium of the voluntary leadership game, $P_{1L}^* < P_{2L}^*$ holds for $c_1 > c_2$.*

This result implies that the agent who has lower opportunity cost is more likely to choose *Leads* than the agent who has higher one does.

In the numerical example of $c_1 = 3, c_2 = 2, \alpha = 8, \beta = 1/2$, we have the mixed strategy equilibrium as $P_{1L}^* = 0.151$ and $P_{2L}^* = 0.175$. This indicates the probability for an effective leadership to be formed is $P_{1L}^* \cdot P_{2F}^* = 0.124$, which is lower than the probability for an ineffective leadership to be formed, $P_{1F}^* \cdot P_{2L}^* = 0.149$. In this case, even worse, the most ineffective Nash team is formed for probability 0.727.

Next, to select one of the two Nash equilibria shown in Table 1, we apply the concept of risk dominance formulated by Harsanyi and Selten (1988). The equilibrium $\{1, 2\} = \{Leads, Follows\}$

risk dominates the equilibrium $\{1, 2\} = \{Follows, Leads\}$ if the former is associated with the larger product of deviation losses Ω : the condition is

$$\Omega \equiv (U_{1L} - U_{1N})(U_{2F} - U_{2N}) - (U_{1F} - U_{1N})(U_{2L} - U_{2N}) > 0.$$

After some manipulations, we have

$$\Omega = (c_1 - c_2) \left\{ \frac{\alpha^3 \beta^7 [128c_1^2 c_2^2 + \alpha \beta (c_1 + c_2) (32c_1 c_2 - \alpha^2 \beta^2)]}{8(8c_1 c_2 - \alpha^2 \beta^2)^3 (16c_1 c_2 - \alpha^2 \beta^2)^2} \right\} > 0 \quad \Leftrightarrow \quad c_1 > c_2$$

Result 12 *A team with efficient leadership is the risk dominant Nash equilibrium of the voluntary leadership game.*

Results 11 and 12 emphasizes the importance of a principal's role in coordinating teams. Result 11 shows that, in the absence of a principal, the efficient leadership team is the least likely to be autonomously formed by the agents. Although Result 12 indicates the efficient leadership team may be evolutionally stable and be voluntarily formed by the agents in the long run, the important thing is that, even if a principal lacks information about the agents' opportunity costs, she can induce the efficient outcome by constructing simple reward schemes.

6.3 Large team

Here, we consider teams of more than two agents to investigate the effect of team size on the equilibrium values. We also show that the optimal team size that maximizes team performance crucially depends on the magnitude of the synergies. Our results provide a theoretical rationale of how team size is determined and why some leaders of larger teams receive higher rewards than other leaders of smaller teams.

We modify some model assumptions to make the analysis tractable. First, we focus on the situation of a Stackelberg team (a leader-follower situation) with $n \geq 2$ agents (one leader and $n - 1$ followers), in which the $n - 1$ followers simultaneously decide their effort after the leader first decides his effort. Second, we modify the team production function (Eq. (1)) as

$$G = g_L + \sum_{i=1}^{n-1} g_{iF} + \alpha(n) \left(g_L \times \sum_{i=1}^{n-1} g_{iF} \right),$$

where g_L and g_{iF} represent, respectively, the leader's and follower i 's efforts on the team project and the function $\alpha(n)$ represents the degree of synergy, which is dependent on the team size. The formulation of the team production function assumes that synergy arises between different positions, i.e., between the leader and followers, and not among followers. The function $\alpha(\cdot)$ captures the effect of the number of followers on the degree of synergy. The synergy function is assumed to have $\alpha' \geq 0$, $\alpha'' \leq 0$ and constant elasticity $\sigma_{\alpha,n} \equiv n\alpha'(n)/\alpha(n) > 0$. The formulation simplifies the analysis, as shown later. Third, agents are assumed to have the same opportunity costs, meaning that they differ only in their position, i.e., leading or following.

The payoffs of the leader and each follower i are given by

$$U_L = \frac{\beta}{n} G - c g_L^2 + b_L \quad \text{and} \quad U_{iF} = \frac{\beta}{n} G - c g_{iF}^2 + b_{iF},$$

where g_L and g_{iL} are, respectively, the leader's and follower i 's efforts and b_L and b_F are, respectively, the position allowances for the leader and each follower. As before, we assume that the position allowances are neutral for the principal such that $b_L + \sum_i^{n-1} b_{iF} = 0$, implying $b_{iF} = -b_L/(n-1)$.

From the first-order conditions for follower i 's problem, we have the following reaction function for follower i :

$$g_{iF} = R_{iF}(g_L) \equiv \frac{\beta [1 + \alpha(n)g_L]}{2cn},$$

which implies that there are no strategic interdependences among followers. The slope of the reaction function has the following property

$$\frac{dR'_{iF}}{dn} = \frac{\alpha(n)\beta}{2cn^2} (\sigma_{\alpha,n} - 1) \gtrless 0 \quad \text{for} \quad \sigma_{\alpha,n} \gtrless 1,$$

where $\sigma_{\alpha,n} \equiv n\alpha'(n)/\alpha(n)$ is the elasticity of the synergy function with respect to the size of the team, which we call the degree of *scale synergy*. If the synergy function is elastic (scale synergy is strong), an increase in team size increases the slope of the followers' reaction function. In the case of constant synergy, as assumed in the basic model (i.e., $\sigma_{\alpha,n} = 0$ or zero scale synergy), a larger team necessarily makes the followers' reaction function flatter, which discourages contribution by the followers.

The following assumption is made in this subsection only to ensure the uniqueness and stability of the equilibrium.

Assumption 2 $\xi \equiv 2c^2n^2 - (n-1)(\alpha(n))^2\beta^2 > 0$

From the first-order condition of the leader's problem, we have the following equilibrium efforts for the leader and followers:

$$g_L^* = \frac{\beta [cn + (n-1)\alpha(n)\beta]}{\xi} \quad \text{and} \quad g_{iF}^* = \frac{\beta [cn + (1/2)\alpha(n)\beta]}{\xi},$$

which indicate that $g_L^* > g_{iF}^*$ holds for all $n \geq 2$.

Then, we have

$$\frac{dg_L^*}{dn} = -\frac{\beta}{n\xi^2} [\eta_1 - \eta_2 \cdot \sigma_{\alpha,n}], \quad (8)$$

$$\frac{dg_{iF}^*}{dn} = -\frac{\beta}{2n\xi^2} [\eta_3 - \eta_4 \cdot \sigma_{\alpha,n}], \quad (9)$$

where

$$\begin{aligned} \eta_1 &\equiv cn [\xi + (n-2)\alpha(n)\beta (2cn + \alpha(n)\beta)] > 0, \\ \eta_2 &\equiv (n-1)\alpha(n)\beta [\xi + 2\alpha(n)\beta (cn + (n-1)\alpha(n)\beta)] > 0, \\ \eta_3 &\equiv \alpha(n)\beta\xi + [\xi + (n-2)(\alpha(n))^2\beta^2] (2cn + \alpha(n)\beta) > 0, \\ \eta_4 &\equiv \alpha(n)\beta [\xi + 2(n-1)\alpha(n)\beta (2cn + \alpha(n)\beta)] > 0. \end{aligned}$$

The signs of Eqs. 8 and 9 depend on the magnitude of the scale synergy. If $\sigma_{\alpha,n}$ is zero (i.e., $\alpha(n) = \text{const.}$), then an increase in team size necessarily decreases all agents' efforts. However,

if the scale synergy is somewhat strong, an increase in team size encourages all agents to devote greater effort.

Result 13 $dg_L^*/dn < (>) 0$ and $dg_{iF}^*/dn < (>) 0$ for small (large) $\sigma_{\alpha,n}$.

Increasing team size has conflicting effects on agents' incentives to exert effort. An increase in the number of followers increases the followers' total efforts and thus strengthens the synergy effect, which incentivizes the leader to contribute more. However, increasing team size reduces the reward from team achievement per agent, which discourages agents from contributing to the team project.¹⁴

One interesting question is how the size of the team affects the relative effort gap between the leader and each follower. We have

$$\frac{d\left(\frac{g_L^*}{g_{iF}^*}\right)}{dn} = \frac{2\alpha(n)\beta [3c + (2n-3)\sigma_{\alpha,n} + \alpha(n)\beta]}{[2cn + \alpha(n)\beta]^2} > 0,$$

which indicates that a leader of a larger team exerts more effort than the followers.

Result 14 $d(g_L^*/g_{iF}^*)/dn > 0$ for all $n \geq 2$.

The equilibrium team achievement G_S is given by

$$G_S = \frac{\beta [cn + 2(n-1)\alpha(n)\beta] (2c^2n^2 + \xi)}{2\xi^2}.$$

Then, we derive the equilibrium position allowances. Because the agents are symmetric, except for their position, the equilibrium position allowance for a leader b_L^* can be determined such that

$$U_L|_{b_L=b_L^*} = U_{iF}|_{b_F=-b_L^*/(n-1)}.$$

Solving the above equation, we have

$$b_L^* = \frac{c(n-1)(2n-3)\alpha(n)\beta^3 [4cn + (2n-1)\alpha(n)\beta]}{4n\xi^2}. \quad (10)$$

We have the following comparative static result.

Result 15 $db_L^*/dn > (<) 0$ for large (small) $\sigma_{\alpha,n}$ and $n \geq 3$.

The proof is in the Appendix. The equilibrium position allowance for the leader is decreasing (increasing) in team size if $n \geq 3$ and $\sigma_{\alpha,n}$ is small (large).

Finally, we derive the optimal team size that maximizes the equilibrium team achievement. From the equilibrium values of g_L^* and g_{iF}^* , we have

$$\frac{dG_S^*}{dn} = \frac{\alpha(n)\beta^2}{2n\xi^3} [\eta_5 \cdot \sigma_{\alpha,n} - \eta_6]$$

¹⁴This result is similar to that of McGinty (2014), who investigates the relationship between equilibrium effort and team size in a team production model with a CES production function. He shows that equilibrium effort is increasing in team size and that effort levels are strategic complements when returns to scale dominate substitutability. The return to scale in his study is similar to the scale synergy in our study.

$$\begin{aligned}\eta_5 &\equiv 2(n-1)[cn + (n-1)\beta\alpha(n)][8c^3n^3 + (\xi + 4c^2n^2)\beta\alpha(n)] > 0, \\ \eta_6 &\equiv 16c^4n^4(n-2) + 2c^3n^3(3-10n+4n^2)\alpha(n)\beta + c(n-1)n(\alpha(n))^3\beta^3 \gtrless 0,\end{aligned}$$

It is shown that $\eta_6 = -2c\alpha(2)^2\beta^3(8c^2 - \alpha(2)^2\beta^2) < 0$ holds when $n = 2$ (the smallest possible team), and $\eta_6 > 0$ necessarily holds for $n \geq 3$. Therefore, $dG_S^*/dn > 0$ holds when $n = 2$. Then, consider the case of $\sigma_{\alpha,n} = 0$ (zero scale synergy). In this case, the optimal team size, n^* , that maximize G_S^* lies in $n^* \in [2, 3]$. Thus, the smallest possible team ($n = 2$ or 3) is optimal, ignoring integer constraints, when $\sigma_{\alpha,n}$ is sufficiently small. Since η_5 is necessarily positive, the larger the value of $\sigma_{\alpha,n}$ is, the larger the optimal team size n^* .

Result 16 $dn^*/d\sigma_{\alpha,n} > 0$.

The result is quite intuitive: a larger team is optimal for the principal when the scale synergy is stronger. Interestingly, a locally optimal team size for maximizing team achievement exists when $\sigma_{\alpha,n}$ has an intermediate value.

Figure 4 confirms the results via numerical simulations for different degrees of scale synergy (zero, weak, and strong scale synergies). As shown in Result 13, the left panels illustrate that the leader's and followers' efforts are decreasing in team size for zero or weak scale synergy (the cases of (i) and (ii)) but may be increasing for strong scale synergy (the case of (iii)). The right panels illustrate the equilibrium position allowances for the leader, as shown in Result 15, where b_L^* has the maximum for $n \in (2, 3)$ for zero or weak scale synergy and is increasing for strong scale synergy. We can see from the three left and right panels that the position allowances for the leader should be paid to fill the gap between the efforts of the leader and the follower.

The middle three panels in Figure 4 illustrate that the equilibrium team achievement is decreasing and increasing in team size for zero and strong scale synergies, respectively, and has an inverted U shape for weak scale synergy. In other words, the smallest (largest) possible team is optimal for the principal when the scale synergy is small (large), and an optimal interior team size exists ($n^* = 5.79 \approx 6$ in this example) for weak scale synergy.

In summary, strong scale synergy increases the optimal team size, which also increases the rewards for the leader by increasing the effort gap between the leader and the followers.

6.4 Leader on a larger project

Here, we consider how the profitability of a team project affects the compensation for a leader. We extend the basic model by incorporating a profitability parameter λ into the team production function given in Eq. (1):

$$G = \lambda \cdot \left(\sum_{i \in \{1,2\}} g_i + \alpha \prod_{i \in \{1,2\}} g_i \right),$$

where $\lambda > 0$ measures the profitability of the team project and $\lambda = 1$ corresponds to the basic model.

The introduction of the profitability of the team project does not change our results qualitatively, so we focus on the effect on the efforts and position allowance in subgame-perfect Nash

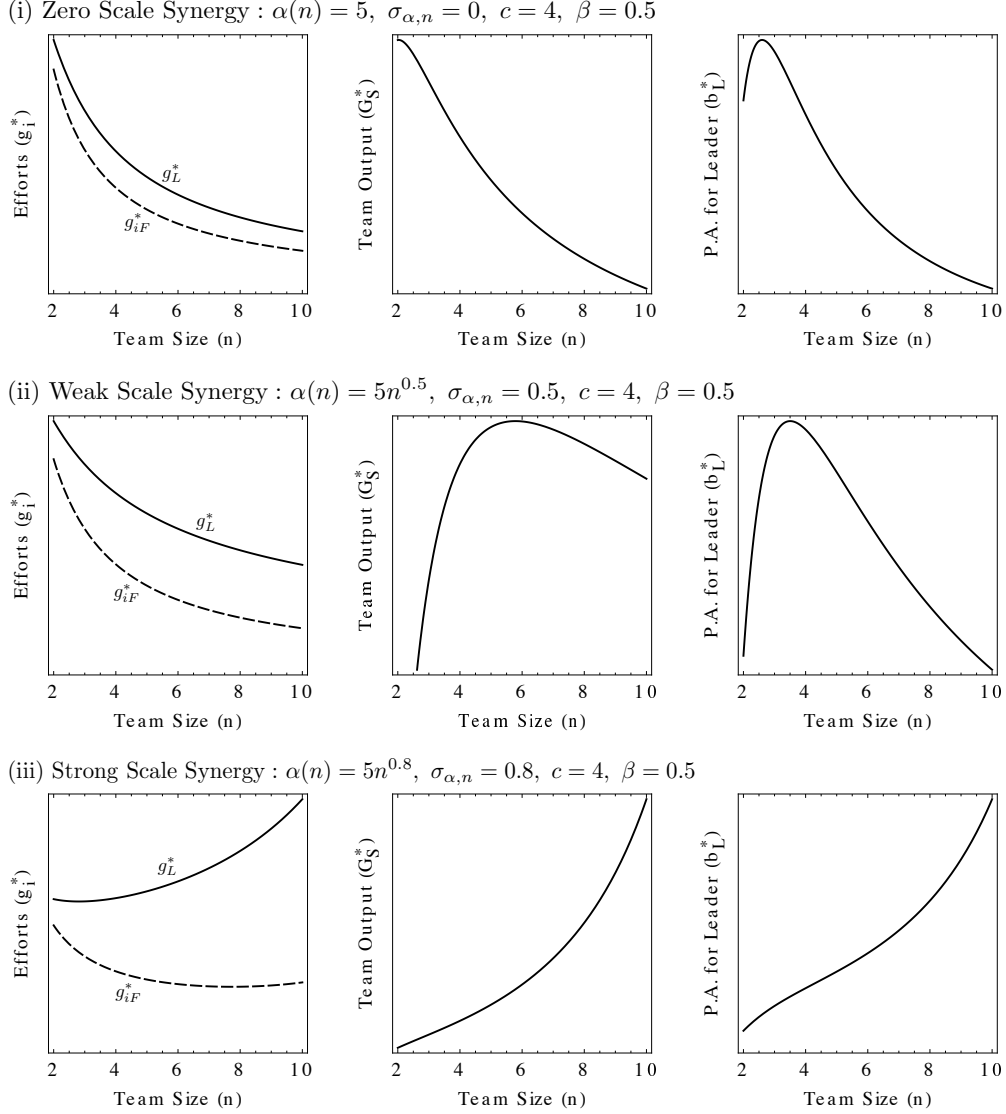


Figure 4: Large team: The case for zero, weak, and strong scale synergies

equilibrium.

If $c_1 > c_2$, then a Stackelberg team under agent 1's leadership is the desirable for the principal, as in the basic case. In this case, the equilibrium efforts are given by

$$g_{1L}^* = \frac{\beta\lambda(2c_2 + \alpha\beta\lambda)}{8c_1c_2 - \alpha^2\beta^2\lambda^2} \quad \text{and} \quad g_{2F}^* = \frac{\beta\lambda(2c_1 + (1/2)\alpha\beta\lambda)}{c_1c_2 - \alpha^2\beta^2\lambda^2},$$

where $8c_1c_2 - \alpha^2\beta^2\lambda^2 > 0$ by assumption. The larger the project is, the more effort both agents will put in. Then, the relative effort gap between the leader and the follower, g_{1L}^*/g_{2F}^* , has the following property

$$\frac{d\left(\frac{g_{1L}^*}{g_{2F}^*}\right)}{d\lambda} = \frac{4\alpha\beta}{(4c_1 + \alpha\beta\lambda)^2}(2c_1 - c_2) > 0,$$

which indicates that the leader exerts more effort than the follower as the project is more profitable.

Second, the equilibrium position allowance for the leader and the follower are given by

$$b_L^* = \frac{\alpha\beta^3\lambda^3 [8c_1c_2(2c_1 + c_2)\alpha\beta\lambda]}{8(8c_1c_2 - \alpha^2\beta^2\lambda^2)^2} \quad \text{and} \quad b_F^* = -b_L^*.$$

Then, we have

$$\begin{aligned} \frac{db_L^*}{d\lambda} &= \frac{2c_1c_2\alpha\beta^3\lambda^2 [24c_1c_2 + \alpha\beta\lambda(8c_1 + 4c_2 + \alpha\beta\lambda)]}{(8c_1c_2 - \alpha^2\beta^2\lambda^2)^3} > 0, \\ \frac{d^2b_L^*}{d\lambda^2} &= \frac{2c_1c_2\alpha\beta^3\lambda [(8c_1c_2 + \alpha^2\beta^2\lambda^2)((2c_2 + \alpha\beta\lambda)(12c_1 + \alpha\beta\lambda) + 4c_2\alpha\beta\lambda) + 32c_1c_2\alpha^2\beta^2\lambda^2]}{(8c_1c_2 - \alpha^2\beta^2\lambda^2)^4} > 0. \end{aligned}$$

Result 17 For $\alpha > 0$, (i) $db_L^*/d\lambda > 0$, (ii) $d^2b_L^*/d\lambda^2 > 0$.

This result indicates that the leader on the twice profitable project will receive more than twice the position allowance, which also increases the wage gap between the leader and the follower.

6.5 Heterogenous skill

In the basic model, the agents are assumed to be heterogeneous only in their opportunity costs. However, in real-world situations of team work, heterogeneity in skill or productivity among agents is also considered to be important for effective team management. This subsection incorporates heterogeneity in skill into the model and checks the robustness of the results obtained in the main body.

The team production function of Eq. (1) is modified as

$$G = \sum_{i \in \{1,2\}} \kappa_i g_i + \alpha \prod_{i \in \{1,2\}} \kappa_i g_i,$$

where $\kappa_i > 0$ represents the skill parameter of agent i .

The following result is worth noting.

Result 18 A Stackelberg team under agent i 's leadership yields greater team achievement than a Stackelberg team under agent j 's leadership if

$$\frac{c_i}{c_j} > \left(\frac{\kappa_i}{\kappa_j} \right)^2.$$

The result implies that, even if agent 1 has higher skills than agent 2 (i.e., $\kappa_1 > \kappa_2$), the Stackelberg team under agent 1's leadership is more effective than the Stackelberg team under agent 2's leadership as long as agent 1 has much higher opportunity cost. For example, when $\kappa_1 = 2$ and $\kappa_2 = 1$, the condition $c_1 > 4c_2$ is sufficient for team LF (with agent 1's leadership) to be the effective leadership style.

In general, leaders' opportunity cost for certain team projects is higher than other members' opportunity cost, which is considered to outweigh the leaders' superiority in skills for the team project. Therefore, our main result that agents with higher opportunity cost should be the leader is also robust when we take heterogeneous skills into account.

7 Concluding remarks

Leadership styles are widely believed to be important for effective team performance. This paper finds that the existence of synergy is crucial for effective leadership. The existence of the synergy makes the team member's efforts complementary, which gives birth to eager leaders. We also find that a principal who does not know the agents' type, can select a good leader by proposing an additional incentive for the leader's position. As a result, the leader's compensation will be greater than that of the follower(s), and the equilibrium wage gap increases when the synergy is more prominent, the project is more profitable, and the follower is more specialized.

Our simple model that incorporates synergy into the traditional model can successfully rationalize real-world observations as an equilibrium outcome: (i) leaders contribute more to team projects than followers do; (ii) many organizations (e.g., firms, sports team, universities) adopt a leadership style; (iii) the leaders of some teams are appointed by upper management and are generally paid more than others when they lead a larger team or are on a larger project; (iv) successful leaders are often involved in other profitable projects, meaning that they face higher opportunity costs for engaging in a certain team project.

An important limitation of our study is that, although we address heterogeneous agents on teams, we do not consider the problem of team formation. For example, consider a case where two types of agents are divided into teams. In this case, it is meaningful to compare the profitability between homogeneous teams consisting of agents of the same type and heterogeneous teams consisting of agents of different types. Such a comparison becomes more interesting when taking into account additional issues, such as voluntary team formation among agents, the optimal payment scheme for selecting effective leaders, and inter-team and intra-team synergy. Another limitation is that our study assumes that there is no incomplete information between a leader and followers (or equivalently, within a team). As examined in Hermalin (1998) and Komai et al. (2007), it is also meaningful to consider information asymmetry between a leader and followers under our team production function with synergy. These issues are worth exploring in future research.

Appendix

A1. Proof for Result 6

Here, we prove Result 6 that characterizes the Nash equilibrium of the game, in which each agent i is required to simultaneously report $\phi_i \in [0, \infty)$ (the amount of compensation required for agent i to be a leader) to the principal and the principal selects the one who reports the lowest values as a leader.

Without loss of generality, we consider a case of $c_1 \geq c_2$. We already know $\bar{b}_1 \leq \bar{b}_2$ if $c_1 \geq c_2$. Now we are looking at agent 2's best response. Clearly, agent 2 never chooses the strategy (report) of $\phi_2 < \bar{b}_2$ for $\phi_1 \leq \bar{b}_2$ because he necessarily worse off from being a leader when $b < \bar{b}_2$. Therefore, agent 2's best response against $\phi_1 < \bar{b}_2$ is any ϕ_2 that satisfies $\phi_2^* \geq \bar{b}_2$. For $\phi_1 > \bar{b}_2$, agent 2's best response is to undercut it, i.e., $\phi_2 = \phi_1 - \epsilon$, with sufficiently small ϵ .

Agent 1 knows that $\phi_2 < \bar{b}_2$ is dominated strategy for agent 2, so his best response is to choose $\phi_1 = \phi_2 - \epsilon$. Because agent 1 strictly prefer surely being a leader by choosing $\phi_1 = \bar{b}_2 - \epsilon$ to facing random assignment by choosing $\phi_1 = \bar{b}_2 = \phi_2$, the only possible equilibrium is that agent 1 surely becomes a leader by reporting $\phi_1 = \bar{b}_2 - \epsilon$. Thus, the equilibrium position allowances are $b_L^* = \bar{b}_2$ and $b_F^* = -\bar{b}_2$, ignoring ϵ .

A2. Proof for Result 10

We compare the optimal labor share of the (desirable) Stackelberg team, β_{LF}^* with that of the Nash team, β_N . From the first-order condition (Eq. 7), we have

$$(1 - \beta_N) = G_N \left(\frac{\partial G_N}{\partial \beta} \right)^{-1} \equiv \Lambda_N.$$

$$(1 - \beta_{LF}^*) = G_{LF} \left(\frac{\partial G_{LF}}{\partial \beta} \right)^{-1} \equiv \Lambda_{LF},$$

Thus, we have

$$\Lambda_N - \Lambda_{LF} = \frac{\alpha\beta^2 (\eta_7\eta_8 + \eta_9)}{32c_1c_2\eta_{10} [\eta_{11} + \alpha^2\beta^2(\eta_{12} + \eta_7)]} > 0,$$

where

$$\begin{aligned} \eta_7 &\equiv 8c_1c_2 - \alpha^2\beta^2 > 0 \quad (\text{from Assumption 1}), \\ \eta_8 &\equiv \alpha^2\beta^2 [\alpha^4\beta^4 + 16c_1\alpha\beta (8c_2^2 + 9c_2\alpha\beta + 2\alpha^2\beta^2) + 16c_1^2 (32c_2^2 + 56c_2\alpha\beta + 9\alpha^2\beta^2)] > 0, \\ \eta_9 &\equiv 2048c_1^3c_2^2 [8(c_1 + c_2)(2c_1 + \alpha\beta)(c_2 + \alpha\beta) + \alpha^2\beta^2(8c_1 + 3c_2 + 5\alpha\beta)] > 0, \\ \eta_{10} &\equiv 2(c_1 + c_2)(16c_1c_2 + 3\alpha^2\beta^2) + \alpha\beta(48c_1c_2 + \alpha^2\beta^2) > 0, \\ \eta_{11} &\equiv 128c_1^2c_2(c_1 + c_2 + 2\alpha\beta) > 0, \\ \eta_{12} &\equiv 16c_1(3c_1 + c_2) > 0. \end{aligned}$$

This proves $(1 - \beta_N) > (1 - \beta_{LF}^*)$, which implies $\beta_{LF}^* > \beta_N$.

A3. The proof for Result 15

Differentiating Eq. (10) in n yields

$$\frac{db_L^*}{dn} = \frac{c\alpha(n)\beta^3}{4n^2\xi^3} [\eta_{13} \cdot \sigma_{\alpha,n} - \eta_{14}],$$

where

$$\begin{aligned} \eta_{13} &\equiv 2(n-1)(2n-3) \left[(2cn + (2n-1)\alpha(n)\beta)\xi \right. \\ &\quad \left. + (4cn + (2n-1)\alpha(n)\beta) \left(2(n-1)(\alpha(n))^2\beta^2 \right) \right] > 0, \\ \eta_{14} &\equiv 8c^3n^3(4n^2 - 15n + 12) + 2c^2n^2(8n^3 - 36n^2 + 44n - 15)\alpha(n)\beta \\ &\quad + 4cn^2(n-1)^2(\alpha(n))^2\beta^2 + (4n^3 - 10n^2 + 9n - 3)(\alpha(n))^3\beta^3 \geq 0. \end{aligned}$$

It can be shown that, the coefficient η_{14} becomes $-(16c + 7\alpha(2)\beta)(8c^2 - (\alpha(2))^2\beta^2) < 0$ if $n = 2$ (the smallest possible team), and it becomes positive for $n \geq 3$. Therefore, the equilibrium position allowance for the leader is decreasing (increasing) in the team size if $n \geq 3$ and $\sigma_{\alpha,n}$ is small (large).

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