Endogenous Public and Private Leadership with Diverging Social and Private Marginal Costs

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Endogenous Public and Private Leadership with Diverging Social and Private Marginal Costs*

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Abstract

We investigate endogenous timing in a mixed duopoly with price competition and with social marginal cost differing from private marginal costs. We find that any equilibrium timing patterns– Bertrand, Stackelberg with private leadership, Stackelberg with public leadership, and multiple Stackelberg equilibria– emerge. When the foreign ownership share in a private firm is less than 50%, public leadership more likely emerges than private leadership. Conversely, private leadership can emerge in a unique equilibrium when the foreign ownership share in a private firm is large. These results may explain recent policy changes in public financial institutions in Japan. We also find a nonmonotone relationship between the welfare advantage of public and private leadership and the difference between social and private marginal costs for a private firm. A nonmonotone relationship does not emerge in profit ranking.

JEL classification numbers: H42, L13

Keywords: public financial institutions, differentiated products, Bertrand, Stackelberg, payoff dominance

highlights

Endogenous roles in a mixed duopoly with price competition are examined.

We allow that social marginal costs are different from private marginal costs.

Any equilibrium patterns emerge depending on cost conditions.

Both private and public leadership can be payoff and risk dominant.

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1 Introduction

In Japan, during the post-war reconstruction and high-growth periods from 1945 to the 1970s, state-owned public enterprises—especially public financial institutions—played a leading role in the Japanese economy. It was widely believed that lending by public financial institutions (such as the Development Bank of Japan) had a pump-priming effect on the lending by private banks. Japan Post was the world largest bank in 1970s and 1980s.[1] Since the 1980s, some public enterprises have undergone major reforms. For example, three major nonfinancial, state-owned public enterprises—the Japan Railway group, Japan Tobacco Incorporated, and Nippon Telegraph and Telephone Corporation—were privatized. In addition, the Japanese flag carrier (Japan Airline) was privatized in 1987. However, the government continued to hold major public financial institutions, and these played dominant roles in the Japanese financial market.

The Koizumi Cabinet (April 2001–September 2006) changed all this, declaring that public financial institutions should play a secondary role to private firms, with private firms leading the market. As a result, there was significant downscaling of the major public institutions. Once again, however, public institutions have begun to lead in Japanese markets (Matsumura and Ogawa, 2017b). Newly established public financial institutions such as the Industrial Revitalization Corporation of Japan, the Enterprise Turnaround Initiative Corporation of Japan, the Regional Economy Vitalization Corporation of Japan, and the Private Finance Initiative Promotion Corporation of Japan play leading roles in current financial markets. The Nikkei, a Japanese newspaper, refers to this phenomenon as"Kiko Capitalism (State Institution Capitalism)" (Nikkei, November 22, 2011). This type of capitalism is still expanding under the current Abe Cabinet (Nikkei, October 8, 2013). For example, the government has tried to establish new public financial institutions, such as the Japan Investment Corporation.[2]

The topic of whether public or private firms should lead markets and how this affects market

[1] See Horiuchi and Sui (1993). It has been observed that, globally, public sectors play an important role in lending markets. See Bose et al. (2014).

[2] This is not limited to Japan. For example, Korea Development Bank plays an important role in Korean industry financing and recently supplied money to rescue Hanjin Heavy Industries (Nikkei, February 13, 2019). In Indonesia, the state-owned bank, Bank Mandiri, are expanding by acquisition of a privately-owned Permata Bank (NNA Asia, April 10,2019).

equilibrium has been actively discussed in the literature on mixed oligopolies.[3] Pal (1998) adopted the observable delay game formulated by Hamilton and Slutsky (1990) and investigated the endogenous role in which public and private firms compete in mixed oligopolies. He showed that public firms should be the followers in welfare and should also become the followers in equilibrium. The literature on endogenous roles in mixed oligopolies is rich and diverse. Matsumura (2003) introduced foreign competition, showing that public firms should be the leaders and that they become the leaders in equilibrium. Nakamura and Inoue (2007) introduced managerial delegation and showed that public firms become the followers. Matsumura and Ogawa (2010) adopted Matsumura's (1998) partial privatization approach and showed that under partial privatization, private leadership is a unique equilibrium or is risk dominant unless the degree of privatization is large.[4] Capuano and De Feo (2010) introduced the shadow cost of public funds and showed that private leadership equilibrium is robust. Tomaru and Saito (2010) considered a subsidized, mixed duopoly and showed that private leadership emerges under an optimal subsidy policy. Amir and De Feo (2014) provide sufficient conditions for the emergence of public and/or private leadership equilibrium under quite general demand and cost conditions. Matsumura and Ogawa (2017a) and Lee and Xu (2018) introduced an environmental problem and showed that the degree of negative externality affects equilibrium roles in mixed duopolies. Matsumura and Ogawa (2017b) introduced product differentiation and showed that public leadership can be risk dominant, although it is worse for welfare than private leadership.[5]

All the papers on endogenous roles in mixed oligopolies mentioned above investigated quantity competition. Until the 1990s, it was difficult to raise funds. In this situation—based on Kreps and Scheinkman (1983) and Friedman (1988)— it would be reasonable to use quantity competition models to analyze the Japanese financial market. Since March of 2001, however, Japanese financial markets have been loosened considerably in response to quantitative monetary-easing policy, and similar situations have prevailed globally. Thus, price competition models may be more appropriate for analyzing the

[3]Recent developments in mixed oligopolies, see Pal and Saha (2014), Amerighi and De Feo (2017), Lin and Matsumura (2018), Escrihuela-Villar and Gutiérrez-Hita (2019), Klumpp and Su (2019) and works cited therein.

[4]For more on the concept of risk dominance, see Harsanyi and Selten (1988).

[5]For the importance of sequential-move games in mixed oligopolies, see Fjell and Heywood (2004), Heywood and Ye (2009a), Ino and Matsumura (2010), Wang and Mukherjee (2012), Gelves and Heywood (2013), and Wang and Lee (2013).

role of public institutions in current financial markets. Moreover, in mixed oligopolies—as Matsumura and Ogawa (2012), Haraguchi and Matsumura (2014), and Din and Sun (2016) have shown— price competition is more natural than quantity competition in endogenous competition structure models.[6] Thus, it is important to investigate endogenous roles in the price competition model.

The literature on endogenous timing with price competition in mixed oligopolies is relatively sparse. Bárcena-Ruiz (2007) investigated price competition and showed that Bertrand emerges in a mixed duopoly as a unique equilibrium, and Din and Sun (2016) showed that his result holds even when the competition structure is endogenized.[7]

In this study, we extend the model of Bárcena-Ruiz (2007) in two directions. One direction is introducing a foreign-ownership share in a private firm.[8] The other direction is allowing private marginal cost to differ from social marginal cost, which appears in various important situations. The social marginal cost is larger than the private marginal cost if a negative externality of production exists, such as pollution. The same is true if a production subsidy is introduced. The social marginal cost is smaller than the private marginal cost if royalty of licensing exists. The same is true if a vertical relationship exists, and there is a double marginalization problem.[9] Thus, our model formulation incorporates many important issues from fields such as industrial organization, public economics, and environmental economics. Especially, the externalities that yield the divergence of social costs from private costs are important for the analysis of mixed oligopolies, because these externalities may serve as the rationale for the existence of public financial institutions.

We find that any distribution of roles, simultaneous-move equilibrium, unique equilibrium with

[6] They used an endogenous competition structure model formulated by Singh and Vives (1984). For more on the topic of welfare and profit ranking over price and quantity competition in mixed duopolies in a simultaneous-move game, see Ghosh and Mitra (2010, 2014), and in a sequential-moves game, see Hirose and Matsumura (2019). See Haraguchi and Matsumura (2016) for an oligopoly version.

[7] In this study, similar to Bárcena-Ruiz (2007) and Din and Sun (2016), we assume that a public firm is a welfare-maximizer, whereas a private firm is a profit-maximizer. Bárcena-Ruiz and Sedano (2011), Matsumura and Ogawa (2014) and Naya (2015) discussed different payoff functions and showed that sequential-move outcomes can emerge in equilibrium.

[8] The literature on mixed oligopolies shows that foreign ownership in private firms often matters. For pioneering works discussing foreign competition in mixed oligopolies, see Corneo and Jeanne (1994), Fjell and Pal (1996), and Pal and White (1998). Foreign ownership is important in the context of public policies in mixed oligopolies. See also Bárcena-Ruiz and Garzón (2005a, b), Heywood and Ye (2009b), Lin and Matsumura (2012), and Wang and Lee (2013).

[9] For discussions on tax-subsidy policy in mixed oligopolies, see Mujumdar and Pal (1998). For discussions on licensing in mixed oligopolies, see Ye (2012) and Kim *et al.* (2018). For discussions on vertical relationship in mixed oligopolies, see Matsumura and Matsushima (2012), Chang and Ryu (2015) and Wu *et al.* (2016).

public leadership, unique equilibrium with private leadership, and multiple equilibria with public and private leadership emerge in equilibrium. This depends on the foreign ownership share in the private firm, the difference between social and private marginal costs, and the degree of product differentiation. Our results suggest that both the Koizumi and Abe Cabinets' policies can be reasonable. Moreover, we find that public leadership more likely emerges in equilibrium when the foreign ownership share in private firms is small, in sharp contrast to the result for quantity competition. Our results may explain the policy shift from the Koizumi to Abe Cabinets, as the presence of foreign financial institutions in the Japanese banking industry became weaker recently.[10]

Next, we investigate welfare and profit ranking, comparing public and private leadership. We find that public leadership is better for social welfare than private leadership when the difference between social and private marginal costs is small. Private leadership becomes better for social welfare than public leadership when the cost difference reaches a threshold value. However, as the cost difference becomes larger and reaches another threshold value, public leadership again becomes better for social welfare than private leadership. In other words, there is a nonmonotone relationship between the advantage of public leadership and the difference between social and private marginal costs.

This nonmonotone relationship does not emerge in the ranking of a private firm's profit. Private (public) leadership yields greater profit for the private firm than public (private) leadership when the difference between social and private marginal costs is small (large).

From these results, we find that public leadership and private leadership can be payoff dominant and risk dominant to private leadership and public leadership, depending on the difference between social and private marginal costs, the degree of product differentiation, and the foreign ownership share in the private firm. Our results highlight the importance of these three factors for the equilibrium role of public firms and the welfare implications.[11]

The rest of this study is organized as follows. Section 2 presents the model. Sections 3 investigates

[10]For example, City Bank exited from the Japanese investment banking market in 2009 and from the retail banking market in 2016. Standard Chartered Bank and HSBC exited from the Japanese private banking market in 2012, and the Royal Bank of Scotland exited in 2017. In 2006, Ripplewood Holdings, which was a dominant stockholder in Shinsei Bank, withdraw their investment.

[11]Matsumura and Ogawa (2009) showed that if one outcome is payoff dominant, this outcome is either the unique equilibrium or the risk-dominant equilibrium in the observable delay game.

fixed timing games. Section 4 shows the equilibrium timing of the observable delay game. Section 5 compares welfare and profit of the private firms. Section 6 concludes the paper.

2 The Model

Firms 0 and 1 produce differentiated commodities, and the inverse demand function is given by $p_i = a - q_i - bq_j$ $(i = 0, 1,\ i \neq j)$, where p_i and q_i are firm i's price and quantity, respectively, a is a positive constant, and $b \in (0, 1)$ represents the degree of product differentiation. A smaller b implies a larger product differentiation. The marginal production costs are constant. Let c_i and s_i denote firm i's marginal private and social costs, respectively. We assume that $a > s_0 > c_1$.[12] Let $\theta \in [0, 1]$ be the foreign ownership share in firm 1.

Firm 0 is a domestic state-owned public firm, and its payoff is the total social surplus given by

$$SW = (p_0 - s_0)q_0 + (p_1 - s_1)q_1 + \left[a(q_0 + q_1) - \frac{q_0^2 + 2bq_0q_1 + q_1^2}{2} - p_0q_0 - p_1q_1 - \theta(p_1 - c_1)q_1\right] \ (:= V_0).$$

The quasi-linear utility function of a representative consumer, $U(q_0, q_1) = a(q_0 + q_1) - (q_0^2 + 2bq_0q_1 + q_1^2)/2 - (p_0q_0 + p_1q_1)$, provides the demand and consumer surplus functions adopted in this study. Firm 1 is a private firm and its payoff is its own profit, $\pi_1 = (p_1 - c_1)q_1$ $(:= V_1)$.

The game runs as follows. In the first stage, each firm i $(i = 0, 1)$ independently chooses whether to move early $(t_i = 1)$ or late $(t_i = 2)$. The basic game is then played using simultaneous play (Bertrand) if both firms choose the same choice, $t_0 = t_1$, and sequential play (Stackelberg) otherwise. See Table 1 for the payoff matrix of the observable delay game in our environment, where V_i^F (res. V_i^L) denotes firm i's equilibrium payoff in the sequential-move game when it is the follower (res. leader), and V_i^B denotes each firm's equilibrium payoff in the simultaneous-move game (Bertrand).

3 Three Fixed Timing Games

In this section, we discuss the second stage game given t_0 and t_1. Let $\Delta_i := s_i - c_i$. If there is a negative (positive) externality of the production of firm i, Δ_i is positive (negative). If there is

[12] If $s_0 \leq c_0$, the public monopoly emerges in equilibrium when the degree of product differentiation is small. To avoid this problem and to simplify the analysis, we assume $s_0 > c_1$.

Table 1: Payoff matrix of the observable delay game.

$0 \backslash 1$	$t_1 = 1$	$t_1 = 2$
$t_0 = 1$	(V_0^B, V_1^B)	(V_0^L, V_1^F)
$t_0 = 2$	(V_0^F, V_1^L)	(V_0^B, V_1^B)

production subsidy (production tax), Δ_i is positive (negative). If firm i pays royalty of licensing and the licenser is a domestic investor, Δ_i is negative. We assume that the following three games have interior solutions.

3.1 Bertrand ($t_0 = t_1 = 1$ or $t_0 = t_1 = 2$)

First, we consider the simultaneous-move game (Bertrand competition). Each firm maximizes its payoff V_i with respect to p_i. The first-order conditions are

$$\begin{aligned}\frac{\partial V_0}{\partial p_0} &= \frac{-p_0 + s_0 - b(p_1 - s_1) - b\theta(p_1 - c_1)}{1 - b^2} = 0,\\ \frac{\partial V_1}{\partial p_1} &= \frac{a(1-b) + c_1 + p_0 - 2p_1}{1 - b^2} = 0.\end{aligned}$$

The second-order conditions are satisfied. From the first-order conditions, we obtain the following reaction functions for firms 0 and 1, respectively:

$$\begin{aligned}R_0(p_1) &= s_0 - b\Delta_1 + b(1-\theta)(p_1 - c_1),\\ R_1(p_0) &= \frac{a(1-b) + c_1 + bp_0}{2}.\end{aligned}$$

Given p_1, firm 0's optimal price does not depend on c_0. Because firm 0 cares about welfare, only social marginal cost s_0 matters. For this reason, Δ_0 does not appear in firm 0's reaction function. However, given p_1, firm 0's optimal price depends on Δ_1. The higher Δ_1 is, the more likely the profit-maximizing price by firm 1 yields excessive output level for welfare. Thus, firm 0 has a greater incentive to reduce firm 1's output when Δ_1 is larger. For this reason, given p_1, firm 0's optimal price is decreasing in Δ_1.

Given p_0, firm 1's optimal price does not depend on s_1 because firm 1 cares only about its own profit.

These reaction functions lead to the following equilibrium prices:

$$p_0^B = \frac{b(1-\theta)(a(1-b)-c_1)+2s_0-2b\Delta_1}{2-b^2(1-\theta)}, \tag{1}$$

$$p_1^B = \frac{a(1-b)+bs_0+(1-b^2(1-\theta))c_1-b^2\Delta_1}{2-b^2(1-\theta)}. \tag{2}$$

The resulting equilibrium outputs are, respectively,

$$q_0^B = \frac{(2-b^2)(a(1-b)-s_0+bs_1)+b(1-b^2)(a-c_1)\theta}{(1-b^2)(2-b^2(1-\theta))}, \tag{3}$$

$$q_1^B = \frac{a(1-b)+bs_0-c_1-b^2\Delta_1}{(1-b^2)(2-b^2(1-\theta))}. \tag{4}$$

The resulting welfare and firm 1's profit are, respectively,

$$V_0^B = \frac{X_1}{2(1-b^2)(2-b^2(1-\theta))^2}, \tag{5}$$

$$V_1^B = \frac{(a(1-b)+bs_0-c_1+b^2\Delta_1)^2}{(1-b^2)(2-b^2(1-\theta))^2}, \tag{6}$$

where the X_1 and the other coefficient X_i are reported in Appendix A.

3.2 Stackelberg with Public Leadership ($t_0 = 1, t_1 = 2$)

Second, we consider the sequential-move game where firm 1 chooses $p_1 = R_1(p_0)$, and firm 0 maximizes its payoff, $V_0(p_0, R_1(p_0))$. We obtain

$$p_0^L = \frac{b(1-2\theta)(a(1-b)-c_1)+2(2-b^2)s_0-2b\Delta_1}{4-3b^2+2b^2\theta}, \tag{7}$$

$$p_1^F = \frac{a(1-b)(2-b^2)+2(1-b^2(1-\theta))c_1+b(2-b^2)s_0-b^2\Delta_1}{4-3b^2+2b^2\theta}. \tag{8}$$

The resulting equilibrium outputs are, respectively,

$$q_0^L = \frac{a(1-b)(4+b-3b^2-b^3)-(2-b^2)^2s_0+b(3-2b^2)c_1+b(2-b^2)\Delta_1+2b(1-b^2)(a-c_1)\theta}{(1-b^2)(4-3b^2+2b^2\theta)},$$

$$q_1^F = \frac{(2-b^2)(a(1-b)+bs_0-c_1)-b^2\Delta_1}{(1-b^2)(4-3b^2+2b^2\theta)}.$$

The resulting welfare and firm 1's profit are, respectively,

$$V_0^L = \frac{X_2}{2(1-b^2)(4-3b^2+2b^2\theta)}, \tag{9}$$

$$V_1^F = \frac{((2-b^2)((1-b)a+bs_0-c_1)-b^2\Delta_1)^2}{(1-b^2)(4-3b^2+2b^2\theta)^2}. \tag{10}$$

3.3 Stackelberg with Private Leadership ($t_0 = 2, t_1 = 1$)

Third, we consider the sequential-move game where firm 0 chooses $p_0 = R_0(p_1)$, and firm 1 maximizes its payoff, $V_1(R_0(p_1), p_1)$. We obtain

$$p_0^F = \frac{(1-\theta)b(a(1-b) - c_1 - bs_0 + b^2\Delta_1) + 2s_0 - 2b\Delta_1}{2(1-b^2(1-\theta))}, \tag{11}$$

$$p_1^L = \frac{a(1-b) + bs_0 + (1-2b^2(1-\theta))c_1 - b^2\Delta_1}{2(1-b^2(1-\theta))}. \tag{12}$$

The resulting equilibrium outputs are, respectively,

$$q_0^F = \frac{2(1-b^2)(a(1-b) - s_0 + bs_1) + b\theta(a(1-b)(1+2b) - bs_0 - (1-2b^2)c_1 + b^2\Delta_1)}{2(1-b^2)(1-b^2(1-\theta))},$$

$$q_1^L = \frac{a(1-b) + bs_0 - c_1 - b^2\Delta_1}{2(1-b^2)}.$$

The resulting welfare and firm 1's profit are, respectively,

$$V_0^F = \frac{X_3}{8(1-b^2)(1-b^2(1-b^2(1-\theta))^2}, \tag{13}$$

$$V_1^L = \frac{(a(1-b) + bs_0 - c_1 - b^2\Delta_1)^2}{4(1-b^2)(1-b^2(1-\theta))}. \tag{14}$$

4 Equilibrium Role

In this section, we discuss the first stage choices and show the equilibrium outcome in the observable delay game. Before presenting our main results (Propositions 1 and 3), we present three supplementary results on the comparison of the equilibrium prices among three fixed timing games (Lemmas 1–3). These lemmas are helpful for understanding the intuition behind our main results.

First, we present a result highlighting the strategic behavior of the leaders.

Lemma 1 *(i) There exists $\tilde{\Delta} > 0$ such that $p_0^B > p_0^L$ if and only if $\Delta_1 < \tilde{\Delta}$. (ii) $\tilde{\Delta}$ is increasing in θ. (iii) $p_1^B \le p_1^L$ and thus $p_0^B \le p_0^F$ regardless of Δ_1, and the equality holds if and only if $\theta = 1$.*

Proof See Appendix B.

First, we explain the intuition behind Lemma 1(i,ii). Suppose that Δ_1 is small. Due to the price-making behavior of firm 1, firm 1's resulting output level is too low (firm 1's price is too high) for welfare, and thus, firm 0 has an incentive to lower firm 1's price. As the leader, firm 0 chooses a lower price than p_0^B because firm 1's reaction curve is upward sloping (strategic complement). Therefore, $p_0^L < p_0^B$ when Δ_1 is small.

Suppose that Δ_1 is large. Because firm 1 chooses its price without considering its high social cost, firm 1's resulting output level is too high (firm 1's price is too low) for welfare, and thus, firm 0 has an incentive to raise firm 1's price. As a leader, firm 0 chooses a higher price than p_0^B. Therefore, $p_0^L > p_0^B$ when Δ_1 is large.

Because firm 1's output is more likely to be excessive for domestic welfare when firm 1 is more foreign (i.e., θ is larger), $p_0^L < p_0^B$ more likely holds when θ is larger (Lemma 1(ii)).

Second, we explain the intuition behind Lemma 1(iii). Firm 1's profit increases with firm 0's price, and firm 0's reaction curve is upward sloping (strategic complement) unless $\theta = 1$ (and firm 0's optimal price is independent of p_1 if $\theta = 1$). Thus, firm 1 chooses a higher price than p_1^B to raise firm 0's price unless $\theta = 1$. Therefore, $p_1^L > p_1^B$, regardless of Δ_1. Because the strategy of firm 0 is strategic complement, $p_0^F > p_0^B$ holds, unless $\theta = 1$.

Next, we present another supplementary result highlighting how Δ_1 affects firm 0's equilibrium prices among the three games.

Lemma 2 $0 > \partial p_0^L/\partial \Delta_1 > \partial p_0^B/\partial \Delta_1$ *and* $0 > \partial p_0^L/\partial \Delta_1 > \partial p_0^F/\partial \Delta_1$.

Proof See Appendix B.

We explain the intuition behind Lemma 2. Lemma 2 states that p_0^B, p_0^L, and p_0^F are decreasing in Δ_1. As Δ_1 increases, the output level of firm 1 is more likely to be excessive for welfare. Thus, firm 0 has a greater incentive to reduce q_1 as Δ_1 increases. Given p_1, a decrease in p_0 reduces q_1. Therefore, firm 0 chooses smaller p_0 as Δ_1 increases in all of three fixed timing games.

Firm 0 has a greater incentive to raise p_1 to reduce q_1 as Δ_1 increases. Thus, as the leader, firm 0 has a greater incentive to raise p_0^L when Δ_1 is larger. This partially cancels out the above price-reducing effect of Δ_1. Such an effect does not exist when firm 0 is the follower or when two firms move simultaneously. Therefore, an increase in Δ_1 reduces p_0^L less significantly than p_0^B and p_0^F.

Finally, we present a result on the comparison of firm 0's price in public leadership as opposed to private leadership.

Lemma 3 *There exists* $\bar{\Delta}$ $(\geq \tilde{\Delta})$ *such that* $p_0^L < p_0^F$ *if and only if* $\Delta_1 < \bar{\Delta}$ *and* $\bar{\Delta} = \tilde{\Delta}$ *if and only if* $\theta = 1$.

Proof See Appendix B.

Regardless of Δ_1, $p_0^F > p_0^B$ (Lemma 1(iii)). When $\Delta_1 < \tilde{\Delta}$, $p_0^L < p_0^B$. Thus, when Δ_1 is small, $p_0^L < p_0^F$. Lemma 2 states that p_0^F decreases more significantly than p_0^L as Δ_1 increases. Thus, $p_0^L > p_0^F$ holds when Δ_1 is large. These yield Lemma 3.

Figure 1 illustrates the relationship between equilibrium prices of firm 0 and Δ_1.

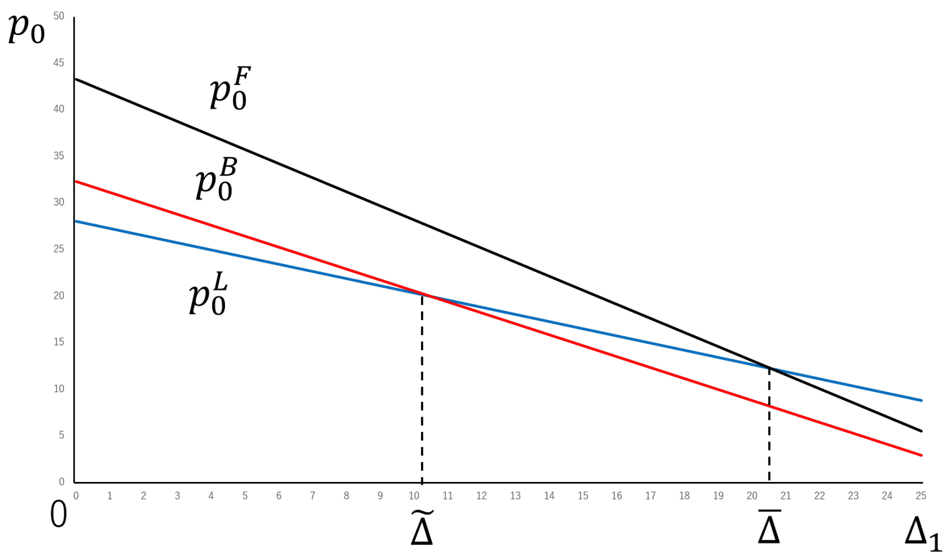

Figure 1: Equilibrium price of public firm.

We now present one of our main results. Proposition 1 describes the equilibrium roles in the observable delay game.

Proposition 1 *Suppose that* $\theta < 1$. *(i) There exists* $\Delta_a > 0$ *such that in equilibrium* $t_0 = t_1 = 1$ $(t_0 \neq t_1)$ *if* $\Delta_1 < \Delta_a$ $(\Delta_1 > \Delta_a)$. *(ii) There exists* $\Delta_b (\geq \Delta_a)$ *such that a unique Stackelberg equilibrium exists (two Stackelberg equilibria exist) if* $\Delta_1 \in (\Delta_a, \Delta_b)$ *(if* $\Delta_1 \geq \Delta_b$*). (iii) Suppose that* $\Delta_1 \in (\Delta_a, \Delta_b)$. *There exists* $\theta_a < 1/2$ *such that in equilibrium* $(t_0, t_1) = (1, 2)$ $((t_0, t_1) = (2, 1))$ *if* $\theta < \theta_a$ $(\theta > \theta_a)$*, and* $\Delta_a = \Delta_b$ *if* $\theta = \theta_a$.

Proof See Appendix B.

Proposition 1(i) states that whether Bertrand or Stackelberg emerges in equilibrium depends on the difference between the social and private marginal costs of firm 1. If there is no difference or the private marginal cost exceeds the social marginal cost (i.e., the cost difference Δ_1 is negative), Bertrand emerges regardless of the foreign ownership share in firm 1 or the degree of product differentiation. However, if the cost difference Δ_1 exceeds the threshold value, Stackelberg emerges. This implies that if there are negative externalities of production or if the private firm's production is subsidized, Stackelberg can emerge in equilibrium. Proposition 1(iii) states that two Stackelberg equilibria (public leadership and private leadership equilibria) emerges if the cost difference Δ_1 exceeds another threshold value. Proposition 1(iii) states that public leadership is more likely to become a unique equilibrium than private leadership when the foreign ownership share in firm 1 is lower. This result may explain the recent revival of public leadership by Japanese public financial institutions, as discussed in the Introduction.

We now explain the intuition behind Proposition 1. Suppose that Δ_1 is small. Lemma 1(i) states that $p_0^L < p_0^B$. Given $t_0 = 1$, firm 1 has an incentive to prevent firm 0's leadership by choosing $t_1 = 1$. Therefore, the public leadership equilibrium does not emerge. Lemma 1(iii) states that $p_1^L > p_1^B$. As we explain in the intuitive explanation for Lemma 1(i), firm 0 prefers a lower firm 1's price when Δ_1 is small. Thus, given $t_1 = 1$, firm 0 has an incentive to prevent firm 1's leadership by choosing $t_0 = 1$. Therefore, private leadership equilibrium does not emerge. Because neither public leadership nor private leadership emerges in equilibrium, the only equilibrium outcome is Bertrand.

Suppose that Δ_1 is large. Lemma 1(i) states that $p_0^L > p_0^B$. Given $t_0 = 1$, firm 1 chooses $t_1 = 2$ to raise p_0. Therefore, the public leadership equilibrium emerges. As we explain in the intuitive explanation for Lemma 1(i), firm 0 prefers a higher p_1 when Δ_1 is large. Thus, given $t_1 = 1$, firm 0 chooses $t_0 = 2$ to raise p_1. Therefore, the private leadership equilibrium emerges.

Suppose that Δ_1 is intermediate and θ is small. Given $t_1 = 1$, choosing $t_0 = 2$ raises p_1. Although a marginal increase in p_1 from p_1^B improves welfare, p_1^L can be too high for welfare, and it is possible that p_1^B is better than p_1^L for welfare. In this case, private leadership fails to result in equilibrium, and the unique equilibrium is public leadership.

Suppose that Δ_1 is intermediate and that θ is large. Firm 1's output level is more likely to be too high (the price level is too low) for welfare when θ is larger because the higher market share of firm 1 increases the outflow of the surplus to foreign investors. Thus, it is less likely that p_1^B ($< p_1^L$) is better than p_1^L for welfare. Therefore, the private leadership equilibrium is more likely to survive. This effect can be so strong that the private leadership equilibrium exists even when the condition for the existence of the public leadership equilibrium is not satisfied. For this reason, the unique equilibrium can involve private leadership when θ is large.

We now present a result when $\theta = 1$, which is not covered by our main result (Proposition 1).

Proposition 2 *Suppose that $\theta = 1$. (i) $V_0^B = V_0^F$ and $V_1^L = V_1^B$. Moreover, $V_0^L = V_0^B = V_0^F$ and $V_1^L = V_1^B = V_1^F$ when $\Delta_1 = \tilde{\Delta}$. (ii) Bertrand is an equilibrium outcome if $\Delta_1 \leq \tilde{\Delta}$. (iii) Public leadership is an equilibrium outcome if $\Delta_1 \geq \tilde{\Delta}$. (iv) Private leadership is an equilibrium outcome regardless of Δ.*

Proof See Appendix B.

From (1), we obtain that the public firm's optimal price does not depend on p_1 when $\theta = 1$. Therefore, as the leader, the private firm cannot affect p_0 and chooses the same price as in the Bertrand case. This leads to Proposition 2(i).

Proposition 2(ii) states that Bertrand equilibrium emerges when Δ_1 is small, while the public leadership equilibrium emerges when Δ_1 is large. These results are like those for Proposition 1(ii). However, Proposition 2(iv) states that the private leadership equilibrium emerges regardless of Δ_1, which is in sharp contrast to Proposition 1(ii). Thus, some readers might perceive that there is a discontinuity with respect to θ at the point $\theta = 1$.

We think that the discrepancy between Propositions 1 and 2 is smaller than what it appears at first glance. In the private leadership equilibrium, both firms adopt weakly dominated strategies when $\Delta_1 < \tilde{\Delta}$. When $\Delta_1 < \tilde{\Delta}$, both Bertrand and the private leadership equilibria exist. However, Bertrand equilibrium is risk dominant. Thus, from the viewpoint of the standard discussion of equilibrium selection (Harsanyi and Selten, 1988), it is natural to focus on Bertrand equilibrium, not the private leadership equilibrium, when Δ_1 is small, a result similar to that for Proposition 1.

This result shows a possible risk of using the model with pure foreign private firms in mixed oligopolies. The result that the private leadership equilibrium always exists holds only when domestic investors own a 0% share in the private firm.

5 Welfare and Profit Ranking

We now present another main result (Proposition 3), which concerns welfare and profit ranking in public and private leadership. Let

$$\begin{aligned}
\Delta_c &:= \frac{(a(1-b)+bs_0-c_1)[(1-b^2)(6-5b^2-2\theta(2-5b^2))-b^2\theta^2(4-7b^2+2b^2\theta))-X_4]}{(4-3b^4)(1-b^2)+8b^4(1-b^2)\theta(1-\theta)+b^4\theta^2(4-b^2-2b^2\theta)}, \\
\Delta_d &:= \frac{(a(1-b)+bs_0-c_1)[5b^4-12b^2+8+8b^2\theta(1-b^2)+4b^4\theta^2-X_5]}{b^2(4b^4\theta^2+12b^2\theta(1-b^2)+9b^4-20b^2+12)}. \\
\Delta_e &:= \frac{(a(1-b)+bs_0-c_1)[(1-b^2)(6-5b^2-2\theta(2-5b^2))-b^2\theta^2(4-7b^2+2b^2\theta))+X_4]}{(4-3b^4)(1-b^2)+8b^4(1-b^2)\theta(1-\theta)+b^4\theta^2(4-b^2-2b^2\theta)}.
\end{aligned}$$

Proposition 3 *Suppose that $\theta < 1$. (i) $\Delta_c, \Delta_d, \Delta_e > 0$. (ii) $\Delta_c < \Delta_e$. (iii) $V_0^L < V_0^F$ if and only if $\Delta_1 \in (\Delta_c, \Delta_e)$. (iv) $V_1^L > V_1^F$ if and only if $\Delta_1 < \Delta_d$.*

Proof See Appendix B.

Suppose that $\theta < 1$. Public leadership is better for social welfare than private leadership ($V_0^L > V_0^F$) when the difference between social and private marginal costs Δ_1 is small (when $\Delta_1 < \Delta_c$). Private leadership becomes better for social welfare than public leadership when the cost difference reaches a threshold value (Δ_c). However, as the cost difference becomes larger and reaches yet another threshold vale (Δ_e), public leadership again becomes better for social welfare than private leadership. In other words, there is a nonmonotone relationship between the advantage of public leadership and the cost difference between social and private marginal costs (Proposition 3(iii)).

Similar nonmonotone relationship does not emerge in the ranking of the private firm's profit (Proposition 3(iv)). The private leadership yields greater profit of firm 1 than public leadership ($V_1^L > V_1^F$) when the cost difference between social and private marginal costs is small (when $\Delta_1 < \Delta_d$). The pub-

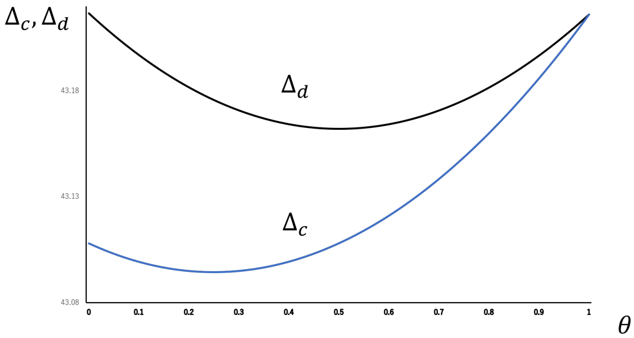

Figure 2-1

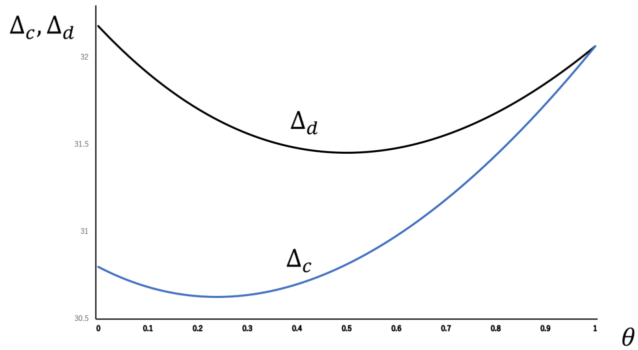

Figure 2-2

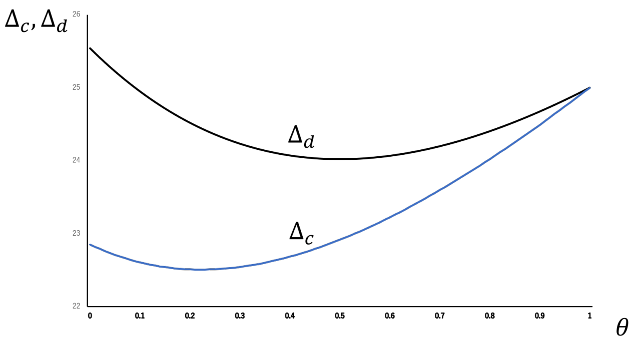

Figure 2-3

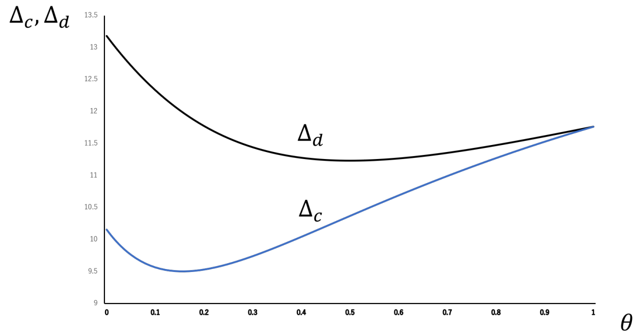

Figure 2-4

Figure 2: Numerical examples of Δ_c and Δ_d where $a = 100$, $s_0 = 10$, and $c_1 = 5$. $b = 0.1, 0.4, 0.6$, and 0.9 in Figures 2-1, 2-2, 2-3, and 2-4, respectively.

lic leadership becomes better for the private firm than the private leadership when the cost difference exceeds a threshold value (Δ_d).

Moreover, Proposition 3 implies that public leadership is payoff dominant to private leadership when $\Delta_1 > \max\{\Delta_d, \Delta_e\}$. In addition, if $\Delta_c < \Delta_d$, private leadership can be payoff dominant to public leadership (private leadership is payoff dominant to public leadership when $\Delta_1 \in (\Delta_c, \min\{\Delta_d, \Delta_e\})$). Although we fail to prove that $\Delta_c \leq \Delta_d$ always hold, we numerically show that the inequality $\Delta_c < \Delta_d$ holds for a wide range of parameter values (Figure 2).[13]

As Matsumura and Ogawa (2009) showed, payoff dominance implies risk dominance in the observable delay game. These results show that both private and public leadership can be risk dominant and payoff dominant. This implies that both can be robust, depending on the cost difference between

[13]We did not find a numerical example in which the inequality $\Delta_c > \Delta_d$ holds.

social and private marginal costs. These results are in sharp contrast to those of Capuano and De Feo (2010) and Matsumura and Ogawa (2010), and they may explain the recent fluctuations in the Japanese government's policy regarding public financial institutions, as discussed in Introduction.

We now explain the intuition behind the results on welfare ranking in Proposition 3. As the leader, firm 0 can choose $p_0 = p_0^B$. This implies that $V_0^L \geq V_0^B$, regardless of Δ_1 and θ. As the leader, firm 1 chooses $p_1 = p_1^L > p_1^B$. We have already explained that p_1^L is too high for welfare when Δ_1 is small. Thus, $V_0^F < V_0^B$. Therefore, $V_0^L > V_0^F$ when Δ_1 is small.

As Δ_1 reaches a threshold value, p_1^L becomes optimal for welfare. Thereafter, p_1^L becomes too low for welfare as Δ_1 increases. As the leader, firm 1 chooses $p_1 = p_1^L > p_1^B$, and welfare advantage of private leadership increases, and eventually $V_0^L < V_0^F$ holds. Note that $p_0^L \neq R(p_1^F)$ and $p_0^F = R(p_1^L)$. In other words, the public firm chooses the optimal price with p_1 as the follower, but it does not choose the optimal price with p_1 as the leader. Therefore, the advantage of private leadership appears as long as p_1^L is close to the optimal price for welfare.

When Δ_1 increases further, the difference between welfare optimal p_1 and p_1^L becomes larger, which worsens the welfare performance of private leadership. Therefore, the advantage of public leadership again emerges when Δ_1 is very large.

We then explain the intuition behind the results on profit ranking in Proposition 3. As the leader, firm 1 can choose $p_1 = p_1^B$. This implies that $V_1^L \geq V_1^B$ regardless of Δ_1 and θ. Moreover, because $p_1^L \neq p_1^B$, $V_1^L > V_1^B$ holds. As we showed, $p_0^L < p_0^B$ and thus $V_1^F < V_1^B$, when Δ_1 is small. Therefore, $V_1^L > V_1^F$ when Δ_1 is small.

As Δ_1 reaches a threshold vale, $p_0^L = p_0^B$ (and thus $V_1^F = V_1^B$) holds, and thereafter, $p_0^L > p_0^B$ (and thus $V_1^F > V_1^B$) holds as Δ_1 increases. The profit advantage of public leadership increases as Δ_1 increases, and eventually $V_1^L < V_1^F$ holds. Note that $p_1^L \neq R(p_0^F)$ and $p_1^F = R(p_0^L)$. In other words, the private firm chooses the optimal price given p_0 as the follower, but it does not choose the optimal price given p_0 as the leader. Therefore, the advantage of public leadership emerges.

We now present a result when $\theta = 1$, which is not covered by our main result (Proposition 3).

Proposition 4 *Suppose that $\theta = 1$ (i)$\Delta_c = \Delta_d = \Delta_e = \tilde{\Delta}$. (ii) $V_0^L \geq V_0^F$ and the equality holds if and* *only if* $\Delta_1 = \Delta_d$. *(iii)* $V_1^L > V_1^F$ *if and only if* $\Delta_1 < \Delta_d$.

Proof See Appendix B.

Proposition 4(iii) states that private leadership yields greater profits for the private firm than public leadership when Δ_1 is small, which is the same result as in Proposition 3(iv). We omit the intuitive explanation, because the intuition behind Proposition 4(iii) is similar to that for Proposition 3(iv).

Proposition 4(ii) states that public leadership always yields greater welfare than private leadership, in contrast to Proposition 3. Proposition 4 is a degenerated version of Proposition 3. Because all Δ_c, Δ_d, and Δ_e converge to the same value when $\theta \to 1$, the range of Δ_1 for the welfare advantage of private leadership over public leadership, (Δ_c, Δ_e), disappears as $\theta \to 1$ (Figure 3). This result again shows a possible risk of using the model with pure foreign private firms in mixed oligopolies. The result that public leadership always yields greater welfare holds only when domestic investors own a 0% share in the private firm.

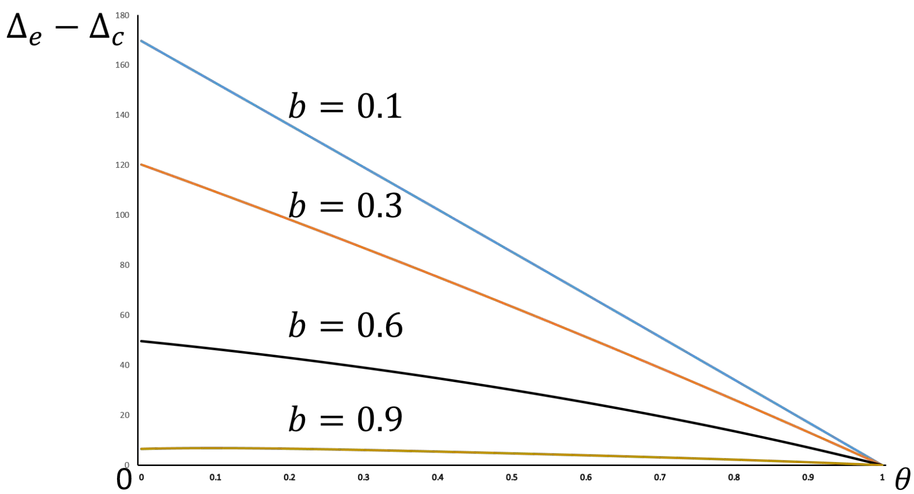

Figure 3: Numerical examples of $\Delta_e - \Delta_c$ where $a = 100$, $s_0 = 10$, and $c_1 = 5$.

The intuition behind Proposition 4(ii) is as follows. As we stated in the previous section, firm 0's optimal price does not depend on p_1. Therefore, Bertrand equilibrium and the private leadership equilibrium yields the same prices, and thus, the same profits and welfare. Because as the leader, firm 0 can choose p_0^B (and then firm 1 chooses p_1^B), the welfare in the public leadership equilibrium is never smaller than that in the private leadership equilibrium.

Proposition 4(i) states that profit and welfare ranking depends on the sign of $p_0^L - p_0^B$. Remember that $p_0^L < (>, =) p_0^B$ if $\Delta_1 < (>, =) \tilde{\Delta}$. Because $V_0^L = V_0^B$ holds only when $p_0^L = p_0^B$, and $V_0^B = V_0^F$ always holds when $\theta = 1$, welfare ranking depends only on whether $p_0^L = p_0^B$ or not. This leads to $\Delta_c = \Delta_e = \tilde{\Delta}$. Because $V_1^F > (<, =) V_1^B$ holds only when $p_0^L > (<, =) p_0^B$ and $V_1^L = V_1^B$ holds, profit ranking depends only on the sign of $p_0^L - p_0^B$. This leads to $\Delta_d = \tilde{\Delta}$. Remember that Δ_d is a threshold value determining the sign of $V_1^L - V_1^F$ and that $\tilde{\Delta}$ is a threshold value determining the sign of $p_0^L - p_0^F$.

6 Concluding Remarks

In this study, we examine an endogenous timing game in mixed duopolies with price competition when social marginal costs are allowed to differ from private marginal costs. We find that any equilibrium timing can emerge, depending on the foreign ownership share in the private firm and the difference between social and private marginal costs. We show that the public firm is more likely to lead when the private competitor is domestic, which may explain the recent change in policies in Japan regarding public financial institutions.

In this study, we consider a single market model. As Haraguchi *et al.* (2018) pointed out, public firms face competitive pressure from neighboring markets, and extending our analysis to a multi-market model is an opportunity for future research.[14]

In this study, we also consider a duopoly model. Extending our analysis to n-firm oligopolies would be quite a tough assignment, and it is beyond the scope of the current study. This is another opportunity for future work.

[14] For discussions on optimal privatization policy in multi-market models, see also Bárcena-Ruiz and Garzón (2017) and Dong *et al.* (2018).

Appendix A

$$
\begin{aligned}
X_1 &:= -(1-b^2)(a-c_1)b^2(a(1-2b)+2bs_0-c_1)\theta^2 - [2b^2(1-b^2)(a(1-b)+bs_0-c_1)\Delta_1 \\
&- 2b^3(1-b^2)\Delta_0^2 + 4b(1-b^2)(a(1+b-b^2)-bc_0-(1-b^2)c_1)\Delta_0 + 2(1-b^2)^2c_1^2 \\
&- 4(1-b^2)^2(a(1-b)+bc_0)c_1 - 2b^2(1-b^2)c_0^2 + 4a(1-b^2)(1+b-b^2)c_0 \\
&+ 2a^2(1-b^2)(1-2b-2b^2+2b^3)]\theta + b^4\Delta_1^2 - 2(2-2b^2+b^4)(a(1-b)+bs_0-c_1)\Delta_1 \\
&+ (2b^4-5b^2+4)\Delta_0^2 - 2((b^5-3b^3+3b)c_1-(2b^4-5b^2+4)c_0-(b^5-2b^4-3b^3+5b^2+3b-4)a)\Delta_0 \\
&+ (b^4-3b^2+3)c_1^2 - 2(3-3b^2+b^4)(a(1-b)+bc_0)c_1 + (2b^4-5b^2+4)c_0^2 \\
&- 2a(1-b)(b^4-b^3-4b^2+b+4)c_0 + a^2(1-b)(2b^4-b^3-7b^2+b+7), \\
X_2 &:= -[4b(1-b^2)(a-c_1)\Delta_0 + 2(1-b^2)c_1^2 - 4(1-b^2)(a(1-b)+bc_0)c_1 + 4ab(1-b^2)c_0 \\
&+ 2a^2(1-b^2)(1-2b)]\theta + b^2\Delta_1^2 - 2(2-b^2)(a(1-b)+bs_0-c_1)\Delta_1 + (2-b^2)^2\Delta_0^2 \\
&+ 2((2b^3-3b)c_1 + (2-b^2)^2c_0 - (b^4+2b^3-4b^2-3b+4)a)\Delta_0 - (2b^2-3)c_1^2 \\
&- 2(3-2b^2)(a(1-b)+bc_0)c_1 + (2-b^2)^2c_0^2 - 2a(1-b)(4+b-3b^3-b^3)c_0 \\
&+ a^2(1-b)(7+b-5b^2-b^3), \\
X_3 &:= [b^6\Delta_1^2 - 2b^4(a(1-b)+bs_0-c_1)\Delta_1 + b^4\Delta_0^2 - 2b^3((4b^2-3)c_1-bc_0+(3+b-4b^2)a)\Delta_0 \\
&+ b^2(4b^2-3)c_1^2 + 2b^2(3-4b^2)(a(1-b)+bc_0)c_1 + b^4c_0^2 - 2ab^3(1-b)(3+4b)c_0 \\
&+ a^2b^2(1-b)(8b^2+3b-3)]\theta^2 + [2b^4(1-b^2)\Delta_1^2 - 8b^2(1-b^2)(a(1-b)+bs_0-c_1)\Delta_1 \\
&+ 6b^2(1-b^2)\Delta_0^2 - 4b(1-b^2)((4b^2-1)c_1-3bc_0+(1+3b-4b^2)a)\Delta_0 - 2(1-b^2)(1-4b^2)c_1^2 \\
&+ 4(1-b^2)(1-4b^2)(a(1-b)+bc_0)c_1 + 6b^2(1-b^2)c_0^2 - 4ab(1-b)^2(1+b)(1+4b)c_0 \\
&- 2a^2(1-b^2)(1+b)(1-b-8b^2)]\theta + b^4(b^2-1)\Delta_1^2 - 2(1-b^2)(2-3b^2)(a(1-b)+bs_0-c_1)\Delta_1 \\
&+ (1-b^2)(4-5b^2)\Delta_0^2 + 2(1-b^2)((4b^3-3b)c_1-(5b^2-4)c_0-(4b^3-5b^2-3b+4)a)\Delta_0 \\
&- (1-b^2)(3-4b^2)c_1^2 - 2(1-b^2)(3-4b^2)(a(1-b)+bc_0)c_1 + (1-b^2)(4-5b^2)c_0^2 \\
&- 2a(1-b)^2(1+b)(4+b-4b^2)c_0 + a^2(1-b)^3(1+b)(7+8b),
\end{aligned}
$$

$$
\begin{aligned}
X_4 &:= 2(1-b^2)(1-\theta)(1-b^2(1-\theta))\sqrt{4-3b^2+2b^2\theta}, \\
X_5 &:= 2(1-b^2)(4-3b^2+2b^2\theta)\sqrt{1-b^2(1-\theta)}, \\
X_6 &:= (1-b^2)(4-3b^2)(a(1-b)+bs_0-c_1)+b^2(1-b^2)\theta(5(a(1-b)+bs_0-c_1)-3(4-b^2)\Delta_1) \\
&+ b^2\theta^2((2+b^2)(a(1-b)+bs_0-c_1)-3b^2(2-b^2)\Delta_1)+b^4\theta^3(a(1-b)+bs_0-c_1-b^2\Delta_1), \\
X_7 &:= [b^4-7b^2+8+4b^2\theta-b^4\theta]((1-b)a+bs_0-c_1)-(6b^2-4b^4+3b^4\theta)\Delta_1, \\
X_8 &:= (-2b^6\theta^3+b^4(7b^2-4)\theta^2+8b^4(1-b^2)\theta+3b^6-3b^4-4b^2+4)\Delta_1^2+2(2b^4\theta^3-b^2(7b^2-4)\theta^2 \\
&+ 2(1-b^2)(2-5b^2)\theta-5b^4+11b^2-6)(a(1-b)+bs_0-c_1)\Delta_1-(2b^2\theta^3+b^2(1-4b^2)\theta^2 \\
&+ 4(1-b^2)(1-2b^2)\theta-(1-b^2)(5-4b^2))(a(1-b)+bs_0-c_1)^2, \\
X_9 &:= b^2(4b^4\theta^2+12b^2(1-b^2)\theta+9b^4-20b^2+12)\Delta_1^2-2(4b^4\theta^2+8b^2(1-b^2)\theta+5b^4-12b^2+8)(a(1-b) \\
&+ bs_0-c_1)\Delta_1+(4b^2\theta^2+4b^2\theta(1-b^2)+4b^2-11b^2+8)(a(1-b)+bs_0-c_1)^2,
\end{aligned}
$$

Appendix B

Proof of Lemma 1

From (1) and (7), we obtain

$$\begin{aligned} p_0^B - p_0^L &= \frac{2b((1-b^2(1-\theta))(a(1-b)+bs_0-c_1)-(2(1-b^2)+b^2\theta)\Delta_1)}{(2-b^2(1-\theta))(4-3b^2+2b^2\theta)} > 0 \\ &\Leftrightarrow \;\; \Delta_1 < \tilde{\Delta} := \frac{(1-b^2(1-\theta))(a(1-b)+bs_0-c_1)}{2(1-b^2)+b^2\theta}. \end{aligned}$$

Because we suppose that $a > s_0 > c_1$, we obtain $a(1-b)+bs_0-c_1 = (a-c_1)-b(a-s_0) > 0$. This implies $\tilde{\Delta} > 0$. These results imply Lemma 1(i).

Differentiating $\tilde{\Delta}$ with respect to θ yields

$$\frac{\partial \tilde{\Delta}}{\partial \theta} = \frac{b^2(1-b^2)(a(1-b)+bs_0-c_1)}{(2(1-b^2)+b^2\theta)^2} > 0.$$

This implies Lemma 1(ii).

Because we assume an interior solution in the price competition stage, from (4), we obtain $a(1-b)+bs_0-c_1-b^2\Delta_1 > 0$. From (2) and (12), we obtain

$$p_1^B - p_1^L = -\frac{(1-\theta)b^2(a(1-b)+bs_0-c_1-b^2\Delta_1)}{2(1-b^2(1-\theta))(2-b^2(1-\theta))} \leq 0, \tag{15}$$

and the equality in (15) holds if and only if $\theta = 1$. From (1) and (11), we obtain

$$p_0^B - p_0^F = -\frac{(1-\theta)^2 b^3(a(1-b)+bs_0-c_1-b^2\Delta_1)}{2(1-b^2(1-\theta))(2-b^2(1-\theta))} \leq 0, \tag{16}$$

and the equality in (16) holds if and only if $\theta = 1$. These imply Lemma 1(iii). ■

Proof of Lemma 2

From (1), (7) and (11), we obtain

$$\frac{\partial p_0^B}{\partial \Delta_1} = -\frac{2b}{2-b^2(1-\theta)} < 0, \tag{17}$$

$$\frac{\partial p_0^L}{\partial \Delta_1} = -\frac{2b}{4-3b^2+2b^2\theta} < 0, \tag{18}$$

$$\frac{\partial p_0^F}{\partial \Delta_1} = -\frac{b(2-b^2(1-\theta))}{2(1-b^2(1-\theta))} < 0. \tag{19}$$

From (17)-(19), we obtain

$$\begin{aligned}
\frac{\partial p_0^B}{\partial \Delta_1} - \frac{\partial p_0^L}{\partial \Delta_1} &= -\frac{4b(1-b^2(1-\theta))}{(2-b^2)(4-3b^2+2b^2\theta)} < 0, \\
\frac{\partial p_0^F}{\partial \Delta_1} - \frac{\partial p_0^L}{\partial \Delta_1} &= -\frac{b(3(1-b^2)^2+2b^2(2(1-b^2\theta)+b^2\theta^2)+1-b^4\theta)}{2(1-b^2(1-\theta))(4-3b^2+2b^2\theta)} < 0.
\end{aligned}$$

These results imply Lemma 2.■

Proof of Lemma 3

From (7) and (11), we obtain

$$\begin{aligned}
p_0^L - p_0^F &= \frac{-(2-b^2-b^2\theta+2b^2\theta)(a(1-b)+bs_0-c_1)+(4-6b^2+3b^4+b^2(4-5b^2)\theta+2b^4\theta^2)s_1}{2(1-b^2(1-\theta))(4-3b^2+2b^2\theta)} < 0 \\
&\Leftrightarrow \;\; \Delta_1 < \bar{\Delta} := \frac{(2-b^2-b^2\theta+2b^2\theta)(a(1-b)+bs_0-c_1)}{4-6b^2+3b^4+b^2(4-5b^2)\theta+2b^4\theta^2}.
\end{aligned}$$

Comparing $\tilde{\Delta}$ and $\bar{\Delta}$, we obtain

$$\bar{\Delta} - \tilde{\Delta} = \frac{b^2(1-b^2)(1-\theta)^2(a(1-b)+bs_0-c_1)(4-3b^2+2b^2\theta)}{(2-2b^2+b^2\theta)(4-6b^2+3b^4+4b^2-5b^4\theta+2b^4\theta^2)} \geq 0.$$

The equality holds if and only if $\theta = 1$. These results imply Lemma 3.■

Proof of Proposition 1

From (5), (6), (9), (10), (13), and (14), we obtain

$$V_0^L - V_0^B = \frac{b^2[(1-b^2(1-\theta))(a(1-b)+bs_0-(c_1+\Delta_1))+(1-b^2)\Delta_1]^2}{2(1-b^2)(2-b^2(1-\theta))^2(4-3b^2+2b^2\theta)} \geq 0, \tag{20}$$

$$V_0^B - V_0^F = \frac{b^2(1-\theta)(a(1-b)+bs_0-c_1-b^2\Delta_1)X_6}{8(1-b^2)(1-b^2(1-\theta))^2(2-b^2(1-\theta))^2}, \tag{21}$$

$$V_1^L - V_1^B = \frac{b^4(1-\theta)^2(a(1-b)+bs_0-c_1-b^2\Delta_1)^2}{4(1-b^2)(1-b^2(1-\theta))(2-b^2(1-\theta))^2} \geq 0, \tag{22}$$

$$V_1^B - V_1^F = \frac{b^2[(1-b^2(1-\theta))(a(1-b)+bs_0-(c_1+\Delta_1))+(1-b^2)\Delta_1]X_7}{(1-b^2)(2-b^2(1-\theta))^2(4-3b^2+2b^2\theta)^2}. \tag{23}$$

The equality in (20) holds if and only if $\Delta_1 = \tilde{\Delta}$. The equality in (22) holds if and only if $\theta = 1$.

Proof of Proposition 1(i)

Bertrand ($t_0 = t_1 = 1$) is an equilibrium if and only if both (21) and (23) are nonnegative, and Bertrand is the unique equilibrium if both are positive.

Equation (21) is positive (negative, zero) if $X_6 > (<, =)0$. Solving the equation $X_6 = 0$ with respect to Δ_1, we obtain

$$\Delta_1 = \hat{\Delta} := \frac{((1-b^2)(4-3b^2) + 5b^2(1-b^2)\theta + b^2(2+b^2)\theta^2 + b^4\theta^3)(a(1-b) + bs_0 - c_1)}{(1-b^2)(b^4 - 8b^2 + 8) + 3b^2(1-b^2)(4-b^2)\theta + 3b^4\theta^2 + b^6\theta^3}.$$

We now show that X_7 is positive. Because we assume an interior solution in the price competition stage, from (4), we obtain

$$a(1-b) + bs_0 - c_1 - b^2\Delta_1 > 0 \Leftrightarrow \Delta_1 < \frac{a(1-b) + bs_0 - c_1}{b^2}.$$

$X_7 > 0$ if

$$\Delta_1 < \frac{(b^4 - 7b^2 + 8 + 4b^2\theta - b^4\theta)(a(1-b) + bs_0 - c_1))}{6b^2 - 4b^4 + 3b^4\theta}.$$

We obtain

$$\begin{aligned} &\frac{a(1-b) + bs_0 - c_1}{b^2} - \frac{(b^4 - 7b^2 + 8 + 4b^2\theta - b^4\theta)(a(1-b) + bs_0 - c_1))}{6b^2 - 4b^4 + 3b^4\theta} \\ &\qquad = -\frac{(1-b^2)(2 - b^2(1-\theta))(a(1-b) + bs_0 - c_1)}{b^2(6 - 4b^2 + 3b^2\theta)} < 0. \end{aligned}$$

These imply $X_7 > 0$.

Because X_7 is positive, equation (23) is positive (negative, zero) if $(1-b^2(1-\theta))(a(1-b)+bs_0-(c_1+\Delta_1))+(1-b^2)\Delta_1 > (<, =)0$. Solving the equation $(1-b^2(1-\theta))(a(1-b)+bs_0-(c_1+\Delta_1))+(1-b^2)\Delta_1 = 0$ with respect to Δ_1, we obtain

$$\Delta_1 = \frac{(1-b^2(1-\theta))((1-b)a + bs_0 - c_1)}{2(1-b^2) + b^2\theta} = \tilde{\Delta}.$$

Therefore, both (21) and (23) are positive if $\Delta_1 < \Delta_a := \min\{\hat{\Delta}, \tilde{\Delta}\}$. Because $(1-b)a + bs_0 - c_1 > 0$, we obtain $\hat{\Delta} > 0$ and $\tilde{\Delta} > 0$. Thus, $\Delta_a > 0$. ■

Proof of Proposition 1(ii)

Two Stackelberg equilibria exist (both $(t_0, t_1) = (1, 2)$ and $(t_0, t_1) = (2, 1)$ are the equilibrium outcomes) if and only if both (21) and (23) are nonpositive. Only one Stackelberg equilibrium exists (either $(t_0, t_1) = (1, 2)$ or $(t_0, t_1) = (2, 1)$ is the equilibrium outcome) if and only if one of (21) and (23) is nonpositive and the other is positive. Let $\Delta_b := \max\{\hat{\Delta}, \tilde{\Delta}\}$. One of (21) and (23) is positive and the other is negative if $\Delta_1 \in (\Delta_a, \Delta_b)$. Both are negative if $\Delta_1 > \Delta_b$. These imply Proposition 1(ii). ■

Proof of Proposition 1(iii)

As we have shown in the proof of Proposition 1(i), private leadership is an equilibrium if $\Delta_1 \geq \hat{\Delta}$, and public leadership is an equilibrium if $\Delta_1 \geq \tilde{\Delta}$. Therefore, if $\hat{\Delta} > \tilde{\Delta}$ ($\hat{\Delta} < \tilde{\Delta}$), the unique Stackelberg is public leadership (private leadership) when $\Delta_1 \in (\Delta_a, \Delta_b)$.

From $\hat{\Delta}$ and $\tilde{\Delta}$, we obtain

$$\hat{\Delta} - \tilde{\Delta} = \frac{b^2(1-b^2)(1-\theta)(2-b^2(1-\theta))[(1-b^2)(1-2\theta)-b^2\theta^2]}{(2-b^2(2-\theta))[(1-b^2)(8-8b^2+b^4)+3b^2(1-b^2)(4-b^2)\theta+3b^4(2-b^2)\theta^2+b^6\theta^3]}. \quad (24)$$

Equation (24) is positive (negative, zero) if $(1-b^2)(1-2\theta)-b^2\theta^2 > (<,=)0$. Solving the equation $(1-b^2)(1-2\theta)-b^2\theta^2 = 0$, we obtain

$$(1-b^2)(1-2\theta)-b^2\theta^2 = 0 \to \theta = \frac{-(1-b^2) \pm \sqrt{1-b^2}}{b^2}.$$

The positive solution is

$$\theta_a := \frac{-(1-b^2)+\sqrt{1-b^2}}{b^2}. \quad (25)$$

Therefore, we obtain $\hat{\Delta} > (<,=)\tilde{\Delta}$ if $\theta < (>,=)\theta_a$.

From (25), we obtain

$$\begin{aligned}
\frac{d\theta_a}{db} &= -\frac{2(1-\sqrt{1-b^2})}{b^3\sqrt{1-b^2}} < 0, \\
\theta_a &= \frac{-(1-b^2)+\sqrt{1-b^2}}{b^2} \to 0 \quad (b \to 1), \\
\theta_a &= \frac{-(1-b^2)+\sqrt{1-b^2}}{b^2} \\
&= \frac{1-b^2}{\sqrt{1-b^2}+1-b^2} \to \frac{1}{2} \quad (b \to 0).
\end{aligned}$$

Thus, $\theta_a \in (0, 1/2)$. These imply Proposition 1(iii). ■

Proof of Proposition 2

Substituting $\theta = 1$ into (21) and (22) we obtain Proposition 2(i).

Bertrand equilibrium emerges if both (21) and (23) are nonnegative. (21) is always zero when $\theta = 1$. (23) is nonnegative if and only if $\Delta_1 \leq \tilde{\Delta}$. These imply Proposition 2(ii).

Public leadership is an equilibrium outcome if (20) is nonnegative and (23) is nonpositive. (20) is always nonnegative. (23) is nonpositive if and only if $\Delta_1 \geq \tilde{\Delta}$. These imply Proposition 2(iii).

Private leadership is an equilibrium if (21) is nonpositive and (22) is nonnegative. Both (21) and (22) are zero when $\theta = 1$. These imply Proposition 2(iv). ■

Proof of Proposition 3(i,ii)

First, we show $\Delta_c > 0$. $\Delta_c > 0$ if the numerator of Δ_c is positive. Let $A := (1-b^2)(6-5b^2-2\theta(2-5b^2)) - b^2\theta^2(4-7b^2+2b^2\theta) - X_4$. We obtain

$$\begin{aligned}
A &= (1-b^2)(6-5b^2-2\theta(2-5b^2)) - b^2\theta^2(4-7b^2+2b^2\theta) - X_4 \\
&= (1-b^2)(6-5b^2-2\theta(2-5b^2)) - b^2\theta^2(4-7b^2+2b^2\theta) \\
&\quad -2(1-b^2)(1-\theta)(1-b^2(1-\theta))\sqrt{4-3b^2+2b^2\theta} \\
&\geq (1-b^2)(6-5b^2-2\theta(2-5b^2)) - b^2\theta^2(4-7b^2+2b^2\theta) - 4(1-b^2)(1-\theta)(1-b^2(1-\theta)) \\
&= (1-b^2)(2-b^2) + 2b^2(1-b^2)\theta + b^4(3-2\theta)\theta^2 > 0.
\end{aligned}$$

Inequality in the fourth line follows from $\sqrt{4-3b^2+2b^2\theta} \in (1,2)$. This implies $\Delta_c > 0$.

Next, we show $\Delta_c < \Delta_e$. Comparing Δ_c and Δ_e, we obtain

$$\Delta_e - \Delta_c = \frac{4(1-b^2)(1-b^2(1-\theta))(1-\theta)(a(1-b)+bs_0-c_1)\sqrt{4-3b^2+2b^2\theta}}{(4-3b^4)(1-b^2)+8b^4(1-b^2)\theta(1-\theta)+b^4\theta^2(4-b^2-2b^2\theta)} > 0. \tag{26}$$

This implies $\Delta_c < \Delta_e$.

We then show $\Delta_d > 0$. $\Delta_d > 0$ if the numerator of Δ_d is positive. Let $B := 5b^4 - 12b^2 + 8 + 8b^2\theta(1-b^2) + 4b^4\theta^2 - X_5$. We obtain

$$\begin{aligned}
B &= 5b^4 - 12b^2 + 8 + 8b^2\theta(1-b^2) + 4b^4\theta^2 - X_5 \\
&= 5b^4 - 12b^2 + 8 + 8b^2\theta(1-b^2) + 4b^4\theta^2 - 2(1-b^2)(4-3b^2+2b^2\theta)\sqrt{1-b^2(1-\theta)} \\
&\geq 5b^4 - 12b^2 + 8 + 8b^2\theta(1-b^2) + 4b^4\theta^2 - 2(1-b^2)(4-3b^2+2b^2\theta) \\
&= b^2(2-b^2) + 4b^2\theta(1-b^2(1-\theta)) > 0.
\end{aligned}$$

Inequality in the third line follows from $\sqrt{1-b^2(1-\theta)} \in (0,1)$. This implies $\Delta_d > 0$. ■

Proof of Proposition 3(iii)

First, we examine the welfare ranking. From (9) and (13), we obtain

$$V_0^L - V_0^F = \frac{b^2 X_8}{8(1-b^2)(1-b^2(1-\theta))^2(4-3b^2+2b^2\theta)}. \tag{27}$$

Equation (27) is positive (negative, zero) if $X_8 > (<,=)0$. Solving the equation $X_8 = 0$ with respect to Δ_1, we obtain

$$X_8 = 0 \to \Delta_1 = \frac{(a(1-b)+bs_0-c_1)[((1-b^2)(6-5b^2-2\theta(2-5b^2)-b^2\theta^2(4-7b^2+2b^2\theta)) \pm X_4]}{(1-b^2)(4-3b^2+b^4\theta(4(2-\theta)+7b^2\theta+2b^2\theta^2))}.$$

Note that Δ_c and Δ_e are solutions of this equation.

Then, we obtain

$$\begin{aligned}
X_8 &< 0 \text{ (and thus } V_0^L < V_0^F), \text{ if } \Delta_1 \in (\Delta_c, \Delta_e),\\
X_8 &= 0 \text{ (and thus } V_0^L = V_0^F), \text{ if } \Delta_1 = \Delta_c \text{ or } \Delta_e,\\
X_8 &> 0 \text{ (and thus } V_0^L > V_0^F), \text{ otherwise.}
\end{aligned}$$

This implies Proposition 3(iii). ■

Proof of Proposition 3(iv)

We examine the profit ranking. If $\Delta_1 < \tilde{\Delta}$, then $p_0^B > p_0^L$. Because $p_1^B = R_1(p_0^B)$, $p_1^F = R_1(p_0^L)$, and $\pi_1(p_0, R_1(p_0))$ is increasing in p_0, we obtain $V_1^F < V_1^B$. Because $V_1^L \geq V_1^B$, we obtain $V_1^F < V_1^L$.

If $\Delta_1 > \bar{\Delta}$, then $p_0^F < p_0^L$. Because $p_1^F = R_1(p_0^L)$, $\pi_1(p_0, R_1(p_0))$ is increasing in p_0, and $\pi_1(p_0, R_1(p_0)) \geq \pi_1(p_0, p_1)$ for any p_1, we obtain $V_1^F > V_1^L$.

We now investigate profit ranking when $\Delta_1 \in [\tilde{\Delta}, \bar{\Delta}]$. From (10) and (14), we obtain

$$V_1^L - V_1^F = \frac{b^2 X_9}{4(1-b^2)(1-b^2(1-\theta))^2(4-3b^2+2b^2\theta)^2}. \tag{28}$$

Equation (28) is positive (negative, zero) if $X_9 > (<,=)0$. We obtain $X_9 = 0$ if $\Delta_1 = \Delta_d \in (\tilde{\Delta}, \bar{\Delta})$. If $\Delta_1 < \Delta_d$, then $X_9 > 0$ holds, and thus, we obtain $V_1^L > V_1^F$. If $\Delta_1 \in [\Delta_d, \bar{\Delta}]$, then $X_9 < 0$ holds, and thus, we obtain $V_1^L < V_1^F$. ■

Proof of Proposition 4

We show $\Delta_c = \Delta_d = \Delta_e$ if $\theta = 1$. Substituting $\theta = 1$ into $\Delta_c(\theta)$, $\Delta_d(\theta)$ and $\Delta_e(\theta)$, we obtain

$$\Delta_c(1) = \Delta_d(1) = \Delta_e(1) = \frac{a(1-b)+bs_0-c_1}{2-b^2} = \tilde{\Delta}(1).$$

This implies Proposition 4(i).

Next, we show that $V_0^L \geq V_0^F$, and the equality holds if and only if $\Delta_1 = \Delta_d$. From (20), we obtain that $V_0^L \geq V_0^B$, and the equality holds if and only if $\Delta_1 = \tilde{\Delta}$. From (21), we obtain that $V_0^B = V_0^F$ when $\theta = 1$. $\Delta_d = \tilde{\Delta}$ when $\theta = 1$. These imply Proposition 4(ii).

In the proof of Proposition 3(iv), we do not use the condition $\theta < 1$. Therefore, Proposition 3(iv) holds when $\theta = 1$. This implies Proposition 4(iii). ■

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