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Human Capital Investment and Growth: A Dynamic Education Model

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Abstract

The paper aims to explicitly determine the distribution of human capital across hierarchic educational stages along the transition process, and to analyse the determinants of its evolution. We apply optimal control principles in a model of endogenous growth with two successive stages of education. We show that with initial relative scarcity of advanced human capital, the duration of studies at the advanced level should increase until reaching its equilibrium level. We also find that, by raising the duration of studies at the advanced schooling level, improvements in the quality of education at this level also enhances the economy's growth rate, both in the transition and in the long-run.

Introduction

The analysis addressed here aims to determine the optimal distribution of human capital across basic and advanced levels of education and along the development process, and to characterize the mechanisms that allow for establishing the equilibrium. The study of the effect of the distribution of human capital on economic growth is legitimate in light of the stylized facts which are raised in the micro-econometric literature regarding the effect of education on individuals' incomes. Psacharopoulos (1994) and Psacharopoulos and Patrinos (2004) provide a good overview of this empirical literature, which points out the supremacy of the social return to the primary school level, relative to those of the secondary and higher education levels.

However, for several reasons, the analysis of rates of return is inappropriate for the determination of an optimal distribution of human capital across the various school levels. First, in almost all countries, primary education is compulsory. Hence, as long as the rates of return associated with the post-primary levels are positive, it is always optimal for these countries to increase the proportion of the population investing in these levels. Unfortunately, this result leaves the optimal proportionality between primary and post-primary stocks of education undetermined. Second, as long as the micro-econometric empirical literature typically focuses on static rates of return, these rates cannot be a reliable policymaking tool in determining the optimal distribution of education along the economy's growth process, because of the dynamical interdependence that exists between the various types of human capital stocks. Finally, beyond these two issues, the estimated structure of rates of return is likely to be biased. An excellent critical evaluation of the rates of return estimations is made by Weale (1993), who argued that there are many biases entering into these estimations [1].

1: In general, rates of return can be biased upwards because of a failure to take into account the influences on incomes of factors other than education (experience, ability, socio-cultural origins, the quality of education…), or they may be depressed because survey-based calculations inevitably fail to take account of most external effects of education (less unemployment among the better educated, technological progress driven by the high skilled workers…). Rates of return such those summarized by Psacharopoulos (1994) or Psacharopoulos & Patrinos (2004) are likely to be highly overestimated. In fact, in these studies, the private return to education in the developing regions are between 13 and 37% for the primary, 13 and 24% for the secondary, and 18 and 28% for the higher-education level, which are likely too high estimates.

Our study applies optimal control theory principles to human capital investment in an endogenous growth model, in order to explore the optimal distribution of human capital across two successive education stages and along the growth process. The accumulation of education starts with the compulsory basic level, which has a uniform duration of studies for all individuals, and ends with the advanced level, where individuals choose their duration of studies. The remaining individuals' time is allocated to working. The individuals' stocks of human capital accumulated at each schooling stage depend principally on the duration of studies spent at this level, and on the relevant schooling quality given by public education expenditures. In addition to human capital, individuals accumulate physical capital, which is used as an input in production, like the two stocks of human capital. Individuals with basic education only choose the optimal path of consumption to maximize their present discounted value of utility, whereas individuals who decide to continue investing in advanced education choose both their optimal levels of consumption and duration of studies.

The model enables us to determine the optimal distribution of these human capital stocks in the long-run, and to study the transitional dynamics toward the equilibrium. We show that, in the symmetric equilibrium, individuals' optimal programs result in non-monotonicities in the transition path toward the long-run equilibrium distribution of human capital. In particular, starting from an initial situation with relative scarcity in advanced human capital, the duration of studies at this level should increase until the optimal distribution of human capital is reached. This equilibrium is found to be a saddle. The optimal growth path of consumption depends on the inter-temporal substitution elasticity parameter of consumption.

We also find that the long-run level of economic growth is increasing in the equilibrium duration of studies at the advanced education, which in turn is increasing in the quality of education received at this stage. The comparative static exercise shows that increasing the quality of education at the advanced level raises the ratio of advanced to basic human capital stocks along the transition path, and in the long-run as well. In addition, this quality improvement not only raises the long-run growth rate of the economy, but also allows reaching this growth rate faster. However, increases in the quality of education at the basic education level drive down the ratio of advanced to basic human capital stocks, both in the transition phase and in the long-run. Raising the quality of education at this level does not affect the long-run economic growth rate, but it speeds the transition to the steady state equilibrium.

Studies employing optimal control theory techniques to determine a dynamical solution for the optimal distribution of human capital are rare. For instance, Driskill and Horowitz (2002) employ optimal control principles in a model with two hierarchical education stages to determine optimal educational investment from a planner perspective. Their main finding is that expenditures on education should be concentrated on the advanced schooling level along the transition phase. Once the optimal human capital distribution is reached, this concentration should switch. However, one important implicit assumption in this study is that human capital investment is exogenous. Thereby, the mechanisms that allow reaching the optimal distribution of human capital are not determined. In addition, while this study analyses optimal investment choices, the long-run economic growth determinants are left unexplored.

The study of Rajhi (1996) is another interesting example, and employs optimal investment program techniques to determine the optimal path of human capital investment. There is only one level of education in Rajhi's (1996) study that allows for the accumulation of human capital. As in our model, consumption and the fraction of time allocated to education are two control variables that permit reaching the equilibrium. However, whereas Rajhi (1996) aims to determine the transitional and long-run proportionality between the stock of physical capital and that of human capital, our analysis focuses on the proportionality between two kinds of human capital stocks.

The remainder of this paper is organized as follows. The first section presents the theoretical model, where individuals' behaviours and the evolution of the aggregate economy are presented. In the second section, we characterise the equilibrium of the economy and determine the transition dynamics toward this equilibrium. Finally, in the third section, comparative static concerning the effects of varying the quality of education at both levels of schooling is discussed.

I- A two-stage education model

We consider a closed economy where the population has a fixed size of unity. There are two sectors. One production sector produces consumption goods, and one education sector forms human capital. Human capital investment is hierarchical, and this hierarchy is modelled as a two-stage process. All individuals firstly invest in the basic education level, which is compulsory and has a uniform duration of studies. Then, while some of these individuals choose to drop out with only a basic level of education, others decide to continue investing in the advanced level. Each individual has one period of time to allocate between education and labour.

I-1 Individuals

Individuals with different choices have different technologies of human capital accumulation and production. We distinguish two types of decisions:
a- A typical individual i who invests in basic education accumulates human capital according to the following technology:

$$\dot{H}^{i}_{Bt} = q_{B}\, l\, H_{At} \tag{1}$$

where subscript t indexes time and $0<l<1$.
There are three inputs in this technology: the compulsory duration of studies at this level, l; the relevant schooling quality, q_B (assumed exogenous and constant) [2], and the average advanced human capital stock, H_{At}. This formulation captures the positive externalities of the economy's average advanced human capital in individuals' accumulation of education at the basic level. In this sense, since the stock of advanced human capital is necessary in the formation of basic human capital, the higher this stock, the more important the individual's accumulation of human capital at the basic level.

With only basic education, an individual becomes an unskilled worker and has an income (or production) function defined by:

$$Y^{i}_{Bt} = \xi\,(K^{i}_{Bt})^{\theta}\left[(1-l)\,H^{i}_{Bt}\right]^{1-\theta} \tag{2}$$

where $0<\theta<1$ and ξ is total factors productivity. K^{i}_{Bt} denotes the individual's stock of physical capital, H^{i}_{Bt} is his accumulated stock of human capital, and $1-l$ is his remaining time devoted to working.
Output can be consumed or invested. If we note by C^{i}_{Bt} the consumption of this individual, we can write his investment in physical capital as follows:

$$\dot{K}^{i}_{Bt} = Y^{i}_{Bt} - C^{i}_{Bt} \tag{3}$$

This unskilled worker has an utility noted by $U(C^{i}_{Bt})$, which is specified by the following iso-elastic function:

$$U(C^{i}_{Bt}) = \frac{(C^{i}_{Bt})^{1-\sigma}-1}{1-\sigma} \tag{4}$$

where σ is the inter-temporal substitution elasticity parameter of consumption.

2: The quality of education in this study can be measured by public expenditures allocated to the education sector.

b- A typical individual i who invests in both levels of education accumulates human capital according to this technology:

$$\dot{H}^{i}_{At} = q_A \,(l + n^{i}_{t})\, H_{At} \tag{5}$$

where $0 < l + n^{i}_{t} < 1$.

Again, there are three inputs: the fraction of time necessary to be highly educated which is captured by the term $(l + n^{i}_{t})$, with n^{i}_{t} is the fraction of time invested in the advanced school level; the quality of education devoted to this level, q_A - which is assumed exogenous and constant-; and the average stock of advanced human capital, H_{At}. Once again, the presence of H_{At} in this equation captures the advanced human capital spillovers in the individual's human capital accumulation technology.

This individual becomes a skilled worker and has an income (or production) function given by:

$$Y^{i}_{At} = \zeta\,(K^{i}_{At})^{\mu} \left[(1 - l - n^{i}_{t})\, H^{i}_{At}\right]^{1-\mu} \tag{6}$$

where $0<\mu<1$, and ζ is total factors productivity. K^{i}_{At} denotes the individual's stock of physical capital, H^{i}_{At} is his accumulated stock of human capital, and $1 - l - n^{i}_{t}$ is his remaining time devoted to working.

This individual accumulates physical capital according to the following relation:

$$\dot{K}^{i}_{At} = Y^{i}_{At} - C^{i}_{At} \tag{7}$$

He derives utility from his consumption, $U(C^{i}_{At})$, given by the following function:

$$U\,(C^{i}_{At}) = \frac{(C^{i}_{At})^{1-\sigma} - 1}{1-\sigma} \tag{8}$$

where σ is the inter-temporal substitution elasticity parameter of consumption.

I-2 Optimal individuals' behaviour

I-2-1 The unskilled individual's program

The program of an individual which decides to dropout from the education system just after the basic level consists in choosing the optimal path of consumption that maximizes his present discounted value of utility. This program may be written as follows:

$$\begin{aligned} \max_{C^{i}_{Bt}} \; W^{i}_{B} &= \int_{0}^{+\infty} \mathrm{U}(\mathrm{C}^{i}_{Bt})\; e^{-\rho t}\, dt \\ s.t \quad \dot{K}^{i}_{Bt} &= \xi\,(K^{i}_{Bt})^{\theta}\left[(1-l)\, H^{i}_{Bt}\right]^{1-\theta} - C^{i}_{Bt} \\ \dot{H}^{i}_{Bt} &= l\; q_B\, H_{At} \end{aligned} \tag{9}$$

where ρ denotes the discount rate.

To characterize the optimal solution, we use Pontryagin's maximum principle. The present-value Hamiltonian for this program is:

$$\boldsymbol{H_1} = U(C^i_{Bt}) + \lambda_{Kt}\left(\xi\,(K^i_{Bt})^{\theta}\left[(1-l)\,H^i_{Bt}\right]^{1-\theta} - C^i_{Bt}\right) + \lambda_{Bt}\; l\; q_B\, H_{At}$$

where λ_{Kt}, and λ_{Bt} are the co-state variables defined as the implicit prices associated with the stocks of physical and basic human capital.

First order conditions yield:

$$\begin{aligned}
&(C^i_{Bt})^{-\sigma} = \lambda_{Kt}\\
&\dot{\lambda}_{Kt} / \lambda_{Kt} = \rho - (f^i_{Kt})_B\\
&\dot{\lambda}_{Bt} / \lambda_{Bt} = \rho - \left(\lambda_{Kt} / \lambda_{Bt}\right) f^i_{Bt}
\end{aligned} \tag{10}$$

where $(f^i_{Kt})_B = \partial Y^i_{Bt} / \partial K^i_{Bt}$ and $f^i_{Bt} = \partial Y^i_{Bt} / \partial H^i_{Bt}$ denote respectively the marginal productivity of physical capital and basic human capital of the unskilled worker. By using the first and second equations of the system (10), we can write the consumption growth rate of the unskilled individual as follows:

$$g_{C^i_B} = \frac{\dot{C}^i_{Bt}}{C^i_{Bt}} = \sigma^{-1}\left((f^i_{Kt})_B - \rho\right) \tag{11}$$

This condition implies that consumption may rise or fall depending on whether the marginal productivity of physical capital exceeds or falls below the rate of time preference.

I-2-2 The skilled individual's program

The program of a representative individual who decides to invest in advanced education is to maximize the present discounted value of utility by choosing the optimal paths of consumption and time devoted to advanced education. That is,

$$\begin{aligned}
&\max_{C^i_{At},\, n^i_t} \; W^i_A = \int_0^{+\infty} \mathrm{U}(\mathrm{C}^i_{At})\; \mathrm{e}^{-\rho\, t}\; \mathrm{d\,t}\\
&s.t \quad \dot{K}^i_{At} = \zeta\,(K^i_{At})^{\mu}\left[(1-l-n^i_t)\,H^i_{At}\right]^{1-\mu} - C^i_{At}\\
&\qquad \dot{H}^i_{At} = q_A\,(l+n^i_t)\,H_{At}
\end{aligned} \tag{12}$$

where ρ denotes the discount rate.

The present-value Hamiltonian for this program is:

$$\boldsymbol{H_2} = U(C^i_{At}) + \upsilon_{Kt}\left(\zeta\,(K^i_{At})^{\mu}\left[(1-l-n^i_t)\,H^i_{At}\right]^{1-\mu} - C^i_{At}\right) + \upsilon_{At}\, q_A\,(l+n^i_t)\,H_{At}$$

where υ_{Kt}, and υ_{At} are the co-state variables associated with physical and advanced human capital stocks. First order conditions yield the following system:

$$
\begin{aligned}
&(C^i_{At})^{-\sigma} = \upsilon_{Kt} \\
&\upsilon_{Kt}\, f^i_{At} = \upsilon_{At}\,(1-l-n^i_t)\, q_A \\
&\dot{\upsilon}_{Kt} / \upsilon_{Kt} = \rho - (f^i_{Kt})_A \\
&\dot{\upsilon}_{At} / \upsilon_{At} = \rho - q_A\,(1-l-n^i_t)
\end{aligned}
\qquad (13)
$$

where $(f^i_{Kt})_A = \partial Y^i_{At} / \partial K^i_{At}$ and $f^i_{At} = \partial Y^i_{At} / \partial H^i_{At}$ denote respectively the marginal productivity of physical capital and advanced human capital of the skilled worker. By combining the first and third equations of the system (13), the growth rate of consumption of a skilled worker may be written as follows:

$$
g_{C^i_A} = \frac{\dot{C}^i_{At}}{C^i_{At}} = \sigma^{-1}\left((f^i_{Kt})_A - \rho\right) \qquad (14)
$$

I-3 The aggregate economy

In this section, we use the individuals' behaviours to determine the growth rates of the main variables of the model at the aggregate level along the balanced growth path. In order to have symmetric equilibrium at all times one needs to assume identical individuals inside each group of workers; and equality between the marginal productivity of physical capital of skilled and unskilled workers, i.e., $(f^i_{Kt})_B = (f^i_{Kt})_A = f_{Kt}$.

Hence, in symmetric equilibrium, Equations (11) and (14) yield the following average growth rate of consumption:

$$
g_{C^i_A} = g_{C^i_B} = g_C = \sigma^{-1}\left(f_{Kt} - \rho\right) \qquad (15)
$$

Let's Y_t note the average production (or income) of the population. We assume that it depends on the average stocks of physical and human capital according to the following Cobb-Douglas technology:

$$
Y_t = \gamma\, K_t^{1-\alpha-\beta} \left[(1-l-n_t)\, H_{At}\right]^{\alpha} \left[(1-l)\, H_{Bt}\right]^{\beta} \qquad (16)
$$

where γ is total factor productivity, K_t, H_{At} and H_{Bt} denote respectively the average stocks of physical capital, advanced human capital and basic human capital. $1-l$ represents the average fraction of time that unskilled workers devote to working, $1-l-n_t$ is the average fraction of time supplied by the skilled workers on the labour market, and $(\alpha, \beta) \in \left]0\ ,\ 1\right[\times \left]0\ ,\ 1\right[$. The average physical capital evolves as follows:

$$
\dot{K}_t = Y_t - C_t \qquad (17)
$$

where C_t denotes average consumption.

If we note by $x_t = C_t / K_t$, the ratio of average consumption to average physical capital, and by $z_t = H_{At} / H_{Bt}$, the ratio of advanced to basic average human capital stocks, we can define from equations (1), (5), (16) and (17) the following growth rates:

$$g_{H_B} = \frac{\dot{H}_{Bt}}{H_{Bt}} = l\, q_B\, z_t \tag{18.a}$$

$$g_{H_A} = \frac{\dot{H}_{At}}{H_{At}} = q_A\, (l+n_t) \tag{18.b}$$

$$g_K = \frac{\dot{K}_t}{K_t} = f_{Kt} \,/\, (1-\alpha-\beta) - x_t \tag{18.c}$$

$$g_Y = \frac{\dot{Y}_t}{Y_t} = (1-\alpha-\beta)\, g_K + \alpha\, g_{H_A} + \beta\, g_{H_B} - \left(\alpha\, n_t \,/(1-l-n_t)\right) g_n \tag{18.d}$$

where g_{H_B}, g_{H_A}, g_K, g_n, and g_Y denote respectively the growth rates of the average basic human capital, the average advanced human capital, the average stock of physical capital, the average duration of advanced education, and the average income.

The balanced growth path is characterized by the constancy of the marginal productivity of the physical capital, f_{Kt}, the constancy of the ratios x_t and z_t, and the constancy of the fraction of time, n_t. Indeed, the constancy of f_{Kt} is necessary to obtain from equation (15) a constant consumption growth rate. Similarly, the constancy of z_t is necessary to get from equation (18.a) a constant growth rate of basic human capital stock. Finally, Equations (18.b) and (18.c) show that f_{Kt}, x_t and n_t must be constant so that the average stocks of physical capital and advanced human capital grow at a constant rate.

The constancy of z_t implies that $g_{H_A} = g_{H_B}$. Similarly, the constancy of x_t implies that $g_C = g_K$. Furthermore, by using the definition of f_{Kt}, it can be deduced from the constancy of n_t and the fact that $g_{H_A} = g_{H_B}$, that $g_K = g_{H_A}$. By using (18.d), it is interesting to show that all the key variables of the model grow in the stationary equilibrium at a same rate given by:

$$g_Y = g_K = g_{H_A} = g_{H_B} = g_C = \sigma^{-1}\left(f_K * - \rho\right) \tag{19}$$

where $f_K *$ will be next determined.

II - Transition dynamics

We show in this section how the proper mechanisms of the model allow re-establishing the long-run proportionality between basic and advanced human capital stocks during the transition dynamics, when the initial condition of such proportionality is violated in terms of relative scarcity or abundance of the advanced human capital. To characterize transition dynamics from an initial condition until the optimal distribution of human capital, $z* = \left(H_A \,/\, H_B\right)*$, is reached, we re-write the dynamical system (18.a)-(18.d) as a function of the variables x_t, z_t, f_{Kt} and n_t, which are constant in the stationary equilibrium, i.e.,:

$$g_x = g_C - g_K = x_t + \left(\frac{\delta-\sigma}{\delta\sigma}\right) f_{Kt} - \frac{\rho}{\sigma} \tag{20.a}$$

$$g_z = g_{H_A} - g_{H_B} = q_A\, (l+n_t) - l\, q_B\, z_t \tag{20.b}$$

$$g_n = \left(\frac{1-l-n_t}{\alpha\, n_t}\right)\left(f_{Kt} - \delta\, x - (1-\alpha)\, q_A\, (l+n_t) + \alpha\, l\, q_B\, z_t\right) \tag{20.c}$$

$$g_{f_K} = g_{H_A} - g_K = q_A(l+n_t) + x_t - f_{Kt}/\delta \tag{20.d}$$

where $\delta = 1-\alpha-\beta$. The expression of g_n is obtained by using the fact that $g_Y = g_{H_A}$ in the stationary equilibrium. By using the definition of the physical capital marginal productivity as well as equation (20.c), it is easy to show that the growth rate of this productivity, g_{f_K}, can be written as in (20.d).

According to the equations (20.a) and (20.d), the constancy of x_t and f_{Kt} implies that:

$$f_{Kt} = \rho + \sigma\, q_A(l+n_t) \tag{21}$$

Since we are focussing on the optimal distribution of two types of human capital, we substitute equation (21) in the system (20.a)-(20.c), and we use first-order conditions in (13) in order to re-write the dynamical model as a function of x_t, z_t, and n_t only. We obtain:

$$g_z = \frac{\rho}{\delta-\sigma} - \frac{\delta}{\delta-\sigma} x_t - l\, q_B\, z_t \tag{22.a}$$

$$g_x = \frac{\delta}{\sigma} x_t + \frac{\delta-\sigma}{\sigma} q_A(l+n_t) - \frac{\rho}{\sigma} \tag{22.b}$$

$$g_n = \frac{1-l-n_t}{\alpha n_t}\left(-\delta x + q_A - (l+n_t)\, q_A(1+\delta)\right) \tag{22.c}$$

As pointed out above, along the balanced growth path, x_t, z_t and n_t are constant, i.e., $g_x = g_z = g_n = 0$. This implies the following steady state equilibrium values:

$$\begin{aligned} z^* &= \frac{(q_A-\rho)}{l\, q_B(1+\sigma)} \\ x^* &= \frac{\rho(1+\delta)+(\sigma-\delta)q_A}{\delta(1+\sigma)} \\ (l+n^*) &= \frac{q_A-\rho}{q_A(1+\sigma)} \end{aligned} \tag{23}$$

Notice that an implicit assumption of the model stemming from the system (23) is that $q_A > \rho$. By using the equilibrium values in (23), the relation (21) yields:

$$\begin{aligned} f_K^* &= \rho + \sigma\, q_A(l+n^*) \\ &= \frac{\rho+\sigma q_A}{1+\sigma} \end{aligned} \tag{24}$$

Substituting (24) in (19) allows re-writing the long-run growth rate as follows:

$$g_Y = g_K = g_{H_A} = g_{H_B} = g_C = \left(\frac{q_A-\rho}{1+\sigma}\right) \tag{25}$$

To determine the stability of this steady state equilibrium, we study the linear differential equation system that approximates (22.a), (22.b) and (22.c) at z^*, x^* and n^*. This linearization yields the following system [3].

$$\begin{bmatrix} \dot{z} \\ \dot{x} \\ \dot{n} \end{bmatrix} = \begin{bmatrix} \phi_z & \phi_x & \phi_n \\ \psi_z & \psi_x & \psi_n \\ \Omega_z & \Omega_x & \Omega_n \end{bmatrix} \begin{bmatrix} z - z^* \\ x - x^* \\ n - n^* \end{bmatrix} \tag{26}$$

where the elements of the Jacobian matrix evaluated at the steady state are:

$$\phi_z = \frac{\partial \dot{z}}{\partial z} = \frac{\rho - q_A}{1+\sigma} < 0, \qquad \phi_x = \frac{\partial \dot{z}}{\partial x} = \frac{\delta}{\sigma - \delta} z^* >?<0, \qquad \phi_n = \frac{\partial \dot{z}}{\partial n} = 0,$$

$$\psi_z = \frac{\partial \dot{x}}{\partial z} = 0, \qquad \psi_x = \frac{\partial \dot{x}}{\partial x} = \frac{\delta}{\sigma} x^* > 0, \qquad \psi_n = \frac{\partial \dot{x}}{\partial n} = \frac{\delta - \sigma}{\sigma} q_A >?<0,$$

$$\Omega_z = \frac{\partial \dot{n}}{\partial z} = 0, \quad \Omega_x = \frac{\partial \dot{n}}{\partial x} = -\frac{\delta(\rho + \sigma q_A)}{\alpha(1+\sigma) q_A} < 0, \quad \Omega_n = \frac{\partial \dot{n}}{\partial n} = -\frac{(1+\delta)(\rho+\sigma q_A)}{\alpha(1+\sigma)} < 0.$$

The notation "$>?<0$" associated with ϕ_x and ψ_n means that the sign of these two elements is either positive or negative depending on the sign of ($\delta - \sigma$).

The eigenvalues of the Jacobian matrix determine the local stability properties of the economy. These eigenvalues are defined as the roots of the characteristic equation generated by the Jacobian matrix. This characteristic equation is given by: $\det(\kappa I - J)$, with κ is an eigenvalue, J is the Jacobian matrix, and '$\det$' is the determinant of the matrix $(\kappa I - J)$. At the steady state, the characteristic equation writes:

$$\kappa^3 - \kappa^2(\phi_z + \psi_x + \Omega_n) + \kappa\left[\phi_z \psi_x + \Omega_n(\phi_z + \psi_x) - \psi_n \Omega_x\right] + \phi_z(\psi_n \Omega_x - \psi_x \Omega_n),$$

which has the following roots:

$$\kappa_1 = \frac{\psi_x + \Omega_n}{2} + \frac{\Delta^{1/2}}{2},$$

$$\kappa_2 = \frac{\psi_x + \Omega_n}{2} - \frac{\Delta^{1/2}}{2},$$

$$\kappa_3 = \phi_z$$

where $\Delta = (\psi_x - \Omega_n)^2 + 4\psi_n \Omega_x$

The sign of κ_3 is negative, while the signs of κ_1 and κ_2 depend on σ and δ. It is worthwhile to distinguish three cases, which after some calculations give the following signs:

3: Linearizing around the steady state permits to study the dynamic and stability of a non-linear system of equations by transforming it to a linear one. This consists on calculating the Jacobian matrix of the system and evaluating it at the steady state equilibrium.

If $\underline{\sigma > \delta}$, we have $\psi_n < 0$ and $\Delta > 0$. It follows that $\kappa_1 > 0$ and $\kappa_2 < 0$.

If $\underline{\sigma < \delta}$, we have $\psi_n > 0$ and $\Delta > 0$. It follows that $\kappa_1 > 0$ and $\kappa_2 < 0$.

If $\underline{\sigma = \delta}$, we have $\psi_n = 0$ and $\Delta > 0$. It follows that $\kappa_1 > 0$ and $\kappa_2 < 0$.

Hence, in all the cases we have $\kappa_1 > 0$, $\kappa_2 < 0$, and $\kappa_3 < 0$, which imply that the stationary equilibrium is a stable saddle-point.

Figures 1-a, 1-b, and 1-c, shown below, present a graphical solution of the transitional dynamics of the model, using phase diagrams for $\sigma < \delta$, $\sigma > \delta$ and $\sigma = \delta$, respectively. In all of the figures, the system (22.a)-(22.c) is represented by the three corresponding curves in phases (I) and (II). In phase (III), we represent a 45° line in the (n_t, n_t) plane in order to relate phase (II) with phase (IV). Finally, phase (IV) represents, once again, the equation $g_z = 0$ in order to display z_t as a function of n_t [4]. This phase relates phase (III) with the other phases of the diagram.

We begin the discussion of the diagrams by assuming that at the initial period, say at $t=0$, the economy is characterised by scarcity in advanced human capital relative to basic human capital, so that the ratio z_0 is under-adjusted relative to its long-run level, z^*. Therefore, z_0 should increase until z^* is reached. The convergence of z_0 to its long-run level is determined by the transition paths of the consumption ratio, x_t, and of the fraction of time allocated to advanced human capital investment, n_t.

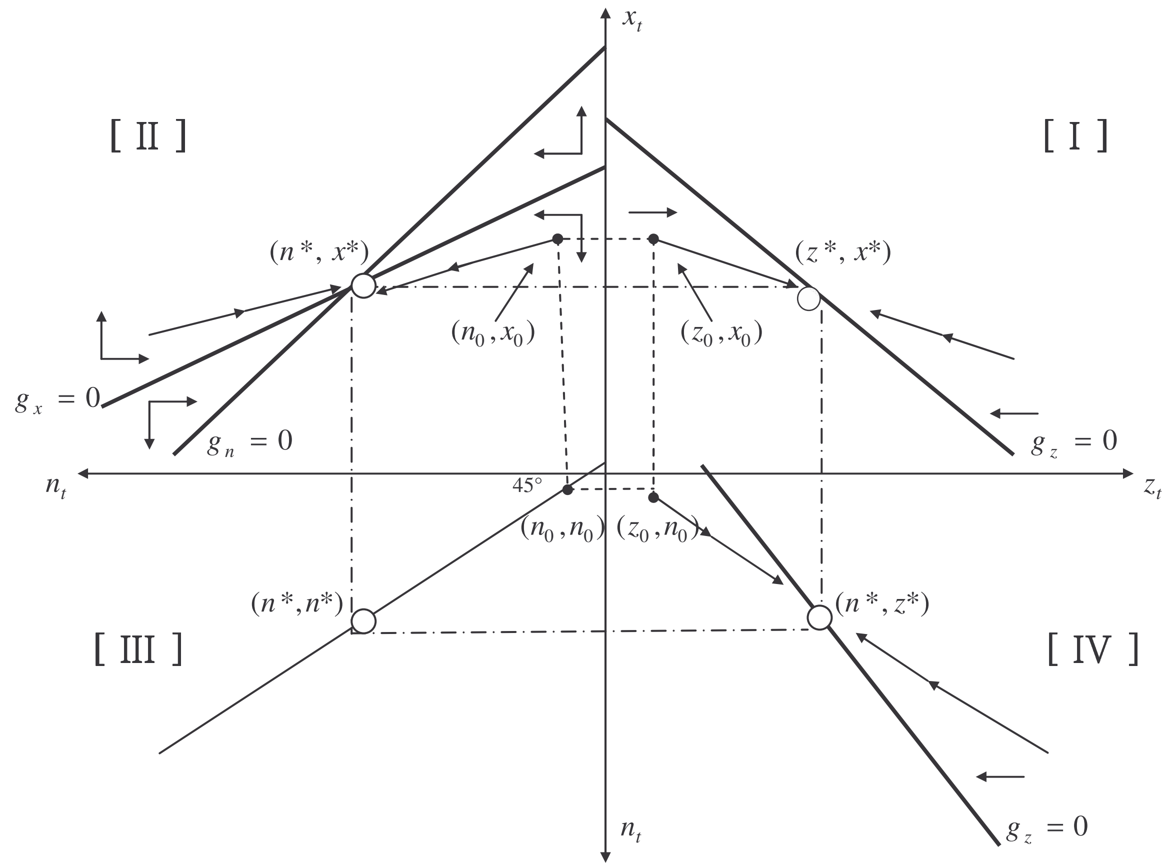
Whether the initial levels of x_0 and n_0 are under-adjusted or over-adjusted relative to their respective stationary levels, x^* and n^*, respectively, depends on the inter-temporal substitution elasticity parameter of consumption, σ. In all of the cases, the diagrams show a unique path that allows reaching the stationary long-run equilibrium.

Case 1: $\sigma < \delta$

This is a situation where consumers permute easily present and future consumption (σ is low). In this case, each of the locus $g_z = 0$, $g_x = 0$ and $g_n = 0$ are represented in phases (I) and (II) by a downward sloping curve. It follows that when z_0 is under-adjusted, the initial ratio of consumption, x_0, is over-adjusted relative to its equilibrium level, x^*, while the initial fraction of time allocated to advanced education, n_0, is under-adjusted relative to its long-run level, n^*.

4 : By using the relation (20.b), it's easy to show that z_t is monotonically increasing in n_t along the transition path. i.e., $z_t = (l + n_t)\, q_A \;/ l\, q_B$.

Figure 1-a: *Transition phase diagram and optimal human capital distribution: case of* $\sigma < \delta$

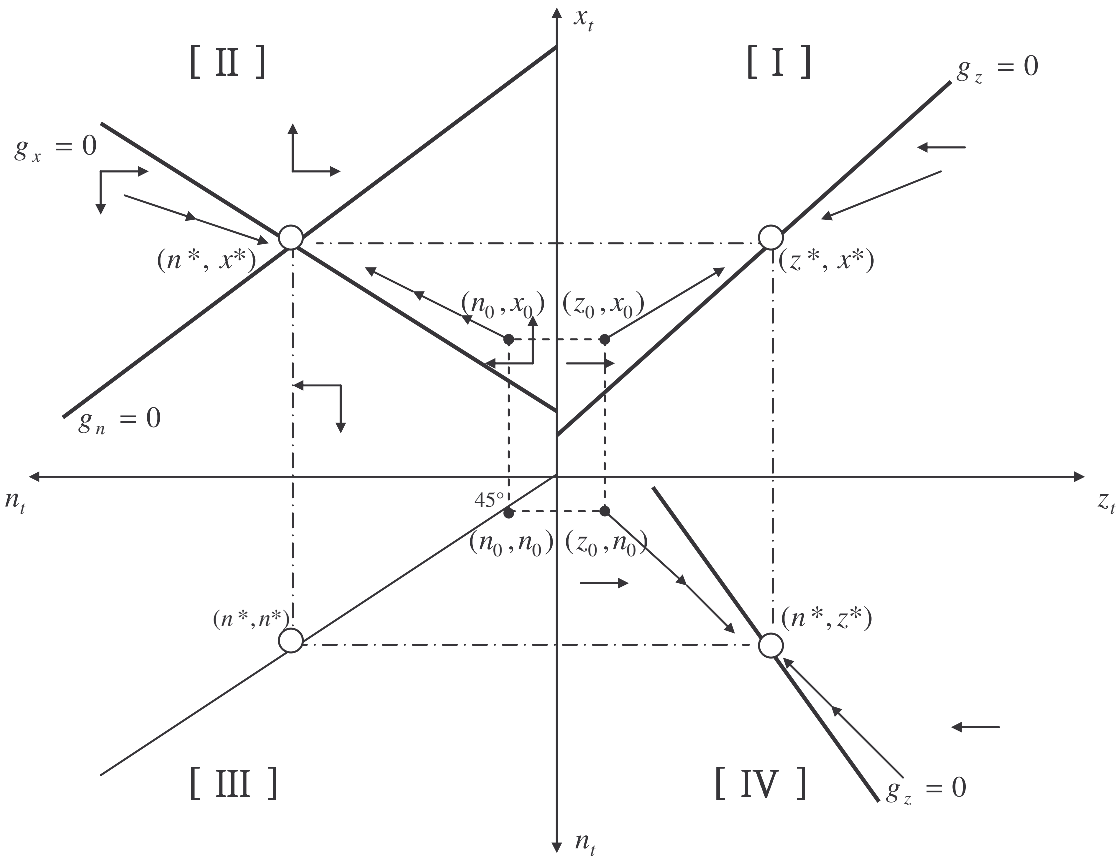


Therefore, the steady state will be approached monotonically, with x_t falling and n_t growing. This adjustment increases the ratio of relative human capital stocks, z_t, until it reaches the equilibrium value z^*. Indeed, the increase in n_t favours the accumulation of human capital at the advanced level and raises the ratio of z_t as shown in phase (IV).

Case 2: $\sigma > \delta$

This is the case of a low degree of substitution between present and future consumption. The locus $g_z = 0$ and $g_x = 0$ are now represented in phases (I) and (II) by two upward sloping curves, while the locus $g_n = 0$ is still represented by the downward sloping curve, as in the previous case. In this situation, for any $z_0 < z^*$, both x_0 and n_0 must increase to reach their stationary values. This adjustment allows increasing the average stock of advanced human capital, and thereby re-establishing proportionality between advanced and basic human capital stocks.

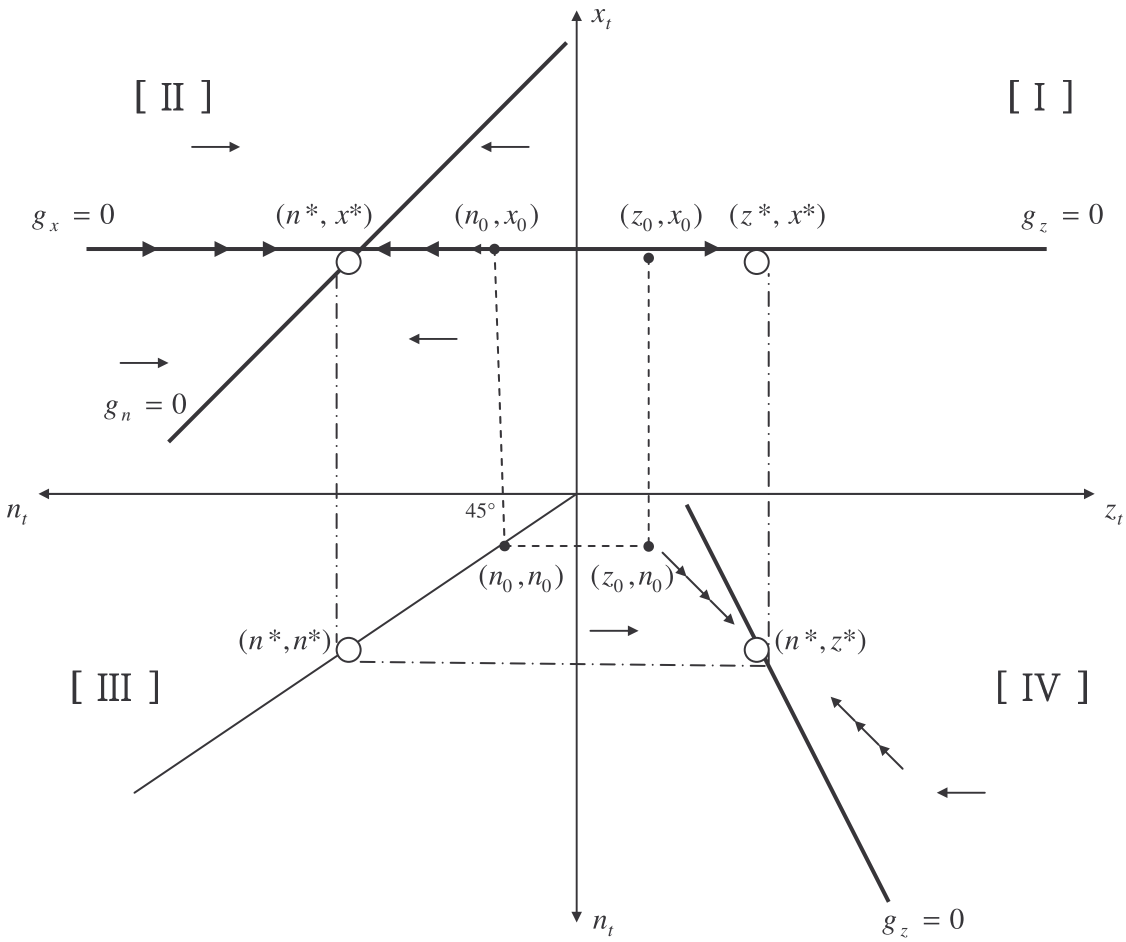
Figure 1-b: *Transition phase diagram and optimal human capital distribution: case of* $\sigma > \delta$



Case 3: $\sigma = \delta$

This case illustrates a situation where σ is moderate. The locus $g_z = 0$ and $g_x = 0$ are illustrated in phases (I) and (II) respectively by two horizontal curves. The ratio of x_0 is well-adjusted relative to its long-run equilibrium level, $x^* = \rho/\sigma$. By contrast, as long as n_t is decreasing in x_t, it follows from the $g_n = 0$ locus that the initial ratio of n_0 is under-adjusted relative to its equilibrium level. Under this condition, the dynamic consists simply of increasing the ratio of n_0 along the curve $g_x = 0$ until n^* is reached. This shift speeds the average growth rate of advanced human capital stock, and brings the ratio of z_0 to its long-run level, z^*, as illustrated in phase (IV) of the figure.

Figure 1-c: *Transition phase diagram and optimal human capital distribution: case of* $\sigma = \delta$



In the following proposition, the optimal transition dynamics of the model are summarized:

Proposition 1

Starting from an initial position with relative scarcity in advanced human capital (z_0 is low), the optimal transition path involves an increase in the duration of advanced education, n_t, to reach its long-run level. The ratio of consumption, x_t, decreases, increases, or remains constant along the transition depending on the level of σ, the inter-temporal substitution elasticity parameter of consumption. The steady state equilibrium is found to be a saddle.

III- Comparative static analysis

Comparative static analysis of the steady state can be made. We focus on the effects of varying the quality of education at the advanced and basic schooling levels, q_A and q_B, respectively, at a given period t of transition. According to the dynamical system (22.a)-(22.c) and the stationary levels given in (23), such a variation has both transitional and long-run effects.

III-1 The impact of improving the quality of education at the advanced level

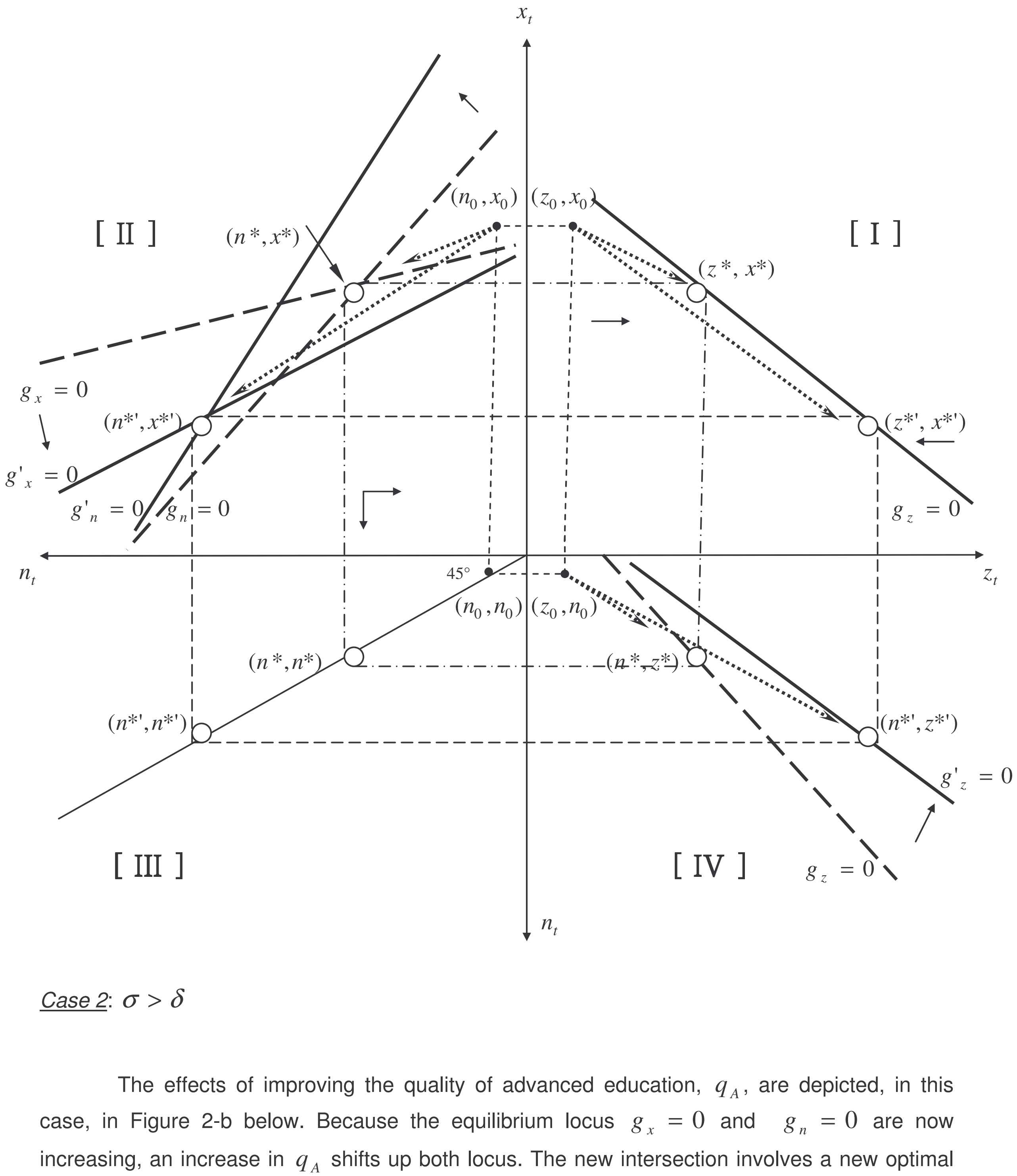
An increase in the quality of education at the advanced schooling level, q_A, shifts the stationary equilibrium curves $g_x = 0$ and $g_n = 0$, and thereby involves new long-run equilibrium values. These effects differ with respect to the cases considered above.

<u>*Case 1*</u>: $\sigma < \delta$

This case is illustrated in Figure 2-a. An improvement in q_A lowers the $g_x = 0$ locus and raises the $g_n = 0$ locus. The new curves are represented in the figure by $g'_x = 0$ and $g'_n = 0$, respectively. The shift in the equilibrium curves involves new long-run stationary values of n^* and x^*. Specifically, the equilibrium ratio of consumption decreases, while the duration of studies in advanced education increases. The new combination is now $(x^{*\prime}, n^{*\prime})$ in the figure. The upward shift in n^* results in an upward shift in the locus $g_z = 0$, as illustrated in phase (IV) of the figure. This, in turn, raises the equilibrium ratio of relative human capital stocks from z^* to $z^{*\prime}$.

The two optimal transition paths before and after varying the quality of advanced education, q_A, are shown in the figure by the dotted arrows. It is worthwhile to point out that one can evaluate the effects of varying q_A on the long-run equilibrium ratios by referring to the relations given in (23). Clearly, it can be found that: $\partial n^* / \partial q_A > 0$, $\partial z^* / \partial q_A > 0$, and $\partial x^* / \partial q_A < 0$ if $\sigma < \delta$, and $\partial x^* / \partial q_A > 0$ if $\sigma > \delta$.

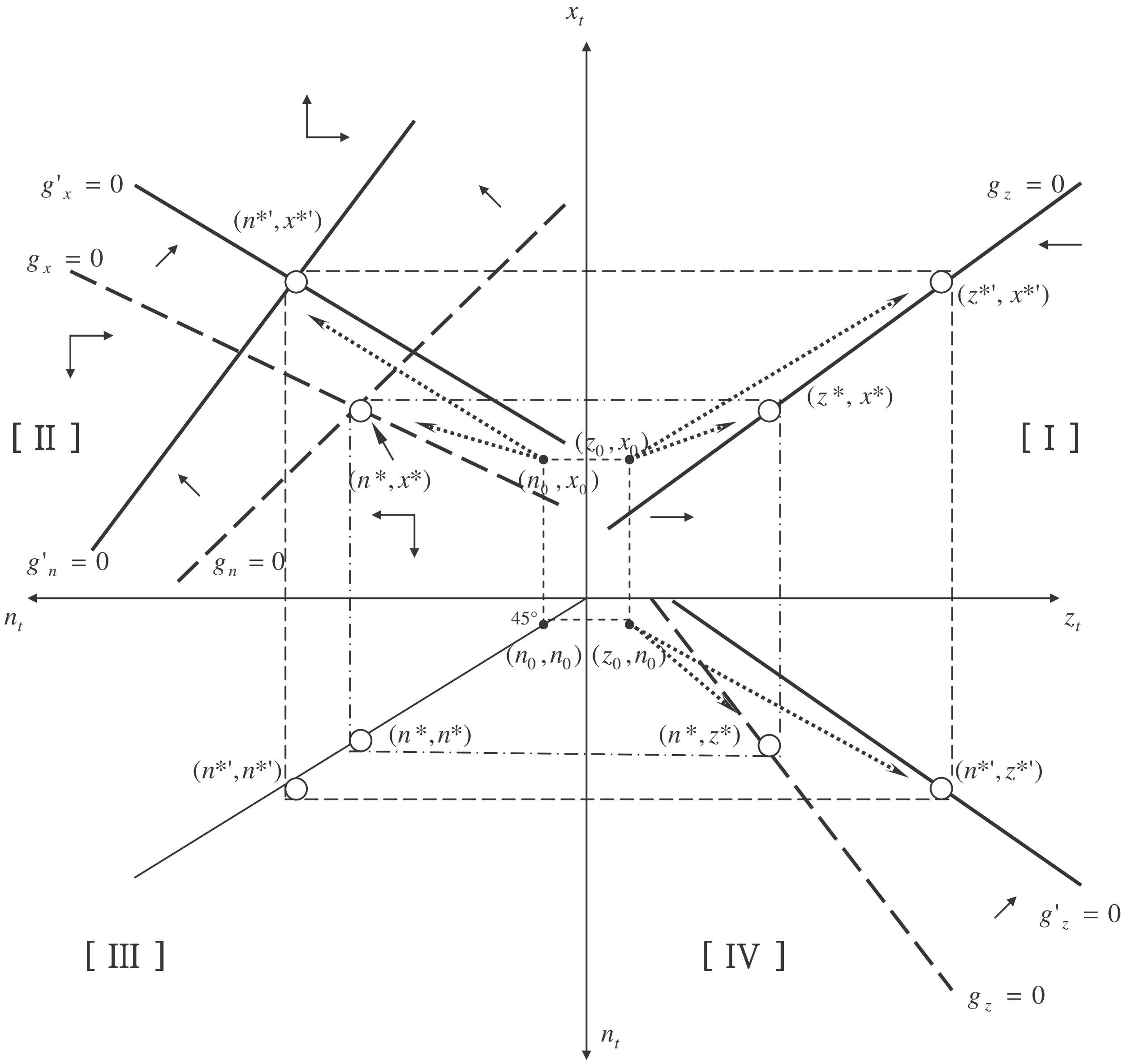
Figure 2-a: *The impact of improving* q_A *: case of* $\sigma < \delta$



<u>Case 2</u>: $\sigma > \delta$

The effects of improving the quality of advanced education, q_A, are depicted, in this case, in Figure 2-b below. Because the equilibrium locus $g_x = 0$ and $g_n = 0$ are now increasing, an increase in q_A shifts up both locus. The new intersection involves a new optimal transition path, which yields a higher equilibrium ratio of consumption, and a longer duration of advanced education. Phase (IV) of the diagram shows that the upward shift in n_t is associated with an upward shift in the ratio of relative human capital stocks, z_t. This adjustment yields a higher equilibrium ratio of z^*, which is noted in the figure by $z^{*'}$.

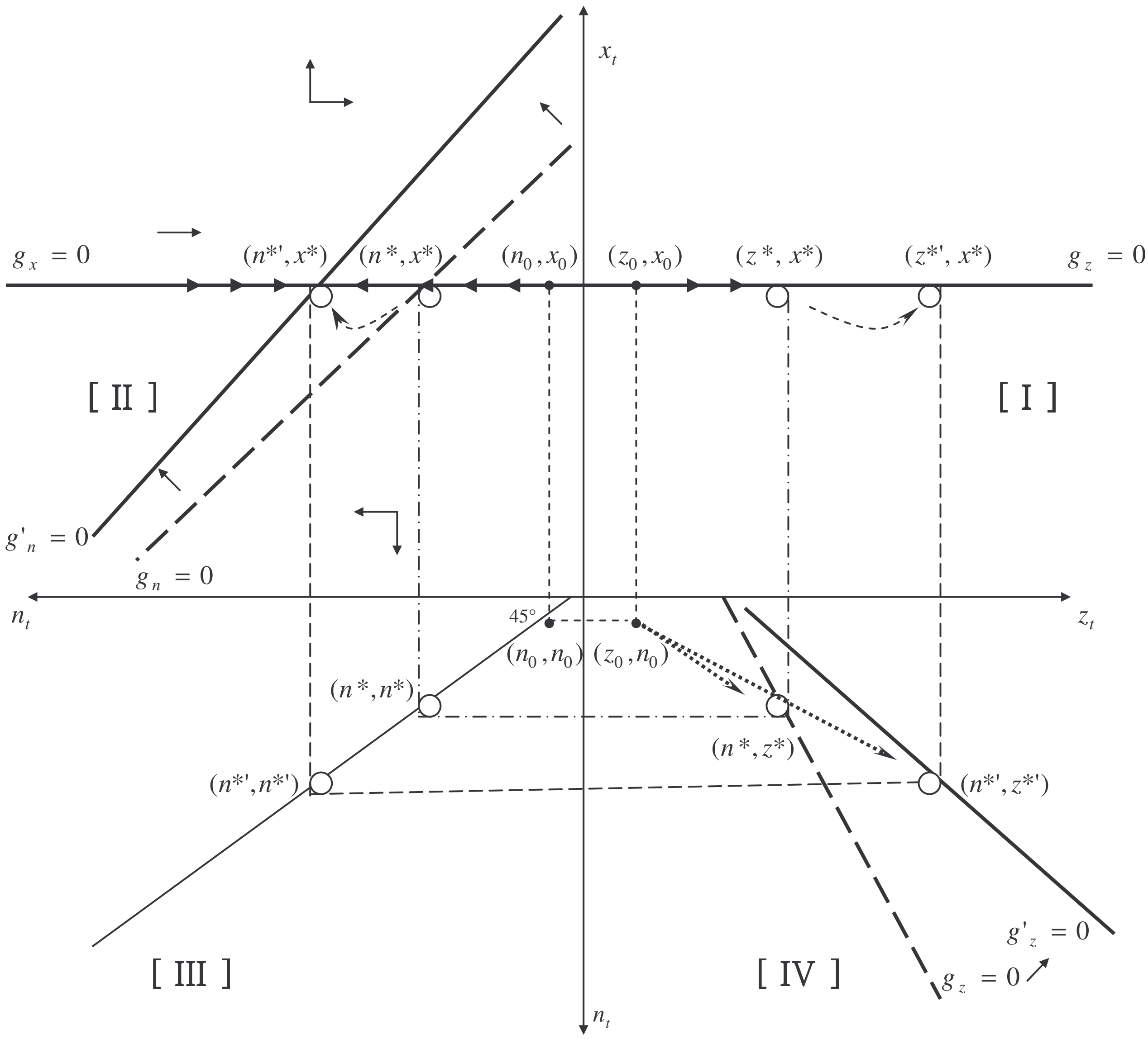
Figure 2-b: The impact of improving q_A : case of $\sigma > \delta$



<u>*Case 3*</u>: $\sigma = \delta$

Under this condition, an improvement in q_A results in an upward shift in the equilibrium locus $g_n = 0$. The $g_x = 0$ locus is still represented in Figure 2-c by a horizontal curve, so that the long-run equilibrium ratio of consumption, x^*, remains unaffected by the variation in q_A. It follows that the average duration of advanced education n_t increases along the curve $g_x = 0$ until reaching the new equilibrium combination, $(n^{*\prime}, x^*)$. The adjustment in n_t is associated with an upward shift in the transition path of the ratio z_t towards its equilibrium value. The latter rises from z^* to $z^{*\prime}$ in the figure.

Figure 2-c: *The impact of improving* q_A*: case of* $\sigma = \delta$



Finally, it is crucial to underline that, independently of σ, Equations (18.b) and (18.d) show that an increase in q_A accelerates the growth rate of g_{H_A}, which, in turn, results in a higher growth rate of the economy along the transition path. This implies a faster convergence toward the long-run equilibrium. In the long-run, the increase in q_A also leads to an increase in the equilibrium growth rate of the economy, as can be deduced from Equation (25), i.e., $\partial g_Y / \partial q_A > 0$. Notice that the growth effect of the improvement in the quality of advanced education may also stem from the increase in the stationary duration of advanced education, n^*, that accompanies such improvement, as shown in Equation (23).

III- 1 The impact of improving the quality of education at the basic level

This section briefly discusses the impacts of improving the educational quality at the basic schooling level on the optimal transition paths and the long-run equilibrium. An increase in q_B fosters the growth rate of basic human capital, g_{H_B}, and leaves unaffected the growth rate of advanced human capital, g_{H_A}, as can be seen from Equations (18.a) and (18.b). Therefore, following Equations (22.a)-(22.c), this policy scheme implies a new transition path along which only the equilibrium locus $g_z = 0$ shifts, while the $g_x = 0$ and $g_n = 0$ curves are left unchanged.

Figures 3-a, 3-b, and 3-c in the Appendix (A) depict the new transition paths and long-run equilibrium combinations associated with this policy for different cases of σ. It is crucial to underline that, although the improvement in q_B has no effect on the long-run growth rate, g_Y, as it comes out from Equation (25), it fosters this growth rate during the transition - as can be deduced from Equations (18.a) and (18.d). That is, the increase in q_B leads to reaching the long-run equilibrium growth rate more rapidly.

<u>Case 1</u>: $\sigma < \delta$

In this case, and according to Equation (22.a), an increase in q_B shifts down the stationary equilibrium curve $g_z = 0$ as illustrated in phase (I) of the diagram. This policy does not affect the equilibrium curves in phase (II), so that the long-run combination of (n^*, x^*) is unchanged. Thereby, the optimal transition path involves an increase in the equilibrium ratio of basic to advanced human capital stocks, or equivalently, a decrease in the ratio of z^* towards $z^{*\prime}$, as illustrated in Figure 3-a.

<u>Case 2:</u> $\sigma > \delta$

The same mechanism of adjustment toward the new steady state equilibrium is involved here. Improving the quality of basic education, q_B, fosters the accumulation rate of basic human capital, and thereby lowers the ratio of z_t. In the long run, this ratio converges to a lower value, $z^{*\prime}$. The long-run values of consumption and duration of advanced education are unaffected by the change in q_B. The adjustment of z_t is represented in Figure 3-b by the upward shift in the $g_z = 0$ locus.

<u>Case 3</u>: $\sigma = \delta$

As in the previous cases, the equilibrium value of z^* is decreased as a result of increasing q_B. This is illustrated in Figure 3-c by the downward shift in the locus $g_z = 0$ in phase (IV) until the new equilibrium of $z^{*\prime}$ is reached. Once again, both optimal long-run ratios of consumption and duration of advanced education are left unchanged. The proposition below sums up the results established in the comparative static analysis:

Proposition 2

a- *An improvement in the quality of advanced education,* q_A*, has both transitional and long-run effects:*

- *In the short-run, it results in an upward shift in the optimal transition path of the duration of advanced education,* n_t*, which in turn is associated with an upward shift in the optimal transition* *path of advanced to basic human capital ratio, z_t. The increase in q_A also accelerates the growth rate of the economy, g_Y, during the transition.*

- In the long-run, the equilibrium values of n^* and z^* are increased, and the economy grows at a faster rate.

b- *An improvement in the quality of basic education, q_B, has the following effects:*

- In the short-run, it promotes the accumulation rate of basic education, and thereby, it shifts down the optimal transition path of the ratio of z_t. It also increases the growth rate of the economy, g_Y, during the transition path.

- In the long-run, the equilibrium value of z^* is decreased while the long-run growth rate of the economy is left unaffected.

Conclusion

The model considered here contributes to the recent macroeconomic literature on the optimal distribution of human capital. It addresses the issue of how to determine the optimal distribution of human capital across basic and advanced stages of education. Its main novelty is its explicit formalization of the economy's transitional dynamics and long-run equilibrium within an endogenous growth framework that employs optimal control principles. As long as basic education is compulsory, the shift in the distribution of human capital toward its equilibrium level is determined by the evolution of the duration of studies at the advanced level, and that of consumption level.

The analysis shows that, with initial situation of scarcity in the advanced human capital, the duration of studies should increase until reaching the equilibrium level, while the evolution of consumption during the transition depends on the inter-temporal substitution elasticity parameter of consumption. We also show that, along the transition and in the long-run as well, the ratio of advanced to basic human capital stocks increases with the quality of education allocated to the advanced level, whereas it decreases in the quality of the basic educational level. Furthermore, we find that the economy's growth rate increases in both the transition phase and the long-run as the quality of advanced education improves. However, an improvement in the quality of education at the basic level does not affect the long-run growth rate, but speeds the transition toward the steady state equilibrium.

Appendix (A): *Phase diagrams associated with the improvement in* q_B

Figure 3-a: *The impact of improving* q_B*: case of* $\sigma < \delta$

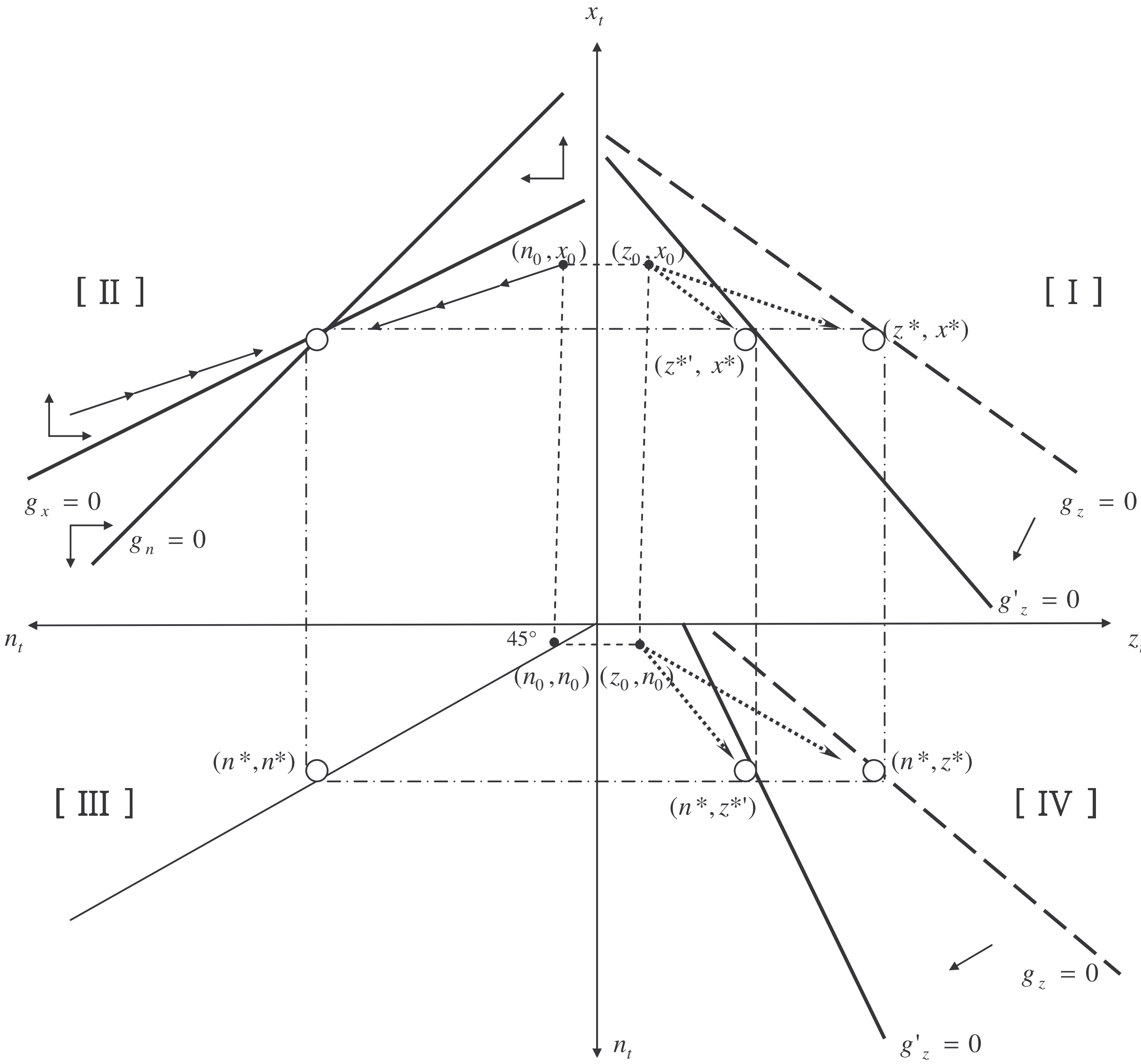


Figure 3-b: *The impact of improving* q_B*: case of* $\sigma > \delta$

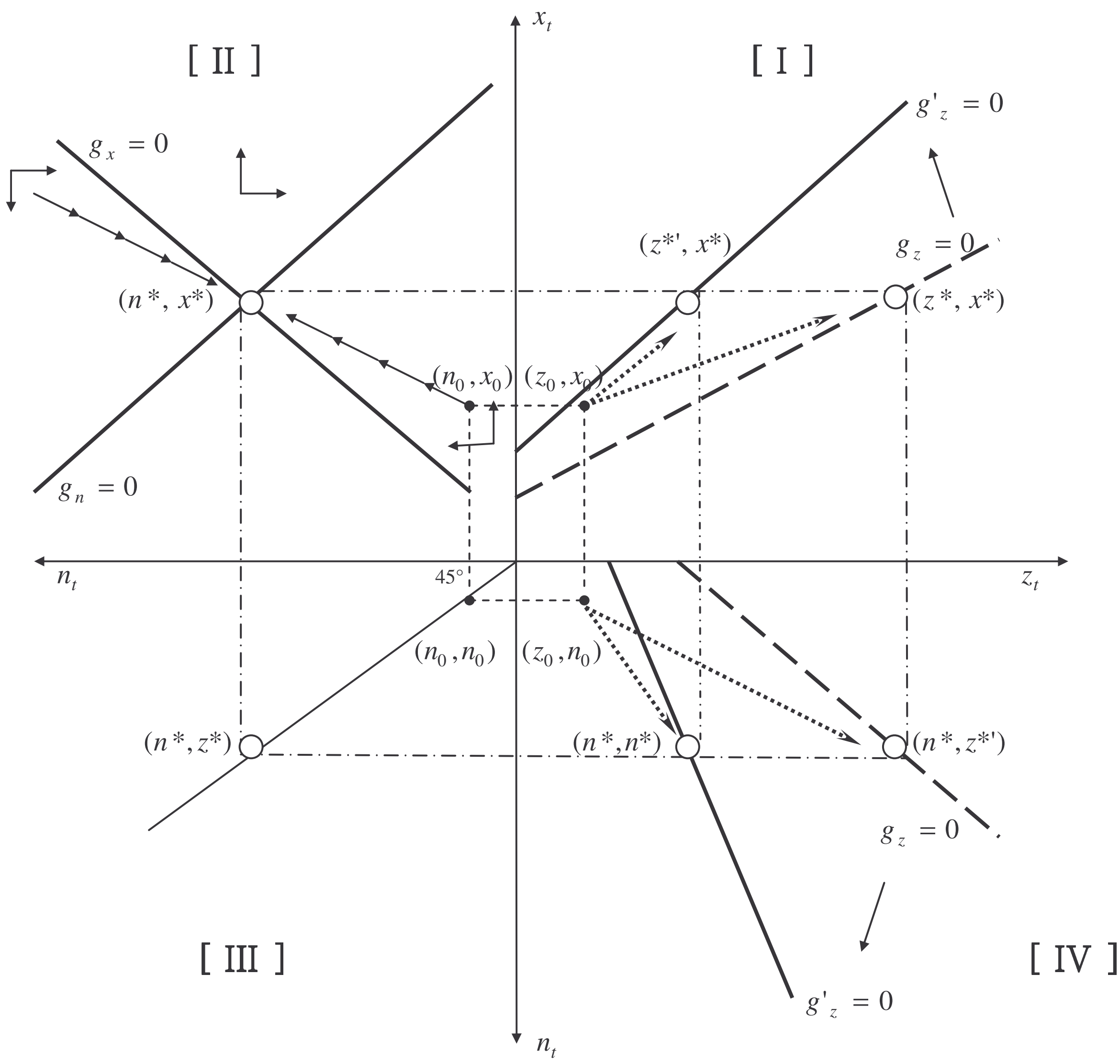
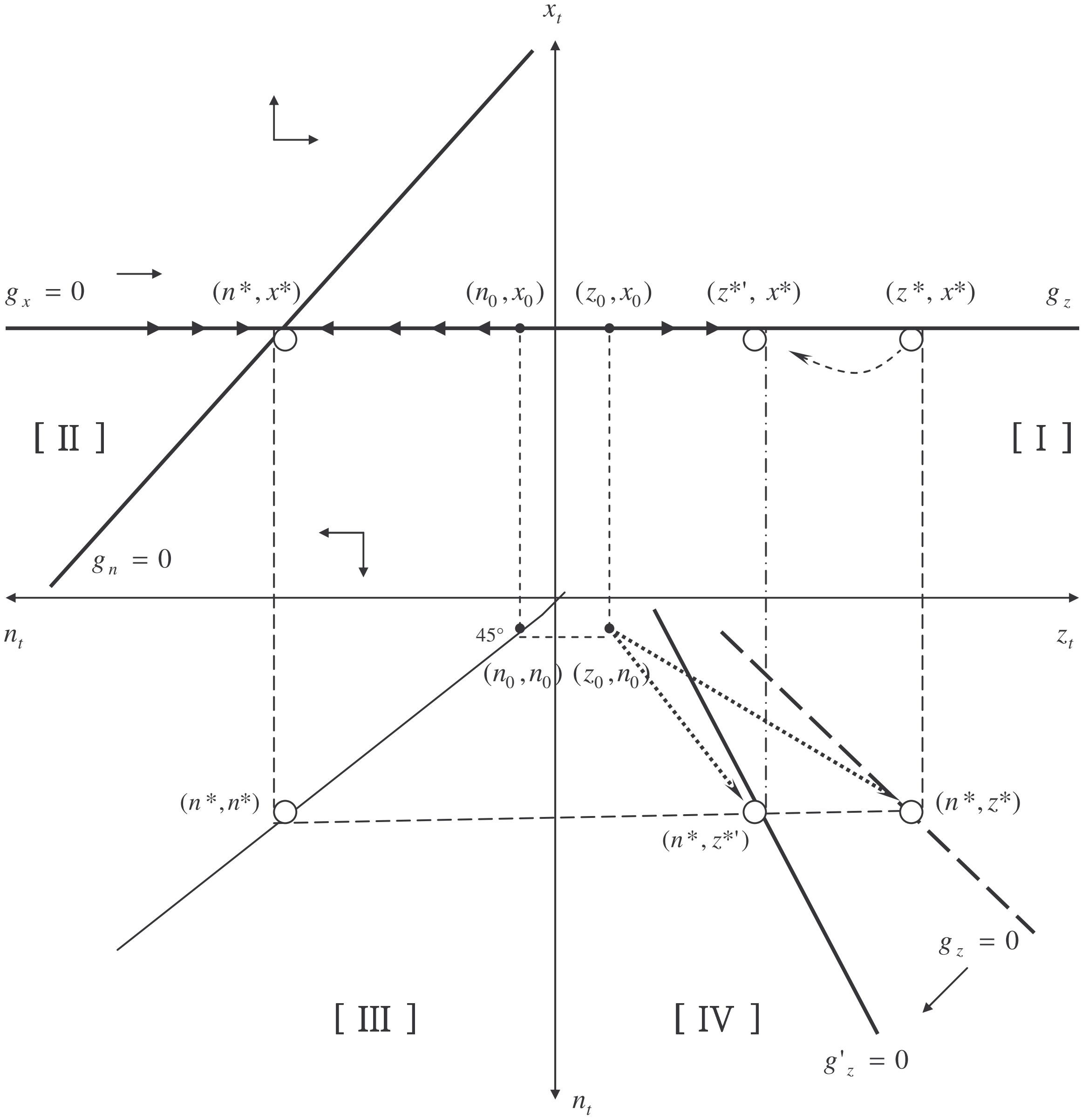


Figure 3-c: *The impact of improving* q_B*: case of* $\sigma = \delta$



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