Household Heterogeneity and the Value of Government Spending Multiplier: an Analytical Characterization

Kopiec, Pawel

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Household Heterogeneity and the Value of Government Spending Multiplier: an Analytical Characterization*

Paweł Kopiec†

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Abstract

This paper provides an analytical formula for the government spending multiplier in economy populated with heterogeneous households that face uninsured idiosyncratic risk. To derive this expression I use the canonical Bewley-Huggett-Aiyagari model extended to capture frictional product market. In my analysis I relax several assumptions that were made in the literature to obtain closed-form solutions in heterogeneous agent models such as: partial equilibrium (e.g., Auclert (2017)), extreme illiquidity (e.g., Krusell et al. (2011)) and constant real interest rates (e.g., Auclert et al. (2018)). The resulting formula for the multiplier is decomposed into interpretable, model-based channels. Calibrated model is used to estimate their magnitude under alternative fiscal and monetary policy rules. Quantitative exploration indicates that household heterogeneity plays a crucial role in the propagation of government expenditures shocks.

JEL Classification: D30, E62, H23, H30, H31

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†Narodowy Bank Polski, Świętokrzyska 11/21, 00-919 Warsaw, Poland. E-mail: pawel.kopiec@nbp.pl.

1 Introduction

The Financial Crisis of 2007-2008 gave rise to a renewed interest in the usefulness of fiscal purchases for spurring a recovery. Attention to that issue has been significantly increased by the fact that in many countries the alternative stabilization tool (i.e., monetary policy) was constrained by the zero lower bound on nominal interest rates. A natural question that emerged concerned the effectiveness of fiscal expenditures in comparison to other government policy measures that could be used to stimulate economic activity.

As pointed by Woodford (2011), much public discussion of this issue had been based on old-fashioned models that ignored the role of intertemporal optimization and expectations. To address this shortcoming, Woodford (2011) used the standard New Keynesian model to explain the key determinants of the government spending multiplier by providing a series of examples that can be solved analytically. His analysis, however, was conducted under a simplifying assumption that households populating the model are identical. This condition seems to be overly restrictive because, as the standard textbook Keynesian-cross logic suggests, multiplier's value depends on the feedback loop between income and aggregate consumption. Properties of the latter, in turn, are heavily affected by household heterogeneity (as documented by Carroll et al. (2014), Jappelli and Pistaferri (2014), Kaplan and Violante (2014), Krueger et al. (2016) among others). Motivated by this observation, my paper seeks to relax the assumption about representative household when analyzing the macroeconomic impact of a rise in government purchases and, by providing an analytical formula for the multiplier, to obtain insights about determinants of fiscal policy effectiveness.

More specifically, I calculate a closed-form expression for the fiscal expenditure multiplier in environment where agents are heterogeneous with respect to income and wealth. To derive this formula, I use the Bewley-Huggett-Aiyagari model extended to capture a decentralized product market featuring search frictions. This departure from the standard framework allows to relax several assumptions that were used in the literature to obtain analytical expressions in the Bewley-Huggett-Aiyagari models, such as: i) extreme illiquidity that eliminates wealth heterogeneity (e.g. Krusell et al. (2011), Werning (2015), Ravn and Sterk (2016), McKay and Reis (2016a)), ii) constant real interest rates or constant prices of goods (e.g., Auclert et al. (2018), Patterson (2018)), iii) partial equilibrium analysis (e.g. Auclert (2017)). In other words, the non-Walrasian formulation of the product market allows to derive an explicit formula for the multiplier in economy that features household heterogeneity with respect to income and wealth, where monetary authority conducts policy using a realistic, Taylor-type rule that affects real interest rates and all general equilibrium effects are taken into account. To my knowledge, this is the first analytical result of this type in the literature.

Frictional product market allows to abstract from assumptions i), ii) and iii) because it enables to summarize all endogenous general equilibrium effects that are taken as given by households in terms of only one variable: product market tightness. This feature turns out to be crucial when computing the reaction of aggregate consumption to change in government spending. Except for that, the assumption about non-Walrasian product market allows to incorporate price-setting mechanisms featuring various levels of price rigidity in a tractable way.

The derived formula for the multiplier depends on distributions of: marginal propensities to consume (henceforth MPC), consumption, income and wealth across households, parameters of the monetary policy rule, parameter describing the comovement of output and prices resulting from a demand shock, aggregate public debt, cross-sectional tax burdens, parameter of the fiscal rule that pins down the way in which additional government purchases are financed and a forward-looking component that captures changes in consumer expectations resulting from the government expenditures shock.

I calibrate the model to match aggregate moments characterizing Italian economy, and most crucially, average MPC of Italian households documented in the SHIW survey. To this end, I follow Jappelli and Pistaferri (2014) and I construct two alternative specifications of the model that enable to mimic a relatively high level of mean MPC observed in the data. First of them assumes that a proportion of households behave like rule-of-thumb consumers. Second version of the model is populated by two groups of consumers with different levels of discount factor (the so-called patient and impatient households). I argue that the latter variant of the model fails to mimic the distributional features of MPC and thus may lead to some erroneous conclusions concerning the impact of fiscal policy. First version, however, circumvents this problem and it is used to evaluate the components of the derived multiplier's formula under several scenarios associated with different variants of fiscal and monetary policy.

This paper is related to several strands of the literature. First, it is associated with works studying the effects of fiscal policy shocks in models with heterogeneous households, in which a significant proportion of agents deviates from the consumption-savings behavior predicted by the permanent income hypothesis and thus exhibits relatively high levels of MPC. There are two main groups of papers within that field: first of them focuses on the role of taxes and transfers (e.g., Oh and Reis (2012), McKay and Reis (2016b), Den Haan et al. (2015)), and the second concentrates on the role of fiscal purchases (e.g., Challe and Ragot (2011), Brinca et al. (2017), Navarro and Ferriere (2016), Hagedorn et al. (2017) and Auclert et al. (2018)). It seems that the closest work to mine is the last one, in which Auclert et al. (2018) characterize analytically the fiscal multiplier using the so-called intertemporal MPCs. Auclert et al. (2018) conduct their analysis under the assumption that monetary policy keeps real interest rates at a constant level. In my paper, I relax this restriction by considering a more general, Taylor-type monetary policy rule.

As already mentioned, the key ingredient in my analysis is frictional product market. The presence of a non-Walrasian market for goods in the model can be motivated by the fact that in reality this market functions in a decentralized manner and features frictions (see Michaillat and Saez (2015)). This formulation goes back to a seminal paper by Diamond (1982) who proposed a model with search frictions in the market for goods that is subject to the so-called thick market externality. More recently, frictional product markets were used, among others, in works by Michaillat and Saez (2015), Petrosky-Nadeau and Wasmer (2015), Kaplan and Menzio (2016), Storesletten et al. (2017) and Michaillat and Saez (2018). My paper is tightly related to the last two. First, as Storesletten et al. (2017), I formalize search costs in terms of disutilty from search effort. Second, similarly to Michaillat and Saez (2018), I study the effects of higher fiscal purchases. The most important difference with respect to Michaillat and Saez (2018) is that I consider a model with heterogeneous agents.

The remaining sections of the paper are organized as follows. Section 2 presents the Bewley-Huggett-Aiyagari model with frictional product market. In Section 3 I derive the analytical formula for the multiplier. Section 4 applies the formula the calibrated model to estimate the magnitude of channels under two alternative scenarios. Section 5 concludes.

2 Model

2.1 Environment

Time is infinite and divided into discrete periods and, to keep the notation simple, for now I concentrate on the stationary equilibrium of the model. The assumption about stationarity is relaxed in Section 2.7. There are two types of agents in the economy: heterogeneous, self-employed households and government that is composed of two branches: central bank and fiscal authority. There are two markets: first of them is a Walrasian market where households trade liquid assets $\tilde{b}$ and the second is the market for consumption goods traded at price p, which is decentralized and features search frictions analogous to those from the standard Diamond-Mortensen-Pissarides model of labor market. By b I denote the value of nominal assets $\tilde{b}$ divided by the price of consumption goods in the previous period.

2.2 Households

The model is populated by a continuum of infinitely lived households of measure one. Households are both consumers and producers at the same time and they manufacture consumption goods using the following technology:

$$z \cdot f$$

where z is an idiosyncratic shock and by f I denote the probability that a unit of good supplied by a household is sold. To put it differently, f can be thought of as capacity utilization. Randomness at the individual level is excluded so f is equal across households. Idiosyncratic shock z follows a Markovian process defined on space Z and it can be interpreted as household's production capacity. By μ I denote the distribution of agents over liquid asset holdings b and shock z.

Similarly to Storesletten et al. (2017), agent preferences are given by the instantaneous utility function $\tilde{u}(c, v)$ where $\tilde{u}_c > 0$, $\tilde{u}_v < 0$, $\tilde{u}_{cc} < 0$, c is consumption and v is search effort exerted by household. Alternatively, v can be seen as a number of visits made by household to purchase goods from other consumers. Values of c and v are related by the following constraint imposed by product market frictions:

$$c = q \cdot v \tag{2.1}$$

where q is the probability with which a visit that is made by household is successful, i.e., it ends with a purchase of a unit of consumption good. Both f and q are endogenous and are determined in equilibrium. Similarly to Diamond (1982), I assume that households cannot consume their own output and they have to search for goods manufactured by others.

Household pays income tax $T(z)$ that depends on its productivity level z. Liquid assets earn nominal interest rate i that is set by monetary authority. Household's choice of next period nominal balances b' is subject to the borrowing constraint:

$$b' \geq -\xi$$

where ξ is a positive constant. To complete the description of household's environment, let us denote by Π the ratio between current price p and its level in the previous period.

The maximization problem of household with real value of liquid balances b and productivity level z can be represented by the following Bellman equation:

$$V(b,z) = \max_{c,v,b'} \left\{ \tilde{u}(c,v) + \beta \mathbb{E}_{z'|z} V\left(b',z'\right) \right\} \tag{2.2}$$

subject to:

$$c + T(z) + \frac{b'}{1+i} = \frac{b}{\Pi} + z \cdot f$$

$$c = q \cdot v$$

$$b' \geq -\xi$$

where V is value function associated with the dynamic problem of agent. Household discounts future utility streams with factor $\beta \in (0,1)$. Consumer optimizes with respect to: budget constraint, product market frictions constraint and liquidity constraint. The resulting policy functions are $c(b,z)$, $v(b,z)$ and $b'(b,z)$.

Finally, let us define:

$$B = [-\xi, +\infty)$$

which is a space to which asset holdings b and asset choices b' belong.

2.3 Government

Government consists of fiscal authority and central bank. Fiscal authority purchases consumption goods G and, since (similarly to households) it operates under product market frictions, its expenditures are subject to the following constraint:

$$G = q \cdot v_G \tag{2.3}$$

where v_G is the number of visits made by government. Government collects taxes and issues bonds $\bar{B}'$ to finance purchases G and repayment of debt $\bar{B}$ issued in the previous period. Consequently, government budget constraint reads:

$$\int_{B\times Z} T\left(z\right) d\mu\left(b,z\right) + \frac{\bar{B}'}{1+i} = \frac{\bar{B}}{\Pi} + G. \tag{2.4}$$

Monetary authority follows a standard Taylor-type rule that depends on deviations: of aggregate output Y and price index Π from their levels in stationary equilibrium (denoted by $\bar{Y}$ and $\bar{\Pi}$, respectively):[1]

$$i = \bar{i} + \phi_Y \cdot \left(\frac{Y - \bar{Y}}{\bar{Y}}\right) + \phi_\Pi \cdot (\Pi - \bar{\Pi})$$

where $\bar{i}$, ϕ_Y and ϕ_Π are positive parameters.

2.4 Matching technology and price-setting

It is assumed that the number of successful matches in the product market is governed by a constant returns to scale matching function M that increases in both arguments and depends on the aggregate number of visits made by households

[1] There is a slight abuse of notation here because I denote the stationary equilibrium values of Y and Π with bars in this part of the paper and I suppress this notational element later.

and government and on the aggregate output capacity:

$$M\left(\int_{B\times Z} v\left(b,z\right)d\mu\left(b,z\right)+v_G, \int_{B\times Z} zd\mu\left(b,z\right)\right).$$

Product market tightness x is given by the ratio between aggregate visits and total production capacity:

$$x\equiv\frac{\int_{B\times Z} v\left(b,z\right)d\mu\left(b,z\right)+v_G}{\int_{B\times Z} zd\mu\left(b,z\right)}. \tag{2.5}$$

Since there is no universal theory that would pin down prices in a decentralized market that features search frictions I will assume that price index Π is a strictly increasing function of x:

$$\Pi=\Pi(x),\ \ \Pi'\left(x\right)>0. \tag{2.6}$$

The assumed relationship relies on the following intuition: price level increases when the ratio between aggregate demand (captured by the aggregate number of visits) and aggregate production capacity (captured by $\int zd\mu$) rises. In other words, Π tends to react positively to demand shocks and negatively to supply shocks. This simple formulation of the price-setting mechanism allows to consider various degrees of price stickiness, which is described by the value of Π'.

2.5 Consistency conditions and market clearing

Probabilities f and q are induced by matching technology M and due to the assumed constant returns to scale they can be expressed as functions of only one argument - x:

$$f\left(x\right)=\frac{M\left(\int_{B\times Z} v\left(b,z\right)d\mu\left(b,z\right)+v_G, \int_{B\times Z} zd\mu\left(b,z\right)\right)}{\int_{B\times Z} zd\mu\left(b,z\right)}=M\left(x,1\right) \tag{2.7}$$

$$q\left(x\right)=\frac{M\left(\int_{B\times Z} v\left(b,z\right)d\mu\left(b,z\right)+v_G, \int_{B\times Z} zd\mu\left(b,z\right)\right)}{\int_{B\times Z} v\left(b,z\right)d\mu\left(b,z\right)+v_G}=M\left(1,\frac{1}{x}\right) \tag{2.8}$$

The market clearing condition for nominal assets reads:

$$\bar{B}' = \int_{B\times Z} b'(b,z)\, d\mu(b,z) \tag{2.9}$$

and the resource constraint for consumption goods is:

$$\int_{B\times Z} c(b,z)\, d\mu(b,z) + G = f(x)\cdot \int_{B\times Z} z d\mu(b,z) \tag{2.10}$$

where the right hand side is defined as aggregate output Y:

$$Y(x) \equiv f(x)\cdot \int_{B\times Z} z d\mu(b,z)\,. \tag{2.11}$$

Observe that aggregate product is demand-driven as aggregate capacity $\int z d\mu$ is fixed and therefore Y depends solely on the probability with which consumers arrive to other households and purchase goods. This is a significant departure from the neoclassical paradigm under which output depends solely on production factors like capital and labor. This assumption, made by Michaillat and Saez (2015) and Storesletten et al. (2017) among others, allows me to isolate the interplay between inequality, aggregate demand and fiscal policy effectiveness from the behavior of the supply-side.

Evolution of the distribution of agents across asset holdings b and shocks z is described by the following equation:

$$\mu'\left(\mathcal{B}', z'\right) = \int_{\{b: b'(b,z)\in\mathcal{B}'\}\times Z} \mathbb{P}(z'|z) d\mu(b,z) \tag{2.12}$$

where $\mathcal{B}'$ is a Borel subset of $[-\xi, +\infty)$ and $\mathbb{P}(z'|z)$ is transition probability between states z and z'. Equation 2.12 defines the law of motion for the distribution of agents and the associated operator Γ:

$$\mu' = \Gamma(\mu)\,. \tag{2.13}$$

Finally, I assume the following standardization:

$$\int_{B\times Z} z d\mu(b,z) = 1 \tag{2.14}$$

i.e., the average productivity z across agents is equal to one.

2.6 Stationary equilibrium

We are in position to define the stationary equilibrium of the model:

Definition. *A stationary equilibrium is: positive numbers x, q, f, i, value function V, policy functions c, v, b', distribution μ such that given $\bar{B}$, G, v_G, Π and T:*

1. Given f, q, i, Π and T function V solves household's maximization problem 2.2 and c, v and b' are associated policy functions.

2. Given $\bar{B}$, G, Π, v_G, q and i equation 2.3 and government budget constraint 2.4 hold.

3. Consistency conditions 2.5, 2.7, 2.8, price-setting relationship 2.6 and resource constraints 2.9, 2.10 are satisfied.

4. Measure μ is a fixed point of operator Γ defined by 2.12 and 2.13.

2.7 Household maximization problem expressed in terms of market tightness x and government purchases G

In what follows I argue that all aggregate objects that are taken as given by households while solving problem 2.2 in period when aggregate shock to fiscal purchases arrives can be expressed as functions of product market tightness x or government purchases G (or both). This feature is crucial when deriving the closed-form expression for the government spending multiplier as it will allow to summarize all general equilibrium effects that affect consumer decisions with only one endogenous variable: x.

Let us start with the consistency condition 2.7 which implies that rate f is simply a function of x. Notice that since equation 2.14 holds we can express output as a function of x because by 2.11 it simply equals f:

$$Y(x) = f(x). \tag{2.15}$$

Combining this with the assumption about price formation (equation 2.6) allows to express central bank policy rate i as:

$$i(x) = \bar{i} + \phi_Y \cdot \left(\frac{Y(x) - \bar{Y}}{\bar{Y}}\right) + \phi_\Pi \cdot \left(\Pi(x) - \bar{\Pi}\right).$$

Observe that since $\Pi'(x) > 0$ (see condition 2.6) and $f'(x) > 0$ (by 2.7 and because M increases in its arguments), function $i(x)$ is strictly increasing:

$$i'(x) \geq 0. \tag{2.16}$$

Tax $T(z)$ paid by household with productivity z can be decomposed into two parts: aggregate tax revenue of government Θ and individual's share $\tau(z)$ in the aggregate tax burden:

$$T(z) \equiv \tau(z) \cdot \Theta \tag{2.17}$$

where $\tau(z) \geq 0$ for all $z \in Z$ are time-invariant and they satisfy:[2]

$$\int_{B\times Z} \tau(z)\, d\mu(b,z) = 1.$$

This obviously means that government's tax income reads:

$$\int_{B\times Z} T(z)\, d\mu(b,z) = \Theta.$$

I use constraint that relates consumption c and visits v to eliminate the latter from the maximization problem 2.2:

$$v = \frac{c}{q(x)}$$

where the relationship between q and x follows from condition 2.8.

Finally, since our main focus is the effect of an aggregate shock to government purchases we have to depart from the assumption about stationary allocation. This, in turn, implies that variables and value functions in the maximization problem become time-dependent. More specifically, it will be assumed that at the beginning of period t economy is in stationary equilibrium and right after-

[2]Navarro and Ferriere (2016) consider the case in which fiscal stimulus is accompanied by a rise in the progressivity of income tax (i.e., $\tau < 0$ for agents with lowest earnings). By imposing the restriction that $\tau \geq 0$, I exclude the possibility of such additional implicit transfers to isolate the sole impact of government purchases.

wards (but still in period t) there is an unexpected rise in fiscal expenditures that jump from the stationary equilibrium level G to G_t. Period t is also referred to as "today".

In what follows, I consider fiscal interventions that can be described by the following rule Λ:

$$\Lambda: \ G_t \rightarrow \left[\left\{ G_s \left(G_t \right) \right\}_{s \geq t}, \left\{ \bar{B}_{s+1} \left(G_t \right) \right\}_{s \geq t} \right] \tag{2.18}$$

which captures both stimulus persistence (path $\left\{ G_s \left(G_t \right) \right\}_{s \geq t}$) and it specifies the way in which stimulus is financed (path $\left\{ \bar{B}_{s+1} \left(G_t \right) \right\}_{s \geq t}$).[3] Moreover, it is assumed that mapping Λ is smooth.[4]

Let us discuss formula 2.18 in a more detailed way. The presence of the sequence of functions $\left\{ G_s \left(G_t \right) \right\}_{s \geq t}$ in the specification of fiscal rule Λ can be interpreted in the following way: future levels of fiscal spending are announced in period t and are pinned down by the size of G_t. In fact, this assumption is very common in the literature: to see that, notice that it nests both a one-time shock, a permanent shock and an autoregressive shock that returns back to stationary equilibrium level G as $s \rightarrow +\infty$ as special cases.

Additionally, formula 2.18 says that together with path of fiscal purchases $\left\{ G_s \left(G_t \right) \right\}_{s > t}$ government announces the path of real public debt $\left\{ \bar{B}_{s+1} \left(G_t \right) \right\}_{s \geq t}$. From now on, I assume that the sequence of functions $\left\{ \bar{B}_{s+1} \left(G_t \right) \right\}_{s \geq t}$ satisfies two conditions. First, it is required that:

$$\forall_{s \geq t} \ \ \frac{d\bar{B}_{s+1}}{dG_t} \geq 0, \tag{2.19}$$

i.e., the rise in fiscal purchases cannot be accompanied with a reduction in public debt on the transition path. This condition imposes a consistency on the fiscal rule Λ as it excludes the coexistence of expansionary government spending with public debt austerity.

Second, it is assumed that $\left\{ \bar{B}_{s+1} \left(G_t \right) \right\}_{s \geq t}$ is such that the implied path of ag-

[3] As we shall see later, the path of taxes will be induced by paths $\left\{ G_s \left(G_t \right) \right\}_{s > t}$ and $\left\{ \bar{B}_{s+1} \left(G_t \right) \right\}_{s \geq t}$ so it is not necessary to report it when discussing the way in which government purchases are financed.

[4] This condition is imposed to guarantee that the forward-looking component in the multiplier's formula is well-defined.

gregate tax burdens (which are specified later with formulas 2.21 and 2.22) satisfies:

$$\forall_{s\geq t}\,\frac{d\Theta_s}{dG_t}\geq 0, \tag{2.20}$$

i.e., the increase of debt during fiscal expansion has an upper limit that prevents from reductions in taxes during expansion. This is required to eliminate the possibility that a rise in G is accompanied with a drop in tax burden (which is equivalent to rise in aggregate transfers). It is imposed because the main object of interest in this paper is fiscal spending multiplier and therefore I want to isolate the impact of government purchases from other types of stimulative fiscal policies (like transfers). In particular, as it is shown in the proof of Theorem, for $s=t$ (the period when expansion begins) bounds 2.19 and 2.20 imply:

$$\frac{d\bar{B}_{t+1}}{dG_t}\in\left[0,1+\bar{i}\right].$$

Finally, given Λ, one can define:

$$\lambda\equiv\frac{d\bar{B}_{t+1}}{dG_t}$$

which will be useful later. The size of λ can be interpreted as a proportion of a rise in government purchases that is financed with the issuance of additional debt in period t when government applies rule Λ.

From what has been said above, given Λ, the budget income from taxes in periods $s>t$ can be expressed as a function of product market tightness x_s and government purchases in period t:

$$\Theta\left(x_s,G_t\right)=\frac{1}{\Pi\left(x_s\right)}\cdot\bar{B}_s\left(G_t\right)-\frac{1}{1+i\left(x_s\right)}\cdot\bar{B}_{s+1}\left(G_t\right)+G_s\left(G_t\right). \tag{2.21}$$

An analogous object in period t is given by:

$$\Theta\left(x_t,G_t\right)=\frac{1}{\Pi\left(x_t\right)}\cdot\bar{B}-\frac{1}{1+i\left(x_t\right)}\cdot\bar{B}_{t+1}\left(G_t\right)+G_t. \tag{2.22}$$

which implies that Λ satisfies government budget constraints for $s\geq t$.

All this means that, given fiscal rule Λ, household's maximization problem in period t can be described by the following Bellman equation:

$$V_t^{\Lambda}\left(b_t, z_t | G_t\right) = \max_{c_t, b_{t+1}} \left\{ u\left(c_t, x_t\right) + \beta \mathbb{E}_{z_{t+1}|z_t} V_{t+1}^{\Lambda}\left(b_{t+1}, z_{t+1} | G_t\right) \right\} \tag{2.23}$$

subject to:

$$c_t + \tau\left(z_t\right) \cdot \Theta\left(x_t, G_t\right) + \frac{b_{t+1}}{1 + i\left(x_t\right)} = \frac{b_t}{\Pi\left(x_t\right)} + z_t \cdot f\left(x_t\right)$$

$$b_{t+1} \geq -\xi$$

where:

$$u(c_t, x_t) \equiv \tilde{u}\left(c_t, \frac{c_t}{q\left(x_t\right)}\right).$$

and where $V_{t+1}^{\Lambda}\left(b_{t+1}, z_{t+1} | G_t\right)$ is value function at the beginning of period $t+1$ that is associated with a perfect foresight equilibrium that follows after fiscal shock of size G_t and where government follows fiscal rule Λ specified in equation 2.18.[5] It is important to highlight the fact that, according to 2.23, solution to consumer problem (i.e., policy functions) will depend on Λ. This occurs because, in the model with uninsured idiosyncratic risk, the Ricardian equivalence ceases to hold and hence the way in which government finances fiscal deficits, resulting from higher purchases in period t, becomes relevant for consumer's consumption-saving behavior.

Formula 2.23 shows that, given Λ, all changes in aggregate objects in period t (i.e.: i_t, Θ_t, Π_t, f_t, q_t and V_{t+1}^{Λ}) that: i) are taken as given by households when solving the maximization problem in period t and ii) are affected by the fiscal shock can be expressed as functions of two variables: x_t and G_t.[6] This implies that the only aggregate determinants of a change in a time-dependent consumption policy in period t, that is driven by the fiscal shock, are x_t and G_t, which is reflected by the following notation: $\left\{c_t^{\Lambda}\left(b, z | x_t, G_t\right)\right\}_{\{b,z\} \in B \times Z}$. This, in turn, means

[5]Note that given the assumption about rational expectations consumers are able to calculate the sequence of future value functions $\{V_s\}_{s>t}$ and recognize their dependence on G_t and Λ. This implies that they know the form of V_{t+1}^{Λ} in period $t+1$ and take it as given when solving the maximization problem in period t.

[6]Observe that the remaining aggregate variable that is taken as given by households and which affects the maximization problem in period t - the distribution of households μ_t - is a state variable that is fixed and equal to its stationary equilibrium value μ and therefore is not affected by a rise in G.

that the impact of the stimulus on aggregate consumption in period t, which is defined as:

$$C_t^{\Lambda}(x_t, G_t) \equiv \int_{B\times Z} c_t^{\Lambda}(b, z|x_t, G_t)\, d\mu_t(b, z) \tag{2.24}$$

can be summarized solely with x_t and G_t and therefore the economy-wide resource constraint in period t can be rewritten as:

$$C^{\Lambda}(x_t, G_t) + G_t = Y(x_t). \tag{2.25}$$

In what follows, I impose the following condition on utility function u:

$$u_{cx} = 0 \tag{2.26}$$

which means that marginal utility from consumption is not affected by the value of product market tightness. The motivation for condition 2.26 and its role in the analysis are discussed in Section 2.8 in greater detail.

Finally, the first order condition associated with problem 2.23 is:

$$u_c(c_t, x_t) \geq (1 + i(x_t)) \cdot \beta \tag{2.27}$$

$$\times \mathbb{E}_{z_{t+1}|z_t} V_{t+1,b}^{\Lambda}\left((1 + i(x_t)) \cdot \left(\frac{b_t}{\Pi(x_t)} + z_t \cdot f(x_t) - c_t - \tau(b_t, z_t) \cdot \Theta(x_t, G_t)\right), z_{t+1}|G_t\right)$$

which is satisfied with equality when $b_{t+1} > -\xi$.

Before proceeding, let us discuss the assumptions that have been made in the analysis so far.

2.8 Discussion about the assumptions

The most significant departure from the canonical Bewley-Huggett-Aiyagari framework in my analysis is the specification of product market that features search frictions. There are several important reasons for which this modification is introduced.

First, there is a long tradition of modeling the productive role of aggregate demand, which is crucial when analyzing the effects of fiscal stimulus, by incorporating search and matching frictions in the product market, which goes back to a seminal contribution by Diamond (1982). By productive role I mean that changes in aggregate demand lead to shifts in capacity utilization and are not entirely absorbed by changes in prices.

Second, it seems that this assumption is not very controversial as trade in product market hardly exhibits Walrasian features: it is rather organized in a decentralized way where frictions play an important role (see Michaillat and Saez (2015)).

Third, I assume this specification of product market because search and matching protocol allows to incorporate various price-setting mechanisms, with perfectly rigid prices and flexible prices as two polar cases, in a tractable manner. Since, as we shall see, in the analyzed framework movements in prices resulting from fiscal expansion will have redistributive effects affecting aggregate demand, it is useful to have a simple way to modify the strength of that channel.

Most importantly, the formulation of product market that is used here allows to represent all general equilibrium effects affecting both the supply side of the product market and the demand side with only one variable - product market tightness x. This property enables to compute the analytic formula for the fiscal multiplier.

One technical remark is in order here. In contrast to Michaillat and Saez (2015) and Michaillat and Saez (2018), who model search costs in terms of goods spent by households while making consumer visits, I assume that these costs are captured by the disutility from search effort (see, e.g., Storesletten et al. (2017)). I follow this convention because it preserves a standard form of the aggregate resource constraint known from the literature (i.e., without goods spent on search activities on the demand side as in Michaillat and Saez (2015) and Michaillat and Saez (2018)) and thus enables to compare my results with a broader set of theoretical outcomes derived in other works.

Finally, let us discuss the role of condition 2.26. Arguably, it is quite intuitive and plausible: it implies that although market conditions (captured with x) influence the amount of effort needed to buy certain amount of goods (see equation 2.1), they do not affect his or her marginal consumption. This can be rationalized by the fact that shopping activities and consumption of products often occur sequentially: during the first stage household collects a basket of consumption goods which requires search effort that is related to the amount of purchases. Subsequently, during the second stage, it consumes goods and neither effort exerted nor market conditions during the first stage influence the utility derived from additional unit. In Section 3 I present functional form of u that satisfies condition 2.26 and, additionally, it excludes wealth effects of search effort, as postulated by Storesletten et al. (2017). In the Appendix I relax condition 2.26 and I derive the multiplier's formula in the situation in which $u_{cx} \neq 0$.[7]

3 Government multiplier: analytical exploration

In this section I study the main problem addressed in this paper, i.e. how household heterogeneity affects the value of government spending multiplier. Recall that I consider an unexpected shock to government expenditures in period t and until its arrival, economy is in stationary equilibrium. The shock is followed by path $\{G_s\}_{s>t}$ which is pinned down by G_t and agents have perfect foresight about aggregate variables in periods $s > t$.

3.1 Preliminary step

Let us start with a preliminary step in which I derive a general formula for the multiplier in the analyzed economy:

Lemma 1. *Suppose that economy is in stationary equilibrium at the beginning of period t and government follows fiscal rule Λ. Then the value of government spending multiplier in period t is:*

$$\frac{dY_t}{dG_t} = \frac{1 + \frac{\partial C_t^{\Lambda}}{\partial G_t}}{1 - \frac{\partial C_t^{\Lambda}}{\partial x_t} \cdot \frac{1}{f'(x_t)}}. \tag{3.1}$$

All proofs are postponed to the Appendix. Although very general, formula 3.1 provides us with some important insights about the determinants of multiplier's magnitude. First, notice that its value is affected by both the reaction of private

[7] Relaxing 2.26 gives rise to a mechanism through which private consumption is either crowded in (when $u_{cx} > 0$) or crowded out (when $u_{cx} < 0$) by government purchases. Nevertheless, this "mechanical" crowding out effect is absent in the vast majority of the literature related to fiscal purchases and hence staying in line with it (and guaranteeing comparability of my analysis with other works) is another argument for excluding the case in which $u_{cx} \neq 0$.

aggregate demand (represented by partial derivatives of C) and the change in capacity utilization $f'(x)$ that drives the response of aggregate output. Second, its magnitude depends on both direct effects of government expenditures on private consumption (i.e., $\frac{\partial C^{\Lambda}}{\partial G}$) and indirect effects associated with general equilibrium forces summarized with a reaction of private demand to change in product market tightness x (i.e., $\frac{\partial C^{\Lambda}}{\partial x}$). Finally, it is important to note that multiplier's value depends implicitly on the underlying fiscal rule Λ that specifies the way in which additional government purchases are financed.

3.2 Analytical formula for the government spending multiplier

3.2.1 Government spending multiplier in economy with heterogeneous households

Before presenting the main result of the paper, I will introduce some additional notation which enables to write down the formula for the multiplier in a heterogeneous agent economy in a concise way.

First, to economize on notation, for now we will suppress the dependence of variables on time and we will use the following aggregation operator $\mathbb{E}_{\mu}$ to denote the expected value of variable m in the population:

$$\mathbb{E}_{\mu} m \equiv \int_{B \times Z} m(b,z)\, d\mu(b,z).$$

where $m(b,z)$ is a variable associated with household that has b of liquid assets and operates with capacity z. Second, we define individual marginal propensity to consume (MPC) and marginal propensity to save (MPS) of a household as:

$$MPC \equiv \frac{dc}{dy},\;\; MPS \equiv \frac{1}{1+i} \cdot \frac{db'}{dy},\;\; \text{where } y \equiv z \cdot f(x) - \tau(z) \cdot \Theta. \tag{3.2}$$

i.e., y is household's disposable income. Similarly to Auclert (2017), let us define the unhedged interest rate exposure as:

$$URE \equiv \frac{b}{\Pi} + z \cdot f - \tau \cdot \Theta - c \tag{3.3}$$

which can be thought of as a difference between maturing assets and maturing liabilities. Moreover, without loss of generality, I standardize:

$$\Pi = 1$$

in stationary equilibrium. I assume that utility $\tilde{u}$ takes the following form:

$$\tilde{u}(c, v) = \begin{cases} \frac{1}{1-\sigma} \cdot \left(c - \frac{\kappa}{\phi} \cdot v^{\phi}\right)^{1-\sigma} & for \;\; \sigma \in (0, +\infty) \setminus \{1\} \\ \log\left(c - \frac{\kappa}{\phi} \cdot v^{\phi}\right). & for \;\; \sigma = 1 \end{cases}$$

taken from Storesletten et al. (2017) where $\kappa > 0$ and $\phi \geq 1$. The specification of $\tilde{u}$ as Greenwood-Hercowitz-Huffman preferences rules out wealth effects in search effort. Re-expressing in terms of utility function u yields:

$$u(c, x) = \frac{1}{1-\sigma} \cdot \left[\left(c - \frac{\kappa}{\phi} \cdot \left(\frac{c}{q(x)}\right)^{\phi}\right)^{1-\sigma} - 1\right]. \tag{3.4}$$

To guarantee that condition concerning the mixed derivative u_{cx} holds (see equation 2.26), I set $\sigma = 1$ and $\phi = 1$ which implies the log utility function where search effort that is linear in terms of the amount of consumed goods.[8]

Finally, let us define the following variable α:

$$\alpha \equiv \frac{\frac{d\Pi}{dx}}{\frac{dY}{dx}} \tag{3.5}$$

Since a rise in x can be interpreted as an increase in aggregate demand in the model (recall that output capacity is fixed and normalized to unity), α can be thought of as a value that characterizes the comovement of prices and output resulting from a positive demand shock.

The following theorem presents the main result of the paper:

Theorem 2. *Suppose that economy is in stationary equilibrium at the beginning of period t, condition 2.26 holds, government follows fiscal rule Λ and agents feature perfect*

[8]It is easy to see that if $\sigma = \phi = 1$ then the value of κ becomes irrelevant for equilibrium allocation. In the Appendix I relax condition 2.26 and I derive the multiplier's formula in a more general case (where, to avoid the indeterminacy of u, I consider the equilibrium in which $q(x) > \kappa$).

foresight about aggregate variables for $s > t$. Under those assumptions the formula for the government spending multiplier is:

$$\frac{dY_t}{dG_t} = \frac{1 + \frac{\partial C_t^{\Lambda}}{\partial G_t}}{1 - \frac{\partial C_t^{\Lambda}}{\partial x_t} \cdot \frac{1}{f'(x_t)}} \tag{3.6}$$

where:

$$\frac{\partial C_t^{\Lambda}}{\partial G_t} \equiv \underbrace{-\left(1 - \frac{\lambda}{1+i}\right) \cdot \mathbb{E}_{\mu}\left(MPC \cdot \tau\right)}_{\textit{Taxation channel}} + \underbrace{\beta \cdot (1+i) \cdot \mathbb{E}_{\mu}\left(MPS \cdot \frac{1}{u_{cc}\left(c\right)} \cdot \mathbb{V}_{bG}^{\Lambda}\right)}_{\textit{Expectations channel}}$$

and:

$$\frac{\partial C_t^{\Lambda}}{\partial x_t} \cdot \frac{1}{f'\left(x_t\right)} \equiv \underbrace{-\frac{\Omega}{1+i} \cdot \mathbb{E}_{\mu}\left(MPS \cdot c\right)}_{\textit{Intertemporal substitution channel}} + \underbrace{\frac{\Omega}{1+i} \cdot \mathbb{E}_{\mu}\left(MPC \cdot URE\right)}_{\textit{Interest rate exposure channel}}$$

$$+ \underbrace{\mathbb{E}_{\mu}\left(MPC \cdot z\right)}_{\textit{Income channel}} \underbrace{- \left(\frac{\Omega}{(1+i)^2} - \alpha\right) \cdot \bar{B} \cdot \mathbb{E}_{\mu}\left(MPC \cdot \tau\right)}_{\textit{Debt service costs channel}} \underbrace{- \alpha \cdot \mathbb{E}_{\mu}\left(MPC \cdot b\right)}_{\textit{Fisher channel}}$$

where Ω and $\mathbb{V}_{bG}^{\Lambda}$ are defined as:

$$\Omega \equiv \phi_{\Pi} \cdot \alpha + \phi_Y.$$

$$\mathbb{V}_{bG}^{\Lambda} \equiv \mathbb{E}_{z_{t+1}|z_t} V_{t+1,bG}^{\Lambda}\left((1+i) \cdot URE_t, z_{t+1} | G_t\right) |_{URE_t = URE,\, G_t = G,\, V_{t+1}^{\Lambda} = V}$$

where variables without time subscripts are evaluated at their stationary equilibrium levels.

There are two additional variables in Theorem 2 that have not been described yet: Ω measures the responsiveness of monetary policy to changes in output and prices caused by a shift in market tightness resulting from the fiscal expansion. Variable $\mathbb{V}_{bG}^{\Lambda}$ measures how expectations about future economic prospects vary with changes in government purchases at the individual level.

Before interpreting the formula in greater detail, let me point to some distinctive features of this result. First, expression 2 bears some resemblance to the result from the seminal work by Auclert (2017) as it contains averaged cross-products of

MPC (or MPS) and individual consumer characteristics. In contrast to his work, however, my result is not a partial equilibrium outcome and it captures general equilibrium effects as well. Second, some works (e.g., Kaplan et al. (2016) and Hagedorn et al. (2017), Kopiec (2018)) use numerical methods to decompose impulse response functions to aggregate shocks in heterogeneous agent economies into model-based channels but they do not explain what are the exact forces behind those mechanisms. Theorem 2 overcomes this unclarity by presenting the determinants of the magnitudes of those channels.

Let us discuss the forces that affect the value of the multiplier in economy with heterogeneous households. The first channel that appears in the numerator is related to the increase in taxation needed to finance additional government spending. Obviously, this channel has negative impact on the value of dY/dG as both MPC and τ are positive for all agents. To minimize this effect, government should either increase the proportion of additional government purchases that is financed with debt (i.e. raise λ) or it should levy larger shares in total tax burden τ on households with lower marginal propensities to consume. The latter coupled with empirical observations that richer consumers tend to exhibit smaller $MPCs$ (see Figure 4.1) implies that to dampen the negative impact of higher taxes on the effectiveness of fiscal stimulus government should apply more progressive taxes. Second channel in the numerator is the so-called expectations channel. Its name is motivated by the presence of aggregated expectations over mixed derivatives $\mathbb{V}_{bG}^{\Lambda}$ of future value functions. Notice if, for instance, $V_{t+1,bG}^{\Lambda} < 0$ then value function "tomorrow" flattens as a result of higher fiscal expenditures "today". This, together with the Euler equation associated with period when expansion starts, can be interpreted as a decline in precautionary motives coming from the current rise in G. If it is the case for a sufficiently large measure of agents then rising consumer confidence crowds private consumption in (recall that $u_{cc} < 0$) and amplifies the effects of higher government purchases. Again, it is important to highlight that the dependence of the multiplier on fiscal rule Λ pursued by government is a consequence of the interplay between market incompleteness and liquidity constraint faced by households which, combined, imply that Ricardian equivalence does not hold and therefore the way in which dG_t is financed becomes relevant for the response of private sector. In Section 4 I demonstrate the relationship between Λ and $\mathbb{V}_{bG}^{\Lambda}$ by comparing two scenarios associated with two different fiscal rules: tax-financed and debt-financed stimulus.

Let us turn to forces that appear in the denominator. First of them is related to intertemporal substitution spurred by monetary policy response during fiscal expansion: if Ω is large then central bank counteracts government stimulus aggressively by raising nominal interest rates and thus creates incentives to save and to reduce private spending which tends to lower the multiplier's value. Monetary authority reaction is prescribed by Taylor rule and takes place because both price level and output rise during expansion. Second force is associated with the unhedged interest rate exposure of households. Notice that it is an outcome of two mechanisms that go in opposite directions for households from different parts of wealth distribution. On the one hand, $MPC \cdot URE$ is negative for indebted agents that roll over their liabilities. On the other hand, this product is positive for agents with high URE. If the former group prevails over the latter then a more responsive monetary policy diminishes the effects of government purchases. In the opposite case, a more aggressive central bank's reaction amplifies the impact of government purchases through wealth effects. Third channel is related to changes in income that accompany fiscal stimulus and it strengthens the effects of expansion. This channel is tightly related to a standard, Keynesian feedback loop between household income and consumption. Fourth force has to do with changes in taxes needed to balance the budget as debt service cost vary.[9] These shifts have two sources: if monetary policy reacts to fiscal stimulus by raising nominal rates significantly then government is forced to issue new debt at lower price and thus has to levy additional taxes to balance the budget. On the other hand, if the stimulation of aggregate demand leads to a substantial rise in prices (captured by parameter α) then the nominal public debt burden, which has to be repaid in the current period, decreases and gives rise to a downward adjustment in taxes. This can be seen as a kind of Fisher channel that is associated with public liabilities. Household balance sheets are affected by the same force: if a consumer has positive nominal wealth then increasing prices impoverish him or her, lead to cut in expenditures which tends to dampen the effects of fiscal stimulus. Contrar-

[9] Observe that this channel is present even if it assumed that stimulus is financed solely with debt. This is because, except for funds needed to finance additional fiscal purchases, government has to balance its budget after changes in i and Π that are induced by its intervention (that can be summarized with the behavior of x). As I assume that the only argument associated with Λ is G_t then, by construction, B_{t+1} does not react to general equilibrium effects captured with x.

ily, for agents with nominal debt, higher prices lower the real value of liabilities and crowd private expenditures in.

3.2.2 Special case: government spending multiplier in economy with identical households

To highlight the role of household heterogeneity in the propagation of fiscal stimulus it is useful to study the extreme case in which inequality across agents is eliminated. For tractability (and to guarantee comparability with the benchmark scenario which is discussed in Section 4.2), for now I will consider a one-time fiscal shock and assume that $\lambda = 0$ which means that stimulus is financed with taxes. Observe that under this assumption, in the representative agent economy we have:

$$b' = b = \bar{B}' = \bar{B}. \tag{3.7}$$

Additionally, since agents are identical:

$$\tau(z) = 1,\, z = 1.$$

Notice that in this situation, the Fisher channel associated with household balance sheet (see formula 3.6) is exactly offset by the impact of inflation on public debt:

$$\underbrace{\alpha \cdot \bar{B} \cdot \mathbb{E}_{\mu}\left(MPC \cdot \tau\right)}_{\text{Debt service costs channel: repayment}} \qquad - \underbrace{\alpha \cdot \mathbb{E}_{\mu}\left(MPC \cdot b\right)}_{\text{Fisher channel}}$$
$$= \alpha \cdot \bar{B} \cdot MPC - \alpha \cdot MPC \cdot b = 0$$

where the last equality follows from condition 3.7. This occurs because on the one hand higher inflation decreases household wealth (which imposes a downward pressure on private consumption) but, one the other hand, it decreases the value of public debt that has to be repaid which leads to reduction in taxes (which stimulates consumption). The fact that both forces cancel out is not surprising because in the representative agent model government liabilities have to be settled by households (so in fact they are household liabilities) and, at the same time government bonds are household assets. This means that any change in the value of liquid assets has no impact on consumer wealth.

Analogously, the impact of a rise in i, that accompanies the fiscal shock, on consumer balance sheet (captured by URE) during fiscal expansion is offset by a symmetric mechanism that affects government that issues new debt $\bar{B}'$. More specifically, an increase in i makes the purchase of assets b' by households cheaper which raises the relative value of their cash-in-hand. At the same time, however, government has to issue new debt at lower price which, automatically, gives rise to budget deficit that under the balanced-budget rule (i.e., when $\lambda = 0$) is covered with a rise in taxes which, in turn, lowers consumer's disposable income. Again, both effects cancel out in the representative agent framework:[10]

$$\underbrace{\frac{\Omega}{1+i} \cdot \mathbb{E}_\mu \left(MPC \cdot URE\right)}_{\text{Interest rate exposure channel}} - \underbrace{\frac{\Omega}{(1+i)^2} \cdot \bar{B} \cdot \mathbb{E}_\mu \left(MPC \cdot \tau\right)}_{\text{Debt service costs channel: issuance}}$$

$$= \frac{\Omega}{(1+i)^2} \cdot MPC \cdot b' - \frac{\Omega}{(1+i)^2} \cdot \bar{B} \cdot MPC = 0.$$

Notice that since the stimulus is assumed to be tax-financed and it is a one-time shock then the representative agent economy is back in stationary equilibrium in period $s = t+1$ and therefore V_{t+1}^{Λ} remains unaffected by the intervention which implies:[11]

$$V_{t+1}^{\Lambda} = V \implies \underbrace{\beta \cdot (1+i) \cdot \mathbb{E}_\mu \left(MPS \cdot \frac{1}{u_{cc}(c)} \cdot \mathbb{V}_{bG}^{\Lambda}\right)}_{\text{Expectations channel}} = 0.$$

The following corollary summarizes those findings and presents the formula for the tax-financed multiplier in economy where agents are identical:[12]

[10] Note that from definition of URE and from household's budget constraint (equation 2.4):

$$URE = \frac{b'}{1+i}.$$

[11] This is because the only aggregate state variable in this special case is $\bar{B}$ which is constant over time. Hence, by backward induction:

$$V = \lim_{t \to +\infty} V_t^{\Lambda} = ... = V_{t+2}^{\Lambda} = V_{t+1}^{\Lambda}.$$

[12] Observe that since debt service cost channel cancels out with Fisher and URE channel and

Corollary 3. *Let us consider a one-time, budget-neutral increase in government purchases in a representative agent economy in period t. The associated government spending multiplier is given by:*

$$\frac{dY_t}{dG_t} = \frac{1}{1 + \frac{\Omega}{1+i} \cdot c}. \tag{3.8}$$

To put it differently, the only channel through which fiscal stimulus interacts with private demand in the representative agent case when $\lambda = 0$ and fiscal shock lasts for one period is the intertemporal substitution mechanism. This outcome bears some analogies to the result presented by Kaplan et al. (2016) for the case of monetary policy in the standard representative agent New Keynesian model.

4 Household heterogeneity and the multiplier's value: empirical assessment

Note, that in contrast to Auclert (2017), we cannot use the so-called sufficient statistic approach to estimate the multiplier's value from expression 3.6. This is because one of the channels - namely the expectations channel - contains unobserved elements that depend on the model's structure. Therefore, the estimation strategy followed in this paper is to calibrate the model to match empirical objects that are relevant from the point of view of the conducted analysis and, on the basis of that model, compute the multiplier. In particular, as formula 3.6 suggests, to obtain a good estimate of the multiplier it will be important to mimic closely the empirical features of MPC.

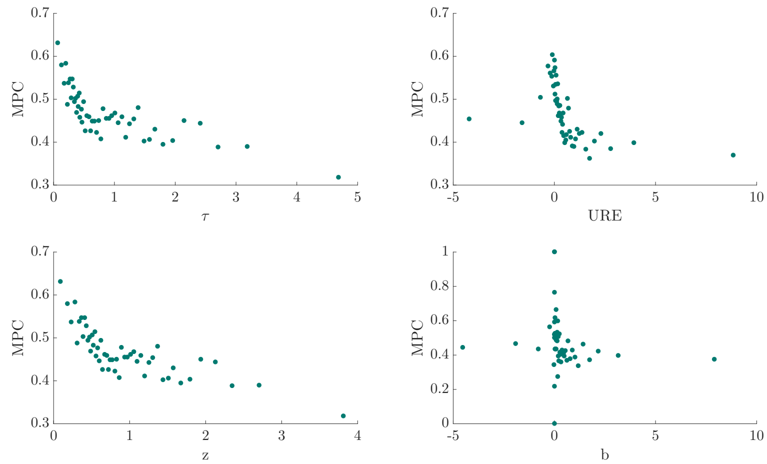
Before moving to the quantitative exercise in which the multiplier's size is calculated, it is instructive to analyze Figures 4.1 and 4.2 showing the relationships between individual-level variables τ, URE, z, b, c and MPC (or MPS) documented in the SHIW survey conducted among Italian households in 2016. In gen-

because $\tau = z = 1$ then formula 3.6 becomes:

$$\frac{dY_t}{dG_t} = \frac{1 - MPC}{1 - MPC + \frac{\Omega}{1+i} \cdot MPS \cdot c}$$

which, because $MPS = 1 - MPC$ is equivalent to equation 3.8.

Figure 4.1: Empirical relationships between MPC and τ, URE, z, b in SHIW 2016



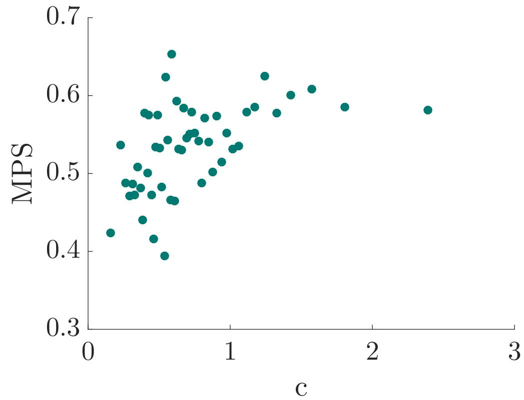
To prepare the plots above I have sorted pairs describing household-level variables that consist of τ (or, alternatively URE, z, b) and MPC in the ascending order with respect to the first coordinate. Second I have grouped those pairs into 50 bins. For each bin I have computed mean values of τ (or, alternatively URE, z, b) and MPC which are represented as dots in the figure. Standardization of τ, URE, z and b is described in the core text.

eral, it can be observed that households with lower income (or lower consumption, lower unhedged interest rate exposure) tend to exhibit higher MPC. Similarly, those, whose share in aggregate tax burden is smaller, have larger MPC. At the same time, Figures 4.1 and 4.2 give us some intuitions about averaged cross-products that constitute channels affecting multiplier's magnitude described in Theorem 2.

4.1 Calibration of the model

The period in the model is equal to one year and the calibration target are moments characterizing Italian economy. I choose Italy because of the availability of MPC data measured at the household level in the SHIW survey. As the average level of MPC documented therein equals 0.475 can be hardly achieved in the standard incomplete market model then, in what follows, I will consider two alternative modifications suggested by Jappelli and Pistaferri (2014) to solve this problem. To match the mean level of empirical MPC, first of them assumes that a proportion μ_{HTM} of households are rule-of-thumb (hand-to-mouth) consumers that feature $MPC = 1$. In the second specification there are two groups of consumers of equal measure: patient households characterized with β_H and impa-

Figure 4.2: Empirical relationship between MPS and c in SHIW 2016



To prepare the plot above I have sorted pairs describing household-level consumption c and MPS in the ascending order with respect to the first coordinate. Second I have grouped those pairs into 50 bins. For each bin I have computed mean values of c and MPS which are represented as dots in the figure. Standardization of c is described in the core text.

tient households with discount factor equal to β_L, where $\beta_H > \beta_L$.[13] Intuitively, raising the value of μ_{HTM} (or, alternatively, lowering β_L) enables to increase the average level of MPC generated by the model. Despite those modifications, formula 3.6 remains valid.[14]

4.1.1 Model with hand-to-mouth agents

Let us turn to calibration of the first version of the model in which proportion μ_{HTM} exhibits $MPC = 1$. As it has been already mentioned, the assumed functional form for u is:

$$u(c,x) = \frac{1}{1-\sigma} \cdot \left[\left(c - \frac{\kappa}{\phi} \cdot \left(\frac{c}{q(x)} \right)^{\phi} \right)^{1-\sigma} - 1 \right]$$

[13]More precisely, Jappelli and Pistaferri (2014) experimented with a uniform decrease in β across all agents to match empirical value of average MPC. This leads to a drastic and unrealistic increase in real interest rates in stationary equilibrium. To avoid this problem, I following Auclert (2017) and split the population into two subgroups: patient and impatient households. The value of β_L is calibrated to match the average value of MPC and the calibration target for β_H is the real interest rate.

[14]The only change that is needed to guarantee that 3.6 holds in economy with patient and impatient households is to take $\beta \in \{\beta_L, \beta_H\}$ under the integral associated with expectations channel.

Table 1: Parameters set without model simulations, identical for models with hand-to-mouth households and heterogeneous discount factors.

Parameter	Name	Value	Target/Source
f	Probability of selling output	0.763	Capacity utilization
Π	Price index	1	Standardization
ϕ_Y	Parameter of Taylor rule	0.125	Galí (2008)
ϕ_Π	Parameter of Taylor rule	1.5	Galí (2008)
$\bar{i}$	Parameter of Taylor rule	0.02	Fisher equation
α	Demand-driven comovement of Y and Π	0.51	SVAR evidence
$\bar{B}$	Real public debt	0.99	Debt to GDP ratio
σ	Risk aversion	1	Condition 2.26
ϕ	Search effort curvature	1	Condition 2.26
$\{\tau(z)\}_{z\in Z}$	Shares in total tax burden	not reported	Italian tax system
G	Government purchases	0.28	Equation 2.4
λ	Stimulus financing rule	$\{0, 1.02\}$	Tax/debt financed dG

and to eliminate wealth effects of search effort (see Storesletten et al. (2017)) and to guarantee that condition 2.26 holds parameters σ and ϕ are equal to one. This, in turn, implies that parameter κ is irrelevant for the equilibrium allocation so we do not need to calibrate its value - it occurs because κ and $q(x)$ do not appear in equations that determine equilibrium when $\sigma = \phi = 1$.[15]

I use the annual real interest rate equal to 2% to pin down the value of β. Since I consider a stationary equilibrium in which $\Pi = 1$ the value of parameter $\bar{i}$ in the Taylor rule equals the real interest rate. Therefore, as it is the case in the literature, while computing stationary allocation I iterate over $\bar{i}$ and keep Π constant. The fact that both Π and f are assumed to be one-argument, monotone functions of x implies that there is a bijective relationship between them. This coupled with the fact that Π is assumed to be constant implies that f can be treated as parameter of the model. Its value is set to be equal to the capacity utilization in Italy in 2016 documented by EUROSTAT. I assume that the two remaining parameters associated with monetary policy rule take standard, textbook values: $\phi_\Pi = 1.5$ and $\phi_Y = 0.125$ (Galí (2008)).

Aggregate public debt in stationary equilibrium $\bar{B}$ is set to be equal to $1.31 \cdot f$

[15]To see that notice that when $\sigma = 1$ and $\phi = 1$ then $u_c(c, x)$ is equal to $\frac{1}{c}$. This means that κ does not affect policy functions $c(b, z)$ and $b'(b, z)$ (see the Euler equation) and thus its value becomes irrelevant for the allocation in equilibrium.

which is the value corresponding to the level of government debt of 131% GDP in 2016. The comovement between price index and output when economy is affected by demand shock which is captured by parameter α is set to be equal 0.51. This value is based on the SVAR model in which demand shocks are identified with sign restrictions, which is presented in the Appendix in a more detailed way.

The calibration target for ξ (parameter that governs the tightness of liquidity constraint) is set to match the ratio between aggregate consumer debt and aggregate positive liquid assets of households calculated from the SHIW survey and equal to 0.44. One comment is in order here: several works analyzing heterogeneous agent economies (e.g., McKay and Reis (2016b), Krueger et al. (2016) and Kopiec (2018)) standardize the parameter that characterizes the liquidity constraint to zero. Formula 3.6 shows why this normalization may lead to a distorted picture of the model's reaction to aggregate shocks: if b (and b') is imposed to be non-negative for all agents then it automatically imposes a restriction on the signs of both the interest rate exposure (notice that from the budget constraint $b' = URE$) and the Fisher channel. In particular, in the context of government expenditures shock analyzed here, this assumption implies that Fisher channel always dampens and interest rate exposure channel always amplifies its impact.

As already mentioned, parameter μ_{HTM} is calibrated to match the average level of MPC in the SHIW survey. I assume that rule-of-thumb consumers constitute an equal proportion of agents across all states.[16]

To pin down vector $\{\tau(z)\}_{z\in Z}$ I first normalize the progressive income tax scale in Italy with respect to average disposable income observed in the data which gives the income tax thresholds in the model. I can now assign the tax rate $\tilde{\tau}$ to each household which is indexed productivity z. Simultaneously, given those thresholds and aggregate output f, I compute the total budget revenues from income tax Θ. To compute the share τ of each household in aggregate tax burden Θ, I divide $\tilde{\tau} \cdot f \cdot z$ (individual amount of tax paid to government) by Θ. Given Θ, $\bar{B}$, Π and $\bar{i}$, I calculate the value of government purchases G in stationary

[16] In other words, conditional distributions of hand-to-mouth and optimizing agents are the same. Jappelli and Pistaferri (2014) assume that that the fraction of rule-of-thumb consumers is 75 percent in deciles 1–3 of cash-in-hand distribution, 40 percent in deciles 4–7, and 30 percent in deciles 8–10. This gives two additional parameters that are used to achieve calibration targets. In my work I set an equal proportion of hand-to-mouth agents and thus I have two parameters less to match the data. Despite that, I manage to mimic empirical observations reasonably well.

Table 2: Parameters calibrated with the simulated model with a proportion of hand-to-mouth agents

Parameter	Name	Value	Target/Source
β	Discount factor	0.9703	Real interest rate
ξ	Liquidity constraint	-2.2	Ratio of debt to assets
μ_{HTM}	Proportion of HTM agents	0.42	Average MPC
σ_T^2	Variance of transitory shocks	0.05	MPC distribution
σ_P^2	Variance of persistent shocks	0.04	MPC distribution
ρ_P	Autocorrelation of persistent component	0.958	MPC distribution

equilibrium from government budget constraint 2.4.[17] Parameter λ determines the way in which government finances the stimulus in period t and is set to be equal to 0 in the benchmark simulation (i.e. additional government purchases are financed solely by taxes) and it equals $1+\bar{i}$ when the alternative scenario (debt-financed stimulus) is considered. More specifically, the benchmark fiscal rule Λ^0 is given by:

$$\Lambda^0 : \; G_t \rightarrow \left[\left\{G_t, G, G, ...\right\}, \left\{\bar{B}, \bar{B}, ...\right\}\right].$$

Let us turn to the calibration of the income process that governs changes in z at the individual level. Similarly to Krueger et al. (2016), I assume that productivity follows a process with transitory and persistent components:

$$\begin{cases} \log z' = s + \epsilon_T \\ s' = \rho_P \cdot s + \epsilon_P \end{cases}$$

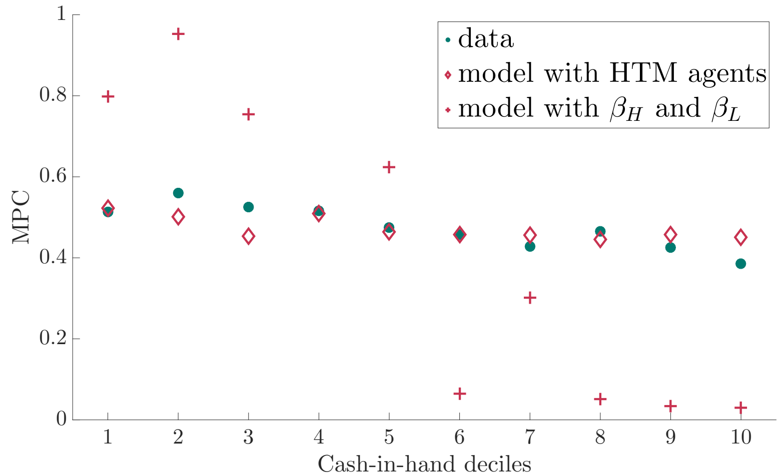
where by ρ_P I denote the autocorrelation of persistent component, ϵ_T is a transitory innovation and ϵ_P is the shock that influences the evolution of persistent component s. Parameters ρ_P, σ_P^2 (variance of the shock to persistent component), σ_T^2 (variance of the shock to transitory component) are calibrated using the Simulated Method of Moments to match the average values of MPC (associated with a transitory change in disposable income) across cash-in-hand deciles.[18] Formula

[17] Recall that we have:

$$\int_{B\times Z} T(z)\, d\mu(b,z) = \Theta.$$

[18] Given ϕ_P, σ_P^2 and σ_T^2 I use the Rouwenhorst algorithm to discretize the persistent component of the process and I apply the Gauss-Hermite quadrature to approximate the transitory shock.

Figure 4.3: SMM estimation of ϕ_P, σ_P^2, σ_T^2 and μ_{HTM}: average MPC across cash-in-hand deciles in the model and in the data.



3.6 clearly shows why this calibration target is crucial when evaluating the value of fiscal multiplier: MPC and $MPS = 1 - MPC$ enter all the channels affecting the size of dY/dG so it is important to mimic their empirical counterparts closely to obtain a plausible estimate of the multiplier. Estimation results for ρ_P, σ_P^2, σ_T^2 are displayed in Figure 4.3.[19]

Calibrated parameters can be divided into two subgroups. First of them contains those calibrated with reference to the literature and to moments which do not require model simulations and is summarized in Table 1. Second group are values pinned down by model simulations (Table 2).

4.1.2 Model with patient and impatient households

In this subsection I follow Auclert (2017) and I describe the calibration of the model where agents are grouped into two populations of measure 0.5: patient and impatient households. The majority of parameter values are set at levels that

Cash-in-hand is defined as: $b/\Pi + f \cdot z$.

[19] Observe that MPC is not monotonically decreasing with respect to cash-in-hand deciles. This may look a bit surprising but is is driven by the fact that under the assumed specification of idiosyncratic income risk it may occur that agent that exhibits low value of persistent shock and high value of transitory innovation has larger cash-in-hand than agent with high value of persistent shock and low value of transitory innovation. The former tends to have higher MPC than the latter which may give rise to locally increasing relationship between cash-in-hand and MPC.

Table 3: Parameters calibrated with the simulated model with heterogeneous discount factors

Parameter	Name	Value	Target/Source
β_H	Discount factor of patient agents	0.9736	Real interest rate
β_L	Discount factor of impatient agents	0.69	Average MPC
ξ	Liquidity constraint	-1.35	Ratio of debt to assets
σ_T^2	Variance of transitory shocks	0.05	MPC distribution
σ_P^2	Variance of persistent shocks	0.04	MPC distribution
ρ_P	Autocorrelation of persistent component	0.958	MPC distribution

are identical with those in the model with hand-to-mouth consumers (see table 1). There are several exceptions: parameters calibrated with the simulated model.

First, there are two discount factors: β_L (associated with impatient households) and β_H (associated with patient households) that satisfy $\beta_L < \beta_H$. The value of discount factor β_H is set to match the level of real interest rate. As mentioned, the calibration target for β_L is the average level of MPC documented in the SHIW survey. Introducing discount factor heterogeneity changes the stationary distribution of households significantly (in comparison to the model with hand-to-mouth consumers) and thus to match the ratio between aggregate debt and aggregate positive assets we need to reparametrize the tightness of liquidity constraint captured with ξ.

The remaining parameters: ρ_P, σ_P^2, σ_T^2 were used to match the distribution of MPC across cash-in-hand deciles in the model with rule-of-thumb consumers. The problem with the variant of the model with heterogeneous discount factor is that it generates excessively large differences in MPC across agents (in comparison to empirical evidence) irrespectively of values assigned to parameters ρ_P, σ_P^2, σ_T^2. Figure 4.3 provides an example of the distribution of MPC generated by the model with β_L and β_H in which ρ_P, σ_P^2, σ_T^2 take values identical to those calibrated for the model with hand-to-mouth agents. As discussed later, the fact that the model with heterogeneous discount factors fails to mimic this pattern has some important consequences for the evaluation of fiscal policy effectiveness.

4.2 Government spending multiplier: benchmark scenario

We are in position to use the formula presented in Theorem 2 and to quantify the impact of channels affecting multiplier's value. This exercise is important because, as discussed previously, several channels are outcomes of forces that have opposite directions and therefore their signs are ambiguous. In what follows, I inspect the effects of fiscal policy shock in both the model with hand-to-mouth agents and the model with patient and impatient households.

As already mentioned, in the benchmark case I assume that: $G_t > G$ and $G_s = G$ for $s > t$ (i.e., fiscal shock lasts for one period) and that λ is equal to zero (i.e., the stimulus is budget neutral).[20] The last assumption is made because if a country is plagued by large debt-to-GDP ratio then issuing additional government bonds can be costly or even impossible. Under such circumstances the only way to increase government consumption is to raise taxes. As the model is calibrated to match the moments of Italian economy - a country that has been under severe fiscal pressure during the Eurozone Crisis and whose ability to issue new debt was limited - then the tax-financed government expansion seems to be a realistic scenario.

In the simulation, government purchases increase by 0.1% of GDP in period t and the shock is followed by a perfect-foresight transition path along which economy converges back to stationary equilibrium.[21]

The value of the multiplier is approximated by the following expression:

$$\frac{dY_t}{dG_t} \approx \frac{Y_t - Y}{G_t - G}.$$

where Y_t is the first element of output transition path.

To estimate the magnitude of multiplier's channels, I will use the following

[20] Observe that I assume that although Italy is a member of the Eurozone, central bank reacts to fiscal shock. This assumption remains plausible if Italy is considered as a large economy among other members of the currency union or if stimulus is coordinated across the Eurozone. The opposite case implies that the ECB does not react to Italian shocks is isomorphic to the situation in which $\phi_\Pi = \phi_Y = 0$ and it is analyzed in Section 4.3.1.

[21] I use a relatively small size of the shock to obtain a better approximation of the multiplier.

approximation of the MPC of household with assets b and productivity z:

$$MPC\left(b,z\right) \approx \frac{c\left(b,z+\epsilon_T\right)-c\left(b,z\right)}{\left[f\cdot\left(z+\epsilon_T\right)-f\cdot z\right]-\left[\tau\left(z+\epsilon_T\right)-\tau\left(z\right)\right]\cdot\Theta}$$

where the numerator is the difference between consumption levels of household with productivity $z+\epsilon_T$ and of households with productivity z in stationary equilibrium. The denominator is the corresponding difference in disposable income. Recall that by ϵ_T I denote the value of a transitory productivity shock. Given $MPC\left(b,z\right)$ and values of c, τ, URE, z, b I compute the magnitudes of: taxation channel, intertemporal substitution channel, interest rate exposure channel, income channel, debt service costs channel and Fisher channel. The size of expectations channel is derived from equation 3.6 given $\frac{dY_t}{dG_t}$ and values of the remaining channels.[22]

Results are reported in Table 4. The budget-neutral government spending multiplier equals 0.69 in the model with a fraction of rule-of-thumb consumers and it is equal to 0.43 in economy with patient and impatient households. For each variant of the model I report two columns of results. First column contains the values of terms appearing in formula 3.6 that describe the magnitudes of channels through which fiscal stimulus affects aggregate demand and, as a consequence, output. To interpret those numbers, second column reports the size of the multiplier under a hypothetical scenario, when a given channel is shut off. If it exceeds the value of $\frac{dY_t}{dG_t}$ then it means that the corresponding channel crowds private consumption out and dampens the impact of government expenditures on output. If, on the other hand, it is lower than $\frac{dY_t}{dG_t}$ then it implies that consumption is crowded in by a given channel and the effects of stimulus are amplified.

Let us turn to the comparison of two versions of the model.[23] The main disadvantage of the variant with hand-to-mouth agents is that a fraction of consumers is assumed to be unable to optimize and, in particular, it cannot make intertemporal choices. This may raise concerns about its ability to measure the expectations channel accurately (as rule-of-thumb agents exhibit $MPS = 0$). Nevertheless, a

[22] I choose this "indirect" method to estimate the size of the expectations channel because of relatively large approximation errors associated with the computation of mixed derivatives V_{bG} when evaluating this channel directly.

[23] Impulse response functions of main aggregate variables in the benchmark simulation for both variants of the model are reported in the Appendix.

comparison with the model populated with patient and impatient households indicates that the potential approximation error is relatively small in absolute terms. On the other hand, the main weakness of the model with heterogeneous discount factors is that it fails to mimic the distribution of MPC across cash-in-hand deciles (see Figure 4.3). This may lead to some erroneous conclusions concerning the aggregate demand reaction to changes in the real value of their liquid positions and URE.[24] Table 4 shows that, indeed, the inability to match the distributional features of MPC by the model with patient and impatient households has some tremendous consequences for the assessment of channels through which fiscal shock operates. In particular, not only do both the Fisher channel (associated with liquid positions) and the interest rate exposure channel (associated with URE) have opposite signs in two versions of the model, but also the difference between those values is large in absolute terms. The fact that the model with rule-of-thumb consumers fits the distribution of MPC incomparably better than the model with heterogeneous discount factors clearly indicates that estimates of interest rate exposure and Fisher channels are more realistic. This implies that the model with a fraction of hand-to-mouth agents significantly overperforms the model with patient and impatient households and hence, in what follows, I will analyze the effects of fiscal stimulus using only this version.

Decomposition shows that only interest exposure channel and income channel amplify the effects of fiscal stimulus. Their impact on the propagation of government expenditures shock is large: in the absence of each of them, the multiplier's value drops by more than 50%. The largest channels that crowd out aggregate consumption and thus dampen the effects of stimulus are: taxation channel and Fisher channel. If closed, each of them generates a rise in the multiplier by approximately 100%. The fact that both Fisher and interest rate exposure channel have a significant impact $\frac{dY_t}{dG_t}$ indicates that wealth heterogeneity is an important determinant of fiscal policy effectiveness.

Observe, that the sign of Fisher channel is somewhat counterintuitive, as in the literature there is a common belief that a rise in prices decreases the real value of loans of indebted households (who, at the same time, exhibit large MPC) and

[24]Notice that, by construction, cash-in-hand is positively correlated with liquid asset holdings and, as in the stationary equilibrium distributions of: URE and liquid assets are the same, it is also positively correlated with the former.

Table 4: Fiscal multiplier: quantitative decomposition, benchmark scenario in two versions of the model

	Model with HTM agents		Model with β_L and β_H	
	Value	Counterfactual $\frac{dY_t}{dG_t}$	Value	Counterfactual $\frac{dY_t}{dG_t}$
Taxation channel	-0.63	1.95	-0.42	0.79
Expectations channel	-0.03	0.76	-0.08	0.50
Intertemporal substitution channel	-0.13	0.94	-0.24	0.54
Interest rate exposure channel	0.56	0.32	-0.50	0.76
Income channel	0.63	0.30	0.43	0.31
Debt service costs channel	-0.22	1.22	-0.14	0.49
Fisher channel	-0.34	2.17	0.29	0.35
MULTIPLIER: $\frac{dY_t}{dG_t}$	0.69		0.43	

thus stimulates private spending. As Figure 4.3 demonstrates, the difference between MPC of the richest and the poorest is not very large. This coupled with the fact that the liquid net worth of Italian households is positive (recall that the ratio between absolute values of debt and positive liquid balances is 0.44) implies that the aggregate positive reaction of consumption of those who benefit from a rise in prices during fiscal expansion is weaker than the aggregate negative reaction of those those whose net worth depreciates in real terms. More precisely, quantitative decomposition of the Fisher channel into responses of debtors and creditors reads:

$$\underbrace{-\alpha \cdot \mathbb{E}_{\mu}\left(MPC \cdot b\right)}_{=-0.34}$$

$$= \underbrace{-\alpha \cdot \mathbb{E}_{\mu}\left(MPC \cdot b | b < 0\right) \cdot \int_{b<0} d\mu\left(b, z\right)}_{=0.28} \underbrace{-\alpha \cdot \mathbb{E}_{\mu}\left(MPC \cdot b | b \geq 0\right) \cdot \int_{b\geq 0} d\mu\left(b, z\right)}_{=-0.62}.$$

The analogous decomposition of interest rate exposure channel is:

$$\underbrace{\frac{\Omega}{1+i} \cdot \mathbb{E}_{\mu}\left(MPC \cdot URE\right)}_{=0.56}$$

$$= \underbrace{\frac{\Omega}{1+i} \cdot \mathbb{E}_{\mu}\left(MPC \cdot URE | b < 0\right) \cdot \int_{b<0} d\mu\left(b, z\right)}_{=-0.49}$$

$$\underbrace{+\frac{\Omega}{1+i} \cdot \mathbb{E}_{\mu}\left(MPC \cdot URE | b \geq 0\right) \cdot \int_{b \geq 0} d\mu\left(b, z\right)}_{=1.05}.$$

Monetary policy reaction to higher output and prices raises real interest rates which discourage agents from consumption in period t but the quantitative importance of this mechanism is limited (see the value that corresponds to intertemporal substitution channel in Table 4). As the amount of public debt in Italy is relatively large then so is the size of the debt service costs channel. It influences the multiplier because as nominal interest rates rise government issues new debt at lower price. Since it is assumed that $\phi_{\Pi} > 1$ in the Taylor rule, this effect is not outweighed by a rise in prices that lower the real value of debt that has to be repaid in period t.

4.3 Government spending multiplier: alternative scenarios

In this part I study the magnitude of the multiplier in the model with rule-of-thumb households under two three alternative scenarios: more active monetary policy, debt-financed stimulus and more persistent fiscal shock.

4.3.1 More active monetary policy

The role of the monetary policy reaction in the propagation of fiscal stimulus has been discussed, among others, by Woodford (2011), who argued that more accommodative monetary policy rule tends to raise the multiplier's size.[25] The intuition behind this result is straightforward: as fiscal policy shock causes a rise of both prices and output then, under a standard parametrization of Taylor rule, nominal interest rates increase. A more aggressive response of monetary policy translates into a more dynamic rise of nominal rates and, when price rigidities are in place, real rates and creates stronger incentives to reduce consumption. I reinvestigate this mechanism in the model with heterogeneous households.

[25] As pointed by Woodford (2011), the situation in which monetary policy is constrained by zero lower bound can be seen as an extreme case of accommodative monetary policy which implies that there is not reaction of central bank to shifts in government purchases. This case, in turn, has been widely discussed in the literature that emerged in the aftermath of the Great Recession (see, e.g., Eggertsson (2011), Christiano et al. (2011)) and the main conclusion from those works is that higher government purchases are more effective in spurring a recovery when economy is in liquidity trap.

Table 5: Alternative scenarios: more active monetary policy, debt-financed stimulus and persistent fiscal shock

Channel\Scenario	Benchmark	More active monetary policy	Debt-financed stimulus	Persistent shock
Taxation channel	-0.63	-0.63	0	-0.63
Expectations channel	-0.03	-0.06	-0.39	-0.15
Intertemporal substitution channel	-0.13	-0.26	-0.13	-0.13
Interest rate exposure channel	0.56	1.12	0.56	0.56
Income channel	0.63	0.63	0.63	0.63
Debt service costs channel	-0.22	-0.75	-0.22	-0.22
Fisher channel	-0.34	-0.34	-0.34	-0.34
MULTIPLIER	0.69	0.52	1.24	0.45

To this end, I compare the benchmark scenario to the one in which both ϕ_Π and ϕ_Y are two times larger (which implies that the value of Ω doubles, too) and other parameters remain unchanged with respect to the benchmark (in particular, government follows fiscal rule Λ^0). Formula 3.6 implies that this automatically increases the magnitude of two channels in the model: intertemporal substitution channel and interest exposure channel by 100% (see Table 5). The former, by creating incentives to save, tends to dampen the effects of fiscal stimulus while the latter amplify it significantly (through wealth effects).

Second, when monetary policy becomes more active, it is more expensive for government to roll over public debt as the price of newly issued bonds is cut severely by central bank's actions. This implies that government has to raise taxes more aggressively to balance its budget which crowds out private spending.

Overall, the joint negative impact of higher debt service cost and stronger incentives for the intertemporal substitution outweigh the rise of the magnitude of interest exposure channel which implies a drop in $\frac{dY_t}{dG_t}$ in comparison to benchmark scenario.

4.3.2 Debt-financed stimulus

Let us turn to the case in which additional fiscal spending is financed entirely with public debt. This implies that I have to set $\lambda = 1 + \bar{i}$ when simulating the model. I assume that the debt-repayment schedule is specified as follows:

$$\begin{cases} \bar{B}_{s+1}\left(G_t\right) = \bar{B} + \left(1+\bar{i}\right) \cdot \left(G_t - G\right) & for\ \ s = t \\ \bar{B}_{s+1}\left(G_t\right) = \bar{B} + \left(1 - \frac{s-t}{t'-t}\right) \cdot \left(1+\bar{i}\right) \cdot \left(G_t - G\right) & for\ \ t < s \leq t' \\ \bar{B}_{s+1}\left(G_t\right) = \bar{B} & for\ \ t' < s \end{cases}$$

i.e., after a rise in period t, government debt is repaid linearly until it attains its stationary equilibrium level in period t' (I set $t' = 5$ in the simulations). This schedule, together with path:

$$\left\{G_s\left(G_t\right)\right\}_{s \geq t} = \left\{G_t, G, G, ...\right\}$$

pins down the fiscal rule Λ^1.

Table 5 reports the results. The fact that government covers a rise in G solely with debt issuance automatically eliminates the taxation channel which was the main factor that decreased the multiplier's value in the benchmark scenario. On the other hand, however, additional debt has to be repaid in the future which implies higher taxes in periods $s \in (t, t')$ which translates into a significant drop in the forward-looking component (i.e., the expectations channel). Its impact on the multiplier's size is less pronounced than the effect of the disappearance of the taxation channel because I consider the model with uninsured idiosyncratic risk and hand-to-mouth consumers which implies that the Ricardian equivalence does not hold. This gives rise to a significant amplification of the stimulus when government purchases are financed with debt: the size of the multiplier almost doubles.

4.3.3 More persistent fiscal shock

In this subsection, I relax the assumption about the one-time fiscal shock and I consider an autoregressive path along which government expenditures return to stationary equilibrium value. The autocorrelation ρ_G of this process is equal to

0.8. Therefore, the associated fiscal rule Λ^2 can be described as follows:

$$\Lambda^2: \; G_t \to \left[\left\{G_t, G + \rho_G \cdot (G_t - G), G + \rho_G^2 \cdot (G_t - G), ...\right\},\right.$$
$$\left.\{\bar{B}, \bar{B}, ...\}\right].$$

This scenario is motivated by Aiyagari et al. (1992) who compared the macroeconomic impact of fiscal spending shocks characterized with different degrees of persistence. In particular, Aiyagari et al. (1992) found that more persistent shocks exhibit higher multiplier values and the mechanism described in their paper was the following: higher government spending require higher taxes that impoverish households and, as a result, increase their labor supply to compensate this loss. Therefore, a more persistent shock raises future numbers of hours worked which, in turn, increases future marginal product of capital and, consequently, creates larger incentives to invest in the current period. This raises both aggregate demand and the value of multiplier. In my analysis, I reconsider the impact of fiscal shock persistence in environment where the supply side is passive and the only source of amplification comes from the demand side and household heterogeneity.

Table 5 indicates that the only channel that differs in comparison to baseline scenario is the one related to consumer expectations. More precisely, persistent shock tends to decrease its value and, as a result, lowers the multiplier's value. This can be explained by the fact that higher government purchases crowd private consumption in the model with a one-time shock and therefore, in case of more persistent stimulus, they are supposed to lower consumption in future periods. This spurs precautionary motives today (captured by a steeper slope of V along argument b) and thus crowds out private spending.

5 Conclusions

This paper presents an explicit formula for the government spending multiplier in the Bewley-Huggett-Aiyagari model extended to capture frictional product market. This modification is needed to solve the model with paper and pencil as it enables to summarize all endogenous general equilibrium effects that affect households with only one variable: product market tightness. At the same time, this departure from the standard framework allows to relax several assumptions that were made in the literature to obtain analytical expressions in the Bewley-Huggett-Aiyagari models, such as: i) extreme illiquidity that eliminates wealth heterogeneity (e.g. Krusell et al. (2011), Werning (2015), Ravn and Sterk (2016), McKay and Reis (2016a)), ii) constant real interest rates or constant prices of goods (e.g., Auclert et al. (2018), Patterson (2018)), iii) partial equilibrium analysis (e.g. Auclert (2017)).

The derived multiplier's formula has a clear, model-based interpretation and entails cross-sectional marginal propensities to consume, distributions of consumption, income and wealth among households, parameters of the monetary policy rule, parameter describing the comovement of output and prices resulting from a demand shock, aggregate public debt, cross-sectional tax burdens and a forward-looking component that captures consumer expectations.

I use the model, calibrated to match moments (and distributions) observed in Italian data, to quantify the channels that determine the multiplier's value under four scenarios. First of them (also referred to as benchmark simulation) assumes that additional fiscal purchases are financed with taxes and monetary policy follows an unconstrained Taylor rule. Second scenario describes the economy in which monetary policy becomes less accommodative. Third case is the debt-financed stimulus and fourth scenario assumes that fiscal policy shock becomes more persistent.

My theoretical and quantitative results indicate that household heterogeneity plays a significant role in the propagation of government expenditures shocks.

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Appendix A: Proofs

Lemma. *Suppose that economy is in stationary equilibrium at the beginning of period t and government follows fiscal rule Λ. Then the value of government spending multiplier in period t is:*

$$\frac{dY_t}{dG_t} = \frac{1 + \frac{\partial C_t^{\Lambda}}{\partial G_t}}{1 - \frac{\partial C_t^{\Lambda}}{\partial x_t} \cdot \frac{1}{f'(x_t)}}.$$

Proof. For clarity I omit time subscripts. First, we use the formula for the derivative of a composite function and the fact that $Y = f$ (see equation 2.15):

$$\frac{dY}{dG} = \frac{dY}{dx} \cdot \frac{dx}{dG} = \frac{df}{dx} \cdot \frac{dx}{dG}.$$

Next, we apply the Implicit Function Theorem to obtain $\frac{dx}{dG}$ from resource constraint 2.25 and plug it into equation above and reformulate:

$$\frac{dY}{dG} = \frac{df}{dx} \cdot \frac{dx}{dG} = \frac{df}{dx} \cdot \left(- \frac{1 + \frac{\partial C^{\Lambda}}{\partial G}}{\frac{\partial C^{\Lambda}}{\partial x} - \frac{df}{dx}} \right)$$

$$= \frac{1 + \frac{\partial C^{\Lambda}}{\partial G}}{1 - \frac{\partial C^{\Lambda}}{\partial x} \cdot \frac{1}{\frac{df}{dx}}}.$$

□

Theorem. *Suppose that economy is in stationary equilibrium at the beginning of period t, condition 2.26 holds, government follows fiscal rule Λ and agents feature perfect foresight about aggregate variables for $s > t$. Under those assumptions the formula for the government spending multiplier is:*

$$\frac{dY_t}{dG_t} = \frac{1 + \frac{\partial C_t^{\Lambda}}{\partial G_t}}{1 - \frac{\partial C_t^{\Lambda}}{\partial x_t} \cdot \frac{1}{f'(x_t)}}$$

where:

$$\frac{\partial C_t^{\Lambda}}{\partial G_t} \equiv \underbrace{- \left(1 - \frac{\lambda}{1+i}\right) \cdot \mathbb{E}_{\mu} \left(MPC \cdot \tau \right)}_{\textit{Taxation channel}} + \underbrace{\beta \cdot (1+i) \cdot \mathbb{E}_{\mu} \left(MPS \cdot \frac{1}{u_{cc}(c)} \cdot \mathbb{V}_{bG}^{\Lambda} \right)}_{\textit{Expectations channel}}$$

and:

$$\frac{\partial C_t^{\Lambda}}{\partial x_t} \cdot \frac{1}{f'(x_t)} \equiv \underbrace{-\frac{\Omega}{1+i} \cdot \mathbb{E}_{\mu}(MPS \cdot c)}_{\text{Intertemporal substitution channel}} + \underbrace{\frac{\Omega}{1+i} \cdot \mathbb{E}_{\mu}(MPC \cdot URE)}_{\text{Interest rate exposure channel}}$$

$$+ \underbrace{\mathbb{E}_{\mu}(MPC \cdot z)}_{\text{Income channel}} \underbrace{- \left(\frac{\Omega}{(1+i)^2} - \alpha\right) \cdot \bar{B} \cdot \mathbb{E}_{\mu}(MPC \cdot \tau)}_{\text{Debt service costs channel}} \underbrace{-\alpha \cdot \mathbb{E}_{\mu}(MPC \cdot b)}_{\text{Fisher channel}}$$

where Ω *and* $\mathbb{V}_{bG}^{\Lambda}$ *are defined as:*

$$\Omega \equiv \phi_{\Pi} \cdot \alpha + \phi_{Y}.$$

$$\mathbb{V}_{bG}^{\Lambda} \equiv \mathbb{E}_{z_{t+1}|z_t} V_{t+1,bG}^{\Lambda}\left((1+i) \cdot URE_t, z_{t+1}|G_t\right)|_{URE_t=URE,\, G_t=G,\, V_{t+1}^{\Lambda}=V}$$

where variables without time subscripts are evaluated at their stationary equilibrium levels.

Proof. For tractability, let us omit time subscripts in the proof. We will derive the formulas for $\frac{\partial C}{\partial x}$ and $\frac{\partial C}{\partial G}$ that appear in the general characterization of the multiplier (equation 3.1) by aggregating individual partial derivatives $\frac{\partial c}{\partial x}$ and $\frac{\partial c}{\partial G}$. This method, that is based on the application of the Implicit Function Theorem to the first order condition 2.27 that holds with equality, can be applied to unconstrained agents only (i.e. to those with $b' > -\xi$). It will be used to obtain derivatives $\frac{dc}{db'}$ and $\frac{dc}{dx}$, respectively. The case of the constrained agents is considered at the end of the proof.
Before moving to $\frac{\partial c}{\partial x}$ and $\frac{\partial c}{\partial G}$, let us make a preliminary step that turns to be very useful later: notice that the Implicit Function Theorem can be used to derive $\frac{dc}{db'}$ from the first order condition 2.27 and after rearranging it yields:

$$u_c(c,x) = (1+i) \cdot \beta \cdot \mathbb{E}_{z'|z} V_b^{\Lambda}\left(b', z'|G\right) \Longrightarrow$$

$$u_{cc}(c,x)\, dc = (1+i) \cdot \beta \cdot \mathbb{E}_{z'|z} V_{bb}^{\Lambda}\left(b', z'|G\right) db'$$

Similarly to Auclert (2017), from the definitions of MPC and MPS (see equation 3.2) we obtain:

$$\frac{dc}{db'} = \frac{1}{1+i} \cdot \frac{MPC}{MPS}.$$

Combining both observations allows to express V_{bb}^{Λ} as a function of u'', MPC,

MPS and i:

$$\mathbb{E}_{z'|z} V_{bb}^{\Lambda}\left(b', z'|G\right) = \frac{1}{\beta \cdot (1+i)} u_{cc}\left(c, x\right) \cdot \frac{dc}{db'} = \frac{1}{\beta \cdot (1+i)^2} \cdot \frac{MPC}{MPS} \cdot u_{cc}\left(c, x\right). \tag{5.1}$$

Notice that when deriving the formula for dY/dG we will be evaluating $V_{bb}^{\Lambda}\left(b', z'|G\right)$ (and other variables) at its stationary equilibrium levels and hence it is equal to the value of $V_{bb}^{\Lambda}\left(b', z'\right)$ in stationary equilibrium. This shows that expressions for MPC and MPS derived here do not depend on changes in aggregate variables and hence they can be compared to the data in the empirical part of the paper.

I apply the Implicit Function Theorem to 2.27 to get $\frac{\partial c}{\partial x}$:

$$\frac{\partial c}{\partial x} = \left[-\frac{1}{u_{cc}\left(c, x\right) + \left(1 + i\left(x\right)\right)^2 \cdot \beta \cdot \mathbb{E}_{z'|z} V_{bb}^{\Lambda}\left(b', z'|G\right)}\right] \cdot \left\{u_{cx}\left(c, x\right) - i'\left(x\right) \cdot \beta \cdot \mathbb{E}_{z'|z} V_{b}^{\Lambda}\left(b', z'|G\right)\right.$$

$$\left. - \left(1 + i\left(x\right)\right)^2 \cdot \beta \cdot \mathbb{E}_{z'|z} V_{bb}^{\Lambda}\left(b', z'|G\right) \cdot \left\{\frac{1}{\left(1 + i\left(x\right)\right)} \cdot i'\left(x\right) \cdot URE - \tau\left(z\right) \frac{\partial \Theta}{\partial x}\left(x, G\right) - \frac{\Pi'\left(x\right)}{\Pi^2\left(x\right)} b + z \cdot f'\left(x\right)\right\}\right\}.$$

where I have used the definition of URE (equation 3.3). I use condition 2.26 and I plug 5.1 into formula for $\frac{\partial c}{\partial x}$ and rearrange to get:

$$\frac{\partial c}{\partial x} = \left[-\frac{1}{u_{cc}\left(c, x\right) + \beta \cdot \left(1 + i\left(x\right)\right)^2 \cdot \frac{1}{\beta \cdot (1+i)^2} \cdot \frac{MPC}{MPS} \cdot u_{cc}\left(c, x\right)}\right] \cdot \left\{-\beta \cdot i'\left(x\right) \cdot \mathbb{E}_{z'|z} V_{b}^{\Lambda}\left(b', z'|G\right)\right.$$

$$-\beta \cdot \left(1 + i\left(x\right)\right)^2 \cdot \frac{1}{\beta \cdot (1+i)^2} \cdot \frac{MPC}{MPS} \cdot u_{cc}\left(c, x\right)$$

$$\left. \cdot \left\{\frac{1}{\left(1 + i\left(x\right)\right)} \cdot i'\left(x\right) \cdot URE - \tau\left(z\right) \frac{\partial \Theta}{\partial x}\left(x, G\right) - \frac{\Pi'\left(x\right)}{\Pi^2\left(x\right)} b + z \cdot f'\left(x\right)\right\}\right\}$$

$$= \left[-\frac{1}{u_{cc}\left(c, x\right) + \frac{MPC}{MPS} \cdot u_{cc}\left(c, x\right)}\right] \cdot \left\{-i'\left(x\right) \cdot \beta \cdot \mathbb{E}_{z'|z}^{\Lambda} V_{b}\left(b', z'|G\right)\right.$$

$$\left. -\frac{MPC}{MPS} \cdot u_{cc}\left(c, x\right) \cdot \left\{\frac{1}{\left(1 + i\left(x\right)\right)} \cdot i'\left(x\right) \cdot URE - \tau\left(z\right) \frac{\partial \Theta}{\partial x}\left(x, G\right) - \frac{\Pi'\left(x\right)}{\Pi^2\left(x\right)} b + z \cdot f'\left(x\right)\right\}\right\}$$

$$= -\left\{\frac{MPS}{u_{cc}\left(c, x\right)} \cdot \left(-i'\left(x\right) \cdot \beta \cdot \mathbb{E}_{z'|z} V_{b}^{\Lambda}\left(b', z'|G\right)\right)\right.$$

$$\left. -MPC \cdot \left\{\frac{1}{\left(1 + i\left(x\right)\right)} \cdot i'\left(x\right) \cdot URE - \tau\left(z\right) \frac{\partial \Theta}{\partial x}\left(x, G\right) - \frac{\Pi'\left(x\right)}{\Pi^2\left(x\right)} b + z \cdot f'\left(x\right)\right\}\right\}$$

where I have used the fact that $MPC = 1 - MPS$. We use first order condition 2.27 and the assumed functional form u (formula 3.4 where $\sigma = 1$ and $\phi = 1$) to simplify the term that contains $\beta \mathbb{E}_{z'|z} V_b^{\Lambda}(b', z'|G)$:

$$\frac{MPS}{u_{cc}(c,x)} \cdot \left(-i'(x) \cdot \beta \cdot \mathbb{E}_{z'|z} V_b^{\Lambda}(b', z'|G)\right)$$

$$= \frac{MPS}{u_{cc}(c,x)} \cdot \left(-\frac{i'(x) \cdot u_c(c,x)}{(1+i(x))}\right)$$

$$= -c \cdot MPS \cdot \left[-\frac{i'(x)}{1+i(x)}\right] = c \cdot MPS \cdot \left[\frac{i'(x)}{1+i(x)}\right]$$

Plugging this result back into $\frac{\partial c}{\partial x}$:

$$\frac{\partial c}{\partial x} = -c \cdot MPS \cdot \left[\frac{i'(x)}{1+i(x)}\right]$$

$$+MPC \cdot \left\{\frac{1}{(1+i(x))} \cdot i'(x) \cdot URE - \tau(z) \frac{\partial \Theta}{\partial x}(x,G) - \frac{\Pi'(x)}{\Pi^2(x)} b + z \cdot f'(x)\right\}.$$

Since partial derivative of aggregate consumption with respect to market tightness is divided by $f'(x)$ (common to all agents) in the formula for general multiplier (equation 3.1), it is useful to calculate:

$$\frac{\frac{\partial c}{\partial x}}{f'(x)} = -c \cdot MPS \cdot \left[\frac{\frac{i'(x)}{f'(x)}}{1+i(x)}\right]$$

$$+MPC \cdot \left\{\frac{1}{(1+i(x))} \cdot \frac{i'(x)}{f'(x)} \cdot URE - \tau(z) \frac{\frac{\partial \Theta}{\partial x}(x,G)}{f'(x)} - \frac{\frac{\Pi'(x)}{f'(x)}}{\Pi^2(x)} b + z\right\}.$$

To proceed with $\frac{\partial c}{\partial x}/f'(x)$ we have to make several observations. From the total derivation of the Taylor rule:

$$di = \phi_{\Pi} d\Pi + \phi_Y dY$$

$$\Longrightarrow i'(x) = \phi_{\Pi} \Pi'(x) + \phi_Y f'(x)$$

and hence:

$$\frac{i'(x)}{f'(x)} = \phi_{\Pi} \frac{\frac{d\Pi}{dx}}{\frac{df}{dx}} + \phi_Y = \phi_{\Pi} \frac{d\Pi}{df} + \phi_Y$$

$$= \phi_{\Pi} \frac{d\Pi}{dY} + \phi_Y = \phi_{\Pi} \cdot \alpha(x) + \phi_Y$$

where I have used the definition of α 3.5 and the fact that $f = Y$. Similarly:

$$\frac{\Pi'}{f'} = \alpha.$$

Let us define:

$$\Omega \equiv \phi_{\Pi} \cdot \alpha + \phi_Y = \frac{i'(x)}{f'(x)}.$$

Moreover, from the government budget constraint 2.22, from the assumed value of price index in stationary equilibrium $\Pi = 1$ and from the results about $\frac{i'(x)}{f'(x)}$ and $\frac{\Pi'(x)}{f'(x)}$ derived above:

$$\frac{\frac{\partial \Theta}{\partial x}(x, G)}{f'(x)} = \frac{1}{f'(x)} \left(\frac{\partial}{\partial x} \left(\frac{1}{\Pi(x)} - \frac{1}{1 + i(x)} \right) \cdot \bar{B} \right)$$

$$= \frac{1}{f'(x)} \left(-\frac{\Pi'(x)}{\Pi^2(x)} + \frac{i'(x)}{(1 + i(x))^2} \right) \cdot \bar{B}$$

$$= \left(-\alpha + \frac{\Omega}{(1 + i(x))^2} \right) \cdot \bar{B}.$$

All these means that $\frac{\partial c}{\partial x} / f'(x)$ can be rewritten as:

$$\frac{\frac{\partial c}{\partial x}}{f'(x)} = -c \cdot MPS \cdot \left[\frac{\frac{i'(x)}{f'(x)}}{1 + i(x)} \right]$$

$$+ MPC \cdot \left\{ \frac{1}{(1 + i(x))} \cdot \Omega \cdot URE + \tau(z) \left(\alpha - \frac{\Omega}{(1 + i(x))^2} \right) \cdot \bar{B} - \alpha \cdot b + z \right\}. \qquad (5.2)$$

Aggregation over all agents yields:

$$\frac{\frac{\partial C}{\partial x}}{f'(x)} = \left[-\frac{\Omega}{1 + i(x)} \right] \cdot \mathbb{E}_{\mu}(MPS \cdot c)$$

$$+ \frac{1}{(1 + i(x))} \cdot \Omega \cdot \mathbb{E}_{\mu}(MPC \cdot URE) - \left(\frac{\Omega}{(1 + i(x))^2} - \alpha \right) \cdot \bar{B} \cdot \mathbb{E}_{\mu}(MPC \cdot \tau)$$

$$-\alpha\mathbb{E}_{\mu}\left(MPC\cdot b\right)+\mathbb{E}_{\mu}\left(MPC\cdot z\right)$$

which is what we wanted to show.

To get the formula for $\frac{\partial C}{\partial G}$, we need to compute $\frac{\partial c}{\partial G}$ and then we need to aggregate it across all agents. The formed is obtained (analogously to $\frac{\partial c}{\partial x}$) by applying the Implicit Function Theorem to 2.27:

$$\frac{\partial c}{\partial G}=-\left[\frac{1}{u_{cc}\left(c,x\right)+\left(1+i\left(x\right)\right)^{2}\cdot\beta\cdot\mathbb{E}_{z'|z}V_{bb}^{\Lambda}\left(b',z'|G\right)}\right]$$

$$\Big(\left(1+i\left(x\right)\right)^{2}\cdot\beta\cdot\mathbb{E}_{z'|z}V_{bb}^{\Lambda}\left(b',z'|G\right)\cdot\tau\left(z\right)\cdot\frac{\partial\Theta}{\partial G}\left(x,G\right)$$

$$-\left(1+i\left(x\right)\right)\cdot\beta\cdot\mathbb{E}_{z'|z}V_{bG}^{\Lambda}\left(b',z'|G\right)\Big).$$

We now apply the relationship between V_{bb}^{Λ} and u_{cc} given by equation 5.1:

$$\frac{\partial c}{\partial G}=-\left[\frac{1}{u_{cc}\left(c,x\right)+\beta\cdot\left(1+i\left(x\right)\right)^{2}\cdot\frac{1}{\beta\cdot(1+i)^{2}}\cdot\frac{MPC}{MPS}\cdot u_{cc}\left(c,x\right)}\right]$$

$$\cdot\Big(\beta\cdot\left(1+i\left(x\right)\right)^{2}\cdot\frac{1}{\beta\cdot\left(1+i\right)^{2}}\cdot\frac{MPC}{MPS}\cdot u_{cc}\left(c,x\right)\cdot\tau\left(z\right)\cdot\frac{\partial\Theta}{\partial G}\left(x,G\right)$$

$$-\left(1+i\left(x\right)\right)\cdot\beta\cdot\mathbb{E}_{z'|z}V_{bG}^{\Lambda}\left(b',z'|G\right)\Big)$$

$$=-\left(1-\frac{\lambda}{1+i}\right)\cdot MPC\cdot\tau\left(z\right)+\frac{MPS}{u_{cc}\left(c\right)}\cdot\left(1+i\left(x\right)\right)\cdot\beta\cdot\mathbb{E}_{z'|z}V_{bG}^{\Lambda}\left(b',z'|G\right)\qquad(5.3)$$

where I have used the fact that $MPS=1-MPC$, the fact that u_{cc} becomes independent of x under assumed preferences and the fact that from 2.22 we obtain:

$$\frac{\partial\Theta}{\partial G}\left(x,G\right)=1-\frac{\lambda}{1+i}.$$

Aggregation of 5.3 over all agents yields the formula for $\frac{\partial C}{\partial G}$.

Let us consider constrained agents now. Observe, that for those households we have $MPC=1$ and $MPS=0$ by the definition. I will argue, that formulas from Theorem 2 continue to apply for consumers with $b'=-\xi$ if we plug $MPC=1$ and $MPS=0$. Individual consumption is determined directly from the budget constraint:

$$c = \frac{b}{\Pi(x)} + z \cdot f(x) - \tau \cdot \Theta(x, G) + \frac{\xi}{1 + i(x)}.$$

Thus, the partial derivative $\frac{\partial c}{\partial x}$ divided by $f'(x)$ reads:

$$\frac{\frac{\partial c}{\partial x}}{f'(x)} = \frac{-\frac{b \cdot \Pi'(x)}{\Pi^2(x)} + zf'(x) - \tau \cdot \left[-\frac{\Pi'(x)}{\Pi^2(x)} + \frac{i'(x)}{(1+i(x))^2}\right] \cdot \bar{B} + URE \cdot \frac{i'(x)}{(1+i(x))^2}}{f'(x)}$$

where I have used the fact that $b' = URE$ (see equation 3.3). Simplifying and using the fact that in stationary equilibrium $\Pi = 1$:

$$\frac{\frac{\partial c}{\partial x}}{f'(x)} = -b \cdot \alpha(x) + z + \tau \cdot \left[\alpha(x) - \frac{\Omega(x)}{(1 + i(x))^2}\right] \cdot \bar{B} + URE \cdot \frac{\Omega(x)}{(1 + i(x))^2}$$

which is identical to formula 5.2 for the unconstrained agents if we plugged $MPC = 1$ and $MPS = 0$. Similarly, for constrained agents:

$$\frac{\partial c}{\partial G} = -\tau \cdot \frac{\partial \Theta}{\partial G} = -\tau \cdot \left(1 - \frac{\lambda}{1 + i}\right)$$

which is identical to formula 5.3 when $MPC = 1$. All this means that formulas for $\frac{\partial c}{\partial x}$ and $\frac{\partial c}{\partial G}$ for constrained agents are special cases of formulas for the unconstrained agents and thus formulas from Theorem 2 capture the case of constrained agents, too. □

Multiplier formula when $u_{cx} \neq 0$

In this part, I present the formula for the multiplier when condition 2.26 is relaxed. This gives rise to an additional, "mechanical" channel through which fiscal purchases affect private consumption.

I proceed analogously to the proof of Theorem. It is easy to see that the only modification that has to be introduced is associated with term $\frac{\partial C}{\partial x}$. First, I apply the Implicit Function Theorem to 2.27 to get $\frac{\partial c}{\partial x}$:

$$\frac{\partial c}{\partial x} = \left[-\frac{1}{u_{cc}(c, x) + (1 + i(x))^2 \cdot \beta \cdot \mathbb{E}_{z'|z} V_{bb}^{\Lambda}(b', z'|G)}\right] \cdot \left\{u_{cx}(c, x) - i'(x) \cdot \beta \cdot \mathbb{E}_{z'|z} V_b^{\Lambda}(b', z'|G)\right.$$

$$-\left(1+i\left(x\right)\right)^{2}\cdot\beta\cdot\mathbb{E}_{z'|z}V_{bb}^{\Lambda}\left(b',z'|G\right)\cdot\left\{\frac{1}{\left(1+i\left(x\right)\right)}\cdot i'\left(x\right)\cdot URE-\tau\left(z\right)\frac{\partial\Theta}{\partial x}\left(x,G\right)-\frac{\Pi'\left(x\right)}{\Pi^{2}\left(x\right)}b+z\cdot f'\left(x\right)\right\}\Bigg\}.$$

I plug 5.1 into formula for $\frac{\partial c}{\partial x}$ and rearrange to get:

$$\frac{\partial c}{\partial x}=\left[-\frac{1}{u_{cc}\left(c,x\right)+\beta\cdot\left(1+i\left(x\right)\right)^{2}\cdot\frac{1}{\beta\cdot\left(1+i\right)^{2}}\cdot\frac{MPC}{MPS}\cdot u_{cc}\left(c,x\right)}\right]\cdot\Big\{u_{cx}\left(c,x\right)-\beta\cdot i'\left(x\right)\cdot\mathbb{E}_{z'|z}V_{b}^{\Lambda}\left(b',z'|G\right)$$

$$-\beta\cdot\left(1+i\left(x\right)\right)^{2}\cdot\frac{1}{\beta\cdot\left(1+i\right)^{2}}\cdot\frac{MPC}{MPS}\cdot u_{cc}\left(c,x\right)$$

$$\cdot\left\{\frac{1}{\left(1+i\left(x\right)\right)}\cdot i'\left(x\right)\cdot URE-\tau\left(z\right)\frac{\partial\Theta}{\partial x}\left(x,G\right)-\frac{\Pi'\left(x\right)}{\Pi^{2}\left(x\right)}b+z\cdot f'\left(x\right)\right\}\Bigg\}$$

$$=\left[-\frac{1}{u_{cc}\left(c,x\right)+\frac{MPC}{MPS}\cdot u_{cc}\left(c,x\right)}\right]\cdot\Big\{u_{cx}\left(c,x\right)-i'\left(x\right)\cdot\beta\cdot\mathbb{E}_{z'|z}^{\Lambda}V_{b}\left(b',z'|G\right)$$

$$-\frac{MPC}{MPS}\cdot u_{cc}\left(c,x\right)\cdot\left\{\frac{1}{\left(1+i\left(x\right)\right)}\cdot i'\left(x\right)\cdot URE-\tau\left(z\right)\frac{\partial\Theta}{\partial x}\left(x,G\right)-\frac{\Pi'\left(x\right)}{\Pi^{2}\left(x\right)}b+z\cdot f'\left(x\right)\right\}\Bigg\}$$

$$=-\Bigg\{\frac{MPS}{u_{cc}\left(c,x\right)}\cdot\left(u_{cx}\left(c,x\right)-i'\left(x\right)\cdot\beta\cdot\mathbb{E}_{z'|z}V_{b}^{\Lambda}\left(b',z'|G\right)\right)$$

$$-MPC\cdot\left\{\frac{1}{\left(1+i\left(x\right)\right)}\cdot i'\left(x\right)\cdot URE-\tau\left(z\right)\frac{\partial\Theta}{\partial x}\left(x,G\right)-\frac{\Pi'\left(x\right)}{\Pi^{2}\left(x\right)}b+z\cdot f'\left(x\right)\right\}\Bigg\}.$$

Let us concentrate on the following functional form that represents preferences:[26]

$$u\left(c,x\right)=\frac{1}{1-\sigma}\cdot\left(c\cdot\left(1-\frac{\kappa}{q\left(x\right)}\right)\right)^{1-\sigma}$$

where $\sigma>0$ and $\sigma\neq1$. This implies that:

$$u_{cc}\left(c,x\right)=-\sigma\cdot\left(c\cdot\left(1-\frac{\kappa}{q\left(x\right)}\right)\right)^{-\sigma-1}\cdot\left(1-\frac{\kappa}{q\left(x\right)}\right)^{2}$$

and:

$$u_{cx}\left(c,x\right)=\left(c\cdot\left(1-\frac{\kappa}{q\left(x\right)}\right)\right)^{-\sigma}\cdot\frac{\kappa\cdot q'\left(x\right)}{q^{2}\left(x\right)}\cdot\left(1-\sigma\right)$$

[26] I impose $\phi=1$ on 3.4 to obtain the characterization of the "mechanical" channel which is a product of two terms: aggregate component that is equal across all households and aggregated cross-products of individual variables. This formulation allows for better interpretability and enables to express that channel in a similar way to other channels.

This means that:

$$\frac{u_{cx}(c,x)}{u_{cc}(c,x)} = \left(-\frac{c}{\sigma}\right)\cdot(1-\sigma)\cdot\underbrace{\frac{\kappa}{q(x)-\kappa}}_{\equiv\Upsilon(x)}\cdot\frac{q'(x)}{q(x)}.$$

Plugging this result into derivations from the proof of Theorem, leads to the following formula for $\frac{\partial C}{\partial x}/f'(x)$:

$$\frac{\frac{\partial C}{\partial x}}{f'(x)} = \left[(1-\sigma)\cdot\frac{\Upsilon(x)}{q(x)}\cdot\frac{q'(x)}{f'(x)} - \frac{\Omega}{1+i(x)}\right]\cdot\frac{1}{\sigma}\cdot\mathbb{E}_\mu(MPS\cdot c)$$

$$+\frac{1}{(1+i(x))}\cdot\Omega\cdot\mathbb{E}_\mu(MPC\cdot URE) - \left(\frac{\Omega}{(1+i(x))^2}-\alpha\right)\cdot\bar{B}\cdot\mathbb{E}_\mu(MPC\cdot\tau)$$

$$-\alpha\mathbb{E}_\mu(MPC\cdot b) + \mathbb{E}_\mu(MPC\cdot z).$$

Given the general formula for the multiplier 3.1 and that $f' > 0$, $q' < 0$ and $q(x) > \kappa$ (and hence $\Upsilon > 0$) leads to conclusion that the additional, "mechanical" channel described with:

$$(1-\sigma)\cdot\frac{\Upsilon(x)}{q(x)}\cdot\frac{q'(x)}{f'(x)}\cdot\frac{1}{\sigma}\cdot\mathbb{E}_\mu(MPS\cdot c)$$

crowds private consumption out when $\sigma > 1$ and it crowds it in when $\sigma \in (0,1)$.

Calibration of parameter α with the SVAR model

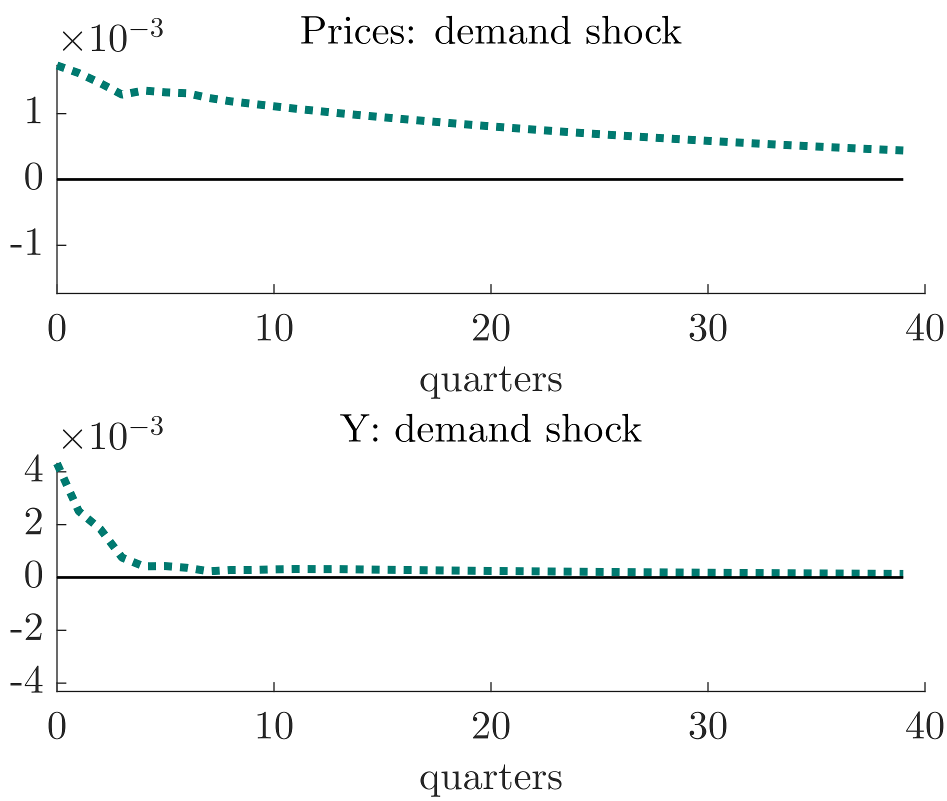
Recall that the value of α is defined as:

$$\alpha \equiv \frac{\frac{d\Pi}{dx}}{\frac{dY}{dx}}$$

and, because x can be thought of as a measure of aggregate demand, α can be interpreted as a measure of comovement of prices and output which results from a positive demand shock. To find an empirical measure of α I use the standard SVAR model (that consists of two variables: output and prices and four lags chosen with standard tests). I estimate the model using quarterly data for Italy from 1985 to 2018 (I take first differences to obtain data used in estimation).

To identify demand shocks I use sign restrictions (it is assumed that a positive demand shock increases both price level and output while a positive supply shock raises output and lowers prices). Parameter α is approximated with the ratio between the value of the impulse response function of price index and the value of the impulse response function of output in period 0. Impulse response functions are reported in Figure 5.1 (I report the median value of all IRFs satisfying the sign restriction at a given date).

Figure 5.1: SVAR simulation: the impact of demand shock on output and prices



Impulse response functions

Figure 5.2: Impulse response functions: baseline scenario, model with HTM agents

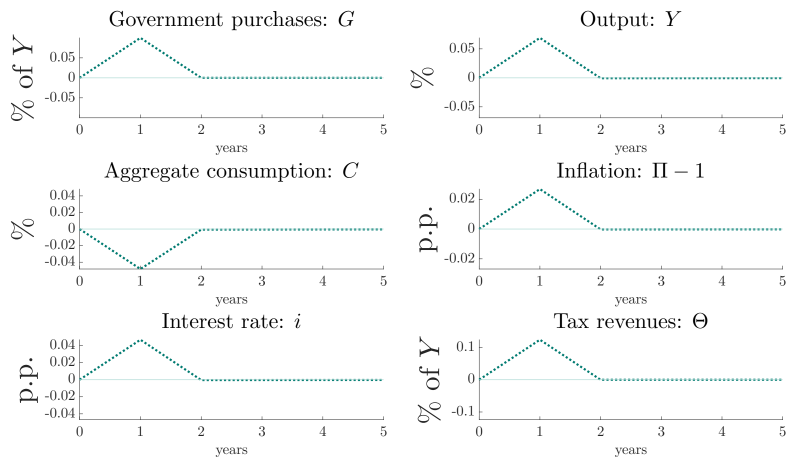


Figure 5.3: Impulse response functions: baseline scenario, model with HTM agents

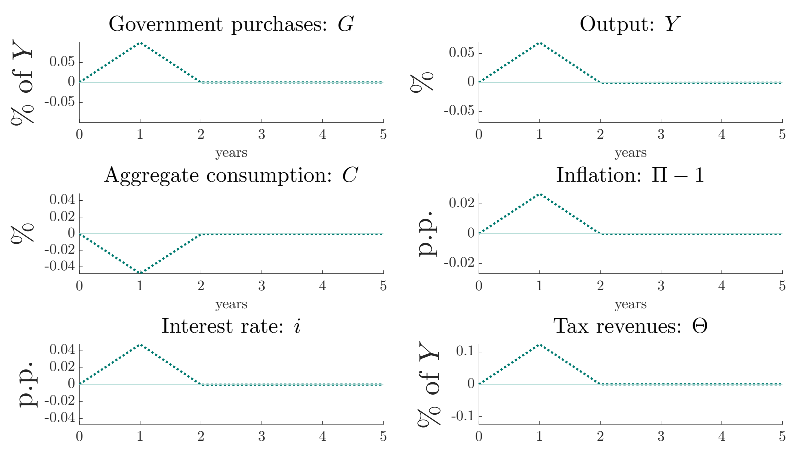


Figure 5.4: Impulse response functions: baseline scenario, model with discount factor heterogeneity

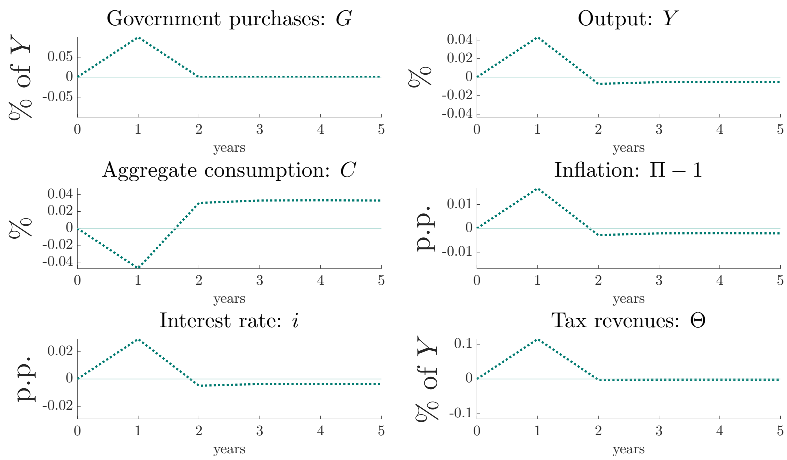


Figure 5.5: Impulse response functions: more active monetary policy, model with HTM agents

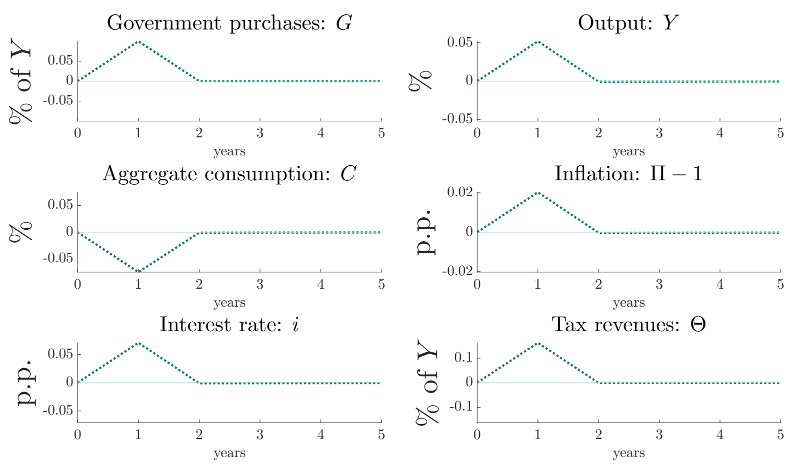


Figure 5.6: Impulse response functions: debt-financed stimulus, model with HTM agents

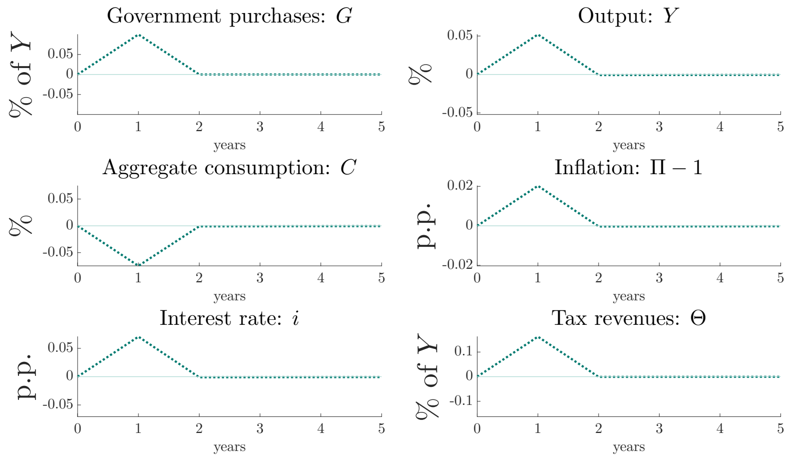


Figure 5.7: Impulse response functions: more persistent shock, model with HTM agents