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Abstract

Estimation of portfolio expected credit loss is required for IFRS9 regulatory purposes. It starts with the estimation of scenario loss at loan level, and then aggregated and summed up by scenario probability weights to obtain portfolio expected loss. This estimated loss can vary significantly, depending on the levels of loss severity generated by the IFSR9 models, and the probability weights chosen. There is a need for a quantitative approach for determining the weights for scenario losses. In this paper, we propose a model to estimate the expected portfolio losses brought by recession risk, and a quantitative approach for determining the scenario weights. The model and approach are validated by an empirical example, where we stress portfolio expected loss by recession risk, and calculate the scenario weights accordingly.

Keywords: Scenario weight, stressed expected credit loss, loss severity, recession probability, Vasicek distribution, probit mixed model

1. Introduction

Estimation of portfolio expected credit loss is required for IFRS9 regulatory purposes ([2], [3]). It starts with the estimation of scenario loss at loan level, and then aggregated and summed over by scenario weights to obtain portfolio expected loss. In general, there are three scenarios under consideration: pessimistic, base, and optimistic.

Let p_1, p_2, and p_3 denote respectively the occurring probabilities for these three scenarios, and y_1, y_2, and y_3 the corresponding portfolio losses. In practice, the expected credit loss is estimated as:

$$EL = w_1^0 y_1 + w_2^0 y_2 + w_3^0 y_3, \quad (1.1)$$

where w_1^0, w_2^0, and w_3^0 are relative weights derived from p_1, p_2, and p_3 as:

$$w_i^0 = \frac{p_i}{p_1+p_2+p_3}, \; 1 \le i \le 3. \quad (1.2)$$

Result for *EL* by (1.1) can vary significantly, depending on the values of p_1, p_2, and p_3 chosen, and the portfolio scenario losses y_1, y_2, and y_3 generated by IFRS9 models. In practice, this number is compared with historical portfolio losses to position its severity (in relative to the historical portfolio loss distribution). An issue arises, when the number calculated by (1.1) is nonintuitive, in which case, practitioners will manually search for weights w_1^0, w_2^0, and w_3^0 to come up with an acceptable number for *EL*. Weights obtained in this way lack quantitative justification, and are somehow arbitrary.

We focus on portfolio loss rate and assume that the exposure for the portfolio is one dollar. We will use interchangeably the words "loss" and "loss rate".

When assuming that the portfolio loss rate y is driven by a latent dynamic s, the expected portfolio loss is equal to the integral $\int_A y(s)p(s)ds$ for the function $y(s)p(s)$ over the range A where s varies, where $p(s)$ denotes the probability density of s. Therefore, the portfolio expected loss is not necessarily equal to weighted sum $[p(s_1)y(s_1) + p(s_2)y(s_2) + p(s_3)y(s_3)]/[p(s_1) + p(s_2) + p(s_3)]$, even when $p(s_1), p(s_2)$, and $p(s_3)$ correspond to the given occurring probabilities.

In this paper, we propose a quantitative approach (see Algorithm 3.4) to finding the weights w_1, w_2, and w_3 (not necessarily the same as w_1^0, w_2^0, and w_3^0 as described above) satisfying equation (1.3) below:

$$EL = w_1 y_1 + w_2 y_2 + w_3 y_3, \quad (1.3)$$

given portfolio expected loss *EL*, the scenario losses y_1, y_2, and y_3, and the corresponding occurring probabilities p_1, p_2, and p_3, respectively for pessimistic, base, and optimistic scenarios.

Forward looking consideration is generally required for expected portfolio loss. Under this setting, portfolio expected loss needs to be assessed under a stressed condition. Scenario weights for the corresponding stressed portfolio loss are required. For this purpose, we propose a model (see model (3.11)) to stress the portfolio expected loss conditional on recession risk. Scenario weights are determined, given the recession probability, occurring probabilities p_1, p_2, p_3, and scenario losses y_1, y_2, y_3.

The paper is organized as follows. In section 2, we demonstrate an equivalent parameterization for Vasicek distribution. An one-factor probit mixed model is defined for each Vasicek distribution. In section 3, we propose a probit-type mixed model for portfolio expected loss conditional on recession risk. A generic algorithm to find scenario weights is proposed in this section. The proposed model and approach are validated in section 4 by an empirical example, where we stress portfolio expected loss by recession risk, and calculate scenario weights accordingly.

2. Preliminaries: Vasicek Distribution and One-Factor Probit Mixed Models

Vasicek distribution ([12], [13]) is skewed and leptokurtic ([13]). It is widely used for modeling of portfolio loss distributions ([4], [5], [6], [7], [11]).

In this section, we demonstrate an alternative formulation for Vasicek distribution by using the mean and standard deviation of the probit form of the variable. An one-factor, probit-type, mixed model is defined by this parameterization. The parameter estimation by maximum likelihood approach for a probit-type mixed model becomes simpler when likelihood is formulated by this parameterization (see Proposition 3.1).

For a variable y $(0 < y < 1)$ that follows a Vasicek distribution, its density is given by ([13]):

$$g(y, p, \rho) = \sqrt{\frac{1-\rho}{\rho}}\ \exp\{-\frac{1}{2\rho}\left[\sqrt{1-\rho}\Phi^{-1}(y) - \Phi^{-1}(p)\right]^2 + \frac{1}{2}\ [\Phi^{-1}(y)]^2\}, \qquad (2.1)$$

where p denotes the mean value of y, and ρ is the asset correlation, which is related to the Asymptotic Single Risk factor model ([1], [5], [8], [13]).

Let Φ denote the cumulative distribution function (CDF) for a standard normal variable and ϕ its density. A random variable y , $0 < y < 1$, is said to follow a Vasicek distribution if its probit form $\Phi^{-1}(y)$ is normally distributed ([9]). Let a and b denote respectively the mean and standard deviation of $\Phi^{-1}(y)$. Then y can be formulated as an one-factor probit-type model:

$$y = \Phi(a + bS),\ S \sim N(0,1). \qquad (2.2)$$

The probability density function is given by proposition below.

Proposition 2.1 ([15]). The density of the variable y in (2.2) is given by

$$f(y, a, b) = U_1/(bU_2), \qquad (2.3)$$

where

$$U_1 = \phi\{[\Phi^{-1}(y) - a]/b\},\ U_2 = \phi[\Phi^{-1}(y)]. \qquad (2.4)$$

□

Proposition 2.2. Densities (2.3) and (2.1) are equivalent under the relationships:

$$a = \frac{\Phi^{-1}(p)}{\sqrt{1-\rho}} \text{ and } b = \sqrt{\frac{\rho}{1-\rho}}. \quad (2.5)$$

Proof. By (2.1), we have:

$$g(y,p,\rho) = \sqrt{\frac{1-\rho}{\rho}} \exp\{-\frac{1-\rho}{2\rho}\left[\Phi^{-1}(y) - \Phi^{-1}(p)/\sqrt{1-\rho}\right]^2 + \frac{1}{2}[\Phi^{-1}(y)]^2\}$$
$$= \frac{1}{b}\exp\left\{-\frac{1}{2}\left[\frac{\Phi^{-1}(y)-a}{b}\right]^2\right\}\exp\{\frac{1}{2}[\Phi^{-1}(y)]^2\}$$
$$= U_1/(bU_2) = f(y,a,b). \ \square$$

By (2.5), we have the following relationships:

$$\rho = \frac{b^2}{1+b^2}, \quad (2.6)$$
$$\frac{1}{\sqrt{(1-\rho)}} = \sqrt{1+b^2}, \quad (2.7)$$
$$a = \Phi^{-1}(p)\sqrt{1+b^2}. \quad (2.8)$$

Denote by $E_e[\Phi(a+be)]$ the expectation of $\Phi(a+be)$ with respect to a random variable e. The following lemma is useful.

Lemma 2.3. ([10]) $E_e[\Phi(a+be)] = \Phi\left(a/\sqrt{1+b^2}\right)$, where $e \sim N(0,1)$. □

The next proposition summaries the properties for Vasicek distribution under (2.2).

Proposition 2.4. The following statements hold;

(a) $E_s[\Phi(a+bs)] = \Phi\left(a/\sqrt{1+b^2}\right) = p$.
(b) Density $f(y,a,b)$ is unimodal if $0 < b < 1$ with mode given by $\Phi\left(\frac{a}{1-b^2}\right)$, and is U-shaped if $b > 1$.
(c) Assume $b = 1$. If $a = 0$ then $f(y,a,b)$ is uniformly distributed over the interval $(0,1)$; it is increasing if $a > 0$ and decreasing if $a < 0$.

Proof. By applying Lemma 2.3 to (2.2) and using (2.8), we have:

$$E_s[\Phi(a+bs)] = \Phi\left(a/\sqrt{1+b^2}\right) = p.$$

This proves (a). Let $z = \Phi^{-1}(y)$. By (2.3), we have:

$$\log\left(\frac{U_1}{U_2}\right) = \frac{-z^2+2az-a^2+b^2z^2}{2b^2} \quad (2.9)$$
$$= \frac{-(1-b^2)\left(z-\frac{a}{1-b^2}\right)^2 + \frac{b^2}{1-b^2}a^2}{2b^2}. \quad (2.10)$$

For statement (c), it follows from the fact that the coefficient of term z^2 in (2.9) is zero. For (b), we have $-(1-b^2) > 0$ when $b > 1$, therefore U_1/U_2 is U-shaped by (2.10). When $0 < b < 1$, we have $-(1-b^2) < 0$, therefore $\log\left(\frac{U_1}{U_2}\right)$ reaches its unique maximum at $z = \frac{a}{1-b^2}$ by (2.10), resulting in a value for the mode at $\Phi\left(\frac{a}{1-b^2}\right)$. □

Remark 2.5. Results in Proposition 2.4 are consistent with results given in [13] where the density distribution is parameterized as (2.1). For most cases, this follows from the fact that $\rho < \frac{1}{2}$ if and only if $b < 1$. Here we show only the consistency for distribution mode. The mode in [13] is given as $\Phi\left(\frac{\sqrt{1-\rho}}{1-2\rho}\Phi^{-1}(p)\right)$, while it is given as $\Phi\left(\frac{a}{1-b^2}\right)$ in Proposition 2.4. First, by (2.6), we have $1-2\rho=\frac{1-b^2}{1+b^2}$. Therefore, by (2.7) and (2.8), we have

$$\frac{\sqrt{1-\rho}}{1-2\rho}\Phi^{-1}(p)=\frac{\sqrt{1+b^2}}{1-b^2}\Phi^{-1}(p)=\frac{a}{1-b^2}$$
$$\Rightarrow \Phi\left(\frac{\sqrt{1-\rho}}{1-2\rho}\Phi^{-1}(p)\right)=\Phi\left(\frac{a}{1-b^2}\right).$$

This proves the consistency for distribution mode between the two formulations.

3. Estimating Portfolio Loss Rate and Weighting Portfolio Scenario Losses

3.1. The Proposed Loss Rate Model

We assume that there exists a latent risk factor S that drives up the portfolio loss, and the loss rate y follows a Vasicek distribution. Thus, by (2.2), portfolio loss rate is given by a probit-type mixed model of the form below:

$$y=\Phi(a+bS),\ b>0, S\sim N(0,1). \tag{3.1}$$

Parameters a and b can be estimated as described in the next proposition below. This is the loss rate model for the portfolio.

Proposition 3.1 ([15]). Given a time series sample $\{y_t\}$, where y_t denotes the loss rate for a portfolio observed at time t, the maximum likelihood estimates for a and b under model (3.1) are the mean and standard deviation for the sample $\{z_t\}$ given by the probit form values of $\{y_t\}$, i.e. $z_t=\Phi^{-1}(y_t)$. □

3.2. Levels of Loss Severity

Given $0<\alpha<1$, the α-quantile value for a loss rate variable y is the loss rate y_α satisfying $P(y\le y_\alpha)=\alpha$. In this case, we say that α is the severity level for the loss rate y_α. The level of severity measures the relative position for the loss rate in its overall loss distribution.

Proposition 3.2. Given a loss rate y_0 under model (3.1), the severity level of y_0 is $\Phi\{[\Phi^{-1}(y_0)-a]/b\}$. Given $0<\alpha<1$, the α-quantile value is given by $y_\alpha=\Phi[a+bS_\alpha]$, where $S_\alpha=\Phi^{-1}(\alpha)$ is the α-quantile value for $S\sim N(0,1)$.

Proof. By model (3.1), $y=\Phi(a+bS),\ S\sim N(0,1)$. The first statement follows as:

$$P(y\le y_0)=P\left\{S\le\frac{\Phi^{-1}(y_0)-a}{b}\right\}=\Phi\left(\frac{\Phi^{-1}(y_0)-a}{b}\right).$$

For the second statement, α-quantile value y_α satisfies the equation below:

$$\alpha=P(y\le y_\alpha)=P[a+bS\le\Phi^{-1}(y_\alpha)]$$
$$\Rightarrow P\{S\le[\Phi^{-1}(y_\alpha)-a]/b\}=\alpha$$
$$\Rightarrow S_\alpha=[\Phi^{-1}(y_\alpha)-a]/b$$
$$\Rightarrow y_\alpha=\Phi[a+bS_\alpha].\ □$$

Remark 3.3. Given an occurring probability α for a scenario for the base or the optimistic scenario, we assume that this occurring probability is given as $P(S<S_\alpha)=\alpha$, under model (3.1). Therefore, it corresponds to a loss at severity level α by Proposition 3.2. While for the pessimistic scenario, the occurring probability α is given as $P(S>S_\alpha)=\alpha$, thus it corresponds to a loss at severity level $(1-\alpha)$. For example, if p_1, p_2, and p_3 are respectively the scenario occurring probabilities for the pessimistic, the base, and the optimistic scenarios, and their values are respectively 10%, 60%, and 30%, then their corresponding loss severity levels are assumed to be 90%, 60%, and 30%.

3.3. Weighting Scenario Losses Given the Noncyclic Expected Loss

By Proposition 2.4 (a), the noncyclic expected loss under model (3.1) is:

$$EL = \Phi\left(a/\sqrt{1+b^2}\,\right). \tag{3.2}$$

Given the noncyclic expected loss EL, and scenario losses y_1, y_2, y_3, as well as the corresponding occurring probabilities p_1, p_2, and p_3, respectively for pessimistic, base, and optimistic scenarios, we need an algorithm to find weights w_1, w_2, and w_3 satisfying the equation below:

$$EL = w_1 y_1 + w_2 y_2 + w_3 y_3. \tag{3.3}$$

3.4. The Proposed Generic Algorithm for Finding Scenario Weights Given Portfolio Expected Loss

We introduce a parameter λ to denote the ratio $\frac{w_2}{1-w_1}$, i.e., $\lambda = \frac{w_2}{1-w_1}$. Then $0 \le \lambda \le 1$, this is because $1 - w_1 = w_2 + w_3$. Thus $\frac{w_3}{1-w_1} = 1 - \lambda$, and we have:

$$w_2 = (1-w_1)\lambda, \quad w_3 = (1-w_1)(1-\lambda). \tag{3.4}$$

Algorithm 3.4 (generic scenario weight algorithm). Suppose the following are given:

(a) The value of noncyclic expected loss EL.
(b) Scenario losses y_1, y_2, y_3, respectively for pessimistic, base, and optimistic scenarios.
(c) The corresponding occurring probabilities p_1, p_2, and p_3.

Follow the steps below to find weights w_1, w_2, and w_3 satisfying (3.3)[1]:

1a. Formulate w_1, w_2, and w_3 as a function of λ. First, solve equation (3.3) for w_1:

$$EL = w_1 y_1 + w_2 y_2 + w_3 y_3$$
$$\Rightarrow EL = w_1 y_1 + (\lambda - \lambda w_1) y_2 + [(1-\lambda) - (1-\lambda) w_1] y_3$$
$$\Rightarrow w_1 = [EL - \lambda y_2 - (1-\lambda) y_3]/[y_1 - \lambda y_2 - (1-\lambda) y_3]. \tag{3.5}$$

Next calculate w_2 and w_3 as:

$$w_2 = (1-w_1)\lambda, \;\; w_3 = (1-w_1)(1-\lambda). \tag{3.6}$$

Note that weights w_1, w_2, and w_3 satisfy (3.3) for each given λ, where $0 \le \lambda \le 1$, as long as:

$$EL \le y_1 \text{ and } y_1 - \lambda y_2 - (1-\lambda) y_3 > 0. \tag{3.7}$$

1b. By (3.7), there exist many values of λ that satisfy (3.3). Calculate w_1^0, w_2^0, and w_3^0 by (1.2), and search by an optimization for a value λ such that the corresponding weights w_1, w_2, and w_3 given by (3.5) and (3.6) are as close as possible in Euclidian distance to the preferable weights w_1^0, w_2^0, and w_3^0.

Remark 3.5. Given components (a), (b) and (c), scenario weights w_1, w_2, and w_3 can be found directly. No model is involved in steps 1a and 1b. Hence, one can obtain the scenario weights for a given stressed portfolio expected loss, by replacing portfolio expected loss EL in (a) with the stressed portfolio expected loss.

[1] Step 1a is proposed by Carlos Lopez

3.5. The Proposed Model for Recession Risk

Assume that there exists a latent risk factor S_R that drives up the recession risk for an economy. Let $p(S_R)$ denote the recession probability conditional on S_R. We assume $p(S_R)$ is given by a probit-type mixed model of the form below for parameters c and d:

$$p(S_R) = \Phi(c + dS_R),\ S_R \sim N(0,1). \qquad (3.8)$$

Given a time series sample $\{R_t\}$, where R_t is an indicator variable with value 1 if a recession occurs at time t or 0 otherwise, the likelihood for observing R_t at time t is:

$$\left(p(S_R)\right)^{R_t} \left(1 - p(S_R)\right)^{1-R_t} = [\Phi(c + dS_R)]^{R_t}[1 - \Phi(c + dS_R)]^{1-R_t}. \qquad (3.9)$$

Here we assume that R_t follows a Bernoulli distribution with probability $p(S_R)$. Parameters c and d can be estimated by maximizing the total sample log-likelihood by using, for example, the SAS procedure PROC NLMIXED ([14]).

3.6. Correlation between Risk Factors S_R and S

Under probit model (3.1), loss rate is driven by the latent risk factor $S \sim N(0,1)$, while for recession probability, it is driven by the latent risk factor $S_R \sim N(0,1)$ under model (3.8). We assume that the pair (S_R, S) is bivariate normal. Let ρ_S denote the correlation between factors S_R and S. Then the latent factor S for loss rate splits into two parts:

$$S = \rho_S\, S_R + \sqrt{1 - \rho_S^2}\ \varepsilon,\ \ \varepsilon \sim N(0, 1),$$

where ε is independent of S_R. Thus by model (3.1), the loss rate y, stressed by recession risk, can be rewritten and transformed to:

$$y_{stressed} = \Phi\left(a + b(\rho_S S_R + \sqrt{1 - \rho_S^2}\ \varepsilon)\right),\ \ \varepsilon \sim N(0,1). \qquad (3.10)$$

3.7. The Proposed Model for Stressed Expected Loss Rate Given Recession Risk S_R

By applying Lemma 2.3 to (3.10), we have the stressed expected loss rate, conditional on S_R, as:

$$EL(S_R)_{stressed} = E_\epsilon[y(S\ |S_R)] = \Phi\left([a + b\rho_S S_R]/\sqrt{1 + b^2(1 - \rho_S^2)}\right). \qquad (3.11)$$

Note that this $EL(S_R)_{stressed}$ differs from the noncyclic expected loss EL. It is the expectation of y given S_R. Given models (3.8) and (3.11), one can stress the expected loss based on a given level of forward looking recession probability as follows: Find the value for S_R corresponding to the given probability β by model (3.8), then use (3.11) to obtain the stressed expected loss.

3.8. The Proposed Methods for Estimating Correlation ρ_S

Given model (3.1) and (3.8) the remaining parameter in (3.11) to be estimated is the correlation parameter ρ_S. By (3.10), the mean and standard deviation of $\Phi^{-1}(y)$, conditional on S_R, are respectively $a + b\rho_S S_R$ and $b\sqrt{1 - \rho_S^2}$. Thus, by Proposition 2.1, the likelihood conditional on S_R for observing loss rate y_t at a time t is $\frac{U_1}{b\sqrt{1-\rho_S^2}\, U_2}$, where:

$$U_1 = \phi[\frac{\Phi^{-1}(y_t)-(a+b\rho_s S_R)}{b\sqrt{1-\rho_s^2}}], \quad (3.12)$$

$$U_2 = \phi[\Phi^{-1}(y_t)]. \quad (3.13)$$

Assume that, for the pair of outcome (R_t, y_t), R_t is independent of y_t, conditional on S_R. Then the joint likelihood for observing the pair of outcome (R_t, y_t) at time t is:

$$LK_t = [p(S_R)]^{R_t}[1 - p(S_R)]^{1-R_t}[\frac{U_1}{b\sqrt{1-\rho_s^2}U_2}]. \quad (3.14)$$

Use (3.12) and (3.13) for U_1 and U_2, and use model (3.8) for $p(S_R)$. Given parameters $a, b, c,$ and d, one can estimate ρ_s by maximizing the total sample likelihood by using, for example, the SAS procedure PROC NLMIXED.

3.9. The Proposed Methods for Determining Scenario Loss Weights for Stressed Expected Loss

Given the stressed expected loss $EL(S_R)_{stressed}$, and scenario losses y_1, y_2, y_3, and the corresponding occurring probabilities p_1, p_2, p_3, one can find weights w_1, w_2, and w_3 by Algorithm 3.4, by replacing EL with $EL(S_R)_{stressed}$, as indicated in Remark 3.5.

4. Empirical Results and Discussions

IFRS9 expected loss evaluation consists of two stages: (a) Stage-one for portfolio expected loss in 12 months, (b) Stage-two for expected lifetime loss for impairment loans after 12 months. We focus only on scenario weights for stage-one loss. For impairment loan losses after 12 months, one could use the scenario weights for 12-month loss and assume that the relative scenario weights keep the same.

4.1. The Loss Rate Model

The loss sample contains the historical loss for a portfolio at each quarter in period 2003Q4-2017Q1. For each quarter, the loss rate is cumulated over four quarters from the beginning of the quarter to get a time series of annual loss rate at quarterly basis. We use this annual loss rate to model 12-month loss rate for IFRS9 stage-one loss. By model (3.1), we assume that the loss rate is of the form:

$$y = \Phi(a + bS), S \sim N(0,1). \quad (4.1)$$

The average loss rate for this sample is 0.34%. This is the noncyclic (unstressed) *EL*. The estimated values for a and b are respectively -2.7243 and 0.1279, estimated respectively as the mean and standard deviation of the probit form of the observed annual loss rates, under Proposition 3.1.

4.2. Results of Generic Algorithm for Noncyclic Expected Loss

Given occurring probabilities p_1, p_2, and p_3, and scenario loss rates y_1, y_2, and y_3, for pessimistic, the base, and the optimistic scenarios, follow the steps below to find w_1, w_2, and w_3:

2a. Calculate the noncyclic expected loss *EL* by (3.2), and set the corresponding relative weights w_1^0, w_2^0, and w_3^0 by (1.2);
2b. Run steps 1a-1b in Algorithm 3.4 to find w_1, w_2, and w_3 satisfying:

$$EL = w_1 y_1 + w_2 y_2 + w_3 y_3. \quad (4.2)$$

The table below shows the scenario weights, with values for p_1, p_2, and p_3 being set at 10%, 60%, and 30%. The corresponding loss severity levels are set at $1 - p_1, p_2$, and p_3 accordingly by Remark 3.3.

Table 1. Scenario loss weights for noncyclic EL [2]

	Scenario Loss Weight			Scenario Occuring Probability			Loss Severity Level				Scenario Loss Rate		
EL	Pestimistic	Base	Optimistic	Pestimistic	Base	Optimistic	EL	Pestimistic	Base	Optimistic	Pestimistic	Base	Optimistic
0.34%	4%	61%	35%	5%	65%	30%	57%	95%	65%	30%	0.60%	0.37%	0.26%
0.34%	10%	60%	30%	10%	60%	30%	57%	90%	60%	30%	0.52%	0.36%	0.26%
0.34%	17%	59%	24%	15%	55%	30%	57%	85%	55%	30%	0.48%	0.34%	0.26%

The column EL under "Loss Severity Level" in the table shows that the severity level for EL (0.34%) is 57%, higher than 50%. We observe that, in these cases, the weights found differ but are close to the occurring probabilities p_1, p_2, and p_3,

4.3. The Model for Expected Loss Given Recession Risk

The historical recession sample contains a recession indicator at each quarter between 1984Q1-2018Q1. To align with the annual loss rate sample, an annual recession indicator is set for the year starting from the beginning of the quarter. For simplicity, the indicator is set to 1 if the number of quarters in recession in the year is above 2, otherwise it is set to 0. There are other ways to set up this recession indicator (see Remark 4.1 below). This gives rise to a time series sample with an annual recession indicator at quarterly basis, which can be joined by the time key to the annual loss rate time series sample, to form a training sample for model (3.11). The average recession rate for this sample is 9.735%.

By model (3.8), we assume that the recession probability is:

$$p(S_R) = \Phi(c + dS_R),\ S_R \sim N(0,1), \tag{4.3}$$

driven by a latent factor $S_R \sim N(0,1)$. The values for c and d are respectively -4.1793 and 3.0636, estimated by maximizing the total sample log-likelihood summed up from the logarithm of (3.9), using SAS procedure PROC NLMIXED.

The remaining parameter in (3.11) to be estimated is the correlation parameter ρ_S between the risk factor S_R for recession probability and factor S for loss rate. By using the overlapped period 2003Q4-2017Q1 between the loss rate sample and the recession indicator sample, we estimate ρ_S by maximizing the total sample log-likelihood summed up from the logarithm of (3.14). The estimate for ρ_S is 0.5797.

4.4. Results of Generic Algorithm for Stressed Periods

Given the occurring probabilities p_1, p_2, and p_3, and scenario loss rates y_1, y_2, and y_3, for pessimistic, the base, and the optimistic scenarios, we follow the steps below to find the scenario loss weights for a given recession probability β:

3a. Set up the reference weights w_1^0, w_2^0, and w_3^0 by (1.2).
3b. For the recession probability β, find the corresponding S_R by setting $S_R = [\Phi^{-1}(\beta) - c]/d$.
3c. Calculate the stressed expected loss $EL(S_R)_{stressed}$ by (3.11). Run 1a-1b in Algorithm 3.4 by replacing EL with $EL(S_R)_{stressed}$ to get weights w_1, w_2, and w_3.

[2] Source: the authors (2018)

The table below shows the results for a series of given recession probabilities. The occurring probabilities, when recession probability is below 99%, are kept as 10%, 60%, and 30%, respectively for pessimistic, base, and optimistic scenarios, and are reset to 5%, 65%, and 30% when recession probability exceeds or equals to 99%. Loss severity levels for three scenarios under "Loss Severity Level" are set by Remark 3.3 at $1 - p_1, p_2$, and p_3. The change of occurring probability for pessimistic scenario is shown in the column under "Loss Severity Level" for pessimistic scenario (highlighted cells).

As expected, the resulting scenario weights differ significantly from the weights given by occurring probabilities, and the weight for pessimistic scenario keeps on increasing when recession probability increases.

Table 2. Scenario loss weights for expected loss stressed by recession probability [3]

	Recession	Stressed	Scenario Loss Weight			Scenario Loss Rate			Loss Severity Level			
No.	Probability	EL	Pestimistic	Base	Optimistic	Pestimistic	Base	Optimistic	Stressed EL	Pestimistic	Base	Optimistic
1	10.00%	0.41%	38.88%	54.96%	6.16%	0.52%	0.36%	0.26%	74.53%	90%	60%	30%
2	20.00%	0.43%	44.84%	52.41%	2.76%	0.52%	0.36%	0.26%	77.11%	90%	60%	30%
3	30.00%	0.44%	50.47%	47.05%	2.48%	0.52%	0.36%	0.26%	78.88%	90%	60%	30%
4	40.00%	0.45%	55.38%	42.39%	2.23%	0.52%	0.36%	0.26%	80.33%	90%	60%	30%
5	50.00%	0.45%	60.04%	37.96%	2.00%	0.52%	0.36%	0.26%	81.62%	90%	60%	30%
6	60.00%	0.46%	64.77%	33.47%	1.76%	0.52%	0.36%	0.26%	82.86%	90%	60%	30%
7	70.00%	0.47%	69.92%	28.57%	1.50%	0.52%	0.36%	0.26%	84.13%	90%	60%	30%
8	80.00%	0.48%	76.06%	22.74%	1.20%	0.52%	0.36%	0.26%	85.53%	90%	60%	30%
9	90.00%	0.50%	84.78%	14.46%	0.76%	0.52%	0.36%	0.26%	87.33%	90%	60%	30%
10	95.00%	0.51%	92.16%	7.45%	0.39%	0.52%	0.36%	0.26%	88.70%	90%	60%	30%
11	99.00%	0.53%	72.45%	26.17%	1.38%	0.60%	0.37%	0.26%	90.97%	95%	65%	30%
12	99.90%	0.56%	85.05%	14.20%	0.75%	0.60%	0.37%	0.26%	93.09%	95%	65%	30%
13	99.99%	0.59%	95.88%	3.92%	0.21%	0.60%	0.37%	0.26%	94.53%	95%	65%	30%

Note that, in both tables 1 and 2, scenario loss rates y_1, y_2, and y_3 are set by the occurring probabilities p_1, p_2, and p_3 at several levels at $1 - p_1, p_2$, and p_3 using loss rate model (4.1) (i.e. model (3.1))). In practice, loss rates y_1, y_2, and y_3 are generated by IFRS9 models for PD, LGD, and EAD, the corresponding loss severity levels may not match up to the severity levels at $1 - p_1, p_2$, and p_3.

In this case, we propose a lookup table for scenario weights, as described below:

> Follow steps 3a-3c to generate a lookup table that contains scenario weights w_1, w_2, and w_3 for each given $EL(S_R)$ and each loss rate triple (y_1, y_2, y_3), where y_i and $EL(S_R)$ vary within an appropriate range, for example, the severity level of y_1 (pessimistic) from 70% to 99%, y_2 (base) from 40% to 60%, and y_3 (optimistic) from 1% to 20%.

When scenario loss rates are generated by IRFS9 models, lookup to the table for a desired level of $EL(S_R)_{stressed}$, and the closest triple (y_1, y_2, y_3) in the table (compared to losses generated by IFRS9 models). The corresponding values for w_1, w_2, and w_3 are then the scenario weights required.

Remark 4.1. The annual recession indicator for a year can also be set in the following way: it has weight $\frac{i}{4}$ for value 1 and weight $(1 - \frac{i}{4})$ for value 0, where i is the number of quarters in recession in the year. This is equivalent to four Bernoulli trials in a year, with i being the number of times that a trial has value 1. Therefore, a sample for an annual recession indicator can be generated to have four observations at each year, where i number of observations (among four) have the value 1 for the indicator.

[3] Source: the authors (2018)

Conclusions. For IFRS9 portfolio expected loss estimation, loan losses generated by IFRS9 models at scenario level are summed up by using the occurring probability weights for the scenarios. Portfolio expected loss estimated in this way can vary significantly, depending on the scenario losses the IFRS9 models generate and the occurring probability weights chosen. The models and approaches proposed in this paper provide a quantitative method for stressing portfolio loss based on recession risk, and a tool for finding the weights for noncyclic portfolio expected loss, and the expected loss under a stressed setting.

Future researches. Models proposed in this paper are limited to probit-type. For future researches, the following questions would be interesting:

(a) How do we perform a similar research, when logit-type mixed models are used (fatter tails than probit form)? In general, the latent recession factor S_R can be non-normal. How do we perform a similar analysis when S_R follows a distribution like log-logistic, Cauchy, or Burr is assumed?
(b) Could a single latent factor be enough to capture the low-probability, high-impact experienced at the recession periods? Would a mixed model with multiple latent risk factors work better?

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Appendix

Proof of Proposition 2.1. The cumulative distribution for y given by

$$\begin{aligned} F(y,a,b) &= P[\Phi(a+bs) \le y] \\ &= P\{s \le [\Phi^{-1}(y) - a]/b\} \\ &= \Phi\{[\Phi^{-1}(y) - a]/b\}. \end{aligned}$$

Since $\Phi[\Phi^{-1}(y)] = y$, the derivative for $\Phi^{-1}(y)$ with respect to y is:

$$\frac{d\Phi^{-1}(y)}{dy} = \frac{1}{\phi[\Phi^{-1}(y)]}.$$

By taking the derivative of $F(y,a,b)$ with respect to y, we have $f(y,a,b) = U_1/(bU_2)$, where U_1 and U_2 are given as (2.4). □

Proof of Proposition 3.1. By (2.3), the log-likelihood at time t is given by $\log(U_1) - \log(b) - \log(U_2)$. By (2.4), we can drop off the term $\log(U_2)$, since its partial derivatives are zero with respect to a and b. The total sample log-likelihood at all times reduces to

$$\begin{aligned} LL &= \textstyle\sum_{1\le t\le T} \; [log(U_1) - log(b)] \\ &= \textstyle\sum_{1\le t\le T} \; [-\frac{(z_t-a)^2}{2b^2} - log(b)]. \end{aligned}$$

Setting the partial derivatives (with respectively to a and b) of LL to zero, we have

$$0 = \frac{\partial LL}{\partial a} = \sum_{1\le t\le T} \frac{2(z_t-a)}{2b^2}$$
$$\Rightarrow a = \frac{1}{T}\sum_{1\le t\le T} z_t \;.$$

$$0 = \frac{\partial LL}{\partial b} = \sum_{1\le t\le T}[\frac{2(z_t-a)^2}{2b^3} - \frac{1}{b}]$$
$$\Rightarrow b^2 = \frac{1}{T}\sum_{1\le t\le T} (z_t - a)^2. \quad □$$