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Abstract. The economics literature on consumer search has focused on inspection goods, the quality of which is observed before purchase. This paper studies a model of experience goods where consumers search for desired varieties but can observe product quality only after consumption. The model yields price and welfare results that are contrary to those for inspection goods. Specifically, we find that equilibrium price may rise even when search intensity is higher and, under plausible conditions, both consumer and social welfare are initially increasing in search cost. Our analysis shows that quality observability is a key determinant of how search markets function.

Keywords: consumer search, experience goods, inspection goods, product quality, search cost

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1. INTRODUCTION

Consumers often conduct costly search in order to find price and product information. The economics literature on consumer search, which is by now extensive, makes the standard assumption that product quality is uncovered from search: the products are "search" or "inspection" goods.[1] However, in many situations, although consumers can find price and variety that match their needs through search, they are unable to observe product quality before purchase. For instance, consumers could be searching for a product with desired price and product features, such as a tour package at a certain destination, a specific restaurant, or a particular-type of furniture; but the quality of the product is learned only after consumption. Despite their prevalence, little is known about how search markets for such "experience" goods operate. The purpose of this paper is to advance our understanding in this regard.

We present a model in which firms produce differentiated product varieties that may also differ in quality. Consumers have heterogeneous preferences for variety but all prefer a high-quality product (H) to a low-quality one (L). Product quality is stochastic: the probability that a firm's product is of high quality is β—which may differ across firms—with a higher-quality firm having a higher β. We thus draw a distinction between firm quality (β) and product quality (q). The market operates for two time periods. At the beginning of period 1, each firm can make a private investment to increase its quality, with heterogeneous investment costs. Average firm quality in period 1 is determined by the portion of firms that make the quality investment. Following the first-period consumers' purchases, firms may establish reputation about their quality (β) to the next generation of consumers.

In each period, once the set of active firms and their average quality is determined, they simultaneously choose prices, followed by consumer search across firms. Each search, which

[1] Starting from the seminal work of Stigler (1961), the economics of consumer search has advanced in the directions of searching for low price among homogeneous sellers (e.g., Stahl, 1989) or for desired variety under horizontal differentiation (e.g., Wolinsky, 1986). More recent search models have considered vertical differentiation where, however, product quality is revealed from search (see discussions later).

costs s, enables a consumer to discover the match value of a firm's product—provided the product is of high quality—and its price. However, high- and low-quality products from the same firm have the same appearance when searched by a consumer,[2] and the quality of the product is detected only after consumption.

We start with a preliminary analysis where the market has a given average firm quality. In addition to its intrinsic interest, this provides the basis for the analysis with endogenous firm quality. We show that there is a uniform-price equilibrium, where consumers conduct (random) sequential search with a reservation value. The equilibrium has interesting similarities and differences as compared to that for inspection goods.[3] The reservation value is determined similarly as in models of search for horizontally differentiated products (e.g., Wolinsky, 1986), adjusting for the fact that a match value is realized only if the product is of high quality, the probability of which is given by the average firm quality in the market. Remarkably, given the average firm quality, consumers have the same reservation value in their search for experience and inspection goods. However, an increase in the average firm quality has opposite effects on the equilibrium price for the two types of goods: for both of them a higher average firm quality will motivate consumers to search more intensively (as if search were more efficient), leading to intensified price competition; but for experience goods it has the additional effect of making the demand for each firm less elastic, and this demand effect dominates the competition effect—under plausible conditions—so that equilibrium prices are higher,[4] in contrast to the outcome for inspection goods.

We next return to the full analysis of our model with endogenous firm quality and reputation. To capture the idea of firm reputation in an especially convenient way, we assume that (some) period-1 consumers will make public their product reviews that are observed

[2] One natural interpretation for this is that an H product has no defect, whereas an L product contains a hidden defect that decreaes the product's value to the consumer.

[3] For inspection goods, a consumer will discover the quality of a firm's product—$q = H$ or L—when searching the firm. An inspection good is otherwise the same as an experience good in our model. In particular, in both cases a high-quality firm has a high β.

[4] As we shall show, despite the higher prices, an increase in the averge firm quality in the market nevertheless will result in higher consumer and social welfare.

by consumers in period 2, which enables the latter to infer the quality of a firm before search. Then, because consumers will have higher expected surplus when searching firms with higher average quality, in period 2 consumers will only search high-quality firms, even though they still cannot detect product quality before purchase. Thus, a high-quality firm will have a higher discounted sum of profits due to its reputation, which provides the incentive (return) for firms to improve quality early on.[5] In equilibrium, a firm will invest to become a high quality producer if and only if its investment cost realization does not exceed some cutoff value. This cutoff determines the average firm quality in period 1, and only high-quality firms will be active sellers in period 2. Consumer search and price competition in both periods are then determined similarly as in the benchmark case. What is most striking about the equilibrium is that search friction has non-monotonic impact on consumer and social welfare: they both first increase and eventually decrease in search cost. An increase in search cost lowers search efficiency (and price competition) under a given average firm quality, but the resulting higher price and profit increase the return to being a high quality firm, motivating more firms to invest in quality, leading to a higher average firm quality in period 1. We demonstrate that, under plausible conditions, the quality effect dominates the search efficiency effect when search cost is (sufficiently) low while the converse is true when search cost is relatively high.

Importantly, in our model if consumers were able to detect product quality before purchase, then both consumer and social welfare would monotonically decrease in search cost. For inspection goods, a higher search cost also increases average firm quality in period 1 by boosting the returns to reputation, despite reducing search efficiency (and price competition). Why, then, is the relationship between search cost and welfare sharply different for the two types of goods? As we illuminate through our analysis, contrary to experience goods, for inspection goods consumers could avoid the utility loss from a low quality product by not purchasing it; hence consumers searching for inspection goods do not gain from a higher firm quality in a way as they would searching for experience goods. On

[5] In experince-goods markets, it is well recognized that reputation can furnish incentives for firms to provide high-quality products (e.g., Choi, 1998; Shapiro, 1983; Wernerfelt, 1988).

the other hand, for both types of goods a higher search cost—despite its indirect positive impact through the higher average firm quality—would reduce search efficiency. Therefore, as in the existing literature, for inspection goods an increase in search cost always harms consumers and social welfare.

We further compare the market provision of product quality with the social optimum. We show that equilibrium investment for product quality is (socially) deficient when search cost is low, which is consistent with the result from the economics literature on experience goods where—without search frictions—firms typically invest too little in product quality (e.g., Riordan, 1986; Shapiro, 1982). However, we also find that investment for product quality can be socially excessive when search cost is relatively high. To understand this result, notice that an increase in the number of firms that make quality investment will impact welfare positively by boosting average firm quality in period 1 but negatively by raising total investment cost. When s is low, consumers will find high match values through search, and they hence benefit more from higher firm quality because a high match value is not realized if product quality turns out to be low; but the private investment incentive is low due to low profit from being a high-quality firm. Consequently, the market under-provides product quality when search cost is low; and the opposite can be true when search cost is relatively high.

We finally extend our model to analyze the role of an intermediary, which can list sellers on its search platform by charging each of them a fixed fee and a percentage of its revenue. The intermediary can improve welfare by screening out low-quality sellers, especially when it can commit to a relatively small listing space on the platform and hence charge a high fixed fee to a listing seller. The high fixed fee deters the low-quality sellers who are unable to earn repeat business, resulting in a separating equilibrium where only high-quality firms will be active in period 1 (and also in period 2).[6] However, if the intermediary is unable to commit to a relatively small listing space on its search platform, then it is possible that both high- and low-quality firms may pay the intermediary's optimally-chosen fees and be

[6] Notice that the intermediary can naturally serve as a carrier of sellers' reputation, recording/publishing consumers' product reviews.

active in period 1, whereas fewer firms will invest in quality because the intermediary's fee lowers sellers' investment return. In this case the intermediary will reduce welfare.

To the best of our knowledge, ours is the first model of sequential search for experience goods. Wolinsky (1986) is an early contribution to the study of consumer search for horizontally differentiated products (for related contributions, see, e.g., Anderson and Renault, 1999; Armstrong et al., 2009; Haan and Moraga-González, 2011; Rhodes, 2011). Recent papers have analyzed consumer search across vertically-differentiated firms (e.g., Athey and Ellison, 2011; Chen and He, 2011), under both horizontal and vertical differentiation (e.g., Eliaz and Spiegler, 2011; Bar-Isaac et al., 2012; Chen and Zhang, 2018), or with investment on product quality (e.g., Fishman and Levy, 2015; Moraga-González and Sun, 2019)[7]. All of these and other studies on consumer search assume that product quality is known before consumers make purchases. Our model advances the literature in an important new direction, and our results provide new perspectives on how search frictions impact market performance.

The Internet, together with new information technology, has drastically reduced search cost for many products. In the existing consumer search literature, reductions in search cost generally benefit consumers and increase social welfare. This is generally true even when a lower search cost sometimes leads to higher market prices (e.g., Chen and Zhang, 2011; Bar-Isaac et al., 2012; Zhou, 2014; Moraga-González, et al., 2017; Choi, et al., 2018), or when it lowers product quality (e.g., Fishman and Levy, 2015; Moraga-González and Sun, 2019). Our model also suggests that there are important consumer and efficiency benefits from reducing search frictions, but it cautions that for experience goods, (further) decreases in search cost can actually reduce consumer and social welfare. In fact, in our model the presence of some search friction is *necessary* in order for either consumer or social welfare to be maximized.[8]

[7] Relatedly, Wolinsky (2005) and Moraga-González and Sun (2018) study consumer search models in which sellers exert costly efforts to create service plans.

[8] This has an interesting connection to the result in Grossman and Stiglitz (1981) on the impossibility of the informationally efficient markets, even though our model and mechanism are very different from theirs.

In the rest of the paper, we describe our model in section 2, analyze the benchmark under a given average firm quality in section 3, and conduct the analysis with endogenous firm quality and reputation in section 4. We extend the model to include a search intermediary in section 5, and conclude in section 6. A numerical example is presented in the Appendix to illustrate results.

2. THE MODEL

The market contains a unit mass of firms and operates for two periods, 1 and 2. A firm's product quality, q, can be either high (H) or low (L). The probability that a firm's product is of high or low quality is respectively β and $1-\beta$, where $\beta \in \{\beta_H, \beta_L\}$ and $0 \leq \beta_l < \beta_h \leq 1$. Initially, all firms have $\beta = \beta_l$; but at the beginning of period 1, each firm can privately make a one-time investment that costs x, to permanently increase its quality from β_l to β_h, where x is a privately-observed random draw from distribution $G(x)$, with density $g(x) > 0$ on $[0, \bar{x}]$ for some $\bar{x} \in (0, \infty)$. Each firm's quality (β) is then determined and remains as the firm's private information. Production cost is normalized to zero.

In each period, a distinct unit mass of consumers are present in the market. Each consumer desires to purchase one unit of the product. A consumer's valuation of an H product is u, which is a random draw from cumulative distribution function $F(u)$, and her valuation of an L product is normalized to zero. Hence firms are differentiated both horizontally and vertically, respectively because each consumer's u is independently drawn across firms and because a high-quality firm $(\beta = \beta_h)$ is more likely to produce a high-quality product. We assume that $F(u)$ has corresponding density $f(u) > 0$ on $[0,\ \bar{u}]$, with $0 < \bar{u} < \infty$.

To focus on experience goods, we assume that an H product and an L product from the same firm have the same appearance. Each consumer needs to search a firm in order to discover the realization of her u for the firm's product—provided its quality is H—and the firm's price, but she learns the quality of the product only after purchase, with the purchased good consumed in the same period. Each search costs the consumer $s > 0$. In each period, firms simultaneously and independently choose prices, after which consumers may conduct sequential search and make purchases. To capture the idea that firms can establish quality reputation, we assume that consumers of period 1 will furnish product reviews about whether $q = H$ or L for each firm's product.[9] In period 2, a new cohort of consumers, who replace the first-period consumers, can observe these product reviews before conducting searches. Values in period 2, when discounted to period 1, have a common discount factor $\delta > 0$.[10]

A firm's strategy specifies its investment decision based on its investment cost x and its prices p_1 and p_2 (possibly contingent on its β) in the two periods. A period-1 consumer's strategy specifies her search and purchase decisions, whereas period-2 consumers may base these decisions also on observed product reviews. At a perfect Bayesian equilibrium, each firm's strategy maximizes its discounted sum of profit, holding beliefs about other firms' and consumers' strategies; each consumer's strategy maximizes her surplus (at any point of her sequential decision process), holding beliefs about firms' qualities and prices; and beliefs are consistent with strategies along the equilibrium path.

One desirable feature of our model is that it can be readily adapted to the study of "inspection goods"; in fact, if consumers were able to observe product quality (q) when searching the firm, our model would become one of search for inspection goods. In the case of inspection goods, we may interpret β as the probability that the firm's product meets each consumer's needs, so that a higher quality firm—whose product possibly has broader appeal to consumers— has a higher β, as in Chen and He (2011). Our formulation allows us to compare results for experience and inspection goods in a unified framework, and to clarify how product quality observability matters for the functioning of search markets.

We analyze our model in two steps. First, as a benchmark, we study in section 3 consumer search and price competition in a single period of our model in which given portions of G and $1 - G$ firms respectively have $\beta = \beta_h$ and $\beta = \beta_l$, for $G \in [0, 1]$. This analysis has its

[9] Our analysis will be the same whether all period-1 consumers or a randomly-drawn portion of them will publically reveal their product experiences. For ease of exposition, we assume all of them will.

[10] We can consider period 2 as combining all possible future periods after period 1 for which firms have established quality reputation, in which case δ could be higher than 1.

independent interest, and it will provide the basis for the full analysis of our model with endogenous G and with two periods in section 4.

3. SEARCH AND PRICE UNDER GIVEN AVERAGE FIRM QUALITY

Consider a single period of our model, in which a given $G \in [0,1]$ portion of firms have $\beta = \beta_h$. The average firm quality in the market is then also given:

$$\gamma = G\beta_h + (1-G)\,\beta_l. \tag{1}$$

For given γ, we first consider consumers' search strategy. As in search models for inspection goods in which firms are horizontally and vertically differentiated (e.g., Eliaz and Spiegler, 2011; Chen and Zhang, 2018), we focus on a uniform-price equilibrium where all firms charge the same price p_γ, and shall we discuss the motivation for this equilibrium when characterizing p_γ later. Each consumer's equilibrium search strategy, holding belief p_γ, solves the following dynamic search problem:

$$V_\gamma = \max_{u_\gamma}\left\{-s + [1-F(u_\gamma)]\,\frac{\int_{u_\gamma}^{\bar{u}}(\gamma u - p_\gamma)\,f(u)\,du}{[1-F(u_\gamma)]} + F(u_\gamma)\,V_\gamma\right\}, \tag{2}$$

where V_γ is a consumer's (maximized) continuation value from searching a randomly-selected firm whose expected quality and price are respectively γ and p_γ. The consumer will sequentially and randomly search sellers, and will purchase when finding a seller whose product's value u reaches her optimal reservation value u_γ (provided the seller's price is indeed p_γ). Each search costs s; and, under reservation value u_γ, the search will lead to a purchase with probability $[1-F(u_\gamma)]$ while the consumer will search again to receive continuation value V_γ with probability $F(u_\gamma)$. The consumer's optimal reservation value u_γ thus satisfies the first-order condition:

$$-(\gamma u_\gamma - p_\gamma)\,f(u_\gamma) + f(u_\gamma)\,V_\gamma = 0.$$

It follows that the consumer's continuation value, which is also the surplus for a consumer to engage in search or to participate in the market, is

$$V_\gamma = \gamma u_\gamma - p_\gamma, \tag{3}$$

and in equilibrium $V_\gamma \geq 0$ for consumers' participation in the market. Combining (2) and (3), we obtain

$$s = -\left[1 - F\left(u_\gamma\right)\right] V_\gamma + \int_{u_\gamma}^{\bar{u}} \left(\gamma u - p_\gamma\right) f\left(u\right) du \ ,$$

which can be re-stated as the following condition for the optimal reservation value in search:

$$\gamma \int_{u_\gamma}^{\bar{u}} \left(u - u_\gamma\right) f\left(u\right) du = s. \tag{4}$$

The left-hand side of equation (4) is the consumer's expected benefit from one more search when she is currently at a seller with u_γ, which decreases in u_γ, while s is the marginal cost of the extra search. The condition extends the optimal search rule for horizontally differentiated products (e.g., Wolinsky, 1986), which is a special case of equation (4) when $\gamma = 1$. As we clarify shortly, when $s < \bar{s}$—which we shall assume—for some positive number $\bar{s}$, there exists a unique $u_\gamma \in (0, \bar{u})$ that solves (4) and indeed $V_\gamma > 0$.

Consider next the pricing strategy by firms. At the proposed uniform-price equilibrium, consumers will have reservation value u_γ at any firm she searches that charges price p_γ, holding the equilibrium belief that all firms have expected quality γ and price p_γ. Now suppose that a firm deviates to a price p. The consumer's purchase decision at this firm will partly depend on her belief about the firm's β, as well as on her belief about other firms' prices and qualities following the deviation. The concept of perfect Bayesian equilibrium, which we adopt, does not constrain beliefs off the equilibrium path, potentially resulting in multiple equilibria. To overcome this well-known difficulty in dynamic games of imperfect information, we assume that consumers hold "passive belief" off the equilibrium path: at the deviating firm with price p, each consumer believes that (i) the firm deviating to price p continues to have the expected quality γ, and (ii) any other firm continues to charge price p_γ with expected quality γ.

Part (ii) of the passive belief follows from the standard assumption in consumer search for differentiated products (e.g., Wolinsky, 1986), where following the deviation by one firm the other firms are expected to continue with the equilibrium price; and the expected quality of any such firm would then continue to be γ. Part (i) of the assumption is motivated by the following consideration. In our model, if a price deviation is profitable for one β

type, it must be equally profitable for the other β type. Thus, if the consumer believes the expected quality of the deviating firm to be, say, $B(p, p_\gamma)$, this belief can be consistent with profitable deviation only if $B(p, p_\gamma) = \gamma$. It is thus reasonable to assume that, observing a deviating price p, consumers will hold belief $B(p, p_\gamma) = \gamma$. In other words, we require consumers' off-equilibrium belief to be consistent with firms' incentives: $B(p, p_\gamma)$ is equal to the expected quality of firms that can (weakly) benefit from the deviation.[11]

Under passive belief, the consumer, who has arrived at a firm with price p and value u, will purchase from the firm if

$$\gamma u - p \geq \gamma u_\gamma - p_\gamma \geq 0.$$

Thus, the demand for the firm with price p from any visiting consumer, given that all other firms charge p_γ, is

$$D(p, p_\gamma) = 1 - F\left(\frac{\gamma u_\gamma + p - p_\gamma}{\gamma}\right),$$

with $D(p_\gamma, p_\gamma) = 1 - F(u_\gamma)$. The profit for a firm of quality β from any visiting consumer, $\pi(p, p_\gamma) = pD(p, p_\gamma)$, is maximized when p satisfies

$$\frac{\partial \pi(p, p_\gamma)}{\partial p} = 1 - F\left(\frac{\gamma u_\gamma + p - p_\gamma}{\gamma}\right) - p\frac{1}{\gamma}f\left(\frac{\gamma u_\gamma + p - p_\gamma}{\gamma}\right) = 0.$$

At the uniform-price equilibrium, $p = p_\gamma$, and

$$p_\gamma = \gamma\frac{1 - F(u_\gamma)}{f(u_\gamma)}. \tag{5}$$

Moreover, if $1 - F(u_\gamma)$ is log-concave, or, equivalently, the inverse hazard rate is (weakly) decreasing:

$$\lambda'(u) \leq 0 \quad \text{for } \lambda(u) \equiv \frac{1 - F(u)}{f(u)}, \tag{6}$$

[11] In the literature on experience goods, firms can sometimes signal their quality through price and other devices (e.g., Choi, 1998; Riordan, 1988; Shapiro, 1983; Wernerfelt, 1988). In our model, given their qualities, firms are symmetric in all other aspects and there exist no signals that could potentially separate them. We will show formally in Proposition 1 below that there can be no "separating" equilibrium in our model for a given γ.

then $\pi(p, p_\gamma)$ is single-peaked at p_γ, the uniform-price equilibrium with $p = p_\gamma$ exists uniquely, and p_γ is (weakly) lower when consumers search more intensively (i.e., u_γ is higher). Moreover, at the unique p_γ,

$$V_\gamma = \gamma u_\gamma - p_\gamma = \gamma u_\gamma - \gamma \lambda(u_\gamma) = \gamma[u_\gamma - \lambda(u_\gamma)].$$

The highest possible search cost ($\bar{s}$) and its corresponding (lowest possible) reservation value (u_0) are defined as

$$\bar{s} \equiv \gamma \int_{u_0}^{\bar{u}} (u - u_0) f(u)\, du, \qquad \text{where} \qquad u_0 \equiv \frac{1 - F(u_0)}{f(u_0)}. \tag{7}$$

Then, for any $s < \bar{s}$, there is a unique $u_\gamma \in (0, \bar{u})$ that solves (4) and $V_\gamma > 0$, so that consumers will indeed engage in search when average firm quality in the market is $\gamma \in [\beta_l, \beta_h]$. We shall maintain assumptions (6) and $s < \bar{s}$ throughout the paper.

In equilibrium, each firm's profit is

$$\pi_\gamma = \sum_i [F(u_\gamma)]^i p_\gamma D(p_\gamma, p_\gamma) = \gamma \lambda(u_\gamma),$$

where $[F(u_\gamma)]^i$ is the number of consumers for whom the seller is their i's visit. We measure consumer welfare and social welfare respectively by aggregate consumer surplus and total surplus. With a unit measure of consumers and of firms, industry profit, consumer welfare and social welfare, for a market with average firm quality γ, are respectively:

$$\Pi_\gamma = \gamma \lambda(u_\gamma); \qquad V_\gamma = \gamma[u_\gamma - \lambda(u_\gamma)]; \qquad W_\gamma = \gamma u_\gamma. \tag{8}$$

The result below summarizes the above discussions and further establishes that there can be no equilibrium in which firms with different β charge different prices. At a potential "separating equilibrium" where β_h and β_l firms respectively charge $p_h \neq p_l$, following a deviating price p in the (small) neighborhoods of p_h or p_l, an assumption analogous to passive belief under the uniform-price equilibrium is that consumers believe the deviation to have been made by a β_h or β_l firm, respectively.

Proposition 1 *There is a unique uniform-price equilibrium in the experience-goods market where average firm quality is γ. At the equilibrium, consumers search sequentially with* *reservation value u_γ and each firm charges price p_γ. Moreover, there can be no equilibrium where β_h and β_l firms respectively charge $p_h \neq p_l$, if consumers believe that a deviating price p in the neighborhoods of p_h or p_l is respectively made by a β_h or β_l firm.*[12]

Proof. It suffices to show that there can be no equilibrium where β_h and β_l firms charge different prices. Suppose, to the contrary, that there is an equilibrium where β_h and β_l firms charge $p_h \neq p_l$. Then the equilibrium profit for the two types of firms must be equal, $\pi_h = \pi_l$, because otherwise a firm of the type with a lower profit, say, β_l, can deviate to p_h and increase its profit. So suppose $p_h \neq p_l$ but $\pi_h = \pi_l$. We show that this leads to a contradiction.

Let each consumer's reservation values be u_h and u_l at a β_h- and β_l-firm, respectively. Then, since the consumer has the same continuation value at both types of firms, we have

$$\beta_h u_h - p_h = \beta_l u_l - p_l. \tag{9}$$

Moreover, reservation values u_h and u_l satisfy the following equation

$$G \int_{u_h}^{\bar{u}} \beta_h \left(u - u_h\right) f\left(u\right) du + \left(1 - G\right) \int_{u_l}^{\bar{u}} \beta_l \left(u - u_l\right) f\left(u\right) du = s, \tag{10}$$

in which the LHS is the expected gain from one more search: When the consumer is currently at a β_h-firm (having u_h and p_h), with probability G she will encounter another β_h-firm with gain $(\beta_h u - p_h) - (\beta_h u_h - p_h) = \beta_h \left(u - u_h\right)$, conditional on her $u > u_h$ from the new firm searched, while with probability $(1 - G)$ the consumer will encounter a β_l-firm with gain $(\beta_l u - p_l) - (\beta_h u_h - p_h)$, which equals $\beta_l \left(u - u_l\right)$ from (9), conditional on $u > u_l$. The argument is similar when the consumer is currently at a β_h-firm (having u_l and p_l).

Next, given consumers' search behavior and the pricing strategies of other firms, if a β_h firm deviates with price p in the neighborhoods of p_h, under our assumption consumers will believe that the deviation is made by the β_h firm. Hence, at the deviating price p, a consumer

[12] Our result that no separating equilibrium can exist also holds if, following a deviating price p at the proposed separating equilibrium, consumers believe that the deviating firm has quality γ, or, more generally, their off-equilibrium beliefs are respectively α_h or α_l for a deviating price p in the neighborhoods of p_h or p_l, with $\alpha_h \geq \alpha_l$ and $\beta_h/\alpha_h \geq \beta_l/\alpha_l$.

with value u at the β_h firm will purchase if $\beta_h u - p \geq (G)[\beta_h u_h - p_h] + (1-G)[\beta_l u_l - p_l] = \beta_h u_h - p_h$. The firm's demand from any visiting consumer is thus $1 - F\left(u_h + \frac{p-p_h}{\beta_h}\right)$. Solving $\max_p p\left[1 - F\left(u_h + \frac{p-p_h}{\beta_h}\right)\right]$, with $p = p_h$ in equilibrium, we obtain $p_h = \beta_h \lambda(u_h)$. Similarly, $p_l = \beta_l \lambda(u_l)$. Therefore

$$\beta_h u_h - p_h = \beta_h [u_h - \lambda(u_h)]; \quad \beta_l u_l - p_l = \beta_l [u_l - \lambda(u_l)],$$

and from (9) we obtain

$$\beta_h [u_h - \lambda(u_h)] = \beta_l [u_l - \lambda(u_l)]. \tag{11}$$

Furthermore:

$$\pi_h = \frac{p_h [1 - F(u_h)]}{1 - (G) F(u_h) - (1-G) F(u_l)}, \qquad \pi_l = \frac{p_l [1 - F(u_l)]}{1 - (G) F(u_h) - (1-G) F(u_l)}. \tag{12}$$

If $p_h > p_l$, then $\pi_h = \pi_l$ implies $u_h > u_l$, which further implies $\beta_h [u_h - \lambda(u_h)] > \beta_l [u_l - \lambda(u_l)]$ since $\lambda'(\cdot) \leq 0$. This contradicts (11). If $p_h = \beta_h \lambda(u_h) < p_l = \beta_l \lambda(u_l)$, then from $\beta_h > \beta_l$ and $\lambda'(\cdot) \leq 0$ we have $u_h \geq u_l$ and hence

$$\beta_h u_h - \beta_h \lambda(u_h) > \beta_l u_l - \beta_l \lambda(u_l),$$

again contradicting (11). ■

A "separating" equilibrium with different prices for different β types cannot exist in our model, because there is nothing to enable such separation. Given average firm quality, the equilibrium in our search model of experience goods is essentially unique and is the uniform-price equilibrium.[13]

3.2 Impacts of Search Cost and Average Firm Quality

We next consider how the equilibrium may vary as search cost s or average firm quality γ changes. From (4), consumers' reservation value, u_γ, increases in γ and decreases in s. Because $p_\gamma = \gamma \lambda(u_\gamma)$ and $\lambda'(\cdot) \leq 0$, it follows from (8) that, given γ, p_γ and Π_γ increase

[13] Search models are known to contain an equilibrium where all firms charge very high prices and no consumer engages in search. We do not consider such "uninteresting" equilibrium.

in s whereas V_γ and W_γ decrease in s. Intuitively, a higher search cost reduces consumer search efficiency, which not only reduces consumers reservation value in search but also lessens competition and raises price. The higher price and lower search efficiency reduce consumer surplus, and the lower search efficiency also reduces social welfare; whereas higher price boosts profit.

From (8), clearly V_γ and W_γ increase in γ, the average quality of firms in the market. The effects of γ on price (and profit) are less obvious, as we can see from

$$\frac{\partial p_\gamma}{\partial \gamma} = \lambda\left(u_\gamma\right) + \gamma\lambda'\left(u_\gamma\right)\frac{\partial u_\gamma}{\partial \gamma},$$

where the first and the second terms on the RHS reflect, respectively, the positive (direct) demand effect and the negative (indirect) search effect on p_γ from an increase in γ. A higher γ lowers the price elasticity of demand for given u_γ[14]:

$$\eta = -\frac{\partial D\left(p, p_\gamma\right)}{\partial p}\frac{p}{D}\bigg|_{p=p_\gamma} = \frac{p_\gamma}{\gamma\lambda\left(u_\gamma\right)},$$

which positively impact price; but it also increases the search reservation value u_γ and negatively impacts p_γ due to $\lambda'\left(u_\gamma\right) \leq 0$. Because

$$\frac{\partial u_\gamma}{\partial \gamma} = \frac{\int_{u_\gamma}^{\bar{u}}\left[1 - F\left(u\right)\right]du}{\gamma\left[1 - F\left(u_\gamma\right)\right]} < \frac{\bar{u} - u_\gamma}{\gamma},$$

a sufficient—but not necessary—condition for $\frac{\partial p_\gamma}{\partial \gamma} > 0$ is

$$\frac{1}{\bar{u} - u_\gamma} \geq -\frac{\lambda'\left(u_\gamma\right)}{\lambda\left(u_\gamma\right)}, \tag{13}$$

which holds, for example, if $F\left(u\right)$ is a uniform or exponential distribution. The proceeding discussions lead to the following:

Corollary 1 *In equilibrium: (i) given average firm quality γ, price and profit increase, while consumer and social welfare decrease, in search cost s; (ii) given s, a higher γ leads to higher price and profit if (13) holds, even though it intensifies search and price competition (i.e., u_γ is higher and $\lambda\left(u_\gamma\right)$ lower); moreover, V_γ and W_γ also increase in γ.*

[14] When γ is higher, the quality-adjusted price $\frac{p}{\gamma}$ is lower and a marginal change in p is associated with less change in $\frac{p}{\gamma}$ and hence leads to less change in the quantity demanded.

With exogenously-given firm quality for experience goods, the effects of search friction on price and welfare are similar to those in search markets for inspection goods.[15] Notably, p_γ increases in γ under (13), despite increased consumer search and price competition; this is in contrast to the result under search for inspection goods, which we turn to next.

3.3 Comparing with Search for Inspection Goods

To make comparison, we now consider inspection goods by assuming that, when searching a firm, a consumer will learn whether the firm's q is H or L, in addition to uncovering its price and u. Everything else is the same as in subsection 3.1. In particular, $\beta \in \{\beta_l, \beta_h\}$ continues to be a firm's quality and remains to be its private information, with γ being the average firm quality in the market as defined in (1). We again look for a uniform-price equilibrium, where each firm charges price p_γ^I. As in subsection 3.1, consumers' optimal search follows a reservation-value strategy, with the optimal reservation value u_γ^I satisfying

$$\gamma \int_{u_\gamma^I}^{\bar{u}} \left(u - u_\gamma^I\right) f(u)\, du = s.$$

Interestingly, this condition is identical to condition (4) for experience goods. This is because when arriving at a firm with $u = u_\gamma^I = u_\gamma$, the expected marginal benefit of an additional search is the same under inspection and experience goods.[16] In other words, given γ and s, $u_\gamma = u_\gamma^I$.

To determine the demand for each firm, suppose a firm deviates with price p. The passive belief assumption is now needed only for its part (ii)—other firms' price is still p_γ^I—because when searching the firm a consumer learns its product quality q. A visiting consumer will purchase from the firm if she finds $q = H$ (which occurs with the firm's probability β) and

$$u - p \geq u_\gamma^I - p_\gamma^I.$$

[15] As we shall show in section 4, under endogenous firm quality and reputation, search costs have rather surprising welfare effects for experience goods, in contrast to those for inspection goods.

[16] However, as we shall see shortly, equilibrium consumer and social welfare are both higher for inspection than for experience goods, because for the former consumers can detect and hence avoid the utility loss from consuming a low quality product. In other words, consuming an L product, which has $u = 0$, is as if the consumer did not search optimally; and hence welfare is lower for experience goods.

The firm's demand from any visiting consumer is thus

$$D^I\left(p, p_\gamma^I\right) = \beta\left[1 - F\left(u^I + p - p_1^I\right)\right],$$

and it chooses p to maximize $pD^I\left(p, p_\gamma^I\right)$, which, in equilibrium, leads to

$$p_\gamma^I = \frac{1 - F\left(u_\gamma^I\right)}{f\left(u_\gamma^I\right)} = \lambda\left(u_\gamma^I\right).$$

Since a random visit by a consumer to a firm will on average result in a purchase with probability $\gamma\left[1 - F\left(u^I\right)\right]$, and since all consumers—whose total mass is one—purchase, the equilibrium output of a firm with quality β is $\frac{D^I\left(p^I, p_\gamma^I\right)}{\gamma[1-F(u^I)]} = \frac{\beta}{\gamma}$, and hence the firm's equilibrium profit is $\pi^I(\beta) = \frac{\beta}{\gamma}\lambda\left(u_\gamma^I\right)$. Thus, a firm will have a higher profit than an average firm if its quality β is higher than the market average, in contrast to the case of experience goods where a firm's equilibrium profit is independent of its β.

Notice that the price elasticity of demand here is independent of γ, in contrast to that for experience goods, which explains why p_γ^I does not depend on γ but p_γ does. Therefore, for inspection goods it is always true that

$$\frac{\partial p_\gamma^I}{\partial \gamma} = \lambda'\left(u_\gamma^I\right)\frac{\partial u_\gamma^I}{\partial \gamma} \leq 0,$$

in contrast to $\frac{\partial p_\gamma}{\partial \gamma} > 0$ for experience goods under condition (13).

In equilibrium, industry profit, consumer surplus, and total welfare are respectively

$$\Pi_\gamma^I = \lambda\left(u_\gamma^I\right); \qquad V_\gamma^I = u_\gamma^I - \lambda\left(u_\gamma^I\right); \qquad W_\gamma^I = u^I\ . \tag{14}$$

Since $u_\gamma^I = u_\gamma$, comparing p_γ^I with p_γ and (14) with (8), we have:

Proposition 2 *Given γ and s, consumers search with the same reservation value for inspection and experience goods, but V, Π, and W are all lower for the latter. Higher γ leads to higher p for experience goods under condition (13) but to lower p for inspection goods. Moreover, a firm's profit increases in its β under inspection goods but is independent of its β under experience goods.*

For inspection goods, a higher average firm quality (γ) in the market implies that consumers will have higher expected benefit from a search, because they are more likely to find an H-product. This boosts consumers' search incentive, as reflected by their higher search reservation value, which increases competition and leads to lower equilibrium price. Because consumers can detect product quality before purchase, an increase in γ will not affect a consumer's demand for a firm. By contrast, for experience goods, product quality can be detected only after consumption, and thus higher γ also increases a consumer's expected utility from the product and hence the demand for it. Consequently, while a higher average firm quality similarly imposes a downward pressure on equilibrium price—by raising consumers' search reservation value—as for inspection goods, it has the additional demand effect that, on balance, results in higher equilibrium price under condition (13).

4. ENDOGENOUS FIRM QUALITY AND REPUTATION

We now return to our model with endogenous firm quality and reputation. Notice that if it is profitable for a firm with a higher x to make the quality investment, it must also be profitable for a firm with a lower x to do so. The equilibrium of our model will thus have the property that, for some threshold t, a firm will invest x to have β_h if $x \leq t$ but will have β_l without the investment if $x > t$. We assume that $\bar{x}$ is high enough so that in equilibrium $t < \bar{x}$; i.e., some firms (with sufficiently high realizations of x) will not incur x.

4.1 Market Equilibrium

For a given t, the average firm quality (β) in the market is

$$\gamma = \gamma(t) \equiv G(t)\,\beta_h + [1 - G(t)]\,\beta_l.$$

The first-period equilibrium is then the same as in our preliminary analysis of section 3 with $\gamma = \gamma(t)$, where consumers conduct sequential search with reservation value u_γ and all firms charge equilibrium price $p_1^* = p_\gamma$.

In the second period, consumers will observe product reviews from period-1 consumers.

For a firm of quality β, a portion β of its period-1 customers experienced quality H for its product. Thus, from the product reviews, period-2 consumers can correctly infer each firm's β.[17] There will thus effectively be two distinguishable segments of competing firms, one having quality β_h and another β_l. Comparing V_γ from (8) for $\gamma = \beta_h$ and $\gamma = \beta_l$, consumers will clearly receive a higher surplus from—and thus only search—the segment of firms with $\beta = \beta_h$. It follows that only β_h firms will be active sellers in the market in period 2, and consumers will search them with reservation value $u_h \equiv u_h(s)$ that uniquely solves

$$\beta_h \int_{u_h}^{\bar{u}} (u - u_h) f(u) \, du = s. \tag{15}$$

Moreover, in equilibrium all β_h firms charge price

$$p_2^* = \beta_h \frac{1 - F(u_h)}{f(u_h)} = \beta_h \lambda(u_h), \tag{16}$$

and each earns profit

$$\pi_2^*(\beta_h) = \frac{\beta_h \lambda(u_h)}{G(t)},$$

where $G(t)$ is the mass of β_h firms in the market. Firms with β_l earn zero profit in period 2.

We next consider the investment choices of firms and determine the threshold t on investment cost x. Given that firms invest x if and only $x \leq t$, if a firm with x acquires β_h at the beginning of period 1, it will earn discounted sum of profit

$$\pi_h = \gamma\lambda(u_\gamma) + \delta \frac{\beta_h \lambda(u_h)}{G(t)} - x. \tag{17}$$

By contrast, if the firm chooses to maintain β_l without the investment, its expected profit is

$$\pi_l = \gamma\lambda(u_\gamma). \tag{18}$$

The equilibrium $t = t^* \equiv t^*(s)$ is determined by the x at which $\pi_h = \pi_l$, or

$$\delta\beta_h \lambda(u_h) = t^* G(t^*). \tag{19}$$

[17] We could allow product reviews to be noisy signals or consumer observations of product reviews in period 2 to be noisy signals as well. Our results will remain valid if the noisy signals are sufficiently accurate.

Because average firm quality

$$\gamma \equiv \gamma(t^*) = \beta_h G(t^*) + \beta_l [1 - G(t^*)] \tag{20}$$

is endogenous, we modify the definition of $\bar{s}$ in (7) by re-defining

$$\int_{u_0}^{\bar{u}} (u - u_0) f(u)\, du = \frac{\bar{s}}{\gamma(t^*(\bar{s}))}, \tag{21}$$

where $u_0 \equiv \lambda(u_0) = \frac{1-F(u_0)}{f(u_0)}$, to ensure consumer participation whenever $s < \bar{s}$.[18] Following the discussions above, we establish the result below by further showing the existence of t^* that solves equation (19).[19]

Proposition 3 *Given $s < \bar{s}$, our model has an equilibrium where a firm has $\beta = \beta_h$ if and only if its $x \leq t^* = t^*(s)$, and the average firm quality in period 1 is $\gamma(t^*)$. Consumers search with reservation value u_γ and pay price p_1^* in period 1, but search only β_h firms with reservation value u_h and pay p_2^* in period 2.*

Proof. The RHS of equation (19) increases in t^*, whereas the LHS of equation (19) is larger than the RHS when $t^* \to 0$. Moreover, define $\bar{t}$ as

$$\delta\beta_h \lambda(u_h(\bar{s})) = \bar{t}G(\bar{t}). \tag{22}$$

Since $\lambda(u_h)$ weakly increases in s, we have $\delta\beta_h\lambda(u_h(s)) \leq \bar{t}G(\bar{t})$ for all $s \in (0, \bar{s})$. Thus, the LHS of equation (19) is no higher than the RHS when $t^* \to \bar{t}$. Therefore, there exists $t^* \in (t, \bar{t})$ that solves equation (19). ■

The second-period industry profit, consumer surplus, and social welfare are respectively

$$\Pi_2^* = \beta_h \lambda(u_h); \qquad V_2^* = \beta_h \phi(u_h); \qquad W_2^* = \gamma u_h,$$

where we define $\phi(u) \equiv [u - \lambda(u)]$, with $\phi(u) > 0$ and $\phi'(u) \geq 1$. Their corresponding

[18] As we shall discuss shortly, $\frac{s}{\gamma(t^*(s))}$ is likely to be monotonically increasing in s. If it is not, there might be multiple s that satisfies (21), in which case we define $\bar{s}$ to be the smallest s among them.

[19] If $\lambda(u)$ is strictly decreasing, then t^* is unique.

discounted sums for the two periods are given by:

$$\Pi^* = \gamma\lambda(u_\gamma) + \delta\beta_h\lambda(u_h) - \int_0^{t^*} xdG(x); \quad (23)$$

$$V^* = \gamma\phi(u_\gamma) + \delta\beta_h\phi(u_h); \quad (24)$$

$$W^* = \gamma u_\gamma + \delta\beta_h u_h - \int_0^{t^*} xdG(x). \quad (25)$$

In equilibrium, each consumer receives positive (expected) surplus from market participation, and all firms receive positive profits, while the more efficient firms (with lower x, for $x < t^*$) receive higher profits.

4.2 Welfare Effects of Search Cost

We now consider the welfare effects of search cost. Utilizing $\frac{\partial u_h}{\partial s} = -\frac{1}{\beta_h[1-F(u_h)]}$ from (15),

$$\frac{\partial p_2^*}{\partial s} = \beta_h\lambda'(u_h)\frac{\partial u_h}{\partial s} = -\frac{\lambda'(u_h)}{[1-F(u_h)]} \geq 0.$$

Thus, as expected, a higher search cost leads to a higher price in period 2. Since

$$\frac{\partial t^*}{\partial s} = \frac{\delta\beta_H\lambda'(u_h)\frac{\partial u_h}{\partial s}}{G(t)+tg(t)} = \frac{-\delta\lambda'(u_h)}{G(t)+tg(t)}\frac{1}{[1-F(u_h)]} \geq 0, \quad (26)$$

and $\frac{\partial\gamma(t^*)}{\partial t^*} = G'(t^*)(\beta_h - \beta_l) > 0$, we have

$$\frac{\partial\gamma(t^*)}{\partial s} = \frac{\partial\gamma(t^*)}{\partial t^*}\frac{\partial t^*}{\partial s} \geq 0.$$

Thus, increases in search cost raise average firm quality.[20] Intuitively, when s is higher, price is higher, and a firm has higher profit in period 2 for being a β_h firm. That is, the return to the reputation of being a high quality firm is higher. This motivates more firms to invest in β_h, so that t^* becomes higher, which boosts γ in period 1.

When γ is given exogenously, a higher s leads to a lower u_γ, which in turn results in higher price and profit. With endogenous γ, changes in s also impact $\gamma = \gamma(t^*)$. While a higher s directly impacts u_γ negatively, it indirectly impacts u_γ positively through a higher

[20] Notice that if $\lambda'(u) = 0$, then $\partial t^*/\partial s = 0$, and hence $\partial\gamma(t^*)/\partial s = 0$. Thus $\lambda'(u) < 0$ is needed in order for average firm quality to (strictly) increase with s.

γ. We expect that the direct effect of s would outweigh its indirect effect through γ, so that $\frac{s}{\gamma}$ is higher with a higher s. Define the elasticity of average seller quality, γ, with respect to search cost as $\varepsilon = \frac{s}{\gamma}\frac{\partial\gamma}{\partial s} = \frac{s}{\gamma}\frac{\partial\gamma}{\partial t^*}\frac{\partial t^*}{\partial s} \geq 0$. Then

$$\frac{d\left(\frac{s}{\gamma}\right)}{ds} = \frac{\gamma - s\frac{\partial\gamma}{\partial s}}{\gamma^2} \geq 0 \quad \Longleftrightarrow \quad \varepsilon \equiv \frac{\partial\gamma}{\partial s}\frac{s}{\gamma} \leq 1.$$

Thus, if $\varepsilon \leq 1$, then

$$\frac{\partial u_\gamma}{\partial s} = \frac{\partial u_\gamma}{\partial (s/\gamma)}\frac{\partial (s/\gamma)}{\partial s} = \frac{\varepsilon - 1}{\gamma\left[1 - F\left(u_\gamma\right)\right]} \leq 0, \tag{27}$$

$$\frac{\partial p_\gamma}{\partial s} = \gamma\lambda'\left(u_\gamma\right)\frac{\partial u_\gamma}{\partial s} \geq 0,$$

and, since $\delta\beta_h\lambda'\left(u_h\right)\frac{\partial u_h}{\partial s} = \left[t^*g\left(t^*\right) + G\left(t^*\right)\right]\frac{\partial t^*}{\partial s}$ from totally differentiating the two sides of (19), we have

$$\frac{\partial\Pi^*}{\partial s} = \frac{\partial p_\gamma}{\partial s} + \delta\beta_h\lambda'\left(u_h\right)\frac{\partial u_h}{\partial s} - t^*g\left(t^*\right)\frac{\partial t^*}{\partial s} = \frac{\partial p_\gamma}{\partial s} + G\left(t^*\right)\frac{\partial t^*}{\partial s} \geq 0.$$

The discussions above lead to:

Remark 1 *$\gamma\left(t^*\right)$ and p_2^* increase in s, and so do p_1^* and Π^*, provided $\lambda'\left(u\right) < 0$ and $\varepsilon \leq 1$.*

Thus, with endogenous firm quality and reputation, search cost continues to be a key indicator of competition intensity, with increases in s leading to less competition and high prices in both periods. However, as we show next, search cost now has unconventional effects on consumer surplus and welfare. The result below refers to assumption

$$-M < \lambda'\left(u\right) < 0 \text{ for some } M > 0 \text{ and for } u \in \left[0, \bar{u}\right], \tag{28}$$

which strengthens condition (6). Condition (28) is satisfied, for instance, if $F\left(u\right)$ is a uniform distribution, but it rules out the boundary case of the exponential distribution.

Proposition 4 *(i) Under condition (28), both V^* and W^* increase in s when s is sufficiently small. (ii) Suppose $\varepsilon \leq 1$. Then, when $s \to \bar{s}$, V^* decreases in s, and so does W^* if $u_0\left(\beta_h - \beta_l\right) \leq \bar{t}$.*

Proof. (i) First, from (24),

$$\frac{\partial V^*}{\partial s} = \frac{\partial \gamma}{\partial s}\phi\left(u_\gamma\right) + \ \gamma\phi'\left(u_\gamma\right)\frac{\partial u_\gamma}{\partial s} + \ \delta\beta_h\phi'\left(u_h\right)\frac{\partial u_h}{\partial s}.$$

Since $\frac{\partial u_\gamma}{\partial (s/\gamma)} = -\frac{1}{[1-F(u_\gamma)]}$ from (4) and from (26):

$$\frac{\partial \gamma}{\partial s} = \frac{\partial \gamma}{\partial t^*}\frac{\partial t^*}{\partial s} = \left(\beta_h - \beta_l\right) g\left(t^*\right) \frac{-\delta\lambda'\left(u_h\right)}{G\left(t^*\right) + t^* g\left(t^*\right)} \frac{1}{\left[1 - F\left(u_h\right)\right]}.$$

With $\frac{\partial u_\gamma}{\partial s} = \frac{\varepsilon - 1}{\gamma[1-F(u_\gamma)]}$ from (27) and $\frac{\partial (s/\gamma)}{\partial s} = \frac{1-\varepsilon}{\gamma}$, we then have

$$\begin{aligned} \frac{\partial V^*}{\partial s} &= \left(\beta_h - \beta_l\right) g\left(t^*\right) \frac{-\delta\lambda'\left(u_h\right)}{G\left(t^*\right) + t^* g\left(t^*\right)} \frac{\phi\left(u_\gamma\right)}{\left[1 - F\left(u_h\right)\right]} + \frac{\phi'\left(u_\gamma\right)\left(\varepsilon - 1\right)}{\left[1 - F\left(u_\gamma\right)\right]} - \delta\frac{\phi'\left(u_h\right)}{\left[1 - F\left(u_h\right)\right]} \qquad (29) \\ &\geq \frac{1}{\left[1 - F\left(u_h\right)\right]}\left[\left(\beta_h - \beta_l\right)\frac{-\delta\lambda'\left(u_h\right)\phi\left(u_\gamma\right)}{\frac{G(t^*)}{g(t^*)} + t^*} - \phi'\left(u_\gamma\right) - \delta\phi'\left(u_h\right)\right], \end{aligned}$$

where the inequality holds because $\varepsilon \geq 0$ and $\left[1 - F\left(u_\gamma\right)\right] \geq \left[1 - F\left(u_h\right)\right]$. When $s \to 0$: $\frac{G(t^*)}{g(t^*)} \to 0$, $u_h \to \bar{u}$, $u_\gamma \to \bar{u}$; $\lambda'\left(\bar{u}\right) < 0$, $\phi\left(u_\gamma\right) \to \bar{u}$; and $\left(\beta_h - \beta_l\right)\frac{-\delta\lambda'(u_h)\phi(u_\gamma)}{\frac{G(t)}{g(t)}+t} \to \infty$. Thus, since $\phi'\left(u\right) = 1 - \lambda'\left(u\right)$ is bounded for any u, we have $\frac{\partial V^*}{\partial s} > 0 \quad \text{as} \quad s \to 0$.

Next, from (25),

$$\begin{aligned} \frac{\partial W^*}{\partial s} &= \frac{\partial \gamma}{\partial s}u_\gamma + \gamma\frac{\partial u_\gamma}{\partial s} + \delta\beta_h\frac{\partial u_h}{\partial s} - t^* g\left(t^*\right)\frac{\partial t^*}{\partial s} \\ &= \left[u_\gamma\left(\beta_h - \beta_l\right) - t\right] g\left(t\right)\frac{\partial t}{\partial s} + \left(\varepsilon - 1\right)\frac{1}{1 - F\left(u_\gamma\right)} - \delta\frac{1}{1 - F\left(u_h\right)} \\ &= \left[u_\gamma\left(\beta_h - \beta_l\right) - t^*\right]\frac{-\delta\lambda'\left(u_h\right)}{t^* + \frac{G(t^*)}{g(t^*)}}\frac{1}{1 - F\left(u_h\right)} + \left(\varepsilon - 1\right)\frac{1}{1 - F\left(u_\gamma\right)} - \delta\frac{1}{1 - F\left(u_h\right)} \\ &> \frac{1}{1 - F\left(u_h\right)}\left\{\left[u_\gamma\left(\beta_h - \beta_l\right) - t^*\right]\frac{-\delta\lambda'\left(u_h\right)}{t^* + \frac{G(t^*)}{g(t^*)}} - 1 - \delta\right\}, \end{aligned}$$

where the last inequality is due to $\varepsilon \geq 0$ and $u_\gamma \leq u_h$. When $s \to 0$, $t^* \to 0$, $\frac{G(t^*)}{g(t^*)} \to 0$, $u_\gamma \to \bar{u}$, and hence $\left[u_\gamma\left(\beta_h - \beta_l\right) - t^*\right]\frac{-\delta\lambda'(u_h)}{t^* + \frac{G(t^*)}{g(t^*)}} \to \infty$. Thus $\frac{\partial W^*}{\partial s} > 0 \quad \text{as} \quad s \to 0$.

(ii) First, $\frac{\partial u_h}{\partial s} < 0$, $\lambda'\left(u\right) \leq 0$, $\frac{\partial u_\gamma}{\partial s} \leq 0$ if $\varepsilon \leq 1$; and, when $s \to \bar{s}$, $\phi\left(u_\gamma\right) = \left[u_\gamma - \lambda\left(u_\gamma\right)\right] \to 0$. Hence, from (29), if $\varepsilon \leq 1$, $\frac{\partial V^*}{\partial s} < 0 \quad \text{as} \quad s \to \bar{s}$.

Next, when $s \to \bar{s}$, $u_\gamma \to u_0$, $t^* \to \bar{t}$, and hence $\frac{\partial W^*}{\partial s} < 0$ if

$$\bar{t} \geq u_0\left(\beta_h - \beta_l\right) \quad \text{and} \quad \varepsilon \leq 1. \qquad (30)$$

■

Therefore, higher search frictions can improve market performance for experience goods. To understand this striking result, notice that the effect of a marginal increase in s on consumer surplus can be decomposed as follows under conditions (28) and $\varepsilon \leq 1$:

$$\frac{\partial V^*}{\partial s} = \underbrace{\frac{\partial \gamma}{\partial s}\phi\left(u_\gamma\right)}_{\text{average firm quality effect } >0} + \underbrace{\gamma\phi'\left(u_\gamma\right)\frac{\partial u_\gamma}{\partial s}}_{\text{search efficiency effect in period 1 } \leq 0} + \underbrace{\delta\beta_h\phi'\left(u_h\right)\frac{\partial u_h}{\partial s}}_{\text{search efficiency effect in period 2 } <0}.$$

An increase in s raises the profit from being a β_h firm, motivating more firms to invest in quality and hence γ is higher in period 1. A higher s thus increases average firm quality in period 1. On the other hand, a higher s reduces u_h and, when $\varepsilon \leq 1$, also reduces u_γ; that is, a higher search cost reduces search efficiency and leads to lower reservation values, which negatively impacts consumer surplus.

When search cost is low, price is low. Thus consumer surplus from an H product, $\phi\left(u_\gamma\right)$, is high, and the number of high quality firms (that incur x) is small. In such situations, although a marginal increase in s raises prices only marginally, the profit increase from becoming a high quality firm is large because a β_h firm will have high sales in period 2. Hence, a marginal increase in s leads to a large increase in the number of high quality firms and in γ (i.e., $\frac{\partial \gamma}{\partial s}$ is high), which means that $\frac{\partial \gamma}{\partial s}\phi\left(u_\gamma\right)$ is high, whereas the effect on search efficiency is more moderate. Thus the average firm quality effect dominates when s is small. On the other hand, when s is large, price is high. Thus $\frac{\partial \gamma}{\partial s}$ and $\phi\left(u_\gamma\right)$ are relatively low, so that the negative search efficiency effect dominates.

We can similarly decompose the effect of search cost on welfare as follows:

$$\frac{\partial W^*}{\partial s} = \underbrace{\frac{\partial \gamma}{\partial s}u_\gamma}_{\text{average firm quality effect } > 0} + \underbrace{\gamma\frac{\partial u_\gamma}{\partial s} + \delta\beta_h\frac{\partial u_h}{\partial s}}_{\text{search efficiency effect } < 0} + \underbrace{-\, t^* g\left(t^*\right)\frac{\partial t^*}{\partial s}}_{\text{investment cost effect } < 0}.$$

In addition to the average firm quality and search efficiency effects, as in the case of consumer surplus, for W^* there is the additional effect of investment cost: a higher search cost increases the total investment cost for β_h, because the higher profit from being a high-quality firm from an increase in s leads to more firms to invest in β_h. But when $s \to 0$,

$t^* \to 0$, and thus the additional effect of investment cost vanishes so that W^* increases in s, similarly as for V^*. On the other hand, when $s \to \bar{s}$, the highest possible value of search cost, $t^* \to \bar{t}$ and $u_\gamma \to u_0$. If $u_0 (\beta_h - \beta_l) < \bar{t}$, then the investment cost effect (alone) dominates the average firm quality effect, and hence W^* decreases in s, similarly as for V^*.

Our finding that both consumer and total welfare are initially increasing in search cost is in sharp contrast to the result in the existing search literature, where consumer and social welfare monotonically decrease as search cost increases. Both endogenous firm quality and the experience nature of goods are important for the non-monotonic result in our model. If average firm quality in the market (γ) is exogenously given, higher search costs would only have the negative effect of reducing search efficiency. In our model, an increase in search cost has the additional effect of inducing a higher γ, which positively impacts consumer and social welfare, and it is the dominant force when search cost is low. However, if the goods were inspection goods, even with endogenous product quality, both consumer and social welfare would decrease with search cost, as we show next.

4.3 Comparing to Welfare for Inspection Goods

For inspection goods, same as in the case of experience goods, for a given t the average firm quality in the market is

$$\gamma = \gamma(t) = G(t)\beta_h + [1 - G(t)]\beta_l.$$

The first-period equilibrium is then the same as in subsection 3.3, with consumers conducting sequential search under reservation value $u_\gamma^I = u_\gamma$ and all firms charging $p_1^I = p_\gamma^I$. Notice that a firm of quality β earns profit $\frac{\beta}{\gamma}\lambda\left(u_\gamma^I\right)$ in period 1.

Suppose also that, as for experience goods, in period 2 consumers can observe first-period consumers' product reviews, which reveal each firm's β.[21] Then, in period 2, consumers will

[21] Since consumers observe $q \in \{H, L\}$ when searching a firm, they will only purchase if $q = H$. A consumer's review in this case is still about whether a firm's product quality q is H or L; even though she does not purchase if the produt quality turns out to be low, the consumer has wasted a costly search if $q = L$.

also only search β_h firms, with reservation value u_h. Moreover, from subsection 3.3, β_h sellers will charge $p_2^I = \lambda(u_h)$, each earning profit $\frac{1}{G(t)}\lambda(u_h)$ in period 2 if the number of β_h firms is $G(t)$. Thus, a β_h seller earns higher profits in both periods.

In equilibrium, a firm will invest in β_h if and only if $x \leq \tau$, where the cutoff value τ is determined by

$$\frac{\beta_h}{\gamma}\lambda\left(u_\gamma^I\right) + \delta\frac{1}{G(\tau)}\lambda(u_h) - \tau = \frac{\beta_l}{\gamma}\lambda\left(u_\gamma^I\right),$$

or

$$\tau = \frac{\beta_h - \beta_l}{\gamma(\tau)}\lambda(u_\gamma) + \delta\frac{1}{G(\tau)}\lambda(u_h). \tag{31}$$

Thus, same as for experience goods, a higher s, which increases $\lambda(u_\gamma)$ and $\lambda(u_h)$, will raise average firm quality $\gamma(\tau)$. Industry profit, consumer surplus, and social welfare for the two periods together are respectively

$$\Pi^I = \lambda(u_\gamma)+\delta\lambda(u_h)-\int_0^\tau xdG(x); \quad V^I = \phi(u_\gamma)+\delta\phi(u_h); \quad W^I = u_\gamma+\delta u_h-\int_0^\tau xdG(x),$$

where we recall $\phi(u) = u - \lambda(u)$.

The effect of search cost on consumer welfare under inspection goods is always negative (provided $\varepsilon \leq 1$ so that $d\left(\frac{s}{\gamma}\right)/ds \geq 0$), because the positive average firm quality effect for experience goods is absent:

$$\frac{\partial V^I}{\partial s} = \underbrace{\phi'(u_\gamma)\frac{\partial u_\gamma}{\partial s}}_{\text{search efficiency effect in period 1 }\leq 0} + \underbrace{\delta\phi'(u_h)\frac{\partial u_h}{\partial s}}_{\text{search efficiency effect in period 2 }<0} < 0.$$

Similarly,

$$\frac{\partial W^I}{\partial s} = \underbrace{\gamma\frac{\partial u_\gamma}{\partial s} + \delta\beta_h\frac{\partial u_h}{\partial s}}_{\text{search efficiency effect }<0}\underbrace{-\tau g(\tau)\frac{\partial \tau}{\partial s}}_{\text{investment cost effect }<0} < 0.$$

We thus have:

Remark 2 *For inspection goods, consumer and total welfare monotonically decrease in search cost, in contrast to the result for experience goods.*

For both inspection and experience goods, an increase in search cost leads to higher price and hence to higher return for quality reputation because only β_h firms sell in period 2. However, consumers can avoid the loss from a low-quality product for inspection goods but not for experience goods. Thus, the marginal benefit from increasing firm quality (γ) due to a higher s, for consumers and for social welfare, is lower for inspection than for experience goods. This explains why a higher s can lead to higher consumer and social welfare through the positive quality effect for experience but not for inspection goods.

4.4 Equilibrium vs. Efficient Quality Investment

We further investigate how the equilibrium quality investment compares with the social optimum, by comparing the cutoff values for quality investment (t) in these two cases. The result below shows that the equilibrium cutoff (t^*) can be higher or lower than the efficient value (t^o) when search cost is sufficiently high or low, respectively.

Proposition 5 *Given $s \in (0, \bar{s})$, there exists $t^o > 0$ that maximizes total welfare. Moreover, provided $t^o < \bar{t}$, there exists a unique $\sigma > 0$ such that $t^* \leq t^o$ if $s \leq \sigma$ but $t^* > t^o$ if $\sigma < s \leq \bar{s}$.*

Proof. Recall $\frac{\partial \gamma}{\partial t} = (\beta_h - \beta_l) g(t)$ and $\frac{\partial u_\gamma}{\partial \gamma} = \frac{s}{\gamma^2} \frac{1}{1-F(u_\gamma)}$. Thus,

$$\begin{aligned} \frac{\partial W^*}{\partial t} &= \frac{\partial (\gamma u_\gamma)}{\partial \gamma} \frac{\partial \gamma}{\partial t} - tg(t) \\ &= \left[\left(u_\gamma + \frac{s}{\gamma} \frac{1}{1 - F(u_\gamma)} \right) (\beta_h - \beta_l) - t \right] g(t). \end{aligned} \tag{32}$$

Clearly $\frac{\partial W^*}{\partial t}|_{t=0} > 0$. Moreover, for given $s > 0$, u_γ is bounded away from $\bar{u}$. Thus, $\frac{\partial W^*}{\partial t} < 0$ if t is sufficiently high. Hence, there exists $t^o \in (0, \bar{x})$ such that W^* is maximized at t^o. Moreover, from (19), t^* increases in s and $t^* \to \bar{t}$ if $s \to \bar{s}$. Therefore, if $t^o < \bar{t}$, there exists a unique σ such that $t^* \leq t^o$ when $s \leq \sigma$, and $t^* > t^o$ when $\sigma < s \leq \bar{s}$. ■

An increase in t results in a higher proportion of firms that invest. This leads to a higher expected quality of sellers and hence higher welfare in the first period, as reflected by a higher γu_γ. On the other hand, investment is costly, and a higher t leads to higher investment cost $\int_0^t x dG(x)$. A socially optimal t^o balances these two opposing forces, with the marginal benefit from a higher γ being equal to the marginal cost of increasing t. From the definition of $\bar{t}$ in (22), we note that $\bar{t} > 0$ is independent of β_l whereas $t^o \to 0$ if $\beta_l \to \beta_h$. Thus $t^o < \bar{t}$ is likely to hold when $(\beta_h - \beta_l)$ is not too large so that the benefit from high quality (β_h) is more limited.

When s is low, consumers have strong search incentives and u_γ is high, so that a higher average firm quality (i.e. a higher t) is more socially desirable, leading to a higher γu_γ. But price—and hence t^*—is low when s is low. Therefore $t^* < t^o$ when s is low. On the other hand, when s is high, u_γ is low and welfare gain from increasing γ is relatively low (so t^o is relatively low), whereas price is high and t^* relatively high, so that t^* tends to exceed t^o.

In the existing literature on experience goods, product quality is usually inefficiently low because it is more costly to induce firms to improve quality when quality is not detectable by consumers before purchase. Our result shows that this can be reversed in the presence of search frictions.[22]

5. THE IMPACT OF AN INTERMEDIARY

In many markets, consumers search their products through an intermediary that serves as a search platform, such as Amazon.com and booking.com. We now extend our model to include such an intermediary.[23] A profit-maximizing intermediary can affect market outcomes by charging sellers fees for being on its platform, which may in turn affect the (average) quality of sellers on the marketplace, search efficiency, and market price.[24]

[22] It can be verified that a similar result also holds for inspection goods. Thus, quality provision is socially deficient when s is low but possibly excessive when s is high, for both experience and inspection goods in search markets.

[23] Athey and Ellison (2011) and Chen and He (2011) study position auctions by search engines, emphasizing their beneficial role as information intermediary. Bagwell and Ramey (1996) pioneered the study of coordination economies in retail market search. Others have shown that search intermediaries need not (optimally) improve search efficiency (e.g. Eliaz and Spiegler, 2011; White, 2013; de Cornière and Taylor, 2014). None of the above analyze experience goods.

[24] In addition to providing a search platform, the intermediary may publish product reviews by customers. The intermediary can thus be a reputation carrier, enabling firms to establish quality reputation when product reviews are otherwise unavailable.

Suppose that the intermediary can charge each seller $(k,\ \mu)$, where $k \geq 0$ is a fixed fee and $\mu \geq 0$ is a percentage of the transaction price. Sellers that pay the fees will have access to consumers associated with the intermediary. We further assume that there is a minimum platform size $\Omega \in (0, 1]$—number of sellers to be listed on the platform—that the intermediary can commit to.[25]

The timing of the extended model is as follows. The intermediary first chooses $(k,\ \mu)$. In period 1, after its realization of x, each seller chooses whether to pay the fees to sell on the platform and decides whether to invest x to become a seller with β_h. Sellers on the platform then set prices, consumers sequentially search sellers on the platform, and transactions are made. In period 2, consumer reviews from previous period are available to the current cohort of consumers. Sellers on the platform set prices, and consumers again sequentially search sellers on the platform and possibly make purchases. Everything else about the model is the same as in section 2.[26] Notice that sellers not on the platform are not active in either period.

Given the average firm quality on the platform, γ, which is endogenously determined by the firms on the platform who will invest in β_h, the firms' pricing and consumers' search strategies are the same as in section 4, unaffected by the values of k and μ. In particular, at a uniform-price equilibrium, the optimal consumer search rule is again given by (21), whereas a seller will choose p to maximize $(1 - \mu)\, pD\,(p, p^*)$, the solution of which does not depend on μ.

There are two possible types of equilibria for a given Ω, depending on its value: (1) a separating equilibrium in which all sellers on the platform are of high quality (β_h), and (2) a pooling equilibrium in which both high and low quality sellers are present on the platform.

First, at a separating equilibrium, the intermediary charges high fees such that only high quality sellers will be able to earn positive profit. Suppose that in equilibrium, there is

[25] A similar assumption is adopted by, for example, Eliaz and Spiegler (2011) under a continum of sellers, or Athey and Ellison (2011) and Chen and He (2011) under a fininte number of sellers.

[26] For convenience, we assume that each search still costs s. The analysis can be easily extended to situations where s becomes lower when consumers search on the platform.

a cutoff value t_k such that only sellers with $x \leq t_k$ choose to invest in β_h and pay to be listed on the platform while other sellers are off the platform and inactive. In this case, in equilibrium the intermediary solves the following problem (P1):

$$\max_{(k,\mu)} \Psi = kG(t_k) + \mu\beta_h\lambda(u_h)(1+\delta),$$

subject to

$$(1-\mu)\frac{1}{G(t_k)}\beta_h\lambda(u_h) - k < 0, \tag{33}$$

$$(1-\mu)\frac{1}{G(t_k)}\beta_h\lambda(u_h)(1+\delta) - k - x \geq 0 \quad \text{for} \quad x \leq t_k, \tag{34}$$

where the first constraint ensures that a seller with β_l has no incentive to be on the platform (being able to sell only in period 1) and the second constraint ensures that sellers with low x find it profitable to acquire β_h and sell on the platform.

Define t_Ω and $\hat{t}$ respectively as

$$G(t_\Omega) = \Omega; \qquad \hat{t} = \frac{1}{G(\hat{t})}\beta_h\lambda(u_h)(1+\delta), \tag{35}$$

and, for $\bar{t}$ defined in (22), we assume $\max\{t_\Omega, \hat{t}\} < \bar{t} < \bar{x}$. Then, exactly Ω firms will be listed on the platform if and only if all firms with $x \leq t_\Omega$ pay (k,μ) and invest x, whereas $G(\hat{t})$ is the mass of firms who will acquire β_h and be on the platform if $k = \mu = 0$ and $\gamma = \beta_h$.

Lemma 1 *Suppose $t_\Omega \leq \hat{t}$. There is a separating equilibrium in which the intermediary optimally sets $\mu^* = 0$ and*

$$k^* = \frac{1}{G(t_\Omega)}\beta_h\lambda(u_h)(1+\delta) - t_\Omega; \tag{36}$$

whereas only firms with $x \leq t_\Omega$ choose to acquire β_h and sell on the platform. Moreover, the presence of the intermediary improves welfare if $t_\Omega \leq t^*$, with t^* defined in (19) and $t^* < \hat{t}$.

Proof. In equilibrium, constraint (34) is binding when $x = t_k$ and thus

$$(1-\mu)\frac{1}{G(t_k)}\beta_h\lambda(u_h)(1+\delta) = k + t_k.$$

Hence,

$$\Psi = \beta_h \lambda (u_h) (1 + \delta) - t_k G (t_k),$$

which decreases in t_k. Thus, the intermediary optimally sets (k^*, τ^*) such that the firm with $x = t_\Omega$ is indifferent between being on and off the platform:

$$k^* = (1 - \mu^*) \frac{1}{G (t_\Omega)} \beta_h \frac{1 - F (u_h)}{f (u_h)} (1 + \delta) - t_\Omega.$$

Moreover, substituting k^* into constraint (33), we have

$$\mu^* < t_\Omega G (t_\Omega) \frac{1}{\delta \beta_h} \frac{f (u_h)}{1 - F (u_h)}.$$

Therefore, $\mu^* = 0$ and k^* solve problem (P1) and induce the separating equilibrium, which improves search efficiency in period 1. If additionally $t_\Omega \leq t^*$, then the total investment cost on quality is not higher in the separating equilibrium than in the equilibrium without the intermediary, and hence social welfare must be higher in the former. ■

Given (relatively small) Ω so that $\hat{t} \geq t_\Omega$, the intermediary can screen out low quality firms by charging high fees and thus organize a platform that contains only high quality sellers. At this equilibrium, search efficiency is higher in period 1 (and is unchanged in period 2) as compared to the market equilibrium without the intermediary; if additionally $t_\Omega \leq t^*$, then the total investment cost on quality is also (weakly) lower—and hence social welfare must be higher—at the separating equilibrium.

We next consider an alternative possible equilibrium, a pooling equilibrium, which arises when $t_\Omega > \hat{t}$. In this equilibrium, there is a cutoff value t_k such that only firms with $x \leq t_k$ choose to acquire β_h, but all firms will pay to be on the platform. The intermediary solves the following maximization problem (P2):

$$\max_{k,\mu} \ \Psi = k + \mu \left[\gamma (t_k) \lambda (u_\gamma) + \delta \beta_h \lambda (u_h) \right],$$

subject to

$$(1 - \mu) \gamma (t_k) \lambda (u_\gamma) - k \geq 0, \tag{37}$$

$$(1 - \mu) \delta \frac{1}{G (t_k)} \beta_h \lambda (u_h) - x \geq 0 \quad \text{for} \quad x \leq t_k, \tag{38}$$

where the two constraints ensure respectively that firms with β_l are willing to pay (k, μ) and that firms with $x \leq t_k$ will additionally choose to acquire β_h. The result below refers to condition

$$\left(\lambda\left(u_\gamma\right) - \lambda'\left(u_\gamma\right)\frac{s}{\gamma}\frac{1}{1-F\left(u_\gamma\right)}\right)\left(\beta_h - \beta_l\right) \leq t^* \tag{39}$$

for $\gamma = \gamma\left(t^*\right)$, which holds if $\left(\beta_h - \beta_l\right)$ is not too large.

Lemma 2 *Suppose $t_\Omega > \hat{t}$ and (39) holds. Then, there exists a pooling equilibrium with $t_k^* \in (0, t^*)$. The intermediary optimally chooses*

$$k^* = \left(1-\mu^*\right)\gamma\left(t_k^*\right)\lambda\left(u_\gamma\right); \qquad \mu^* = 1 - t_k^* G\left(t_k^*\right)\frac{1}{\delta\beta_h}\lambda\left(u_h\right);$$

and all firms choose to be on the platform. However, only firms with $x \leq t_k^*$ choose to acquire β_h.

Proof. Constraint (38) is binding when $x = t_k$, with

$$t_k = (1-\mu)\,\delta\frac{1}{G\left(t_k\right)}\beta_h\lambda\left(u_h\right). \tag{40}$$

Since RHS of (40) decreases in t_k and μ, it follows that t_k decreases in μ. In equilibrium, (37) is binding. Moreover, from (40),

$$t_k G\left(t_k\right) = (1-\mu)\,\delta\beta_h\lambda\left(u_h\right).$$

Thus, the intermediary's objective function becomes, for $\gamma = \gamma\left(t_k\right)$,

$$\Psi = \gamma\left(t_k\right)\lambda\left(u_\gamma\right) - t_k G\left(t_k\right) + \delta\beta_h\lambda\left(u_h\right). \tag{41}$$

Since $\frac{\partial u_\gamma}{\partial \gamma} = \frac{1}{\left[1-F\left(u_\gamma\right)\right]}\frac{s}{\gamma^2}$ and $\frac{\partial \gamma}{\partial t_k} = \left(\beta_h - \beta_l\right)g\left(t_k\right)$, we have

$$\begin{aligned}
\frac{\partial \Psi}{\partial t_k} &= \left(\lambda\left(u_\gamma\right) + \gamma\lambda'\left(u_\gamma\right)\frac{\partial u_\gamma}{\partial \gamma}\right)\frac{\partial \gamma}{\partial t_k} - G\left(t_k\right) - t_k g\left(t_k\right) \\
&= \left(\lambda\left(u_\gamma\right) - \lambda'\left(u_\gamma\right)\frac{s}{\gamma}\frac{1}{1-F\left(u_\gamma\right)}\right)\left(\beta_h - \beta_l\right)g\left(t_k\right) - G\left(t_k\right) - t_k g\left(t_k\right) \\
&= \left[\left(\lambda\left(u_\gamma\right) - \lambda'\left(u_\gamma\right)\frac{s}{\gamma}\frac{1}{1-F\left(u_\gamma\right)}\right)\left(\beta_h - \beta_l\right) - t_k\right]g\left(t_k\right) - G\left(t_k\right).
\end{aligned} \tag{42}$$

Since $\lambda'(u_\gamma) \leq 0$, we have $\frac{\partial \Psi}{\partial t_k}|_{t_k \to 0} > 0$. Also, under (39), $\frac{\partial \Psi}{\partial t_k}|_{t_k \to t^*} < 0$. Therefore, there exists $t_k^* < t^*$ that maximizes Ψ, with $\mu^* > 0$. ■

When the minimum platform size Ω is relatively large and $(\beta_h - \beta_l)$ relatively small, there is a pooling equilibrium in which the intermediary finds optimal to accommodate both high and low quality firms, with positive k and μ. Due to $\mu^* > 0$, however, $t_k^* < t^*$ and the average firm quality in period 1 is lower than when the intermediary is absent. The intermediary can thus lower welfare if it leads to a pooling equilibrium, because the market provision of quality may be already too low without the intermediary.

Combining Lemma 1 and Lemma 2, noting $t^* < \hat{t}$ and recalling from Proposition 5 that $t^* < t^o$ if $s < \sigma$, we have

Proposition 6 *For the extended model with an intermediary, assume* $\max\{t_\Omega, \hat{t}\} < \bar{t}$. *(i) If* $t_\Omega < \hat{t}$, *then it is an equilibrium for firms with* $x \leq t_\Omega$ *to acquire* β_h *and be listed by the intermediary, with the intermediary improving social welfare. (ii) If* $t_\Omega > \hat{t}$, *then it is an equilibrium for all firms to be listed by the intermediary but only those with* $x \leq t_k^* < t^*$ *to acquire* β_h; *and if* $s < \sigma$, *then* $t_k^* < t^* < t^o$, *so that the market provision of quality is further below the social optimum.*[27]

The presence of a profit-maximizing search intermediary can thus either increase or reduce welfare. Notice that $t_\Omega < t^*$ is more likely to hold if s is relatively large, while $t_\Omega > \hat{t}$ and $s < \sigma$ are more likely to hold if s is relatively small. Therefore, the presence of the intermediary is more likely to increase welfare when the intermediary can commit to a relatively small minimum listing size, or under relatively large search cost; but it can reduce welfare when the minimum listing space on the search platform is relatively large or there is relatively high search cost.[28]

[27] In this case, social welfare, same as W^* from (25), is likely—but not necessarily—lower under t_k^* than under t^*. If W^* is monotonically increasing in t for $t < t^o$, which for example is true when $F(\cdot)$ and $G(\cdot)$ are uniform distributions, then W^* is unambiguously lower under t_k^* than under t^* if $t_k^* < t^* < t^o$.

[28] We have not established the uniqueness of equilibrium in either case. Thus, this conclusion needs the qualification that the separating and the pooling equilibrium will prevail respectively when $t_\Omega < \hat{t}$ and when $t_\Omega > \hat{t}$.

6. CONCLUSION

This paper has studied consumer search and price competition for experience goods. In contrast to results for inspection goods, which has been the focus of the existing search literature, we find that a higher average firm quality tends to raise market price despite intensifying search and competition; and, more strikingly, both consumer and social welfare are initially increasing in search cost under endogenous firm quality and reputation. Our results suggest that the observability of product quality (before purchase) plays an important role for understanding how search markets function. We also find that equilibrium firm quality is inefficiently low when search cost is small but can be excessively high when search cost is relatively large. Moreover, if a search intermediary can commit to a relatively small space to list sellers, it tends to improve welfare; otherwise it is likely to reduce welfare.

APPENDIX. A NUMERICAL EXAMPLE

The numerical example illustrates Proposition 4 (V^* and W^* change with s non-monotonically), Proposition 5 (comparing t^* and t^o), and Proposition 6 (an intermediary's impact on welfare).

Suppose that $F(u) = \frac{u}{\Delta}$ for $u \in [0, \Delta]$. Given γ, we have

$$\int_{u_\gamma}^{\Delta} (u - u_\gamma) \frac{1}{\Delta} du = \frac{s}{\gamma} \implies u_\gamma = \Delta - \sqrt{\frac{2\Delta s}{\gamma}}, \qquad p_\gamma = \sqrt{2\Delta s \gamma}.$$

Moreover, $u_h = \Delta - \sqrt{\frac{2\Delta s}{\beta_h}}$ and $p_h = \sqrt{2\Delta s \beta_h}$. Suppose $G(x) = bx$ and $g(x) = b$ for $x \in [0, \frac{1}{b}]$. Then $G(t) = bt$,

$$\gamma(t) = \beta_l + (\beta_h - \beta_l) bt.$$

From (19),

$$\delta \beta_h \lambda(u_h) = t^* G(t^*), \qquad \implies \qquad t^* = \left(\frac{\left(2\Delta \delta^2 s \beta_h\right)^{0.5}}{b} \right)^{0.5}.$$

Hence, with $\gamma = \gamma(t^*)$,

$$\begin{aligned}
\Pi^* &= \sqrt{2\Delta s\gamma} + \delta\sqrt{2\Delta s\beta_h} - \int_0^{t^*} bxdx; \\
V^* &= \gamma\left(\Delta - 2\sqrt{\frac{2\Delta s}{\gamma}}\right) + \delta\beta_h\left(\Delta - 2\sqrt{\frac{2\Delta s}{\beta_h}}\right); \\
W^* &= \gamma\left(\Delta - \sqrt{\frac{2\Delta s}{\gamma}}\right) + \delta\beta_h\left(\Delta - \sqrt{\frac{2\Delta s}{\beta_h}}\right) - \int_0^{t^*} bxdx.
\end{aligned}$$

Let $\beta_h = 0.8$, $\beta_l = 0.3$, $\delta = 0.8$, $\Delta = 100$, $\frac{1}{b} = 50$. Figures 1A and 1B below show that both V^* and W^* exhibit an inverted-U shape in s, illustrating Proposition 4.

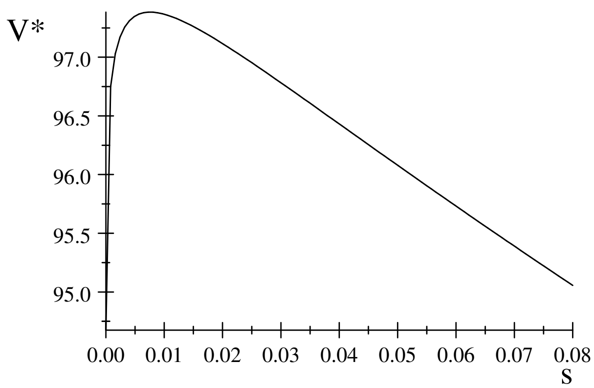

Figure 1A

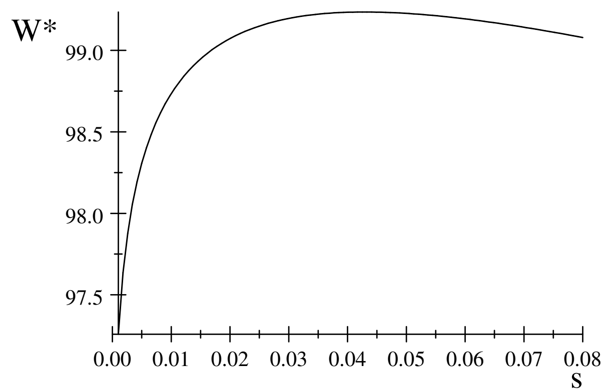

Figure 1B

We next illustrate Proposition 5. Let $\beta_h = 0.8$, $\beta_l = 0.3$, $\delta = 0.8$, $\Delta = 20$ and $\frac{1}{b} = 100$. Note that from (32), t^o solves

$$\frac{\partial W^*}{\partial t} = \left(\left(\Delta - \sqrt{\frac{2\Delta s}{\gamma(t)}} + \frac{s}{\gamma(t)}\frac{1}{1 - \frac{u_\gamma}{\Delta}}\right)(\beta_h - \beta_l) - t\right)b = 0.$$

We can show that there exists $\sigma = 0.04$ such that $t^* \leq t^o$ if $s \leq \sigma$ but $t^* > t^o$ if $\sigma < s$. Therefore, quality investment is deficient (excessive) when s is small (large). For example, if $s = 0.03$, $t^* = 8.8535 < t^o = 9.5355$; and if $s = 0.05$, $t^* = 10.059 > t^o = 9.3998$.

We finally illustrate Proposition 6. Suppose $s = 0.03$ and $\Omega = 0.1$. Then, from (35) and (22), $t_\Omega = 10 < \hat{t} = 13.28 < \bar{t} = 21.363$. In this case, social welfare with the intermediary is

$$W^*(t_\Omega) = \beta_h\left(\Delta - \sqrt{\frac{2\Delta s}{\beta_h}}\right) + \delta\beta_h\left(\Delta - \sqrt{\frac{2\Delta s}{\beta_h}}\right) - \int_0^{t_\Omega} bxdx = 26.536,$$

which is higher than welfare without the intermediary:

$$W^{*}\left(t^{*}\right)=\gamma\left(t^{*}\right)\left(\Delta-\sqrt{\frac{2\Delta s}{\gamma\left(t^{*}\right)}}\right)+\delta\beta_{h}\left(\Delta-\sqrt{\frac{2\Delta s}{\beta_{h}}}\right)-\int_{0}^{t^{*}}bxdx=17.867.$$

However, if $\Omega = 0.2$ and thus $t_{\Omega} = 20 > \hat{t}$, then from (42), t_k^* solves

$$\frac{\partial\Psi}{\partial t_{k}}=\left[\left(\sqrt{\frac{2\Delta s}{\gamma}}+\frac{s}{\gamma}\frac{\Delta}{\sqrt{\frac{2\Delta s}{\gamma}}}\right)\left(\beta_{h}-\beta_{l}\right)-t_{k}\right]b-b\left(t_{k}\right)=0$$

and $t_k^* = 7.45\,38$. Hence, we have $t_k^* < t^* < t^o$. In this case, social welfare in the presence of the intermediary is

$$W^{*}\left(t_{k}^{*}\right)=\gamma\left(t_{k}^{*}\right)\left(\Delta-\sqrt{\frac{2\Delta s}{\gamma\left(t_{k}^{*}\right)}}\right)+\delta\beta_{h}\left(\Delta-\sqrt{\frac{2\Delta s}{\beta_{h}}}\right)-\int_{0}^{t_{k}^{*}}bxdx=17.848.$$

Therefore, if $t_{\Omega} > \hat{t}$, the intermediary reduces welfare by inducing an equilibrium quality that is further below the social optimum.

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