Decomposition of intra-household disparity sensitive fuzzy multi-dimensional poverty index: A study of vulnerability through Machine Learning

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DECOMPOSITION OF INTRA-HOUSEHOLD DISPARITY SENSITIVE FUZZY MULTI-DIMENSIONAL POVERTY INDEX: A STUDY OF VULNERABILITY THROUGH MACHINE LEARNING

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1. Introduction

The traditional multi-dimensional measures have failed to properly project the vulnerability of human-beings towards poverty. Some of the reasons behind this inability may be the failure of the existing measures to recognise the graduality inside the concept of poverty and the disparities within the household in wealth distribution. So this work wants to develop a measure to estimate the vulnerability of households in becoming poor in a multidimensional perspective through incorporating the intra-household disparities and graduality within the causal factors. Dimensional decomposition of the developed vulnerability measure is also under the purview of this work. To estimate the vulnerability and dimensional influences with the help of artificial intelligence an integrated mathematical framework is developed.

2. Review of literature

One of the major impedement of the well accepted Multi-dimensional Poverty Indices (Alkire & Foster, Counting and Multidimensional Poverty Measurement, 2009) (Alkire & Foster, 2011) (Alkire, Kanagaratnam, & Suppa, The Global Multidimensional Poverty Index (MPI):2018 Revision, 2018) is that they have tried to distinguish the poor from the non-poor through the classical Boolean logic. But the idea of poverty suffers from vagueness and naturally cannot be defined through a well defined cut-off. So discussing the multidimensional poverty through ordinary proposition is not correct (Qizilbash, 2006).

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The graduality within a vague concept can well be represented through fuzzy logic (Zadeh, 1965). Cerioli and Zani first attemted to use the fuzzy logic on the measurement of multidimentional poverty (MP) (Cerioli & Zani, 1989). They have estimated the strength of poverty in each dimension through a membership function. Then the strength of all dimensions are added and normalised to get the household level multidimentional poverty. Their idea is improved subsequently through a voluminous research (Chelli & Lemmi, 1995) (Martinetti, 2006) (Betti, Cheli, Lemmi, & Verma, 2006) (Chakravarty, 2006).

Another important drawback of the conventional multidimentional poverty indices is that all of these measures have accepted that the poverty status of the individuals within a household are equall. Thus these indices have accepted the household as a homogenous unit and such that have failed to capture the intra-household differences. But it is an established fact that the different members within a family enjoys varied endowment as well as bargaining power (Agarwal, 1997) (Duflo, 2003). So computing poverty measures taking households as the basic unit leads to improper estimation. The earliest work to put importance on the individuals instead of the household was carried out by Haddad and Kanbur (Haddad & Kanbur, 1990). Vijaya *et. al.* (Vijaya, Lahoti, & Swaminathan, 2014) and Klasen *et. al.* (Klasen & Lahoti, 2016) have developed individual sensitive measures in the multi-dimensional framework.

Apart from measuring the composite effect of the multi-dimensional poverty a large volume of research appeared on the decomposition of composite index. Using the properties of sub-group decomposability Alkire *et. al.* have decomposed Alkire-Foster Adjusted Headcount Ratio (Alkire, Roche, & Vaz, 2017). Deutch and Silber have tried to decompose the fuzzy multidimentional poverty index through Shapley method to find the importantance of the causal factors (Deutsch & Silber, 2006). The Shapley Value Decomposition determines the average of the marginal contributions of a factor under different combinations. To that respect, the concerned factor is first withdrawn from the model and the rest of the factors are permuted to form different distributions. Gradually, the withdrawn factor is added to each of the combination and the marginal contribution of the added factor in a specific distribution is counted. The average of marginal contributions of the stated factor from all the distributions is the influence of that very factor on the composite variation (Shorrocks, 2013).

To decompose the multi-dimensional poverty index machine learning can be used. Machine Learning(ML) is a technique of data analytics that instructs computer to learn from experience. Machine Learnings algorithms use computational methods to "learn" information directly from data without depending on a pre-set equation as a model (Kubat, 2017) (Theobald, 2017). Shapley Value Machine Learning can successfully implement Shapley Decomposition of MP. A framework called SHapley Additive exPlanation (SHAP) executes this through Local Interpretable Machine-agnostic Explanation (LIME) algorithms (Lundberg & Lee, 2017).

So to improve the poverty estimation incorporation of fuzzy logic and intra-household disparities are needed. Dimensional decomposition of the improved estimate can examine the influence of individual causal factors on the vulnerability of the households to become poor. Shapley Value Machine Learning can play an important role in this decomposition. Thus the specific objectives of this study are the following.

3. Objectives

- Measuring the vulnerability to become poor multi-dimentionaly incorporating the ideas of graduality and intra-household disparities.
- Development of appropriate machine learning process with the help of artificial importance to examine the dimensional influence on household level vulnerability.

4. Findings

Let there are n individual, expressed as i=1,2,..n and k dimensions expressed as j=1,2,…k. The performance level of each individual on each dimension can be expressed as a n×k real valued non-negative matrix. Each row vector $y_{i}=\{y_{ij}\}$interprets the performance vector of ith individual.

Let, z is a vector of dimensional thresholds when $z=\{z_j\}$.

Then matrix $g_i^0=\{1,0\}$ can be created where

$g_{ij}^0=1$, when $y_{ij}<z_j$ and

$g_{ij}^0=0$, when $y_{ij}\geq z_j$

g_{ij}^0 is 1 if the ith individual is poor in the jth dimension and otherwise. Now let $c_i=|g_{ij}^0|$ and vector c shows the number of dimensions where each individual is lying below the established dimensional thresholds. If d is the aggregative cut-off to become poor then ith individual will become multi-dimensionally poor when $c_i \geq d$ where $1 < d < k$.

This identification has failed to consider the vagueness of poverty and the existence of intra-household disparities. To rectify this let us assume that each of the households consists of q individuals where q is a positive integer.

Let the grade of membership to the poor set of the qth member of the ith household in a specific dimension is expressed through the dimension specific individual membership function

$$\mu_p^q(\mathrm{i}) = 1 \text{ if } 0 \leq y_j^q \leq y'_j \quad \text{and}$$

$$\mu_p^q(\mathrm{i}) = 0 \text{ if } y_j^q > y''_j$$

An individual is definitely poor if his achievement in a particular dimension j is from 0 upto y'_j. On the other hand if individual achievement is above y''_j then the individual is not poor on dimension j. For individual achievement between y'_j and y''_j the membership function thakes on values in [0,1]. More clearly it can be interpreted that if

- $\mu_p^q(\mathrm{i}) = 0$ if the ith individual is certainly not poor in the jth dimension.
- $\mu_p^q(\mathrm{i}) = 1$ if the ith individual completely belongs to the poor set corresponding to jth dimension.
- $0 < \mu_p^q(\mathrm{i}) < 1$ if the ith individual shows a partial membership to the poor set p of jth dimension.

The strength of membership of all the individuals of a particular household in a particular dimension can be added and deflated by the number of household members to get the collective strength of household membership in a particular dimension. Thus the individual sensitive grade of membership of ith household in jth dimension can be represented as

$$\mu_p(ij) = \frac{\sum \mu_p^q}{q}$$

The grade of membership of the ith individual to the multi-dimensional poor set M can be defined as

$$\mu_M(i) = \frac{\sum_{j=1}^{k} \mu_p(ij)}{k}$$

Where $0 \leq \mu_M(i) \leq 1$ and $\mu_M(i)$ is the vulnerability of ith household to become multi-dimensionally poor. If weight w_j is assigned to the jth dimension then it can be written that

$$\mu_M(i) = \frac{\sum_{j=1}^{k} y_{ij.w_j}}{\sum_{j=1}^{k} w_j}$$

This work is interested to examine the dimensional influences on the household level vulnerability through necessary decomposition. Let the household level vulnerability is λ_i. It is quite natural that the desired value of λ_i is 0. Thus, the difference between desired and observed vulnerability is λ_i. To decompose λ_i Shapley value decomposition has been used. This method calculates the average of marginal contributions of each dimension to the level of vulnerability. To find the contribution of j^{th} dimension we would find different combination of K-1 dimensions. So, the total no of combinations without the jth dimension is –

$$^{(K-1)}C_1 + {}^{(K-1)}C_2 + {}^{(K-1)}C_3 + {}^{(K-1)}C_4 + \ldots . + {}^{(K-1)}C_{K-1}$$

$$= \sum_{h=1}^{K-1} (k-1)_{C_h}$$

$$= \theta$$

Subsequently, the j^{th} dimension is added to each of θ combinations to find the marginal effect of j^{th} dimension from that particular combination. Naturally, we would get θ marginal contributions. According to Shapley decomposition, the contribution of j^{th} dimension is the average of all the marginal contributions of jth dimension. Let, the marginal contribution of j^{th} dimension from s^{th} combination is φ_s. So, the set of marginal contributions of the j^{th} dimension is-

$$H_j = (\varphi_1^j, \varphi_2^j, \ldots, \varphi_\theta^j)$$

Then average of marginal contribution of the jth dimension is

$$CON_j = \frac{1}{\theta} \sum_{s=1}^{\theta} \varphi_s^j$$

As we have θ contributions under each dimension then it can be said that these marginal contributions have been generated through θ^j functions. The polinomial form of sth combination under the jth dimension can be chosen from the set of ψ alternative polinomials ψ^{sj}. To that respect the machine learning process will find the observed value of the composite dependent variable through a particular polinomial from set ψ^{sj} and compare that observed value with the expected value to find the error. Learning from the successive errors within ψ^{sj} the machine learning process will choose that polinomial from ψ^{sj} which will minimise the error. In this way θ^{j} functional forms can be determined under jth dimension. From these θ functions we can find the θ^{j} incremental influences. The average of the θ^{j} incremental influences will deliver CON_{j}.

This estimation of dimensional contributions can be executed through Local Interpretable Machine-agnostic Explanation (LIME) algorithms. LIME deliberately perturbs a combination by accepting input variables from the neighbourhood and counts the effect of that perturbation on the output. Finally the relevance of the particular input is determined through the average of deviation in the output due to the perturbations. Following this logic in our decomposition model LIME will first find θ^{j} functions from the θψ alternatives through perturbation. After that the algorithm proceeds to estimate the dimensional contribution. Technically LIME initiates the process to locate

$$\min E_{\pi_{sj}} = \min[\mathrm{g}(F_u) - \mathrm{f}(F_v)]_{\pi_{sj}} \rightarrow F^{*}_{sj} \rightarrow \varphi^{j}_{s}$$

where u≠ v and u,v=1,2…..ψ. π_s is the neighbourhood of sth functional form under jth dimension. g(F_u) is the expected value and f(F_v) is the observed value of the multi-dimensional poverty from a particular polinomial related to π_{sj}. F^{*}_{sj} is the chosen polynomial from ψ^{sj}. This process will continue for all the combinations under θ^{j} to find θ^{j} incremental influences. Finally the average of the θ^{j} incremental incluences determines

$$CON_{\mathrm{j}} = \frac{1}{\theta} \sum_{s=1}^{\theta} \varphi_{\mathrm{s}}^{\mathrm{j}}$$

5. Conclusion

The mathematical framework developed under the current work will deliver a better estimate of vulnerability as well as dimensional influences to household level multi-dimensional poverty through accommodating fuzzy logic and intra-household disparities. The use of artificial importance as demonstrated within the model will make the model easy to execute. This estimation of vulnerability and decomposition will help the planners to locate the role of different dimensions behind the vulnerability of human beings to become poor more precisely.

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