Age matters

Guo, Danqiao and Boyle, Phelim and Weng, Chengguo and Wirjanto, Tony

University of Waterloo, Wilfrid Laurier University, University of Waterloo, University of Waterloo

1 May 2019

Age Matters

Danqiao Guo[a], Phelim Boyle[a,b], Chengguo Weng[a], Tony Wirjanto[a,c]

[a] *Department of Statistics and Actuarial Science, University of Waterloo*
[b] *Lazaridis School of Business and Economics, Wilfrid Laurier University*
[c] *School of Accounting and Finance, University of Waterloo*

Abstract

This paper starts from examining the performance of equally weighted $1/N$ stock portfolios over time. During the last four decades these portfolios outperformed the market. The construction of these portfolios implies that their constituent stocks are in general older than those in the market as a whole. We show that the differential performance can be explained by the relation between stock returns and stock age. We document a significant relation between age and returns. Since 1977 stock returns have been an increasing function of age apart from the oldest ages. For this period the age effect completely dominates the size effect.

Keywords: Bootstrapped portfolio, rebalanced portfolio, age effect, size effect
JEL Classification: G10, G11

1. Introduction

Financial economists have long been interested in the empirical distribution of individual stock returns. These returns provide the raw inputs for the evaluation of portfolio strategies as well as a testing ground for asset pricing theories. Indeed Markowitz (1952) in his classic paper on portfolio selection advocated the use of the empirical distribution of historical stock returns as the first step in providing parameter estimates for his optimization algorithms. More recently Bessembinder (2018) has conducted an extensive analysis of individual stocks using returns from the Center for Research in Securities Prices (CRSP) database.

Portfolios of individual stocks are attractive to risk averse investors because of their potential diversification benefits. The equally weighted $1/N$ strategy has been widely studied in the finance

Email addresses: danqiao.guo@uwaterloo.ca (Danqiao Guo), pboyle@wlu.ca (Phelim Boyle), chengguo.weng@uwaterloo.ca (Chengguo Weng), twirjanto@uwaterloo.ca (Tony Wirjanto)

[1] We gratefully acknowledge the comments from Hendrik Bessembinder, Michael Brennan, Andrew Karolyi, Jay Ritter, Raman Uppal, and Xiaofei Zhao. All remaining errors are ours alone. This research was supported by the Natural Sciences and Engineering Research Council of Canada (RGPIN-2014-04 and RGPIN-2016-04001). Wharton Research Data Services (WRDS) was used in preparing this manuscript. This service and the data available thereon constitute valuable intellectual property and trade secrets of WRDS and/or its third-party suppliers.

literature. It is a very simple strategy since it involves no estimation, no optimization, no short positions and has relatively little turnover compared to other strategies. Benartzi and Thaler (2001) document that it is widely used in practice by participants of defined contribution pension plans as a heuristic method for choosing asset classes. Despite its naive construction this strategy has been shown to outperform most alternative strategies. For example DeMiguel et al. (2009) examine the performance of the $1/N$ rule using a variety of datasets. They attribute the superior performance of the equally weighted strategy to the presence of estimation risk and its well known perverse interaction with optimization.

Brennan and Torous (1999) also demonstrate the superior performance of an equally weighted $1/N$ rebalanced portfolio over the value weighted market portfolio. They use individual stock returns from the CRSP database for the period 1926-1997 to construct their equally weighted portfolio. They attribute this outperformance mainly to the small firm effect:

> "because of higher returns on small firms, an equally weighted portfolio of as few as five randomly chosen firms can provide the same level of expected utility as the value weighted market portfolio."

Plyakha et al. (2015) show that equally weighted portfolios outperform value weighted portfolio based on samples of individual stocks in the S&P indices. They show that the major source of the extra alpha in the equally weighted portfolio is due to the contrarian nature of the strategy.

Our paper examines the performance of equally weighted portfolios and we show that there is an additional reason for their superior returns. It is worth emphasizing that our portfolios are made up of individual stocks from the entire CRSP database. In contrast the datasets used by DeMiguel et al. (2009) where the N components of the equally weighted portfolios are themselves portfolios[1] or indices for seven of their datasets. Their eighth dataset is based on simulated stock returns for a single factor model. Our equally weighted portfolios are constructed as in Brennan and Torous (1999). We use the same comprehensive dataset as Bessembinder (2018) since it facilitates comparisons with his results.

This approach, where the components of the equally weighted portfolios are individual securities

[1] For example their first data set consists of ten sector portfolios of the S&P plus the US equity market portfolio for the period 1981-2002.

rather than other portfolios, is better suited to our purpose. It permits us to keep track of the time series properties of the individual stocks and in particular their ages. Another difference between using individual stocks and portfolios is that when a stock is delisted it disappears from the equally weighted portfolio. If this happens it is replaced with another stock drawn at random from the available pool. This newly added stock will be representative of the market as a whole in particular in terms of age. The other stocks in the portfolio will age by one period so that the portfolio as a whole will grow older.

A simple and effective way to compare the performance of the equally weighted portfolio with that of the market is to use comparably sized portfolios which contain N equally weighted stocks at the start of each period. These portfolios are routinely liquidated at the end of each period and a new set of N stocks is selected at random from the available pool. By means of this construction these portfolios are representative of the market as a whole. Bessembinder (2018) used the same type of construction except that his portfolios were value weighted instead of equally weighted. Following his convention we refer to these portfolios as equally weighted bootstrapped portfolios or just bootstrapped portfolios. We denote the traditional $1/N$ portfolios as equally weighted rebalanced portfolios or just rebalanced portfolios. One important property of the bootstrapped portfolios is that because of the periodic rebalancing they have the same[2] exposure to reversals as the traditional $1/N$ portfolios.

The current paper compares the returns on the rebalanced portfolio with the returns on the bootstrapped portfolio over the 1926-2016 period spanned by the CRSP data base. While there is some secular variation in the relative performance of the two types of portfolio over time, our most striking finding is that the rebalanced portfolio yields higher realized returns than the bootstrapped portfolio during the most recent forty-year period: 1977-2016. This finding is robust to the portfolio size and to the choice of different starting dates and to the investment horizon within this period. We contend that this difference is not due to the rebalanced portfolio benefitting from reversals since the bootstrapped portfolio will also benefit to the same extent. During the 1926-1976 period the returns on the rebalanced portfolio are very similar to the returns on the bootstrapped portfolio

There are two arguments for why one might not expect the rebalanced portfolio to outperform

[2] We show this in Appendix B.

the market. The first has to do with delisting since holding a stock until it disappears from the market does not seem to be a smart strategy. Actually, the popular belief that being delisted is bad news is somewhat misleading. This is because a stock can disappear from the market for reasons other than bankruptcy. For example the most frequent reason for delisting is merger and acquisition, which often reflects the past success of a company. Even for the stocks that exit from the market due to unfavorable reasons investors rarely lose all of their investment. The second reason is that our results appear to run counter to the size[3] effect. If on average portfolios of small stocks outperform portfolios of large stocks and if it is true that older firms in general tend to be larger than younger firms and if age is positively correlated with firm size, the stocks in the rebalanced portfolios being older than average will be larger than the stocks in the bootstrapped portfolios. We explain in the paper why the situation is much more nuanced than this and we disentangle the intertwined effects of size and age.

In this paper we argue that the reason for the performance difference between the rebalanced portfolios and the bootstrapped portfolios stems from the relation between stock age and stock return. The age distribution of the stocks in the bootstrapped portfolio will be very similar to that of the stock universe whereas the age distribution of the stocks in the rebalanced portfolio will typically be older than those in the stock universe. Thus the age profile of the N stocks in the rebalanced portfolio will be older than those of the N stocks in the bootstrapped portfolio. If stock return is related to age this will impact the relative performance of the two types of portfolios. We show in the paper that there is a significant positive relation between stock return and stock age during the period 1977-2016 and that the relation is much weaker[4] during the first fifty years from 1926 to 1976.

This positive relation between age and return is consistent with the underperformance of IPO's documented by Ritter (1991). He finds that newly listed firms perform worse on average than a matched sample of older firms during the first five years after listing. Updated tables providing

[3]There is considerable evidence that the importance of the size effect has declined in recent years. See Horowitz et al. (2000), Alquist et al. (2018).

[4]Barry and Brown (1984) examined the relation between stock return and stock age over a period roughly corresponding the first half of our sample period. They report a mildly negative relation between stock age and return. There are some differences between our approaches. Their data period is 1930-1980 whereas ours is 1926-1976. They only consider stocks that have been listed on the exchange for 60 months and so omit many stocks that we include. In addition their method of computing the delisting returns differs from ours.

data on the long run performance of new issues are available from Jay Ritter's website[5]. During the period 1980-2016 IPO firms have underperformed matched (by size) firms by an average of 3.3% per annum during the first five years. Brennan and Torous (1999) made this connection[6] between the poor returns on new listings and the composition of the equally weighted portfolios.

The relation between firm age and stock return is also consistent with the recent model of Lin et al. (2018) who analyze the conditions under which firms adopt new technology. In their setup firms differ in their capacity to adopt new (and costly) technology which will make them more efficient. The authors define the concept of capital age as the length of time since the last adoption of a new technology. Capital age is used to measure different levels of technical efficiency. Young capital age firms are closer to the technological frontier than old capital age firms. Young capital age firms are predicted to be more productive and less risky than old capital age firms. Hence they earn lower expected returns than old capital age firms and this is confirmed empirically. Our measure of calendar age bears a similar relation to expected return.

To better understand the age effect and the connection between the age effect and the size effect we construct 16 portfolios that are doubly sorted into four age groups and four size groups and compare their performance. We focus on the 1977-2016 period and report comparable results for the 1926-1976 period in Appendix A. The age effect is clearly observed in all size groups, and the size effect is evident in all age groups. When we divide stocks into four age groups we find that returns are increasing with age over the first three groups but are flat or drop a little for the oldest group. That is the age effect is not monotone. It holds over the bulk of a firm's life but may be reversed in the oldest age group. Hence our age effect is not inconsistent with the finding that firms are less profitable at older ages (see for instance Loderer and Waelchli (2010)).

This leads us to conclude that the age and size are not spanned by a common underlying factor. Moreover the age effect seems to be in conflict with the size effect, since stock age and size are positively correlated but explain the stock returns in the opposite direction. To further resolve

[5] `https://site.warrington.ufl.edu/ritter/ipo-data/`

[6] Brennan and Torous (1999) note on page 138 that "The main difference between the randomly selected portfolios and the EW CRSP index portfolio is that the securities included in the former are all listed at the beginning of the 10- or 20- year period, whereas the constituents of the CRSP portfolio are continuously updated to reflect new listings. Therefore the superior performance of the randomly selected portfolios is consistent with the abnormal returns to new listings that have been documented by Loughran and Ritter (1995)."

this puzzle we divide stocks into decile groups based on their age or size and calculate the return statistics within each decile group. The results suggest that the observed small firm effect is a result of the extremely positive return skewness in the smallest 10% of the stocks (This has been noted in Bessembinder (2018), and the general return skewness problem is discussed in Heaton et al. (2017) for instance). If the within-group median return is used as the performance measure, the direction of how the two factors affect stock returns turns out to be the same.

This paper makes the following contributions to the literature. First, we acquire deeper understanding of why the rebalanced portfolio outperforms the bootstrapped portfolio so impressively over the period from 1977 to 2016. We show that this is caused by a combination of the older age profile of the rebalanced portfolio and the relation between stock returns and firm age. Second, we empirically document an age effect: an asset pricing anomaly that is entangled with but quite distinct from the size effect. Third, our results provide a possible opportunity for investment management. An institution could in principle structure a portfolio to exploit the age effect.

The remaining part of this paper is organized as follows. Section 2 analyzes the performance of the equally weighted bootstrapped portfolio and the equally weighted rebalanced portfolio and highlights the performance gap. Section 3 relates the performance gap to the difference in age distribution between the two portfolios and discusses some aspects of the age effect. We provide a detailed analysis of the these phenomena for the period 1977-2016 and give a summary of the results for the first 50 years of data in Appendix A. Section 4 discusses economic explanations for the age effect. Section 5 concludes the paper.

2. Bootstrapped versus Rebalanced Portfolios

In this section we compare the realized returns on our two basic portfolio strategies. These are the conventional $1/N$ equally weighted strategy[7] that has been studied by DeMiguel et al. (2009) and the equally weighted bootstrapped strategy. We use the same data as Bessembinder (2018). The data is available from the Center for Research in Securities Prices (CRSP) monthly stock return database. As in Bessembinder (2018) only common stocks with share codes 10, 11, and 12 are included in the study. The entire period runs from June 1926 to December 2016 and

[7] It is denoted in this paper as the rebalanced strategy.

includes 26,051 distinct CRSP permanent numbers (PERMNOs). The monthly returns are inclusive of reinvested dividends.

We construct the bootstrapped portfolio by picking N stocks at the start of each month and investing equal amounts in each stock. We hold this portfolio for one month before liquidating the portfolio and starting this process all over again for the next month. By compounding all the monthly[8] returns we obtain the holding period return of the bootstrapped portfolio. The rebalanced portfolio is constructed by selecting N random stocks at inception and investing equal amounts in each stock. Each month the weights are adjusted to obtain equal investments in each stock. If a stock in this portfolio is delisted in a particular month it is replaced by another stock selected at random from the available pool of actively traded CRSP stocks at that time.

2.1. Relative Performance

We compare the performance of the bootstrapped and rebalanced N-stock portfolios in Table 1 and find that on average the returns on the rebalanced portfolios exceed those on the bootstrapped portfolios. These results are based on simulations of 20,000 portfolios of each type for $N = 5$, 25, 50, or 100. As noted previously in the Introduction these results may appear counterintuitive. They provide the motivation for investigation of the age effect in the next section.

Comparing the mean annualized returns in the same rows, we notice that the rebalanced portfolios outperform the bootstrapped portfolios for all four values of N. For $N = 5$ the performance gap is 1.23% per annum. This pattern becomes even more obvious when we look at the percentage out of the 20,000 portfolios that outperform the equally weighted portfolio of the whole market. For the bootstrapped portfolios, as the portfolio size increases, this percentage increases toward 50%[9]. However the proportion of rebalanced portfolios that outperform the equally weighted market gradually increases to be over 80%. Note we have not yet taken transaction costs into account when

[8]It is worth pointing out that some stocks enter the bootstrapped portfolio in their last trading month and are delisted during the month. These stocks are associated with a code of delisting reason and a delisting return. The delisting return is calculated by comparing the security's Amount After Delisting with its price on the last day of trading. In such a case we adjust the stock return by incorporating the delisting return to reflect the actual return an investor would obtain when holding the stock till it is delisted. There are a few occasions where the delisting reason is specified but the delisting return is missing. In such occasions we follow the method proposed in Shumway (1997) to fill the delisting return according to the delisting reason .

[9]Actually the limit of this percentage as N increases is not exactly 50%, because the return of the equally weighted market is only the expected value instead of the 50% quantile of the return distribution of the bootstrapped portfolios

Table 1: Summary of annualized returns of 20,000 bootstrapped and rebalanced N-stock portfolios. In a bootstrapped portfolio the indicated numbers of stocks are selected at random for each month. In a rebalanced portfolio the indicated numbers of stocks are selected at random at the beginning of investment horizon, the same stocks are adjusted to have equal weights each month unless one or more stocks are picked at random to make up for the delisted one(s). Equally weighted portfolio returns are computed each month and are linked over the horizon from July 1926 to December 2016. Annualized return is recorded for each of 20,000 simulations of each portfolio type. Mean, median, and skewness of the 20,000 annualized returns are reported, as well as the percentage out of the 20,000 returns that are positive, greater than the return on Treasury bill, and greater than the return on an equally weighted portfolio of the whole market.

	Bootstrap portfolios			Rebalanced portfolios		
	Mean	Median	Skew	Mean	Median	Skew
			N = 5			
Holding return	0.0978	0.0976	0.1131	0.1101	0.1108	-0.1365
% > 0	100.00%			100.00%		
% > T-bill	99.99%			100.00%		
% > EW mkt	14.89%			23.32%		
			N = 25			
Holding return	0.1179	0.1178	0.0725	0.1239	0.1239	-0.0037
% > 0	100.00%			100.00%		
% > T-bill	100.00%			100.00%		
% > EW mkt	31.76%			53.05%		
			N = 50			
Holding return	0.1205	0.1204	0.0703	0.1258	0.1258	-0.0090
% > 0	100.00%			100.00%		
% > T-bill	100.00%			100.00%		
% > EW mkt	36.32%			67.08%		
			N = 100			
Holding return	0.1220	0.1220	0.0404	0.1269	0.1269	0.0174
% > 0	100.00%			100.00%		
% > T-bill	100.00%			100.00%		
% > EW mkt	40.87%			82.25%		

calculating the returns. That is, since the rebalanced portfolios have much less turnover compared with the bootstrapped ones, the former will be more favourable if transaction costs were included.

Table 2: Summary of annualized returns of 20,000 bootstrapped and rebalanced 100-stock portfolios over three shorter holding periods: July 1926 - December 1976, January 1977 - December 2016, and January 2007 - December 2016. Construction of bootstrapped and rebalanced portfolios is the same as described in Table 1 except that the monthly returns of equally weighted portfolios are linked over indicated investment horizons and that the portfolio size is fixed at $N = 100$. Annualized return is recorded for each of 20,000 simulations of each portfolio type. Mean, median, and skewness of the 20,000 annualized returns are reported, as well as the percentage out of the 20,000 returns that are positive, greater than the return on Treasury bill, and greater than the return on an equally weighted portfolio of the whole market.

July 1926 - December 1976						
	Bootstrapped portfolios			Rebalanced portfolios		
	Mean	Median	Skew	Mean	Median	Skew
Holding return	0.1167	0.1167	-0.0058	0.1159	0.1159	-0.0050
% > 0	100.00%			100.00%		
% > T-bill	100.00%			100.00%		
% > EW mkt	45.01%			35.07%		
January 1977 - December 2016						
	Bootstrapped portfolios			Rebalanced portfolios		
	Mean	Median	Skew	Mean	Median	Skew
Holding return	0.1286	0.1284	0.0913	0.1518	0.1517	0.0605
% > 0	100.00%			100.00%		
% > T-bill	100.00%			100.00%		
% > EW mkt	41.36%			99.36%		
January 2007 - December 2016						
	Bootstrapped portfolios			Rebalanced portfolios		
	Mean	Median	Skew	Mean	Median	Skew
Holding return	0.0579	0.0575	0.1110	0.0728	0.0728	-0.0302
% > 0	99.91%			100.00%		
% > T-bill	99.69%			100.00%		
% > EW mkt	45.44%			78.28%		

While Table 1 demonstrates that the returns on the rebalanced portfolios are consistently higher than those on the bootstrapped portfolios, the differences for $N = 50$ and $N = 100$ do not seem large at around fifty basis points. However recall that these results are based on the entire 90 year period from 1926 to 2016 and that there were relatively few stocks at the start of this period. We obtain more interesting and more dramatic results when we divide the period up into smaller subperiods. We redo the same calculations as in Table 1 but based on shorter investment horizons. The first period is from July 1926 to December 1976 which leads to a holding period of about 50 years. The second period from January 1977 to December 2016 coincides with a typical time period that would be currently used for asset pricing empirical tests. The third period is from January 2007 to December 2016, leading to a 10-year holding period. In addition we set $N = 100$.

Table 2 reports the performance of the bootstrapped and rebalanced portfolios over these sub-periods. There is a substantial difference in the relative performance of the two portfolios in the first fifty years and in the last forty years. During the earlier period the returns are very close with the bootstrapped portfolio being marginally better by 0.08% per annum. However during the most recent forty years the returns on the rebalanced portfolio are on average 2.32% per annum higher than those on the bootstrapped portfolio. For the most recent decade (2007-2016) the rebalanced portfolio return is 1.49% per annum higher than the return on the bootstrapped portfolio. We recall from Table 1 that over the entire 90 year period with $N = 100$ that the mean return on the rebalanced portfolio exceeds the mean return on the bootstrapped portfolio by 0.49% per annum. This suggests something quite different is happening in the last four decades as compared to the first five decades.

2.2. How Bad is Being Delisted?

In the Introduction we mentioned that some observers tend to think that the rebalanced portfolio would perform poorly because it holds a stock until it disappears from the market. However it is sometimes overlooked that being delisted is not necessarily bad news. We refer readers to Table 2B in Bessembinder (2018) for a detailed summary of lifetime buy-and-hold returns by final delisting status. The results suggest that the majority of stocks that are finally delisted due to Merger, Exchange, or Liquidation yield a lifetime buy-and-hold return exceeding that of the one-month Treasury bill. Even for the stocks that are delisted by the exchange, the mean lifetime buy-and-hold return is -0.8%, which is far from a devastating outcome. However it should be noted that this is thanks to the diversification effect - the median lifetime buy-and-hold return is much more negative. In addition more stocks were delisted due to Merger, Exchange, or Liquidation than any other reasons. These results together explain why delisting does not unduly penalize the returns on the rebalanced portfolios.

2.3. Comparison with Value Weighted Bootstrapped Portfolio

We can gain additional insight by comparing the returns on equally weighted strategies with the returns on value weighted strategies. Specifically we compare the performance of equally weighted bootstrapped portfolios with similar value weighted bootstrapped portfolios. Bessembinder (2018) has already computed the returns on value weighted bootstrapped portfolios and we compared his results with our equally weighted bootstrapped portfolios. In our comparison we use the same set of stocks in each comparison pair so that the portfolios differ only by their respective weights. We find[10] that the returns on the equally weighted bootstrapped portfolios are on average 2.24% per annum higher than the returns on the value weighted bootstrapped portfolios. Since both portfolios have the same age distribution this performance cannot be explained by an age effect. It is due to the contrarian nature of the equally weighted portfolio and the small firm effect. As we will see in the next section the equally weighted rebalanced portfolio and the equally weighted bootstrapped portfolio have quite different age distributions and this can impact their relative performance.

3. Stock Age and Cross-sectional Returns

In this section we demonstrate that stock age is an important determinant of returns. In particular we show that portfolio age is a key difference between the bootstrapped portfolios and the rebalanced ones and that this difference leads to the performance gap between these two portfolio types. The numerical analysis presented in this section is based on the period 1977-2016. This is because the recent 40-year period is more relevant to the current financial market. For completeness we report the corresponding results for the period 1926 to 1976 in Appendix A. The age effect is observable but much weaker during this earlier period.

3.1. A Probabilistic View on Age Distribution

In this subsection we explain using a probabilistic argument why the rebalanced portfolio will have an older age distribution than the bootstrapped portfolio. Consider a rebalancing date when there are M stocks available in the stock universe. Then each of the M stocks has a probability of N/M of being included in the N-stock bootstrapped portfolio. If K stocks that were in the rebalanced portfolio in the previous period leave the portfolio because of delisting, then the $N - K$ stocks that already exist in the rebalanced portfolio will remain in the portfolio with a probability of one. Moreover each of the remaining $M - (N - K)$ stocks in the pool will be selected into the rebalanced portfolio with a probability of $K/(M - N + K)$ ($<N/M$). From this perspective

[10] The results are available on request.

an important difference between the two portfolio types rests squarely on the rebalanced portfolio favouring *seasoned* stocks by assigning them a much higher probability of staying in the portfolio. In other words the component stocks in the rebalanced portfolio become mature in terms of age as time elapses. In contrast, the bootstrapped portfolio does not take into account the age of the stocks. This means that the average age of the rebalanced portfolio will increase over time, whereas the average age of the bootstrapped portfolio will be similar to that of the stock universe.

Figure 1 shows the profile of the stock population in the CRSP data from 1977 to 2016. The red portion in each bar represents the number of stocks that entered the universe in the current calendar year. The blue portion represents the number of stocks that have existed in the universe at the beginning of each calendar year. In demographic parlance the new listings correspond to births and the delistings correspond to deaths. The crude birth rate is the total number of births in a given year divided by the size of the population. The crude death rate is the total number of deaths in a given year divided by the size of the population. For this data the average crude birth rate for the period 1977-2016 is 10% while the crude death rate is 9%. These numbers are similar[11] to those obtained by Loderer and Waelchli (2010) but higher[12] than those obtained by Doidge et al. (2017) who focus only on domestic US stocks. A delisting rate of 10% implies that in the rebalanced portfolio about 90% of the constituent stocks remain in place each year and as a result they age by one year. The other ten percent that are added to the portfolio will have an age distribution similar to that of the stock universe. The age distribution of the bootstrapped portfolio reflects the age distribution of the stock universe. Hence the average age of the stocks in the rebalanced portfolio increases[13].

3.2. Age Distribution in Bootstrapped and Rebalanced Portfolios

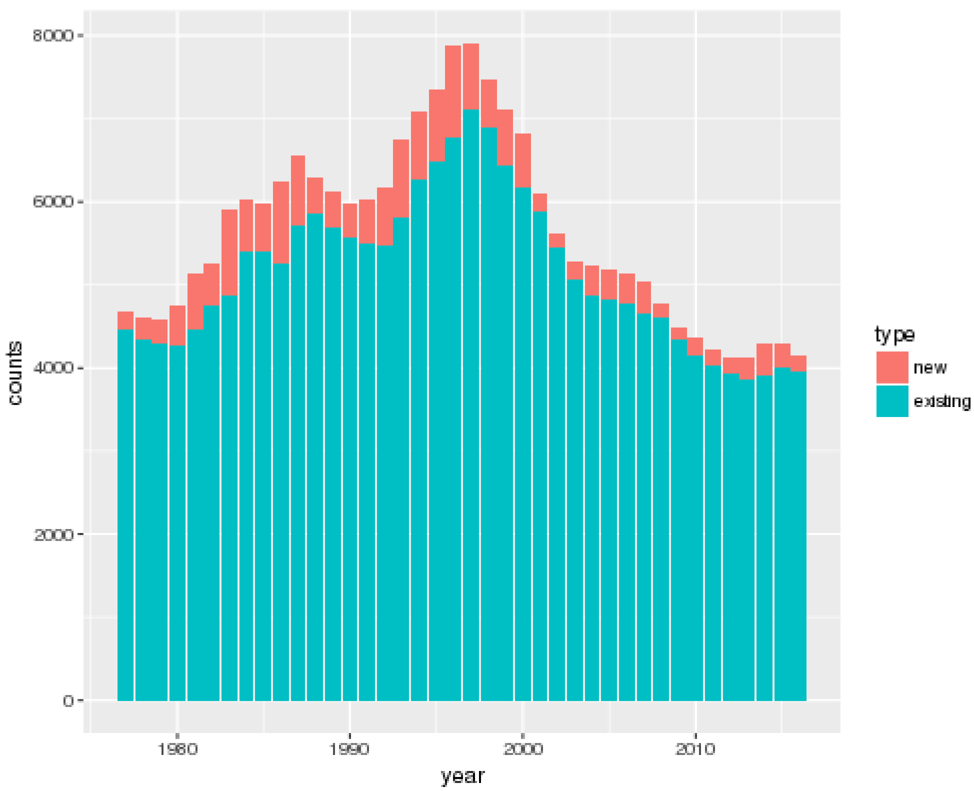
Before presenting the age distributions for different portfolios, it is useful to clarify the calculation of age in our study. On any given month the age of a stock is the number of months that have elapsed since the first month the stock appeared in the CRSP database divided by twelve. Our empirical

[11]Based on the period 1978-2004, Loderer and Waelchli (2010) obtained 10.3% for the crude birth rate and 9.9% for the crude death rate.

[12]Doidge et al. (2017) estimate an average crude birth rate of 7.5% and an average crude death rate of 8.2% based on the period 1975-2012. They just focus on US stocks whereas we follow Bessembinder (2018) and retain securities with share codes 10, 11 and 12. Hence our rates are higher.

[13]We can show that if the stock universe is stationary over time, then the average age in the rebalanced portfolio keeps increasing until it reaches an asymptotic limit.

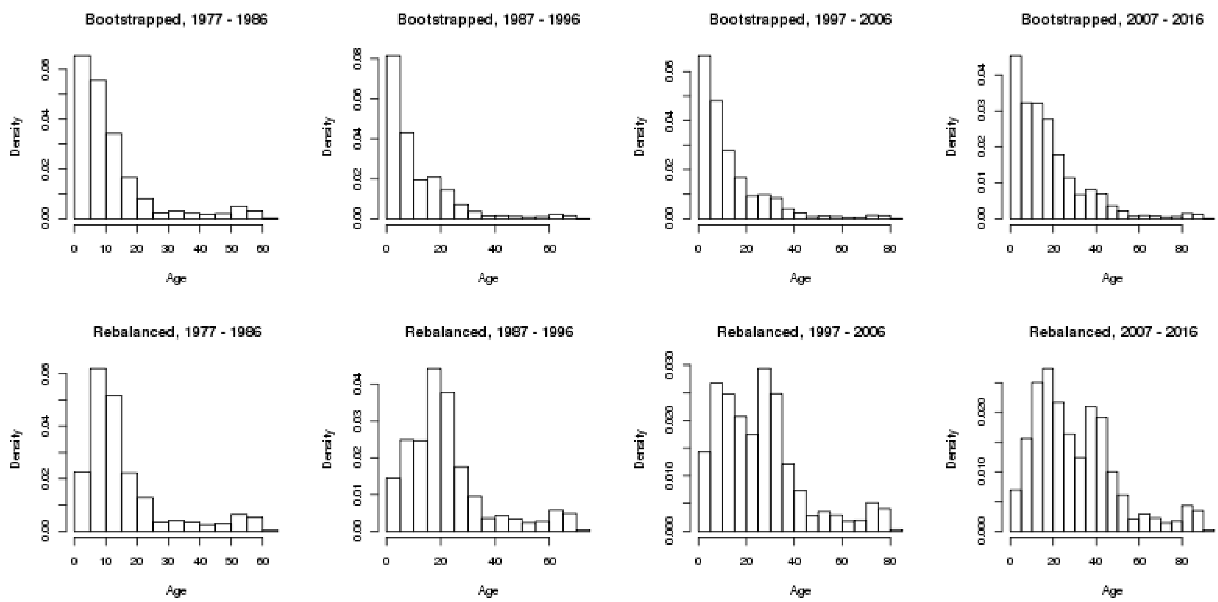
Figure 1: Population of stocks in the CRSP database: existing stocks and new listings. The figure shows the change in the stock population in the CRSP database from 1977 to 2016. The red portion in each bar represents the number of stocks that entered the universe in the indicated calendar year. The blue portion represents the number of stocks that have existed in the universe at the beginning of the indicated calendar year.



data confirm our predictions about the difference in age distribution between the bootstrapped and rebalanced portfolios. We consider the holding period from January 1977 to December 2016 and simulate 1000 bootstrapped portfolios as well as 1000 rebalanced portfolios, each containing 100 component stocks. In each month we record the age of all component stocks in each simulated portfolio. We report the empirical age distribution in both portfolios over each decade in the 40-year horizon according to the aggregate result across 1000 simulations. The aim of this decade-by-decade breakdown is to highlight the evolution of the age distribution in the two portfolios as time passes. Figure 2 presents the age distribution of the two representative portfolios over time. Furthermore, based on the 1000 simulations, we report the average age of component stocks in each portfolio type over the four non-overlapping decades. The result is shown in Table 3.

Some institutional background is helpful in interpreting these graphs. The average age of the entire stock universe is increasing over these four decades for two main reasons. The first is due to the aging of the large influx of Nasdaq stocks that entered the database as a group in 1972. Their entry is shown clearly in Figure A.1 of Appendix A. The second is due to the decline[14] in new listings since the mid 1990's as evidenced by Figure 1. From Figure 2 we see that the histograms of the age distribution of both the bootstrapped portfolio and the rebalanced portfolio move to the right over the four decades. However if we compare the histograms of the two portfolios across each decade we see that the age histogram of the rebalanced portfolio is consistently further to the right of the corresponding histogram of the bootstrapped portfolio. Table 3 confirms this observation. The age difference is 4.3 years for the period 1977-1986 and averages 12 years over the next three decades. The fact that the stock universe is replenished each year with newly listed stocks and the way in which the rebalanced portfolio is constructed leads to the difference in the age distributions between the two portfolio types.

Figure 2: Age distribution in 100-stock bootstrapped and rebalanced portfolios over different time periods. The distributions are based on age of components of one thousand 100-stock bootstrapped and rebalanced portfolios held over the period from January 1977 to December 2016. We report the empirical age distribution in both portfolios over each decade in the 40-year horizon.



3.3. Age Effect

We have identified the age distribution as a key difference between the bootstrapped and rebalanced portfolios. The next step is to investigate whether age can explain the cross-section of stock

[14]See also Doidge et al. (2017).

Table 3: Average age of 100-stock bootstrapped and rebalanced portfolios over different time periods. The average ages are calculated based on age of components of one thousand 100-stock bootstrapped and rebalanced portfolios held over the period from January 1977 to December 2016. We report the average age in both portfolios over each decade in the 40-year horizon.

Portfolio Type	1977 - 1986	1987 - 1996	1997 - 2006	2007 - 2016
Bootstrapped	11.60	11.24	12.90	17.35
Rebalanced	15.90	22.11	25.61	29.89

returns. If age is a significant predictor of stock returns, then the performance gap between the rebalanced and bootstrapped portfolios can be explained by the difference in the age distributions.

To study the age effect we include each stock that has ever appeared in the stock universe from January 1977 to December 2016 in our analysis. Each month we record the age and monthly return (annualized by multiplying by 12) of each stock in the entire universe. Let R_{it} denote the annualized return[15] of the ith stock in the tth month and A_{it} denote the age of the ith stock in the tth month. Now that the data has both time-series and cross-section dimensions, panel data modeling techniques are used to estimate the parameter of interest, which is the effect of age on asset returns. Equation (1) is a simple (time effects) model relating the stock's return to its age[16].

$$R_{it} = \beta A_{it} + \gamma_t + \epsilon_{it} \tag{1}$$

The parameter β quantifies the change in the cross-sectional stock return when the stock age increases by one unit. The parameter γ_t characterizes the level of average cross-sectional returns in the tth month. The time effect term is included because the market movement from month to month could make it problematic to pool samples across time and thus affect the estimation of the age effect. In addition it can be argued that the age effect in the panel-data regression in equation (1) is potentially confounded by vintage years. A vintage year is the year in which a company receives its first influx of investment capital. It is the year when capital is contributed by a venture capital, a private equity fund or a partnership drawing down from its investors. A vintage year at the peak or bottom of a business cycle can potentially affect subsequent returns on initial investment as the company undergoes over or under-valuation at the time. The introduction of the time effect

[15]The purpose of annualizing the monthly returns is to bring the estimate of the model coefficient to a more visible scale.

[16]The estimate of β will be biased if size and possibly other firm characteristics affect asset returns and are correlated with age.

in the panel-data regression in equation (1) can be viewed as a rough measure to control for this potentially omitted confounding influence. Lastly, ϵ_{it} is the error term.

Subtracting the cross-sectional average $\bar{R}_t = \frac{1}{N_t}\sum_i R_{it}$, where N_t is the number of existing stocks in the tth month, from the initial model equation (1) becomes

$$\ddot{R}_{it} = \beta \ddot{A}_{it} + \ddot{\epsilon}_{it}, \tag{2}$$

where $\ddot{R}_{it} = R_{it} - \bar{R}_t$, $\ddot{A}_{it} = A_{it} - \bar{A}_t$ and $\ddot{\epsilon}_{it} = \epsilon_{it} - \bar{\epsilon}_t$. Note that $\bar{A}_t$ and $\bar{\epsilon}_t$ are defined in the same way as $\bar{R}_t$. The first row in Table 4 summarizes the estimation and hypothesis testing results of the time effects model. The sign of the $\hat{\beta}$ and the highly significant p-value for the t-test confirm age has a significantly positive impact on the cross-sectional return of a stock. In other words, stocks that are older tend to outperform younger stocks.

Table 4: Empirical results for time effects model: January 1977 - December 2016. In the first row, the estimate, standard error, t-statistics, and the associated p-value for the overall time effects model are reported. In each month, the age group each stock belongs to is determined based on the cross-sectional ranking of the stock's current age. The breakpoints between age groups are the first quartile, median, and third quartile of the cross-sectional age distribution. All stock-month observations are divided into four age groups in this way. In each of the second to fifth rows, model fitting results for the indicated age group are reported.

Age Group	Estimate	Std. Error	t-value	p-value
All ages	0.0007	0.0001	7.7808	0.0000
Infant	0.0061	0.0026	2.3621	0.0182
Youth	0.0068	0.0019	3.6444	0.0003
Adult	0.0008	0.0010	0.7864	0.4316
Senior	-0.0006	0.0001	-4.8058	0.0000

We fit the same regression model with sub-groups of the data to provide additional robustness to our result. At the beginning of each month in the investment horizon, each stock in the universe is labeled with one of the four age groups, Infant, Youth, Adult, and Senior, according to their current age. The breakpoints between adjacent age groups are the first quartile, median, and third quartile of the cross-sectional age distribution[17]. In this way we add an additional categorical feature to each stock-month observation. Then we divide the data into four sub-groups according to the age group label and fit the model in equation (1) using each of the four subsets. The estimation and hypothesis testing results are also presented in Table 4. Within each of the youngest two age groups there is a

[17]This grouping method leads to a dynamic group membership. Size of different groups may be different because there may be multiple stocks at the breakpoint ages.

significant and positive relationship between stock age and stock return. The age-return relation in the second oldest age group is insignificant. A significant and negative age effect is observed in the oldest age group which represents a downturn in performance when a stock gets really old. However the magnitude of the coefficient in the two younger age groups is nine times larger than that in the two older age groups. The sub-group analysis allows us to acquire a deeper understanding of the nature of the age effect at different stages of a firm's life cycle.

We claim that the significant relationship between a stock's age and its return, together with the difference in age distribution between the bootstrapped and rebalanced portfolios, explains the performance gap between these two types of portfolios. We show in Appendix B that the returns on the rebalanced and bootstrapped portfolios have the same exposure to mean reversion so that any performance difference cannot be attributed to mean reversion. This does not contradict the results of Plyakha et al. (2015) which show that the performance of the rebalanced $1/N$ portfolio does benefit from mean reversion. The performance of the bootstrapped $1/N$ portfolio benefits from mean reversion to the same extent. When we compare returns on the two portfolios, the impact of mean reversion cancels out so that the difference in returns on the two portfolios cannot be accounted for by mean reversion.

3.4. Age Effect vs. Size Effect

The small firm effect is a well-known pricing anomaly in finance which holds that smaller firms, or those companies with a small market capitalization (as a product of price and number of outstanding shares), outperform larger companies. This effect has been documented by many researchers (see Van Dijk (2011) for a review) but the consensus is (cf Alquist et al. (2018)) that it has become less important in more recent years. In results not reported here we confirmed its presence in our data as well.

In the previous subsection we showed that senior firms generally outperform junior firms. The "senior firm effect" and the "small firm effect" seem to be complementary (and not substituting) effects, because a stock's age and its market capitalization are positively correlated measures of scale of the issuing company. However these two measures explain the cross-sectional stock returns in opposite directions since age and size are positively correlated. We confirm the presence of this correlation in our data. We record the age and market capitalization of all available stocks in each current month as well as the ranking (according to age and size respectively) of each stock within the current stock universe. Pooling the records across months we obtain vectors of stock age, size, rank by age, as well as rank by size. The correlation coefficient between the raw values of age and size is 0.23, and the correlation coefficient between the rank of age and rank of size is 0.29.

The finding that two positively correlated stock characteristics explain the cross-sectional stock returns in opposite directions is puzzling at first. To give a more detailed picture of how the age and size factors affect stock returns, we construct quartile portfolios which are doubly sorted according to both the age and size factors. At the beginning of each month all stocks in the universe are divided into four roughly equal-size age groups, i.e., Infant, Youth, Adult, and Senior, according to their current age. The breakpoints between adjacent groups are the first quartile, median, and third quartile of the stock age distribution in the particular month[18]. Within each of the four age groups, the stocks are further divided into four size groups, i.e., Tiny, Small, Medium, and Big, according to their current market capitalization. The doubly sorting procedure yields 16 roughly equal-size stock groups. For each of the 16 groups we construct an equally weighted portfolio. At the beginning of each month all these doubly sorted factor portfolios are liquidated and reconstructed to reflect the change in group members. The portfolio construction date in our study is the beginning of January 1977. All of the portfolios formed on age and size are rebuilt each month until the end of December 2016. Table 5 summarizes three performance measures of these 16 portfolios, namely the annualized return, the standard deviation, and the Sharpe ratio. The riskless rate used in the calculation of Sharpe ratios is downloaded from the Kenneth French website[19]. A comparison among these doubly sorted quartile portfolios reveals how each factor affects cross-sectional stock returns.

Table 5 displays two main features in the returns. It confirms the existence of both an age effect and a size effect. We focus initially on the age effect since the size effect is already well documented in the literature. The age effect is quite pronounced but it is not uniformly monotonic across all age groups. The average returns generally increase as the age group moves through the first three age groups. There is a slight decrease in returns as we move from the third age group to the oldest age group for three of the four size groups. An exception occurs for the third largest

[18]This grouping method leads to a dynamic group membership. Size of different groups may be different because there may be multiple stocks at the breakpoint ages.

[19]`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`.

Table 5: Performance of sixteen doubly sorted equally weighted portfolios formed on age and size. Starting from January 1977, at the beginning of each month each available stock is assigned to one of sixteen factor portfolios based on its cross-sectional ranking of age and size. The breakpoints between adjacent age/size groups are the first quartile, median, and third quartile of the age/size distribution. the returns on equally weighted portfolio of all stocks in each factor portfolio are calculated. All factor portfolios are held until December 2016. The average annualized return, standard deviation, and Sharpe ratio of all sixteen portfolios are reported.

Age Group	Size Group	Return	Std Dev	Sharpe
Infant	Tiny	13.61%	26.75%	0.34
	Small	8.89%	23.52%	0.18
	Medium	10.66%	24.80%	0.25
	Big	12.66%	23.97%	0.34
Youth	Tiny	17.58%	24.08%	0.54
	Small	11.05%	21.21%	0.31
	Medium	12.24%	21.21%	0.36
	Big	12.17%	20.22%	0.38
Adult	Tiny	21.20%	23.32%	0.71
	Small	16.47%	21.32%	0.56
	Medium	15.81%	20.27%	0.55
	Big	14.23%	17.97%	0.54
Senior	Tiny	20.43%	22.92%	0.69
	Small	14.62%	19.62%	0.51
	Medium	16.12%	18.80%	0.61
	Big	13.79%	15.38%	0.60

size group which we have denoted as the Medium sized group. However we notice that with the size group fixed, the returns of the two oldest age groups (Adult and Senior) are very close to each other, and each is much higher than the returns of the two youngest age groups. The closeness in performance between the oldest two age groups explains why the rebalanced portfolios outperform the bootstrapped ones notwithstanding the apparent non-monotonicity of the age effect. It is also worth pointing out that the size factor is not monotone either. Within all of the age groups, the size group Tiny always outperforms the size group Big, yet the performance does not deteriorate monotonically with size. The important implication of our findings is that stocks that are both mature and small tend to outperform the market. These two features, although seemingly having opposite effects, when appearing together can lead to profitable returns.

The Sharpe ratio results in Table 5 provide an even more striking demonstration of the age effect. For each size group the Sharpe ratio of the oldest age group is typically double the Sharpe ratio of the youngest age group. For the first three size groups the Sharpe ratio of the most senior age group is at least twice the Sharpe ratio of the youngest age group. For the largest size group the Sharpe ratio of the oldest age group is 1.8 times the Sharpe ratio of the youngest age group. On the other hand if we hold age fixed the Sharpe ratios of the different size portfolios are much less disperse.

Table 6: Returns of sixteen doubly sorted equally weighted portfolios: by decade. Starting from January 1977, at the beginning of each month each available stock is assigned to one of sixteen factor portfolios based on its cross-sectional ranking of age and size. The breakpoints between adjacent age/size groups are the first quartile, median, and third quartile of the age/size distribution. The average return on an equally weighted portfolio of all stocks in each factor portfolio is calculated. All factor portfolios are held until December 2016. Annualized returns over the four non-overlapping decades of each factor portfolio are reported.

	January 1977 - December 1986					January 1987 - December 1996			
	Tiny	Small	Medium	Big		Tiny	Small	Medium	Big
Infant	23.40%	17.71%	16.86%	17.64%	Infant	11.68%	4.22%	10.59%	16.50%
Youth	22.38%	13.66%	14.52%	14.35%	Youth	21.38%	7.79%	10.96%	13.54%
Adult	26.00%	25.84%	22.95%	30.12%	Adult	26.17%	11.04%	12.29%	15.64%
Senior	30.12%	21.80%	22.31%	17.23%	Senior	20.72%	10.35%	13.50%	14.94%
	January 1997 - December 2006					January 2007 - December 2016			
	Tiny	Small	Medium	Big		Tiny	Small	Medium	Big
Infant	18.70%	10.13%	8.70%	8.90%	Infant	0.65%	3.48%	6.49%	7.61%
Youth	21.88%	15.75%	14.89%	11.47%	Youth	4.68%	7.00%	8.58%	9.30%
Adult	23.56%	18.92%	16.35%	12.52%	Adult	9.08%	10.09%	11.65%	9.11%
Senior	20.41%	15.14%	17.01%	13.32%	Senior	10.45%	11.16%	11.65%	9.67%

Since the age effect is the key finding in our paper, we explore its robustness across different periods. Table 6 contains a more detailed decade-by-decade breakdown of the doubly sorted portfolios. This breakdown shows clearly that the age effect is both strong and persistent across all four decades. For all decades the returns are generally increasing in age. The age group Youth has in general higher returns than age group Infant with the average difference being 1.80% over all size groups and decades. In turn age group Adult has in general higher returns than age group Youth with the average difference being 4.33% over all size groups and decades. Age groups Adult and Senior represent the two oldest groups. The difference between the two oldest age groups is somewhat lower and negative. Over all the 16 combinations, the average return for age group Senior is lower than the average return for age group Adult. The average difference is 1.35% per annum which is small relative to the other differences. These results are consistent with our earlier regression results in Table 4.

We obtain a more compelling demonstration of the impact of age when we combine the two youngest age groups by taking their average and the two oldest age groups in the same way. The group containing the two youngest age groups is labelled Junior and the group containing the two oldest age groups is labelled Senior. The left hand side of Table 7 compares the returns on these age sorted portfolios over four size groups for each of the four decades. Differences between the return of age group Senior and return of age group Junior are reported as SMJ which is a short

Table 7: Analysis of portfolios formed on age and size: for each decade. The sixteen factor portfolios are re-organized into eight by combining the youngest (smallest) two age (size) groups and the oldest (biggest) two age (size) groups with equal weights for each of the four size (age) groups. Annualized returns of the merged portfolios over each decade between January 1977 and December 2016 are reported. Confounding the size group, the average return differences between the two combined age groups are reported as SMJ. Confounding the age group, the return differences between the two merged size groups are reported as SMB.

January 1977 - December 1986										
Age group	Tiny	Small	Medium	Big	Average	Size Group	Infant	Youth	Adult	Senior	Average
Infant +Youth	22.89%	15.69%	15.69%	15.99%		Tiny+Small	20.56%	18.02%	25.92%	25.96%	
Adult+Senior	28.06%	23.82%	22.63%	23.68%		Medium+Big	17.25%	14.44%	26.54%	19.77%	
SMJ	5.17%	8.13%	6.94%	7.68%	6.98%	SMB	3.31%	3.58%	-0.62%	6.19%	3.12%
January 1987 - December 1996											
Age group	Tiny	Small	Medium	Big	Average	Size Group	Infant	Youth	Adult	Senior	Average
Infant +Youth	16.53%	6.01%	10.78%	15.02%		Tiny+Small	7.95%	14.59%	18.61%	15.54%	
Adult+Senior	23.45%	10.70%	12.90%	15.29%		Medium+Big	13.55%	12.25%	13.97%	14.22%	
SMJ	6.92%	4.69%	2.12%	0.27%	3.50%	SMB	-5.59%	2.34%	4.64%	1.32%	0.67%
January 1997 - December 2006											
Age group	Tiny	Small	Medium	Big	Average	Size Group	Infant	Youth	Adult	Senior	Average
Infant +Youth	20.29%	12.94%	11.80%	10.18%		Tiny+Small	14.42%	18.82%	21.24%	17.78%	
Adult+Senior	21.99%	17.03%	16.68%	12.92%		Medium+Big	8.80%	13.18%	14.44%	15.16%	
SMJ	1.69%	4.09%	4.88%	2.74%	3.35%	SMB	5.62%	5.63%	6.80%	2.62%	5.17%
January 2007 - December 2016											
Age group	Tiny	Small	Medium	Big	Average	Size Group	Infant	Youth	Adult	Senior	Average
infant +Youth	2.67%	5.24%	7.54%	8.46%		Tiny+Small	2.07%	5.84%	9.59%	10.81%	
Adult+Senior	9.76%	10.63%	11.65%	9.39%		Medium+Big	7.05%	8.94%	10.38%	10.66%	
SMJ	7.10%	5.39%	4.11%	0.94%	4.38%	SMB	-4.99%	-3.10%	-0.79%	0.14%	-2.19%
Average					4.55%						1.69%

notation for “Senior minus Junior". The most striking result from Table 7 is that the return on the Senior portfolios exceeds the return on the Junior portfolios for each size group within each decade. Furthermore the average difference in these portfolio returns over all sixteen combinations is 4.55% which is very significant. According to the same set of results presented in Appendix A for the period before 1977, the SMJ is also positive for each size group within each decade, but the average return difference is only 1.79%.

It is instructive to conduct a similar grouping based on size to compare the relative importance of the age effect and the size effect. We group the two smallest size groups together (Tiny+Small) and the two largest size groups together (Medium+Big) and calculate the returns on these size sorted portfolios over all four age groups for each of the four decades. Differences between return of these two coarser size groups are reported as SMB which is a short notation for “Small minus Big" in the right half of Table 7. It should be clarified that SMB represents the return difference between the smaller half (Tiny+Small) and the bigger half (Medium+Big) rather than that between the size groups Small and Big. It turns out that the average SMB return over the sixteen age-decade combinations is 1.69% which is much lower then the average SMJ return. In fact the magnitude of the age effect in this framework based on these calculations is 2.7 times as large as the size effect. Over the early years before 1977 the average SMB return is 5.10% (see Table 7(b) in Appendix A). Furthermore, over the most recent decade, the average SMB across the four age groups is negative. The disappearance of the size effect over more recent periods has been documented in the literature (see Horowitz et al. (2000) and Alquist et al. (2018)) and our findings are consistent with this evidence.

3.5. Effect of Aging on Size Distribution

In this subsection we examine the relation between age and size. We use longitudinal data techniques to study this problem. Specifically we identify cohorts of stocks and follow their evolution over time. The use of longitudinal data has the advantage of reducing heterogeneity from two sources typically associated with the use of cross-sectional data, i.e., they include stocks that were created at different times and subject to different selection processes. We track three cohorts of stocks over a seven-year period in order to study the effect of aging on stock size distribution over time. The first cohort of stocks were issued in 1984 and were followed until 1991; the second cohort issued in

1994 and tracked until 2001; and the last one issued in 2004 and tracked until 2011. The effect of aging is evaluated by comparing the market capitalization distributions of the set of stocks in each issuance year and the corresponding end-of-tracking year. In the first cohort, for example, from the 632 stocks identified as new in 1984, only 240 were still active in 1991. This leads to three different distributions of interest. The first one is the distribution of all entrants in 1984; the second one, the distribution of survivors in 1991; and the third one, the size distribution in 1984 of those stocks that survived until 1991.

Table 8 reports the median, mean, standard deviation, and quartile coefficient of dispersion of nine distributions of interest (each of the three cohorts is associated with three distributions) mentioned in the previous paragraph. The quartile coefficient of dispersion is a robust measure of dispersion and is defined as $(Q_3 - Q_1)/(Q_3 + Q_1)$, where Q_1 and Q_3 are the first and the third quartiles of each dataset. Figure 3 presents the kernel density functions of the nine sets of log-transformed market capitalization, each panel corresponding to one of the three cohorts. Comparing the solid curves (All, 1984/1994/2004) with the dashed curves (Survivors, 1984/1994/2004), we find that stocks that survived through the seven-year tracking period tend to have larger market capitalization going back in the year of issuance. Comparing the dashed curves with the dotted curves (Survivors, 1991/2001/2011), we observe that the market capitalization distribution moves to the right and becomes more dispersed as stocks become mature. The increasing dispersion in market capitalization distribution is also evident according to the last column in Table 8. Note that the graphs in Figure 3 use the log of size and the actual dispersion in dollar terms is considerably greater. Therefore we conclude that the effect of aging on market capitalization distribution is two-fold. The market capitalization on average becomes larger as time passes. However the market capitalization distribution also becomes significantly more disperse over time as well. Therefore it is possible for a stock to be both senior in terms of age and small in terms of market capitalization.

3.6. Return Skewness within Age and Size Decile Groups

In this subsection we examine the skewness of the stock return distribution for different age and size deciles and discuss the implications for our results. Bessembinder (2018) studied the distributions of monthly buy-and-hold stock returns in different size decile groups. According to his Table 3A the *median* return in each size group increases (non-strictly) monotonically as we move

Table 8: Summary of three market capitalization distributions for three cohorts of stocks. Market capitalization of three cohorts of stocks, namely those entered the CRSP database in 1984, 1994, and 2004, are tracked over a seven-year period after their entrance. Each cohort is associated with three market capitalization distributions of interest. The first one is the distribution among all entrants at the beginning of the tracking period; the second one is the distribution among survivors at the end of the tracking period; the third one is the distribution at the beginning of the tracking period of those stocks that survived until the end of the tracking period. We report the mean, median, standard deviation, and quartile coefficient of dispersion of each distribution for each cohort.

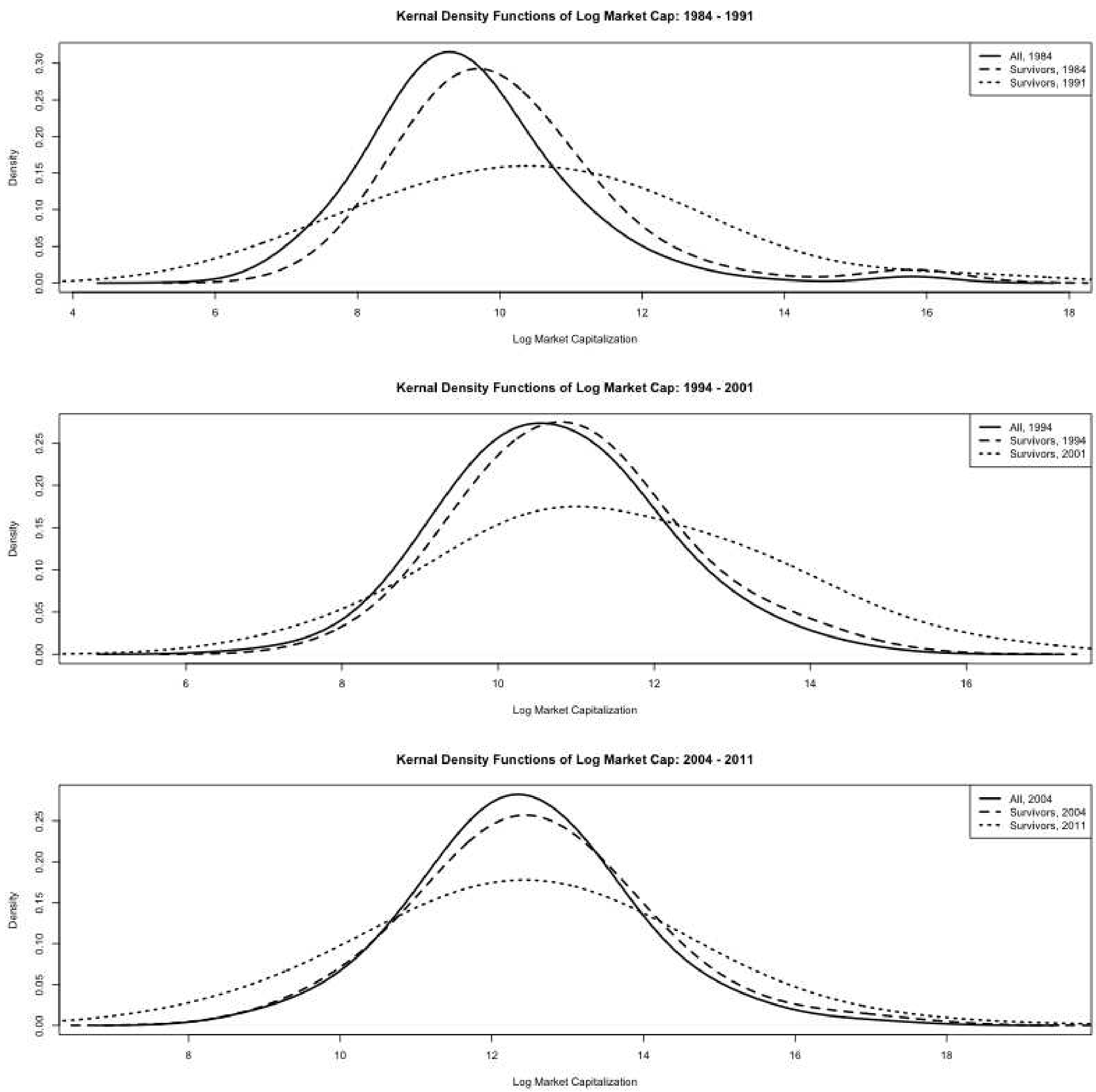
	Median	Mean	Std Dev.	Dispersion
New entrants in 1984, in 1984	12,159	125,383	834,804	0.63
Survivors through 1991, in 1984	18,755	291,655	1,338,064	0.66
Survivors through 1991, in 1991	31,149	717,302	3,200,902	0.89
New entrants in 1994, in 1994	42,513	124,181	292,084	0.71
Survivors through 2001, in 1994	55,147	163,698	385,767	0.69
Survivors through 2001, in 2001	78,300	726,510	2,529,198	0.88
New entrants in 2004, in 2004	232,365	799,082	2,648,895	0.67
Survivors through 2011, in 2004	257,991	1,104,531	3,678,639	0.70
Survivors through 2011, in 2011	231,092	2,263,304	13,310,190	0.86

from small to big groups. However the *mean* return in each size group does not show a clear pattern except that the smallest group yields a mean return much higher than any other size group. This observation implies that the observed "small firm effect" is to a large extent a result of the extreme positive skewness in the smallest 10% of firms. Another important implication of this finding is that heterogeneity in the smallest 10% of firms in terms of return is unmatchable by that in any other size group. Since stock age is a key feature in our study, it is also of interest to explore the pattern in the within-group skewness when stocks are grouped by age.

The leftmost columns of Table 9 report the mean, median, and skewness of monthly buy-and-hold stock returns grouped by size and the rightmost columns report the the set of statistics when the stocks are grouped by size. For each month during the period from January 1977 to December 2016 each available stock is assigned to a size (age) decile group based on its market capitalization (age since issuance) at the end of the last month. A group number closer to 10 means a more senior group or a larger-cap group. In this way each stock-month combination is tagged with an age group number and a size group number. Each decile group contains roughly 10% of the stock-month observations. Each stock-month observation is associated with a buy-and-hold return[20] of the particular stock over the particular month. The reported statistics are calculated based on all annualized monthly returns that belong to each decile group. The first four columns in Table 9 report similar information to that reported in Table 3A - Panel A of Bessembinder (2018). The

[20]Any delisting return is included into the calculation.

Figure 3: Kernal Density Functions of Log Market Capitalization. Market capitalization of three cohorts of stocks, namely those entered the CRSP database in 1984, 1994, and 2004, are tracked over a seven-year period after their entrance. Each cohort is associated with three market capitalization distributions of interest. The first one is the distribution among all entrants at the beginning of the tracking period; the second one is the distribution among survivors at the end of the tracking period; the third one is the distribution at the beginning of the tracking period of those stocks that survived until the end of the tracking period. Each panel in the figure corresponds to an indicated cohort, and the kernel density functions of the three distributions of log market capitalization are shown.



minor discrepancy between the two sets of results appears to have originated from the difference in the data used[21].

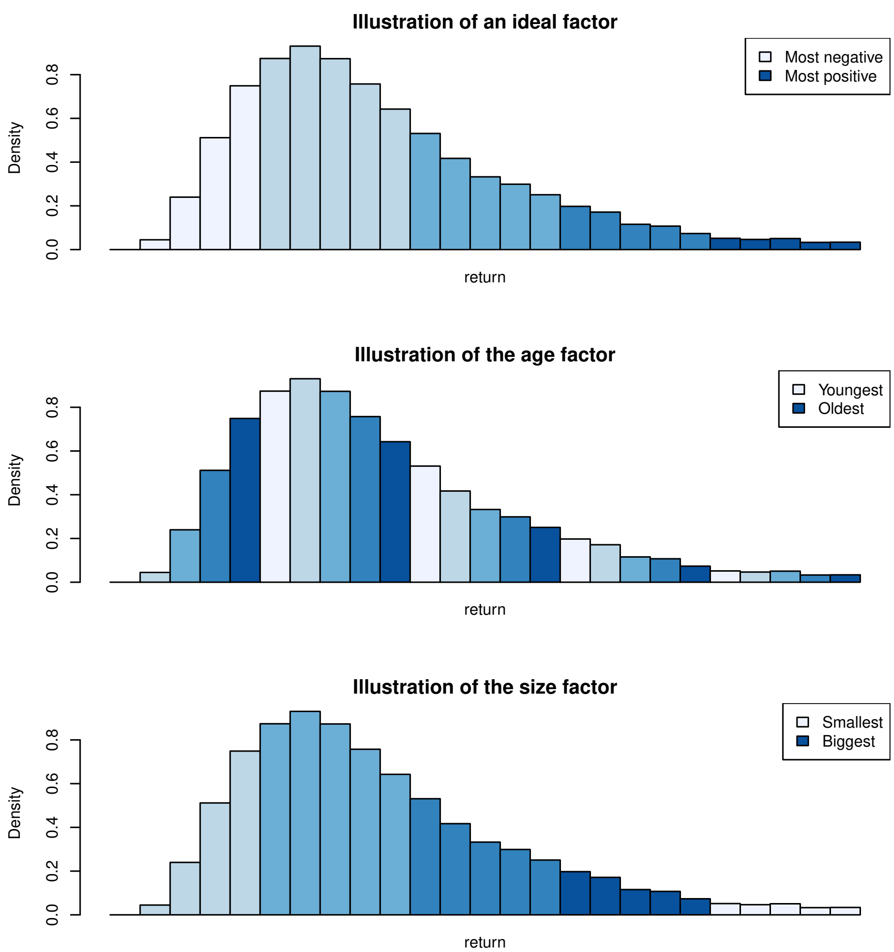
Despite the difference in source data the pattern in return skewness across different decile groups shown in Table 9 is similar to that reported in Bessembinder's study: extremely positive skewness is observed in the smallest decile group, and there is a decreasing trend in the within-group skewness as we move from small to big size groups. The last four columns in Table 9 show the return statistics in different age decile groups. Compared with what we can observe from the within-group mean returns in different size groups, here we can see a more distinctly increasing trend in the within-group mean return as we move from young to senior age groups. However it is worth noting that the mean returns in the youngest and in the second youngest decile groups are reversely ordered compared with the general trend; this is also true in the comparison between the oldest and second oldest decile groups. A potential explanation for both observations can be found from the industrial organization literature. Fichman and Levinthal (1991) explain why firms face an initial "honeymoon" period in which they are buffered from a sudden exit by their initial stock of resources. Barron et al. (1994) argue that old firms are prone to suffer from a "liability of obsolescence" and also a "liability of senescence". This effect will be discussed in more detail in Section 4.

There is a marked difference in the skewness patterns in the size decile and in the age decile. For the size decile the return skewness decreases as we move in the direction of increasing firm size with the smallest decile having a skewness of 6.08 and the largest decile having a skewness of just 0.41. For the age decile the return skewness is remarkably stable with an average value of 5.07 across all age groups.

If we assign stocks to decile groups based on an ideal (hypothetical) factor, then the conditional return distribution in different groups should be clearly distinct. The overall return distribution is a mixture of ten distinct conditional distributions. In such a scenario we should expect a monotonic trend in the within-group mean and reduced (compared with skewness of unconditional return distribution) within-group skewness. Figure 4 shows a hypothetical unconditional return distribution and illustrates the effect of grouping according to different factors on the within-group (conditional) mean and skewness. The effect of grouping according to an ideal factor on within-group mean and

[21] It is important to point out that Bessembinder (2018) uses the period 1926-2016, while our calculations are based on the period 1977-2016.

Figure 4: All three panels show a same histogram of hypothetical stock returns. We intend to illustrate the effect of factor-based grouping on within-group mean and skewness. In each panel, different bins are colored differently to reflect the level of factor. A darker blue color represents a more favorable factor group in the top panel, a more senior age group in the middle panel, and a bigger size group in the bottom panel.



skewness is illustrated in the top panel of Figure 4. Here a darker color stands for a more favorable factor group. According to Table 9 neither factor appears to be that ideal. For the age factor monotonicity is roughly observed but the conditional distributions are quite skewed. The middle panel of Figure 4 illustrates such a possibility. Here a darker color represents a more senior age group. For the size factor the monotonicity is violated. However most within-group skewness is reduced compared with skewness of the unconditional distribution of 5.13. This effect is illustrated in the bottom panel of Figure 4. Our key results depend more on the monotonicity feature and are not affected much by the skewness feature since skewness can be diversified away in portfolios.

Table 9: Statistics of one-month buy-and-hold returns in different size and age decile groups. For each month during the period from January 1977 to December 2016, each available stock is assigned to a size (age) decile group based on its market capitalization (age since issuance) at the end of the last month. A group number closer to 10 means a more senior group or a larger-cap group. Each stock-month observation is associated with a buy-and-hold return of the particular stock over the particular month. The reported statistics are calculated based on all annualized monthly returns that belong to each decile group.

Size Group	Mean	Median	Skew	Age Group	Mean	Median	Skew
1	0.2883	0.0000	6.0821	1	0.0988	0.0000	2.8856
2	0.1008	0.0000	3.3851	2	0.0825	0.0000	5.5759
3	0.0981	0.0000	3.1559	3	0.1131	0.0000	5.8414
4	0.1109	0.0000	3.5532	4	0.1543	0.0000	4.4354
5	0.1208	0.0000	2.4979	5	0.1254	0.0000	5.2705
6	0.1259	0.0000	1.8118	6	0.1766	0.0000	5.8495
7	0.1313	0.0593	1.3073	7	0.1589	0.0000	4.2460
8	0.1384	0.0945	1.2735	8	0.1624	0.0027	5.0487
9	0.1344	0.1126	0.7520	9	0.1575	0.0550	4.5711
10	0.1249	0.1173	0.4136	10	0.1400	0.0938	6.9448

4. Understanding Stock Age Effects

Many economic theories tend to treat firm size and firm age as capturing the same fundamental information. For example Greiner (1989) presents his “stages of growth" model of organizational change in growing firms, in which size is linearly related to age. Other scholars have nonetheless made specific predictions about how firm performance changes with age. In this section we review these theoretical predictions in terms of three categories: selection effects, learning-by-doing effects, and inertia effects.

It is worth pointing out that the stock age used in our study is calculated based on the date on which a stock appears in the CRSP database for the first time. The three effects to be reviewed are from the industrial organization literature and characterize firms' earning ability in different stages. There is a gap between this stock age and the actual firm age. However we find the three effects capable of explaining the stock age effects we have discovered and therefore briefly discuss them since they may help explain some of the key findings in this paper.

4.1. Selection Effects

Selection effects arise when selection pressures progressively eliminate the weakest firms, and result in an increase in the average productivity level of surviving firms, even if the productivity levels of individual firms do not change with age. This situation corresponds to an influential model in Industrial Organization in Economics proposed by Jovanovic (1982). According to this model, firms are born with fixed productivity levels, and learn about their productivity levels as time passes. In this model, low productivity firms are observed to exit, while high productivity firms remain in business. As a result the average productivity of the cohort increases with the cohort ages, even if the productivity levels of individual firms remain constant over time. Therefore selection effects provide a potential explanation for the age effect uncovered in our study.

4.2. Learning-by-doing Effects

Learning-by-doing effects occur when firms increase their productivity as they learn about more productive production techniques and incorporate these improvements in their production routines. (See Ulen and Newman (1998) for a survey of the learning-by-doing concept.) Learning-by-doing effects can be expected to be particularly relevant for young firms. According to Garnsey (1998): "New firms are hampered by their need to make search processes a prelude to every new problem they encounter. As learning occurs benefits can be obtained from the introduction of a repertoire of problem-solving procedures ... eliminating open search from the problem-solving response greatly reduces the labor and time required to address recurrent problems."

Furthermore older firms may benefit from their greater business experience, established contacts with customers, and easier access to resources. For example, Sørensen and Stuart (2000) point out that entrepreneurs often lack detailed information about their jobs, firms and even the environments until they are active in the market. After a firm's creation, an intense learning process starts and contributes to the firm's growth and survival in the long-term. Also Chang et al. (2002) provide evidence on the existence of microeconomic "learning-by-doing" effects with positive effects on the aggregate output. The learning-by-doing effects may provide an explanation to the positive association between stock age and expected returns reported in this paper.

4.3. Inertia Effects

As firms get older they might become less productive if they become increasingly inert and inflexible. Barron et al. (1994) argue that old firms are prone to suffer from a "liability of obsolescence" (because they do not fit in well to the changing business environment) and also a "liability of senescence" (according to which they become ossified by accumulated rules, routines, and organizational structures). At a theoretical level Hannan and Freeman (1984) justify inertia effects as "an outcome of an ecological evolutionary process". The idea is that firms are not able to change as fast as their environment would dictate. Firms with inertia effects can survive by adopting strategies such as the creation of new firms designed specifically to take advantage of new opportunities. However, if firms are not able to adapt to the changes in the business environment, new entrants will enter the industry. Accordingly it is the changes in the business environment, which favor some bundles of firm resources over others, that lead to differences in firm performance. Inertia effects provide a potential explanation to the non-monotonicity in the age effect particularly in the most senior age groups documented in our study.

5. Conclusion

We have documented a persistent performance gap between the equally weighted rebalanced portfolio and the equally weighted bootstrapped portfolio during the last forty years. Both portfolios involve randomly picked stocks. In the bootstrapped portfolio N stocks are randomly selected and assigned equal weight at the beginning of each month. In the rebalanced portfolio N stocks are randomly chosen at inception and the portfolio is rebalanced to be equally weighted at the beginning of each month. If a stock is delisted in the rebalanced portflio it is replaced with a randomly picked stock from the available universe.

The key feature that differentiates these two types of portfolios is the age distribution of the portfolio constituents. There is considerable turnover in the composition of the stock universe. During the last forty years, the average annual rate of new listings was around ten percent and the average annual rate of withdrawals was about the same magnitude. This means that the bootstrapped portfolio will in general include a sizable proportion of younger stocks. In contrast any stock that is included in the rebalanced remains there until it is delisted. As a consequence the rebalanced portfolio contains more senior stocks.

We analyzed the effect of stock age on the cross-sectional stock return and showed that the difference in age distribution is the primary reason behind the performance gap between the bootstrapped and rebalanced portfolios. Our regression results show that stock age has a significantly positive effect on stock return during the last four decades. This finding supports our conjecture that age explains the performance gap. We doubly sort portfolios by age and by size to assess the effect of age particularly when the size factor is controlled for. The age effect is observed in all of the four size groups. In addition a decade-by-decade breakdown of the returns on the sixteen factor portfolios indicates that the age effect dominates the size effect over the period from 1977 to 2016. It is robust when we consider the decade by decade results. Prior to 1977 the age effect was found to be weaker than it was after 1977. In addition the age effect while present in the earlier time period was dominated by the size effect.

The size effect is a well-known asset pricing anomaly. However the age effect that we have uncovered here appears to be in conflict with it, since age and size are generally positively correlated measures. To resolve this puzzle we monitored the size of three cohorts of stocks and analyzed the effect of aging on the market capitalization distribution within each cohort. The results suggest that the average market capitalization of a cohort of stocks increases as the stocks are aging. However the dispersion in market capitalization rises much more dramatically. So it is not uncommon for a stock to remain or to become a small cap as it ages. In an experiment where stocks are divided into decile groups according to age or size, we find that when the within-group median return is used as the performance measure, the direction of the size effect and that of the age effect are consistent with our intuition. Larger-size groups and more senior groups outperform; however the extremely positive skewness in the returns of the smallest 10% stocks causes the mean return of the smallest stocks to be much higher than the median return.

References

Alquist, R., Israel, R., and Moskowitz, T. J. (2018). Fact, fiction, and the size effect. *The Journal of Portfolio Management*, 2018(1):082.

Barron, D. N., West, E., and Hannan, M. T. (1994). A time to grow and a time to die: Growth and mortality of credit unions in new york city, 1914-1990. *American Journal of Sociology*, 100(2):381–421.

Barry, C. B. and Brown, S. J. (1984). Differential information and the small firm effect. *Journal of Financial Economics*, 13(2):283–294.

Benartzi, S. and Thaler, R. H. (2001). Naive diversification strategies in defined contribution saving plans. *American Economic Review*, 91(1):79–98.

Bessembinder, H. (2018). Do stocks outperform treasury bills? *Journal of Financial Economics*, 129(3):440–457.

Brennan, M. J. and Torous, W. N. (1999). Individual decision making and investor welfare. *Economic Notes*, 28(2):119–143.

Chang, Y., Gomes, J. F., and Schorfheide, F. (2002). Learning-by-doing as a propagation mechanism. *American Economic Review*, 92(5):1498–1520.

DeMiguel, V., Garlappi, L., and Uppal, R. (2009). Optimal versus naive diversification: How inefficient is the 1/n portfolio strategy? *The Review of Financial Studies*, 22(5):1915–1953.

Doidge, C., Karolyi, G. A., and Stulz, R. M. (2017). The U.S. listing gap. *Journal of Financial Economics*, 123(3):464–487.

Fichman, M. and Levinthal, D. A. (1991). Honeymoons and the liability of adolescence: A new perspective on duration dependence in social and organizational relationships. *Academy of Management Review*, 16(2):442–468.

Garnsey, E. (1998). A theory of the early growth of the firm. *Industrial and Corporate Change*, 7(3):523–556.

Greiner, L. E. (1989). Evolution and revolution as organizations grow. In *Readings in strategic management*, pages 373–387. Springer.

Hannan, M. T. and Freeman, J. (1984). Structural inertia and organizational change. *American Sociological Review*, pages 149–164.

Heaton, J., Polson, N., and Witte, J. H. (2017). Why indexing works. *Applied Stochastic Models in Business and Industry*, 33(6):690–693.

Horowitz, J. L., Loughran, T., and Savin, N. (2000). The disappearing size effect. *Research in Economics*, 54(1):83–100.

Jovanovic, B. (1982). Selection and the evolution of industry. *Econometrica: Journal of the Econometric Society*, pages 649–670.

Lin, X., Palazzo, B., and Yang, F. (2018). The risks of old capital age: Asset pricing implications of technology adoption. *Working paper, SSRN 2535409*.

Loderer, C. F. and Waelchli, U. (2010). Firm age and performance. *Available at SSRN 1342248*.

Loughran, T. and Ritter, J. R. (1995). The new issues puzzle. *The Journal of Finance*, 50(1):23–51.

Markowitz, H. (1952). Portfolio selection. *The Journal of Finance*, 7(1):77–91.

Plyakha, Y., Uppal, R., and Vilkov, G. (2015). Why do equal-weighted portfolios outperform value-weighted portfolios? *Working paper, SSRN 2724535*.

Ritter, J. R. (1991). The long-run performance of initial public offerings. *The Journal of Finance*, 46(1):3–27.

Shumway, T. (1997). The delisting bias in CRSP data. *The Journal of Finance*, 52(1):327–340.

Sørensen, J. B. and Stuart, T. E. (2000). Aging, obsolescence, and organizational innovation. *Administrative Science Quarterly*, 45(1):81–112.

Ulen, T. S. and Newman, P. (1998). *The New Palgrave Dictionary of Economics and the Law*. Palgrave Macmillan UK.

Van Dijk, M. A. (2011). Is size dead? a review of the size effect in equity returns. *Journal of Banking & Finance*, 35(12):3263–3274.

Appendix A. Main Results Based on Data from July 1926 to December 1976

Figure A.1 presents the profile of stock population over the period from 1927 to 1976. A comparison between Figure A.1 and Figure 1 further justifies the need of analyzing the earlier 50-year period and the more recent 40-year period separately. There is a marked difference in the activity level of the market, in terms of number of listed stocks and frequency of enters and exits, between the two periods. Further the CRSP database went through drastic changes in 1962 due to the inclusion of AMEX stocks and in 1972 due to the inclusion of Nasdaq stocks. The stock population is much more stable in the more recent 40 years.

Figure A.1: Population of stocks in the CRSP database: existing stocks and new listings. The figure shows the change in the stock population in the CRSP database from 1927 to 1976. The red portion in each bar represents the number of stocks that enter the universe in the indicated calendar year. The blue portion represents the number of stocks that have existed in the universe by the beginning of the indicated calendar year.

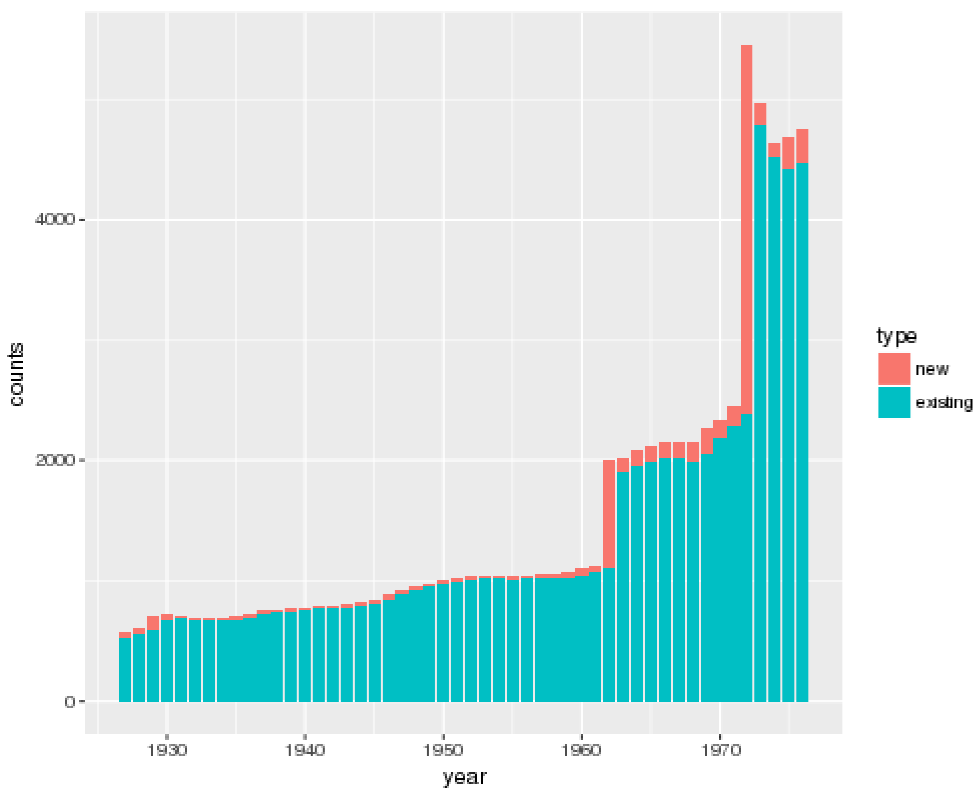
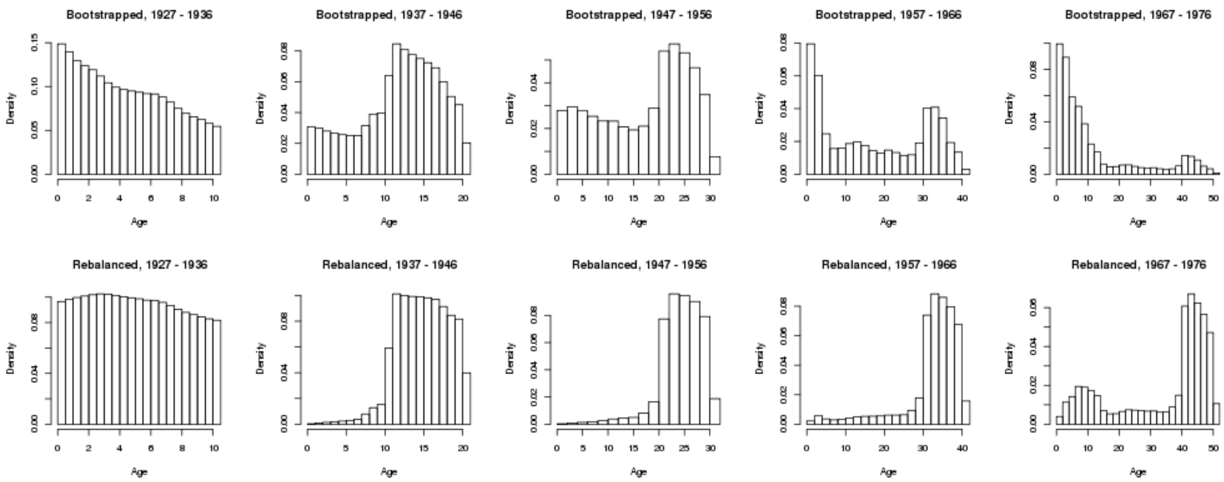


Figure A.2 and Table 3(b) contain similar information as Figure 2 and Table 3 except that the time range of the data used here is from July 1926 to December 1976. As in Table 3 we can see a clear increasing trend in the average age in the rebalanced portfolio from decade to decade. In addition there is an increasing age gap between the bootstrapped and rebalanced portfolios. This is

Figure A.2: Age distribution in 100-stock bootstrapped and rebalanced portfolios over different time periods. The distributions are based on age of components of one thousand 100-stock bootstrapped and rebalanced portfolios held over the period from July 1926 to December 1976. We report the empirical age distribution in both portfolios over each decade in the horizon of about 50 years.



unsurprising because the age gap comes from the mechanism of portfolio construction and should not depend on the data period. As long as there are listings and delistings in the market we should be able to see such an age gap. The average age in bootstrapped portfolios can be viewed as a proxy for the average age of all available stocks. According to Table 3(b), the average age of bootstrapped portfolios varies substantially from decade to decade, compared with the much more stable average age reported in Table 3. The volatile average age across time again implies that the stock universe prior to 1977 is highly unstable.

Table 3(b): Average age of 100-stock bootstrapped and rebalanced portfolios over different time periods. The average ages are calculated based on age of components of one thousand 100-stock bootstrapped and rebalanced portfolios held over the period from July 1926 to December 1976. We report the average age in both portfolios over each decade in the horizon of around 50 years.

Portfolio Type	1926 - 1936	1937 - 1946	1947 - 1956	1957 - 1966	1967 - 1976
Bootstrapped	4.49	11.98	17.20	17.54	11.98
Rebalanced	5.16	15.25	24.80	32.31	31.38

Table 4(b) reports the estimation and hypothesis testing results of the time effects model in equation (1). The model is fitted with stock-month data from July 1939 to December 1976. We use July 1939 as the starting time because prior to this month, the first quartile, median, and third quartile of the cross-sectional age distribution are not three distinct numbers so that we have difficulty in assigning stocks to the four age groups.

Regression over the full dataset yields a significant and positive coefficient that represents the age effect. However, compared with the coefficient estimate of 0.001 in Table 4, the magnitude of the age effect over the earlier years is much smaller. In addition the t-test is much less significant according to the p-value. As for model fitting within each age group, the age effect is significant and positive in three out of the four age groups, but the p-values are relatively high compared with those in Table 4.

Table 4(b): Estimation and testing result of time effects model: July 1939 - December 1976. In the first row, the estimate, standard error, t-statistics, and the associated p-value for the overall time effects model are reported. In each month, the age group each stock belongs to is determined based on the cross-sectional ranking of the stock's current age. The breakpoints between age groups are the first quartile, median, and third quartile of the cross-sectional age distribution. All stock-month observations are divided into four age groups in this way. In each of the second to fifth rows, model fitting results for the indicated age group are reported.

Age Group	Estimate	Std. Error	t-value	p-value
All ages	0.0003	0.0001	2.5544	0.0106
Infant	0.0036	0.0017	2.1480	0.0317
Youth	-0.0025	0.0018	-1.3851	0.1660
Adult	0.0020	0.0009	2.3186	0.0204
Senior	0.0007	0.0003	2.1954	0.0281

Table 5(b) is a replicate of Table 5 with data over the earlier years. Again the starting time of the period over which portfolio returns are calculated is July 1939 because there are overlappings in the first quartile, median, and third quartile of the cross-sectional age distribution prior to this month. We can see how strong the small firm effect was during the earlier years in this table. Regardless of the age group the Tiny group always outperforms the other three size groups in terms of return. Compared with the notable size effect the age effect is less visible. Among the four Tiny portfolios the Youth-Tiny portfolio achieves the highest return, closely followed by Adult and Senior. Among the four Small portfolios or the four Medium portfolios the Senior age group always has the best return. Among the four Big portfolios the Adult group performs the best. Although the more senior age groups in general perform better the return difference between a more senior group and a less senior one is quite small compared with the return different between the Tiny portfolios and the remaining ones.

Table 6(b) provides a decade-by-decade breakdown of the performance of the sixteen doubly sorted factor portfolios discussed earlier. To better visualize the magnitude of the age and size effects as well as the change in the magnitude across time, we perform the same merging procedure as what we did in Section 3. The decade-by-decade returns on these merged groups, as well as return

Table 5(b): Performance of sixteen doubly sorted equally weighted portfolios formed on age and size. Starting from July 1939, at the beginning of each month each stock is assigned to one of sixteen factor portfolios based on its cross-sectional ranking of age and size. The breakpoints between adjacent age/size groups are the first quartile, median, and third quartile of the age/size distribution. All factor portfolios are held until December 1976. The annualized average return, standard deviation, and Sharpe ratio of all sixteen portfolios are reported.

Age Group	Size Group	Return	Std Dev	Sharpe
Infant	Tiny	18.91%	25.10%	0.65
	Small	12.42%	20.43%	0.48
	Medium	11.77%	18.50%	0.49
	Big	11.26%	15.79%	0.54
Youth	Tiny	21.79%	27.54%	0.69
	Small	15.06%	21.59%	0.57
	Medium	13.86%	18.61%	0.60
	Big	11.49%	16.14%	0.55
Adult	Tiny	20.21%	28.81%	0.61
	Small	14.17%	21.30%	0.54
	Medium	14.28%	19.24%	0.60
	Big	12.24%	15.60%	0.61
Senior	Tiny	20.52%	29.67%	0.60
	Small	15.98%	22.17%	0.60
	Medium	14.30%	19.38%	0.60
	Big	12.29%	15.37%	0.63

Table 6(b): Returns of sixteen doubly sorted equally weighted portfolios: by decade. Starting from July 1939, at the beginning of each month each stock is assigned to one of sixteen factor portfolios based on its cross-sectional ranking of age and size. The breakpoints between adjacent age/size groups are the first quartile, median, and third quartile of the age/size distribution. All factor portfolios are held until December 1976. Annualized average returns over the four non-overlapping decades of each factor portfolio are reported.

	Jul 1939 - Dec 1946					Jan 1947 - Dec 1956			
	Tiny	Small	Medium	Big		Tiny	Small	Medium	Big
Infant	37.66%	22.47%	18.76%	14.08%	Infant	14.74%	13.74%	14.63%	14.53%
Youth	40.90%	25.02%	19.69%	14.69%	Youth	13.74%	15.53%	15.59%	15.34%
Adult	36.30%	20.61%	18.99%	42.97%	Adult	14.16%	13.01%	15.12%	18.10%
Senior	42.97%	26.13%	20.06%	14.13%	Senior	14.28%	15.00%	14.62%	15.15%
	Jan 1957 - Dec 1966					Jan 1967 - Dec 1976			
	Tiny	Small	Medium	Big		Tiny	Small	Medium	Big
Infant	11.44%	9.70%	8.59%	9.79%	Infant	16.78%	6.42%	6.92%	7.35%
Youth	18.45%	12.16%	12.82%	10.37%	Youth	18.94%	10.15%	8.83%	6.36%
Adult	15.97%	13.11%	12.30%	10.56%	Adult	18.69%	11.65%	11.96%	7.04%
Senior	12.16%	12.39%	11.26%	10.24%	Senior	18.63%	13.08%	12.78%	10.11%

differences between merged groups, are summarized in Table 7(b). The SMJ return characterizes the magnitude of age effect, and the SMB return represents the size effect.

The average SMJ return over sixteen combinations of four size groups and four decades is 1.02%, much lower than the average SMJ return over the recent 40 years. The positive age effect despite its magnitude provides additional robustness to our main findings in the paper. In contrast the average SMB return over sixteen combinations of four age groups and four decades is 4.44%, much greater than that over the recent 40 years. The size effect clearly dominates the age effect in the earlier years.

It is comforting to see that in the decade from 1967 to 1976, the average SMJ across the four size groups is higher than that in the earlier three decades. This decade can be viewed as a period of transition after which the age effect becomes dominant. In addition, even in the earlier decades when the age effect is weak, the sign of SMJ is always positive. In contrast the size effect peaks with an average SMB of 11.09% in the earliest period from 1939 to 1946 and becomes much weaker or even negative in the subsequent decades. Combining the results in Table 7 and Table 7(b) we conclude that the size effect is vanishing and highly variable, while the age effect is gradually become more dominating and has been robust with respect to time period.

Table 7(b): Return difference between portfolios formed on age and size: by decade. The sixteen factor portfolios are re-organized into eight by merging the youngest (smallest) two age (size) groups and the oldest (biggest) two age (size) groups with equal weights for each of the four size (age) groups. Annualized returns of the merged portfolios over each decade between July 1939 and December 1976 are reported. Confounding the size group, the return differences between the two merged age groups are reported as SMJ. Confounding the age group, the return differences between the two merged size groups are reported as SMB.

					July 1939 - December 1946						
Age group	Tiny	Small	Medium	Big	Average	Size Group	Infant	Youth	Adult	Senior	Average
Infant+Youth	39.28%	23.75%	19.22%	14.38%		Tiny+Small	30.06%	32.96%	28.45%	34.55%	
Adult+Senior	39.63%	23.37%	19.53%	28.55%		Medium+Big	16.42%	17.19%	30.98%	17.10%	
SMJ	0.35%	-0.38%	0.30%	14.17%	3.61%	SMB	13.64%	15.78%	-2.53%	17.45%	11.09%
					January 1947 - December 1956						
Age group	Tiny	Small	Medium	Big	Average	Size Group	Infant	Youth	Adult	Senior	Average
Infant +Youth	14.24%	14.63%	15.11%	14.93%		Tiny+Small	14.24%	14.64%	13.58%	14.64%	
Adult+Senior	14.22%	14.00%	14.87%	16.63%		Medium+Big	14.58%	15.46%	16.61%	14.89%	
SMJ	-0.02%	-0.63%	-0.24%	1.70%	0.20%	SMB	-0.34%	-0.83%	-3.03%	-0.24%	-1.11%
					January 1957 - December 1966						
Age group	Tiny	Small	Medium	Big	Average	Size Group	Infant	Youth	Adult	Senior	Average
Infant+ Youth	14.94%	10.93%	10.71%	10.08%		Tiny+Small	10.57%	15.30%	14.54%	12.28%	
Adult+Senior	14.07%	12.75%	11.78%	10.40%		Medium+Big	9.19%	11.60%	11.43%	10.75%	
SMJ	-0.88%	1.82%	1.07%	0.32%	0.58%	SMB	1.38%	3.71%	3.11%	1.53%	2.43%
					January 1967 - December 1976						
Age group	Tiny	Small	Medium	Big	Average	Size Group	Infant	Youth	Adult	Senior	Average
Infant +Youth	17.86%	8.28%	7.88%	6.85%		Tiny+Small	11.60%	14.54%	15.17%	15.85%	
Adult+Senior	18.66%	12.36%	12.37%	8.58%		Medium+Big	7.14%	7.60%	9.50%	11.45%	
SMJ	0.80%	4.08%	4.49%	1.72%	2.77%	SMB	4.46%	6.95%	5.67%	4.41%	5.37%
Average					1.79%						4.44%

Appendix B. Portfolio Performance in Presence of Negative Serial Correlation

In this section we suppose that monthly stock returns have negative serial correlation and examine the effects of this return dynamic on the expected return of the bootstrapped and rebalanced portfolios. We assume an AR(1) model to reflect the negative serial correlation in stock returns. Let r_{it} denote the simple rate of return of stock i over the tth month. The following AR(1) model with negative values of ϕ_i is able to capture the negative serial correlation.

$$(r_{it} - \mu_i) = \phi_i(r_{i(t-1)} - \mu_i) + \epsilon_{it}, \tag{B.1}$$

where $|\phi_i| < 1$ and ϵ_{it}'s are i.i.d. normal residuals with mean 0 and variance σ_i^2. Next we consider a fixed stock universe case where there are no new listings and delistings at all since the portfolio construction time as well as a dynamic stock universe case where in each month there are an equal number of new listings and delistings. We show that in both cases the negative serial correlation does not lead to outperformance of rebalanced portfolios (RP) over bootstrapped portfolios (BP).

Case 1: Fixed universe

Consider a fixed stock universe (no listings and delistings) of M stocks and a holding period of T months. The log holding period returns of our N-stock bootstrapped and rebalanced portfolios are:

$$\sum_{t=1}^{T} \log\left\{1 + \frac{1}{N}\sum_{i=1}^{M} \mathbb{I}_{\{i \text{ in BP in } t\text{th month}\}} r_{it}\right\} \tag{B.2}$$

and

$$\sum_{t=1}^{T} \log\left\{1 + \frac{1}{N}\sum_{i=1}^{M} \mathbb{I}_{\{i \text{ in RP in } t\text{th month}\}} r_{it}\right\} \tag{B.3}$$

respectively, where $\mathbb{I}_{\{A\}}$ is an indicator function valued at 1 if event A occurs and 0 otherwise. To show that the dynamic of r_{it} does not lead to a difference in expected portfolio returns, it suffices to show that for each $t = 1, 2, \ldots, T$,

$$E\log\left\{1 + \frac{1}{N}\sum_{i=1}^{M} \mathbb{I}_{\{i \text{ in BP in } t\text{th month}\}} r_{it}\right\} = E\log\left\{1 + \frac{1}{N}\sum_{i=1}^{M} \mathbb{I}_{\{i \text{ in RP in } t\text{th month}\}} r_{it}\right\}. \tag{B.4}$$

Since the stock picking is completely random, the random variables $\sum_{i=1}^{M} \mathbb{I}_{\{i \text{ in BP in } t\text{th month}\}} r_{it}$ and $\sum_{i=1}^{M} \mathbb{I}_{\{i \text{ in RP in } t\text{th month}\}} r_{it}$ are identically distributed for $t = 1, 2, \ldots, T$. Therefore equation (B.4) must hold, which means that negative serial correlation in stock return time series does not lead to any difference in the expected portfolio returns. Actually what is affected by the stock return dynamic is the variance of the portfolios returns.

Case 2: Dynamic universe

Now we move to the dynamic stock universe case. We assume for the sake of simplicity that there are always M available stocks in the investment universe. We further assume that at the beginning of each month $K(> N)$ out the M stocks are delisted from the market and the same number of new stocks are immediately added to the stock universe. In addition in each month all available stocks have an equal probability of being delisted. Let $\mathcal{M}_t$ denote the set of available stocks at the beginning of the tth month. The log holding period returns of the two portfolios are:

$$\sum_{t=1}^{T} \log \left\{ 1 + \frac{1}{N} \sum_{i \in \mathcal{M}_t} \mathbb{I}_{\{i \text{ in BP in } t\text{th month}\}} r_{it} \right\} \tag{B.5}$$

and

$$\sum_{t=1}^{T} \log \left\{ 1 + \frac{1}{N} \sum_{i \in \mathcal{M}_t} \mathbb{I}_{\{i \text{ in RP in } t\text{th month}\}} r_{it} \right\} \tag{B.6}$$

respectively. If the rebalanced portfolio seeks to exploit the negative serial correlation in stock return time series, it should pick stocks in a way such that $E[\mathbb{I}_{\{i \text{ in RP in } t\text{th month}\}}]$ is higher for a lower $r_{i(t-1)}$ and vice versa.

We examine the probability that a stock from the available stock pool at the beginning of the tth month is a component of the rebalanced portfolio over the tth month, i.e. $E[\mathbb{I}_{\{i \text{ in RP in } t\text{th month}\}}]$ for some $i \in \mathcal{M}_t$. If the stock i has been in the market before the portfolio construction date[22], i.e.

[22]This implies that the stock i is in the investment universe in each period up to the tth.

$i \in \bigcap_{k=1}^{t} \mathcal{M}_k$,

$$\begin{aligned}
&E\left[\mathbb{I}_{\{i \text{ in RP in } t\text{th month}\}} \middle| i \in \bigcap_{k=1}^{t} \mathcal{M}_k\right] \\
&= 1 - \Pr\{i \text{ not in RP in } t\text{th month}\} \\
&= 1 - \Pr\{i \text{ not in RP in 1st month}\} \\
&\quad \times \Pr\{i \text{ not in RP in 2nd month} | i \text{ not in RP in 1st month}\} \times \dots \\
&\quad \times \Pr\{i \text{ not in RP in } t\text{th month} | i \text{ not in RP in } (t-1)\text{th month}\} \\
&= 1 - \frac{M-N}{M} p^{t-1},
\end{aligned}$$

where

$$p = \sum_{k=0}^{N} \frac{\binom{K}{k}\binom{M-1-K}{N-k}}{\binom{M-1}{N}} \frac{M-N}{M-N+k}. \tag{B.7}$$

Note that p is the conditional probability that a stock is not included in the rebalanced portfolio in the current month given that it was not in the rebalanced portfolio in the previous month. The k involved in the expression for p represents the number of delisted stocks within the rebalanced portfolio.

If the stock i first appears in the investment universe at the beginning of the sth month ($s > 1$), i.e., $i \notin \bigcup_{k=1}^{s-1} \mathcal{M}_k$ and $i \in \bigcap_{k=s}^{t} \mathcal{M}_k$,

$$\begin{aligned}
&E\left[\mathbb{I}_{\{i \text{ in RP in } t\text{th month}\}} \middle| i \notin \bigcup_{k=1}^{s-1} \mathcal{M}_k, i \in \bigcap_{k=s}^{t} \mathcal{M}_k\right] \\
&= 1 - \Pr\{i \text{ not in RP in } t\text{th month}\} \\
&= 1 - \Pr\{i \text{ not in RP in } s\text{th month} | i \text{ not in RP in } (s-1)\text{st month}\} \times \dots \\
&\quad \times \Pr\{i \text{ not in RP in } t\text{th month} | i \text{ not in RP in } (t-1)\text{th month}\} \\
&= 1 - p^{t-s+1}
\end{aligned}$$

where p is given in equation (B.7).

In summary we have

$$E[\mathbb{I}_{\{i \text{ in RP in } t\text{th month}\}}|i \text{ was listed at } s] = \begin{cases} 1 - \frac{M-N}{M}p^{t-1} & \text{, if } s = 1 \\ 1 - p^{t-s+1} & \text{, if } 1 < s \leq t \end{cases} . \tag{B.8}$$

It is easy to check that $\frac{M-N}{M} < p$. Therefore $1 - \frac{M-N}{M}p^{t-1} > 1 - p^t$ and $E[\mathbb{I}_{\{i \text{ in RP in } t\text{th month}\}}]$ is decreasing in s (increasing in age of stock i). In contrast $E[\mathbb{I}_{\{i \text{ in BP in } t\text{th month}\}}]$ is equal to $\frac{N}{M}$ for $\forall i \in \mathcal{M}_t$. This leads us to conclude that the difference in stock picking scheme between the bootstrapped and rebalanced portfolios only comes through the age factor.