Reference Dependence and Market Competition

Jidong Zhou

University College London

May 2008

Reference Dependence and Market Competition[*]

Jidong Zhou
Department of Economics
University College London

First Version: October 2007. This Version: May 2008

Abstract

This paper studies the implications of consumer reference dependence in market competition. If consumers take some product (e.g., the first product they have considered) as the reference point in evaluating others and exhibit loss aversion, then the more "prominent" firm whose product is taken as the reference point by more consumers will randomize its price over a high and a low one. All else equal, this firm will on average earn a larger market share and a higher profit than its rival. The welfare impact is that consumer reference dependence could harm firms and benefit consumers by intensifying price competition. Consumer reference dependence will also shape firms' advertising strategies and quality choices. If advertising increases product prominence, *ex ante* identical firms may differentiate their advertising intensities. If firms vary in their prominence, the less prominent firm might supply a lower-quality product even if improving quality is costless.

Keywords: advertising, competition, loss aversion, product quality, reference dependence
JEL classification: D11, D43, L13, M37

1 Introduction

Economists have recently shown great interest in studying the market implications of human behavioral biases (see, for example, Ellison (2006)). A branch of this literature investigates how consumers' reference-dependent preferences (Kahneman and Tversky (1979,91)) influence the firm's behavior in the market. A main finding is that consumer loss aversion can cause price stickiness (Heidhues and Kőszegi (2007), for instance). Complementary to this view, this paper will present a model to argue that consumer loss aversion can also give rise to price variation in a competitive market and help explain sales in the market.

Our model is motivated by the fact that people often encounter and consider options sequentially, and lots of evidence has shown that the option which is considered (or tried)

[*]I am indebted to Steffen Huck and Mark Armstrong for their guidance and many helpful discussions. I also thank Jean-Charles Rochet, Ran Spiegler, Michael Waterson, and Xiaojian Zhao for comments. Financial support from Overseas Research Students Awards (UK), the KC Wong Education Foundation, and the Economic and Social Research Council (UK) is gratefully acknowledged. Contact information: jidong.zhou@ucl.ac.uk.

first could be favored disproportionately even if there is little cost involved in moving to options considered later.[1] One explanation, based on reference dependence, is that people tend to regard the first option as the reference point when they come to value later ones, and they display loss aversion in the sense later options' relative disadvantages are weighed more than their relative advantages. Thus, all else equal, the early option may outperform later ones.[2]

Specifically, we consider a duopoly model with differentiated products where consumers consider or try products sequentially. We suppose that consumers take the first product as the reference point when they value the second one, and they exhibit loss aversion in both the price dimension and the product dimension: they are *excessively* averse to paying a price higher than the reference price or to having a product less well matched than the reference product. Another ingredient of our model is that one firm might be more "prominent" than the other in the sense that the prominent product is considered first and so taken as the reference product by more consumers. The prominent product could be the default option, the product which is more heavily advertised, the product which is recommended or displayed more visibly in the store, and the product which enters the market earlier and consumers first hear of. The main question we investigate is how firms will strategically adjust their prices and product attributes to manipulate consumers' reference points in a competitive environment, and what the impact of this strategic behavior on the market is.

Sections 2 and 3 investigate the pricing and welfare implications of consumer reference dependence. Section 2 considers the case where *all* consumers take the same product as the reference product. Such a relatively simple setting helps illustrate the key feature of our model: firms' price choices have a direct bearing on consumers' price sensitivity. If the reference firm charges a lower price than its rival, loss aversion in the price dimension makes consumers *more* price sensitive; if the reference firm charges a higher price, loss aversion in the product dimension makes the marginal consumer who must have a strong state for the reference product *less* price sensitive. Graphically, the reference firm's demand curve has an inward kink at its rival's price. In contrast, the other firm's demand curve has an outward kink. With this new function for price, the reference firm has an incentive to randomize its price. It will either charge a lower price than its rival to earn a large market share or charge a higher price to focus on high-value consumers. But the other firm will charge a medium price constantly. We further show that, all else equal, the

[1] For example, Madrian and Shea (2001) identify a default effect with employee savings plans. They find that participation in such schemes is significantly higher under automatic enrollment, and a substantial fraction of participants under automatic enrollment choose both the default contribution rate and fund allocation even though few employees hired before automatic enrollment picked this particular outcome. A similar default effect in automobile insurance purchases is documented by Johnson et al. (1993). Ho and Imai (2006) and Meredith and Salant (2007) observe that being listed first on the ballot paper can significantly increase a candidate's vote share. See also relevant experimental evidence in, for example, Samuelson and Zeckhauser (1988) and Kahneman et al. (1991).

[2] The reference-dependent effect does not necessarily require that people possess the reference product physically for a long time, though it might be more pronounced in that case. For example, in most of experimental studies on the status quo bias and the endowment effect, the time of possessing the object is rather short and sometimes subjects only possess the object mentally. However, even in such situations, subjects seem to be attached to the object strongly.

reference product is on average more expensive and occupies a larger market share, and the reference firm also earns a higher profit. Section 3 considers a more flexible setting which allows for heterogenous reference products among consumers. There the prominent firm plays the same role as the reference firm and similar results hold.

One implication of our price result is, if sellers in the market are not equally prominent, the more prominent one (for instance, the seller advertising more heavily or that having an advantageous location in a shopping area) may charge more volatile prices (for example, put its product on sales more frequently). This offers a new justification for the existence of sales in the market:[3] sales can be used to manipulate consumers' price sensitivity when a majority of consumers consider a certain product first in their market search process.[4]

The main welfare findings are: (i) More severe loss aversion could intensify price competition, this harming firms and benefiting consumers. However, it usually leads to lower total welfare in our setting with inelastic demand. This is mainly because more severe loss aversion tends to enlarge the price difference between products and thus induce a worse matching of consumers along the personal taste dimension. (ii) Although the prominent product is on average more expensive, it may be better for consumers to consider it first, because doing so will prevent them from being over "addicted" to the low price at the expense of taste satisfaction.

Section 4 considers endogenous prominence and justifies asymmetric prominence as an equilibrium outcome of an extended game. We first show that a greater prominence difference between firms can boost *each* firm's profit. The intuition is, when a firm becomes relatively more prominent, it will rely more on those strong-taste consumers and so charge the high price more frequently, which will relax the price competition. Based on this result, we argue that if firms sell their products through a platform, the platform has an incentive to make the two products unequally prominent; if firms have an advertising competition and if advertising only increases product prominence, then *ex ante* identical firms may differentiate their advertising intensities.

Section 5 studies the case with asymmetric product qualities. We show that a relative increase of the prominent firm's product quality could benefit *both* firms. This is because letting consumers consider a higher quality product first can make them in aggregate less price sensitive and thus relax price competition. Therefore, if there is a quality choice stage prior to the price competition, the less prominent firm may have an incentive to supply a lower-quality product than its prominent rival even if it is *costless* to improve quality. This offers an alternative explanation for quality differentiation in the market.[5]

---

[3] Sales can also be explained as a result of intertemporal price discrimination (Sobel (1984), for instance) or price discrimination across captive and non-captive consumers (Varian (1980), for instance) in the single-product market, or as the loss-leader pricing strategy in the multi-product market (Lal and Matutes (1994)). See, for example, Hosken and Reiffen (2004) for a recent empirical study of sales.

[4] In particular, our price prediction is also consistent with the observation that the prices of national brands in supermarkets are often more volatile than the prices of private brands, if consumers tend to regard the national brand as the reference point. For example, Slade (1998) documents that the private-label prices of saltine crackers in U.S. are less volatile than major-manufacturer prices. Muller et al. (2006) provide similar evidence that the average number of price changes is significantly smaller for private label products than for national brands.

[5] In the conventional literature on vertical product differentiation (Shaked and Sutton (1982), for instance), due to a different reason (consumers' heterogeneous preferences over quality), firms also want to

Related Literature:

Since the seminal work by Kahneman and Tversky (1979), it has been well established that people's preferences are often influenced by some reference point and characterized by loss aversion.[6] This behavioral regularity has been extensively applied to explain many economic anomalies such as the endowment effect, the status quo bias, and small-stake risk aversion (see, for example, Kahneman and Tversky (2000), and Della Vigna (2007)). However, relatively fewer articles investigate the firm's response to consumer reference dependence. Most of the existing works study how a monopoly firm makes its dynamic pricing decision when consumers tend to regard the historical price as the reference price (see, for example, Fibich et al. (2007) and references cited therein).[7] A main result is, if consumers display loss aversion, then the firm should charge a constant price.[8]

Nevertheless, competition is important for investigating the market implications of consumer biases.[9] Our work makes a step in this direction. A related recent paper is Heidhues and Kőszegi (2007). Following Kőszegi and Rabin (2006), they use consumers' rational expectations of possible transaction results as the reference point. They then argue that loss aversion might give rise to "focal pricing" in the sense that firms may not adjust their prices even if their costs have changed and firms with different costs may charge the same price. A simple argument can go as follows. Suppose that consumers expect to pay some fixed price before they enter the market. Then, due to loss aversion, each firm's demand will become *more* price responsive if its actual price is *higher* than that expected fixed price, and so at that price the demand curve has an outward kink. This could drive all firms to actually charge that fixed price for a range of cost conditions.[10] The main difference between their model and ours is that consumers in their model take the expectation as the reference point, so no individual firm's *actual* decision can influence it; while our reference point is some real product in the market, so firms can manipulate it directly. This is the key reason why two models have different pricing predictions. Since the formation of reference points is usually context dependent, it is highly desirable to examine how different assumptions of reference points could lead to different market implications. In addition, we also investigate the impacts of consumer reference dependence on firms' advertising and product quality choices.[11]

---

differentiate their qualities to soften price competition.

[6] Besides reference-dependent preferences and loss aversion, the other two elements of Kahneman and Tversky's prospect theory are diminishing sensitivity (which implies risk aversion in the domain of gains and risk seeking in the domain of losses) and nonlinear decision weights (in the environment with uncertainty), but they are irrelevant in our model.

[7] Putler (1992) is an early theoretical attempt to introduce the reference-price effect into consumer demand theory. Gilboa and Schmeidler (2001) study the effect of satisficing behavior and adjustable aspiration levels on consumers' dynamic choice, which bears some resemblance to the effect of reference-dependent preferences. However, neither paper studies how firms might respond to this non-standard consumer behavior.

[8] In contrast, if the effect of a gain is greater than a same-size loss, the firm should price cyclically.

[9] In addition, it is also desirable to take into account reference dependence in non-price product dimensions. Some empirical research (Hardie et al. (1993), for instance) suggests that loss aversion is even more severe in the product dimension than in the price dimension.

[10] See also their companion paper Heidhues and Koszegi (2005) which, among other results, shows a similar price-stickiness result in a static monopoly setting.

[11] Other recent papers which study the implications of the reference-dependence effect (in a broader sense)

The reference-dependence effect in our model can be regarded as a particular kind of switching costs: moving from the reference product to the other involves psychological costs if the latter is relatively inferior in some aspects. But it is rather different from the traditional switching costs in both specifications and consequences (see, for example, Farrell and Klemperer (2007)). We will further discuss this difference in the end of Section 5.

Broadly, this paper also contributes to the emerging literature on behavioral industrial organization. For instance, Della Vigna and Malmendier (2004), Eliaz and Spiegler (2006), Gabaix and Laibson (2006), and Grubb (2006) study how firms might take advantage of consumers' limited abilities to forecast their future preferences. Armstrong and Chen (2007), Rubinstein (1993), and Spiegler (2006a,b) investigate how the heuristic decision making of consumers could induce firms to confuse consumers. Chen et al. (2005) and Shapiro (2006) examine the market implications of consumers' limited memory.

2 Single Reference Product

2.1 Model

There are two firms (1 and 2) in an industry, each supplying a distinct product at constant common unit cost, which we normalize to zero. They set prices p_1 and p_2 simultaneously. Consumers have diverse tastes for different products. We model this scenario via the Hotelling linear city. A consumer's taste is represented by the parameter x which is distributed on the interval $[0, 1]$ according to a cumulative distribution function $F(x)$ which is differentiable and has a positive density $f(x)$. Firm 1 is exogenously located at the endpoint $x = 0$ and firm 2 is at the other endpoint $x = 1$. For a consumer at x, the match utility of product 1 is $v - x$, and that of product 2 is $v - (1 - x)$, where v is the gross utility of the product and is assumed to be sufficiently large such that the whole market is covered in equilibrium.[12] Consumers have unit demand for one product, and the number of consumers is normalized to one.

We introduce consumer reference dependence by considering a sequential-consideration scenario: consumers consider or try products one by one, and a product's price and match utility are discovered when it is considered or tried. We assume that consumers will take the first product they consider as the reference point. When they come to the second one, they will value its relative advantage (lower price or higher match utility) in the standard way, while they will over weigh its relative disadvantage (higher price or lower match utility) in the spirit of loss aversion.[13] We also assume that consumers do not

include, for example, Compte and Jehiel (2003) (prior offers as reference points in sequential bargaining), Eliaz and Spiegler (2007) (the default alternative as the reference point in forming consideration sets), Hart and Moore (2007) (contracts as reference points in *ex post* trading relationship), and Rosenkranz and Schmitz (2007) (reserve prices in auctions as reference points in deciding on bidding strategies).

[12] Alternatively, we can assume that a product's match utility is a random draw from some common distribution and its realization is independent across consumers and products. Our following analysis still applies to this setting by modifying notation slightly.

[13] In our model, the reference point is an individual product. An alternative specification of the reference point could be a weighted average of all products a consumer has considered before making a purchase decision. A product is more prominent if consumers put more weight on it. Our main results carry over

intentionally choose the order in which they consider products, and they may just follow some natural presentation order of products or be guided by firms' marketing activities. (See a discussion about more sophisticated consumers in Section 3.) In this section, for simplicity we further suppose that *all* consumers will consider product 1 first (which is the default option, for instance), and we call it the reference product. (We will treat a more flexible setting in Section 3.)[14]

One point deserves mention before proceeding. Although sequential consideration is a reasonable scenario to think about reference dependence, our model is actually not restricted to this interpretation. What we need is that consumers somehow take some product in the market as the reference point when they evaluate others. The details on why some product becomes the reference point is not crucial to most of our following analysis.

Consumer preferences are specified as follows. Given the prices p_1 and p_2, a consumer at x values product 1, the reference product, in the standard way:

$$v - x - p_1;$$

her valuation of product 2 is

$$v - (1 - x) - p_2 - (\lambda - 1)\max\{0, p_2 - p_1\} - (\lambda - 1)\max\{0, 1 - 2x\},$$

where the first three terms represent the standard intrinsic surplus of product 2 and the other two terms capture the potential reference-dependent "loss utility" in each dimension. $\lambda > 1$ is the loss-aversion parameter and measures the strength of the reference-dependence effect.[15] If $\lambda = 1$, we return to the orthodox Hotelling model. An implicit assumption here is that the reference-dependent "loss utility" occurs separately in the price dimension and the product dimension. It is psychologically reasonable and has been well supported in the literature of prospect theory (see, for example, Tversky and Kahneman (1991)). For simplicity, we have assumed the same degree of loss aversion in both dimensions. Considering asymmetric degrees of loss aversion in the two dimensions will not affect most of our main results qualitatively.[16]

To highlight how reference dependence could benefit the reference product, we first focus on the case with a symmetric distribution of consumers (i.e., $F(1 - x) = 1 - F(x)$).

to that case qualitatively. However, the assumption that the reference point is from the market rather from outside (for example, some "ideal" product in a consumer's mind before she enters the market) is important.

[14] Our one-shot model may be more suitable for infrequently purchased products or for frequently purchased products but with poor-memory consumers. Otherwise, the reference point might also be influenced by the historical purchase.

[15] The strength of the reference-dependence effect could be affected by the time lag between considering options. If the time lag is too long, people may have forgotten the first option when they value the second one; if it is too short, people may have not adapted themselves to the first option when the second one comes. Hence, the effect might be most pronounced when the time lag is appropriate. Presumably, the effect would be also more pronounced if consumers encounter alternative options somehow unexpectedly. If people have been expecting to consider other options when they encounter the first one, they may not attach to it too much and so the reference dependence effect might be weak.

[16] Our welfare results could be affected if the degrees of loss aversion in the two dimensions differ sufficiently. We will discuss this issue in Section 3.

That is, there is no systematic quality difference between the two products. (We will discuss the impact of asymmetric qualities in Section 5.) We also assume away any possible explicit costs involved in moving from one product to the other. Introducing such costs will bring firm 1 with an extra advantage.

Now we are ready to derive each firm's demand function. We claim that if firm 1 charges $p_1 < p_2$, its demand function is

$$q_1(p_1 < p_2) = F\left(\frac{1}{2} + \frac{\lambda}{2}(p_2 - p_1)\right). \tag{1}$$

This is because all consumers with $x \leq \frac{1}{2}$ will definitely buy product 1, and those with $x > \frac{1}{2}$ will buy product 1 only if the gain from product 2's higher match utility is less than the loss (including the psychological part) from its higher price, i.e., only if $2x - 1 < \lambda(p_2 - p_1)$, which leads to

$$x < \frac{1}{2} + \frac{\lambda}{2}(p_2 - p_1).$$

It is ready to see that consumers are now more price sensitive than in the orthodox model (which applies when $\lambda = 1$). This is because the attractiveness of firm 1's lower price has been amplified by consumers' loss aversion in the price dimension.

When firm 1 charges $p_1 > p_2$, all consumers with $x > \frac{1}{2}$ will buy product 2, and those with $x < \frac{1}{2}$ will choose product 1 only if the loss (including the psychological part) from product 2's lower match utility exceeds the gain from its lower price, i.e., only if $\lambda(1 - 2x) > p_1 - p_2$. Now those consumers with $x < \frac{1}{2}$ become less price sensitive, because the unattractiveness of firm 1's higher price has appeared less important relative to the unattractiveness of product 2's lower match utility. The corresponding demand function is

$$q_1(p_1 > p_2) = F\left(\frac{1}{2} + \frac{1}{2\lambda}(p_2 - p_1)\right). \tag{2}$$

(1) and (2) imply that, around p_2, firm 1's demand is more price responsive at $p_1 < p_2$ than at $p_1 > p_2$, and hence the demand curve has an *inward* kink at $p_1 = p_2$ (see Figure 1 below which illustrates the case with uniform x).

Firm 2's demand is $q_2 = 1 - q_1$. Explicitly, using the symmetry of distribution, we have

$$q_2(p_2 > p_1) = F\left(\frac{1}{2} + \frac{\lambda}{2}(p_1 - p_2)\right); \quad q_2(p_2 < p_1) = F\left(\frac{1}{2} + \frac{1}{2\lambda}(p_1 - p_2)\right). \tag{3}$$

When $p_2 > p_1$, the unattractiveness of firm 2's higher price will be amplified by loss aversion since consumers regard p_1 as the reference price. When $p_2 < p_1$, the attractiveness of its lower price to the marginal consumer at $x < \frac{1}{2}$ will be reduced by her extra aversion to product 2's lower match utility. Clearly, around p_1, firm 2's demand is more price responsive at $p_2 > p_1$ than at $p_2 < p_1$, which implies that q_2 has an *outward* kink at $p_2 = p_1$ (see Figure 1 below).

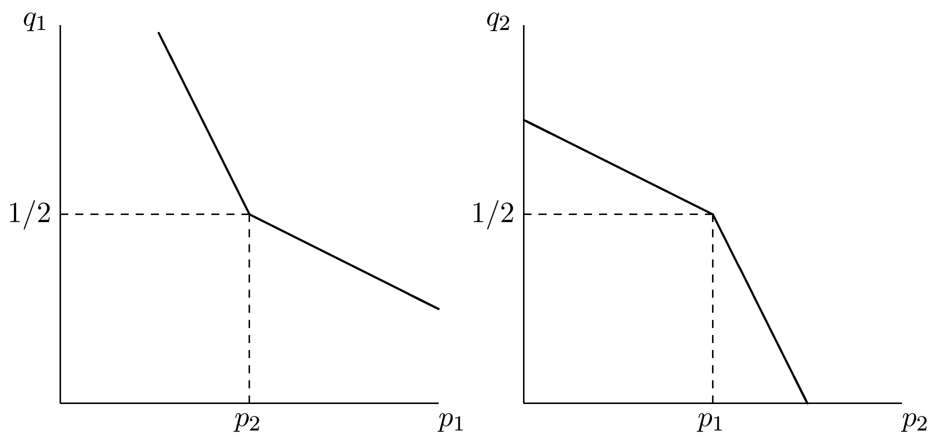

Figure 1: An Illustration of Demand Curves

In sum, compared to the orthodox case, consumers will become more (less) price sensitive if the reference product is cheaper (more expensive) than the other. Moreover, consumer reference dependence benefits the reference firm but harms the other in the sense that, at any pair of prices $p_1 \neq p_2$, q_1 increases but q_2 decreases relative to the orthodox case.

Two additional properties of the demand function deserve mention. First, $q_1 > q_2$ if and only if $p_1 < p_2$. In effect, reference dependence does not affect each firm's demanded quantity if they charge the same price. However, it still affects the price sensitivity at that point. Second, at any fixed price pair $p_1 \neq p_2$, both firms' demand curves have the same slope given $q_2 = 1 - q_1$.[17] In the following, we denote by $\pi_i(p_1, p_2) = p_i \cdot q_i(p_1, p_2)$ the profit function of firm i.

2.2 Equilibrium

Now we derive the Nash equilibrium of the price competition. First of all, both firms charging the same price is not an equilibrium. Given a positive price of firm 2, firm 1's demand has an inward kink at this price, which means that its profit function has a local minimum at this point. Hence, charging the same price will never be firm 1's best response. Second, there is no asymmetric pure-strategy equilibrium either. Suppose $p_1 \neq p_2$ were an equilibrium. Since both firms face the same demand slope at this hypothetical equilibrium point, in equilibrium the firm charging the higher price should have a higher demand.[18] But that is impossible in our symmetric environment. We formalize the above argument in the following proposition. All omitted proofs are presented in the Appendix.

Proposition 1 *Given a symmetric distribution of consumers, the price competition has no pure-strategy Nash equilibrium.*

We will then show that, under regularity conditions, the game has a mixed-strategy equilibrium in which firm 1 charges a low price p_1^L with probability $\mu \in (0, 1)$ and a high

[17] This property does not depend on the assumption of a symmetric distribution of consumers.

[18] This is because, in a hypothetical asymmetric equilibrium, each firm's demand function is smooth around its own equilibrium price, and so we have the first-order conditions: $q_1 + p_1 \frac{\partial q_1}{\partial p_1} = q_2 + p_2 \frac{\partial q_2}{\partial p_2} = 0$.

price p_1^H with probability $1-\mu$, and firm 2 charges a medium price p_2 for sure. Given firm 1's mixed pricing strategy, let

$$q_2^e(p;\mu,p_1^L,p_1^H)=\mu\cdot q_2(p_1^L,p)+(1-\mu)\cdot q_2(p_1^H,p) \tag{4}$$

be firm 2's expected demand function. It has two outward kinks at p_1^L and p_1^H which divide it into three segments (see Figure 2 below). The regularity conditions are:

Assumption 1 *(i) $f(x)$ is logconcave;*[19] *(ii) for $\mu\in(0,1)$ and $0<p_1^L<p_1^H$, each segment of q_2^e is regular such that the corresponding part of firm 2's profit function is quasi-concave.*[20]

Notice that the uniform distribution ($F(x)=x$) satisfies Assumption 1 since then each segment of q_2^e is linear.

Proposition 2 *Given a symmetric distribution of consumers and Assumption 1,*[21] *there exists a mixed-strategy equilibrium as specified in the above where the quadruplet (μ,p_1^L,p_1^H,p_2) satisfies the following conditions:*

(i) $p_2=\arg\max_p p\cdot q_2^e(p;\mu,p_1^H,p_1^L)$;

(ii) $p_1^L=\arg\max_{p\le p_2} p\cdot q_1(p,p_2)$, and $p_1^H=\arg\max_{p\ge p_2} p\cdot q_1(p,p_2)$;

(iii) $\pi_1(p_1^L,p_2)=\pi_1(p_1^H,p_2)$.

Conditions (i) and (ii) define each firm's best response given its rival's strategy, and condition (iii) means that firm 1 is indifferent between charging the low and the high price. A potential complication is, if λ is sufficiently large, firm 1 may occupy the whole market when it charges the low price p_1^L. As we discuss in Appendix A.2, such an equilibrium with a corner solution can actually occur. However, in the main text of this paper (except in Section 5), we focus on the interior-solution equilibrium in which no firm captures all consumers (which requires relatively small λ).

We illustrate the equilibrium in Figure 2 below which is based on the uniform-distribution case, where π_i is firm i's iso-profit curve. The intuition of this mixed-strategy equilibrium is as follows. Given firm 2's price, firm 1 can either charge a lower price to make consumers more price sensitive and then earn a large market share, or charge a higher price to exploit those consumers who have a strong taste for its product and will thus become less price sensitive due to loss aversion in the taste dimension.[22] Although these two strategies are

[19] The logconcavity condition is satisfied by many well-known (truncated if necessary) distributions. See, for example, Bagnoli and Bergstrom (2005) for a detailed discussion.

[20] Since logconcave f implies logconcave F, firm 1's demand in each side of its kink is logconcave such that the corresponding part of its profit function is logconcave (so quasi-concave). (Remember that firm 1's whole profit function will never be quasi-concave.) However, a weighted average of two logconcave functions may fail to be logconcave. In our case, though $q_2(p_1^i,p)$ is logconcave under (i), $q_2^e(p)$ defined in (4) may not be. That is why we need (ii). But one can show that (i) implies (ii) if λ is close to one or if $\frac{f'(0)}{f(0)}<\frac{4}{1+\lambda p_1^H}$. (The latter condition actually implies concave profit function of firm 2.)

[21] If Assumption 1 fails to be satisfied, we may have other types of mixed-strategy equilibrium. But note that the general existence of equilibrium is no problem according to the Glicksberg Theorem, since each firm's profit function is continuous and we can restrict each firm's feasible prices to a compact interval.

[22] This argument does not apply to firm 2. Given fixed p_1, if firm 2 charges a higher price, consumers will become more price sensitive, which will drive firm 2 to lower its price; if firm 2 charges a lower price than p_1, the marginal consumer who has a strong taste for product 1 will become less price sensitive, which will drive firm 2 to raise its price.

equally profitable in equilibrium, firm 1 will not adopt either strategy predictably. Otherwise, firm 2 would either be attempted to charge a low price to steal business when p_1^H applies, or be forced to match p_1^L to protect its own market share. Either situation will lower firm 1's profit, so firm 1 has an incentive to randomize its price and keep firm 2 guessing.

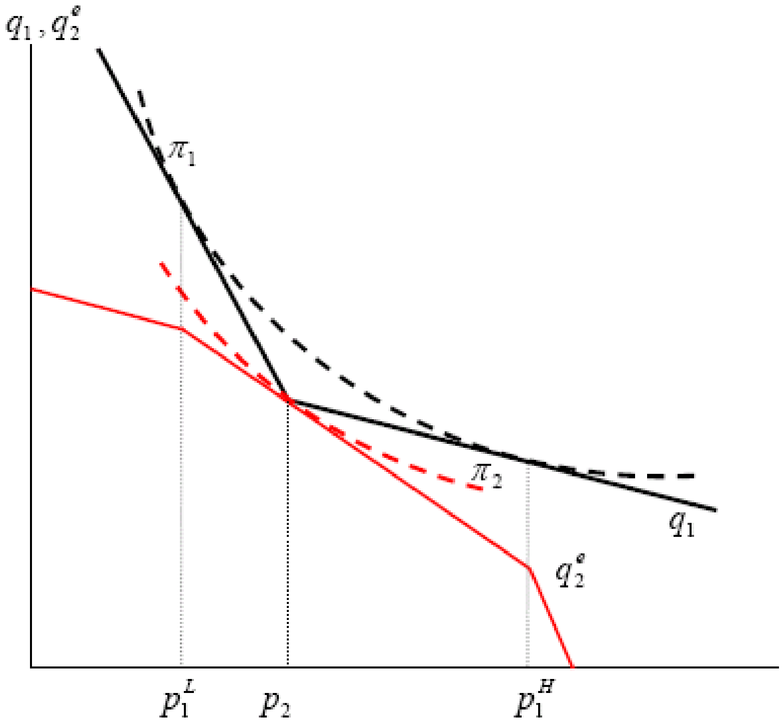

Figure 2: An Illustration of the Equilibrium

- ***The robustness of equilibrium.*** Readers may wonder whether there are other types of mixed-strategy equilibrium in our model. A sufficient condition for the uniqueness of our equilibrium is, on top of Assumption 1, for any possible mixed pricing strategy of firm 1, firm 2's expected profit function will be globally quasi-concave. We do not have primitive conditions for this, but it is satisfied at least by the uniform distribution as we will show in Section 3.

Our equilibrium is robust to heterogenous reference points among consumers. For example, when product 1 is more heavily advertised than product 2, more than half consumers may notice and consider product 1 first and others may notice product 2 first. We will investigate such a general setting in Section 3, and there we will show that a similar equilibrium exists provided that the two products are not equally noticeable. It is also not difficult to extend our model to the case with more than two firms, if consumers still take some product as the reference point in evaluating others.[23] No fundamental changes will take place since how firms' price choices affect the price sensitivity of consumers remains unchanged.

[23] However, if in the sequential-consideration scenario consumers' reference points evolve as the search process goes on, then the situation could become complicated, depending on how the evolution process is specified.

Another issue is about the assumption of symmetric distributions. From the proofs of Propositions 1 and 2, we can see that this assumption can be replaced by a weaker condition: $F(\frac{1}{2}) = \frac{1}{2}$. Beyond this, will our mixed-strategy equilibrium still persist? The following proposition tells us that, given the degree of loss aversion, our equilibrium continues to hold provided that the distribution is not too skewed to either endpoint.

Proposition 3 *Given Assumption 1,*

(i) for fixed λ, there exists $\varepsilon > 0$ such that, when $\left|F(\frac{1}{2}) - \frac{1}{2}\right| < \varepsilon$, there is no pure-strategy equilibrium, and a similar mixed-strategy equilibrium as before exists;

(ii) for fixed $\left|F(\frac{1}{2}) - \frac{1}{2}\right| > 0$, there exists $\lambda^* > 1$ such that, when $\lambda < \lambda^*$, there is only a pure-strategy equilibrium with $p_1 > p_2$ if $F(\frac{1}{2}) > \frac{1}{2}$ and $p_1 < p_2$ if $F(\frac{1}{2}) < \frac{1}{2}$.

Part (ii) of this proposition means that, given an asymmetric distribution, if the degree of loss aversion is sufficiently low, pure-strategy equilibrium will emerge. We will further illustrate this result in Section 5 when we discuss asymmetric product qualities (which is a special case of asymmetric distributions).

• ***The benefit of selling the reference product.*** We then investigate whether the reference firm enjoys an advantage over its rival merely due to consumer reference dependence. As we can see from the demand function, if the reference firm charges a higher price than its rival, the shrink of its market share will be mitigated by consumers' loss aversion in the product dimension; and if it charges a lower price, the expansion of its market share will be amplified by consumers' loss aversion in the price dimension. In either case, consumer reference dependence favors the reference firm. Thus, we should expect that the reference firm will earn more the other. In addition, we will also compare the two firms' average prices and market shares, of which the results are not easy to predict in advance given the mixed-strategy equilibrium.

Proposition 4 *Given a symmetric distribution of consumers and Assumption 1, in the mixed-strategy equilibrium we have identified,*

(i) firm 1 charges the high price more frequently ($\mu < \frac{1}{2}$) and product 1 is on average more expensive than product 2 ($p_1^e = \mu p_1^L + (1-\mu)p_1^H > p_2$);

(ii) on average firm 1 occupies a (weakly) larger market share than firm 2 ($q_2^e \leq \frac{1}{2}$), and they share the market equally if and only if the distribution is uniform;

(iii) firm 1 earns strictly higher profit than firm 2.

There are two other questions deserving investigation. First, how will price and welfare vary with the degree of loss aversion? Second, given firms' equilibrium pricing strategies, if consumers realize their own biases and can choose the consideration order freely, is it really in their own interests to consider the reference product first? Due to the tractability issue, we discuss them in the uniform-distribution case in next section.

3 Heterogeneous Reference Products

This section allows for heterogenous reference products among consumers. Specifically, we now suppose that $\frac{1}{2}+\theta$ of consumers will consider product 1 first and take it as the reference product, while $\frac{1}{2} - \theta$ of consumers will take product 2 as the reference product. Without loss of generality, let $\theta \in [0, \frac{1}{2}]$. When $\theta > 0$, we say product 1 is more "prominent" than product 2, and θ indicates the prominence difference between the two products. This flexible setting will allow us to discuss endogenous prominence through advertising competition in Section 4.

3.1 The general case

Each firm now has two demand sources: those consumers regarding its product as a reference point and those regarding its rival's product as a reference point. When firm 1 charges $p_1 < p_2$, its demand function becomes

$$q_1(p_1 < p_2) = (\frac{1}{2} + \theta) F\left(\frac{1}{2} + \frac{\lambda}{2}(p_2 - p_1)\right) + (\frac{1}{2} - \theta) F\left(\frac{1}{2} + \frac{1}{2\lambda}(p_2 - p_1)\right).$$

The first part is the same as before, and the second part is because, among consumers who regard product 2 as the reference product, the marginal consumer is now at $x > \frac{1}{2}$ and she will become less price sensitive due to her extra aversion to the less well matched product 1. Similarly,

$$q_1(p_1 > p_2) = (\frac{1}{2} + \theta) F\left(\frac{1}{2} + \frac{1}{2\lambda}(p_2 - p_1)\right) + (\frac{1}{2} - \theta) F\left(\frac{1}{2} + \frac{\lambda}{2}(p_2 - p_1)\right).$$

For $\theta \in (0, \frac{1}{2})$, we have

$$\begin{aligned} & \lim_{p_1 \to p_2^-} q_1'(p_1 < p_2) = -\frac{1}{2} f(\frac{1}{2}) \left[(\frac{1}{2} + \theta)\lambda + (\frac{1}{2} - \theta)\frac{1}{\lambda} \right] \\ < & \lim_{p_1 \to p_2^+} q_1'(p_1 > p_2) = -\frac{1}{2} f(\frac{1}{2}) \left[(\frac{1}{2} - \theta)\lambda + (\frac{1}{2} + \theta)\frac{1}{\lambda} \right], \end{aligned}$$

which means that q_1 has an inward kink at $p_1 = p_2$. Firm 2's demand function can be treated similarly and it has an outward kink at $p_2 = p_1$. Therefore, compared to the single-reference-product case, we should not expect any qualitative changes to take place. The counterparts of Propositions 1–3 can be proved similarly but with heavier notation. Although the counterpart of Proposition 4 has not been established completely, we conjecture it would hold. We will verify it in the uniform-distribution case below, and we can also verify it when λ is close to one.[24]

[24] When $\lambda = 1 + \varepsilon$ and ε tends to zero, equilibrium prices can be approximated as

$$\begin{aligned} p_1^L &\approx p\left[1 - \theta\varepsilon + \left(3\theta^2 - \frac{1-\theta}{2} - A\right)\varepsilon^2\right], \\ p_1^H &\approx p\left[1 + \theta\varepsilon + \left(3\theta^2 - \frac{1+\theta}{2} - A\right)\varepsilon^2\right], \\ p_2 &\approx p\left[1 + (2\theta^2 - \frac{1}{2})\varepsilon^2\right], \ \mu \approx \frac{1}{2} - (\frac{\theta}{2} + \frac{A}{2\theta})\varepsilon, \end{aligned}$$

where $p = 1/f(\frac{1}{2})$ is the equilibrium price in the standard Hotelling model and $A = \frac{\theta^2 p^3}{16} f''(\frac{1}{2})$. The complication in this limit analysis is that we need the second-order price approximations to proceed welfare analysis. This is because, when $\varepsilon \to 0$, we find λ has no first-order effect on all welfare variables. (But it turns out that, for μ, the first-order approximation is enough.) All details about the limit analysis in this paper are available from the author.

A notable exception is $\theta = 0$ (in which case the two products are equally prominent). In this symmetric case, the demand functions are smooth and we have a symmetric pure-strategy equilibrium with

$$p^* = \frac{2}{(\lambda + 1/\lambda) f(1/2)}.$$

Compared to the standard Hotelling model where $\lambda = 1$, loss aversion leads to lower equilibrium price since $\lambda + \frac{1}{\lambda} > 2$. However, any extent of prominence difference between the two firms will overturn this equilibrium.

3.2 The uniform-distribution case

In the following, we will proceed our analysis in the uniform-distribution case with asymmetric prominence (i.e., $\theta > 0$). One justification for asymmetric prominence will be provided in Section 4 where we consider endogenous prominence through advertising competition.

We first introduce two pieces of notation:

$$h = (\frac{1}{2} + \theta)\lambda + (\frac{1}{2} - \theta)\frac{1}{\lambda}, \quad l = (\frac{1}{2} - \theta)\lambda + (\frac{1}{2} + \theta)\frac{1}{\lambda}.$$

Clearly, $h > l$ when $\theta > 0$. Then the two firms' demand functions can be written as

$$q_1 = \frac{1}{2} + \frac{i}{2}(p_2 - p_1), \quad q_2 = \frac{1}{2} + \frac{i}{2}(p_1 - p_2),$$

where $i = h$ if $p_1 < p_2$ and $i = l$ if $p_1 > p_2$. So the demand is more price responsive when the prominent product is relatively cheaper. When firm 1 uses the mixed strategy as in Section 2, firm 2's expected demand function is

$$q_2^e(p_2) = \frac{1}{2} + \frac{\mu h}{2}(p_1^L - p_2) + \frac{(1-\mu)l}{2}(p_1^H - p_2), \tag{5}$$

for $p_2 \in [p_1^L, p_1^H]$.

In this uniform setting, the following condition guarantees the mixed-strategy equilibrium with an interior solution:

$$1 < r = \sqrt{\frac{h}{l}} < 3. \tag{6}$$

In particular, when $\theta = \frac{1}{2}$, we have $h = \lambda$ and $l = \frac{1}{\lambda}$, so (6) requires $\lambda < 3$. For smaller θ, (6) is easier to hold. For example, when θ tends to zero, h and l will coincide, and so (6) is always true.

We first derive equilibrium. (Note that, in the uniform setup, all following necessarily conditions are also sufficient.) Given p_2, firm 1's best responses imply

$$p_1^L = \frac{1}{2h} + \frac{p_2}{2}; \quad p_1^H = \frac{1}{2l} + \frac{p_2}{2}.$$

And the indifference condition requires

$$p_1^L \left[\frac{1}{2} + \frac{h}{2}(p_2 - p_1^L)\right] = p_1^H \left[\frac{1}{2} + \frac{l}{2}(p_2 - p_1^H)\right].$$

From them, we solve

$$\begin{aligned}
p_1^L &= \frac{1}{2}\left[\frac{1}{h}+\frac{1}{\sqrt{hl}}\right]=\frac{1+r}{2h},\\
p_1^H &= \frac{1}{2}\left[\frac{1}{l}+\frac{1}{\sqrt{hl}}\right]=\frac{r(1+r)}{2h},\\
p_2 &= \frac{1}{\sqrt{hl}}=\frac{r}{h}.
\end{aligned}$$

With the expected demand function in (5), firm 2's best response implies

$$2\left[\mu h+(1-\mu)l\right]p_2 = 1+\mu h p_1^L+(1-\mu)l p_1^H, \tag{7}$$

from which we get

$$\mu=\frac{1}{1+r}.$$

It is ready to see that $p_1^L < p_2 < p_1^H$ and $\mu \in (0, \frac{1}{2})$. For this solution to be a real equilibrium, we further need that no firm captures all consumers. Simple calculation shows that firm 1's demands are

$$q_1^L=\frac{1+r}{4},\quad q_1^H=\frac{1+r}{4r}$$

when it charges p_1^L and p_1^H, respectively. So (6) indeed guarantees an interior-solution equilibrium.

Moreover, this is the unique equilibrium. Since firm 2's demand function is strictly concave for any fixed p_1 in this uniform setting, its expected demand function is also strictly concave for any mixed pricing strategy of firm 1. Firm 2 will therefore never randomize its price. On the other hand, for any fixed p_2, firm 1's demand function is linear in each side of its kink, so it is impossible for firm 1 to have more than two best replies.

We then study the properties of equilibrium. Define

$$\triangle_L = p_2-p_1^L=\frac{r-1}{2h};\quad \triangle_H=p_1^H-p_2=\frac{r(r-1)}{2h}.$$

Then

$$\frac{\triangle_H}{\triangle_L}=r>1, \tag{8}$$

which means that, relative to firm 2's price, p_1^H deviates more than p_1^L. On the other hand, firm 1 charges the high price p_1^H more frequently since $\mu < \frac{1}{2}$. These two observations imply that product 1 is on average more expensive than product 2:

$$p_1^e=\frac{1+r^2}{2h}>p_2=\frac{r}{h}.$$

One can check that firm 1's expected demand is just $\frac{1}{2}$. That is, on average both firms share the market equally though firm 1 is more prominent.[25] But firm 1 earns more than firm 2:

$$\pi_1=\frac{1}{8}\left[\frac{1}{\sqrt{h}}+\frac{1}{\sqrt{l}}\right]^2=\frac{(1+r)^2}{8h}>\pi_2=\frac{p_2}{2}=\frac{r}{2h}. \tag{9}$$

[25] This result is not a general property as Proposition 4 has suggested.

Thus, similar results as in Proposition 4 have been established in this heterogenous-reference-product setting with the uniform distribution.

We now answer those two questions proposed in the end of Section 2.

• ***Which product should consumers consider first?*** In our model, the order in which consumers consider product is specified exogenously or determined by firms' marketing activities such as advertising. We have not yet consider the question that, if consumers realize their own behavioral biases and can choose their own consideration orders freely, how they will behave given that firms adopt the above equilibrium pricing strategies and they are distinguishable in the market.

We first set the welfare criterion. Throughout this paper, we take the view that the psychological "loss utility" only occurs in the decision process, and it affects the ultimate welfare status of consumers only through influencing their choices. Hence, we use the orthodox welfare measurement.[26] Also remember that consumers do not know a product's match utility until they consider it, so consumers are *ex ante* identical. Since $p_1^e > p_2$, people may conjecture that those considering product 2 first would obtain higher surplus. This conjecture, however, is *not* true.

Define

$$x_L = \frac{1}{2} + \frac{\lambda}{2}\triangle_L; \quad x_H = \frac{1}{2} - \frac{1}{2\lambda}\triangle_H. \tag{10}$$

For a consumer considering product 1 first, if its price is p_1^i, she will buy product 1 if and only if her location is less than x_i, so her expected surplus is $v - \alpha_i$, where

$$\alpha_i = \int_0^{x_i} (x + p_1^i)dx + \int_{x_i}^1 (1 - x + p_2)\, dx$$

is the sum of expected taste loss and expected payment. Thus, given firms' strategies, the expected surplus of a consumer who considers product 1 first is $v - \mu\alpha_L - (1-\mu)\alpha_H$. Similarly, define

$$y_L = \frac{1}{2} + \frac{1}{2\lambda}\triangle_L; \quad y_H = \frac{1}{2} - \frac{\lambda}{2}\triangle_H. \tag{11}$$

Then, the expected surplus of a consumer who considers product 2 first is $v - \mu\beta_L - (1 - \mu)\beta_H$, where

$$\beta_i = \int_0^{y_i} (x + p_1^i)dx + \int_{y_i}^1 (1 - x + p_2)dx.$$

Therefore, considering product 1 first is better if

$$\mu\alpha_L + (1-\mu)\alpha_H < \mu\beta_L + (1-\mu)\beta_H. \tag{12}$$

One can verify

$$\alpha_L - \beta_L = \underbrace{\int_{y_L}^{x_L} (2x-1)dx}_{\text{Increase of Taste Loss}} - \underbrace{\int_{y_L}^{x_L} \triangle_L dx}_{\text{Saving of Payment}} = \frac{M}{4}\triangle_L^2 > 0,$$

[26] Even if we add the "loss utility" to welfare calculation, our following result still holds if λ is relatively small.

where $M = (\lambda - 1/\lambda)(\lambda + 1/\lambda - 2) > 0$. This means that, when firm 1 is charging the low price, considering its product first is actually worse than considering product 2 first. Two conflicting forces work here. On the one hand, if a consumer considers the cheaper product first, she will become excessively averse to paying a higher price due to loss aversion, and so she will be more likely to buy product 1 (i.e., $x_L > y_L$), which of course can save expected payment. On the other hand, relative to the socially optimal situation in which consumers should buy the most matched product regardless of the price difference, $x_L > y_L > \frac{1}{2}$ implies that considering the cheaper product 1 first will result in more severe product-choice distortion and so involve a greater expected taste loss. It turns out that the latter negative effect dominates in our setting. Similarly, we have

$$\alpha_H - \beta_H = \int_{y_H}^{x_H} (2x - 1 + \triangle_H)dx = -\frac{M}{4}\triangle_H^2 < 0.$$

That is, when firm 1 is charging the high price, considering its product first is actually better.

Now (12) becomes $\mu\triangle_L^2 < (1-\mu)\triangle_H^2$, which, according to (8), is equivalent to

$$\frac{\mu}{1-\mu} < r^2.$$

But this inequality must be true since $\mu < \frac{1}{2}$ and $r > 1$.

Therefore, in the uniform-distribution setting with the orthodox welfare criterion, considering the more prominent product 1 first will yield higher expected consumer surplus. The reason is that considering the more expensive product first will prevent consumers from being over "addicted" to the low price at the expense of taste satisfaction.[27,28]

• ***The impact of loss aversion.*** We now turn to examine how price and welfare vary with the degree of loss aversion. We will show that, when the prominence difference between the two firms is relatively small, more severe loss aversion will intensify price competition, harm firms and benefit consumers (which is consistent with the observation at $\theta = 0$); when the prominence difference is relatively large, more severe loss aversion will tend to increase industry profit and harm consumers. In either case, more severe loss aversion leads to lower total welfare in our inelastic-demand setting.

We first present some useful observations (remember $r = \sqrt{\frac{h}{l}}$):

	h	l	hl	r
λ	+	?	+	+

[27] We conjecture that this result would even hold for a general distribution, which can be verified at least in the limit case with λ close to one. However, if the extent of loss aversion in the price dimension is rather weak, then the result could be reversed. In particular, if there is no loss aversion in the price dimension, then $M = (1 - 1/\lambda)(1 + 1/\lambda - 1) < 0$ and so considering product 2 first is better.

[28] One implication of our result is that, if consumers are sophisticated and they are able to choose their consideration order freely, our game with *ex ante* symmetric firms has three equilibria. In the symmetric pure-strategy equilibrium (with $\theta = 0$), expecting both firms are charging the same price, consumers will consider products in a random order, which will further sustain the symmetric equilibrium. In the other two asymmetric equilibria (with $\theta = \frac{1}{2}$ and $-\frac{1}{2}$, respectively), expecting one firm is charging a random but on average higher price, consumers will visit this firm first, which will also further sustain the asymmetric equilibrium.

In this table, "+" means the variable in the row increases with the parameter in the column, and "?" means a possible non-monotonic relationship.[29] (Note that, when $\theta = \frac{1}{2}$, $hl = 1$ is independent of λ. This caveat applies to all following analysis.)

Price. It is ready to see that both $p_2 = 1/\sqrt{hl}$ and $p_1^L = \frac{1}{2h} + \frac{p_2}{2}$ fall with λ. But p_1^H may vary with λ non-monotonically. Figure 3 below is a numerical example with $\theta = 0.3$ where the thin line is p_2.

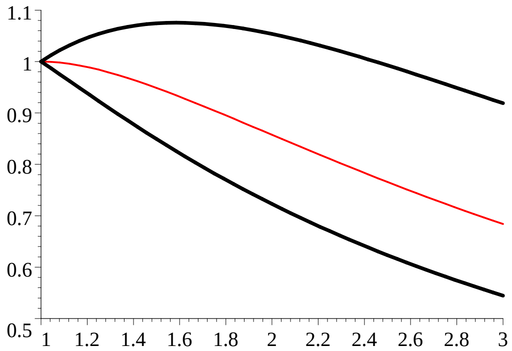

Figure 3: Prices and λ

In particular, from $p_1^H = \frac{1}{2l} + \frac{p_2}{2}$, we can see that p_1^H will fall with λ if l increases with λ, which is true if (see footnote 29)

$$\lambda^2 > \frac{1/2 + \theta}{1/2 - \theta}. \tag{13}$$

This condition is easier to hold for higher λ and lower θ. For example, when θ tends to zero, it is always true; when θ tends to $\frac{1}{2}$, it fails for sure.[30] Roughly speaking, for smaller θ, firm 1's profit from exploiting those strong-taste consumers by charging a high price will go down since now fewer of them will visit firm 1 first.

Profit. It is clear that $\pi_2 = p_2/2$ decreases with λ. But π_1 could vary with λ non-monotonically.[31] In particular, since $\pi_1 = \frac{1}{8}(h^{-1/2} + l^{-1/2})^2$ (see (9)), a sufficient condition for π_1 to be decreasing with λ is also (13) given h increases with λ. Therefore, it is possible (at least under (13)) that more severe loss aversion will intensify competition and harm both firms.[32]

Consumer surplus and welfare. Remember that our welfare measurement does not include the psychological "loss utility". Let W be total welfare and $W = v - T$, where

$$T = \frac{1}{4}\left[1 + \mu A_L \triangle_L^2 + (1-\mu) A_H \triangle_H^2\right]$$

(with $A_L = h(\lambda + 1/\lambda) - 1$ and $A_H = l(\lambda + 1/\lambda) - 1$) is the overall taste loss. Notice that, if both firms charge the same price, then each consumer will buy the product she most

[29] It is ready to show $\frac{\partial l}{\partial \lambda} = \frac{1}{2} - \theta - (\frac{1}{2} + \theta)\lambda^{-2}$. Thus, l increases with λ if and only if $\lambda^2 > \frac{1/2+\theta}{1/2-\theta}$ for $\theta < \frac{1}{2}$.

[30] For $\theta = \frac{1}{2}$, p_2 is a constant, and so p_1^H increases with λ.

[31] One can show that $\frac{\partial \pi_1}{\partial \lambda}$ has the sign of $\frac{\theta(r^3-1)}{2}(1+\lambda^{-2}) - \frac{r^3+1}{4}(1-\lambda^{-2})$. Thus, when θ tends to zero, $\frac{\partial \pi_1}{\partial \lambda} < 0$; when θ tends to $\frac{1}{2}$, $\frac{\partial \pi_1}{\partial \lambda} > 0$; and for intermediate θ, π_1 could be non-monotonic with λ.

[32] However, it is also possible that more severe loss aversion will boost industry profit. This will happen at least when θ is close to $\frac{1}{2}$, because at $\theta = \frac{1}{2}$, π_2 is a constant and π_1 increases with λ.

likes, which is socially optimal and leads to the minimum taste loss $\frac{1}{4}$. When consumers exhibit reference dependence and $\theta > 0$, there exists price difference between products, which will cause distortion in product choice. Specifically, when firm 1 charges p_1^i and firm 2 charges p_2, the efficiency loss is

$$(\frac{1}{2}+\theta)(x_i-\frac{1}{2})^2+(\frac{1}{2}-\theta)(y_i-\frac{1}{2})^2=\frac{1}{4}A_i\triangle_i^2$$

where x_i and y_i have been defined in (10) and (11). Consumer surplus is $V = v-T-\pi_1-\pi_2$.

We consider the simple case with $\theta = \frac{1}{2}$ first. In this case, one can check $4T-1 = (\lambda-1)^2/4\lambda$ which goes up with λ, and so more severe loss aversion is detrimental to total welfare (and consumer surplus in the light of footnote 32). This welfare result is mainly driven by the fact that $\triangle_i$ increases with λ and larger price gaps imply greater product-choice distortion and so lower efficiency. For $\theta < \frac{1}{2}$, numerical simulations suggest that W still decreases with λ (see Figure 4 below where from the bottom to the top θ ranges from 0.1 to 0.5), but how V varies with λ depends on θ.[33] Figure 5 below indicates that more severe loss aversion will be beneficial to consumers themselves when θ is relatively low. This is because, when θ is lower, it is more likely that higher λ will decrease all prices (see (13)).

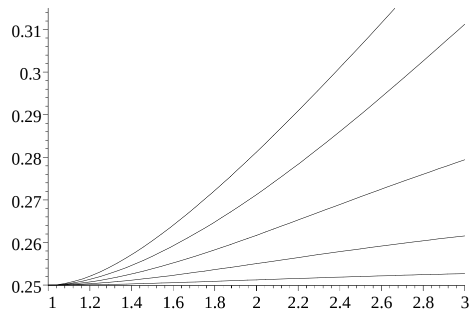

Figure 4: T and λ

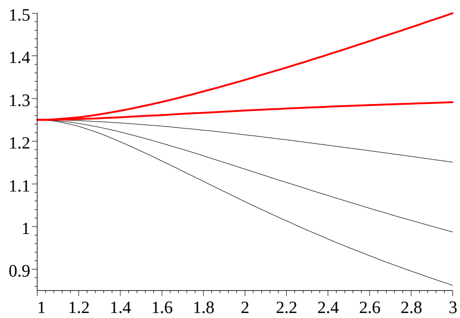

Figure 5: $T+\pi_1+\pi_2$ and λ

Discussion:

Now we discuss how asymmetric degrees of loss aversion in the two dimensions will affect our results. The influence can be seen from two extreme cases. (i) If loss aversion occurs only in the price dimension, all analysis applies if we use $h = (\frac{1}{2}+\theta)\lambda+(\frac{1}{2}-\theta)$ and $l = (\frac{1}{2}-\theta)\lambda+(\frac{1}{2}+\theta)$. Clearly, both h and l increases with λ. One can verify that more severe loss aversion will then always intensify price competition such that all prices and profits decrease with λ and consumer surplus increases with λ. So loss aversion in the price dimension is pro-competitive. (ii) If loss aversion occurs only in the taste dimension, all analysis also applies as long as we use $h = (\frac{1}{2}+\theta)+(\frac{1}{2}-\theta)/\lambda$ and $l = (\frac{1}{2}-\theta)+(\frac{1}{2}+\theta)/\lambda$. Now both h and l decreases with λ. It is not difficult to check that more severe loss version will now always soften price competition such that all prices and profits increase with λ and consumer surplus decreases with λ. So loss aversion in the taste dimension is anti-competitive. In either case, total welfare in our inelastic-demand setting still goes

[33] We should be cautious of this total welfare result, because our unit-demand model does not reflect output efficiency. If higher λ could also give rise to lower prices even in an elastic-demand model, more severe loss aversion could lead to higher total output and so higher efficiency.

down with λ. Our model with symmetric degrees of loss aversion is just a combination of these two extreme cases.

4 Endogenous Prominence: Platform and Advertising Competition

In this section, we discuss two possible ways to endogenize prominence. One story is to consider a platform (for example, a supermarket) on which firms sell products. Suppose firms can set product prices by themselves and the platform can charge each of them a fee proportional to their profit. Then does the platform have an incentive to manipulate consumers' consideration order through adjusting the relative prominence among products (for example, displaying some product more visibly or recommending some product to consumers)? The other story is to consider advertising competition. For example, a consumer might first notice and then consider the product whose adverts first come to her attention, and then the more heavily advertised product is more prominent in the market.[34] Then what is the equilibrium of advertising competition?

The key step of our analysis is to know how θ affects each firm's profit. Without loss of generality, we focus on $\theta > 0$. We first consider the uniform setting in Section 3. It is easy to see that $\pi_2 = p_2/2$ increases with θ by noticing $p_2 = 1/\sqrt{hl}$ and hl falls with θ. One can also show $\pi_1 = \frac{1}{8}(h^{-1/2} + l^{-1/2})^2$ increases with θ.[35] Therefore, a greater prominence difference between firms will benefit *both* firms. The intuition is, when more consumers consider product 1 first, firm 1 will rely more on those strong-taste consumers and so charge the high price more frequently, which will further relax the price competition in an environment where prices are strategic complements. Figure 6 below is a numerical example with $\lambda = 2$. For a general distribution, we have the same result at least in the limit case with λ close to one.[36]

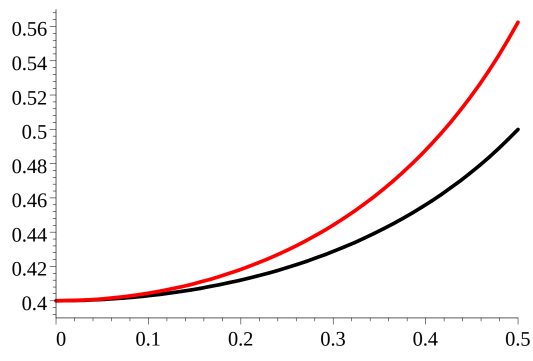

Figure 6: Profit and θ

[34] The advertising story also applies to the case in which the platform can sell prominent placements to firms.

[35] This is because the derivative of the bracket term with respect to θ has the sign of $\frac{1}{l\sqrt{l}} - \frac{1}{h\sqrt{h}} > 0$. One can further show that both profit functions are convex in θ.

[36] When $\lambda = 1 + \varepsilon$ and ε tends to zero, using the approximations in footnote 24, we can approximate equilibrium profits as $\pi_1 \approx \frac{p}{2} + (\frac{3}{2}\theta^2 - \frac{1}{4})p\varepsilon^2$ and $\pi_2 \approx \frac{p}{2} + (\theta^2 - \frac{1}{4})p\varepsilon^2$. Both of them are increasing and convex in θ.

Given such a result, the answer to the platform story is easy to see: the platform will make one product more prominent than the other. Now let us consider the advertising-competition story. If advertising competition occurs prior to price competition and the advertising technology is symmetric among firms, then two observations immediately follow. First, both firms advertising at the same positive level is not an equilibrium outcome. This is because otherwise each firm could then improve profit by reducing advertising unilaterally. Second, if the two firms advertise at different intensities, then the firm advertising less must actually not advertise at all. Otherwise, it could always increase profit by reducing advertising and further enlarging the prominence difference. Thus, if we focus on pure-strategy advertising equilibrium, the two firms will either differentiate their advertising intensities (i.e., one advertises and the other does not) or both not advertise. Furthermore, if advertising is not too costly, the latter can neither be an equilibrium outcome, which leaves asymmetric advertising as the only possible pure-strategy equilibrium. Hence, consumer reference dependence might give rise to endogenous asymmetric prominence as a result of advertising competition.[37]

5 Reference Dependence and Product Qualities

We now return to the setting with exogenous prominence and explore how consumer reference dependence could shape firms' product quality choices. We will first study the properties of equilibrium with two products differing in their qualities. The main finding is, when mixed-strategy equilibrium occurs, a relative increase of the prominent product's quality will soften price competition and benefit both firms. We then deduce that the less prominent firm may want to choose a lower quality level than its prominent rival even if improving quality is costless.

Let v_i be product i's gross utility, and define $\triangle = v_1 - v_2$ to be the quality difference between the two products. We assume that firm 1 is exogenously more prominent and $\frac{1}{2} + \theta$ of consumers will consider it first. Denote by $\hat{x}$ the solution to $v_1 - x = v_2 - (1 - x)$. Then the consumer at

$$\hat{x} = \frac{1}{2} + \frac{\triangle}{2}$$

is indifferent between the two products if there is no price difference. To make the situation interesting, we focus on mild quality difference $\triangle \in (-1, 1)$ such that $\hat{x} \in (0, 1)$. That is, no firm will occupy the whole market if they charge the same price. For tractability, we focus on the uniform setting again. One can check that now the demand functions become[38]

$$q_1 = \hat{x} + \frac{i}{2}(p_2 - p_1), \quad q_2 = 1 - \hat{x} + \frac{i}{2}(p_1 - p_2),$$

where $i = h$ if $p_1 < p_2$ and $i = l$ if $p_1 > p_2$.

[37] It is straightforward to write down a formal model of advertising competition in which we can show the existence of pure-strategy advertising equilibrium under certain conditions. We can also discuss the properties of possible mixed-strategy advertising equilibrium. The details are available from the author.

[38] We are implicitly assuming that consumers regard personal taste and product quality together as the product dimension. This assumption is reasonable when consumers only have an overall impression of product satisfaction.

We derive equilibrium first. Now the equilibrium will be either pure-strategy or mixed-strategy depending on the magnitude of quality difference relative to the strength of reference dependence. If there is no consumer reference dependence and $\triangle > 0$, clearly firm 1 will charge a higher price than firm 2. When the psychological bias emerges, charging a lower price than its rival will also become an attractive strategy to firm 1, because that will expand its market share substantially (remember firm 1 is more prominent). Hence, we expect that, for fixed quality difference, when the psychological bias becomes stronger gradually, the equilibrium should evolve from a pure-strategy one to a mixed-strategy one. Let us keep the notation $r = \sqrt{\frac{h}{l}}$ which increases with λ and θ, and define

$$a(x) = \frac{2-x}{x}; \quad b(x) = 3\sqrt{\frac{1-x}{5-5x-x^2}}.$$

Then we have the following result:

Proposition 5 *With the uniform distribution and the quality difference* $\triangle \in (-1, 1)$,

(i) we have the mixed-strategy equilibrium with an interior solution (i.e., $q_1^L < 1$*) if*

$$\frac{1}{r} < \frac{a(\hat{x})}{3} < r < a(\hat{x}) \text{ for } \hat{x} \in (0, \sqrt{3}-1), \tag{14}$$

and we have the mixed-strategy equilibrium with a corner solution (i.e., $q_1^L = 1$*) if*

$$\begin{cases} r > a(\hat{x}) & \text{for } \hat{x} \in (0, \sqrt{3}-1) \\ r > b(\hat{x}) & \text{for } \hat{x} \in (\sqrt{3}-1, \tilde{x}), \end{cases} \tag{15}$$

where $\tilde{x} \approx 0.854$ *is the solution to* $5 - 5x - x^2 = 0$.

(ii) We have the pure-strategy equilibrium with $p_1 > p_2$ *if*

$$\begin{cases} r < 3/a(\hat{x}) & \text{for } \hat{x} \in (1/2, \sqrt{3}-1) \\ r < b(\hat{x}) & \text{for } \hat{x} \in (\sqrt{3}-1, \tilde{x}) \\ \text{any } r & \text{for } \hat{x} > \tilde{x}, \end{cases} \tag{16}$$

and we have the pure-strategy equilibrium with $p_1 < p_2$ *if*

$$r < \frac{a(\hat{x})}{3} \text{ for } \hat{x} \in (0, 1/2). \tag{17}$$

Figure 7 below describes the relationship between equilibrium and the parameter pair $(r, \hat{x})$, where the horizontal axis is $\hat{x}$ and the vertical axis is r.

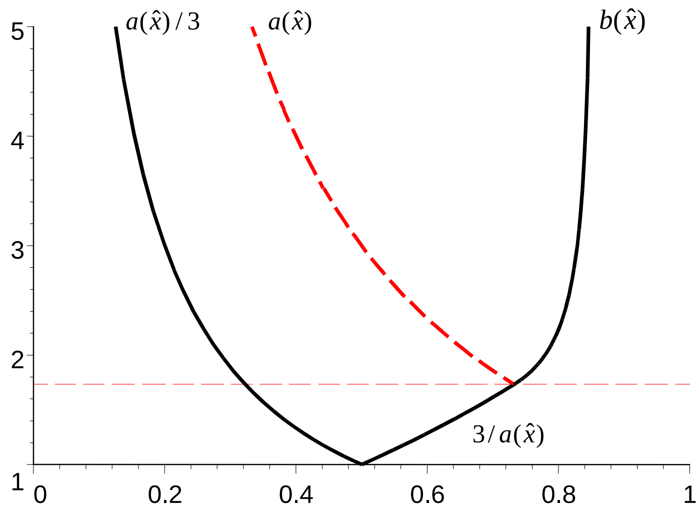

Figure 7: Equilibrium and $(r, \hat{x})$

In the area below the solid lines, we have pure-strategy equilibrium. In the area above them, we have mixed-strategy equilibrium. This area is further divided by the dashed line $a(\hat{x})$. Below that, we have the interior-solution equilibrium, and above that, we have the corner-solution equilibrium. When $r < \sqrt{3}$ (i.e., below the horizontal dashed line), there is no mixed-strategy equilibrium with a corner solution for any $\hat{x}$. We observe that, for fixed r, mixed-strategy equilibrium is more likely to occur for smaller quality difference; and for fixed $\hat{x} < \tilde{x} \approx 0.854$, mixed-strategy equilibrium is more likely to occur for greater r (so for greater θ or λ). (In particular, when there is no quality difference (i.e., when $\hat{x} = \frac{1}{2}$), we only have mixed-strategy equilibrium.) These two observations illustrate Proposition 3.

We now turn to investigate the properties of equilibrium. In the pure-strategy equilibrium, we can show

$$\begin{aligned} p_1 &= \frac{2}{i} \cdot \frac{1+\hat{x}}{3}, \quad \pi_1 = \frac{2}{i}\left(\frac{1+\hat{x}}{3}\right)^2; \\ p_2 &= \frac{2}{i} \cdot \frac{2-\hat{x}}{3}, \quad \pi_2 = \frac{2}{i}\left(\frac{2-\hat{x}}{3}\right)^2, \end{aligned}$$

where $i = l$ if $\hat{x} > \frac{1}{2}$ and $i = h$ if $\hat{x} < \frac{1}{2}$. It is clear that p_1 and π_1 increase with $\hat{x}$ while p_2 and π_2 decrease with $\hat{x}$. Thus, a relative increase of firm 1's quality will benefit firm 1 but hurt firm 2. This is consistent with the result in the orthodox model with $\lambda = 1$.

However, the situation is very different in the mixed-strategy equilibrium. Our discussion is based on the interior-solution case. (In Appendix A.5 we establish the similar results in the corner-solution case.) First, from (33) and (34), we see that *all* prices increase with $\hat{x}$ and μ decreases with $\hat{x}$ (i.e., firm 1 will charge the high price more frequently when its relative quality rises). Second, following the proof of Proposition 5, simple calculation yields

$$\pi_1 = \frac{(1+r)^2}{2h}\hat{x}^2, \quad \pi_2 = \frac{2r}{3h}(2-\hat{x})\hat{x}.$$

It is clear that *each* firm's profit increases with $\hat{x}$ since $\hat{x} < 1$. That is, a relative increase of the prominent firm's quality will benefit *both* firms.

The results concerning firm 1 are not surprising, and here we try to understand the results concerning firm 2. When firm 1's relative quality increases, several forces affect firm 2's pricing incentive. Let us see its first-order condition $p_2 = q_2^e/(-\frac{\partial q_2^e}{\partial p_2})$, where q_2^e is its expected demand defined in (32). (i) Given prices, higher $\hat{x}$ reduces q_2^e directly since firm 2 is then relatively less favored by consumers. This will drive firm 2 to lower its price. (ii) Higher $\hat{x}$ causes higher prices of firm 1 and lower μ, which will enhance q_2^e and give firm 2 an incentive to raise its price. (Note that this is the standard strategic effect in an environment where prices are strategic complements.) (iii) Lower μ also decreases $-\frac{\partial q_2^e}{\partial p_2} = \frac{1}{2}(\mu h + (1-\mu)l)$ (i.e., makes firm 2's expected demand less price responsive). This is because consumers in aggregate are less price sensitive when firm 1 charges the high price. This will further motivate firm 2 to raise its price. The first two effects are standard, but the third one is only present in the mixed-strategy equilibrium caused by consumers reference dependence. Our result implies that the latter two positive effects together outweigh the first negative one. In light of this price result, the profit result is not difficult to understand.

We further illustrate the above results in the following two graphs which are based on a numerical example with $\theta = \frac{1}{2}$ and $\lambda = 2$.

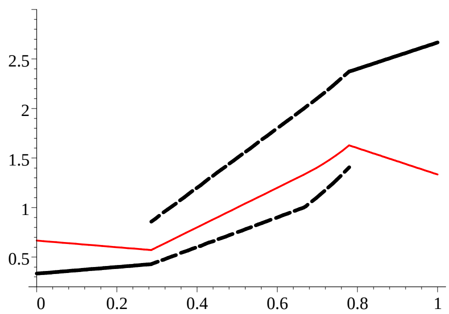

Figure 8: Equilibrium Prices and $\hat{x}$

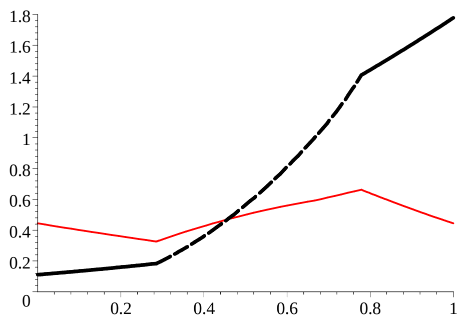

Figure 9: Equilibrium Profit and $\hat{x}$

They describe how equilibrium prices and profit vary with $\hat{x}$, respectively. (The thick lines correspond to firm 1, and the dashed parts correspond to the mixed-strategy equilibrium.)

The main implication of the above results is, when the quality difference between the two products is not too large (such that the mixed-strategy pricing equilibrium occurs), the less prominent firm has *no* incentive to improve its quality slightly even if it is costless to do so. This is because it does not want to trigger the prominent firm to charge a low price more frequently. Put differently, the less prominent firm even has an incentive to reduce its quality even if doing so does not save any costs. We then deduce, if one firm is more prominent than the other and if there is a (simultaneous) quality choice stage before the price competition, choosing the same positive quality level will never be an equilibrium outcome. This is because at $\hat{x} = \frac{1}{2}$ we must have the mixed-strategy equilibrium and then π_2 increases with $\hat{x}$ (i.e., reducing quality is profitable for firm 2). This also implies, at least when the range of feasible quality levels is restricted such that there is no possibility for pure-strategy pricing equilibrium, the less prominent firm will choose a lower quality level than its prominent rival even if improving quality is costless.[39,40]

We summarize the main results in the following proposition:

Proposition 6 *(i) With the uniform distribution and product 1's relative quality "advantage"* $\triangle \in (-1, 1)$*, the prominent firm 1's prices and profit always increase with* $\triangle$*. Firm 2's price and profit decrease with* $\triangle$ *in the pure-strategy equilibrium, but increase with* $\triangle$ *the mixed-strategy equilibrium.*

(ii) Suppose there is a quality choice stage prior to the price competition. Then in equilibrium the two firms will never choose the same quality level. Moreover, at least when the range of feasible quality levels is relatively narrow, the less prominent firm will choose a lower quality level than its prominent rival even if improving quality is costless.

Discussion:

The reference-dependence effect in our model can be regarded as a kind of switching cost. But it occurs only if the second product is relatively inferior to the first one in at least one aspect. Readers may wonder whether the results we have derived in this paper could be replicated by using an exogenous cost involved in moving from one product to the other. In that setting, all else equal, the prominent firm will also earn more than the other, but we cannot establish other main results. Suppose the cost is s. Then, with the same notation, the demand functions are:

$$q_1(p_1) = \hat{x} + \theta s + \frac{p_2 - p_1}{2}; \quad q_2(p_2) = 1 - (\hat{x} + \theta s) + \frac{p_1 - p_2}{2}.$$

Since they are smooth functions, no firm will randomize its price. If s is appropriate such that we have an interior-solution equilibrium, then the equilibrium prices and profits are:

$$\begin{aligned} p_1 &= \frac{2}{3}(1 + \hat{x} + \theta s), \quad \pi_1 = \frac{2}{9}(1 + \hat{x} + \theta s)^2; \\ p_2 &= \frac{2}{3}(2 - \hat{x} - \theta s), \quad \pi_2 = \frac{2}{9}(2 - \hat{x} - \theta s)^2. \end{aligned}$$

Clearly, making one firm more prominent or improving its product quality will benefit this firm but harm the other, so our results on advertising and quality choice will not emerge either.

6 Conclusion

This paper has examined the impacts of consumer reference dependence on market competition. In particular, if consumers take some *real* product in the market as the reference point and exhibit loss aversion, the firm whose product is more likely to be taken as the

[39] This result could even hold for a broader range of quality levels. Let us see the following simple example. Suppose the free quality feasible set is $v_i \in [v, v+1]$ (so $\triangle \in [-1, 1]$). Then it is clear that the prominent firm 1 will always pick the highest quality level $v_1 = v + 1$. Firm 2's problem is thus to choose $\hat{x}$ between $[\frac{1}{2}, 1]$. From Figure 9, we see that it will choose $x_r > \frac{1}{2}$ (i.e., $v_2 < v_1$), where x_r is the upper limit value of $\hat{x}$ such that we have a mixed-strategy pricing equilibrium at r.

[40] Another implication of our profit results is that, in the case with a platform as we have discussed in Section 4, for any $\triangle \in (-1, 1)$, the platform will display the higher-quality product more prominently because that will lead to higher industry profit. The proof is available from the author.

reference point will randomize its price over a high and a low one. This offers a new explanation of sales. The welfare impact is that consumer reference dependence could harm firms and benefit consumers by intensifying price competition. We also find that a greater prominence difference between firms can soften price competition, and so *ex ante* identical firms may tend to differentiate their advertising intensities; a relative increase of the prominent product's quality can also soften price competition, and so the less prominent firm might supply a lower-quality product even if improving quality is costless. By comparing to the existing research in the literature, our price result indicates that the market implications of reference dependence may be sensitive to the specification of reference points. In particular, it is crucial whether consumers' reference points are independent of or influenced by firms' *actual* decisions.

Some related topics deserve future studies. First, it is desirable to explore the impact of consumer reference dependence in a dynamic competition setting if consumers purchase the product frequently. The impact could be different because there the historical purchase may influence the reference point. Second, it is also interesting to investigate how a firm supplying several (vertically) differentiated products could benefit from manipulating the order in which consumers consider or try products (for example, by recommending some products or by adjusting the product launch strategy), especially when consumers are overconfident that trying some product first would not influence their subsequent preferences.

Our work also intends to delivery the message that the order in which people consider options deserves more attention in economics, even if there are no explicit costs involved in moving from one option to the other.[41] In traditional economics, the consideration order has no impact on people's choices as long as they face the same choice set eventually. However, with some behavioral biases, it may become an important choice determinant factor. Reference dependence is such a bias and this paper has studied its market implications. Other biases also deserve research. For example, when processing informative signals sequentially, people with confirmatory bias tend to stick to the opinion formed in the early stage (Rabin and Schrag (1999), for instance), and so the early signals might be over weighted. This may motivate some market players to manipulate the order in which other players receive signals.

A Appendix

A.1 Proof of Proposition 1

We prove a more general result:

Claim 1 *If the distribution of consumers satisfies $F(\frac{1}{2}) = \frac{1}{2}$ (of which the symmetric distribution is a special case), then the price competition has no pure-strategy Nash equilibrium.*

[41] See Rubinstein and Salant (2006) and Salant (2007) for some research about choice from a list (i.e., an ordered choice set) in the decision theory context.

Define

$$q_i'(p_1,p_2) = \frac{\partial q_i(p_1,p_2)}{\partial p_i}; \quad \pi_i'(p_1,p_2) = \frac{\partial \pi_i(p_1,p_2)}{\partial p_i}.$$

If $p_1 = p_2 = \hat{p} > 0$ were an equilibrium, we must have

$$\lim_{p_1 \to \hat{p}^+} \pi_1'(p_1,\hat{p}) \le 0 \le \lim_{p_1 \to \hat{p}^-} \pi_1'(p_1,\hat{p})$$

where

$$\pi_1'(p_1,\hat{p}) = q_1(p_1,\hat{p}) + p_1 q_1'(p_1,\hat{p}).$$

However, we have

$$\lim_{p_1 \to \hat{p}^-} q_1'(p_1,\hat{p}) = -\frac{\lambda}{2} f(\frac{1}{2}) < \lim_{p_1 \to \hat{p}^+} q_1'(p_1,\hat{p}) = -\frac{1}{2\lambda} f(\frac{1}{2})$$

which leads to

$$\lim_{p_1 \to \hat{p}^-} \pi_1'(p_1,\hat{p}) < \lim_{p_1 \to \hat{p}^+} \pi_1'(p_1,\hat{p}).$$

This is a contradiction, so $p_1 = p_2 = \hat{p}$ cannot be an equilibrium.

Now suppose $p_1 > p_2 > 0$ were an equilibrium. First of all, it is impossible that $q_2 = 1$, since firm 1 would then choose $p_2 - \varepsilon$ to earn a positive profit. Since each firm's demand function is smooth around its own equilibrium price in such an asymmetric equilibrium, we must have

$$q_1 + p_1 q_1'(p_1,p_2) = q_2 + p_2 q_2'(p_1,p_2) = 0.$$

However, it is always true that $q_1'(p_1,p_2) = q_2'(p_1,p_2)$ for $p_1 \ne p_2$. Therefore, we have

$$\frac{q_1(p_1,p_2)}{q_2(p_1,p_2)} = \frac{p_1}{p_2}.$$

On the other hand, when $p_1 > p_2$, $q_1(p_1,p_2) < \frac{1}{2} < q_2(p_1,p_2)$ since $F(\frac{1}{2}) = \frac{1}{2}$. This again leads to a contradiction, so $p_1 > p_2$ can neither be an equilibrium. Using the same logic, we can also exclude the possibility of $p_1 < p_2$.

A.2 Proof of Proposition 2

We show a more general result:

Claim 2 *In the single-reference-product case, given Assumption 1 and $F(\frac{1}{2}) = \frac{1}{2}$, there exists a mixed-strategy equilibrium in which firm 1 randomizes over p_1^L and p_1^H and firm 2 charges a constant price p_2.*

Define

$$z_L = \frac{1}{2} + \frac{\lambda}{2}(p_2 - p_1^L); \quad z_H = \frac{1}{2} + \frac{1}{2\lambda}(p_2 - p_1^H). \tag{18}$$

They are the locations of consumers who are indifferent between the two products when firm 1 charges p_1^L and p_1^H, respectively. Then the demand functions are

$$q_1(p_1^i, p_2) = F(z_i),\ i = L, H; \quad q_2^e = \mu\left[1 - F(z_L)\right] + (1-\mu)\left[1 - F(z_H)\right].$$

Let $F_i = F(z_i)$ and $f_i = f(z_i)$. Then, in the interior-solution case, condition (i) requires

$$\mu(1 - F_L) + (1 - \mu)(1 - F_H) = \frac{p_2}{2}\left(\mu\lambda f_L + \frac{1-\mu}{\lambda} f_H\right). \tag{19}$$

Condition (ii) requires

$$F_L = \frac{\lambda}{2} p_1^L f_L \Leftrightarrow \frac{F_L}{f_L} + z_L = \frac{\lambda}{2} p_2 + \frac{1}{2}, \tag{20}$$

$$F_H = \frac{1}{2\lambda} p_1^H f_H \Leftrightarrow \frac{F_H}{f_H} + z_H = \frac{1}{2\lambda} p_2 + \frac{1}{2}, \tag{21}$$

where we have used $p_1^L = p_2 + (1 - 2z_L)/\lambda$ and $p_1^H = p_2 + \lambda(1 - 2z_H)$ from (18). The indifference condition (iii) is

$$p_1^L F_L = p_1^H F_H. \tag{22}$$

(19)–(22) define an equilibrium if **(a)** they have a solution (μ, p_2, z_L, z_H) with $\mu \in (0, 1)$ and $1 \geq z_L > \frac{1}{2} > z_H \geq 0$, and **(b)** no firm has global profitable deviation given its rival's strategy.

We now show that (a) and (b) are indeed satisfied given $F(\frac{1}{2}) = \frac{1}{2}$ and Assumption 1. Since our proof may involve asymmetric distributions, we modify firm 2's demand function first:

$$q_2(p_2 < p_1) = 1 - F\left(\frac{1}{2} + \frac{1}{2\lambda}(p_2 - p_1)\right), \quad q_2(p_2 > p_1) = 1 - F\left(\frac{1}{2} + \frac{\lambda}{2}(p_2 - p_1)\right).$$

One complication is, when firm 1 charges p_1^L, it may occupy the whole market (i.e., the corner-solution case). Thus, we need to deal with several cases separately. Define $k_1 = F(\frac{1}{2})/f(\frac{1}{2})$ and $k_2 = F(1)/f(1)$. For logconave F, $k_1 < k_2$.

(1) If $\lambda^2 < (k_2 + \frac{1}{2})/k_1$, we have the mixed-strategy equilibrium with an interior solution. First, since p_2 will never be negative, $z_H \geq 0$ is no problem according to (21). We then prove $1 > z_L > \frac{1}{2} > z_H$, which is true if

$$\frac{2}{\lambda} k_1 < p_2 < 2\lambda k_1.$$

Note that $z_L < 1$ requires $p_2 < \frac{2}{\lambda}(k_2 + \frac{1}{2})$, which will be implied by $p_2 < 2\lambda k_1$ given the condition $\lambda^2 \leq (k_2 + \frac{1}{2})/k_1$. Now we show that, given (20)–(21), (22) does have a solution $p_2 \in (\frac{2}{\lambda}k_1, 2\lambda k_1)$. If $p_2 = \frac{2}{\lambda}k_1$, then (20) and (21) require $z_L = \frac{1}{2}$ (i.e., $p_1^L = p_2$) and $0 < z_H < \frac{1}{2}$, so

$$\pi_1(p_1^H, p_2) = \arg\max_{p \geq p_2} p q_1(p, p_2) > p_2 q_1(p_2, p_2) = \pi_1(p_1^L, p_2).$$

Similarly, if $p_2 = 2\lambda k_1$, then $z_H = \frac{1}{2}$ (i.e., $p_1^H = p_2$) and $1 > z_L > \frac{1}{2}$, so

$$\pi_1(p_1^L, p_2) = \arg\max_{p \leq p_2} p q_1(p, p_2) > p_2 q_1(p_2, p_2) = \pi_1(p_1^H, p_2).$$

Then the continuity of the profit function implies our result. The last step is to show that (19) has a solution $\mu \in (0, 1)$. According to (31) in the proof of Proposition 4 below, it is actually true given $z_L > \frac{1}{2} > z_H$ and $F(\frac{1}{2}) = \frac{1}{2}$.

We then discuss condition (b). Under Assumption 1, $F(z_i)$ is logconcave in p_1^i, so for firm 1, the necessary conditions in (20) and (21) are also sufficient for optimization. For firm 2, there is no profitable deviation on $[p_1^L, p_1^H]$ since Assumption 1 guarantees that its profit function on this interval is quasi-concave. But does it have any profitable deviation to $p_2 < p_1^L$ or $p_2 > p_1^H$? Under Assumption 1, its profit function is also quasi-concave in either case. (Note that this does not mean that firm 2's whole profit function is quasi-concave.) Thus, a sufficient condition for neither case to be a profitable deviation is that

$$\left.\frac{\partial \pi_2^e}{\partial p_2}\right|_{p_2 \to (p_1^L)^-} > 0 \text{ and } \left.\frac{\partial \pi_2^e}{\partial p_2}\right|_{p_2 \to (p_1^H)^+} < 0,$$

where $\pi_2^e = pq_2^e(p)$. They are actually true because Assumption 1 and $p_2 \in (p_1^L, p_1^H)$ imply

$$\left.\frac{\partial \pi_2^e}{\partial p_2}\right|_{p_2 \to (p_1^L)^+} \geq 0 \text{ and } \left.\frac{\partial \pi_2^e}{\partial p_2}\right|_{p_2 \to (p_1^H)^-} \leq 0,$$

and the two kinks of q_2^e are both outward.

(2) We also have the mixed-strategy equilibrium with an interior solution when $(k_2 + \frac{1}{2})/k_1 < \lambda^2 < k_2 f(\bar{z})/F(\bar{z})^2$ if $f(1) \leq 2$, where $\bar{z} \leq \frac{1}{2}$ is the solution to

$$\frac{k_2}{k_2 + 1/2} = \frac{F(\bar{z})^2}{F(\bar{z}) + (\bar{z} - 1/2)f(\bar{z})}. \tag{23}$$

One can verify that the solution $\bar{z} \leq \frac{1}{2}$ exists and $(k_2 + \frac{1}{2})/k_1 \leq k_2 f(\bar{z})/F(\bar{z})^2$ if and only if $f(1) \leq 2$.[42] The same proof as in the above still applies except that now we need to show $p_2 \in (\frac{2}{\lambda}k_1, \frac{2}{\lambda}(k_2 + \frac{1}{2}))$. When $p_2 = \frac{2}{\lambda}k_1$, the same proof as before implies $\pi_1(p_1^L, p_2) < \pi_1(p_1^H, p_2)$. When $p_2 = \frac{2}{\lambda}(k_2 + \frac{1}{2})$, (20) implies $z_L = 1$, so $p_1^L = p_2 - \frac{1}{\lambda} = \frac{2}{\lambda}k_2$ and we get $\pi_1(p_1^L, p_2) = \frac{2}{\lambda}k_2$. Meanwhile, (22) implies $\pi_1(p_1^H, p_2) = 2\lambda\frac{F_H^2}{f_H}$, where z_H now satisfies

$$\frac{F_H}{f_H} + z_H - \frac{1}{2} = \frac{1}{\lambda^2}(k_2 + \frac{1}{2}). \tag{24}$$

Then $\pi_1(p_1^L, p_2) > \pi_1(p_1^H, p_2)$ if $k_2 > \lambda^2\frac{F_H^2}{f_H}$ which is further equivalent to

$$\frac{k_2}{k_2 + 1/2} > \frac{F_H^2}{F_H + (z_H - 1/2)f_H}$$

by using (24). This is true if $z_H > \bar{z}$, which is implied by $\lambda^2 < k_2 f(\bar{z})/F(\bar{z})^2$ by appealing to (24) again.

(3) We have the mixed-strategy equilibrium with a corner solution if $\lambda^2 \geq (k_2 + \frac{1}{2})/k_1$ (given $f(1) > 2$) or if $\lambda^2 \geq k_2 f(\bar{z})/F(\bar{z})^2$ (given $f(1) \leq 2$). Under Assumption 1, we have the mixed-strategy equilibrium with a corner solution if the following conditions are satisfied:

(i) $p_1^H = \arg\max_{p \geq p_2} p \cdot q_1(p > p_2)$;

(ii) p_1^L is determined by $z_L = 1$, i.e., $p_1^L = p_2 - \frac{1}{\lambda}$;

[42] Let z_0 satisfy $F(z_0) + (z_0 - \frac{1}{2})f(z_0) = 0$. Then the right-hand side of (23) is a decreasing and positive function on $(z_0, \frac{1}{2})$ (which varies from ∞ to $\frac{1}{2}$) and a decreasing and negative function on $(0, z_0)$. Therefore, $\bar{z} \in (z_0, \frac{1}{2}]$ when $k_2 \geq \frac{1}{2}$ (i.e., $f(1) \leq 2$).

(iii) p_1^L is the best response to p_2: $\frac{\partial pq_1(p<p_2)}{\partial p} \leq 0$ at $p = p_1^L$;
(iv) The indifference condition: $p_1^L = \pi_1(p_1^H, p_2)$;
(v) p_2 is the best response to (p_1^L, p_1^H, μ):

$$\frac{p_2}{2}\left[\mu\lambda f(1) + \frac{1-\mu}{\lambda} f(z_H)\right] = q_2^e = (1-\mu)(1-F_H). \tag{25}$$

We need to show that the above conditions have a solution with $p_1^L > 0$, $p_2 \leq p_1^H$ and $\mu \in (0,1)$.

First, under Assumption 1, (i) is again equivalent to (21), so we need $z_H \leq \frac{1}{2}$ (i.e., $p_2 \leq 2\lambda k_1$) for $p_2 \leq p_1^H$. Second, (iii) requires $1 - \lambda f(1) p_1^L/2 \leq 0$. Using condition (ii), this requires $p_2 \geq \frac{2}{\lambda}(k_2 + \frac{1}{2})$ (which also implies $p_1^L > 0$). Therefore, we need $p_2 \in [\frac{2}{\lambda}(k_2 + \frac{1}{2}), 2\lambda k_1]$. (Note that this interval is not empty given our conditions.) When p_2 tends to $2\lambda k_1$, $z_H = \frac{1}{2}$ (i.e., $p_1^H = p_2$) and condition (iii) is satisfied, so $p_1^L = \max_{p \leq p_2} pq_1(p < p_2) > p_2 q_1(p_2, p_2) = \pi_1(p_1^H, p_2)$. When p_2 tends to $\frac{2}{\lambda}(k_2 + \frac{1}{2})$, we want to have $p_1^L < \pi_1(p_1^H, q_2)$, i.e., $k_2 < \lambda^2 F_H^2 / f_H$, where z_H is again determined by (24). Reversing the proof in case (2) can prove this inequality.

Finally, we prove that condition (v) has a solution $\mu \in (0,1)$. When $\mu = 1$, the left-hand side of (25) is positive but the right-hand side is zero. When $\mu = 0$, the left-hand side is $\frac{p_2}{2\lambda} f_H$ and the right-hand side is $1 - F_H$. Since $p_2 < 2\lambda k_1$, the former is smaller if $k_1 < (1 - F_H)/f_H$. This is of course true given $z_H < \frac{1}{2}$ and $F(\frac{1}{2}) = \frac{1}{2}$.

A.3 Proof of Proposition 3

We prove Proposition 3 in the following two claims:

Claim 3 *In the single-reference-product case, given Assumption 1, for fixed $\lambda > 1$, there is $\varepsilon_1 > 0$ such that, when $\left|F(\frac{1}{2}) - \frac{1}{2}\right| < \varepsilon_1$, there exists a similar mixed-strategy equilibrium as that defined in Proposition 2.*

As we have seen, $F(\frac{1}{2}) = \frac{1}{2}$ is only used in proving $\mu \in (0,1)$. Hence, we only need to revisit that step. (i) The interior-solution case. For $\mu \in (0,1)$, we need $g_H = -g_L > 0$ in (31). Let $\hat{z}$ satisfy

$$1 - 2F(\hat{z}) - f(\hat{z})(\hat{z} - \frac{1}{2}) = 0.$$

Then we are done if $z_L > \hat{z} > z_H$, since $\frac{1-2F(x)}{f(x)}$ is a decreasing function given logconcave f. For fixed $\lambda > 1$, the solution (p_2, z_L, z_H) from (20)–(22) must satisfy $z_L > \frac{1}{2} > z_H$. Thus, what we need is that $\hat{z}$ is close to $\frac{1}{2}$, which is true if $F(\frac{1}{2})$ and $\frac{1}{2}$ are close to each other enough. (ii) The corner-solution case. For fixed $\lambda > 1$, we have $z_H < \frac{1}{2}$ and so $(1 - F_H)/f_H > \left(1 - F(\frac{1}{2})\right)/f(\frac{1}{2})$. The latter tends to k_1 if $F(\frac{1}{2})$ is close to $\frac{1}{2}$.

Claim 4 *In the single-reference-product case, given Assumption 1, (i) for fixed $\lambda > 1$, there exists $\varepsilon_2 > 0$ such that, when $\left|F(\frac{1}{2}) - \frac{1}{2}\right| < \varepsilon_2$, there is no pure-strategy equilibria. (ii) For fixed $\left|F(\frac{1}{2}) - \frac{1}{2}\right| > 0$, there exists $\lambda^* > 1$ such that, when $\lambda < \lambda^*$, there is a pure-strategy equilibrium with $p_1 > p_2$ if $F(\frac{1}{2}) > \frac{1}{2}$ and $p_1 < p_2$ if $F(\frac{1}{2}) < \frac{1}{2}$.*

(i) We only deal with the case with $F(\frac{1}{2}) > \frac{1}{2}$. (The other one is similar.) First of all, it is ready to check that $p_1 \le p_2$ cannot even satisfy the first-order conditions given the demand functions for $p_1 < p_2$. If $p_1 > p_2$, the demand functions are

$$q_1 = F\left(\frac{1}{2} + \frac{1}{2\lambda}(p_2 - p_1)\right),\ q_2 = 1 - F\left(\frac{1}{2} + \frac{1}{2\lambda}(p_2 - p_1)\right).$$

If there exists an equilibrium with $p_1 > p_2$, then the necessary conditions are $q_i + p_i \frac{\partial q_i}{\partial p_i} = 0$, which imply

$$\frac{F(z)}{1 - F(z)} = \frac{p_1}{p_2},\ \ \frac{F(z)}{f(z)} = \frac{p_1}{2\lambda},$$

where $z = \frac{1}{2} + \frac{1}{2\lambda}(p_2 - p_1)$. Using $p_2 = p_1 - \lambda(1 - 2z)$, we get

$$\frac{F(z)}{1 - F(z)} = \frac{1}{1 - (1 - 2z)f(z)/2F(z)}. \tag{26}$$

The necessary conditions define a pure-strategy equilibrium with $p_1 > p_2$ if (a) the equation (26) has a solution $z < \frac{1}{2}$ and (b) given p_i, firm j has no global deviation. In the following, we will show that condition (a) is always true given $F(\frac{1}{2}) > \frac{1}{2}$, while condition (b) will fail if $F(\frac{1}{2})$ is too close to $\frac{1}{2}$.

Logconcave F implies that $\frac{f(z)}{F(z)}$ decreases with z. Then the right-hand side of (26) has the following shape: there exists $z_0 \in (0, \frac{1}{2})$ satisfying $1 - (1 - 2z_0)f(z_0)/2F(z_0) = 0$, such that it decreases from 0 to $-\infty$ when $z \in (0, z_0)$, and it decreases from $+\infty$ to 1 when $z \in (z_0, \frac{1}{2})$. Meanwhile, the left-hand side of (26) is an increasing function of z, and when $F(\frac{1}{2}) > \frac{1}{2}$, we have $\frac{F(1/2)}{1-F(1/2)} > 1$. Thus, (26) must have a solution $z \in (z_0, \frac{1}{2})$. If $F(\frac{1}{2}) \to (\frac{1}{2})^+$, then the solution z to (26) tends to $\frac{1}{2}$ and so $p_1 \to p_2$. For fixed $\lambda > 1$, then firm 1 must have a profitable deviation due to its inward demand kink.

(ii) When will (b) be satisfied? We only need to worry about firm 1's possible deviation. For fixed $F(\frac{1}{2}) - \frac{1}{2} > 0$, $z < \frac{1}{2}$ is fixed, and so $p_1 - p_2 = \lambda(1 - 2z)$ is bounded away from zero. If $\lambda \to 1$, then firm 1's demand curve tends to be smooth everywhere and so the necessary conditions should also be sufficient under Assumption 1.

A.4 Proof of Proposition 4

We first need to prove the following lemma:

Lemma 1 *Suppose $f(x)$ is symmetric and logconcave on $[0, 1]$.*

(i) The function

$$\phi(x) = \frac{1/2 - F(x)}{(1/2 - x)f(x)}$$

is symmetric on $[0, 1]$. For the uniform distribution, $\phi(x) = 1$. Beyond this special case, $\phi(x)$ strictly decreases on $[0, \frac{1}{2})$ and strictly increases on $(\frac{1}{2}, 1]$.

(ii) The function

$$A(x) = \frac{F(x)^2}{F(x) + (x - 1/2)f(x)}$$

decreases on $(z_0, \frac{1}{2})$ *and increases on* $(\frac{1}{2}, 1]$*, where* z_0 *satisfies* $F(z_0)+(z_0-1/2)f(z_0)=0$*. For any* $\varepsilon \in (0, \frac{1}{2}-z_0)$*,* $A(\frac{1}{2}-\varepsilon) > A(\frac{1}{2}+\varepsilon)$*.*

(iii) The function

$$B(x) = F(x)^2\,[2\phi(x)+1]$$

increases on $[0,1]$*.*

Proof. (i) The symmetry is easy to see since $f(x)$ is symmetric. Now we prove $\phi(x)$ is strictly decreasing on $[0, \frac{1}{2}]$ for non-uniform distributions. Since $f(x)$ is logconcave and symmetric, it must increase on $[0, \frac{1}{2}]$, and so $F(x)$ is convex on $[0, \frac{1}{2}]$. One can check that, when $x < \frac{1}{2}$, $\phi'(x)$ has the sign of

$$(F-\frac{1}{2})\left(\frac{1}{x-1/2}+\frac{f'}{f}\right)-f$$

which is negative if

$$(\frac{1}{2}-F)\frac{f'}{f}+f > \frac{1/2-F}{1/2-x}.$$

The right-hand side is increasing since F is convex on $[0, 1/2]$. When $x < \frac{1}{2}$, the derivative of the left-hand side has the sign of $ff''-f'^2$ which must be negative since f is logconcave, and so the left-hand side is decreasing. Moreover, when $x \to \frac{1}{2}$, both sides tend to $f(\frac{1}{2})$. Therefore, the above inequality must hold for $x < \frac{1}{2}$.

(ii) $A(x)$ is positive on $(z_0, 1]$. One can verify that $A'(x)$ has the sign of $(x-\frac{1}{2})(2f^2 - Ff')$. Since the second term must be positive given logconcave F, $A(x)$ decreases on $(z_0, \frac{1}{2}]$ and increases on $[\frac{1}{2}, 1]$. Second, notice

$$A(\frac{1}{2}-\varepsilon) = \frac{(1/2-\sigma)^2}{1/2-\sigma-\varepsilon f}, \quad A(\frac{1}{2}+\varepsilon) = \frac{(1/2+\sigma)^2}{1/2+\sigma+\varepsilon f},$$

where $\sigma = \frac{1}{2} - F(\frac{1}{2}-\varepsilon) = F(\frac{1}{2}+\varepsilon) - \frac{1}{2} > 0$ and $f = f(\frac{1}{2}-\varepsilon) = f(\frac{1}{2}+\varepsilon)$. Then $A(\frac{1}{2}-\varepsilon) > A(\frac{1}{2}+\varepsilon)$ if and only if

$$2(\frac{1}{4\sigma}+\sigma)(1+\frac{\varepsilon f}{\sigma}) > 1.$$

Since $\frac{1}{4\sigma}+\sigma \geq 1$, this inequality must be true.

(iii) When $x > \frac{1}{2}$, we have known that $\phi(x)$ is increasing in x, so $B(x)$ is increasing as well. When $x < \frac{1}{2}$, $\phi(x) > 1$ since $\phi(\frac{1}{2}) = 1$. One can check that $B'(x) > 0$ if $(2\phi+1)f/F > -\phi'$. Notice that, for $x < \frac{1}{2}$,

$$-\phi' = \frac{\phi-1}{x-1/2} + \phi\frac{f'}{f} < \phi\frac{f'}{f}.$$

Thus, it suffices to show $2 + \frac{1}{\phi} > Ff'/f^2$, which however must be true since logconcave F implies $Ff' < f^2$. ∎

We then continue to prove the main result. We first show a preliminary result:

$$\frac{1}{2} - z_H < z_L - \frac{1}{2}, \tag{27}$$

where z_i is defined in (18). To prove this, we rewrite the indifference condition (22) as

$$\frac{F_L^2/f_L}{F_H^2/f_H} = \lambda^2 \tag{28}$$

by using (20) and (21). On the other hand, (20) and (21) also imply

$$\frac{F_L/f_L + (z_L - 1/2)}{F_H/f_H + (z_H - 1/2)} = \lambda^2. \tag{29}$$

From (28) and (29), we have $A(z_L) = A(z_H)$. (27) then follows from result (ii) in Lemma 1 given $z_H > z_0$ (which is further implied by (21) and the definition of z_0).

(i) Using (20) and (21), we rewrite (19) as

$$\mu \left[1 - 2F_L - (z_L - \frac{1}{2})f_L\right] + (1-\mu)\left[1 - 2F_H - (z_H - \frac{1}{2})f_H\right] = 0. \tag{30}$$

Let $g(x) = 1 - 2F(x) - (x - \frac{1}{2})f(x)$. Then we solve

$$\mu = \frac{g_H}{g_H - g_L}, \tag{31}$$

where $g_i = g(z_i)$. Notice that, for any $\varepsilon \in [0, \frac{1}{2}]$, $g(\frac{1}{2} - \varepsilon) = -g(\frac{1}{2} + \varepsilon) > 0$. Hence, (27) implies $\mu < \frac{1}{2}$. We now prove $p_1^e > p_2$. It is equivalent to $(1-\mu)(p_1^H - p_2) > \mu(p_2 - p_1^L)$, which holds if and only if

$$\frac{-g_L}{z_L - 1/2}\lambda^2 > \frac{g_H}{1/2 - z_H}$$

by using (31) and (18). Furthermore, using (28) and the definition of g_i, we can rewrite this inequality as $B(z_L) > B(z_H)$, which is true given result (iii) in Lemma 1 and $z_L > z_H$.

(ii) $q_2^e = 1 - \mu F_L - (1-\mu)F_H \le \frac{1}{2}$ if and only if

$$(F_L - \frac{1}{2})g_H \ge -(\frac{1}{2} - F_H)g_L.$$

Using the definition of g_i, this is further equivalent to $\phi(z_H) \le \phi(z_L)$. For the uniform distribution, this inequality must be binding and so $q_2^e = \frac{1}{2}$. Beyond this special case, result (i) in Lemma 1 and (27) imply that this inequality holds strictly.

(iii) Given p_2, firm 1 can at least earn $\frac{p_2}{2}$ by charging p_2. Thus, $\pi_1 > \frac{p_2}{2} \ge \pi_2$ since $q_2^e \le \frac{1}{2}$.

A.5 Proof of Proposition 5

(i) We first deal with *the mixed-strategy equilibrium with an interior solution.* If firm 1 uses the mixed strategy, then for $p \in (p_1^L, p_1^H)$ firm 2's expected demand function is

$$q_2^e(p) = 1 - \hat{x} + \frac{\mu h}{2}(p_1^L - p) + \frac{(1-\mu)l}{2}(p_1^H - p). \tag{32}$$

Similar treatment as in Section 3 leads to

$$p_1^L = \frac{1+r}{h}\hat{x}, \quad p_1^H = \frac{r(1+r)}{h}\hat{x}, \quad p_2 = \frac{2r}{h}\hat{x}. \tag{33}$$

Now (7) becomes

$$2\left[\mu h + (1-\mu)l\right]p_2 = 2(1-\hat{x}) + \mu h p_1^L + (1-\mu)l p_1^H.$$

Then

$$\mu = \frac{1}{r^2-1}\left(\frac{2-\hat{x}}{3\hat{x}}r - 1\right). \tag{34}$$

For μ to be between zero and one, we need

$$\frac{1}{r} < \frac{2-\hat{x}}{3\hat{x}} < r.$$

Since $q_1^L = \hat{x} + \frac{h}{2}(p_2 - p_1^L) = \frac{\hat{x}}{2}(1+r)$, the condition for having an interior solution is $r < \frac{2-\hat{x}}{\hat{x}}$, which is the counterpart of (6) in this asymmetric-quality case. Finally, these two conditions themselves require $\hat{x} < \sqrt{3}-1$.[43]

We then consider *the mixed-strategy equilibrium with a corner solution.* It exists if the following conditions are satisfied:

(i) $p_1^H = \arg\max_{p \geq p_2} p \cdot q_1(p > p_2)$;

(ii) p_1^L is determined by $q_1^L = \hat{x} + \frac{h}{2}(p_2 - p_1^L) = 1$;

(iii) p_1^L is the best response to p_2: $\frac{\partial p q_1(p<p_2)}{\partial p} \leq 0$ at $p = p_1^L$;

(iv) The indifference condition: $p_1^L = p_1^H \cdot q_1(p_1^H)$;

(v) p_2 is the best response to (p_1^L, p_1^H, μ):

$$\frac{p_2}{2}[\mu h + (1-\mu)l] = q_2^e = (1-\mu)\left[1 - \hat{x} + \frac{l}{2}\left(p_1^H - p_2\right)\right].$$

We prove the existence of a solution which satisfies $p_1^L < p_2 < p_1^H$ and $\mu \in (0,1)$ when (15) holds. First of all, from (i) and (ii), we have

$$p_1^L = p_2 - \frac{2}{h}(1-\hat{x}),\ p_1^H = \frac{\hat{x}}{l} + \frac{p_2}{2}.$$

Then the indifference condition (iv) requires

$$p_2 - \frac{2}{h}(1-\hat{x}) = \frac{1}{2l}\left(\hat{x} + \frac{l}{2}p_2\right)^2.$$

One can show that this equation has a solution

$$p_2 \in \left(\frac{2}{h}(2-\hat{x}), \min\left\{\frac{2\hat{x}}{l}, \frac{2}{l}\frac{2-\hat{x}}{3}\right\}\right) \tag{35}$$

by noting that $\frac{2-\hat{x}}{\hat{x}}$ and $3\sqrt{\frac{1-\hat{x}}{5-5\hat{x}-\hat{x}^2}}$ cross at $\hat{x} = \sqrt{3}-1$. Explicitly,

$$p_2 = \frac{2}{l}\left[2 - \hat{x} - 2\sqrt{(1-\hat{x})(1-1/r^2)}\right]. \tag{36}$$

Next, we prove that condition (v) has a solution $\mu \in (0,1)$. Simple algebra shows that condition (v) can be rewritten as

$$\frac{\mu}{1-\mu} = \frac{2-\hat{x}}{hp_2} - \frac{3}{2r^2}. \tag{37}$$

[43] This is because $\frac{1}{r} < \frac{2-\hat{x}}{3\hat{x}}$ implies $r > \frac{3\hat{x}}{2-\hat{x}}$. Then we need $\frac{3\hat{x}}{2-\hat{x}} < \frac{2-\hat{x}}{\hat{x}}$ which implies $\hat{x} < \sqrt{3}-1$.

To prove that the right-hand side of (37) is positive, we consider the case $\hat{x} < \frac{1}{2}$ and $\hat{x} > \frac{1}{2}$ separately. In the former case, $\frac{2\hat{x}}{l} < \frac{2}{l}\frac{2-\hat{x}}{3}$, so $p_2 < \frac{2\hat{x}}{l}$ follows from (35). In the latter case, $p_2 < \frac{2}{l}\frac{2-\hat{x}}{3}$. Both of them imply that the right-hand side of (37) is positive. The last step is to check (iii). At $p = p_1^L$,

$$\frac{\partial pq_1(p < p_2)}{\partial p} = 1 - \frac{h}{2}p_1^L < 0$$

since $p_2 > \frac{2}{h}(2-\hat{x})$ from (35) implies $p_1^L > \frac{2}{h}$.[44]

(ii) We now consider *the pure-strategy equilibrium.* Let us consider the case with $\hat{x} > \frac{1}{2}$ first. In this case, we have an equilibrium with $p_1 > p_2$ if (i) $p_1 = \arg\max_{p \geq p_2} p \cdot q_1(p > p_2)$, (ii) $p_2 = \arg\max_{p \leq p_1} p \cdot q_2(p < p_1)$, and (iii) $p_1 \cdot q_1(p_1 > p_2) \geq \max_{p \leq p_2} p \cdot q_1(p < p_2)$. Condition (iii) means that firm 1 does not want to deviate to a price lower than p_2. (We do not need to worry about firm 2 since its demand function is concave.) From (i) and (ii), it is ready to solve

$$p_1 = \frac{2}{l}\frac{1+\hat{x}}{3} > p_2 = \frac{2}{l}\frac{2-\hat{x}}{3}$$

given $\hat{x} > \frac{1}{2}$. Then

$$\pi_1 = \frac{2}{l}\left(\frac{1+\hat{x}}{3}\right)^2; \ \pi_2 = \frac{2}{l}\left(\frac{2-\hat{x}}{3}\right)^2.$$

Now consider firm 1's potential deviation to $p_1' < p_2$. Let π_1' be the corresponding deviation profit. Given p_2, if we do not consider any constraint, firm 1's optimal response associated with the demand function $q_1(p < p_2)$ is

$$p_1' = \frac{p_2}{2} + \frac{\hat{x}}{h}.$$

But this price could be greater than p_2 or too low such that the corresponding demand is greater than 1. This causes complications and we need to discuss the following three cases separately. (a) When $p_1' \geq p_2$ (which happens when $r \leq \sqrt{\frac{3\hat{x}}{2-\hat{x}}}$), the optimal deviation price should be p_2, so the deviation must be unprofitable. (b) When p_1' is too low such that the demand is greater than 1 (which happens when $r \geq \sqrt{3}$), the optimal deviation price should be

$$p_2 - \frac{2}{h}(1-\hat{x}),$$

which just makes firm 1 win the whole market. Then

$$\pi_1' = p_2 - \frac{2}{h}(1-\hat{x}) = 2\left(\frac{2-\hat{x}}{3l} - \frac{1-\hat{x}}{h}\right),$$

[44] Now we discuss the properties of this equilibrium. First, one can show that p_2 in (36) goes up with $\hat{x}$ under (15). This is because $\frac{\partial p_2}{\partial \hat{x}} > 0$ if and only if $r^2 > \frac{1}{\hat{x}}$. If $\hat{x} > \frac{1}{3}$, it is no problem because $r > \sqrt{3}$ from (15) (see also Figure 7). If $\hat{x} < \frac{1}{3}$, (15) implies $r^2 > (\frac{2-\hat{x}}{\hat{x}})^2 > \frac{1}{\hat{x}}$. Then it is clear that firm 1's prices also increase with $\hat{x}$. Second, firm 1's profit is just equal to p_1^L and so increases with $\hat{x}$, and firm 2's profit is

$$\begin{aligned}\pi_2 &= \frac{1-\mu}{2}\left[(2-\hat{x})p_2 - \frac{l}{2}p_2^2\right] \\ &= \frac{hp_2^2}{2}\left[1 - \frac{r^2-1}{(2-\hat{x})/lp_2 + r^2 - 3/2}\right],\end{aligned}$$

where the second equality follows from (37). A lengthy proof shows π_2 also rises with $\hat{x}$.

which is lower than π_1 if and only if $r^2(5-5\hat{x}-\hat{x}^2) < 9(1-\hat{x})$. (c) When p_1' is appropriate (i.e., when $\sqrt{\frac{3\hat{x}}{2-\hat{x}}} < r < \sqrt{3}$), the deviation profit is

$$\pi_1' = \frac{1}{2h}(\hat{x} + \frac{h}{2}p_2)^2 = \frac{2}{h}\left[\frac{r^2}{3} + (\frac{1}{2} - \frac{r^2}{6})\hat{x}\right]^2.$$

One can check that $\pi_1' < \pi_1$ if and only if $r < \frac{3\hat{x}}{2-\hat{x}}$. The conditions derived in (a)–(c) can be rewritten as (16) with the help of Figure 7.

The case with $\hat{x} < \frac{1}{2}$ can be similarly treated. The candidate pure-strategy equilibrium prices are

$$p_1 = \frac{2}{h}\frac{1+\hat{x}}{3} < p_2 = \frac{2}{h}\frac{2-\hat{x}}{3}.$$

Then

$$\pi_1 = \frac{2}{h}\left(\frac{1+\hat{x}}{3}\right)^2; \ \pi_2 = \frac{2}{h}\left(\frac{2-\hat{x}}{3}\right)^2.$$

When we consider firm 1's potential deviation to $p_1' > p_2$, we will not encounter the situation like the above (b), so it is much simpler to derive (17). The calculation is straightforward and so omitted.

References

Armstrong, M., and Y. Chen (2007): "Inattentive Consumers and Product Quality," mimeo.

Bagnoli, M., and T. Bergstrom (2005): "Log-Concave Probability and its Applications," *Economic Theory*, 26(2), 445–469.

Chen, Y., G. Iyer, and A. Pazgal (2005): "Limited Memory and Market Competition," mimeo.

Compte, O., and P. Jehiel (2003): "Bargaining with Reference Dependent Preferences," mimeo.

Della Vigna, S. (2007): "Psychology and Economics: Evidence from the Field," mimeo.

Della Vigna, S., and U. Malmendier (2004): "Contract Design and Self-Control: Theory and Evidence," *Quarterly Journal of Economics*, 119(2), 353–402.

Eliaz, K., and R. Spiegler (2006): "Contracting with Diversely Naive Agents," *Review of Economic Studies*, 73(3), 689–714.

——— (2007): "Consideration Sets and Competitive Marketing," mimeo.

Ellison, G. (2006): "Bounded Rationality in Industrial Organization," in *Advances in Economics and Econometrics: Theory and Applications: Ninth World Congress of the Econometric Society*, ed. by R. Blundell, W. Newey, and T. Persson. Cambridge University Press, Cambridge, UK.

FARRELL, J., AND P. KLEMPERER (2007): "Coordination and Lock-In: Competition with Switching Costs and Network Effects," in *Handbook of Industrial Organization (Vol. 3)*, ed. by M. Armstrong, and R. Porter. North-Holland, Amsterdam.

FIBICH, G., A. GAVIOUS, AND O. LOWENGART (2007): "Optimal Price Promotion in the Presence of Asymmetric Reference-Price Effects," *Managerial and Decision Economics*, 28(6), 569–577.

GABAIX, X., AND D. LAIBSON (2006): "Shrouded Attributes, Consumer Myopia, and Information Suppression in Competitive Markets," *Quarterly Journal of Economics*, 121(2), 505–540.

GILBOA, I., AND D. SCHMEIDLER (2001): "Reaction to Price Changes and Aspiration Level Adjustment," *Review of Economic Design*, 6(2), 215–223.

GRUBB, M. (2006): "Selling to Overconfident Consumers," mimeo.

HARDIE, B., E. JOHNSON, AND P. FADER (1993): "Modelling Loss Aversion and Reference Dependence Effects on Brand Choice," *Marketing Science*, 12(4), 378–394.

HART, O., AND J. MOORE (2007): "Contracts as Reference Points," *Quarterly Journal of Economics*, forthcoming.

HEIDHUES, P., AND B. KŐSZEGI (2007): "Competition and Price Variation When Consumers are Loss Averse," *American Economic Review*, forthcoming.

HEIDHUES, P., AND B. KOSZEGI (2005): "The Impact of Consumer Loss Aversion on Pricing," mimeo.

HO, D., AND K. IMAI (2006): "Estimating Causal Effects of Ballot Order from a Randomized Natural Experiment: The California Alphabet Lottery, 1978-2002," mimeo.

HOSKEN, D., AND D. REIFFEN (2004): "Patterns of Retail Price Variation," *Rand Journal of Economics*, 35(1), 128–146.

JOHNSON, E., J. HERSHEY, J. MESZAROS, AND H. KUNREUTHER (1993): "Framing, Probability Distortions, and Insurance Decisions," *Journal of Risk and Uncertainty*, 7(1), 35–51.

KAHNEMAN, D., J. KNETSCH, AND R. THALER (1991): "Anomalies: The Endowment Effect, Loss Aversion, and Status Quo Bias," *Journal of Economic Perspectives*, 5(1), 193–206.

KAHNEMAN, D., AND A. TVERSKY (1979): "Prospect Theory: An Analysis of Decision under Risk," *Econometrica*, 47(2), 263–291.

——— (2000): *Choices, Values, and Frames.* Cambridge University Press.

KŐSZEGI, B., AND M. RABIN (2006): "A Model of Reference-Dependent Preferences," *Quarterly Journal of Economics*, 121(4), 1133–1166.

Lal, R., and C. Matutes (1994): "Retail Pricing and Advertising Strategies," *Journal of Business*, 67(3), 345–370.

Madrian, B., and D. Shea (2001): "The Power of Suggestion: Inertia in 401(K) Participation and Savings Behavior," *Quarterly Journal of Economics*, 116(4), 1149–1187.

Meredith, M., and Y. Salant (2007): "The Causes and Consequences of Ballot Order-Effects," mimeo.

Putler, D. (1992): "Incorporating Reference Price Effects Into a Theory of Consumer Choice," *Marketing Science*, 11(3), 287–309.

Rabin, M., and J. Schrag (1999): "First Impressions Matter: A Model of Confirmatory Bias," *Quarterly Journal of Economics*, 114(1), 37–82.

Rosenkranz, S., and P. Schmitz (2007): "Reserve Prices in Auctions as Reference Points," *The Economic Journal*, 117(520), 637–653.

Rubinstein, A. (1993): "On Price Recognition and Computational Complexity in a Monopolistic Model," *Journal of Political Economy*, 101(3), 473–484.

Rubinstein, A., and Y. Salant (2006): "A Model of Choice from Lists," *Theoretical Economics*, 1(1), 3–17.

Salant, Y. (2007): "Procedural Analysis of Choice Rules with Applications to Bounded Rationality," mimeo.

Samuelson, W., and R. Zeckhauser (1988): "Status Quo Bias in Decision Making," *Journal of Risk and Uncertainty*, 1(1), 7–59.

Shaked, A., and J. Sutton (1982): "Relaxing Price Competition Through Product Differentiation," *Review of Economic Studies*, 49(1), 3–13.

Shapiro, J. (2006): "A 'Memory Jamming' Theory of Advertising," mimeo.

Sobel, J. (1984): "The Timing of Sales," *Review of Economic Studies*, 51(3), 353–368.

Spiegler, R. (2006a): "Competition over Agents with Boundedly Rational Expectations," *Theoretical Economics*, 1(2), 207–231.

——— (2006b): "The Market for Quacks," *Review of Economic Studies*, 73(4), 1113–1131.

Tversky, A., and D. Kahneman (1991): "Loss Aversion in Riskless Choice: A Reference-Dependent Model," *Quarterly Journal of Economics*, 106(4), 1039–1061.

Varian, H. (1980): "A Model of Sales," *American Economic Review*, 70(4), 651–659.