Economic profit, NPV, and CAPM: Biases and violations of Modigliani and Miller's Proposition I

Magni, Carlo Alberto

University of Modena and Reggio Emilia

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Carlo Alberto Magni

Università di Modena e Reggio Emilia, Dipartimento di Economia Politica
viale Berengario 51, 41100 Modena, Italy
tel. +39-059-2056777, fax +39-059-2056997, Email: magni@unimo.it

Abstract. For one–period projects under certainty, the notion of Net Present Value (NPV) formally translates the notion of economic profit, where the discount rate is the cost of capital. Under uncertainty, the cost of capital is the expected rate of return of an equivalent-risk alternative that the investor might undertake and is often found by making recourse to the Capital Asset Pricing Model. This paper shows that the notions of *disequilibrium* NPV and economic profit for risky one-period projects are not equivalent: NPV-minded agents are open to framing effects and to arbitrage losses, which imply violations of Modigliani and Miller's Proposition I. The notion of *disequilibrium* (present) value, deductively derived from the CAPM by several authors and widely used in applied corporate finance, should therefore be dismissed.

Keywords. Capital Asset Pricing Model, Net Present Value, economic profit, disequilibrium, framing effects, arbitrage, Modigliani and Miller's Proposition I.

JEL codes. G11, G12, G31, M21, M41

1 Introduction

Economic profit on one side, (net) present value on the other side. The former is one of the building blocks of economic theory, the latter is a cornerstone in financial economics.

Economic profit is a fundamental notion in economic theory since Marshall (1890). It represents the "excess profit that is gained from an investment over and above the profit that could be obtained from the best alternative foregone" (Rao, 1992, p. 87). That is, economic profit from an investment is the difference between profit from that investment and profit from the best alternative foregone. In other terms, the foregone alternative's profit acts as an opportunity cost (see Buchanan, 1969). As known, many synonyms have been coined to mean 'economic profit': 'excess profit' (Preinreich, 1938), 'excess realizable profit' (Edwards and Bell, 1961), 'excess income' (Peasnell, 1982), 'abnormal earnings' (Ohlson, 1995), 'supernormal profit' (see Begg, Fischer, and Dornbusch, 1984, p. 121), 'residual income' (Solomons, 1965; Biddle, Bowen, and Wallace, 1999; Martin, Petty and Rich, 2003), 'economic value added' (Stewart, 1991). The concept of 'Goodwill' (e.g., Preinreich, 1936) is also strictly related to that of excess profit.[1]

Net Present Value (NPV) is a fundamental notion in finance since Fisher (1930), although "the technology of discounting is not an invention of twentieth century" (Miller and Napier, 1993, p. 640): Discounted-cash-flow analysis was known and (sometimes) employed since eighteenth century (Brackenborough, McLean and Oldroyd, 2001. See also Parker, 1968; Edwards and Warman, 1981). As known, the NPV is a function of the discount rate, and the latter is often found by making use of the classical Capital Asset Pricing Model (CAPM) (Sharpe, 1964; Lintner, 1965; Mossin, 1966), which puts into effect the NPV methodology.

The notions of economic profit and NPV are often viewed as two sides of the same medal: The NPV is just economic profit disguised in present terms. The common idea of economic profit maximization is then equivalent to the idea of net present value maximization: "The firm attempts to maximize the present value of its net cash flow over an infinite horizon" (Abel, 1990, p. 755) and "the net present value rule is also the basis for the neoclassical theory of investment" (Dixit and Pindyck, 1994, p. 5). Decision making is straightforward with such equivalent notions. Rubinstein (1973) shows that if the CAPM assumptions are met, a project is worth undertaking if its expected rate of return is greater than the *disequilibrium* required rate of return. His maximization rule is as follows:

> The firm should accept the project with the highest excess expected internal rate of return weighted by its cost (p. 174)
>
> This result ... is equivalent to accepting the project with the highest net present value (*ibidem*, footnote 14).

The first quotation just focusses on maximization of economic profit, the second one suggests to maximize net present value. Magni (2008) shows that the use of disequilibrium values is standard in corporate finance and is widespread in academic papers as well as in textbooks. This paper shows that, contrary to what Rubinstein seems to imply, the alleged equivalence of (disequilibrium) NPV and economic profit does not hold. Such a NPV does not represent economic profit and, in addition, it is a biased measure because it is nonadditive; the same holds for the notion of disequilibrium value.[2] In

[1] See Magni, 2000a, 2000b, 2003, 2005, 2007c for a nonclassical way of formalizing economic profit.

[2] Problems in the *equilibrium* NPV may be found as well (see Magni, 2007a, 2008).

particular, decision makers abiding by the standard NPV+CAPM methodology give inconsistent answers to the same problem differently framed. In other terms, they are trapped in a sort of mental accounting (Thaler, 1985, 1999) so that their evaluations differ depending on whether outcomes are seen as aggregate or disaggregate quantities. This amounts to saying that their valuations and choice behaviours do not comply with the principle of description invariance, which prescribes that valuations and decisions must be invariant under changes in description of the same asset. Violations of this principle are known as framing effects (Tversky and Kahneman, 1981; Kahneman and Tversky, 1984; Soman, 2004). This bias bears significant relations to the violation of the principle of arbitrage, which is a well-established principle of economic rationality implying that rational decision makers do not incur arbitrage losses (see Nau and McCardle, 1991; Nau, 1999). In the field of corporate valuation this violation reduces to an infringement of the classical Modigliani and Miller's Proposition I.

The paper is structured as follows. In section 2 it is shown that NPV and economic profit bear a strong formal relation in that the former is the present value of the latter. Section 3 shows an example highlighting the fact that NPV does not represent economic profit, is not additive and does not fulfill the principle of description invariance (i.e. implies framing effects). In contrast, economic profit is additive and frame-independent. Section 4 shows the same results in more formal terms. In section 5 it is shown, on the basis of the previous results, that value itself is nonadditive. Section 6 shows that NPV-minded decision makers incur arbitrage losses. Section 7 shows that the association of CAPM and NPV does not comply with Modigliani and Miller's Proposition I. In particular, the choice behaviour of a potential NPV-minded buyer is not invariant under changes in the firm debt-equity ratio.

2 Economic profit and NPV as companions

Let W^0 be an investment cost and denote with W^1 the final payoff at time 1. Consider the profit $W^1 - W^0$, which we can reformulate as rW^0, with $r = \dfrac{W^1 - W^0}{W^0}$ being the rate of return. Consider also an alternative business for the investor and let i be the relative rate of return. The corresponding profit is $W^0(1+i) - W^0 = iW^0$ and represents an opportunity cost, a foregone return. The economic (i.e. excess) profit is given by the difference between the *factual* profit the entrepreneur receives and the *counterfactual* profit she would receive if she invested in the alternative business. Denoting economic profit with π we have:

$$\pi = rW^0 - iW^0. \tag{1}$$

Note that the above equation may also be stated as a difference between two future values:

$$\pi = W^1 - W^0(1+i). \tag{2}$$

From a financial perspective, π is the Net Future Value. In finance, it is common to work with present values so the notion of Net Present Value (NPV) is introduced, which is given by the discounted algebraic sum of all cash flows involved in the business. In our simplified one-period case, we have

$$NPV = -W^0 + \frac{W^1}{1+i}. \tag{3}$$

Economic profit and NPV bear a strong formal relation: NPV is the present value of (1) (or, equivalently, the present value of (2)):

$$NPV = \frac{\pi}{1+i} = \frac{1}{1+i}(rW^0 - iW^0). \tag{4}$$

For one-period proejcts, economic profit and net future value are different names for the same notion, whereas net present value is the present value of economic profit. It is worthwhile noting that eqs. (3)-(4) preserve the sign of eqs. (1)-(2) (as long as $i > -1$, as will be assumed here). Decision-making implications of this formal equivalence are straightforward: A business is worth undertaking if and only if the economic profit (the NPV) is positive.

Under uncertainty, the rates r and i are expected values and the two rates refer to alternatives equivalent in risk, so that eqs. (1) and (3) are measures of expected excess profit (in final and present terms respectively). What 'equivalent in risk' means depends on the model selected. The classical and sophisticated CAPM is the most common tool for measuring an asset's risk, which is given by its beta:

$$\beta = \frac{\mathrm{cov}(\tilde{r}, \tilde{r}_m)}{\sigma_m^2} = \frac{\mathrm{cov}(\tilde{W}^1, \tilde{r}_m)}{W^0 \sigma_m^2} \tag{5}$$

where $\tilde{r}_m$ and σ_m^2 denotes the market rate of return and its variance (a tilde on a symbol will henceforth highlight randomness).

To calculate excess profit (and NPV) under uncertainty one just has to use the fundamental equation of the CAPM, known as the Security Market Line (SML). Under suitable assumptions, the latter individuates the required rate of return of the business under examination; such a rate is the (opportunity) cost of capital, i.e. the expected rate of return of the counterfactual alternative available to the entrepreneur. We have

$$i = r_f + \beta(r_m - r_f) \tag{6}$$

where r_f is the risk-free rate and r_m is the expected market rate of return. Applying this security valuation relation to capital budgeting we have a simple rule: A project should be undertaken if and only if

$$r > r_f + \beta(r_m - r_f) \tag{7}$$

i.e. if and only if its expected rate of return exceeds the cost of capital (see Rubinstein, 1973, p. 171) or, in terms of NPV, if and only if its risk-adjusted NPV is positive:

$$-W^0 + \frac{W^1}{1 + r_f + \beta(r_m - r_f)} > 0 \tag{8}$$

where W^1 is the expected value of $\tilde{W}^1$.

Remark 1 It is worth noting that the beta in (5) is a disequilibrium beta, so that the NPV in (8) is a disequilibrium NPV. Beside Rubinstein (1973), the legitimacy of the disequilibrium beta for computing the cost of capital has been deducted from the CAPM by several authors (see Senbet and Thompson, 1978; Magni, 2007b, for a review).

3 Nonadditivity and framing effects: An example

Consider the security market described in Table 1, where a risky asset and a risk-free asset are traded and two possible states may occur, conventionally labeled 'good' and 'bad', with probability 0.8 and 0.2 respectively. The market is complete, is assumed to be in equilibrium (all marketed assets lie on the SML) and arbitrage is not possible.[3] Let us imagine an economic agent runs across the opportunity of investing in a business A composed of two sub-projects. The first one, say A_1, consists of an outlay of 15500 euros and generates an outcome of $\widetilde{W}^1_{A_1}$, equal to 58000 in good state and 3000 in bad state. The second one, say A_2, consists of an outflow of 70000 euros and a final risk-free inflow of 72000 at time 1. Suppose also that this two-project business is to be *fully* accepted or *fully* rejected (no sub-project may be undertaken alone). To decide, the investor computes the NPV of the business. The rates of return of A_1 are $58000/15500 - 1 = 2.7419$ and $3000/15500 - 1 = -0.8064$ in good and bad state respectively; the expected rate of return is $r_{A_1} = (2.7419)(0.8) + (-0.8064)(0.2) = 2.0322$. The covariance of $\widetilde{r}_{A_1}$ with $\widetilde{r}_m$ is $\text{cov}(\widetilde{r}_{A_1}, \widetilde{r}_m) = 0.5677$ and the risk is therefore $\beta_{A_1} = 0.5677/0.16 = 3.5484$. The cost of capital is $i_{A_1} = r_f + \beta_{A_1}(r_m - r_f) = 0.15 + 3.5484(0.45 - 0.15) = 1.2145$. The economic profit is then

$$W^0_{A_1}(r_{A_1} - i_{A_1}) = 15500(2.0322 - 1.2145) = 12675 \tag{9}$$

while the NPV is

$$NPV_{A_1} = \frac{12675}{1 + i_{A_1}} = \frac{12675}{1 + 1.2145} = 5723. \tag{10}$$

As for A_2, its rate of return is $0.02857 = 72/70 - 1$ in both states. As the project is riskless, the cost of capital is $r_f = 0.15$, so the excess profit is

$$W^0_{A_2}(r_{A_2} - r_f) = 70000(0.0285 - 0.15) = -8500 \tag{11}$$

and the NPV is

$$NPV_{A_2} = \frac{-8500}{1 + r_f} = \frac{-8500}{1 + 0.15} = -7391. \tag{12}$$

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[3] As Dybvig and Ingersoll (1982) show, if (i) the CAPM pricing relation holds for all securities in the market, (ii) the market is complete, (iii) the probability that $\widetilde{r}_m > r_m + \dfrac{\sigma^2_m}{r_m - r_f}$ is positive, then arbitrage opportunities arise. But in our market of Table 1 condition (iii) is not satisfied.

Table 1. The security market

	Security				
	risky	risk-free	**Market**		
Outstanding shares	10		10		
				state	probability
Cash Flow	165	115	1650	s_1	0.8
	65	115	650	s_2	0.2
Rate of return (%)	65	15	65	s_1	0.8
	−35	15	−35	s_2	0.2
Expected rate of return (%)	45	15	45		
Covariance with the market rate of return	0.16	0	0.16		
Beta	1	0	1		
Value	100	100	1000		

Consider now a business B that can be undertaken with an expenditure of 85500 euros whereby the investor will obtain $\widetilde{W}_{B_1}$, equal to 130000 or 75000 in good and bad state respectively. The rate of return of B is $130000/85500-1=0.5204$ and $75000/85500-1=-0.1228$ in good and bad state respectively so that the expected rate of return is $r_B=(0.8)0.5204+(0.2)(-0.1228)=0.39181$. It is easy to see that the risk of B is $\beta_B=0.6432$ and the cost of capital is therefore $i_B=0.15+0.6432(0.45-0.15)=0.34298$. The excess profit is

$$W_B^0(r_B-i_B)=85500(0.39181-0.34298)=4175 \tag{13}$$

and the NPV is

$$NPV_B=\frac{4175}{1+i_B}=\frac{4175}{1+0.34298}=3108. \tag{14}$$

It is worthwhile noting that the NPV of business B differs from the NPV of business A, which is $5723-7391=-1668$. Yet, the two businesses represent the same course of action described in two

different ways, because both share the same total investment outlay (15500+75000=85500) and the same final outcomes in good and bad state (58000+72000=130000 and 3000+72000=75000). We have then $A_1 + A_2 = B$. This is a significant result. From a financial perspectives, it means that the NPV is nonadditive (because $\text{NPV}_{A_1} + \text{NPV}_{A_2} \neq \text{NPV}_{A_1+A_2}$); from a cognitive and behavioural outlook, it means that an NPV-minded economic agent incurs framing effects in decision making, because the alternative $A_1 + A_2$ is rejected (its NPV is negative) and the logically equivalent alternative B is accepted (its NPV is positive). By contrast, note that the economic profit as translated in (1) gives univocal results: Economic profit from B is 4175, which coincides with economic profit from the two-project business A (=12675 − 8500).

4 Nonadditivity and framing effects: A simple formalization

In general, consider an investment whose initial outlay is W^0 and whose final payoff is the random sum $\widetilde{W}^1$, available at time 1. This investment may always be viewed as a portfolio of two investments, one risky and one risk-free, whose outlays are $W^0 - h$ and h respectively and whose outcomes are $\widetilde{W}^1 - k$ and k respectively, with $h, k \in \mathbb{R}$. The economic profit of the investment may be formalized as the sum of these two investments' excess profits. In order to avoid framing effects, description invariance must be guaranteed, which means that economic profit must be invariant under changes in h and k. Indeed, considering π and i as functions of h and k, we have

$$\pi(h,k) = \overbrace{[(W^1 - k) - (W^0 - h) - i(h,k)(W^0 - h)]}^{\text{risky excess profit}} + \overbrace{(k - h - r_f h)}^{\text{risk-free excess profit}} \tag{15}$$

with

$$i(h,k) = r_f + \frac{r_m - r_f}{\sigma_m^2} \cdot \text{cov}\left(\frac{\widetilde{W}^1 - k}{W^0 - h} - 1, \widetilde{r}_m \right).$$

Substituting the latter in (15) we obtain

$$\begin{aligned}
\pi(h,k) &= W^1 - W^0 - \left[r_f + \frac{r_m - r_f}{\sigma_m^2} \text{cov}\left(\frac{\widetilde{W}^1 - k}{W^0 - h} - 1, \widetilde{r}_m \right) \right] \cdot (W^0 - h) - r_f h \\
&= W^1 - W^0 - \left[r_f + \frac{r_m - r_f}{(W^0 - h)\sigma_m^2} \text{cov}(\widetilde{W}^1, \widetilde{r}_m) \right] \cdot (W^0 - h) - r_f h \\
&= W^1 - W^0 (1 + r_f) - \frac{r_m - r_f}{\sigma_m^2} \text{cov}(\widetilde{W}^1, r_m).
\end{aligned}$$

It is then evident that $\frac{\partial \pi(h,k)}{\partial h} = \frac{\partial \pi(h,k)}{\partial k} = 0$ for all h and k, which means that economic profit does not change whatever the way the investment is partitioned (i.e., regardless of aggregation or disaggregation of cash flows).

As for the NPV, seen as a function of h and k, things are different:

$$NPV(h,k) = -(W^0 - h) + \frac{W^1 - k}{1 + r_f + \frac{r_m - r_f}{\sigma_m^2}\mathrm{cov}\left(\frac{\tilde{W}^1 - k}{W^0 - h} - 1, \tilde{r}_m\right)} + \left[-h + \frac{k}{1 + r_f}\right] \qquad (16)$$

whence

$$NPV(h,k) = -W^0 + \frac{W^1 - k}{1 + r_f + \frac{r_m - r_f}{\sigma_m^2 (W^0 - h)}\mathrm{cov}(\tilde{W}^1, \tilde{r}_m)} + \frac{k}{1 + r_f}. \qquad (17)$$

It is evident that, in general, $\frac{\partial NPV(h,k)}{\partial h} \neq 0$ as well as $\frac{\partial NPV(h,k)}{\partial k} \neq 0$. Therefore NPV changes as h and/or k change, and it is not true that $\mathrm{NPV}(h_1,k_1) = \mathrm{NPV}(h_2,k_2)$ for all h_1, h_2, k_1, k_2, as the principle of description invariance requires. As a particular case, the example above described has shown that

$$\pi(70000,72000) = \pi(0,0) = 4175$$

whereas

$$NPV(70000,72000) = -1668 \neq NPV(0,0) = 3108;$$

in the latter case choice behaviour depends on the choice of the pair (h,k), in the former case it is irrelevant.

5 Value is nonadditive

As a consequence, the notion of value in this context is severely undermined. The value V of an asset is given by $V = \mathrm{NPV} + W^0$ (where W^0 is the cost to be paid by investors for undertaking it). Referring to the numerical example above where $B = A_1 + A_2$ and bearing in mind the previous results about NPV, we have

$$\begin{aligned} & V_{A_1} + V_{A_2} = NPV_{A_1} + W_{A_1}^0 + NPV_{A_2} + W_{A_2}^0 \\ & = NPV_{A_1} + NPV_{A_2} + W_B^0 \\ & \neq NPV_B + W_B^0 = V_B = V_{A_1 + A_2}. \end{aligned}$$

with obvious meaning of $W_{A_1}^0$, $W_{A_2}^0$, W_B^0. Putting it differently, value is a function of h and k:

$$V(h,k) = NPV(h,k) + W^0 \qquad (18)$$

whose partial derivatives are not identically zero (see eq. (17)), and thus value is not invariant under

changes in the description of valuation process.

6 Arbitrage Losses

The nonadditivity of value and net present value is full of implications. In addition to the framing effect above mentioned we have that our NPV-minded investor is subject to arbitrage losses. To see why, let us refer to the example in section 3. Suppose an economic agent (whom we can call the arbitrageur) asks the investor for a borrowing of 89000 euros whereby he will repay the amount $\widetilde{W}_B^1$ after one period (the investor accepts to lend money, given that the NPV is easily found to be 112.326). At the same time, the arbitrageur offers our investor two financings: A loan of 15500 whereby the investor will repay $\widetilde{W}_{A_1}^1$ and a loan of 70000 whereby the investor will reimbursed 72000 euros at the end of the period. These financings are to be accepted or rejected conjunctly. Our investor evidently accepts, given that $-(\text{NPV}_{A_1}+\text{NPV}_{A_2})$ is positive (=1668). As a result of this choice behaviour, our NPV-minded investor receives a sure loss of 3500 euros. (Table 2 shows the NPV-minded investor's payoffs. Those for the arbitrageur are the same with opposite sign).

Table 2. Arbitrage loss for a NPV-minded agent

	Payoffs	
	Time 0	Time 1
Borrowing $(=-A_1)$	15500	$-\widetilde{W}_{A_1}^1$
Borrowing $(=-A_2)$	70000	-72000
Lending	-89000	$\widetilde{W}_B^1$
Net Payoffs	-3500	$\widetilde{W}_B^1-\widetilde{W}_{A_1}^1-72000=0$

7 Violation of Modigliani and Miller's Proposition I

Let us now focus on a world *à la* Modigliani and Miller (1958) where Proposition I holds, so that firm value is not affected by the mix equity-debt. Consider an example of two firms. Firm U is unlevered and all the stocks are owned by entrepreneur U; firm L is levered and all the stocks and bonds are owned by agent L.[4] Let P be a potential buyer and suppose that:

[4]Agents U and L are therefore representative agents (for sake of simplicity) but one may equivalently consider agents holding only *some* shares and bonds in a convenient ratio.

- the two firms will generate the same total cash flow $\widetilde{W}^1$
- agent U is ready to sell his stocks in exchange of W^0 euros
- agent L is ready to sell his entire endowment in firm L selling the stocks in exchange of W^0 euros but giving free his bonds to the buyer of the firm
- the debt of firm L is risk-free
- agent P is a CAPM enthusiast and selects alternatives via NPV rule.

As a result of the above assumptions, investor P computes the value of both firms as follows. The value of firm U is[5]

$$V_U = \frac{W^1}{k_U} \tag{19}$$

where $k_U = r_f + \beta_U (r_f - r_m)$ is the (unlevered) cost of capital. Denoting with $\widetilde{r}_U$ the rate of return for firm U's stockholders, the unlevered beta is given by

$$\beta_U = \frac{\text{cov}(\widetilde{r}_U, \widetilde{r}_m)}{\sigma_m^2}. \tag{20}$$

As firm U is sold at W_0 and will generate payoff $\widetilde{W}^1$, the rate of return for the buyer is $\widetilde{r}_U = \frac{\widetilde{W}^1}{W^0} - 1$, and (20) becomes

$$\beta_U = \frac{\text{cov}(\frac{\widetilde{W}^1}{W^0} - 1, \widetilde{r}_m)}{\sigma_m^2} = \frac{\text{cov}(\widetilde{W}^1, \widetilde{r}_m)}{W^0 \sigma_m^2}. \tag{21}$$

The value of firm L is easily found. Denoting with I the cash flow to debt, the equity cash flow is $\widetilde{W}^1 - I$. Bearing in mind that the cost of debt equals the risk-free rate we have

$$V_L = \frac{W^1 - I}{k_e} + \frac{I}{r_f} \tag{22}$$

where $k_e = r_f + \beta_e (r_f - r_m)$. Denoting with $\widetilde{r}_e$ the rate of return for firm L's stockholders, the beta of equity is given by

$$\beta_e = \frac{\text{cov}(\widetilde{r}_e, \widetilde{r}_m)}{\sigma_m^2}. \tag{23}$$

As equity is sold at W_0, the rate of return is $\widetilde{r}_e = \frac{\widetilde{W}^1 - I}{W^0} - 1$, so that (23) becomes

[5]The relations presented in this section may be interpreted in two ways: Perpetuity of constant cash flows may be assumed, as usual, or (for coherence with the above sections) one may think of a one-period firm so that $\widetilde{W}^1$ is the final free cash flow, the rates r_f, r_m, k_U, k_e are capitalization factors (i.e. 1 plus rate), and I represents interest+principal repayment.

$$\beta_e = \frac{\operatorname{cov}\left(\frac{\widetilde{W}^1 - I}{W^0} - 1, \widetilde{r}_m\right)}{\sigma_m^2} = \frac{\operatorname{cov}(\widetilde{W}^1 - I, \widetilde{r}_m)}{W^0 \sigma_m^2}. \tag{24}$$

But $\operatorname{cov}(\widetilde{W}^1 - I, \widetilde{r}_m) = \operatorname{cov}(\widetilde{W}^1, \widetilde{r}_m)$ for I is a real number. Consequently we have

$$\beta_e = \frac{(\widetilde{W}^1, \widetilde{r}_m)}{W^0 \sigma_m^2} = \beta_U \tag{25}$$

which implies

$$k_e = r_f + \beta_e (r_f - r_m) = r_f + \beta_U (r_f - r_m) = k_U \tag{26}$$

whence

$$V_L = \frac{W^1 - I}{k_U} + \frac{I}{r_f} = \frac{W^1}{k_U} - \frac{I}{k_U} + \frac{I}{r_f} \neq \frac{W^1}{k_U} = V_U .$$

This result contradicts Modigliani and Miller's Proposition I. This just means what we already know: Valuation is not invariant under changes in framing. In this case, we have two financially equivalent firms paying off the same total cash flows: viewing the latter either as an aggregate quantity or as the sum of two quantities of different nature makes valuation nonequivalent.[6] Analogously, choice behaviour may differ. Whenever agent P finds that

$$NPV_L = V_L - W^0 < 0 < V_U - W^0 = NPV_U$$

then firm U is purchased and firm L is not. In the opposite case

$$NPV_U = V_U - W^0 < 0 < V_L - W^0 = NPV_L$$

it is firm L to be purchased.[7] Again, this is a bias in the behaviour of our NPV enthusiast.

In contrast, economic profit leads to a correct decision: Economic profit from U is

$$(W^1 - W^0) - k_U W^0,$$

economic profit from L is

$$\left[(W^1 - I) - W^0 - k_e \cdot W^0\right] + \left[(I - 0) - r_f \cdot 0\right]$$

which are equal since $k_e = k_U$, as shown in (26).

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[6] We have assumed that agent L gives free his holdings of bonds. This is not restrictive, as the numerical example in section 3 shows: Assume A_1's cash flow is the equity cash flow of a levered firm, A_2's cash flow is the cash flow to debt, A_1 and A_2's outlays are just the price at which agent L is ready to sell equity and bonds respectively; suppose also B's cash flow is the capital cash flow of an unlevered firm and B's outlay is the price at which agent U is ready to sell the firm. Then, the values of the two firms differ, as seen.

[7] Obviously, in this case agent P becomes, at the same time, stockholder and bondholder.

8 Conclusions

For one-period investments under certainty, the Net Present Value (NPV) of an investment represents economic profit. Under uncertainty, a widespread method to value an asset (and thus to compute a NPV) is to discount cash flows with a disequilibrium cost of capital calculated via CAPM (for the use of an *equilibrium* NPV see Magni, 2007a, 2008). The (disequilibrium) NPV of an investment is formally given by the present value of excess profit (value is then computed as the present value of excess profit plus cost). The disequilibrium NPV is validly deducted from the CAPM, as Rubinstein (1973) shows. However this paper, focussing on such a *disequilibrium* NPV shows that:

- while it is true that this NPV is calculated by discounting economic profit (and value is found by adding cost), it does not represent economic profit
- the disequilibrium NPV is nonadditive, which also implies that the disequilibrium value is nonadditive
- NPV-minded decision makers incur framing effects in both valuation (different values and NPVs) and choice behaviour (accepting and rejecting the same investment)
- NPV-minded agents are open to arbitrage losses
- the standard disequilibrium NPV+CAPM valuation procedure is not consistent with Modigliani and Miller's Proposition I.

Consequently, the association of disequilibrium NPV and CAPM is a flawed methodology and should not be used for project valuation and selection, given that it does not fulfill the principle of description invariance (valuation and judgment must not depend on framing) and the principle of arbitrage (rational decision makers do not incur arbitrage losses).

References

Abel, A.B. (1990). Consumption and Investment. In B. M. Friedman and F. H. Hahn, Eds., *Handbook of Monetary Economics*. Amsterdam: North-Holland.

Biddle, G.C., Bowen, R. M., and Wallace, J. S. (1999). Evidence on EVA. *Journal of Applied Corporate Finance*, **12**, 69--79.

Begg, D., Fischer, S., and Dornbusch, R. (1984). *Economics*. Mc-Graw Hill, British Edition.

Brackenborough, S., McLean, T. and Oldroyd, D. (2001). The emergence of discounted cash flow analysis in the Tyneside coal industry c.1700-1820. *British Accounting Review*, **33**, 137−155.

Buchanan, J. (1969). *Cost and Choice. An Inquiry in Economic Theory*. Chicago: Markham. Republished as Midway reprint, Chicago: University of Chicago Press, 1977.

Dybvig, P.H. and Ingersoll, J.E. (1982). Mean-variance theory in complete markets. *Journal of Business*, **55** (2), 233−250.

Dixit, A. and Pindyck, R. (1994). *Investment under Uncertainty*. Princeton NJ: Princeton University Press.

Edwards, E. and Bell, P. (1961). *The Theory and Measurement of Business Income*. Berkeley: University of California Press.

Edwards, J.R. and Warman, A. (1981). Discounted cash flow and business valuation in a nineteenth century merger: A note. *The Accounting Historians Journal*, **8** (2), 37−50.

Fernández, P. (2002). *Valuation Methods and Shareholders Value Creation*. San Diego: Academic Press.

Fisher, I. (1930). *The Theory of Interest*. New York: Macmillan; (reprinted) Clifton, NJ: Augustum M. Kelley Publishers, 1974.

Kahneman, D. and Tversky, A. (1984). Choices, values and frames. *American Psychologist*, **39**, 341−350.

Lintner, J. (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics*, **47**, 13−37.

Magni, C.A. (2000a). Decomposition of a certain cash flow stream: differential systemic value and net final value, *Proceedings of the XXIV AMASES Conference* (Association for Mathematics Applied to Economic and Social Sciences) AMASES, Padenghe, September, pp. 206−217.

Magni, C.A. (2000b). Systemic Value Added, Residual Income and Decomposition of a Cash Flow Stream, Working paper n. 318, Department of Economics, University of Modena and Reggio Emilia. Available at SSRN: <http://ssrn.com/abstract=1032011>.

Magni, C.A. (2003). Decomposition of Net Final Values: Systemic Value Added and Residual Income. *Bulletin of Economic Research* **55** (2), 149−176.

Magni, C.A. (2004). Modelling excess profit. *Economic Modelling*, **21**, 595−617.

Magni, C.A. (2005). On Decomposing Net Final Values: EVA, SVA and Shadow Project. *Theory and Decision*, **59** (1), 51−95.

Magni, C.A. (2007a). CAPM and capital budgeting: present versus future, equilibrium versus disequilibrium, decision versus valuation. Available at SSRN: <http://ssrn.com/abstract=1024716>.

Magni, C.A. (2007b). Project selection and equivalent CAPM-based investment criteria, *Applied Financial Economics Letters*, **3** (3) (May), 165−168.

Magni, C.A. (2007c). Measuring performance and valuing firms: In search of the lost capital. Available at SSRN: <http://ssrn.com/abstract=1029244>.

Magni, C.A. (2008). Correct or incorrect application of CAPM? Correct or incorrect decisions with CAPM? *European Journal of Operational Research*, forthcoming. Available at SSRN: <http://ssrn.com/abstract=1025998>.

Marshall, A. (1890). *Principles of Economics*. London: MacMillan.

Martin, J.D., Petty, J.W., and Rich, S.P. (2003). An analysis of EVA and other measures of firm performance based on residual income. Hankamer School of Business Working paper. Available at SSRN: <http://papers.ssrn.com/sol3/papers.cfm?abstract_id=412122>.

Miller, P. and Napier, C. (1993). Genealogies of calculation. *Accounting, Organizations and Research*, **18** (7/8), 631−647.

Modigliani, F. and Miller, M. H. (1958). The cost of capital, corporation finance and the theory of investment. *The American Economic Review*, **48**, 261−297.

Mossin, J. (1968). Equilibrium in a Capital Asset Market, *Econometrica*, **34** (4) 768−783.

Nau, R. (1999). Arbitrage, incomplete models, and other people's brains. In M. Machina and B. Munier, Eds., *Beliefs, Interactions, and Preferences in Decision Making*, Kluwer Academic Press.

Nau, R. and McCardle K. (1991). Arbitrage, rationality, and equilibrium. *Theory and Decision*, **31**, 199−240.

Ohlson. J.A. (1995). Earnings, book values, and dividends in equity valuation. *Contemporary Accounting Research*, 11(2) (Spring), 661−687.

Parker, R.H. (1968). Discounted cash flow in historical perspective. *Journal of Accounting Research*, **6** (1), 58−71.

Peasnell, K. (1981). On Capital Budgeting and Income Measurement. *Abacus*, **17**, 52−67.

Peasnell, K.V. 1982. Some formal connections between economic values and yields and accounting numbers. *Journal of Business Finance & Accounting*, **9** (3), 361−381.

Preinreich, G. (1936). The Law of Goodwill. *The Accounting Review*, **11** (4) (December), 317−329.

Preinreich, G. (1937). Valuation and amortization. *The Accounting Review*, **12** (3), 209−226, September.

Preinreich, G. (1938). Annual Survey of Economic Theory: The Theory of Depreciation. *Econometrica*, **6** (1) (January), 219−241.

Rao, R. (1992). *Financial Management*, MacMillan.

Rubinstein, M.E. (1973). A mean-variance synthesis of corporate financial theory. *Journal of Finance*, **28**, 167−182.

Senbet, L.W. and Thompson, H.E. (1978). The equivalence of mean-variance capital budgeting models, Journal of Finance, **23**, 395−401.

Sharpe, W. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance*, **19**, 425−442.

Solomons, D. (1965). *Divisional Performance: Measurement and Control*. Homewood, IL: Richard D. Irwin.

Soman (2004). Framing, loss aversion, and mental accounting. In D. J. Koehler and N. Harvey, Eds., *Blackwell Handbook of Judgment and Decision Making*, pp. 379−398. Oxford, UK: Blackwell.

Thaler, R.H. (1985) . Mental accounting and consumer choice. *Marketing Science*, **4** (3), 199−214

Thaler, R.H. (1999). Mental accounting matters. *Journal of Behavioral Decision Making*, **12**, 183−206.

Tversky, A. and Kahneman, D. (1981). The framing of decisions and the psychology of choice. *Science*, **211**, 453−458.