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Abstract

We introduce a novel commodity tax mechanism in oligopolies that improves upon the standard tax policies. The government (i) announces an excise tax rate τ and (ii) auctions-off a number of tax exemptions. Namely, it invites the firms in a market to acquire the right to be exempted from the excise tax. The highest bidders are exempted paying the government their bids; and all other firms remain subject to τ. Depending on the characteristics of the market, the mechanism we suggest has a number of desirable features. First, it allows the government to collect more revenues than the standard commodity tax policies (this is due to the competition among the firms to acquire the exemptions). Second, for markets where firms have informational advantage over the government, the mechanism allows for information revelation (via the firms' bids in the auction). Third, it impedes collusive activities in the market (as the mechanism creates an artificial asymmetry among the firms, which hinders collusion). Lastly, the mechanism is voluntary, namely the firms participate in the auction only if they wish and hence they are free to choose how to be taxed.

Keywords: excise tax; tax exemption; auction; asymmetric information; collusion

JEL Classification: H25, L13.

1 Introduction

Is there a commodity tax policy that can address simultaneously the following important issues: raise enough tax revenues; circumvent the informational disadvantage that the tax authorities often have when imposing a tax policy; impede collusion among firms in a market and, finally, be voluntary in the sense that the firms may choose the form of their taxation. The goal of the current paper is to propose such a mechanism. The mechanism

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combines an excise tax policy with a tax-exemption auction: the government imposes an excise tax rate τ that the firms in a market have to pay per unit of production, and also announces the auctioning of a number of tax exemptions. Namely, it invites the firms to bid in order to acquire, if they wish, the right to be excluded from paying τ. The firms that place the highest bids win this right, pay the government their bids, and are not subject to the excise tax. All other firms remain subject to it.

Via this mechanism, each firm in the market essentially has to decide whether to transform a part of its marginal cost (i.e., the per unit tax) into a fixed cost (i.e., its bid in the tax exemption auction). If it does so, it may obtain a cost-efficiency advantage over its competitors in the product market. In this sense, the government acts as a patent holder, which sells licenses to the use of a marginal cost-reducing "innovation" (the innovation being the avoidance of the per unit tax).

Depending on market characteristics, the suggested mechanism has the desirable features mentioned in the beginning of the introduction. Namely:

(i) It can create *more revenues* for the government in comparison to the standard excise tax policy.[1] This is due to the competition among the firms to acquire the exemptions and avoid competing with a marginal cost disadvantage. Among other things, this implies that if the government wants to collect revenues of magnitude, say, $R(\tau)$, it can do so by announcing a tax rate $\tau' < \tau$ and by inviting the firms to bid for tax exemptions. Such a lower tax rate creates lower distortions in the market. Moreover, announcing low(er) tax rates is often more attractive from a political view point.

(ii) In markets where the government *lacks information* about the characteristics of the firms, the tax-exemption auction can serve as an information eliciting mechanism: the bids of the firms reveal, under certain conditions, their types to the government. The latter fact can then be used by the tax authorities appropriately (e.g., to fight tax evasion).

(iii) In markets where *collusion and cartel formation* is an issue, the mechanism can serve as a cartel impeding device: depending on the number of tax exemptions auctioned-off and the pre-auction profile of marginal costs, the mechanism may result on magnified cost asymmetries among the firms, a fact which, in general, impedes cartels from forming.

(iv) The above take place within a *voluntary mechanism*, namely the firms choose whether to participate in the auction or not. This may address a potential criticism against the mechanism: why bother with setting up an auction and not impose mandatory lump-sum tax policies.

To fully capture the merits of the mechanism, which require competition among firms, we introduce it for markets operating as oligopolies where firms' interactions are most prevalent. We choose the simplest possible such structure, a duopoly with homogeneous

[1] Notice that the extra revenues may suffice to finance any costs resulting from setting up the tax-exemption auction.

goods.[2] To boost competition for the exemption, we assume that the government auctions-off a sole tax exemption. Our results, which are essentially spelled out above, can be summarized as follows. We first analyze the mechanism under symmetric information, where the government knows the characteristics of the firms. We show that if firms are symmetric, the auction mechanism always generates higher tax revenues than the standard excise tax policy, i.e., the policy where all firms pay the excise tax. When firms are (cost) asymmetric, we show that: (i) the efficient firm wins the tax exemption in the auction; (ii) the resulting tax revenues surpass the revenues from the standard excise tax policy provided that the cost asymmetry is not very high. Under both cases of symmetry and asymmetry, the mechanism enhances the welfare in the market vis-a-vis the excise tax policy.

We also examine the problem under asymmetric information, where the government does not know with certainty the characteristics of the firms, and in particular, their cost structure. As noted before, the auction now serves one more purpose: it may provide information to the government. Indeed, we show that in an environment with symmetric firms, the bids of the firms in the tax-exemption auction fully reveal their types to tax authorities. This can then be used by the latter to prevent tax evasion: among other things, true revelation of types implies that the firm that loses the auction will not be able to under-report its demand. On top of fighting tax evasion, the government collects higher expected revenues compared to the standard excise tax policy.

Finally, we test our mechanism in markets where the firms have the possibility to collude. We consider an infinitely repeated market environment where the cartel, if it forms, manipulates the market price. We derive two results. First, we show that the mechanism is more effective in impeding the formation of the cartel (compared again to the standard excise tax policy): the winner of the auction becomes more efficient than the rival firm and, as a result, has a high incentive to violate a collusive agreement. This implies that the cartel forms for lower ranges of the value of the discount factor (vis-a-vis the standard excise tax policy). Second, the government's revenues (weakly) surpass its revenues under the excise tax policy.

The paper connects to the literature on commodity taxation in oligopoly. The roots of this literature go back to the mid 80's when economists started analyzing the design and optimality of various tax policies outside the two antipodean cases of perfect competition and monopoly.[3] The main issues examined include tax incidence (Katz and Rozen 1985; Seade 1985; Stern 1987; Hamilton 1999; Anderson et al. 2001); comparison between ad valorem and excise taxes (Dierickx et al. 1988; Delipalla and Keen 1992; Skeath and Trandel 1994; Denicolo and Matteuzzi 2000); taxation in oligopolies under general equilibrium (Myles 1989; Reinhorn 2005; Collie 2015); the optimal structure of excise and ad valorem taxes when firms maximize their joint profits (Kay and Keen 1983); the sustainability of collusion under ad valorem vs. excise taxation (Colombo and Labrecciosa 2013; Azacis and Collie 2018), the sustainability of collusion in international duopolies (Haufler and Schjelderup 2004); the comparison between ad valorem and excise taxation under uncertainty and/or asymmetric information (Dickie and Trandel 1996; Goerke 2011; Goerke et al. 2014; Kotsogiannis and Serfes 2014).

[2]We choose a duopoly structure as we want to introduce the mechanism in a simple framework. The analysis can -in principle- be extended for the case of more than two firms.

[3]See Dierickx et al. (1988) for a review of the early literature on taxation and imperfect competition.

Not all aspects related to taxation in oligopoly are, of course, examined in the current paper. For example, the paper only focuses on excise and not on ad valorem taxation (including both cases would require doubling the size of the paper). Further, the market structure is fixed and simple, i.e., duopoly with homogeneous goods. So we don't examine how entry in the market, product differentiation, etc., affect the performance of the mechanism. Further, commodities in our model are not "sin" goods, so a potential increase in their consumption, due to our mechanism, is not a concern. Despite the restrictions though, our framework provides, we believe, an adequate first step towards introducing the tax mechanism and pointing out its merits.

Finally some comments on the "applicability" or "practicality" of the mechanism are in order. First, our paper does not suggest that a tax-exemption auction mechanism should be built for all markets (that fit our theoretical framework). This would be practically impossible. What we suggest is that a government may choose a small number of markets, that are well organized, with financially sound firms,[4] etc, and therein run its auction(s).

Secondly, in order to highlight the relevance of the mechanism for the real world we note that auctions are already used in a related policy issue in the field of international trade: the auctioning of import licenses. Via this policy the government of the country that imposes quotas on trade can uncover some of the rents associated with the restriction. The mechanism has been used by countries like Australia, New Zealand, etc., (see Tan 2001) and its success depends on the structure of product markets (see Krishna 1988, 1991, 1993; Tan 2001). So, if an auction-backed government policy can work in international markets then a related mechanism could perhaps work in the current paper's context.

The paper is organized as follows. Section 2 presents the basic ingredients of the mechanism and the market. Sections 3 analyzes the symmetric information case under one-shot market competition. Section 4 analyzes the mechanism in a repeated market with collusion. Section 5 introduces asymmetric information and the last section concludes.

2 Preliminaries

We consider a duopoly with firms 1 and 2. The quantity and price of firm i are denoted by x_i and p_i, respectively, $i = 1, 2$. The marginal cost of i is constant at the level of c_i; fixed costs are set equal to zero. The firms produce a homogeneous commodity (differentiated commodities can be easily incorporated in the analysis).

There is also a government which taxes the market. The tax mechanism consists of an excise or per unit tax τ and a tax-exemption auction. Namely, the government announces the tax rate τ that each firm pays per unit of production and also invites the firms to bid in order to acquire the (unique) exemption from τ. If a firm acquires the exemption, it pays the government its bid (we assume a first-price auction) and is exempted from τ, while its rival remains subject to it. If no firm decides to participate in the auction, then both are subject to τ.

The interaction evolves in time as follows:

(i) The government announces the excise tax rate and the rules of the tax-exemption auction.

[4] Such elements are silently assumed to hold true in our paper.

(ii) The firms decide whether to participate or not in the auction; the auction takes place (if any firm decides to participate); the winner is determined and pays the government its bid.

(iii) The firms compete in the product market, given the outcome of the auction. Namely, firm i operates with marginal cost c_i, if it has acquired the exemption, or with $c_i + \tau$, otherwise.

3 One-shot market competition

We begin the analysis by focusing on the assumption that the firms compete in quantities in one-shot fashion. We posit that an equilibrium always exists in the product market for all values of τ. We denote by $x_1(\tau, 0)$ and $x_2(\tau, 0)$ the equilibrium quantities of the firms when firm 1 only is subject to the per unit tax; and by $x_1(0, \tau)$ and $x_2(0, \tau)$ the equilibrium quantities if firm 2 only is subject to the tax. Given our homogeneous-good environment, there will be a common market price, p. The equilibrium price is denoted by $p(\tau, 0)$, if firm 1 only pays the per unit tax, or $p(0, \tau)$, if firm 2 only pays it. Moreover, $\pi_1(\tau, 0)$ and $\pi_2(\tau, 0)$ denote the equilibrium profits if firm 1 is subject to the per unit tax, and $\pi_1(0, \tau)$ and $\pi_2(0, \tau)$ denote profits if firm 2 is subject to the tax. Finally, we denote equilibrium quantities, price and profits under the standard excise policy, where both firms pay τ, by $x_i(\tau, \tau)$, $p(\tau, \tau)$ and $\pi_i(\tau, \tau)$, $i = 1, 2$.

Given the above, we can now focus on the auction. Consider first the case of identical marginal costs, $c_1 = c_2 = c$ (which means that firm i competes in the product market with essential marginal cost $c + \tau$ or c). Each firm will place a bid in the auction which is equal to the difference between its profit when it is the sole firm exempted from the per unit tax and its profit when its opponent only is exempted. In other words, the bids of firms 1 and 2 are

$$b_1(\tau) = \pi_1(0, \tau) - \pi_1(\tau, 0), \;\; b_2(\tau) = \pi_2(\tau, 0) - \pi_2(0, \tau) \tag{1}$$

By symmetry of firms, the above bids are equal. Assuming firm, say, 1 is given the exemption, the total revenues for the government are $B(\tau) = b_1(\tau) + \tau x_2(0, \tau)$.

On the other hand, under the standard tax policy, again with per unit tax rate τ, the tax revenues are $T(\tau) = \tau x_1(\tau, \tau) + \tau x_2(\tau, \tau)$. We are interested in comparing the revenues of the two policies for a fixed per unit tax rate.

Proposition 1 *If $c_1 = c_2$ then $B(\tau) \geq T(\tau)$.*

Proof We will assume that $\tau < p(0, \tau) - c$, which implies that the firm without the tax exemption, namely firm 2, survives in the market.[5]

By the preceding analysis we need to show that

$$\pi_1(0, \tau) - \pi_1(\tau, 0) + \tau x_2(0, \tau) \geq \tau x_1(\tau, \tau) + \tau x_2(\tau, \tau)$$

Using the ex ante symmetry between the firms, the above is equivalent to

[5] The opposite case, $\tau \geq p(0, \tau) - c$, which would imply that firm 2 exits the market, could be considered undesirable from a political view point. Nonetheless, we briefly discuss this case after the current proof.

$$\big(p(0,\tau)-c\big)(x_1(0,\tau)-x_2(0,\tau)) \geq \tau(x_1(\tau,\tau)+x_2(\tau,\tau)-2x_2(0,\tau))$$

Using the upper bound on the tax rate, it suffices to show that

$$\big(p(0,\tau)-c\big)(x_1(0,\tau)-x_2(0,\tau)) \geq \big(p(0,\tau)-c\big)(x_1(\tau,\tau)+x_2(\tau,\tau)-2x_2(0,\tau))$$

or simply that $x_1(0,\tau)+x_2(0,\tau) \geq x_1(\tau,\tau)+x_2(\tau,\tau)$. But this inequality holds since the l.h.s is total output when one firm only is charged the per unit tax, whereas the r.h.s. is total output when both firms are charged the per unit tax.[6] ■

Let's now see briefly the case $\tau \geq p(0,\tau)-c$. Then $\pi_2(0,\tau)=0$ and also $b_1(\tau)=\pi_m$, where π_m denotes the monopoly profit. Recalling that under constant returns the maximum industry profit is attained when the market is a monopoly, our mechanism then gives the upper bound for the revenues of the government.

Remark 1 We note that the two firms end up with the same net equilibrium profits. These net profits are $\pi_1(0,\tau)-b_1(\tau)=\pi_1(\tau,0)$ for firm 1 and $\pi_2(0,\tau)$ for firm 2, where $\pi_1(\tau,0)=\pi_2(0,\tau)$. So although only one firm is tax-exempted, the mechanism does not create ex post (i.e., after tax) asymmetries.

We now move to the case of cost-asymmetric firms where, say, $c_1<c_2$. The first question that arises is whether the efficient or inefficient firm wins the auction. To address this, notice that the maximum amount firm 1 is willing to bid in the auction is $b_1^{max}(\tau)=\pi_1(0,\tau)-\pi_1(\tau,0)$; likewise the maximum amount for firm 2 is $b_2^{max}(\tau)=\pi_2(\tau,0)-\pi_2(0,\tau)$. The firm with the highest such amount will win the auction by biding the other firm's maximum amount.

Lemma 1 *Assume that $c_1<c_2$. Then firm 1 wins the auction biding $b_2^{max}(\tau)$.*

Proof We need to show that $b_1^{max}(\tau) \geq b_2^{max}(\tau)$. This holds iff $\pi_1(0,\tau)+\pi_2(0,\tau) \geq \pi_1(\tau,0)+\pi_2(\tau,0)$. The l.h.s of this inequality is industry profit at the marginal cost profile $(c_1, c_2+\tau)$ and the r.h.s is industry profit at the marginal cost profile $(c_1+\tau, c_2)$. The former profile has higher variance than the latter (this holds as $c_1<c_2$). Then we can use Proposition 1 of Van Long and Soubeyran (2001) that industry profit (at a Cournot equilibrium where all firms are active) is an increasing function of the variance of individual marginal costs. ■

The second task is to compare the revenues under the policies of tax exemption and excise tax. To facilitate the comparison we restrict attention to a linear inverse demand function, $p=a-X$, where $X=x_1+x_2$.

Remark 2 We assume that $c_2<(a+c_1)/2$ which guarantees that, in the absence of any government policy, the less efficient firm is active in the market.

As in the symmetric case, we will assume that the tax rate is such that the firm that loses the auction remains active in the market. For the linear demand case this requires that

[6] This inequality holds under all "regular" comparative statics results in Cournot competition.

$\tau < (a - 2c_2 + c_1)/2.$[7]

Proposition 2 *Assume that $p = a - X$. The following hold:*

(i) If $c_2 \leq (a + 7c_1)/8$ then $B(\tau) \geq T(\tau)$.

(ii) If $c_2 \geq (a + 4c_1)/5$ then $T(\tau) \geq B(\tau)$.

(iii) If $(a + 7c_1)/8 < c_2 < (a + 4c_1)/5$ then $B(\tau) \geq T(\tau)$ for $\tau \leq a + 4c_1 - 5c_2$; and $B(\tau) < T(\tau)$ for $a + 4c_1 - 5c_2 < \tau$.

Proof Appears in the Appendix. ■

If c_2 is sufficiently small (but always higher than c_1) then we are "close" to the symmetric case, so the result of Proposition 1 is essentially repeated. On the other hand, if c_2 is large enough, namely if the cost disadvantage of firm 2 is large, then firm 1 outbids firm 2 relatively easy. As a result, the government collects relatively low revenues from the auction and the standard policy outperforms it. For these two polar cases, the role of c_2 is dominant, so the tax rate plays no role in the comparison of the policies. Naturally, for intermediate values of c_2 other factors, such us τ, come into place. Essentially Proposition 2 (iii) repeats parts (i) and (ii) by saying that the auction mechanism (the standard policy) generates more revenue than the standard policy (the auction mechanism) if cost asymmetry adjusted by τ is low (high). This we can see by re-writing $\tau \lesseqgtr a + 4c_1 - 5c_2$ (which combines the two inequalities in the statement of Proposition 2 (iii)) as $\tau + 5c_2 \lesseqgtr a + 4c_1$.

Remark 3 It is straightforward to show that under either cost symmetry ($c_1 = c_2$) or asymmetry ($c_1 \neq c_2$), social welfare in the market is higher under the tax-exemption mechanism than under the standard excise tax policy.

4 Collusion

In this section we demonstrate another desirable feature of our mechanism: the impediment of collusive activities in the market. As we will show, this occurs without any reduction in the revenues of the government (compared to the standard excise tax policy). To elaborate, assume that the two firms compete in the market for an infinite number of periods. The firms are symmetric ($c_1 = c_2 = c$) and try to collude by selecting the market *price* and by allocating the consumers between them.[8] For simplicity, this allocation splits the market equally between the two firms. We note that we don't allow for side payments between the two firms.

In every period each firm has to pay a tax τ per unit of its production. The government auctions-off an exclusive tax exemption in the beginning of the first period. The winner of the auction, which we will assume to be firm 1, pays the government its bid in every period and is not subject to τ; and firm 2 remains subject to τ, in every period too.

[7] Imposing this bound on τ further guarantees, in conjunction with Remark 2, that the thresholds presented in Proposition 2, and their relations, are well-defined.

[8] The analysis goes through if we assume that firms collude in quantities. See the discussion after Proposition 3.

Denote by $p^m(0,\tau)$ the collusive price when firm 2 only pays the per-unit tax and by $\pi_i^m(0,\tau)$ the corresponding collusive profit of firm i. Denote by $\pi_i^d(0,\tau)$ the profit of firm i if i breaks the agreement, and by $\pi_i^b(0,\tau)$ its profit in the punishment phase. Finally let $b_1(\tau)$ be the winning bid of firm 1 (we will return to this later on).

Consider firm 1. Notice that $\pi_1^m(0,\tau) = (p^m(0,\tau) - c)\frac{X(p^m(0,\tau))}{2}$, where, with a slight abuse of notation, $X(p^m(0,\tau))$ denotes industry output at price[9] $p^m(0,\tau)$. If 1 breaks the agreement, it charges a price slightly below the collusive price, takes all market demand (as goods are homogeneous), and has a profit which is approximately equal to $\pi_1^d(0,\tau) = (p^m(0,\tau) - c)X(p^m(0,\tau))$. Finally, the price of firm 1 at the punishment stage is $p_1(0,\tau) = c + \tau$ and hence $\pi_1^b(0,\tau) = (c+\tau-c)X(c+\tau) = \tau X(c+\tau)$. Firm 1 sticks to the collusive agreement as long as[10] $\pi_1^m(0,\tau) \geq (1-\delta)\pi_1^d(0,\tau) + \delta\pi_1^b(0,\tau)$ or as long as

$$\delta \geq \frac{\pi_1^d(0,\tau) - \pi_1^m(0,\tau)}{\pi_1^d(0,\tau) - \pi_1^b(0,\tau)} = \frac{\pi_1^m(0,\tau)}{2\pi_1^m(0,\tau) - \pi_1^b(0,\tau)} = \delta_1(0,\tau)$$

Consider now firm 2. Using a reasoning similar to the above, we have $\pi_2^m(0,\tau) = (p^m(0,\tau) - c - \tau)\frac{X(p^m(0,\tau))}{2}$, $\pi_2^d(0,\tau) = (p^m(0,\tau) - c - \tau)X(p^m(0,\tau))$ and $\pi_2^b(0,\tau) = 0$. Firm 2 does not deviate from the collusive agreement as along as $\pi_2^m(0,\tau) \geq (1-\delta)\pi_2^d(0,\tau) + \delta\pi_2^b(0,\tau)$ or

$$\delta \geq \frac{\pi_2^d(0,\tau) - \pi_2^m(0,\tau)}{\pi_2^d(0,\tau) - \pi_2^b(0,\tau)} = \frac{1}{2} = \delta_2(0,\tau)$$

Notice that $\delta_1(0,\tau) > \delta_2(0,\tau)$, namely the winner of the auction has lower incentive to join the cartel. This is because it is less hurt by a possible retaliation than its rival.

The above ex post asymmetry disappears under the standard per unit tax policy, as in this case the punishment payoffs are zero for both firms. Namely, denoting this payoff by $\pi_i^b(\tau,\tau)$ for firm i, we have $\pi_1^b(\tau,\tau) = \pi_2^b(\tau,\tau) = 0$. Let $p^m(\tau,\tau)$ be the collusive price under the excise tax policy and $\pi_i^m(\tau,\tau)$ and $\pi_i^d(\tau,\tau)$ the collusive and deviation profits of i. Then $\pi_i^m(\tau,\tau) = (p^m(\tau,\tau) - c - \tau)X(p^m(\tau,\tau))/2$ and $\pi_i^d(\tau,\tau) = (p^m(\tau,\tau) - c - \tau)X(p^m(\tau,\tau))$. Then firm, say, 1 does not deviate from the agreement as long as

$$\delta \geq \frac{\pi_1^d(\tau,\tau) - \pi_1^m(\tau,\tau)}{\pi_1^d(\tau,\tau) - \pi_1^b(\tau,\tau)} = \frac{1}{2} = \delta_1(\tau,\tau) = \delta_2(\tau,\tau)$$

It is easy to see that $\delta_1(0,\tau) > \delta_1(\tau,\tau)$; moreover $\delta_2(0,\tau) = \delta_2(\tau,\tau)$. So we conclude the following.

Lemma 2 *Cartel formation occurs less often under the tax-exemption auction mechanism than under the standard excise tax policy.*

Finally we will compare the revenues of the government under the two policies as a function of the discount factor δ and of τ. To do so, we first need to go back to the tax exemption auction. Recall that firm 1 wins the auction. The value of the winning

[9] Similar notation is used for industry output at any other price.

[10] The winning bid does not play a role in the decision of firm 1 to collude, so we don't include it in the analysis for the time being.

bid depends on whether collusion or not occurs afterwards in the market. In particular, $b_1(\tau) = \pi_1(0,\tau) - \pi_1(\tau,0)$ under no collusion; and $b_1(\tau) = \pi_1^m(0,\tau) - \pi_1^m(\tau,0)$ under collusion. In the former case, $B(\tau) = b_1(\tau)$ and in the latter, $B(\tau) = b_1(\tau) + \tau X(p^m(0,\tau))/2$. On the other hand, tax revenues under the standard policy are $T(\tau) = \tau X(c+\tau)$ under no collusion; and $T(\tau) = \tau X(p^m(\tau,\tau))$ under collusion.

The comparison of $B(\tau)$ and $T(\tau)$ will use the following mild assumption:

A1 The collusive price is a weakly increasing function of the marginal cost of each firm.

Proposition 3 *Assume A1 holds. Then $B(\tau) \geq T(\tau)$ for all δ.*

Proof Let first $\delta < 1/2$. Then the cartel does not form under either of the policies. Under the tax exemption policy one firm will operate with marginal cost c and its rival with $c+\tau$. Hence, the market price that will result from the ensuing price competition is $c+\tau$. Going back to the auction, $b_1(\tau) = \pi_1(0,\tau) - \pi_1(\tau,0) = (c+\tau-c)X(c+\tau) - 0 = \tau X(c+\tau) = B(\tau)$. Under the standard per unit tax policy, both firms operate with marginal cost $c+\tau$, which will be the market price. Hence the government collects, as we said before, $T(\tau) = \tau x_1(\tau,\tau) + \tau x_2(\tau,\tau) = \tau X(c+\tau)$. We conclude that the two policies are equivalent for this range of δ.

Let next $1/2 < \delta < \delta_1(0,\tau)$. For this range the cartel forms under the standard tax policy only. Hence $T(\tau) = \tau X(p^m(\tau,\tau))$ and $B(\tau) = \tau X(c+\tau)$, where the latter is derived in the previous paragraph. But $p^m(\tau,\tau) \geq p^m(0,\tau)$, by A1. Moreover, notice that $p^m(0,\tau) > c+\tau$. Thus $X(c+\tau) \geq X(p^m(\tau,\tau))$ and $B(\tau) \geq T(\tau.)$ Let finally $\delta > \delta_1(0,\tau)$. In this range the cartel forms under both tax policies. Under the auction mechanism we have $B(\tau) = \pi_1^m(0,\tau) - \pi_1^m(\tau,0) + \tau X(p^m(0,\tau))/2 = \tau X(p^m(0,\tau)) \geq \tau X(p^m(\tau,\tau)) = T(\tau)$, where the inequality uses again A1. ■

We have assumed that collusive activities are based on price competition. We note that we would reach to similar conclusions (from a qualitative view point) in an environment of quantity competition. For example, using a quantity competition model with linear demand and quadratic costs we can show that: (i) the winner of the auction has higher incentive to deviate from a collusive agreement than the opponent; (ii) the critical discount factor for the formation of the cartel is an increasing function of the excise tax parameter; (iii) the government collects higher revenues under the suggested mechanism for all values of δ.

Furthermore we note that, as in previous cases, the net profits of the two firms are equal. This might address a potential criticism: why implement an auction mechanism and not simply charge the firms two different excise tax rates, τ_1 and τ_2 and still impede the formation of the cartel (as asymmetry would again arise). In such a case, the two firms would ex post be treated asymmetrically in the sense that they would end up with different net profits. To the extent that this is undesirable, the government would then need to compensate the firm that pays the higher excise rate, which makes the whole mechanism cumbersome.[11] Furthermore, such a policy loses the voluntary property of the auction mechanism.

Notice also that a possible reaction of the firms against the mechanism is that they collude not only in the product market, but also over not participating in the tax exemption

[11]Notice also that due to the compensation, the net revenues of the government might not be as high.

auction. A deviation from the collusive agreement would then mean that the deviant firm participates in the auction and wins the exemption. This might strengthen the sustainability of the cartel. Anticipating this, the government could announce a sufficiently high rate τ so as to make a deviation more attractive (breaking the agreement by participating in the auction is more tempting under a relatively high τ).

5 Asymmetric information

We will now analyze the last feature of the mechanism, its ability to convey information to tax authorities. We will analyze a market with symmetric firms in which the government does not have complete information about the characteristics of the firms. In particular, it does not know the exact value of the marginal cost of production; it knows though that the firms are symmetric. There are a couple of ways to justify this information structure. For example, it might be that the two firms face the same input supplier who provides a homogeneous input to them (at a uniform price), and this is known to the government. Still the government does not know the actual value of the input price. We assume that the government believes that the cost of each firm is c^L with probability q and c^H with probability $1-q$, where $c^L < c^H$. We let q be the prior probability attached to the event that the two firms have marginal cost c^L and $1-q$ the probability attached to the event that they have marginal cost c^H.

The government again auctions-off a single exemption from τ. As in the previous sections, the government commits to the value of τ before it observes the bids. Then, one means via which the government benefits from a potential information revelation is that a low-cost firm cannot be perceived as being high-cost and be taxed for a smaller quantity.[12] We will return to this in Remark 4 below.

We will analyze the mechanism under a quantity competition framework (the analysis could be easily extended to the price competition case as well). We begin by introducing some notation. Let $k \in \{L, H\}$. Denote by $x_i^k(\tau, 0)$ the equilibrium quantity of firm i when firms are of cost c^k and firm 1 only is subject to the per unit tax; and by $x_i^k(0, \tau)$ the corresponding equilibrium quantity of i if firm 2 only is subject to the tax. Equilibrium profits will be denoted in a similar fashion. Namely, $\pi_i^k(\tau, 0)$ denotes the profit of firm i if firm 1 is subject to the per unit tax, $\pi_i^k(0, \tau)$ denotes the profit of firm i if firm 2 is subject to the tax, and so on. Finally let $b^k(\tau) = \pi_1^k(0, \tau) - \pi_1^k(\tau, 0) = \pi_2^k(\tau, 0) - \pi_2^k(0, \tau)$.

We will assume the following.

A2 $b^L(\tau) > b^H(\tau)$*, for any* τ*.*

Assumption A2 will hold throughout the section and we won't refer to it again (it is easy to show that A2 holds when the demand is, for example, linear). A uniform ranking of $b^L(\tau)$ and $b^H(\tau)$, namely a ranking that holds for all τ, simplifies considerably the number of steps that we have to check in order to find an equilibrium in the auction under the current section's information framework.

The first observation states that any bid above $b^H(\tau)$ is a weakly dominated action for a

[12] Committing to the value of τ is by no means a necessary action. However, the merits of our mechanism can be identified even under this assumption.

high-cost firm. For this observation we do not need to assume that the government knows that the firms are symmetric.

Lemma 3 *$\tilde{b} > b^H(\tau)$ is weakly dominated by $b^H(\tau)$ for a high-cost firm.*

Proof Take the high-cost type of, say, firm 1. If 1 wins the auction with bid $\tilde{b}$, its net profit is $\pi_1^H(0,\tau) - \tilde{b} < \pi_1^H(\tau,0)$. If it loses, its net profit is $\pi_1^H(\tau,0)$, if it is perceived as a high-cost firm, or $\pi_1^H(\tau,0) - \tau(x_1^L(\tau,0) - x_1^H(\tau,0))$, if it is perceived as a low-cost firm. Let now firm 1 deviate from $\tilde{b}$ to $b^H(\tau)$. If it wins the auction, its net profit is $\pi_1^H(0,\tau) - b^H(\tau) = \pi_1^H(\tau,0)$. If it loses, its net profit is as in the case of bidding $\tilde{b}$. We conclude that $\tilde{b}$ is weakly dominated by $b^H(\tau)$. ■

In what follows we will assume that players do not use weakly dominated actions. Given this, Lemma 3, although a straightforward observation, will play an important role in the sequel. If the government ever observes a bid above $b^H(\tau)$ it must conclude that this bid comes from a low-cost firm. But since the government knows that firms are symmetric, it will further deduce that both firms are low-cost firms. On the other hand, if the government observes two bids less than or equal to $b^H(\tau)$, it will deduce that the firms are of high cost. Indeed the competition of the low cost firms will result in bids above[13] $b^H(\tau)$ (the high cost firms cannot do so).

Denote by $\mu(c^L|(b_1,b_2))$ the posterior probability that the government assigns to the event that the two firms are of low cost given that it observes bids (b_1,b_2); $\mu(c^H|(b_1,b_2))$ is defined in a similar way. Given the above, consider the following posterior beliefs of the governments:

(a) $\mu(c^L|(b_1,b_2)) = 1$, if $b_i > b^H(\tau)$, for at least one i.

(b) $\mu(c^H|(b_1,b_2)) = 1$, if $b_i \leq b^H(\tau)$, for $i = 1,2$.

Proposition 4 *The following constitute an equilibrium in the tax-exemption auction:*

(i) *Each low-cost firm bids $b^L(\tau)$; and each high-cost firm bids $b^H(\tau)$.*

(ii) $\mu(c^L|(b^L(\tau),b^L(\tau))) = 1$, $\mu(c^H|(b^H(\tau),b^H(\tau))) = 1$.

(iii) *Beliefs off-equilibrium: see (a) and (b).*

Proof Take the low-cost type of, say, firm 1. If it sticks to $b^L(\tau)$, its bid is equal to that of firm 2. So it is either given the exemption and thus have a net profit $\pi_1^L(0,\tau) - b^L(\tau) = \pi_1^L(\tau,0)$ or it is not, in which case it pays the per unit tax and have profit again equal to $\pi_1^L(\tau,0)$. Notice that in the latter case it will be perceived as a low-cost firm (given the posterior beliefs of the government).[14]

If the low-cost type of firm deviates to $\tilde{b} > b^L(\tau)$ it will win the auction but with net payoff $\pi_1^L(0,\tau) - \tilde{b} < \pi_1^L(\tau,0)$. If it deviates to $\tilde{b} < b^L(\tau)$ it will lose the auction, without

[13]This can be easily shown.

[14]If it was perceived as a high-cost firm its profit -in the event of not receiving the exemption- would be $\pi_1^L(\tau,0) + \tau(x_1^L(\tau,0) - x_1^H(\tau,0))$.

changing the beliefs of the government (since the other firm sticks to its bid). Hence its net profit in the latter case will be $\pi_1^L(\tau, 0)$. So a low-cost firm does not deviate.

Take now the high-cost type, again of firm 1. If it sticks to $b^H(\tau)$ it is again either given the exemption and thus have a net profit $\pi_1^H(0,\tau) - b^H(\tau) = \pi_1^H(\tau, 0)$ or it is not, in which case it pays the per unit tax and have profit $\pi_1^H(\tau, 0)$. If it deviates to $\tilde{b} > b^H(\tau)$, it will win the auction with net payoff $\pi_1^H(0,\tau) - \tilde{b} < \pi_1^H(\tau, 0)$ (see also Lemma 3). If it deviates to $\tilde{b} < b^H(\tau)$ it will lose the auction, without again changing the beliefs of the government. Hence its net profit will be $\pi_1^H(\tau, 0)$. So a high-cost firm does not deviate too. Finally, the beliefs of the government stated above are consistent with the equilibrium actions of the firms. ■

Remark 4 Since the bids of the firms reveal their types, the low-cost type of the firm that loses the auction cannot claim that is of high-cost so as to be taxed for a smaller quantity. In this sense the mechanism has a tax-evasion avoidance property (which was mentioned before in the paper).[15]

Consider now the standard per-unit tax policy where both firms pay τ. The expected revenues for the government are $q[\tau(x_1^L(\tau,\tau) + x_2^L(\tau,\tau))] + (1-q)[\tau(x_1^H(\tau,\tau) + x_2^H(\tau,\tau))]$. Proposition 1 has already shown that for each c, the revenues under our mechanism are higher than the revenues under the standard per unit tax policy. Combining this with Proposition 4 we conclude that the mechanism generates higher revenue also under asymmetric information.

Proposition 5 *Under the outcome described in Proposition 4, the government achieves higher expected revenues compared to the standard excise tax policy.*

We note that, given the beliefs of the government specified above, there could be no other equilibrium bids. Of course, introducing other (less plausible) beliefs can result into other outcomes. For example, if the government believes that a low bid of, say, firm 1 means 1 is of c^H type irrespective of the bid of the other firm, then the c^L type of firm 1 will have incentive to deviate from $b^L(\tau)$ so as to lose the auction, be perceived as high-cost, and have net payoff[16] $\pi_1^L(\tau, 0) + \tau(x_1^L(\tau, 0) - x_1^H(\tau, 0)) > \pi_1^L(\tau, 0)$. However, we exclude this kind of beliefs on behalf of the government.

6 Conclusions

This paper has introduced a novel tax mechanism in oligopoly. Essentially, the suggested mechanism works complementarily to the existing commodity tax policies in several aspects (namely, in revenue-raising, in resolving issues of asymmetry of information and in the impediment of collusive activities). One of the merits of the mechanism is that it is not mandatory: each firm may choose how to be taxed. In this sense, the mechanism improves also upon the policy of mandatory lump-sum taxes on firms' profits.

[15] Clearly, the mechanism's merits increase if we allow the government to optimize over τ after observing the bids of the firms.

[16] The inequality holds as, clearly, $x_1^L(\tau, 0) > x_1^H(\tau, 0)$.

One can think of a number of potential extensions of the current work (see also the discussion in the Introduction). The analysis of the mechanism under an ad valorem tax policy seems to be of immediate interest. Allowing for more than two firms in the market and endogenizing the number of tax exemptions that the government auctions-off, may shield more light on how far the mechanism can go in real markets. Further, allowing for transferable exemptions, in the sense that the winner of the auction is free to to sell, if it wishes, the tax exemption to his rivals is also worth to be examined. Finally, relaxing the assumption that the firms know each others' marginal cost in section 5 could enhance the significance of that part of analysis.

Appendix

Proof of Proposition 2 We have $T(\tau) = \tau(\frac{a-2c_1+c_2-\tau}{3} + \frac{a-2c_2+c_1-\tau}{3})$ and $B(\tau) = \tau\frac{2a-4c_2+2c_1-\tau}{3} + \tau\frac{a-2c2-+c_1-2\tau}{3}$. It is easy to show that $B(\tau) > T(\tau)$ iff $a+4c_1-5c_2-\tau > 0$. We notice that if $c_2^{max} > c_2 > (a+4c_1)/5 \equiv c_2'$ then $B(\tau) < T(\tau)$ for all τ. And if $c_2 < c_2'$ then $B(\tau) > T(\tau)$ iff $\tau < a + 4c_1 - 5c_2$. We conclude the proof noting $a + 4c_1 - 5c_2 > (a - 2c_2 + c_1)/2$ iff $c_2 < (a + 7c_1)/8$. ■

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