Equalized Factor Price and Integrated World Equilibrium

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Abstract – The Heckscher-Ohlin model is with structure capacity to demonstrate general trade equilibrium of multiple factors and multiple commodities for multiple countries. However, the Heckscher-Ohlin theory still does not attain a price-trade equilibrium even for the simplest 2x2x2 model. This paper derives a general trade equilibrium of the Heckscher-Ohlin 2x2x2 model. The equalized factor price is determined by world factor endowments. The equalized factor price at equilibrium has three important features. The first one is that it makes sure that countries participating in free trade gain from trade. The second is that it is the Dixit-Norman price that the prices remain the same when the allocation of factor endowments changes within the IWE box. The factor price is the function of world factor endowments. It implies that world factor endowments determine world price (common commodity price and factor price). The last oneis that the capital/labor ratio equals the world-labor/world-capital ratio. It is not related to technologies and not related to commodity price. The paper processed two approaches to present the equilibrium. One is by geographic method on the IWE diagram. Another is by using a utility function to simulate market mechanism for international trade. The results are same.

Keywords:
Factor content of trade; factor price equalization; General equilibrium of trade; Integrated World Equilibrium; IWE

Introduction

Essentially the Heckscher-Ohlin theorem and the factor-price equalization (FPE) theorem paved the road toward general equilibrium. The general equilibrium and the FPE are the same issues by different angles. McKenzie (1955)'s cone of diversification of factor endowments is a good concept to understand FPE and trade from production supply constraints. He provided a mathematical demonstration of the existence of the FPE for many factors and many goods.

Vanek(1968)'s HOV model promoted the usability of Heckscher-Ohlin theories on empirical trade analyses. The share of GNP in the HOV model engaged prices with trade and consumption. It also resulted in the application issue on how to convert the assumption of homothetic taste into consumption balance. Woodland (2013) summarized the general equilibriums of trade and reviewed all important parts about trade equilibriums.

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The Integrated World Equilibrium (Dixit and Norman, 1980) is remarkable to illustrate equalized factor price by trade. It provided a strong clue for what a price-trade equilibrium should be and what an equalized factor price is. Helpman and Krugman (1985) normalize the assumption of integrated equilibrium, which presented equilibrium analyses in a simple way. Deardroff (1994) derived the conditions of the FPE for many goods, many factors, and many countries by using the IWE approach. He discussed the FPE for all possible allocations of factor endowments.

The one focus of studies on the general equilibrium with constant returns and perfect competition is by the social utility function and direct and indirect trade utility function (offer curve). It is another research direction for equilibriums.

Woodland (2013, pp39) described the importance of the general equilibrium, “General equilibrium has not only been important for a whole range of economics analyses, but especially so for the study of international trade” The Heckscher-Ohlin theories still do not achieve this important goal, even for the simplest 2x2x2 model. The Stolper-Samuelson theorem and the Rybczynski theorem are post-equilibrium analyses. Due to no equilibrium, the Stolper-Samuelson theorem assumed holding commodity outputs unchanged; the Rybczynski theorem assumed holding prices unchanged. Without the result of equilibrium, the Heckscher-Ohlin theory cannot show comparative advantages and gains from trade fully for a giving model.

Guo (2005) investigated the price structure of the FPE by a very specific condition. He was to try to reach a general equilibrium by a new approach, the analyses of the redistributable share of GNP.

This study derived a price-trade equilibrium for the Heckscher-Ohlin model and demonstrated that the equalized factor price and common commodity price at the equilibrium depended directly on world factor endowments. The result is consistent with the insight inference that Dixit and Norman made four decades ago. This is the first study to answer how equalized factor price is formulated.

The study provides two approaches to derive the equilibrium. One is using geometric analysis inside the IWE diagram. Another is using a utility function to simulate market processing.

Dixit (2010) mentioned, “The Stolper-Samuelson and factor price equalization papers did not actually produce the Heckscher-Ohlin theorem, namely the prediction that the pattern of trade will correspond to relative factor abundance, although the idea was implicit there. As Jones (1983, 89) says, ‘it was left to the next generation to explore this 2×2 model in more detail for the effect of differences in factor endowments and growth in endowments on trade and production patterns.’ That, plus the Rybczynski theorem which arose independently, completed the famous four theorems." The equalized factor price at the equilibrium of this study presented the Heckscher-Ohlin theorem. Jones expected the result of the next generation probably is the trade equilibrium, which explores the last secret of the Heckscher-Ohlin model. Guo (2019) provide a trade effect analyses based on the equilibrium of this paper, it displayed that the trade effect of changes of factor endowments is a chain effect of the Rybczynski' trade effect triggering the Stolper-Samuelson' trade effect. The Rybczynski theorem will not arise lonely. The equilibrium solution put all of the four-core theorems together.

This paper is divided into four sections. Section I introduces the equilibrium solution of IWE by a geometric method. Section II presents the equilibrium by using a utility function to simulate market mechanism. Section III provides a way to estimate the autarky price and calculate gains from trade. Section IV discusses the equilibrium and autarky price.

I . The Equilibrium by Geometric Analyses

We take the following normal assumptions of the Heckscher-Ohlin model in this study: (1) identical technology across countries, (2) identical homothetic taste, (3) perfect competition in the commodities and factors markets, (4) no cost for international exchanges of commodities, (5) factors are completely immobile across countries but that can move costlessly between sectors within a country, (6) constant return of scale and no factor intensity reversals (7) full employment of factor resources.

We denote the Heckscher-Ohlin model in the following way, for the convenience of analyses of this paper.

a. The production constraint of full employment of factor resources are

$$AX^h = V^h \qquad (h = H, F) \qquad (2\text{-}1)$$

where A is the 2x2 technology matrix (matrix of direct factor inputs), X^h is the 2 x1 vector of commodities of country h, V^h is the 2x1 vector of factor endowments of country h. The elements of matrix A is $a_{ki}(w/r), k = K, L, i = 1,2$. We assume that A is not singular.

b. The zero-profit unit cost condition

$$A'W^h = P^h \qquad (h = H, F) \qquad (2\text{-}2)$$

where $W^h$is the 2x1 vector of factor prices, its elements are r rental for capital and w wage for labor , P^h is the 2x1 vector of commodity prices.

Factor price equalization means (assuming it was equalized completely),

$$P^* = P^H = P^F \qquad (2\text{-}3)$$

$$W^* = W^H = W^F \qquad (2\text{-}4)$$

$$A'W^* = P^* \qquad (h = H, F) \qquad (2\text{-}5)$$

Both P^* and W^* are world prices when factor price equalization reached.

c. The definition of the share of GNP of country h to world GNP,

$$s^h = P'\, X^h / P'\, X^W \qquad (h = H, F) \qquad (2\text{-}6)$$

or

$$s^h = W'\,V^h / W'\,V^W \qquad (h = H, F) \tag{2-7}$$

d. The export specification for the home country is

$$T^H = (1 - s)X^H - sX^F \tag{2-8}$$

e. The factor content of trade for the home country is

$$F^H = (1 - s)V^H - sV^F \tag{2-9}$$

f. The constraint of the cone of diversification of factor endowments

$$\frac{a_{K1}}{a_{L1}} > \frac{K^H}{L^H} > \frac{a_{K2}}{a_{L2}} \quad , \quad \frac{a_{K1}}{a_{L1}} > \frac{K^F}{L^F} > \frac{a_{K2}}{a_{L2}} \tag{2-10}$$

This condition makes sure that the commodity outputs obtained from production equation (1) are positive.

g. The constraint of the cone of commodity price

$$\frac{a_{K1}}{a_{K2}} > \frac{p_1^*}{p_2^*} > \frac{a_{L1}}{a_{L2}} \tag{2-11}$$

This condition will make sure that the factor rewards from cost equation (2) are positive. Fisher (2011) proposed this insight concept and called it as “goods price diversification cones”.

Figure 1 is a regular IWE diagram. The dimensions of the box represent world factor endowments. The origin for country home is the lower left corner, for country foreign is the right upper corner.
ON and OM are the rays of the cone of diversification. Any point within parallelogram $ONO'M$ is an available allocation of factor endowments of two countries. Suppose that an allocation of the factor endowments is at point E, where the home country is capital abundant.

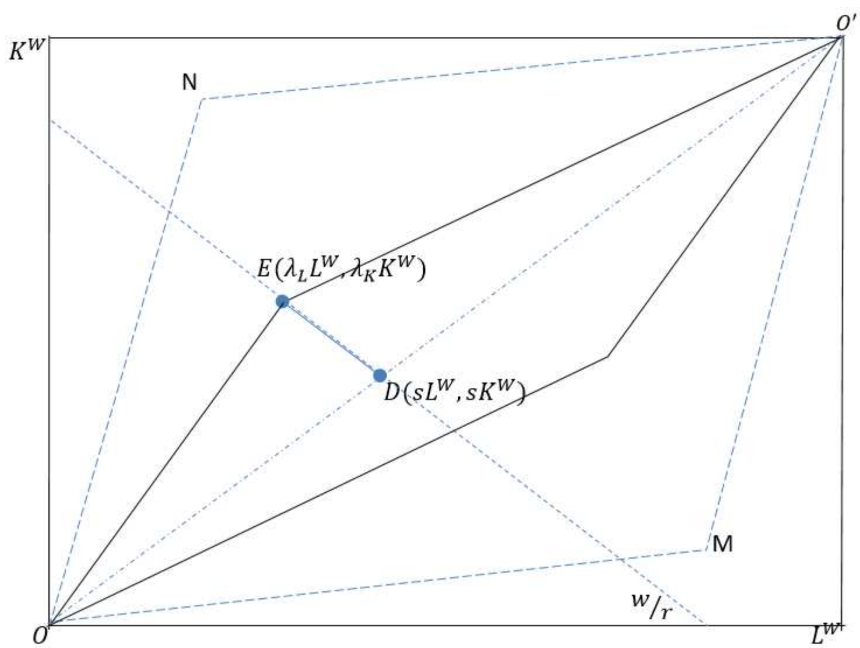

Figure 1 IWE Diagram

Dixit and Norman (1980) proofed the constant equalized factor price (FPE) when the allocation of factor endowments of two countries changes. It implied price-trade equilibrium. It provided a strange clue for what price structure is for equalized factor price and what trade equilibrium is the Heckscher-Ohlin model.

By introducing two parameters, which are the shares of home factor endowments to their corresponding world factor endowments

$$0 \le \lambda_L \le 1 \tag{2-12}$$

$$0 \le \lambda_K \le 1 \tag{2-13}$$

we denote

$$L^H = \lambda_L L^w \tag{2-14}$$

$$K^H = \lambda_K K^w \tag{2-15}$$

When λ_L and λ_K changes, they can present any point in the box. The allocation of Point E is $E(\lambda_L L^w, \lambda_K K^w)$.

The factor contents of trade are

$$F_K^H = K^H - sK^w \tag{2-16}$$

$$F_L^H = L^H - sL^w \tag{2-17}$$

Using trade balance of factor contents yields

$$\frac{r^*}{w^*} = -\frac{L^H - sL^w}{K^H - sK^w} \tag{2-18}$$

Substituting (2-14) and (2-15) into (2-18) yields

$$\frac{r^*}{w^*} = -\frac{\lambda_L L^w - sL^w}{\lambda_K K^w - sK^w} = \frac{(s-\lambda_L)L^w}{(\lambda_K - s)K^w} \tag{2-19}$$

Introduce a constant C as

$$C = \frac{(s-\lambda_L)}{(\lambda_K - s)} \tag{2-20}$$

Substituting it into (2-19) yields

$$\frac{r^*}{w^*} = C\frac{L^w}{K^w} \tag{2-21}$$

The factor price ratio (r^*/w^*) and factor price are unchanged or fixed within parallelogram $ONO'M$ on IWE diagram. That was proofed by Dixit and Norman (1980) and other following studies. Therefore, C should be a constant. Equation (2-21) illustrates that the rent/wage ratio is the function of the world factor endowments. This is why the rent/wage ratio is constant when the allocation of factor endowments changes within parallelogram $ONO'M$ in the IWE diagram.

We have interesting to know what value C takes. We denote an allocation of factor endowment at D, which is a point at the diagonal line of the IEW box. At that point, $D(sL^w, sK^w)$, its two parameters of factor endowment ratios to their world factor endowments are $\lambda_{Ld} = s$ and $\lambda_{Kd} = s$, where s is country home' share of GNP. There is no trade at this point.

We now suppose that allocation E is nearby to D. Imagine point E moves to close to point D.

Taking $\lambda_L \to s$ and $\lambda_k \to s$ yields

$$\lim_{\substack{\lambda_L \to s \\ \lambda_k \to s}} \frac{(s-\lambda_L)}{(\lambda_K - s)} = 1 = C \tag{2-22}$$

We see that constant C equals to 1. From (2-20), we have the share of GNP for equilibrium as

$$s = \frac{1}{2}(\lambda_L + \lambda_K) = \frac{1}{2}\left(\frac{K^H}{K^W} + \frac{L^H}{L^W}\right) \tag{2-23}$$

In addition, equation (2-10) is reduced as

$$\frac{r^*}{w^*} = \frac{L^w}{K^w} \tag{2-24}$$

This is true for every allocation of factor endowments within parallelogram $ONO'M$.

Is it properly to use point $D(sL^W, sK^W)$ to illustrate $1 = C$? Helpmand and Krugman (1985, pp16) thought that the point, like *D*, was a right point for trade equilibrium, they write, "the FPE is not empty because it always contains the diagonal OO'." At point *D*, there is no trade but price.

Dixit and Norman (1980, p112) used a numerical example as $\lambda_L = 1/3$, and $\lambda_K = 1/2$ in their original study to illustrate how their IWE works. The share of GNP for their example is 5/12 by equation (2-12). Let convince that this result is true. The rest of factor endowments should generate the rest of the share of GNP. The rest of factor endowments is $\lambda_L = 2/3$, and $\lambda_K = 1/2$. The rest share of GNP now is 7/12 by equation (2-12). This demonstratethat all the derivation above are right.

With equilibrium share of GNP (2-23) and the rent/wage ratio (2-24), we now obtain the whole equilibrium solution of the Heckscher-Ohlin model as

$$r^* = \frac{L^w}{K^w} \tag{2-25}$$

$$w^* = 1 \tag{2-26}$$

$$p_1^* = a_{k1}\frac{L^w}{K^w} + a_{L1} \tag{2-27}$$

$$p_2^* = a_{k2}\frac{L^w}{K^w} + a_{L2} \tag{2-28}$$

$$F_K^h = \frac{1}{2}\frac{K^h L^w - K^w L^h}{L^w}, \qquad F_L^h = -\frac{1}{2}\frac{K^h L^w - K^w L^h}{K^w}, \quad (h = H, F) \tag{2-29}$$

$$T_1^h = x_1^h - \frac{1}{2}\frac{K^h L^w + K^w L^h}{K^w L^w} x_1^w, \qquad T_2^h = x_2^h - \frac{1}{2}\frac{K^h L^w + K^w L^h}{K^w L^w} x_2^w, \quad (h = H, F) \tag{2-30}$$

We assumed $w^* = 1$ by using Walras' equilibrium condition to drop one market clear condition.

The price solution above illustrates that Dixit-Norman price more stable. The technology matrix A keeps unchanging no matter $A = A(w/r)$ or $A = A(w/p)$.

II. The General Equilibrium by Market Simulation

Trade Box

We suppose here that the home country is capital-abundant as

$$\frac{K^H}{L^H} > \frac{K^F}{L^F} \tag{3-1}$$

Trades redistribute national welfares, which are measured by GNP. This is a major trade consequence.

Figure 2 is an IWE diagram added with a trade box. The dimensions of the box represent world factor endowments. Everything in Figure 2 is as same as in Figure 1, except a trade box added.

The boundaries of the share of GNP corresponding the cone of commodity price (1-11) are

$$s_b^H(p) = s(p\left(\frac{a_{K1}}{a_{K2}}, 1\right)) = \frac{a_{K1}x_1 + a_{K2}x_2}{a_{K1}x_1^W + a_{K2}x_2^W} = \frac{K^H}{K^F + K^H} \tag{3-2}$$

$$s_a^H(p) = s(p\left(\frac{a_{L1}}{a_{L2}}, 1\right)) = \frac{a_{L1}x_1 + a_{L2}x_2}{a_{L1}x_1^W + a_{L2}^H x_2^W} = \frac{L^H}{L^F + L^H} \tag{3-3}$$

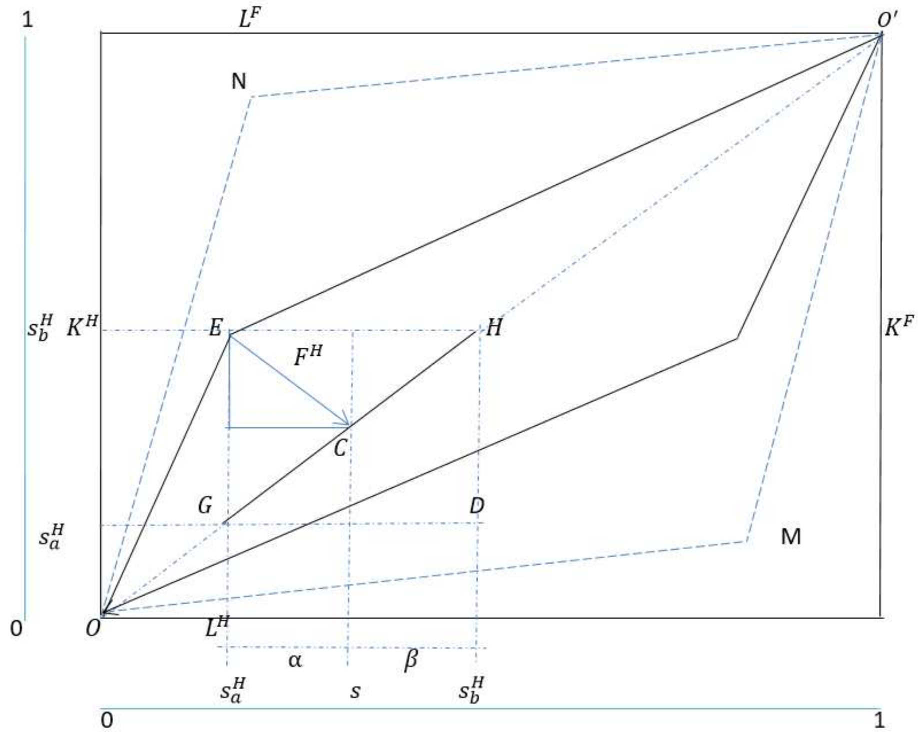

Figure 2 IWE with Trade Box

They identify the trade box $EHDG$ in Figure 2. If a commodity price lies in the cone of commodity price, the share of GNP will lie in the trade box.

Marketing Simulation

The share of GNP of the home country s divides the trade box into two parts in Figure 2. Their lengths are α and β separately. The lengths of α and β can be expressed as

$$\alpha = \left(\frac{K^H}{K^W} - s\right), \qquad \beta = \left(s - \frac{L^H}{L^W}\right) \tag{3-4}$$

The α and β are under constraint

$$\alpha + \beta = \left(\frac{K^H}{K^W} - \frac{L^H}{L^W}\right) \tag{3-5}$$

When α increases, the share of GNP of the home country increases, the share of GNP of the foreign country decreases, and vice versa. In trade competitions, the both countries want to reach their maximum GNP share through free trade.

We notice that the trade box not only is the trade area but also is the redistributable area of the share of GNP for the two countries. Outside the box, they are not redistributable by trade (the trade outside of the trade box will course a factor payment being negative). Therefore, α is redistributable part of the share of GNP of the home country; β is redistributable part of the share of GNP of the foreign country.

For reaching a competitive price-trade equilibrium of the model, we set a utility function as the product of redistributable shares of GNP of the two countries as

$$u = \alpha\beta \tag{3-6}$$

which is under constraint (3-5).

This simple utility function reflects that the market mechanism that each country is trying to reach its larger share of GNP and commodity market needs clear and factor market needs to be clear. One country cannot obtain gains without trade-off from another country.

Substituting (3-4) into (3-6) yields

$$u = (s - \frac{L^H}{L^W})(\frac{K^H}{K^W} - s) \tag{3-7}$$

We are interested in maximizing the utility function μ, so we take differential of (3-8) with respect to s yields

$$\frac{du}{ds^H} = -2s + \left(\frac{K^H}{K^W} + \frac{L^H}{L^W}\right) \tag{3-8}$$

By first order condition, we obtain

$$s = \frac{1}{2}\left(\frac{K^H}{K^W} + \frac{L^H}{L^W}\right) \tag{3-9}$$

Assume

$$w^* = 1 \tag{3-10}$$

the share of GNP now is,

$$s = \frac{K^H r^* + L^H}{K^W r^* + L^W} \tag{3-11}$$

Using (3-9) and (3-11) together yields

$$\frac{1}{2}\left(\frac{K^H}{K^W} + \frac{L^H}{L^W}\right) = \frac{K^H r^* + L^H}{K^W r^* + L^W} \tag{3-12}$$

Solving it, we have

$$r^* = \frac{L^W}{K^W} \tag{3-13}$$

The equilibrium share of GNP (3-9) and the rent/wage ratio (3-13) are the same as the result of the last section.

Competitive Price

From the factor content of trade (2-29), we see that when $\frac{K^H}{L^H} > \frac{K^w}{L^w}$, then $F_K^H > 0$. This just states the Heckscher-Ohlin theorem.

The equalized factor price (2-26) display that the relative factor price, rent/wage, in reversely, is proportional to their world factor endowments. It does not relate to technologies. Moreover, it does not relate to commodity prices.

Dixit and Norman (1980) illustrated that when the allocation of the factor endowments changes, the factor price and the commodity price will remain the same. Their major argument is that the new allocation of factor endowments of the two countries leaves the same world supply of goods and, hence incomes unchanged and so supplies will still match the unchanged world demand. We call the equilibrium price the Samuelson-Dixit-Norman price.

The changes of allocations of factor endowments within parallelogram $ONO'M$ in the IWE box will cause changes of shares of GNP and the changes of trade volumes of two countries. This does not affect world commodity output and world prices.

Why the equilibrium share of GNP lies in the middle of the trade box? In the trade box, when the relative commodity price closes to a_{K1}/a_{K2}, the home country, which is capital abundant, dominates the trade. There is no reward for labor. This is a hurt for both countries. On the contrary, when the relative commodity price closes to a_{L1}/a_{L2}, , the foreign country, which is labor abundant, dominates the trade. There is no reward to capital for the both countries. This is also a hurt for both countries. When the share of GNP s moves toward the middle from the left, labor begins to get its reward and begins to play a role in determining the world price. In the middle point C, two factors symmetrically play equal roles fully to determine prices. Only at this point, prices are a function of world resources, as

$$p_i^* = p_i(L^w, K^w) \tag{3-15}$$

$$r^* = r(L^w, K^w) \tag{3-16}$$

Free trade is a fair trade. The constant relative factor prices mean that there is no room to adjust factor rewards with the reallocation of factor endowments of the two countries.

III. Autarky Price and Comparative Advantage

It is hard to know autarky prices before free trade for countries. Therefore, it is not easy to show comparative advantages and gains from trade for the Heckscher-Ohlin model. We now propose a way to estimate autarky prices.

By the logic, that world factor resource determines world price in the last section, we imagine a country with an isolated market, its "autarky" price can be determined by its "autarky" factor endowments.

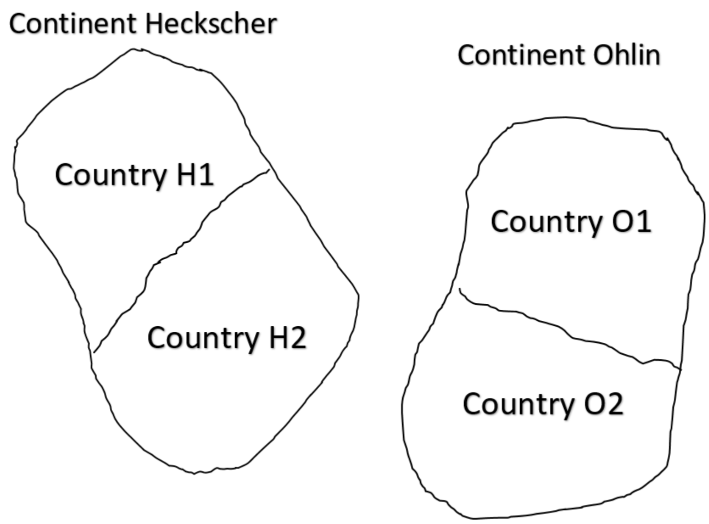

Figure 3 Autarky Price Formation

A good case to explain the estimation of autarky price is by Figure 3. There are two geographic continents, Heckscher and Ohlin, separated by an ocean. Continent Heckscher is with two free trade countries, H1 and H2. In addition, Continent Ohlin is with two free trade countries O1 and O2. Two continents start to free trade by no-cost shipping. Knowing the total factor endowments of each continent, we can estimate the prices of each continent by the expression of world price (2-14) through (4-17).

The IWE diagram itself supports the logic that autarky factor resources determine autarky price analytically. Assuming that one country shrinks to very small, another country's autarky price is then the world price of the current trade. Mathematically, when $V^H \to 0$, inside the IWE box, then $V^F \to V^W$ and the relative factor price r^* after trade will close to the relative autarky factor price of the foreign country,

$$r^* = \frac{L^W}{K^W} = \frac{L^H + L^F}{K^H + K^F} \to r^{Fa} = \frac{L^F}{K^F} \tag{4-1}$$

Moreover, the common commodity price will close to the foreign country's autarky commodity price. Therefore, we proved the autarky price formation mathematically.

We need to add an assumption that the technology matrix A at autarky does not change too much from autarky to trade as that $A = A(r^*/w^*) \cong A(r^{ah}/w^{ah})$.

Based on the above discussion, we present the autarky prices of countries that participate in free trade as

$$r^{ha} = \frac{L^h}{K^h} \qquad (h = H, F) \tag{4-2}$$

$$w^{ha} = 1 \qquad (h = H, F) \tag{4-3}$$

$$p_1^{ha} = a_{k1}\frac{L^h}{K^h} + a_{L1} \qquad (h = H, F) \tag{4-4}$$

$$p_2^{ha} = a_{k2}\frac{L^h}{K^h} + a_{L2} \qquad (h = H, F) \tag{4-5}$$

where superscript ha is used to indicate the autarky price of country h.

Assuming the home country is capital abundant, we have:

$$\frac{p_1^{Ha}}{p_2^{Ha}} = \frac{a_{k1}L^H + a_{L1}K^H}{a_{k2}L^H + a_{L2}K^H} < \frac{p_1^*}{p_2^*} = \frac{a_{k1}L^W + a_{L1}K^W}{a_{k2}L^W + a_{L2}K^W} < \frac{p_1^{Fa}}{p_2^{Fa}} = \frac{a_{k1}L^F + a_{L1}K^F}{a_{k2}L^F + a_{L2}K^F} \tag{4-6}$$

$$\frac{w^{Ha}}{r^{Ha}} = \frac{K^H}{L^H} > \frac{w^*}{r^*} = \frac{K^W}{L^W} > \frac{w^{Fa}}{r^{Fa}} = \frac{K^F}{L^F} \tag{4-7}$$

Inequalities (4-6) and (4-7) are the necessary and sufficient condition of gains from trade. They show the trade reason and the source of comparative advantage. Moreover, inequality (4-7) is the price definition of capital abundance.

The Heckscher-Ohlin model brings another source of comparative advantage, differences in factor endowments across countries. Its gains from trade are measured by

$$-W^{ha\prime}F^h > 0 \qquad (h = H, F) \tag{4-8}$$

$$-P^{ha\prime}T^h > 0 \qquad (h = H, F) \tag{4-9}$$

We add the negative sign in inequalities above since we expressed trade by net export, T^h . In most other literatures, they express trade by net import. Appendix B is the proof of the gain from trade by inequality (4-8). It implies that the equalized factor price always makes sure that the countries gain from trade.

The analyses of this section demonstrate that the world prices at the equilibrium will ensure the gains from trade for both countries, by the autarky prices inference.

The result of gains from trade is another good side effect of the equilibrium of trade. It is one important property of the equilibrium and the FPE.

Theorem – The comparative advantage theorem

At the equilibrium, the world prices (equalized factor price and common commodity price) are the Samuelson-Dixit-Norman price. The world factor endowments, fully employed, determine world prices that assure the gains from trade for countries participated in trade.

Proof

The solution (2-25) through (2-28) shows how the world prices are determined and why it remains the same with mobile factor endowments in the IWE box. The solution is unique for a giving IWE box.

Appendix B proved the gains from trade as inequality (4-8).

End Proof

The equilibrium shows the unification of the Heckscher-Ohlin theorem, The FPE theorem, gains from trade, and Dixit-Norman price. Each of them means other of it.

IV. Discussions of the equilibrium and Autarky Price.

The geometrical approach for the equilibrium in section 2 depends on the assumption that the factor price at the IWE is fixed. The utility function approach does not depend on this assumption. It shows that the design of the utility function senses. When analyzing multiple factors and multiple commodities, the utility function is more flexible.

The solution is a Walrasian equilibrium. Every country's consumption maximizes its utility given prices. It reached markets clear: the total demand for each commodity just equals the aggregate endowment. It is also Pareto Optimal since the utility function shows how social trade-off played. It also is a typical problem of Nash non-cooperative game. It involves two players in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only their own strategy. It reflects the two-country competitive relations in a pair of commodity trades. It reached a possible win-win solution as each country takes its best strategy.

Samuelson (1949) made arguments about factor price equalization and outlines his description of autarky trade equilibrium. He reasoned that an angel's recording geographer device notified some fraction of all factor endowments, one is called American, the rest to be Europeans. "Obviously, just giving people and areas national label does not alter anything; it does not change commodity or factor prices or production patterns, but with identical real wage and rents and identical modes of commodity production. ... [W]hat will be the result? Two countries with quite different factor proportions, but with identical real wages and rents and identical modes of commodity production (but with different relative importance of food and clothing industries). ... Both countries must have factor proportions intermediate between the proportions in the two industries. The angel can create a country with proportions not intermediate between the factor intensities of food and clothing. But he cannot do so by following the above-described procedure, which was calculated to leave prices and production unchanged." He mentioned, "to leave prices and production unchanged" with emphasis. He implies that the autarky price of the kingdom is the world price of countries by artificial map or labels of the recording geographer device. This is the earliest thought about the estimation of autarky price. It is consistent with the estimation of autarky price by the logic that autarky factor endowments determine autarky price.

At the equilibrium, the ratio of factor content of trade equals to consumption ratio. It reflected Leamer theorem (Leamer, 1980). We provide a chain of inequalities that includes the Heckscher-Ohlin theorem, the Leamer theorem, the Factor Price Equalization theorem, and the Dixit and Norman IWE price, as the follows,

$$\frac{a_{K1}}{a_{L1}} > \frac{K^H}{L^H} > \frac{K^H - F_K^H}{L^H - F_L^H} = \frac{K^w}{L^w} = \frac{w^*}{r^*} = \left|\frac{F_K^H}{F_L^H}\right| = \frac{K^F - F_K^F}{L^F - F_L^F} > \frac{K^F}{L^F} > \frac{a_{K2}}{a_{L2}} \tag{5-1}$$

It is a mathematical brief statement for the Heckscher-Ohlin theorem, the Leamer theorem, the Factor Price Equalization theorem, and the Dixit and Norman IWE price principle, which arrive together at equilibrium.

Conclusion

The paper attained the general equilibrium of trade in the 2 x 2x 2 standard Heckscher-Ohlin model. The equilibrium addresses the Heckscher-Ohlin theorem with trade volume, the factor-price equalization theorem with price structure, and comparative advantage with gains from trade.

The study explored the principle that world factor resources determine world prices.

The paper made an inference of autarky prices by using the principle that world factor resources determining world price. The study provided proof of gains from trade by the equilibrium price.

Appendix A

We express the gains from trade for country H as

$$-W^{Ha\prime}F^H > 0 \tag{A-1}$$

Adding trade balance condition $W^{*'}F^H = 0$ on (A-1) yields

$$-(W^{Ha\prime} - W^{*'})F^H > 0 \tag{A-2}$$

We see

$$W^{Ha} = \begin{bmatrix} \frac{L^H}{K^H} \\ 1 \end{bmatrix} \quad , \qquad W^* = \begin{bmatrix} \frac{L^W}{K^W} \\ 1 \end{bmatrix}$$

Substituting them into (A-2) yields,

$$-\begin{bmatrix}\frac{L^H}{K^H}-\frac{L^W}{K^W} & 0\end{bmatrix}\begin{bmatrix}\frac{1}{2}\frac{K^hL^w-K^wL^h}{L^w}\\ -\frac{1}{2}\frac{K^hL^w-K^wL^h}{L^w}\end{bmatrix}>0 \tag{A-3}$$

It can be reduced to

$$-(\frac{L^H}{K^H}-\frac{L^W}{K^W})\times\frac{1}{2}\frac{K^hL^w-K^wL^h}{L^w}>0 \tag{A-4}$$

It means

$$-\left(\frac{L^H}{K^H}-\frac{L^W}{K^W}\right)\times\frac{1}{2}\frac{\frac{L^W}{K^W}-\frac{L^H}{K^H}}{L^w}K^wK^H=\left(\frac{L^H}{K^H}-\frac{L^W}{K^W}\right)^2\times\frac{1}{2L^w}K^wK^H>0 \tag{A-5}$$

It is true. So that (A-1) holds.

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