Too Little Lending: A Problem of Symmetric Information

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Abstract
In a simple model of the consumer credit market, we show that asymmetric information may enhance welfare relative to full information. The advantage of hidden types is that solvency and default constraints are relaxed, allowing beneficial lending. Prohibiting the use of observable information may therefore be efficient. It is also shown that, rather than the nature of borrower heterogeneity, whether asymmetric information involves adverse or advantageous selection depends on the magnitude of default costs. Even when selection is adverse, lending and welfare may be higher under asymmetric information than under symmetric information.

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1 Introduction

Following Akerlof (1970), asymmetric information has generally been thought to impair market functioning. We show that in competitive consumer-loan markets, asymmetric information may raise welfare. A second contribution concerns the determinants of whether selection under asymmetric information is advantageous or adverse. Whereas with business lending it is the nature of individual heterogeneity that distinguishes cases (compare Stiglitz and Weiss, 1981, with de Meza and Webb, 1987), here it is the magnitude of default costs. Whether selection is adverse or advantageous, asymmetric information relaxes a solvency constraint that prevents high default types borrowing under symmetric information. The high-default types excluded from the market may be unprofitable, but such loans may nevertheless be beneficial to the recipients. Asymmetric information may therefore make socially desirable lending possible. Regulations that prohibit the use of certain kinds of information may consequently be efficiency enhancing.

The problem with symmetric information is that repayment is limited by available income. Even in the absence of a legal limited liability constraint, there is therefore a cap on the effective interest rate. For individuals likely to have low realizations, this “solvency” constraint may bind under symmetric information. That is, at the highest feasible interest rate, lenders cannot expect to recover their advance from individuals unlikely to be in a position to repay, so they are excluded from the loan market. Yet it does not follow that such lending is socially wasteful. Loans that default have typically financed valuable consumption, albeit that a low-income realization prevents repayment. When information is asymmetric, these high-default types cannot be identified, so they do obtain loans. As a result, welfare may be higher under asymmetric information, but this is not certain. Although there could be too little lending under symmetric information, with asymmetric information lending may be excessive. This is the advantageous selection case which occurs when default costs are high. Individuals knowing they are likely to default then stay out of the market. Still, the presence of good risks provides a cross subsidy potentially drawing in some high-default types that contribute negative social surplus. Whether the symmetric or asymmetric information distortion is the more harmful is ambiguous. In the adverse selection case, it is the low default individuals that stay out of the market, preferring to postpone consumption till they can self-finance thereby avoiding actuarially unfair interest rates. The analysis is now more complex as it is not only the volume of lending that is wrong but also the mix of lending. Nevertheless, asymmetric information allows entry by those solvency constrained under symmetric information and the latter effect may dominate selection effects, enhancing efficiency.
The theoretical literature on the impact of asymmetric information on market performance is well known. Akerlof (1970) famously shows that asymmetric information may lead to inefficient market shrinkage, as also in the models of insurance by Rothschild and Stiglitz (1976), and of business loans by Stiglitz and Weiss (1981). The possibility that asymmetric information may lead to overexpansion in business lending was explored by de Meza and Webb (1987) and by de Meza and Webb (2001), and Einav, Finkelstein, and Cullen (2010) for insurance markets. Mahoney and Weyl (2017) show that advantageous selection, by expanding the loan market, may counteract the contractionary effect of imperfect competition in the banking sector. Jaffee and Russell (1976) seem to be the first to examine the impact of asymmetric information on the market for consumption loans. They present two models. In one, borrowers are identical but differ in default cost, which is private information. In the main analysis, only a single contract is allowed. It is shown that credit rationing may arise, in the sense that everyone wishes to borrow more at the ruling interest rate than they can. In the second model, future income is stochastic. Everyone is identical, ex ante, and optimistically but incorrectly think that low realizations are impossible. It is again shown that credit rationing may emerge. Welfare implications are not explicitly examined in either formulation. In our paper, income is stochastic but distributions are heterogeneous, and this is potentially private information. When default costs are low, credit rationing is possible, but the main results concern welfare. Asymmetric information may be preferable to symmetric and the nature of selection effects and appropriate policies depend on the size of default costs.

Zinman (2014) provides a useful survey of empirical work on consumer credit markets. With regard to asymmetric information, there is no decisive evidence nor accepted theoretical model with which to evaluate it. Five notable contributions come to different conclusions. Ausubel (1999) and Agarwal, Chomsisengphet and Liu (2010) find, in market experiments, that default is more likely when credit-card borrowers are offered worse terms, suggesting adverse selection. The field experiment of Karlan and Zinman (2009) analyze a sample of micro-borrowers in South Africa offered different terms. There is no significant selection effect. Einav et al. (2012) look at the market for used car loans (an example of subprime lending). Marginal buyers (and hence borrowers) are more likely to default, suggesting advantageous selection.

\[1\] It is noted that a single contract is not an equilibrium if multiple contracts are allowed, but the separating equilibrium is not identified.

\[2\] Credit rationing does not imply underlending, as noted by de Meza and Webb (2000) in the context of business lending. The same conclusion applies for consumer lending, as will be noted later.

\[3\] Mahoney and Weyl (2016) estimate that this implies the marginal borrower is in receipt of a
in payday lending.\footnote{All of these papers examine the selection effect on borrowers of being charged higher interest rates. This does not directly answer what would happen were information symmetric. As we show, whatever the effect of interest rates on the return to lending, symmetric information may lower welfare relative to asymmetric information.} All of these papers examine the selection effect on borrowers of being charged higher interest rates. This does not directly answer what would happen were information symmetric. As we show, whatever the effect of interest rates on the return to lending, symmetric information may lower welfare relative to asymmetric information.

The rest of the paper is as follows. Section 2 sets out the model. Sections 3 and 4 derive the equilibria and consider the welfare implications of shifting information regime for high and low default cost cases respectively. Section 5 concludes.

\section{The model}

Consumers wish to purchase an indivisible durable good, but initially have insufficient wealth to use as collateral or down-payment.\footnote{Taking a payday loan does not seem to affect general creditworthiness, according to Bhutta, Skiba and Tobacman (2015). This suggests the decision to take such loans is rational.} Taking a loan enables the purchase of the durable good to be brought forward. Future income is random, but individuals differ in their probability of income realization. Under asymmetric information, lenders only know the population distribution of types, whereas borrowers know their own type. There is a continuum of individuals (or households), a large number of lenders and three time periods, $t = 0, 1, 2$. Utility in each period is linear in income, except that consumers can buy an indivisible durable good at normalized price 1. The utility derived from the good in the first period is $\theta u$, while the utility received in the second period is $u$. The good does not yield utility after $t = 2$. Individuals have no endowment in $t = 0$. In $t = 1$, they receive a stochastic income, either 0 or $M > 1$. Individuals are indexed by their probability of receiving high income, $p \in [p_L, p_H]$, where $0 \leq p_L < p_H \leq 1$, so they can be ordered in the sense of first-order stochastic dominance. The distribution function of the probability of high income is $F(p)$. To consume at $t = 0$ requires a financial loan. Income realizations are not observable. If a loan is obtained, default is inevitable should the income realization be 0. If the realization is positive, borrowers might choose to default even though they could repay. If all borrowers choose strategic default, no lending could take place. Borrowers do though incur costs of $D$ if default occurs for whatever reason. These costs, arising even if default is not deliberate, may
involve exclusion from future borrowing, hassle costs or psychic costs even if default is not deliberate. In most jurisdictions, debt recovery is highly regulated. The main sanction is generally credit file endorsement.\textsuperscript{6} Even if the creditor has some discretion over how actively to seek recovery, it may not be possible to credibly precommit to limit penalties. Default costs are therefore taken to be exogenous.\textsuperscript{7} Dishonest default is assumed to involve greater psychic cost than obligatory default, taking the total to $D + \delta$.\textsuperscript{8} The risk-free rate is zero, and risk-neutral lenders compete by setting the interest rate, $r$, on the loan contract $(1, r)$. The price of the good is the same in all periods.

In what follows, both symmetric and asymmetric information are considered. In the latter case, whether there is adverse or advantageous selection depends on the magnitude of default cost. In particular, the cost threshold turns out to be $D = (1 + \theta)u \equiv \hat{D}$. In the following two sections, we consider two distinct cases: in Section 3, default costs are high, that is $D \geq \hat{D}$; in Section 4, default costs are low, $D < \hat{D}$.

Figure 1 previews the nature of equilibrium configurations. The bars are the

\textsuperscript{6}Historically, penalties were draconian, such as jail or slavery, though presumably enforcement rates were low.

\textsuperscript{7}If the creditor can precommit to default costs there may be separating contracts.

\textsuperscript{8}Repossession of the good may also be allowed in the event of default. As to be discussed, for simplicity it will be assumed that in equilibrium this sanction is not feasible.
intervals over which lending occurs, assuming that solvency does not bind. Under
symmetric information the equilibrium cutoff is $p_S$, and under asymmetric infor-
mation it is $p_A$. It is always the highest $p_i$ types that obtain loans under symmetric
information. When default costs are high, the lending pattern under asymmetric
information is the same as symmetric information, that is selection is advantageous
with the volume of lending greater under asymmetric information. When $D$ is low,
the lowest $p_i$ types receive loans. How the volume of lending compares to the sym-
metric information regime is ambiguous. It is possible that the solvency constraint
binds under symmetric information, lowering lending in that regime, but does not
bind under asymmetric information.

3 High default cost

In this section $D \geq \bar{D} = (1 + \theta)u$, that is default cost is higher than the sum of the
first- and second-period utilities.

3.1 Equilibrium under symmetric information

In this case, the individual’s $p_i$ is known to the borrower as well as to the lender.
There are four possible strategies for each consumer.

Plan 1. Do not borrow and do not buy the good at $t = 1$ even if income is high

The expected utility of individual $i$, under plan 1, is then

$$E[U_{i1}]^S = p_i M.$$  \hspace{1cm} (1)

Plan 2. Do not borrow and buy the good at $t = 1$ if income is high

The expected utility with plan 2 is

$$E[U_{i1}]^S = p_i (M + u - 1).$$  \hspace{1cm} (2)

We only consider the interesting case in which $u \geq 1$, so plan 2 dominates plan 1.

Plan 3. Borrow, buy the good at $t = 0$, and repay if income is high

Under plan 3, expected utility is

$$E[U_{i0}]^S = p_i [M + \theta u + u - (1 + r_i)] + (1 - p_i)(\theta u + u - D).$$  \hspace{1cm} (3)
The interpretation of (3) is as follows: with probability \( p_i \), the borrower obtains a positive income, repays the loan and receives the first- and second-period consumption utility from the good; with probability \( 1 - p_i \), the borrower defaults, loses \( D \), but obtains consumption benefits \( \theta u \) and \( u \).\(^9\)

**Plan 4. Borrow, buy the good at \( t = 0 \), and default at \( t = 1 \), even if income is high**

If such strategic default is preferred, lending is rendered unprofitable. For lending to occur, plan 4 must therefore be inferior to plan 3. Writing the extra cost of a dishonest declaration as \( \delta \), the condition for repayment in the high realization state, given no repossession, is

\[
M + u - (1 + r_i) \geq M + u - D - \delta. \tag{4}
\]

That is, the net income of borrower \( i \) must be higher when the loan is repaid rather than when the two default costs are incurred. From (4), plan 4 is dominated for borrowers satisfying

\[
p_i \geq \frac{1}{D + \delta} \equiv p_i^d. \tag{5}
\]

Thus, strategic default is more attractive to low-\( p_i \) types as they face higher interest rates. Although the strategic default constraint in (5) is a valid determinant of borrowing (similar in effect to the solvency constraint, shortly to be derived), for simplicity, it will be assumed that it does not bind in any of the cases to be considered. All that is required is that \( \delta \) is sufficiently high that, even at \( p_i = p_i^L \), (5) does not bind.

Under plan 3, the break-even interest rate charged to borrower \( i \) depends on their probability, \( p_i \), of obtaining a positive income. If borrower \( i \) has enough income to fully repay in the good state, the (type-dependent) equilibrium interest rate, \( r_i \), satisfies the bank’s zero-profit condition,

\[
E[R_i]_S = p_i (1 + r_i) = 1. \tag{6}
\]

where \( E[R_i]_S \) is the expected return per loan on borrower \( i \). Thus, for borrower \( i \), the competitive interest rate is

\[
r_i = \frac{1 - p_i}{p_i}. \tag{7}
\]

Individual \( i \) chooses to borrow under plan 3, rather than self-finance under plan 2 if, using \( r_i \) from (7),

\[
E[U_i]_S^0 - E[U_i]_S^1 = \theta u - (1 - p_i)(1 + D - u) \geq 0. \tag{8}
\]

\(^9\)For simplicity, repossession of the good is assumed not to be feasible or worthwhile, but the qualitative results would be similar if it is part of default penalties.
From (8), the threshold for wanting to borrow under symmetric information, given that an actuarially fair interest rate is available, is

\[ p_t \geq 1 - \frac{\theta u}{1 + D - u} \equiv p_S. \]  
(9)

If \( p_S < p_L \), all want to borrow. Thus, the higher the probability of obtaining a high income, the greater is the incentive to borrow. The benefit of a loan is that it allows first-period consumption, which is independent of \( p_t \). According to (9), individuals with a high enough \( p_t \) will always want to borrow.

Although \( \delta \) has been assumed sufficiently high that plan 3 dominates plan 4, it is possible that the probability of default is so high that the repayment required for the lender to break-even exceeds the ability to pay in the good state. This solvency constraint for individual \( i \) is \( M \geq 1 + r_t \) or, using (7),

\[ p_t \geq \frac{1}{M} \equiv p^M_S. \]  
(10)

If \( p_t < p^M_S \), the solvency constraint binds, and it is not possible to get a loan. This means that that there is an upper bound, \( M - 1 \), on the effective interest rate. As a result, there may be a probability threshold below which a loan is unobtainable because even with the lender claiming all \( M \) in the event of solvency, they cannot expect to cover their costs. Note that, as \( M \) increases from 1, \( p^M_S \) declines from unity towards zero. It is therefore possible that \( p^M_S \) exceeds \( p_S \) or the reverse.

**Remark 1** Under symmetric information, there is a cut-off probability below which borrowing does not occur. This threshold is \( \max\{p_S, p^M_S\} \).

It will be assumed that there is always some lending under symmetric information. This requires that: a) the solvency constraint does not bind the best borrower with \( p_t = p_H \) that is \( p_H M > 1 \); b) the best borrower wishes to borrow if solvency does not bind and an actuarially fair interest rate is offered, so \( p_H > p_S \). There may be individuals with \( p_t > p_S \), so they want to borrow, but for whom \( p_t M < 1 \). Hence, at the capped interest rate, it is not possible to cover costs.

**Remark 2** Under symmetric information, there is credit rationing in the sense that individuals with \( p_t < 1/M \) are unable to obtain a loan despite having positive surplus at the highest possible market rate of \( M - 1 \).
3.2 Equilibrium under asymmetric information

Under asymmetric information, individuals know their own type, while lenders only know the distribution of types. If lending takes place, it must be at a common interest rate, $r$. Since $u \geq 1$, we restrict attention to plans 2, 3 and it will again be assumed that $\delta$ is sufficiently high to block plan 4 (strategic default). Throughout, it is also assumed that, under symmetric information, not everyone obtains a loan, even if solvency and strategic default constraints do not bind.

**Plan 2. Individuals do not borrow and buy the good at $t = 1$**

The expected utility of individual $i$ is, as in the symmetric information case,

$$E[U_i|A] = p_i(M + u - 1) = E[U_i|S]. \quad (11)$$

**Plan 3. Borrow, buy the good at $t = 0$, and repay if income is high**

If individual $i$ borrows at $r$ and buys the good, the expected utility is

$$E[U_i|A] = p_i(M + \theta u + u - (1 + r)) = (1 - p_i)(\theta u + u - D). \quad (12)$$

Taking a loan is preferable to delaying consumption if

$$E[U_i|A] - E[U_i|S] = \theta u - p_i r - (1 - p_i)(D - u) \geq 0. \quad (13)$$

A marginal borrower is therefore characterized by repayment probability

$$p_i = \frac{D - \theta u - u}{D - r - u} = \frac{D - \bar{D}}{D - r - u} \equiv p_A(r). \quad (14)$$

If we denote by $E[R]|A$ the expected return per loan under asymmetric information, the zero-profit condition is

$$E[R]|A = \frac{(1 + r) \int_{p_A}^{p_H} p d F(p)}{1 - F(p_A)} = 1. \quad (15)$$

We now show that, as with symmetric information, it is the high probability types that borrow (if there is any lending). Denote by $r_H$ the interest rate that would make an individual with $p_i = p_H$ indifferent whether or not to borrow. Such a rate must satisfy

$$E[U_i|A] - E[U_i|S] = \theta u - p_H r_H - (1 - p_H)(D - u) = 0. \quad (16)$$
By assumption, $D > \tilde{D} = u(\theta + 1)$, hence (16) implies $p_H(D - r_H) > 0$. If there is lending under symmetric information, at $r_H$, the lender makes positive profits. As the break-even equilibrium rate, denoted by $\bar{r}$, is below $r_H$, this implies that $D - \bar{r} - u > 0$. According to (13), the gain from borrowing is then increasing in $p_i$. It is therefore individuals with $p_i \geq p_A$ that borrow.

**Remark 3** If $D \geq \tilde{D}$, under asymmetric information, there is a cut-off probability below which borrowing does not occur.

To see the possible equilibrium relationships between the level of lending under symmetric and asymmetric information, consider Figure 2. The upward sloping locus is the gross expected return per loan under asymmetric information, $E[R_{\bar{r},A}]$. The expected return is unambiguously increasing in $r$, as individuals with the lowest $p_i$ drop out, and there is higher repayment from the remaining borrowers. Equilibrium equates the expected gross return per loan to the cost, and occurs at $\bar{r}$. The solvency constraint means the highest effective interest rate that can be charged is $M - 1$, as at this repayment exhausts income. In the figure, this solvency constraint does not bind. At the equilibrium pooling rate, $\bar{r}$, marginal borrowers have success probability $p_A$. If $p_A \geq p_S$, there is a contradiction. At $r_S$, the break-even interest rate, under symmetric information, paid by the marginal individual with $p_i = p_S$, all better borrowers, under asymmetric information, generate positive expected profit. Similarly, if $\bar{r} > r_S$. So, if there is any lending under asymmetric information, it exceeds the level under symmetric information. When $M$ is low, $M - 1$ lies to the left of $\bar{r}$, as shown in Figure 2. This implies that there is no lending under asymmetric information. If $\bar{r} = M - 1$, and is not too much below $\bar{r}$, the ability to pick the applicants with higher $p_i$ still allows some profitable lending under symmetric information.

**Proposition 1** If $D \geq \tilde{D}$, and if there is lending under asymmetric information, it exceeds the level under symmetric information. If there is no lending under asymmetric information, there may be lending under symmetric information.

As returns are monotonically rising in the interest rate, if there is excess demand, it is always profitable to raise the interest rate.

**Remark 4** If $D \geq \tilde{D}$, under asymmetric information, Stiglitz-Weiss credit rationing cannot arise.

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10 Under symmetric information, the borrower’s surplus increases in $p_i$. Hence, borrowers with $p_i = p_H$ obtain positive surplus when charged the actually fair rate. So, to make them indifferent, the rate must exceed the break-even level, implying positive profits for lenders.
3.3 Welfare

It has been shown that the thresholds for borrowing under symmetric and asymmetric information are, respectively, \( \max\{p_S, p_S^M\} \) and \( p_A \). In this section, we again assume that \( p_H > p_S > p_L \), and that \( \delta \) is always sufficiently high that strategic default does not occur. These restrictions do not affect the qualitative welfare results of this sub-section.

To recap: under symmetric information, borrowers are those with \( p_t \in [\max\{p_S, p_S^M\}, p_H] \); under asymmetric information, borrowers are those with \( p_t \in [p_A, p_H] \) or, if \( T > M - 1 \), there is no lending.

Welfare implications depend on whether the solvency constraint binds. If it does not bind, the standard result applies and asymmetric information diminishes welfare. As symmetric information is first-best, if the asymmetric information equilibrium is different, in whichever direction, welfare must be lower. When the solvency constraint binds, neither symmetric nor asymmetric information are first best and asymmetric information may yield higher welfare.\(^{11}\) Both cases are analyzed below, in which \( E[W|A], E[W]|S \), and \( E[W]|S^M \) are the aggregate welfare of borrowers in \([p_A, p_H], [p_S, p_H]\), and \([p_S^M, p_H]\), respectively.

3.3.1 Solvency constraint does not bind under symmetric information

In this case, \( p_A < p_S^M \leq p_S \), so marginal borrowers with symmetric information are characterized by a probability of success equal to \( p^S \).

\(^{11}\)As in Bar-Isaac, Jewitt and Leaver (2018), our welfare criterion is expected aggregate surplus. In their model, full information always yields maximal welfare.
Expected social welfare under symmetric information is then

\[ E[W]_S = \int_{p_L}^{p_S} p(M + u - 1)dF(p) + \int_{p_S}^{p_H} [\theta u + u + pM - 1 - (1 - p)D]dF(p). \]

The first integral on the first line is the aggregate utility of individuals who do not borrow, and the second integral is that of individuals who do borrow.

With asymmetric information, aggregate welfare is

\[ E[W]_A = \int_{p_L}^{p_A} p(M + u - 1)dF(p) + \int_{p_A}^{p_H} [\theta u + u + pM - 1 - (1 - p)D]dF(p), \]

where individuals with \( p_t \geq p_A \) are those who borrow.

In Appendix A, we show that the difference between aggregate welfare under symmetric and asymmetric information can be written as

\[ E[W]_S - E[W]_A = - \int_{p_A}^{p_S} [p + \theta u - 1 - (1 - p)(D - u)]dF(p). \quad (17) \]

So, \( E[W]_S - E[W]_A > 0 \) if \( p + \theta u - 1 - (1 - p)(D - u) < 0 \), or \( p < 1 - \theta u/(1 + D - u) = p_S \). This is true as, in (17), \( p \in [p_A, p_S] \), so we obtain the following result.

**Proposition 2** If \( D \geq \bar{D} \), and the solvency constraint does not bind, welfare is higher under symmetric information.

### 3.3.2 Solvency constraint binds under symmetric information

In this case, \( p_A < p_S < p_S^M \), so the solvency constraint is binding with symmetric information and marginal borrowers have success probability \( p_S^M \). This means that \( M \) is sufficiently low that \( p_S < p_S^M \). The excluded borrowers would have contributed to net social surplus were they to obtain a loan, decreasing the social surplus under symmetric information and opening up the possibility that asymmetric information yields higher welfare. Rewriting the difference between aggregate welfare under symmetric and asymmetric information in (17), replacing \( p_S \) with \( p_S^M \),

\[ E[W]_S^M - E[W]_A = - \int_{p_A}^{p_S^M} [p + \theta u - 1 - (1 - p)(D - u)]dF(p). \quad (18) \]

We have that \( E[W]_S^M - E[W]_A > 0 \) if \( p < 1 - \theta u/(1 + D - u) = p_S \). Since, in this sub-case, \( p_A < p_S < p_S^M \), this means that \( E[W]_S^M - E[W]_A > 0 \) for all \( p_t \in [p_A, p_S] \), and \( E[W]_S^M - E[W]_A < 0 \) for all \( p_t \in [p_S, p_S^M] \). This suggests that aggregate welfare
may be higher under asymmetric information. To show this explicitly, in Appendix B, we show that, when a uniform distribution with support $[0, 1]$ is assumed, and if solvency does not bind, asymmetric is to be best when $M$ is above a certain threshold. Figure 3 provides a numerical example of the uniform case. In the example, $E[W|S] - E[W|A] < 0$ when $M \in (1.12, 1.21)$.

**Proposition 3** If $D \geq \bar{D}$, and if the solvency constraint binds, welfare may be higher under asymmetric information.

Figure 4 summarizes the welfare results in the case of a uniform distribution. Under symmetric information with the solvency constraint not binding, the marginal borrower has success probability $p_S$ and obtains zero expected surplus at the break-even interest rate. There is more lending under asymmetric information with the marginal borrower having success probability $p_A$. All the extra lending under asymmetric information involves welfare losses. The aggregate loss from asymmetric information is the area of the triangle $p_A p_S \equiv \Delta_A$. When $M$ is sufficiently low that the solvency constraint binds under symmetric information, the cutoff success probability is higher at $p_M$. Relative to the unconstrained equilibrium at $p_S$, welfare is lower by the area of the triangle $p_S p_M \equiv \Delta_S$. If $\Delta_S > \Delta_A$, which it will for low enough $M$, asymmetric information yields higher welfare.

## 4 Low default cost

This section analyses the case of $D < \bar{D} = (1 + \theta)u$. Again, symmetric information is considered first, and then asymmetric information.
4.1 Equilibrium under symmetric information

The analysis of symmetric information is as in sub-section 3.1. Again, individuals with higher \( p_i \) are more likely to take a loan, and given the strategic default constraint is assumed not to bind because \( \delta \) is high, the threshold for borrowing is \( \max\{p_L, p_S, p_S^M\} \).

4.2 Equilibrium under asymmetric information

Under asymmetric information, when default cost is low, that is, \( D < \bar{D} \), the selection effects of sub-section 3.2 are reversed. Under plan 3, individual \( i \) borrows and buys the good at \( t = 0 \), yielding expected utility

\[
E[U_{i}^{0}]_A = p_i [M + \theta u + u - (1 + r)] + (1 - p_i)(\theta u + u - D).
\]

Plan 2 is equivalent to that of sub-section 3.2. Taking a loan is preferable to delaying consumption if

\[
E[U_{i}^{0}]_A - E[U_{i}^{1}]_A = \theta u - p_i r - (1 - p_i)(D - u) \geq 0.
\]

The marginal borrower satisfies

\[
p_i = \frac{D - \bar{D}}{D - r - u} = p_A(r). \tag{19}
\]

The issue is whether it is those above or below \( p_A(r) \) that seek loans. Consider again the interest rate, \( r_H \), at which the individual with \( p_i = p_H \) is indifferent whether to borrow. This rate satisfies

\[
E[U_{H}^{0}]_A - E[U_{H}^{1}]_A = \theta u - p_H r_H - (1 - p_H)(D - u) = 0. \tag{20}
\]
An implication of (20) is that, as $D - \bar{D} < 0$, then $D < r_H$. This in turn implies that, at $r_H$, the net benefit of borrowing is decreasing in $p_i$. Hence, at $r_H$, everyone wishes to borrow. As $r$ rises above $r_H$, it is borrowers with the highest $p_i$ that drop out first. In general, those wishing to borrow are those with $p_i \leq p_A$, where

$$p_A = \begin{cases} \frac{D - \bar{D}}{D - r_A}, & \text{if } r > r_H; \\ p_H, & \text{if } r \leq r_H. \end{cases}$$

**Remark 5** If $D < \bar{D}$, under asymmetric information, there is a cut-off probability above which borrowing does not occur.

The reason for this property is that high-default types are not so concerned about interest rates as these are relatively rarely paid. When combined with low default costs, which are often incurred, borrowing is more attractive for low default types. To consider the features of the equilibrium, note that increasing the interest rate above $r_H$ decreases the attraction of borrowing to all, but it is the high-$p_i$ types that drop out first. This makes it ambiguous whether the return per loan goes up or down. Figure 5 shows a possible configuration. Under the assumption that the worst types are not funded under symmetric information, the gross-return function must end up below the cost of the loan. In the figure, the solvency constraint does not bind under symmetric information, and the asymmetric equilibrium occurs with the best types choosing not to borrow. Comparison with the symmetric information equilibrium is not straightforward. As the selection effects are opposite, it is not just the volume of lending that matters.

To simplify, we restrict the following discussion to the case in which $p_S > p_L$, $p_A < p_H$, and $\delta$ such that $p_S^u > p_S^d$. Thus, if $D < \bar{D}$, the bank zero-profit condition
is

\[ E[R]_A = \frac{(1 + r) \int_{p_L}^{p_A} pdF(p)}{F(p_A)} = 1. \] (21)

Therefore: under symmetric information, as in Section 3, borrowers are those with \( p_i \in [\max\{p_S, p_S^M\}, p_H] \); under asymmetric information, borrowers are those with \( p_i \in [p_L, p_A] \).

**Remark 6** If \( D < \tilde{D} \), symmetric and asymmetric information equilibria are characterized by either: i) \( p_S^M \leq p_S \leq p_A \); ii) \( p_S < p_S^M \leq p_A \). It is not possible that \( p_S > p_A \).

The reason for the last part of Remark 6 is that, under asymmetric information, the average success probability is below \( p_A \), so the break-even interest rate, \( \tilde{r} \), must exceed what a \( p_A \) type would be charged under symmetric information. This borrower, therefore, obtains a strictly positive surplus under symmetric information. The same must be true of a slightly worse \( p_i \)-type borrower.

Despite knowing that the marginal borrower is more likely to default under asymmetric information, it does not follow that there is less lending than under symmetric information. This is because, under symmetric information, lending is to those above the threshold and, under asymmetric information, to those below.

**Proposition 4** If \( D < \tilde{D} \), and there is no lending under asymmetric information, there may be lending under symmetric information. It is also possible that the volume of lending may exceed that under symmetric information.

As returns do not monotonically rise in the interest rate, if the supply of funds is upward sloping, there may be excess demand at the turning point, but it would not be profitable to raise the interest rate.

**Remark 7** If \( D < \tilde{D} \), under asymmetric information, Stiglitz-Weiss credit rationing may arise if the supply of funds is upward sloping.

### 4.3 Welfare

#### 4.3.1 Solvency constraint does not bind under symmetric information

In this case, \( p_S^M \leq p_S \leq p_A \) and, under symmetric information, borrowers are those with \( p_i \geq p_S \), while under asymmetric information, those with \( p_i \leq p_A \).
Under symmetric information, expected social welfare is
\[ E[W]_S = \int_{p_L}^{p_S} p(M + u - 1)dF(p) + \int_{p_S}^{p_H} [\theta u + u + pM - 1 - (1 - p)D]dF(p). \]

Under asymmetric information, aggregate welfare is
\[ E[W]_A = \int_{p_L}^{p_A} [\theta u + u + pM - 1 - (1 - p)D]dF(p) + \int_{p_A}^{p_H} p(M + u - 1)dF(p), \]
where, in this case, individuals with \( p_i \leq p_A \) borrow, and individuals with \( p_i > p_A \) do not borrow.

In Appendix C, we show that the sign of \( E[W]_S - E[W]_A \) depends on the sign of
\[ -\int_{p_L}^{p_S} [p + \theta u - 1 - (1 - p)(D - u)]dF(p), \tag{22} \]
which is positive when \( p + \theta u - 1 - (1 - p)(D - u) \leq 0 \) or \( p \leq 1 - \theta u/(1 + D - u) = p_S \). This last inequality holds as, in (22), \( p \in [p_L, p_S] \). Hence, \( E[W]_S - E[W]_A \geq 0 \).

**Proposition 5** If \( D < \tilde{D} \), when the solvency constraint does not bind, welfare is higher under symmetric information.

### 4.3.2 Solvency constraint binds under symmetric information
If \( p_S < p_S^M \leq p_A \), borrowers under symmetric information are those with \( p_i \geq p_S^M \), and under asymmetric information, still those with \( p_i \leq p_A \). In Appendix D, we show that the sign of \( E[W]_S^M - E[W]_A \) depends on the sign of
\[ -\int_{p_L}^{p_S^M} [p + \theta u - 1 - (1 - p)(D - u)]dF(p), \tag{23} \]
which, in this case, is ambiguous, as it is positive if \( p \leq p_S \), that is for all \( p \in [p_L, p_S] \), but negative if \( p > p_S \), that is for \( p \in [p_S, p_S^M] \). This implies that aggregate welfare can be higher under asymmetric information. This can be easily shown for the case of a uniform distribution. The expected return per loan is
\[ E[R]_A = \frac{(1 + r)(D - \tilde{D})}{2(D - r - u)}. \tag{24} \]
So,

\[
\frac{dE[R]}{dr} = \frac{(D - \hat{D})(1 + D - u)}{2(r + u - D)^2} < 0. 
\]

(25)

The signing follows because, for the threshold for borrowing under symmetric information in (9), \( p_S = 1 - \theta u/(1 + D - u) \), to be lower than 1 (that is, if there is lending under symmetric information), we need that \( 1 + D - u > 0 \). The relationship between the interest rate and returns per loan is illustrated in Figure 6.\(^{12}\)

**Remark 8** With a uniform distribution, when the interest rate is sufficiently high that not everyone borrows, the expected return per loan is decreasing in the interest rate. The implication is that, if there is any lending under asymmetric information, everyone borrows.

To show that the asymmetric information equilibrium can be welfare superior to symmetric information, first the condition for an equilibrium in which all borrow is obtained and then the condition for this to be a welfare maximum is derived. Finally, it is noted that under symmetric information, the solvency constraint must bind so there is less lending than under asymmetric information and hence lower welfare.

For an asymmetric information equilibrium with borrowing, when the distribution of types is uniform and the support is \([0, 1]\), for costs to be covered, it is necessary that \((1 + r_H)/2 \geq 1\). From (24), this requires

\[
r_H = \theta u \geq 1. 
\]

(26)

\(^{12}\)Notice that, if the supply of funds is upward sloping, there could be a rationing equilibrium at \( r_H \).
This condition assumes the repayment is feasible and hence that

\[ M \geq 1 + \theta u. \tag{27} \]

If even the individual with \( p_i = 0 \) contributes to social welfare by borrowing, \( E[U_0]^0_A - E[U_0]^1_A = \theta u + u - D \geq 1 \), or

\[ \theta u + u \geq 1 + D. \tag{28} \]

Under symmetric information, there cannot be lending to the individual with \( p_i = 0 \), as they never repay. In fact, the lowest type to obtain a loan has \( p_i = 1/M \). As the three conditions, (26), (27), (28) are consistent, welfare can be higher under asymmetric information.

**Proposition 6** If \( D \leq \bar{D} \), asymmetric information leads to adverse selection, but lending and welfare may be higher than under asymmetric information.

## 5 Discussion

This paper presents a simple model of a consumption loan market in the presence of unidimensional heterogeneity, an exogenous probability of ability to repay. Banning the use of privately profitable signals in effect shifts from symmetric to asymmetric information. Our analysis shows that it is possible that the consequences of such a policy are not just (arguably) equitable, but also efficiency enhancing. When information is symmetric, the problem is that those known to be likely to default will not be given loans even though the benefits of an advance are similar to borrowers likely to be solvent. Under asymmetric information the unobservable default factors cannot prevent lending. Welfare may therefore be higher when lenders are uninformed or prevented from using information. In addition, whether the market equilibrium under asymmetric information involves advantageous or adverse selection depends on the level of default cost. Low-default individuals are more likely to repay and are charged an unfair rate. This discourages borrowing but the offset is the probability of incurring default costs are lower. Hence, whether a higher interest rate causes lower or higher default types to drop out depends on the level of default cost. In either case, welfare may be higher under asymmetric information due to the weakening of the solvency constraint for high default types. Whether or not there is credit rationing, the asymmetric information equilibrium may involve more or less lending than under symmetric information. This reflects the possibility that average and marginal selection effects work in opposite directions, as noted by de Meza and
Webb (2017) and Azevedo and Gottlieb (2017), although there, multidimensional heterogeneity is required.

Given the information regime, subsidies or taxes on interest rates may be appropriate. Under symmetric information, a subsidy is desirable when the solvency constraint binds, as it must for low-income groups. In the presence of asymmetric information, matters depend on default costs. When they are high lending is excessive given these costs are real so a lending tax is appropriate. If default costs are high, marginal borrowers are more likely to repay than other borrowers, and a lending subsidy is needed. When default costs are low simple tax/subsidy policies cannot generally create a first-best outcome under asymmetric information, because the problem is not just the volume of lending but also its composition. A subsidy draws in low-default types, which is beneficial but the high default types remain. In principle, income redistribution from individuals experiencing the high realisation to those with a low realisation could also enhance efficiency under symmetric information. In the event of default, lenders can reclaim the transfer making it more attractive to lend to low-$p_k$ types. Income redistribution is though administratively costly, especially as it would have to apply even to those with no interest in borrowing because they do not wish to buy the durable.

The model could obviously be extended. Most generalizations complicate the analysis without affecting the insights. Making default cost heterogeneous does though have interesting implications. Suppose two groups, high and low cost. Now selection effects go in both directions. The net effect could be that the default rate does not vary with the interest rate. Using conventional tests, it would then be concluded that asymmetric information is not present and the market is efficient. This though is not the case. Under symmetric information some individuals would be offered better interest rates and some worse. High-default types would not be able to borrow at all. Again, both the composition and the volume of lending would change, with ambiguous effects on welfare. Passing an interest rate test does not imply that all is well. Perhaps the main message for applied work is that even observables are relevant for market efficiency.

Appendix

A High default cost - solvency constraint does not bind under symmetric information:
Under symmetric information, the expected social welfare can be rewritten as
\[
E[W]_S = (M + u - 1) \int_{p_L}^{p_H} p dF(p) + \int_{p_S}^{p_H} [p + \theta u - 1 - (1 - p)(D - u)] dF(p). \quad (A1)
\]

With asymmetric information,
\[
E[W]_A = (M + u - 1) \int_{p_L}^{p_H} p dF(p) + \int_{p_A}^{p_H} [p + \theta u - 1 - (1 - p)(D - u)] dF(p). \quad (A2)
\]

Since, in this case, \( p_A < p_S \), then
\[
E[W]_S - E[W]_A = - \int_{p_A}^{p_S} [p + \theta u - 1 - (1 - p)(D - u)] dF(p), \quad (A3)
\]

implying \( E[W]_S - E[W]_A > 0 \) if \( p + \theta u - 1 - (1 - p)(D - u) < 0 \), or \( p < 1 - \theta u/(1 - D - u) = p_S \). As in (A3), \( p \in [p_A, p_S] \), this inequality holds.

**B) High default cost - solvency constraint binds under symmetric information:**

With a uniform distribution with support \([0, 1]\), if solvency does not bind, the (positive) solution for the pooling interest rate under asymmetric information, deriving from the condition in (15), is
\[
\bar{\alpha} = \frac{1 + 2(D - u) - \theta u - \beta}{2},
\]

where \( \beta = \sqrt{[1 + 2(D - u) - \theta u]^2 - 4\theta u} \). In this case,
\[
E[W]_S^M - E[W]_A = \frac{2(D - u)M^2 - 4[1 + D - u][1 + \theta - u - (D - u)M^2 + [2(D - u)^2 - \theta u^2 - 1]M^2 - M^2\beta + \theta u M^2 \beta - 2(1 + D - u)^2]}{4(1 + D - u)M^2}.
\]

Aggregate welfare is higher under asymmetric information, that is \( E[W]_S^M - E[W]_A < 0 \), if
\[
M < \frac{1}{1 - \frac{\theta u}{1 + D - u} + \frac{\sqrt{[1 + D - u]^2 \theta u^2 - \theta u (4(1 + D - u) - \gamma) + [1 + 2(D - u)][1 + 2(D - u) - \gamma]}}{\sqrt{2(1 + D - u)^2}}} \equiv M_1, \text{ and}
\]
\[
M > \frac{1}{1 - \frac{\theta u}{1 + D - u} - \frac{\sqrt{[1 + D - u]^2 \theta u^2 - \theta u (4(1 + D - u) - \gamma) + [1 + 2(D - u)][1 + 2(D - u) - \gamma]}}{\sqrt{2(1 + D - u)^2}}} \equiv M_2,
\]

where \( \gamma = \sqrt{\theta u^2 + [1 + 2(D - u)]^2 - 2[3 + 2(D - u)]\theta u} \). In addition, it is required that \( \bar{\alpha} \leq M - 1 \), which is satisfied when
\[
M \geq \frac{3 + 2(D - u) - \theta u - \beta}{2} \equiv \tilde{M}.
\]
The lowest value of $M$ at which the solvency constraint does not bind, that is for which $p_S = p_S^M$, is

$$M = \frac{1 + D - u}{1 + D - \theta u - u} \equiv M_S.$$  

So, if asymmetric information is to be best, $M$ must be below this threshold. Figure 3 provides a numerical example of the uniform case. In the example, $E[W]_S^M - E[W]_A < 0$ when $M \in (\tilde{M}, M_1) = (1.12, 1.21)$.

C) Low default cost - solvency constraint does not bind under symmetric information:

In this case, $p_S^M \leq p_S \leq p_A$. Under symmetric information, expected social welfare can be rewritten as

$$E[W]_S = (M + u - 1) \int_{p_L}^{p_H} pdF(p) + \int_{p_S}^{p_H} [p + \theta u - 1 - (1 - p)(D - u)]dF(p).$$

Under asymmetric information,

$$E[W]_A = (M + u - 1) \int_{p_L}^{p_H} pdF(p) + \int_{p_L}^{p_A} [p + \theta u - 1 - (1 - p)(D - u)]dF(p).$$

Since, in this case, $p_S \leq p_A$, then $E[W]_S - E[W]_A$ is equal to

$$\int_{p_S}^{p_H} [p + \theta u - 1 - (1 - p)(D - u)]dF(p) - \int_{p_L}^{p_A} [p + \theta u - 1 - (1 - p)(D - u)]dF(p). \quad (C1)$$

The expression in (C1) can be rewritten as

$$\int_{p_S}^{p_H} [p + \theta u - 1 - (1 - p)(D - u)]dF(p) - \int_{p_S}^{p_H} [p + \theta u - 1 - (1 - p)(D - u)]dF(p)$$

$$- \int_{p_S}^{p_A} [p + \theta u - 1 - (1 - p)(D - u)]dF(p). \quad (C2)$$

In (C2), the difference between the first and third integral is

$$\int_{p_S}^{p_H} [p + \theta u - 1 - (1 - p)(D - u)]dF(p) - \int_{p_S}^{p_A} [p + \theta u - 1 - (1 - p)(D - u)]dF(p) > 0, \quad (C3)$$

The second integral in (C2), that is

$$- \int_{p_L}^{p_S} [p + \theta u - 1 - (1 - p)(D - u)]dF(p), \quad (B4)$$

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is positive when \( p + \theta u - 1 - (1 - p)(D - u) \leq 0 \) or \( p \leq 1 - \theta u/(1 + D - u) = p_S \), which holds for all \( p \in [p_L, p_S] \). Hence, \( E[W]^M_S - E[W]^A \geq 0 \).

D) Low default cost - solvency constraint binds under symmetric information:
In this case, \( p_S < p^M_S \leq p_A \). Under symmetric information, expected social welfare can be rewritten as

\[
E[W]^M_S = (M + u - 1) \int_{p_L}^{p_H} pdF(p) + \int_{p_S^M}^{p_H} [p + \theta u - 1 - (1 - p)(D - u)]dF(p).
\]

Under asymmetric information, aggregate welfare is

\[
E[W]^A = (M + u - 1) \int_{p_L}^{p_H} pdF(p) + \int_{p_L}^{p_A} [p + \theta u - 1 - (1 - p)(D - u)]dF(p).
\]

In this case, \( E[W]^M_S - E[W]^A \) is equal to

\[
\int_{p_S^M}^{p_H} [p + \theta u - 1 - (1 - p)(D - u)]dF(p) - \int_{p_L}^{p_A} [p + \theta u - 1 - (1 - p)(D - u)]dF(p). \tag{D1}
\]

Thus, (D1) can be written as

\[
\int_{p_S^M}^{p_H} [p + \theta u - 1 - (1 - p)(D - u)]dF(p) - \int_{p_L}^{p_S^M} [p + \theta u - 1 - (1 - p)(D - u)]dF(p)
- \int_{p_S^M}^{p_A} [p + \theta u - 1 - (1 - p)(D - u)]dF(p). \tag{D2}
\]

The difference between the first and third integral in (D2) is

\[
\int_{p_S^M}^{p_H} [p + \theta u - 1 - (1 - p)(D - u)]dF(p) - \int_{p_S^M}^{p_A} [p + \theta u - 1 - (1 - p)(D - u)]dF(p) > 0, \tag{D3}
\]

Since \( p_S < p^M_S \), the sign of the second integral in (D2), that is,

\[
- \int_{p_L}^{p_S^M} [p + \theta u - 1 - (1 - p)(D - u)]dF(p), \tag{D4}
\]

is ambiguous, as it is positive if \( p \leq p_S \), that is for all \( p \in [p_L, p_S] \), but negative if \( p > p_S \), that is for \( p \in [p_S, p^M_S] \). This implies that aggregate welfare can be higher under asymmetric information.
References


