Price-directed Consumer Search

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Abstract. We extend Stahl's (1989) model to a setting with differentiated products to study the effects of price-directed consumer search. Consumers engage in costly search to find out whether products meet their needs. Consumer search is directed by prices when they are observable before search, in contrast to the case in which prices are discovered only after search, where search is naturally random. The equilibrium under price-directed search differs substantially from that under random search, despite certain similarities. We show that as search costs decrease, sales become more likely and firms earn higher expected profits under price-directed search, whereas the opposite holds under random search. Moreover, compared with random search, under price-directed search firms' expected profits are always lower, but consumer surplus and total welfare are higher provided that the search cost is sufficiently small.

Keywords: Consumer search, Observable price, Search cost
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1 Introduction

Consumers in many markets now have easy access to price information due to the rapid development of Internet technology. For instance, through price search engines or price comparison websites, a consumer can easily obtain a list of prices for various products. With prices in hand, consumers may examine, in a deliberate order, whether products match their preferences. In other words, consumer search may be directed by prices. Although it is becoming more common for consumers to have access to price information before engaging in costly search, the market implications of price-directed consumer search have not been well studied in the literature, which typically assumes that consumers search in a random order to acquire both product and price information (e.g. Wolinsky, 1986; Anderson and Renault, 1999).

This paper develops an oligopolistic model to study situations where consumers may have access to price information before engaging in sequential search among differentiated products. To this end, we modify and extend Stahl's (1989) model by introducing simple and tractable product differentiation. In particular, each consumer has a need and each firm provides a product or service that meets the consumer's need with some exogenous probability. Consumers derive a positive utility only if their needs are met. As in Varian (1980) and Stahl (1989), there are two groups of consumers: informed and uninformed. Informed consumers have both product and price information and thus purchase from a matched seller with the lowest price. Uninformed consumers have to engage in costly sequential search to examine whether products meet their needs and to find out prices if these are not observable before search. Firms simultaneously set their prices by taking consumer search strategies into account.

We consider both non-observable and observable price scenarios. When prices are not observable before search, the uninformed consumers discover the product prices (together with the product match) as they proceed with their costly search. Because there is no ex-ante differentiation among firms, consumers naturally visit firms in a random order. When prices are observable before search, firms are no longer ex-ante symmetric because the order of consumer search is influenced by prices. We refer to the former scenario as the case of random search and the latter scenario as the case of price-directed search.

Our model has some desirable features that are lacking in recent models of price-directed search (e.g. Armstrong and Zhou, 2011; Haan, Moraga-González, and Petrikaite, 2015; Choi, Dai, and Kim, 2016). As discussed in Armstrong and Zhou (2011) and Haan, Moraga-González, and Petrikaite (2015), if prices are observable in search models with differentiated products such as Wolinsky (1986), there will be no pure-strategy equilibrium because a firm has an incentive to slightly undercut its price so as to attract all consumers to visit it first. Moreover, mixed-strategy equilibria are not tractable. To overcome these difficulties, most existing price-directed search models build in ex-ante product differentiation, which permits equilibria in pure-strategies to exist. In contrast, our model assumes no ex-ante product differentiation among firms and focuses on equilibria in price mixing.[1] We believe that our model is a better fit for market settings where product attributes are not known by some consumers without costly search or there is no difference among the product attributes that can be observed for free.[2]

We derive an optimal search rule for consumers and characterize a unique symmetric equilibrium under random search and price-directed search, respectively. Under random search, as in Stahl's model, the uninformed consumers search randomly with an optimal *reservation price*: they stop and buy if the product turns out to be a match and the price of the product is below their reservation price. Under price-directed search, the uninformed consumers search only firms with prices below a certain price which we call *threshold price*, and proceed from low- to high-priced firms. As the search

[1]Armstrong and Zhou (2011) also study an equilibrium that involves price mixing in a Hotelling model where a consumer's product values are perfectly negatively correlated.

[2]For example, a person located in Denver planning a trip to Beijing for the first time may need to check (on a price-comparison website) which airline provides a round-trip flight between Denver and Beijing that fits his travelling schedule. Some studies (e.g. Golan, Karp and Perloff, 2002) find evidence that mixed strategies in pricing are quite common in the airline industry.

cost decreases, the reservation price under random search decreases because the uninformed consumers demand a better price deal. However, under price-directed search, the threshold price increases as the search cost decreases, because for a lower search cost, the uninformed consumers are willing to continue searching among firms that charge higher prices. The behavior of the reservation and threshold prices in opposite directions with respect to the search cost is a crucial driver of many of the results in this paper.

The price equilibrium under price-directed search has similarities with and differences from that under random search. First, under both random and price-directed search, each firm charges a price randomly drawn from a price distribution. This is because firms have an incentive to charge low prices to attract the informed consumers who can compare prices directly without search but at the same time they also have an incentive to charge high prices to exploit the uninformed consumers who have to search among firms. Like in Stahl's model, these two forces are balanced when firms randomize their prices. Second, unlike in Stahl's model, the symmetric equilibrium price distributions in our model may have a non-convex support with a low-price and a high-price interval. This occurs when the search cost is low under random search or when the search cost is high under price-directed search. In particular, under random search, on the one hand, the informed consumers who match only with one product will accept a very high price, up to the product value. On the other hand, a firm will not be able to sell to the uninformed consumers at a price above their reservation price. When the reservation price is far below the product value, which occurs when the search cost is low, firms optimally swing between targeting the matched informed consumers and attracting the uninformed ones. Thus, the equilibrium price distribution has both a low-price and a high-price interval, the latter being above the reservation price. Similarly, under price-directed search, when the threshold price is sufficiently low because of a high search cost, the support of the equilibrium price distribution is non-convex.

If one regards prices in the high-price interval as "regular prices" and those in the low-price interval as "sales prices", the two-interval pricing strategies are qualitatively consistent with the empirical evidence on supermarket pricing (Chevalier et al. 2003; Kehoe and Midrigan, 2008) and from E-retailers on the Internet (Baye, Morgan, and Scholten, 2004). We further show that when the search cost decreases, a sale at a price in the low-price interval becomes more likely under price-directed search and less likely under random search. Under random search, for a lower search cost, the uninformed consumers are pickier and choose a lower reservation price. Thus, firms increase the probability of charging a price from the high-price interval that targets more the matched consumers. However, under price-directed search, the uninformed consumers use a higher threshold price for a lower search cost. Thus, firms optimally increase the probability of charging a price from the low-price interval.

We obtain some additional interesting results by comparing the effects of search cost under random and price-directed search. First, as the search cost decreases, the expected market price decreases when the search cost is large but then increases when the search cost is small under both random and price-directed search. However, the reasons for the non-monotonic relationship between the expected market price and the search cost are quite different. For instance, under random search and for a small search cost, firms adopt the two-interval pricing strategy. A lower search cost leads to a lower reservation price, which induces firms to charge a price from the high-price interval more frequently. The increased probability of the high-price interval is a dominant force that results in a higher expected market price. Conversely, under price-directed search and for a small search cost, the entire equilibrium price distribution is below the threshold price of the uninformed consumers. A decrease in the search cost raises the threshold price and thus reduces price competition among firms, resulting in higher equilibrium prices.[3] As we will discuss in detail in Section 2, our results relate to

[3]For a large search cost, the opposite forces prevail in the model. In particular, the equilibrium price distribution is below the reservation price under random search and shifts down with a lower search cost because of a lower reservation price. Under price-directed search, firms adopt the two-interval pricing strategy. More weight is placed on the low-price interval for a higher threshold price due to the lower search cost. Hence, the expected market price is lower for a lower search cost.

but differ from the existing studies (e.g. Stahl, 1989; Wolinsky, 1986; Armstrong and Zhou, 2011; Haan, Moraga-González, and Petrikaite, 2015; Choi, Dai, and Kim, 2016) that examine the effect of search cost on market price.[4] Second, as the search cost decreases, firms' expected profits weakly decrease under random search because the uninformed consumers search more aggressively. However, when prices are observable before search, the uninformed consumers will continue searching among firms that charge higher prices if the search cost is lower. Consequently, firms' expected profits increase as search costs decrease.

We also compare the welfare implications of random and price-directed search. Our analysis suggests that firms' profits are always higher under random search, while consumer surplus and total welfare are higher under price-directed search when the search cost is small. Intuitively, if prices are observable before search, they influence the order in which consumers search and thus the demand for firms. Hence, price competition among firms is intensified, resulting in lower expected profits. Moreover, comparing price-directed search with random search, the uninformed consumers search more efficiently in the sense that they do not search firms whose posted prices are higher than their threshold price, and always search firms with lower prices first. Therefore, total welfare and consumer surplus tend to be higher under price-directed search than under random search when the search cost is small. It is worthy of noticing that, in our model, total welfare can be higher under random search than under price-directed search when the search cost is large. This is because when the search cost is large, firms always price below the reservation price under random search while they price above the threshold price with some positive probability under price-directed search. Hence, the uninformed consumers are less likely to make purchases under price-directed search since they will not continue searching among firms that charge prices above the threshold price. This may result in lower total welfare under price-directed search.

[4]Specifically, existing studies find that equilibrium prices go up in search costs under random search (Stahl, 1989; Wolinsky 1986) but go down under price-directed search (Armstrong and Zhou, 2011; Haan, Moraga-González, and Petrikaite, 2015; Choi, Dai, and Kim, 2016)

The remainder of this paper is organized as follows. In Section 2, we review the related literature. In Section 3, we describe the model and conduct equilibrium analysis under random and price-directed search, respectively. In Section 4, we compare the effects of lower search costs on market performance and welfare under random and price-directed search. Section 5 concludes the paper. All proofs and diagrams are relegated to Appendix A and B, respectively.

2 Related literature

Our model is related to two classical models of sequential consumer search. Focusing on consumer search for the best price among competing homogeneous sellers, Stahl (1989) develops a remarkable model to show how price dispersion arises in a homogeneous-product market due to the presence of consumers who engage in costly search to discover market prices.[5] Emphasizing consumer search for the best value among horizontally differentiated sellers, Wolinsky (1986) provides an influential model in which consumers are initially uninformed about their valuation of each product or its price, but may learn this information through costly search.[6] Our model incorporates these two important features, price dispersion and product differentiation. In particular, in our model, price dispersion arises in equilibrium because firms adopt a mixed strategy in pricing and consumers actively search for a product that meets their needs and a lower price.

Several papers have studied the impact of non-random consumer search on market performance. Arbatskaya (2007) examines a model where consumers engage in sequential search among firms selling homogenous goods. In her model, consumers have to

[5]Studies extend Stahl's model in various directions. For example, Janssen, Moraga-González, and Wildenbeest (2005) relax the assumption that consumers obtain the first price quotation for free; Stahl (1996) and Chen and Zhang (2011) allow for the heterogeneity of search costs and of searchers, respectively; and Janssen and Parakhonyak (2014) consider the possibility of costly revisits.

[6]Anderson and Renault (1999) further extend Wolinsky's model by studying the effect of consumer taste for diversity on equilibrium prices.

visit firms in an exogenous pre-determined order. In equilibrium, prices charged by firms decline in the order of consumer search. A consumer with a higher (lower) search cost buys from the firm at the top (bottom) of the order paying a higher (lower) price. Armstrong, Vickers, and Zhou (2009) investigate a model of differentiated products where consumers visit a prominent firm before turning to non-prominent ones. They show that the prominent firm charges a lower price and the non-prominent firms charge higher prices, compared to the situation with random search.[7] Differently, in the case of price-directed consumer search we consider in our model, the order of consumer search is endogenously determined by prices. Thus, the prices chosen by firms also affect their profit by influencing the order of consumer search.

Our paper is closely related to recent papers that study situations where consumer search is directed by prices.[8] Armstrong and Zhou (2011) explore a price-directed search model in which consumers know the prices charged by firms before they engage in sequential search to discover product match utilities. In their model, two firms are located at the ends of a Hotelling line and thus a consumer's match utilities for the two firms are perfectly negatively correlated.[9] They find that equilibrium prices decrease

[7]Zhou (2011) further extends Armstrong, Vickers, and Zhou (2009) to a completely ordered search model and shows that prices rise in the order of consumer search. Rhodes (2011) shows that a prominent retailer earns significantly more profit than others, even when the cost of gathering product and price information is essentially zero.

[8]Studies also examine the situation where firms can strategically influence the order of consumer search. Chen and He (2006) investigate a model where firms differing in relevance of matching the needs of consumers bid for the advertised position for their products and consumers are initially uncertain about whether the product matches their needs but can learn this through costly search. Wilson (2010) studies a model where a firm can choose to obfuscate itself by increasing the cost that some consumers must incur to search for its product price. Haan and Moraga-González (2011) consider an attention-directed consumer search model where consumers first visit the firm whose advertising is most salient. Garcia and Shelegia (2015) examine directed consumer search with observational learning.

[9]Shen (2015) also studies a Hotelling model in which consumers with heterogeneous ex-ante "brand" preferences obtain the match values through costly search.

with search costs because a higher search cost increases a firm's profit from being prominent, and thus provides a higher incentive for the firm to charge a lower price. Haan, Moraga-González, and Petrikaite (2015) examine a consumer search model a la Wolinsky, but allow firms to influence the search order by adjusting prices and providing match-value information. They find that higher search costs intensify price competition on first visits, but relax it after first visits. Overall, the first effect may dominate and result in unusual comparative statics. They also study price advertising decisions and show that both firms choose to advertise prices even though equilibrium prices and firms' profits are lower when prices are advertised. Choi, Dai, and Kim (2016) study an oligopoly model in which consumer search is based on partial product information and advertised prices. They show that market prices increase with a reduction in search cost but may increase or decrease when more information is provided to consumers. Our price-directed search model differs from the above models in that price is the only observable product attribute before consumers search, and consumer utilities, conditional on matching, are independently distributed across $N \geq 2$ firms. Moreover, the nature of the equilibrium price distribution in our model is quite different from those of the other models. Finally, an increase in search cost may also lead to lower expected prices in our model, but for a rather different reason. In particular, in the above models, a higher search cost intensifies price competition for first visitors and thus leads to lower expected prices, while in our model, the uninformed consumers do not know whether a product meets their needs before searching and thus they have to be compensated by a lower observable price if the search cost is higher.[10]

[10]Our paper also relates to Moraga-González, Sándor, and Wildenbeest (2017), who show in a Wolinsky-type model how search costs affect both the intensive search margin (or search intensity) and the extensive search margin (or the decision to search or not) and how equilibrium price formation is determined by these two margins. Our price-directed search model has elements relating to the extensive search margin in the sense that the uninformed consumers search only if observable prices are low enough, and may be priced out of the market when the search cost is large enough.

3 The model and equilibrium analysis

There is a continuum of consumers, each with a need. A consumer derives utility V if her need is met and zero otherwise. There are $N \geq 2$ firms in the market, each of which carries a product that meets the consumer's need with a certain probability. We call it a "match" if a product meets the consumer's need. The probability of a match is assumed to be independent and identical for all firms and consumers, and is denoted as $\theta \in [0, 1]$.[11] The marginal cost is constant and normalized to zero.

Similar to Varian (1980) and Stahl (1989), there are two groups of consumers: a fraction $\mu \in (0, 1)$ of informed consumers who are fully aware of product and price information (without searching) and a fraction $1 - \mu$ of uninformed consumers who engage in costly search to find a product that matches their needs, and its price if not observable before search. The search cost for the uninformed consumers is denoted as s. Following the standard consumer search literature, we assume that the first search is free.

We consider two alternative situations where prices are either non-observable or observable before searching, respectively. In the situation where prices are not observable before searching, consumer search is random because the uninformed consumers discover whether a product is a match and obtain its price only through search. In the situation where prices are observable before search, consumer search is directed by prices because the uninformed consumers rationally examine the products of firms charging lower prices first. We refer to the former situation as the random search case and to the latter as the price-directed search case.

As in the literature, we focus on a symmetric Nash equilibrium, that is, a price dis-

[11] This could arise when consumers have specific needs and the available products are so broad that consumers have to verify whether a product can satisfy their needs. Alternatively, one can think of a product as a bundle of characteristics. Each consumer values only a few product attributes and is satisfied as long as a product embeds these wanted characteristics. Athey and Ellision (2011), Chen and He (2011), and Chen and Zhang (2018) similarly assume that a firm provides a product that either matches a consumer's need or not.

tribution function and a search strategy for the uninformed consumers such that, given the pricing strategy of firms and the search strategies of consumers, it is optimal for each firm to price according to the price distribution, and given the price distribution, the search strategy is optimal for the uninformed consumers.

3.1 Consumer search

We begin with consumers' optimal search strategies. As the informed consumers observe all prices and whether the products match their needs, they purchase from the matched firm (if one exists) with the lowest price in the market. The uninformed consumers decide on a search strategy to maximize their expected payoffs.

Random search

In the case of random search, the optimal stopping rule for an uninformed consumer is as follows. Given a price distribution $\Phi(p)$, the uninformed consumer will randomly sample firms and stop searching if she finds a matched product and the price of that product is below a reservation price ω which is determined by the equation:

$$\theta \int_{p \leq \omega} (\omega - p) d\Phi(p) = s. \tag{1}$$

Note that the left-hand side is the expected benefit of a single additional search. If the uninformed consumer has searched all firms without finding a matched product priced below ω, she will return to the firm that provides a matched product and charges the lowest price provided that the price is below V or exit the market otherwise. Note that ω decreases as s decreases because the expected marginal benefit is lower for a lower ω.

Price-directed search

When prices are observable before search, the order of search by the uninformed consumer is no longer random, but directed by prices. Given that a firm charges a price p, the uninformed consumer will search the firm if the expected search benefit is higher than the search cost: $\theta(V-p) \geq s$ or

$$p \leq r = V - \frac{s}{\theta}. \tag{2}$$

We call the threshold price of the uninformed consumer r. Note that r increases as s decreases because the uninformed consumer is willing to continue searching among firms that charge higher prices when the search cost is lower.

Note that the uninformed consumer receives a higher expected payoff by visiting the firm that offers a lower price. Moreover, the uninformed consumer will buy from a firm she has visited if that firm offers a matched product or move on to search another firm in case of no match. Therefore, an uninformed consumer's strategy consists of (i) a search order from low- to high-price firms; (ii) a threshold price r: searching only if $p \leq r$; and (iii) a purchase decision: purchasing from the first matched firm, if one exists, during search.

3.2 Equilibrium price distribution

We now turn to the analysis of firms' pricing in a symmetric Nash equilibrium. Note that, as in Stahl's model, our model has no pure-strategy equilibrium and the equilibrium price distribution must be atomless on its entire support.[12] As we will show in Propositions 1 and 2, the symmetric equilibrium price distributions under random and price-directed search have either a convex support over a continuous interval or a non-convex support with a low-price interval and a high-price interval, depending on the search cost. Here, we explain intuitively the driving forces behind these results

[12] To see why, note that in equilibrium a firm earns a positive expected profit, by charging $p = V$ and selling only to those consumers whose needs are satisfied only by the firm. This implies that equilibrium prices exceed zero. Now, if some price p was charged with positive probability in a symmetric equilibrium, there would be a positive probability of a tie at p. Therefore, it would be profitable for a firm to charge $p - \epsilon$, because the deviating firm would give up an arbitrarily small profit to gain a positive measure of informed consumers who have more than one match. Thus, competition would tend to drive prices down to marginal cost zero and the profit of every firm would also become zero, which yields a contradiction.

before we formally characterize the equilibria. First, note that each firm has a positive measure of informed consumers who only match with the product of the firm in question and can sell to them at a price up to V. At the same time, firms also compete for the uninformed consumers. However, firms hardly sell to the uninformed consumers if they charge a price above ω or r as the uninformed consumers may return only after visiting all other firms in the case of random search or do not visit them at all in the case of price-directed search. Hence, when ω or r is high and close to V, firms optimally charge a price below ω or r to sell to both groups of consumers. Therefore, the equilibrium price distribution is similar to that of Stahl's model: the entire price distribution is below ω or r with convex support. When ω or r is low and far below V, firms optimally swing between targeting their captive matched informed consumers and attracting the uninformed consumers. Thus, the equilibrium price distribution has a low-price interval and a high-price interval with the latter above ω or r: each firm randomly chooses a price from either the low-price or the high-price interval with some probability. Moreover, the shape of the equilibrium price distribution depends on the values of ω or r, which in turn crucially depend on s.

To ensure that the uninformed consumers engage in search, we assume that s is not too high. Moreover, for easier presentation, we define the following auxiliary variables:

$$\phi = \int_{1-\theta}^{1} \frac{\mu N\theta(1-\theta)^{N-1} + (1-\mu)[1-(1-\theta)^N]}{\mu N\theta y^{N-1} + (1-\mu)[1-(1-\theta)^N]} dy$$

and

$$\eta = \int_{1-\theta(1-\alpha)}^{1} \frac{\mu N\theta(1-\alpha)(1-\theta(1-\alpha))^{N-1} + (1-\mu)\left[1-(1-\theta(1-\alpha))^N\right]}{\mu N\theta(1-\alpha)y^{N-1} + (1-\mu)\left[1-(1-\theta(1-\alpha))^N\right]} dy,$$

where α will be defined later. We next move to the discussion of random and price-directed search.

Random search

Under random search, we show that if s is relatively high, the entire equilibrium price distribution is below the reservation price of the uninformed consumers. However, if s is relatively low, the equilibrium price distribution, in contrast, has a non-convex support with a high-price interval and a low-price interval. Formally, we divide the possible values of s into $(0, s_1)$ and $[s_1, s_2]$ where

$$s_1 = \frac{N\theta(1-\theta)^{N-1}}{N\theta\mu(1-\theta)^{N-1} + (1-\mu)\left[1-(1-\theta)^N\right]}(\theta-\phi)V \qquad \text{and} \qquad s_2 = (\theta-\phi)V.$$

Note that we can verify that $s_1 < s_2$ if $\mu < 1$. We obtain the following results.[13]

Proposition 1. *In the case of random search, there exists a unique symmetric equilibrium as follows.*

(i) If $s \in [s_1, s_2]$, each firm prices according to a mixed strategy

$$F(p) = \frac{1}{\theta}\left\{1 - \left[\frac{\omega_2}{p}(1-\theta)^{N-1} + \frac{1-\mu}{\mu}\left(\frac{\omega_2}{p}-1\right)\frac{1-(1-\theta)^N}{N\theta}\right]^{\frac{1}{N-1}}\right\} \tag{3}$$

$$\text{if} \qquad p \in \left[\frac{N\theta\mu(1-\theta)^{N-1} + (1-\mu)\left[1-(1-\theta)^N\right]}{N\theta\mu + (1-\mu)\left[1-(1-\theta)^N\right]}\omega_2, \omega_2\right],$$

and the uninformed consumers search randomly with a reservation price $\omega_2 = \frac{s}{\theta-\phi}$. In this equilibrium, each firm earns expected profit

$$\omega_2\theta\left[\mu(1-\theta)^{N-1} + (1-\mu)\frac{1-(1-\theta)^N}{N\theta}\right];$$

(ii) If $s \in (0, s_1)$, each firm prices according to a mixed strategy

$$G(p) = \begin{cases} (1-\alpha)G_l(p) & \text{if } \frac{\mu N\theta(1-\alpha)(1-\theta+\theta\alpha)^{N-1} + (1-\mu)\left[1-(1-\theta+\theta\alpha)^N\right]}{\mu N\theta(1-\alpha) + (1-\mu)\left[1-(1-\theta+\theta\alpha)^N\right]}\omega_1 \le p \le \omega_1 \\ (1-\alpha) & \text{if } \omega_1 < p < \left(\frac{1-\theta}{\theta\alpha+1-\theta}\right)^{N-1}V \\ (1-\alpha) + \alpha G_h(p) & \text{if } \left(\frac{1-\theta}{\theta\alpha+1-\theta}\right)^{N-1}V \le p \le V \end{cases}$$

where

$$G_h(p) = 1 - \frac{1-\theta}{\theta\alpha}\left[\left(\frac{V}{p}\right)^{\frac{1}{N-1}} - 1\right],$$

$$G_l(p) = \frac{1}{\theta(1-\alpha)}\left\{1 - \left[\frac{\omega_1}{p}(\theta\alpha+1-\theta)^{N-1} + \frac{1-\mu}{\mu}\left(\frac{\omega_1}{p}-1\right)\frac{1-(1-\theta+\theta\alpha)^N}{N\theta(1-\alpha)}\right]^{\frac{1}{N-1}}\right\},$$

[13] In Appendix B (Figures 5-8), we provide examples of equilibrium probability density functions (p.d.f.s) corresponding to the equilibrium price distributions in Proposition 1. The c.d.f.s are denoted by capital letters while the corresponding p.d.f.s are represented by lower case letters.

and the unique α is determined by

$$\frac{\mu N\theta(1-\alpha)(\theta\alpha+1-\theta)^{N-1}+(1-\mu)\left[1-(1-\theta+\theta\alpha)^{N}\right]}{N\theta(1-\alpha)\left[\theta(1-\alpha)-\eta\right]}=\frac{V(1-\theta)^{N-1}}{s}, \qquad (4)$$

and the uninformed consumers search randomly with a reservation price $\omega_1 = \frac{s}{\theta(1-\alpha)-\eta}$. In this equilibrium, each firm earns expected profit $\theta(1-\theta)^{N-1}V$.

The case of random search in our model is an extension of Stahl (1989). In fact, if $\theta \to 1$, our model under random search degenerates to Stahl's model with unit demand.[14] However, the equilibrium properties substantially differ from those in Stahl's model when the search cost is low (i.e. $s \in (0, s_1)$) and $\theta < 1$. Specifically, unlike in Stahl's model, the equilibrium price distribution has a non-convex support with two disconnected intervals.[15] This is because a firm sells to at least $\theta(1-\theta)^{N-1}$ informed matched consumers even if it prices above ω_1. Moreover, by charging a price higher than ω_1, the firm may lose sales to the uninformed consumers. Thus, all prices in the support of the high-price interval, $G_h(p)$, must be discretely higher than ω_1 because when a firm raises its price above ω_1 its demand jumps down, which must be exactly offset by a jump-up of the price so that all prices in the equilibrium support yield the same expected profit.[16]

Another interesting feature of this equilibrium that contrasts with Stahl's model is that recalls may occur. This is because the uninformed consumers search for both prices and matches in our model. Even if the product matches their needs, the uninformed consumers do not purchase immediately if the price is above ω_1. If the uninformed

[14] To see this, note that $s_1 \to 0$ when $\theta \to 1$. Thus, $F(p)$ is the equilibrium price distribution. Moreover, we can show that $F(p)$ asymptotically approaches the equilibrium price distribution in Stahl's model with unit demand as $\theta \to 1$.

[15] Note that $\omega_2 \to 0$ as $s \to 0$ because the reservation price increases with s. When $s = s_1$, $\alpha = 0$ and $\omega_2 = \omega_1$. Thus, the price distribution with non-convex support converges to the one with convex support as $s \to s_1$.

[16] Chen and Zhang (2011) also find an equilibrium price distribution with two disconnected intervals. However, in their model, firms sell homogenous products to shoppers, local searchers who buy at random, and global consumers who search optimally. Their price distribution with two disconnected interval arises when the valuation of local searchers is sufficiently high.

consumers do not find a matched product with a price below ω_1 after sampling all firms, they may return to buy from the matched firm with the lowest price, and thus recall occurs.

Price-directed Search

We will show that, under price-directed search, an equilibrium price distribution with support over two disconnected intervals arises when s is high, in contrast to the case of random search. The reason is that under price-directed search, the threshold price of the uninformed consumers, r, decreases with s, while under random search, the reservation price, ω, increases with s. Hence, a smaller s pushes r closer to V but drives ω away from V. Therefore, the equilibrium price distributions behave quite the opposite as s changes. We divide the possible values of s into $(0, s_1')$ and $[s_1', s_2']$:

$$s_1' = (1-\mu)\theta V \qquad \text{and} \qquad s_2' = [1 - \mu(1-\theta)^{N-1}]\theta V.$$

The next proposition characterizes the equilibrium under price-directed search.

Proposition 2. *In the case of price-directed search, there exists a unique symmetric equilibrium as follows.*

(i) If $s \in (0, s_1')$, each firm prices according to a mixed strategy

$$R(p) = 1 - \frac{1-\theta}{\theta}\left[\left(\frac{r}{p}\right)^{\frac{1}{N-1}} - 1\right], \text{ with } p \in [(1-\theta)^{N-1}r, r] \tag{5}$$

and the uninformed consumers search sequentially with a threshold price r from the low-price to the high-price firms. In this equilibrium, each firm earns an expected profit

$$\theta(1-\theta)^{N-1}\left(V - \frac{s}{\theta}\right);$$

(ii) If $s \in [s_1', s_2']$, each firm prices according to a mixed strategy

$$Q(p) = \begin{cases} (1-\alpha')Q_l(p) & \text{if } \mu(1-\theta)^{N-1}V \le p \le r \\ (1-\alpha') & \text{if } r < p < \frac{r}{\mu} \\ (1-\alpha') + \alpha' Q_h(p) & \text{if } \frac{r}{\mu} \le p \le V \end{cases} \tag{6}$$

where

$$Q_h(p) = 1 - \frac{1-\theta}{\theta\alpha'}\left[\left(\frac{V}{p}\right)^{\frac{1}{N-1}} - 1\right], \tag{7}$$

$$Q_l(p) = \frac{1}{1-\alpha'} - \frac{1-\theta}{\theta(1-\alpha')}\left[\left(\frac{\mu V}{p}\right)^{\frac{1}{N-1}} - 1\right], \tag{8}$$

$$\text{and} \qquad \alpha' = \frac{1-\theta}{\theta}\left[\left(\frac{\mu V}{r}\right)^{\frac{1}{N-1}} - 1\right], \tag{9}$$

and the uninformed consumers search only firms whose prices are below r from the low-price to the high-price firms. Each firm earns an expected profit $\mu\theta(1-\theta)^{N-1}V$.

Similar to the case of random search, the threshold price of the uninformed consumers influences the pricing strategy of firms under price-directed search. In particular, when r is close to V, which occurs when s is low, their pricing strategy is driven mainly by the consideration of the threshold price of the uninformed consumers. Thus, the entire equilibrium price distribution is below r. In this equilibrium, the uninformed consumers always find a matched product, if one exists, when searching from the low-price to the high-price firms. When r is far below V, which occurs when s is high, the support of the equilibrium price distribution consists of two price intervals, with corresponding c.d.f.s $Q_h(p)$ and $Q_l(p)$, played with probability α' and $1-\alpha'$, respectively. In equilibrium, with probability α', each firm targets only the matched informed consumers as its price is above r according to the c.d.f. $Q_h(p)$. With probability $1-\alpha'$, each firm charges a price below r according to the c.d.f. $Q_l(p)$, trying to sell to both informed and uninformed consumers. In this equilibrium, the uninformed consumers may not find a matched product even if it exists as they will not visit firms that charge prices above r.

4 Random search v.s. price-directed search

In this section, we first study and compare the effects of lowering search costs on the expected market price, firms' profits and total welfare under random search and price-directed search. As we illustrate below, there are several interesting results in the comparative statics of lower search costs. Then, we compare the welfare levels under the two cases.

4.1 The effects of lower search costs

As shown in Section 3, firms may adopt a two-interval pricing strategy under random and price-directed search. We refer to prices in the low-price interval as "sales", which occur with probability $(1-\alpha)$ under random search and with probability $(1-\alpha')$ under price-directed search. The next result provides an interesting comparison.

Corollary 1 of Propositions 1 and 2. *As the search cost decreases, sales are less likely under random search (i.e.* $\frac{\partial(1-\alpha)}{\partial s} > 0$*) but more likely under price-directed search (i.e.* $\frac{\partial(1-\alpha')}{\partial s} < 0$*).*

Interestingly, a lower search cost decreases the likelihood of sales under random search but increases it under price-directed search. The intuition is as follows. Note that a sales price is primarily used to target the uninformed consumers. In the case of random search, when s decreases, searching for a good price becomes less costly so that the uninformed consumers lower their reservation price. To lock in more uninformed consumers, a firm has to lower sales prices at the opportunity cost of a lower mark-up for other consumers. Therefore, sales become more costly for firms as s decreases. Consequently, firms reduce the probability of choosing a price from the low-price interval. In the case of price-directed search, as s decreases, r increases. This implies that the uninformed consumers are willing to continue searching among firms that charge higher prices (from the low-price interval) when they do not find a match from firms that charge lower prices. This provides incentives for firms to increase the likelihood of the low-price interval to target more uninformed consumers.

The next proposition shows the effects of lowering the search cost under random search and price-directed search.

Proposition 3. *As the search cost decreases, (i) the expected market price in-*

creases when the search cost is small, but decreases when the search cost is large under both random and price-directed search; (ii) a firm's expected profit weakly decreases under random search but weakly increases under price-directed search, and (iii) total welfare increases under price-directed search and may increase or decrease under random search.

Under both random and price-directed search, the expected market price exhibits similar patterns as the search cost decreases. In particular, when s is small, the expected market price increases as the search cost decreases under both random and price-directed search. However, the reasons are quite different. In the case of random search, firms adopt a two-interval pricing strategy when s is small. A decrease in s imposes two opposing forces on the expected market price. On the one hand, a lower search cost pushes down the prices in the low-price interval because of a lower ω. On the other hand, to compensate, firms increase the probability of charging a price from the high-price interval. It turns out that the latter force dominates and thus the expected market price increases as the search cost decreases. In the case of price-directed search, when s is small, r is close to V and thus the entire equilibrium price distribution is below r. Because r decreases with s, firms charge higher prices. Similarly, when s is large, under random search, ω is close to V and thus the entire equilibrium price distribution is below ω. Hence, as s decreases, the entire price distribution shifts down and, thus, the expected market price decreases. Under price-directed search, firms adopt a two-interval pricing strategy because r is low when s is large. A decrease in s results in a higher r. Consequently, firms more frequently draw prices from the low-price interval, thereby targeting the captive matched uninformed consumers. This has a dominant effect and the expected market price declines.

As s decreases, firms' expected profits weakly decrease under random search but weakly increase under price-directed search. Intuitively, under random search, a lower search cost increases the net benefit of additional search. Thus, the uninformed consumers demand a better price deal from a matched firm. Hence, under random search, firms charge lower prices and earn less profits when s decreases. Under price-directed search, a lower s leads to a higher r. Thus, the uninformed consumers continue searching among firms that charge higher prices. This reduces competition among firms, which allows firms to earn higher profits.[17]

It is easy to see that under price-directed search, total welfare increases as s decreases. This is because when s is small, a consumer will end up finding a matched product (if one exists) because the equilibrium prices are below r. In other words, a decrease in s does not affect the probability of a transaction. This implies an increase in total welfare due to a decrease in the search cost. When s is large, total welfare also increases as s decreases because firms charge a price more frequently from the low-price interval. Hence, the probability of a transaction increases and thus total welfare increases because the uninformed consumers search only those firms that charge a price from the low-price interval.

The effect of the search cost on total welfare under random search is more subtle. Similar to the situation under price-directed search, total welfare increases as s decreases when the entire equilibrium price distribution is below ω, which occurs when s is large. However, when s is small and decreases, firms more frequently charge a price from the high-price interval and consequently, the uninformed consumers incur more search costs since they search more. This negative effect on consumer welfare due to a higher expected number of visits can potentially dominate the benefit of a decrease in the search cost. Therefore, under random search, total welfare may increase with s.[18]

[17] One additional effect is present when firms adopt a two-interval pricing strategy: a decrease in ω (increase in r) caused by a decrease in s reduces the likelihood of a low-price (high-price) interval. Nevertheless, firms' expected profit remains unchanged as s changes. This is because a firm earns the expected profit $\theta\,(1-\theta)^{N-1}\,V$ when it charges $p = V$ and by the property of a mixed strategy, the profit must be the same at every price in support.

[18] This can happen when θ is sufficiently large. Note that α increases from 0 to 1 as s decreases from s_1 to 0 monotonically. Moreover, $s_1 \to 0$ as $\theta \to 1$. Thus, if θ is sufficiently close to 1, α has to increase from 0 to 1 very fast ($|\frac{\partial\alpha}{\partial s}|$ is very large because s_1 is close to 0), which dominates the positive effect of a decrease in s. For example, if $\theta = 0.94$, $\mu = 0.65, N = 2$, and $V = 1$, then total welfare decreases from 0.995 to 0.993 when s decreases from 0.13 to 0.04.

In our model, under both random and price-directed search, as $s \to 0$, the equilibrium price distributions do not degenerate to marginal cost. This is because each individual firm still enjoys monopoly power over those consumers who find that the firm is their only match. Thus, each firm sells to its own captive buyers and simultaneously competes for consumers who have more than one match. In this situation, all consumers are in effect informed. Following the literature (e.g. Varian, 1980), we can show that there exists a symmetric equilibrium such that each firm prices according to a distribution $K(p)$.[19,20]

Corollary 2 of Propositions 1 and 2. *Under both random and price-directed search, as $s \to 0$, the equilibrium price distribution converges to*

$$K(p) = 1 - \frac{1-\theta}{\theta}\left[\left(\frac{V}{p}\right)^{\frac{1}{N-1}} - 1\right], \text{ with } p \in [(1-\theta)^{N-1}V, V]. \tag{10}$$

4.2 Welfare comparison

We are also interested in comparing welfare levels under random and price-directed search.

Proposition 4. *Compared with random search, a firm's expected profit is always lower but consumer surplus and total welfare are higher under price-directed search when the search cost is small.*

From Proposition 3, as the search cost decreases, a firm's expected profit weakly decreases under random search but weakly increases under price-directed search. Moreover, by Corollary 2, firms earn the same expected profits under random and price-directed search when the search cost approaches zero. Hence, firms earn lower expected

[19] As s approaches 0, $\alpha \to 1$ under random search and $r \to V$ under price-directed search, respectively. Thus, it can be shown that both $G(p)$ and $R(p)$ approach $K(p)$.

[20] Another interesting asymptotic result is when $N \to +\infty$. Under random search, it can be verified that $s_2 \to 0$ as $N \to +\infty$, which indicates both types of equilibria can not be supported with positive search costs. Under price-directed search, the probability of high price interval $\alpha' \to 0$ as $N \to +\infty$. Moreover, in both types of equilibria, the expected price and profit approach zero because market competition increases with N when prices are observable.

profits under price-directed search when the search cost is positive. Intuitively, if prices are observable, they influence the order in which consumers search and thus the demand for firms. Hence, price competition among firms is intensified, resulting in lower expected profits.

When the search cost is small, by Propositions 1 and 2, the entire equilibrium price distribution is below the threshold price under price-directed search while firms price above the reservation price with positive probability. Hence, the probability of a transaction is higher under price-directed search than under random search. Moreover, comparing price-directed search with random search, the uninformed consumers search more efficiently in the sense that they start searching at the low-price firms and do not search firms whose posted prices are higher than the threshold price. Therefore, total welfare and consumer surplus tend to be higher under price-directed search than under random search when the search cost is small. It is worthy of noticing that when the search cost is large, total welfare can be higher under random search as suggested by Figure 3 in Appendix B. Intuitively, for a large search cost, the entire equilibrium price distribution is below the reservation price under random search while there is a positive probability that firms price above the threshold price under price-directed search. Thus, total welfare can be higher under random search due to a higher chance of a transaction, even though consumers may search more efficiently under price-directed search. The results in Proposition 4 are illustrated in the following figures in which blue (red) lines denote variables under random (price-directed) search.

[Insert **Figure 1. Expected Price** here]

[Insert **Figure 2. Expected Profit** here]

[Insert **Figure 3. Total Welfare** here]

[Insert **Figure 4. Consumer Surplus** here]

5 Conclusion

To study the implications of price-directed search, we introduce tractable horizontal product differentiation into Stahl (1989)'s classical model and allow the uninformed consumers to actively search for a product that meets their needs. When prices are observable before searching, the uninformed consumers first visit firms that charge lower prices, provided that they are below a threshold price. In contrast, when prices are not observable before searching, the uninformed consumers engage in random search with a reservation price. We find that the reservation price decreases while the threshold price increases when the search cost declines.

We show that the equilibrium price distribution under random (price-directed) search has a convex support over a continuous interval when the search cost is high (low) and a non-convex support with both a high-price interval and a low-price interval when the search cost is low (high). As the search cost decreases, sales are more likely and firms earn higher expected profits under price-directed search while the opposite is true under random search. Our analysis also suggests that compared with random search, under price-directed search firms' expected profits are always lower, but consumer surplus and total welfare are higher for a small search cost.

In future research, it would be interesting to examine the effects of price-directed consumer search in the case of multi-product sellers (e.g. Zhou, 2014 and Rhodes, 2015). In particular, suppose that horizontally differentiated firms sell both low- and high-quality products and consumers have heterogeneous tastes for product quality. Assume that each firm can advertise only its lowest price for its products because, in reality, limited information can be conveyed through advertisements. In this multi-product consumer search framework, firms have extra incentives to advertise a lower price for their low-quality products to attract consumers to search their high-quality products sooner.

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Appendix A

Proof of Proposition 1.

We will show that $F(p)$ and $G(p)$ constructed in Proposition 1 are equilibrium price distributions when $s \in [s_1, s_2]$ and $s \in (0, s_1)$, respectively. We will also establish the uniqueness of the equilibria.

(i) The case of high search cost $(s \in [s_1, s_2])$**.**

Step 1. In this case, the price distribution is below ω_2 and consumers will purchase if a product meets their needs. Moreover, ω_2 is determined by the following optimal stopping rule:

$$\theta \int_{p \leq \omega_2} (\omega_2 - p) dF(p) = s.$$

Step 2. Note that in a mixed strategy, a firm's profits must be the same at every p in the support of $F(p)$.

(a) If $p = \omega_2$,

$$
\begin{aligned}
\pi &= \omega_2\mu\theta(1-\theta)^{N-1} + \omega_2(1-\mu)\theta[\frac{1}{N}\sum_{i=0}^{N-1}(1-\theta)^i] \\
&= \omega_2\mu\theta(1-\theta)^{N-1} + \omega_2(1-\mu)\theta\frac{1-(1-\theta)^N}{N\theta} \qquad (11)
\end{aligned}
$$

where the first (second) term is the firm's profit derived from the informed (uninformed) consumers.

(b) If $p < \omega_2$,

$$
\begin{aligned}
\pi &= p\mu\theta[\sum_{i=0}^{N-1}\binom{N-1}{i}(\theta(1-F(p)))^i(1-\theta)^{N-1-i}] + p(1-\mu)\theta[\frac{1}{N}\sum_{i=0}^{N-1}(1-\theta)^i] \\
&= p\mu\theta\{1-\theta+\theta[1-F(p)]\}^{N-1} + p(1-\mu)\theta\frac{1-(1-\theta)^N}{N\theta}. \qquad (12)
\end{aligned}
$$

Equating (11) and (12) leads to (3) in Proposition 1. Then, we set $F(p) = 0$ and $F(p) = 1$ to solve for the lower and upper bounds of $F(p)$, respectively.

Step 3. In the equilibrium, the reservation price ω_2 is implicitly determined by the optimal stopping rule. We apply a similar technique as in Janssen et al. (2005) to solve for ω_2 explicitly. From (3),

$$
p = \omega_2\frac{(1-\theta)^{N-1} + \frac{1-\mu}{\mu\theta}\frac{1-(1-\theta)^N}{N}}{[1-\theta F(p)]^{N-1} + \frac{1-\mu}{\mu\theta}\frac{1-(1-\theta)^N}{N}}. \qquad (13)
$$

By the definition of $E(p)$,

$$
\theta E(p) = \theta\int_{\underline{p}}^{\omega_2} p dF(p) = \int_{\omega_2}^{\underline{p}} p d(1-\theta F(p)).
$$

Let $y = 1 - \theta F(p)$. We plug (13) into $E(p)$. Then

$$
\theta E(p) = \int_{1-\theta}^{1} p dy = \omega_2\int_{1-\theta}^{1}\frac{(1-\theta)^{N-1} + \frac{1-\mu}{\mu\theta}\frac{1-(1-\theta)^N}{N}}{y^{N-1} + \frac{1-\mu}{\mu\theta}\frac{1-(1-\theta)^N}{N}}dy = \omega_2\phi.
$$

From the optimal stopping rule, we can solve ω_2 by replacing $\theta E(p)$. We then have

$$
s = \theta[\omega_2 - E(p)] = \theta\omega_2 - \omega_2\phi
$$

and thus,

$$\omega_2 = \frac{s}{\theta - \phi}.$$

Clearly, ω_2 increases with s in the equilibrium. The upper bound of ω_2 is V, which determines the largest s that ensures the existence of the equilibrium. In particular,

$$s_2 = (\theta - \phi)V.$$

Step 4. To complete the proof, we need to show prices that are not in the support of $F(p)$ will yield lower expected profits. Note that $p > \omega_2$ would lead to zero demand and any price less than the lower bound of $F(p)$ would lead to the same amount of demand but at a lower price. Hence, all other prices will yield a lower profit and $F(p)$ is an equilibrium price distribution. Finally, the uniqueness of the equilibrium can be established since the reservation price ω_2 is uniquely determined by the optimal stopping rule, which indicates that $F(p)$ is unique.

(ii) The case of low search cost $(s \in (0, s_1))$.

We first assume that the support of the price distribution is non-convex. Then, we show this is indeed an equilibrium and determine the maximum value s_1 that ensures the existence of the equilibrium.

Step 1. To start with, we assume that the c.d.f. of the equilibrium price distribution has the following form:

$$G(p) = \begin{cases} (1-\alpha)G_l(p) & \text{if } \underline{p}_l \le p \le \omega_1 \\ (1-\alpha) & \text{if } \omega_1 < p < \underline{p}_h \\ (1-\alpha) + \alpha G_h(p) & \text{if } \underline{p}_h \le p \le V \end{cases}.$$

Again, profits must be the same at every p in the support.

(a) If $p = V$,

$$\pi = V\mu\theta(1-\theta)^{N-1} + V(1-\mu)\theta(1-\theta)^{N-1} = V\theta(1-\theta)^{N-1} \tag{14}$$

where $V(1-\mu)\theta(1-\theta)^{N-1}$ is the firm's profit derived from the uninformed consumers who have searched all other firms without finding another matched product.

(b) If $\omega_1 < p < V$,

$$\begin{aligned}
\pi &= p\mu\theta[\sum_{i=0}^{N-1} \tbinom{N-1}{i} (\theta\alpha(1-G_h(p)))^i (1-\theta)^{N-1-i}] \\
&\quad + p(1-\mu)\theta[\sum_{i=0}^{N-1} \tbinom{N-1}{i} (\theta\alpha(1-G_h(p)))^i (1-\theta)^{N-1-i}] \\
&= p\theta[\theta\alpha(1-G_h(p)) + 1 - \theta]^{N-1} \qquad (15)
\end{aligned}$$

where the first (second) term is the firm's expected profit derived from the informed (uninformed) consumers.

(c) If $p = \omega_1$,

$$\begin{aligned}
\pi &= \omega_1\mu\theta[\sum_{i=0}^{N-1} \tbinom{N-1}{i} (\theta\alpha)^i (1-\theta)^{N-1-i}] + \omega_1(1-\mu)\theta[\frac{1}{N}\sum_{i=0}^{N-1} (1-\theta(1-\alpha))^i] \\
&= \omega_1 \left[\mu\theta(1-\theta+\theta\alpha)^{N-1} + (1-\mu)\theta\frac{1-(1-\theta+\theta\alpha)^N}{N\theta(1-\alpha)}\right]. \qquad (16)
\end{aligned}$$

Note that if $p = \omega_1$, the uninformed consumers purchase immediately if the product is a match (which occurs with probability θ) and the price is below ω_1 (which occurs with probability $1-\alpha$). Otherwise, they will move to the next firm (which occurs with probability $1-\theta(1-\alpha)$).

(d) If $p < \omega_1$,

$$\begin{aligned}
\pi &= p\mu\theta[\sum_{i=0}^{N-1} \tbinom{N-1}{i} [\theta(1-(1-\alpha)G_l(p))]^i (1-\theta)^{N-1-i}] \\
&\quad + p(1-\mu)\theta[\frac{1}{N}\sum_{i=0}^{N-1} (1-\theta(1-\alpha))^i] \\
&= p\left[\mu\theta[1-\theta(1-\alpha)G_l(p)]^{N-1} + (1-\mu)\theta\frac{1-(1-\theta+\theta\alpha)^N}{N\theta(1-\alpha)}\right]. \qquad (17)
\end{aligned}$$

The expressions of $G_h(p)$ and $G_l(p)$ in Proposition 1 come from the equal-profit conditions from (14) and (15), (16) and (17), respectively.

Step 2. There are two endogenous variables that we need to determine in the equilibrium, ω_1 and α. We first solve for ω_1. Setting (14) equal to (16), we have

$$\omega_1 = \frac{(1-\theta)^{N-1}}{\mu(1-\theta+\theta\alpha)^{N-1} + (1-\mu)\frac{1-(1-\theta+\theta\alpha)^N}{N\theta(1-\alpha)}} V. \qquad (18)$$

On the other hand, ω_1 is endogenously determined by the optimal stopping rule:

$$\theta \int_{p \leq \omega_1} (\omega_1 - p) d(1-\alpha) G_l(p) = s.$$

By definition, $E_l(p) = E[G_l(p)]$ and thus,

$$\theta(1-\alpha) E_l(p) = \theta(1-\alpha) \int_{\underline{p}}^{\omega_1} p dG_l(p) = \int_{\omega_1}^{\overline{p}} pd[1-\theta(1-\alpha)G_l(p)].$$

Let $y = 1 - \theta(1-\alpha)G_l(p)$. Inserting p into the above equation, we have

$$\theta(1-\alpha)E_l(p) = \int_{1-\theta(1-\alpha)}^{1} pdy = \omega_1 \int_{1-\theta(1-\alpha)}^{1} \frac{(1-\theta+\theta\alpha)^{N-1} + \frac{1-\mu}{\mu}\frac{1-(1-\theta+\theta\alpha)^N}{N\theta(1-\alpha)}}{y^{N-1} + \frac{1-\mu}{\mu}\frac{1-(1-\theta+\theta\alpha)^N}{N\theta(1-\alpha)}} dy = \omega_1 \eta.$$

By the optimal stopping rule,

$$s = \theta(1-\alpha)[\omega_1 - E_l(p)] = \theta(1-\alpha)\omega_1 - \omega_1 \eta$$

and thus,

$$\omega_1 = \frac{s}{\theta(1-\alpha) - \eta}. \tag{19}$$

Step 3. From (18) and (19), we can derive (4). Next, we show that there is a unique $\alpha \in [0,1]$ that satisfies (4). Obviously, the RHS of (4) is not affected by α. We will show that the LHS of (4) is monotone with α and thus there is at most one solution. Given that

$$\frac{1-(1-\theta+\theta\alpha)^N}{N\theta(1-\alpha)} = \frac{1}{N}\sum_{i=0}^{N-1}(1-\theta+\theta\alpha)^i,$$

the numerator of LHS of (4) is increasing with α. Consider the denominator of LHS of (4). Note that

$$\theta(1-\alpha) - \eta = \int_{1-\theta(1-\alpha)}^{1} \frac{y^{N-1} - (1-\theta(1-\alpha))^{N-1}}{y^{N-1} + \frac{1-\mu}{\mu}\frac{1-(1-\theta+\theta\alpha)^N}{N\theta(1-\alpha)}} dy.$$

By chain rule,

$$\frac{\partial \left[\int_{1-\theta(1-\alpha)}^{1} \frac{y^{N-1}-(1-\theta(1-\alpha))^{N-1}}{y^{N-1}+\frac{1-\mu}{\mu}\frac{1-(1-\theta+\theta\alpha)^N}{N\theta(1-\alpha)}} dy \right]}{\partial \alpha} = \frac{\partial \left[\int_{1-\theta(1-\alpha)}^{1} \frac{y^{N-1}-(1-\theta(1-\alpha))^{N-1}}{y^{N-1}+\frac{1-\mu}{\mu}\frac{1-(1-\theta+\theta\alpha)^N}{N\theta(1-\alpha)}} dy \right]}{\partial \theta(1-\alpha)} \cdot \frac{\partial \theta(1-\alpha)}{\partial \alpha}.$$

Moreover,

$$\frac{\partial\left[\int_{1-\theta(1-\alpha)}^{1}\frac{y^{N-1}-(1-\theta(1-\alpha))^{N-1}}{y^{N-1}+\frac{1-\mu}{\mu}\frac{1-(1-\theta+\theta\alpha)^{N}}{N\theta(1-\alpha)}}dy\right]}{\partial\theta(1-\alpha)}=0-0+\int_{1-\theta(1-\alpha)}^{1}\frac{\partial\left[\frac{y^{N-1}-(1-\theta(1-\alpha))^{N-1}}{y^{N-1}+\frac{1-\mu}{\mu}\frac{1-(1-\theta+\theta\alpha)^{N}}{N\theta(1-\alpha)}}\right]}{\partial\theta(1-\alpha)}dy.$$

Note that $y^{N-1}-(1-\theta(1-\alpha))^{N-1}$ is positive and increases with $\theta(1-\alpha)$, and

$$y^{N-1}+\frac{1-\mu}{\mu}\frac{1-(1-\theta(1-\alpha))^{N}}{N\theta(1-\alpha)}=y^{N-1}+\frac{1-\mu}{\mu N}\sum_{i=1}^{N-1}(1-\theta(1-\alpha))^{i}$$

is positive and decreases with $\theta(1-\alpha)$. Hence,

$$\frac{\partial\left[\int_{1-\theta(1-\alpha)}^{1}\frac{y^{N-1}-(1-\theta(1-\alpha))^{N-1}}{y^{N-1}+\frac{1-\mu}{\mu}\frac{1-(1-\theta+\theta\alpha)^{N}}{N\theta(1-\alpha)}}dy\right]}{\partial\theta(1-\alpha)}=\int_{1-\theta(1-\alpha)}^{1}\frac{\partial\left[\frac{y^{N-1}-(1-\theta(1-\alpha))^{N-1}}{y^{N-1}+\frac{1-\mu}{\mu}\frac{1-(1-\theta+\theta\alpha)^{N}}{N\theta(1-\alpha)}}\right]}{\partial\theta(1-\alpha)}dy>0.$$

Since $\frac{\partial\theta(1-\alpha)}{\partial\alpha}<0$ we have

$$\frac{\partial\left[\int_{1-\theta(1-\alpha)}^{1}\frac{y^{N-1}-(1-\theta(1-\alpha))^{N-1}}{y^{N-1}+\frac{1-\mu}{\mu}\frac{1-(1-\theta+\theta\alpha)^{N}}{N\theta(1-\alpha)}}dy\right]}{\partial\alpha}<0.$$

In summary, the LHS numerator of (4) increases with α and the denominator decreases with α, which implies that the LHS of (4) increases monotonically with α. Therefore, if (4) holds, α must be unique.

Step 4. We then pin down the proper range of s that ensures the existence of the equilibrium. Given the monotonicity, we need to check only two points, $\alpha=0$ and $\alpha=1$. When $\alpha=0$, we derive the largest search cost s_1 that supports the equilibrium. In particular,

$$s_1=\frac{N\theta(1-\theta)^{N-1}}{N\theta\mu(1-\theta)^{N-1}+(1-\mu)\left[1-(1-\theta)^{N}\right]}(\theta-\phi)V.$$

If $\alpha\to 1$, $s\to 0$ such that both sides of (4) approach infinity. Therefore, if $s\in(0,s_1)$, the equilibrium can be supported.

Step 5. To see the uniqueness of the equilibrium, note that any p below $\underline{p}_l$ will lead to a lower profit for the firm since a lower price will not bring in more demand. In addition, any p between ω_1 and $\underline{p}_h$ will also result in a lower profit for the firm. Hence, other prices that are not in the support of $G(p)$ will be charged with zero probability. Therefore, the equilibrium is unique since both α and ω_1 are unique.

Proof of Proposition 2

As in Proposition 1, we will show that the proposed is an equilibrium and establish its uniqueness.

(i) The case of low search cost $(s \in (0, s_1'))$.

Note that given $R(p)$, the uninformed consumers search optimally. To show that the proposed is an equilibrium, we thus only need to show that given r and other firms choose $R(p)$, each firm optimizes choosing any $p \in [(1-\theta)^{N-1}r, r]$. For any such price, the firm's expected profit is as follows.

(a) If $p = r$, in a symmetric equilibrium, no other firm would price above r, and thus,

$$\pi = r\theta(1-\theta)^{N-1}.$$

(b) If $p < r$,

$$\begin{aligned}\pi &= p\theta[\sum_{i=0}^{N-1} \tbinom{N-1}{i}(\theta(1-R(p)))^i(1-\theta)^{N-1-i}] \\ &= p\theta[1-\theta+\theta(1-R(p))]^{N-1}.\end{aligned}$$

The equal profit condition yields $R(p)$ in (5). Note that $p > r$ would lead to zero demand and any $p < (1-\theta)^{N-1}r$ would result in the same amount of demand as $p = (1-\theta)^{N-1}r$ but at a lower price. Therefore, the firm is maximizing its profit by choosing its price from $R(p)$. Note that this is also the only symmetric equilibrium because the reservation price r and price distribution $R(p)$ are uniquely determined.

(ii) The case of high search cost $(s \in [s_1', s_2'])$**.**

We show that each firm optimizes following $Q(p)$, given that other firms choose prices according to $Q(p)$ and the threshold price of the uninformed consumers is r. Note that a firm can only sell to a consumer if its price is the lowest among the consumer's matched firms. Moreover, the firm has to set a price no higher than r to sell to the uninformed consumers. The expected profit when the firm chooses p is:

(a) If $p = V$,

$$\pi = V\mu\theta(1-\theta)^{N-1}$$

where the firm sells to the informed consumers with only one match.

(b) If $V > p > r$,

$$\begin{aligned}\pi &= p\mu\theta[\sum_{i=0}^{N-1}\binom{N-1}{i}(\theta\alpha'(1-Q_h(p)))^i(1-\theta)^{N-1-i}] \\ &= p\mu\theta[1-\theta+\theta\alpha'(1-Q_h(p))]^{N-1}\end{aligned}$$

where the firm can only sell to the informed consumers.

(c) If $p = r$,

$$\begin{aligned}\pi &= r\theta[\sum_{i=0}^{N-1}\binom{N-1}{i}(\theta\alpha')^i(1-\theta)^{N-1-i}] \\ &= r\theta[1-\theta+\theta\alpha']^{N-1}\end{aligned}$$

where the firm sells to both the informed and the uninformed consumers if it charges the lowest price among the firms that match the consumers' needs.

(d) If $p < r$,

$$\begin{aligned}\pi &= p\theta[\sum_{i=0}^{N-1}\binom{N-1}{i}(\theta(1-(1-\alpha')Q_l(p)))^i(1-\theta)^{N-1-i}] \\ &= p\theta[1-\theta+\theta(1-(1-\alpha')Q_l(p))]^{N-1}\end{aligned}$$

where the firm can sell to both the informed and the uninformed consumers.

Equal profit from (a) and (c) yields (9). Moreover, (7) and (8) come from the equal-profit conditions of (a) and (b), and of (c) and (d), respectively. Similarly, it can be proved that other prices are charged with zero probability. Therefore, the firm optimally chooses a price according to $Q(p)$. The values of s_1' and s_2' are derived by setting α' equal to 0 and 1, respectively. This is a unique equilibrium because both r and α' are unique.

Proof of Corollary 1

Under random search, from the proof of Proposition 1, the LHS of (4) increases with α. Thus, as s decreases, α needs to increase so that (4) holds, $\frac{\partial(1-\alpha)}{\partial s} > 0$.

Under price-directed search, (2) and (9) imply that $\frac{\partial(1-\alpha')}{\partial s} < 0$.

Proof of Proposition 3

We show the effect of search cost on the expected market price, firms' profits and total welfare under random and price-directed search, respectively.

(i) Random search

We first consider the expected market price. If $s \in [s_1, s_2]$, from the proof of Proposition 1, we have $E(p) = \frac{\omega_2}{\theta}\phi$. Because ω_2 increases with s, $E(p)$ decreases as s decreases. If $s \in (0, s_1)$, the p.d.f of the high-price interval is

$$\alpha g_h(p) = \frac{1}{p(N-1)} \frac{1-\theta}{\theta} \left(\frac{V}{p}\right)^{\frac{1}{N-1}}.$$

Both the density function and its upper bound V are not affected by α. However, if s decreases, α also increases as indicated by Corollary 1, which raises the measure of $G_h(p)$ by adding weight to its tail. Essentially, firms reduce the weight of $G_l(p)$ and raise that of $G_l(p)$. We have shown that the upper bound of $G_l(p)$ is discretely less than the lower bound of $G_h(p)$. Therefore, $E(p)$ must increase as s decreases.

Second, from Proposition 1, firms' expected profits weakly decrease as s decreases because $\frac{\partial \omega_2}{\partial s} > 0$.

Finally, we investigate the effect of s on total welfare. If $s \in [s_1, s_2]$, total welfare is

$$\begin{aligned} W_{rd} &= \mu[1-(1-\theta)^N]V + (1-\mu)\theta \sum_{i=1}^{N-1} (1-\theta)^i (V - is) \qquad (20) \\ &= [1-(1-\theta)^N]V - (1-\mu)[\sum_{i=1}^{N-1} (1-\theta)^i - (N-1)(1-\theta)^N]s. \end{aligned}$$

On the RHS of (20), the first term is welfare of the informed consumers and the second term is that of the uninformed consumers. Clearly, W_{rd} increases as s decreases.

If $s \in (0, s_1)$, total welfare is

$$\begin{aligned} W_{rd} &= \mu[1-(1-\theta)^N]V + (1-\mu)\{\theta(1-\alpha)\sum_{i=0}^{N-2}[1-\theta(1-\alpha)]^i(V-is) \\ &\quad + [(1-\theta(1-\alpha))^{N-1}-(1-\theta)^N][V-(N-1)s]\} \\ &= [1-(1-\theta)^N]V - (1-\mu)[\sum_{i=1}^{N-1}(1-\theta+\theta\alpha)^i - (N-1)(1-\theta)^N]s. \qquad (21) \end{aligned}$$

In this case, it is not clear whether W_{rd} will increase or decrease as s decreases since α is higher for a lower s. We can numerically verify that W_{rd} increases with s in a certain range. For example, if $\theta = 0.94$, $\mu = 0.65, N = 2$, and $V = 1$, then W_{rd} decreases from 0.995 to 0.993 when s decreases from 0.13 to 0.04.

(ii) Price-directed search

We start with the expected market price. Note that if $s \in (0, s_1')$, from Proposition 2, $R(p)$ stochastically decreases with r. From (2), r decreases with s. Moreover, both the limits of upper bound and lower bound of $R(p)$ decrease with s. Therefore, the price distribution $R(p)$ stochastically decreases and thus the equilibrium prices are stochastically higher as s decreases. Consequently, each firm earns a higher expected profit for a lower s. If $s \in (s_1', s_2')$, substituting (7), (8) and (9) into (6), we can rewrite the equilibrium price distribution as

$$Q(p) = \begin{cases} 1 - \frac{1-\theta}{\theta}\left[\left(\frac{\mu V}{p}\right)^{\frac{1}{N-1}} - 1\right] & if \quad V - \frac{s}{\theta} \geq p \geq (1-\theta)^{N-1}\mu V \\ 1 - \frac{1-\theta}{\theta}\left[\left(\frac{\mu V}{V-\frac{s}{\theta}}\right)^{\frac{1}{N-1}} - 1\right] & if \quad \frac{V-\frac{s}{\theta}}{\mu} > p > V - \frac{s}{\theta} \\ 1 - \frac{1-\theta}{\theta}\left[\left(\frac{V}{p}\right)^{\frac{1}{N-1}} - 1\right] & if \quad V \geq p \geq \frac{V-\frac{s}{\theta}}{\mu} \end{cases}$$

which weakly increases with s. Moreover, $\frac{V-\frac{s}{\theta}}{\mu}$ and $V - \frac{s}{\theta}$ increase as s decreases. Therefore, the price distribution $Q(p)$ stochastically increases, and thus the equilibrium prices are stochastically lower as s decreases.

Second, from Proposition 2, firms' expected profits weakly increase as s decreases.

Finally, we consider the effect of s on total welfare. If $s \in (0, s_1')$, a decrease in s does not affect the probability of transaction since the entire support of $R(p)$ is below

r. This implies an increase in total welfare due to a decrease in the search cost. If $s \in (s_1', s_2')$, as s decreases, total welfare increases because the low-price distribution will be played with a higher probability resulting in a higher probability of transaction.

Proof of Proposition 4

For firms' expected profits, Proposition 3 states that a firm's profit reaches its minimum at $s = 0$ under random search, while it reaches its maximum at $s = 0$ under price-directed search. In addition, by Corollary 2, firms' profits are the same at $s = 0$ in both cases. Hence, firms' expected profits must be higher under random search.

If $s \in (0, s_1')$, total welfare under price-directed search is

$$\begin{aligned} W_{pd} &= \mu[1-(1-\theta)^N]V + (1-\mu)\theta \sum_{i=0}^{N-1} (1-\theta)^i (V - is) \\ &= [1-(1-\theta)^N]V - (1-\mu)[\sum_{i=1}^{N-1} (1-\theta)^i - (N-1)(1-\theta)^N]s. \end{aligned} \tag{22}$$

Comparing (22) with (20) and (21), we have $W_{pd} \geq W_{rd}$. Hence, total welfare is higher under price-directed search when search cost is small. This also implies consumer surplus is weakly higher under price-directed search for a small search cost because firms' profits are lower under price-directed search.

Appendix B

All diagrams are graphed by taking $\theta = 0.7$, $\mu = 0.8$, $V = 1$, $N = 2$.

Figure 1: Expected Price

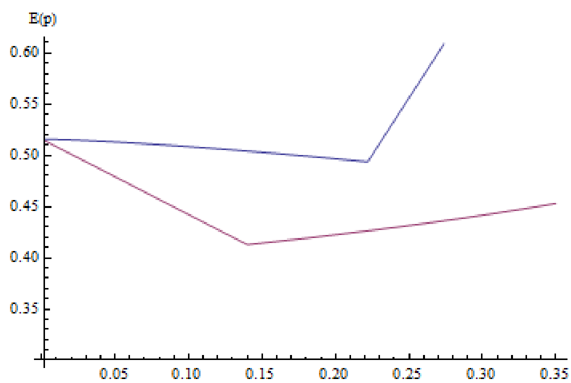

Figure 2: Expected Profit

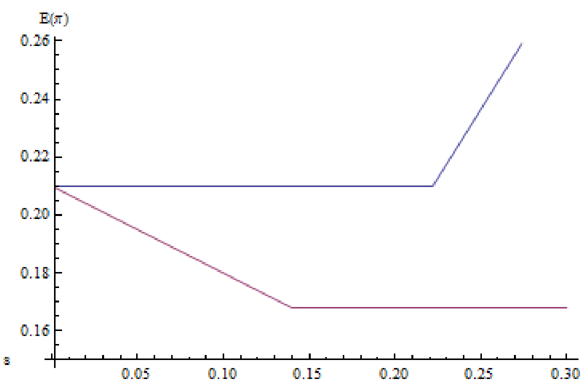

Figure 3: Total Welfare

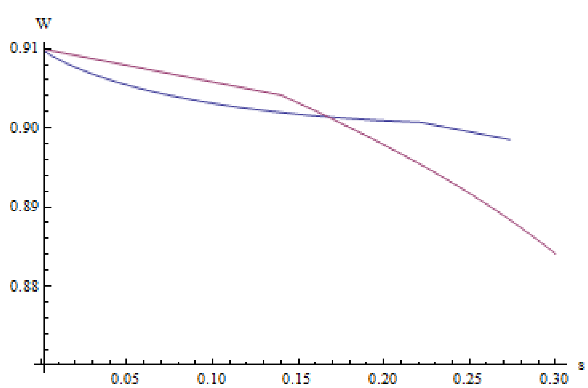

Figure 4: Consumer Surplus

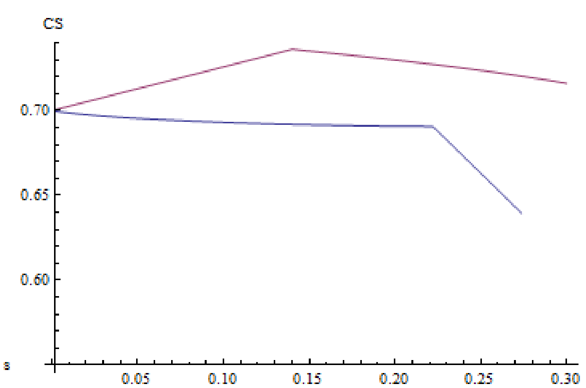

Figure 5: $f(p)$ with $s = 0.25$

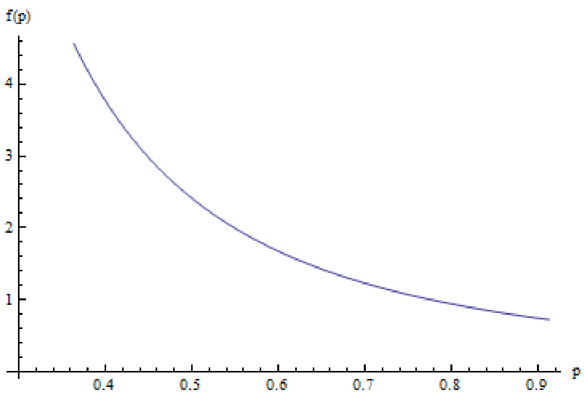

Figure 6: $g(p)$ with $s = 0.05$

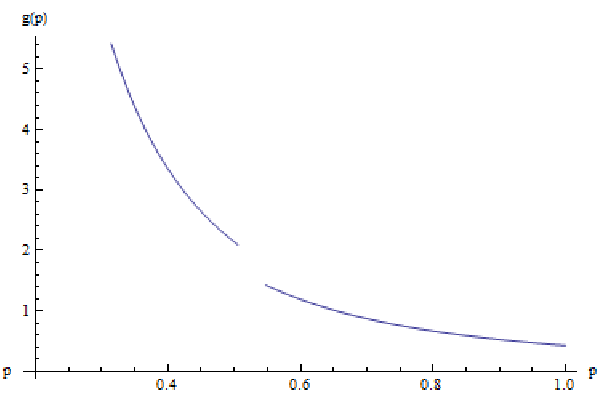

Figure 7: $r(p)$ with $s = 0.1$

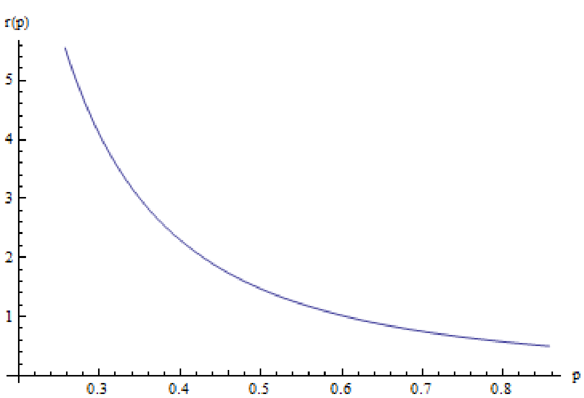

Figure 8: $q(p)$ with $s = 0.25$