A safe Minimum Standard, an Elasticity of Substitution, and the Cleanup of the Ganges in Varanasi

Xing, Shiqi and Batabyal, Amitrajeet

Rochester Institute of Technology, Rochester Institute of Technology

9 May 2019

A Safe Minimum Standard, an Elasticity of Substitution, and the Cleanup of the Ganges in Varanasi[1]

by

Shiqi Xing[2]

and

Amitrajeet A. Batabyal[3]

---

[1] For their helpful comments on a previous version of this paper, we thank the Editor-in-Chief Shandelle Henson, an Associate Editor, two anonymous reviewers, and participants in (i) the RSAI Japan Section Annual Conference, Sapporo, Japan, in October 2018, (ii) a Graduate School of International Service seminar, Sookmyung Women's University, Seoul, South Korea, in October 2018, (iii) an AIEFS session in the ASSA conference, Atlanta, Georgia, in January 2019, and (iv) the SRSA Annual Conference, Arlington, Virginia, in April 2019. This paper is based on an independent study project conducted by Xing under the supervision of Batabyal. Batabyal acknowledges financial support from the Gosnell endowment at RIT. The usual disclaimer applies.

[2] Department of Economics, Rochester Institute of Technology, 92 Lomb Memorial Drive, Rochester, NY 14623-5604, USA. E-mail: sx7599@rit.edu

[3] Department of Economics, Rochester Institute of Technology, 92 Lomb Memorial Drive, Rochester, NY 14623-5604, USA. E-mail: aabgsh@rit.edu

A Safe Minimum Standard, an Elasticity of Substitution, and the Cleanup of the Ganges in Varanasi

Abstract

Despite repeated calls for a thorough cleanup of water pollution in the Ganges river, there are only two papers in the social sciences by Batabyal and Beladi (2017, 2019) that have shed *theoretical* light on this cleanup problem and its connection to the sustainability of tourism in Varanasi. Hence, we extend the above mentioned analyses and focus on two specific questions. First, we introduce the notion of a safe minimum standard (SMS) into the study and show how to analyze a probabilistic model of the Ganges cleanup problem when the SMS is accounted for. Second, for a representative citizen of Varanasi, we study how the magnitude of the elasticity of substitution between a composite consumption good and water quality in the Ganges---modeled by the SMS---affects the tradeoff between consumption and water quality maintenance.

Keywords: Ganges river, Safe Minimum Standard, Tourism, Uncertainty, Water Pollution

JEL Codes: Q53, L83

Recommendations for Resource Managers

1. Successful cleanup policies entail high costs but probabilistically successful cleanup policies involve lower costs.
2. If citizens of Varanasi are moderate environmentalists, then a decline in the water quality of the Ganges will lead only to a temporary loss of utility.
3. If citizens of Varanasi are strict environmentalists then a decline in the water quality of the Ganges will lead to a permanent loss of utility.

1. Introduction

The longest river in India is the Ganges and this river occupies an essential place in the Hindu religion. Hindus generally consider the Ganges to be sacred and therefore millions of them standardly visit the holy city of Varanasi in the state of Uttar Pradesh in India to perform a purification ritual that involves, *inter alia*, bathing in the river. The city of Varanasi is significant not only for what Rinschede (1992) calls "religious tourism," but also because it is one of the oldest populated cities in the world. Alley (1992) and Chitravanshi (2014) point out that in modern times, in addition to being a salient center for both domestic and foreign tourism, Varanasi is also prominent for its art, culture, and music.

Unfortunately, the Ganges has now become a dumping ground for all kind of pollutants. For instance, in Varanasi one can find animal carcasses, partially cremated corpses, and the material offerings of Hindu devotees in the river. In this regard, Dhillon (2014) maintains that 32,000 bodies are cremated every year in Varanasi and that this process results in 300 tons of ash and 200 tons of half-burnt human flesh being dumped into the Ganges. Given this extremely unhealthy state of the river, questions are now regularly being asked about the sustainability of the tourism industry in Varanasi.

There have now been several calls for a thorough cleanup of the Ganges. Even so, Das and Tamminga (2012) rightly point out that these calls have led to little or no change in the extremely polluted status of the river. However, as noted by Batabyal and Beladi (2019), the Ganges now appears to have a champion in the current Indian Prime Minister Mr. Narendra Modi. After becoming Prime Minister, Mr. Modi promised to convert Varanasi into a vibrant city for tourists by initiating a major campaign to clean the Ganges.

This cleanup campaign has received a lot of publicity. From both environmental and touristic standpoints, it is clearly important that the campaign succeed. In this regard, Srinivas *et al.* (2017) have pointed out the damaging impacts that trace metals have on aquatic life in the Ganges. Srivastava *et al.* (2017) contend that that it is very important to maintain the quality of the water in the Ganges by controlling pollution. *Inter alia*, these researchers demonstrate that the presence of polycyclic aromatic hydrocarbons (PAHs) in the Ganges poses a significant health risk to humans. Finally, Srinivas *et al.* (2018) use what they call a geo-statistical overlay analysis to describe priority zones such as the Kanpur-Varanasi stretch in the Ganges where they believe that the implementation of sustainable management policies is particularly salient.

Even though the studies mentioned in the preceding paragraph have improved our understanding of the nature of water pollution in the Ganges from a physical and a chemical sciences perspective, our central claim is that there are virtually *no* social science studies of the Ganges cleanup problem that are both *dynamic* and *stochastic* in terms of the methodology employed to analyze the underlying cleanup problem.[4] In fact, to the best of our[5] knowledge, there are only two papers by Batabyal and Beladi (2017, 2019) that have engaged in probabilistic modeling and thereby shed light on the economic aspects of the Ganges cleanup problem and its connection to the sustainability of tourism in Varanasi. Given the paucity of intertemporal and probabilistic explorations of the Ganges cleanup problem, we extend the analyses in Batabyal and

---

[4] We emphasize that the principal goal of this paper is to construct and analyze a *mathematical model* of the Ganges pollution cleanup problem. In particular, our aim here is *not* to conduct either a case study based or empirical analysis of this problem. Therefore, issues related to data collection, sampling, the identification and survey of experts, and potential problems stemming from a limited number of observations are irrelevant. The reader should note that our mathematical model based analysis is entirely consistent with the aims and scope of *Natural Resource Modeling*. In this regard, the web page of the journal clearly says that *Natural Resource Modeling* "is an international journal devoted to mathematical modeling of natural resource systems."

[5] We use the first person---and hence first person pronouns---in our paper because this usage is very common in the literature on natural resources and the environment. In addition, *Natural Resource Modeling* has frequently published papers written in the first person in the past. Two recent examples are Batabyal and Beladi (2019) and Ackleh *et al.* (2019).

Beladi (2017, 2019) by concentrating on two specific research questions.

First, we introduce the notion of a safe minimum standard (SMS)[6] into the analysis and then show how to construct and analyze a probabilistic model of the Ganges cleanup problem when the SMS is explicitly accounted for. Note that the SMS can be thought of as a minimally acceptable threshold for the provision of important ecosystem services. As such, we use this SMS notion in our analysis because it provides us with a straightforward way of modeling the idea that when managing pollution in the Ganges, the ability of this salient ecological-economic system to provide vital ecosystem services cannot be allowed to fall below a minimally acceptable threshold.

Second, for a representative citizen of Varanasi, we study how the magnitude of the elasticity of substitution between a composite consumption good and water quality in the Ganges---modeled by the SMS---influences the tradeoff between consumption and the maintenance of water quality.

The remainder of this paper is organized as follows. Section 2.1 presents a theoretical framework that is adapted from Antelman and Savage (1965) and Batabyal and Nijkamp (2005) and that is used to analyze a probabilistic model of the Ganges cleanup problem in Varanasi. Section 2.2 derives analytic expressions for the long run expected cleanup cost incurred by a city authority (CA) who uses two different policies to undertake the cleanup task. Section 2.3 discusses a key criterion that determines which policy leads to lower long run expected cleanup costs. Section 3.1 presents a different theoretical framework that is adapted from Gueant *et al.* (2012) and that is used to examine the tradeoff between consumption and water quality faced by a

---

[6] More than sixty years ago, Ciriacy-Wantrup (1952) suggested that when uncertainty and irreversibility are issues in natural resource management, the management function ought to pay attention to the establishment of what he called a "safe minimum standard (SMS)." The idea here is to manage an ecological-economic system such as the Ganges river so that this system's ability to provide humans with a flow of ecosystem services does not fall below a particular standard, namely, the SMS.

representative citizen of Varanasi. Section 3.2 discusses the behavior of this representative citizen's utility under alternate assumptions about the magnitude of a key elasticity of substitution. Section 3.3 comments on the policy implications of the results obtained in section 3.2. Section 4 concludes and then discusses two ways in which the research delineated in this paper might be extended.

2. A Stochastic Model of the Ganges Cleanup Problem

2.1. The theoretical framework

Citizens of Varanasi and both religious and non-religious tourists visiting this city obtain a variety of goods and ecosystem services from the Ganges. These include, but are not limited to, fish for food, the performance of Hindu religious rituals, and the availability of boatable, bathable, and drinkable water. Now, consider a fixed zone along the Ganges in Varanasi with an inspection point at the end of this zone. A city authority (CA)[7] inspects the water quality of the Ganges at this particular point.

Because of the continued deposit of effluents into the Ganges from a variety of non-point sources, many of which are specific to Varanasi (see section 1), the water quality of the Ganges deteriorates *probabilistically* over time.[8] Put differently, the *state* of the water quality in the Ganges declines stochastically over time. Following Batabyal and Nijkamp (2005), we model this feature by supposing that the state of the quality of water in the Ganges changes in accordance with a Brownian motion process with drift $\delta > 0$.[9] This is shown in figure 1 by the function that

---

[7] In Varanasi, an example of such a CA would be the Municipal Commissioner who is in charge of the Varanasi Municipal Corporation.

[8] In the setting of this paper, the words "probabilistic" and "stochastic" are synonyms and therefore there is no difference between the notions of probabilistic deterioration and stochastic deterioration.

[9]

begins at the origin and then zig-zags downward and rightward.[10] When the state of the water

Figure 1 about here

quality in the Ganges crosses $\mathbb{S}$, this river is so polluted that it is unable to *safely* provide the citizens of Varanasi and tourists with the goods and services we alluded to in the preceding paragraph. From the perspective of the CA, state $\mathbb{S}$ is the *least* acceptable state of water quality in the Ganges. In the remainder of this second section, we shall think of this state $\mathbb{S}$ as one in which water quality is at the level of the so called SMS. This SMS state of water quality is shown by the horizontal red line in figure 1. In addition, when the state of the water quality in the Ganges is $\mathbb{S}$, the CA incurs a very high cost $C_{\mathbb{S}} > 0$ to remove all pollutants from the designated zone of the Ganges and thereby bring the water quality state of the river to state 0. State 0 is at the origin of the graph depicted in figure 1. It is also the best possible state of the Ganges in the sense that water quality in this state is as high as possible. From the CA's standpoint, a policy that involves essentially doing nothing and cleaning up the Ganges only when its water quality state reaches the SMS state is the *passive policy*. This policy is denoted by the letters PP in red in figure 1.

Because water quality in the Ganges crossing state $\mathbb{S}$ is the least acceptable outcome for the CA, it is certainly possible that this individual will want to clean up the designated zone of the Ganges *before* the water quality state of the river reaches the SMS state $\mathbb{S}$. To this end, we keep the cleanup problem interesting by supposing that if the water quality state of the Ganges is s---shown in blue and measured along the vertical axis in figure 1---and the CA attempts to rid the

---

See Ross (2014, pp. 607-644) for a textbook treatment of Brownian motion processes. Note that for our subsequent mathematical analysis to be valid, we need the drift $\delta > 0$ to be time-independent.

[1010] According to Harris (2000, p. 156), a flow diagram is a graphic representation of the physical route or flow of people, materials, paperwork, vehicles, or communication associated with a process, procedure, plan or investigation. In this paper, the probabilistic decline of water quality over time (the flow) and the task of cleaning up pollution in the Ganges (the procedure) are clearly shown in figure 1.

designated zone of the river of all pollutants then this attempt will succeed with probability $p(s) > 0$ and fail with complementary probability $1 - p(s)$. If the cleanup is successful, then the water quality state of the Ganges returns to state 0 (the best possible state) and if it is unsuccessful then we suppose that the water quality state of the Ganges reaches $\mathbb{S}$, the least acceptable state. The cost of attempting a cleanup is denoted by $C_s > 0$ and we have $C_{\mathbb{S}} > C_s > 0$. In contrast to the passive policy delineated in the preceding paragraph, we shall call a policy in which the CA attempts a cleanup in state s, an *active policy*. This active policy is denoted by the letters AP in blue in figure 1.

The reader should note the following aspect of our modeling strategy. The CA's passive policy involves a *higher* cost $C_{\mathbb{S}} > 0$ and hence this policy is assumed to be successful with probability one. In contrast, the active policy involves a *lower* cost $C_s > 0$ and therefore this policy is assumed to be successful only with some positive probability that is strictly less than one. With this description of the model out of the way, our next task is to use renewal theory[11] to derive closed-form expressions for the long run expected cleanup cost incurred by the city authority (CA) when he pursues first the active policy and then the passive policy.

2.2. The long run expected cost expressions

We begin by deriving the long run expected cost stemming from the use of the active policy or $LREC_{AP}$ in which the CA attempts a cleanup when the water quality state of the designated zone of the Ganges is s and $0 < s < \mathbb{S}$. When the CA attempts a cleanup of the Ganges in water quality state s and this cleanup is successful, the state (water quality) of the the Ganges returns to state 0 which is the best possible state with the highest possible water quality. This return of the

---

[11] See Tijms (2003, pp. 33-79) or Ross (2014, pp. 409-479) for lucid textbook accounts of renewal theory.

water quality state of the Ganges to state 0 constitutes what is known as a renewal and therefore we can use the well-known “renewal-reward theorem”[12] to compute the $LREC_{AP}$.

If we say that a cycle is completed every time a renewal occurs then the renewal-reward theorem tells us that with probability one, the long run expected reward is given by the expected return received in a cycle divided by the length of time it takes to complete this same cycle. The “return” referred to in the preceding sentence can either be negative such as a cost or positive such as profit. Therefore, the renewal-reward theorem applies to our problem where the return is a cost but it would also apply to problems in which the return is profit. Adapting the renewal-reward theorem to our problem, we deduce that the $LREC_{AP}$ we seek is given by

$$LREC_{AP} = \frac{E[cost\ \ per\ \ cycle]}{E[length\ \ of\ \ cycle]}, \qquad (1)$$

where $E[\cdot]$ denotes the expectation operator. To compute the numerator of the ratio on the right-hand-side (RHS) of equation (1), observe that this numerator is given by a weighted sum of the two costs C_s and $C_{\mathbb{S}}$ where the weights account for the fact that when the CA attempts a cleanup in state s, his attempt will fail with probability $1-p(s)$. Putting these two pieces of information together, we get

$$E[cost\ \ per\ \ cycle] = C_s + \{1-p(s)\}C_{\mathbb{S}}. \qquad (2)$$

The computation of the expected length of a cycle is more elaborate and in what follows, we proceed as in Batabyal and Nijkamp (2005). Let us represent the expected amount of time it takes the water quality state of the Ganges---that we are modeling with a Brownian motion

---

[12] See Tijms (2003, p. 41) or Ross (2014, p. 427) for textbook expositions of the renewal-reward theorem.

process---to reach state s with the function $h(s)$. Now, from definition 10.1 in Ross (2014, p. 608) we know that a Brownian motion process has independent and stationary increments. Therefore, for any two states s and t, we can write

$$h(s+t) = h(s) + h(t). \tag{3}$$

Equation (3) and the two aforementioned properties of a Brownian motion process together tell us that the function $h(s)$ is linear with the form $h(s) = \alpha s$ where α is a constant. Following the procedure used in Batabyal and Nijkamp (2005, p. 46), we infer that $h(s) = s/\delta$. This last result tells us that

$$E[length\ \ of\ \ cycle] = \frac{s}{\delta}. \tag{4}$$

Using equations (2) and (4) together, we obtain an expression for the $LREC_{AP}$ that we seek. Specifically, we get

$$LREC_{AP} = \frac{\delta[C_s + \{1 - p(s)\}C_{\mathbb{S}}]}{s}. \tag{5}$$

Equation (5) tells us that the long run expected cost of cleaning up the Ganges with the active policy is given by the ratio of the weighted sum of the two cost terms C_s and $C_{\mathbb{S}}$ to the state $s, 0 < s < \mathbb{S}$, in which the cleanup is initiated.

Our next task is to ascertain the long run expected cost of cleaning up the Ganges with the passive policy or $LREC_{PP}$. Some thought ought to convince the reader that for this policy, the expected cost per cycle or $E[cost \;\; per \;\; cycle] = C_{\mathbb{S}}$. Following the logic of the derivation that led to equation (4), we infer that $E[length \;\; of \;\; cycle] = \mathbb{S}/\delta$. Putting these last two pieces of information together, we get

$$LREC_{PP} = \frac{\delta C_{\mathbb{S}}}{\mathbb{S}}. \tag{6}$$

In words, equation (6) tells us that the long run expected cost of cleaning up the Ganges using the passive policy is given by the product of the drift parameter δ of the Brownian motion process and the cost $C_{\mathbb{S}}$ to the SMS state $\mathbb{S}$. We now discuss the criterion that plays a major role in determining whether the active policy or the passive policy leads to lower long run expected cleanup costs for the CA.

2.3. The probability function

Inspection of equations (5) and (6) tells us that if we know that $LREC_{AP} < LREC_{PP}$ then it is rather straightforward to use calculus to determine the optimal water quality state in which the CA ought to begin the cleanup process to minimize the $LREC_{AP}$. On the other hand, if we know that $LREC_{AP} > LREC_{PP}$ then it is clear that our CA should commence no cleanup activities until the water quality state in the Ganges reaches the SMS state $\mathbb{S}$.

The problem for the CA lies in determining whether the active policy or the passive policy results in lower long run cleanup costs. Once again, inspection of equations (5) and (6) tells us that the probability function $p(s)$ is the *key* criterion that determines whether the CA ought to pursue the active or the passive policy in cleaning up the Ganges. To see this clearly, consider the following example in which the probability function is given by $p(s) = 1 - ns/\mathbb{S}$ where $n \geq 1$.

In this case, it is clear that $1 - p(s) = ns/\mathbb{S}$. Substituting this last expression in equation (5) and then simplifying the resulting expression, we obtain

$$LREC_{AP} = \frac{C_s}{s} + \frac{nC_{\mathbb{S}}}{\mathbb{S}} > \frac{C_{\mathbb{S}}}{\mathbb{S}} = LREC_{PP}. \tag{7}$$

The strict inequality in (7) tells us that when the probability function $p(s) = 1 - ns/\mathbb{S}$, it is *never* optimal for the CA to clean up the Ganges using the active policy and that he is better off pursuing the passive policy which involves doing nothing until the water quality state in the Ganges declines to the SMS state $\mathbb{S}$. Having said this, note that this "be passive" result may well be replaced by a "be active" result for alternate functional forms for the probability function $p(s)$. We now proceed to examine how the magnitude of the elasticity of substitution between a composite consumption good and water quality in the Ganges---modeled by the SMS---influences the tradeoff between consumption and the maintenance of water quality faced by a representative citizen of Varanasi.

3. The Tradeoff between Consumption and Water Quality

3.1. The theoretical framework

When concluding their analysis of the Ganges cleanup problem, Batabyal and Beladi (2019) point out that it would be interesting to generalize their analysis by studying social welfare in Varansi in the context of the removal of water pollutants from the Ganges. We undertake this task by studying how a representative citizen of Varanasi values consumption and water quality in the Ganges, in the long run. This citizen understands that his ancient city is famous, at least in part, because the Ganges runs through it and because its status as a holy site is responsible for attracting large numbers of religious and non-religious tourists to it.

Now, to fix ideas, let the representative Varanasi citizen's utility at any time t be defined over a composite consumption good C_t and environmental quality. The specific kind of environmental quality we have in mind here is the water quality of the Ganges. Water quality in general at time t is denoted by W_t and the specific level of W_t that we shall be interested in is the *fixed* SMS level of water quality or $\mathbb{S}$. Therefore, in what follows, we shall set $W_t = \mathbb{S}$. Next, let σ denote the constant elasticity of substitution between the composite consumption good and water quality in the Ganges. This elasticity measures how easy it is to compensate for decreased water quality in the Ganges by increasing the consumption of the composite good. Finally, we suppose that consumption in the Varanasi economy grows at a constant rate.

The representative Varanasi citizen's utility function is given by

$$U(C_t, W_t) = \{C_t^{\frac{\sigma-1}{\sigma}} + \mathbb{S}^{\frac{\sigma-1}{\sigma}}\}^{\frac{\sigma}{\sigma-1}}. \tag{8}$$

The reader should note that when the elasticity of substitution $\sigma \to \infty$, there is *perfect* substitutability between the composite consumption good C_t and water quality $\mathbb{S}$. In contrast, when $\sigma \to 0$, there is *no* substitutability at all between C_t and $\mathbb{S}$. With this description of the theoretical framework out of the way, we now proceed to analyze the behavior of the utility function in equation (8) under alternate assumptions about the magnitude of the elasticity of substitution σ.

3.2. The salience of the magnitude of the elasticity of substitution

From equation (8), it is clear that the representative citizen of Varanasi cares about water quality in the Ganges. But how much does this individual care about water quality? To answer this question meaningfully, we need to have a way of quantifying the representative citizen's caring. This quantification can be undertaken in a straightforward manner with the elasticity of substitution or σ parameter. Therefore, in what follows, we use σ to delineate two broad cases which correspond to two very different levels of caring about water quality in the Ganges. That said, the reader should note that we are using two broad values of σ $(\sigma > 1 \; and \; \sigma < 1)$ because these values help us model and comprehend two different levels of caring about water quality in the Ganges. Finally, because σ is an exogenously given parameter, we are *not* interested in determining optimal values for this parameter.

The first case we study is the scenario in which the elasticity of substitution $\sigma > 1$. This is the case where the representative citizen of Varanasi cares about the water quality of the Ganges but *not* excessively so. In other words, this is the case where the representative citizen of Varanasi is a "modest environmentalist." In this setting, we want to study the limiting behavior of the representative citizen's utility as time $t \to \infty$. To this end, let us express the representative Varanasi citizen's utility in terms of consumption C_t and water quality divided by consumption or $\mathbb{S}/C_t$. We get

$$U(C_t, W_t) = C_t \left[1 + \left\{\frac{\mathbb{S}}{C_t}\right\}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}. \tag{9}$$

By assumption, we know that consumption C_t is growing at a constant rate and that $\sigma > 1$. Putting these two pieces of information together, we reason that in the long run, i.e., in the limit as time $t \to \infty$, the ratio $\{\mathbb{S}/C_t\}^{\frac{\sigma-1}{\sigma}}$ approaches zero. This last finding tells us that when $\sigma > 1$, the utility of the representative citizen of Varanasi (the modest environmentalist) will grow at the *same rate* as consumption of the composite good. Note that for this line of reasoning to be valid, we need the SMS state $\mathbb{S}$ to be constant.

Next, let us analyze the case where the elasticity of substitution $\sigma < 1$. This corresponds to the case where the representative citizen of Varanasi cares *greatly* about water quality in the Ganges and hence this citizen can reasonably be thought of as a "serious environmentalist." In contrast to the $\sigma > 1$ case, we now write the utility function in terms of the SMS water quality $\mathbb{S}$ and the ratio $\mathbb{S}/C_t$. We get

$$U(C_t,\ \mathbb{S}) = \mathbb{S}\left[1 + \left\{\frac{\mathbb{S}}{C_t}\right\}^{\frac{1-\sigma}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}. \tag{10}$$

Because $\sigma < 1$ by assumption, it is clear that $(1-\sigma)/\sigma > 0$. This last result tells us that in the limit as time $t \to \infty$, the ratio $\{\mathbb{S}/C_t\}^{\frac{1-\sigma}{\sigma}}$ approaches zero. This last finding tells us that when $\sigma < 1$, the utility of the representative citizen of Varanasi (the serious environmentalist) will approach the constant value of $\mathbb{S}$. In other words, there now exists a *limit* on the maximum utility that the representative Varanasi citizen can attain. Let us now discuss the policy implications of the results we have obtained thus far in this section.

3.3. Policy implications

In the first case studied in section 3.2 where the elasticity of substitution $\sigma > 1$, the representative Varanasi citizen's level of utility grows at the same rate as the consumption of the composite good even though the water quality of the Ganges is fixed at the SMS level $\mathbb{S}$. This finding tells us that a decline in the quality of the water in the Ganges *can* be compensated by an increase in the consumption of the composite good. In other words, this finding tells us that because the representative Varanasi citizen's long run utility depends essentially on consumption and consumption is assumed to be growing at an exogenously given constant rate, even when water quality in the Ganges declines, the associated loss in utility stemming from this decline is *short-lived* or *temporary*.[13] This is because eventually the representative Varanasi citizen's consumption will have grown sufficiently to compensate for the reduction in water quality.

In contrast, in the second case analyzed in section 3.2 where $\sigma < 1$, there is a clear *limit* on the utility that can be attained by the representative citizen of Varanasi. Therefore, in this case, a lowering of water quality in the Ganges will reduce this limit and hence lead to a permanent *reduction* in the utility of the representative citizen of Varanasi. Therefore, from the standpoint of the CA, it is a lot *more* important to avoid a decline in the water quality of the Ganges when the representative Varanasi citizen's elasticity of substitution σ between consumption and water quality is less than one. This completes our discussion of a SMS, an elasticity of substitution, and the cleanup of the Ganges in Varanasi.

4. Conclusions

In this paper, we expanded upon the analyses in Batabyal and Beladi (2017, 2019) about the cleanup of the Ganges and focused on two particular research questions. First, we introduced the notion of a SMS into the analysis and then showed how to construct and analyze a probabilistic model of the Ganges cleanup problem when the SMS was explicitly accounted for. Second, we

---

[13] Note that this temporary decline in utility is not a definitional issue.

studied how the magnitude of the elasticity of substitution between a composite consumption good and water quality in the Ganges, modeled by the SMS, influenced the tradeoff between consumption and water quality for a representative citizen of Varanasi.

Here are two suggestions for extending the research delineated in this paper. First, it would be interesting to see the extent to which one can obtain analytic results when working with a stochastic model in which changes in the water quality state of the Ganges are modeled not with a Brownian motion process but instead with a more general semi-Markov process. Second, it would be useful to determine the extent to which the work of Beladi *et al.* (2013) can be used to study the impact that price and/or quantity controls used by the CA---with the aim of improving water quality in the Ganges---have on the representative Varanasi citizen’s long run utility. Studies of the cleanup of the Ganges that incorporate these aspects of the problem into the analysis will provide additional insights into the ways in which river water pollution affects the welfare of both tourists visiting and citizens living in Varanasi.

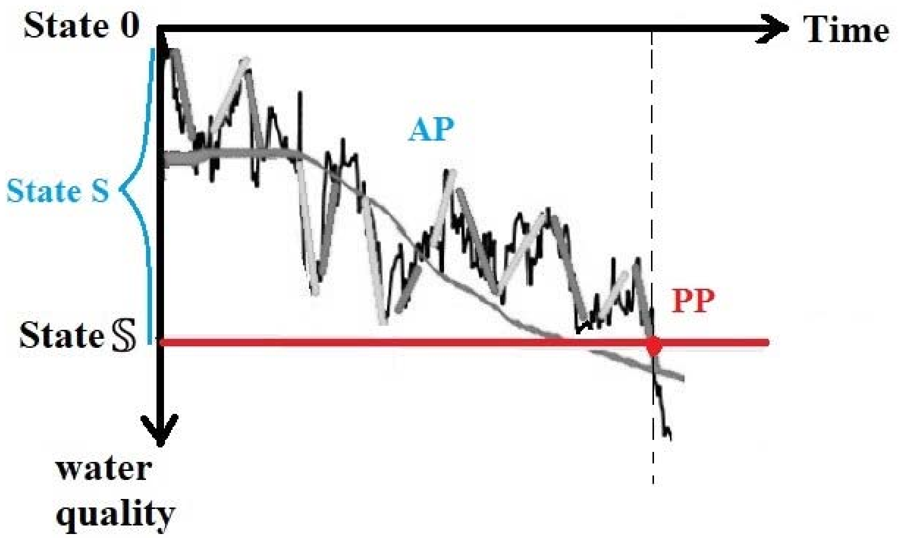

Figure 1: Main features of the theoretical model

References

1. Ackleh, A.S., Caswell, H., Chiquet, R.A., and Tang, T. 2019. Sensitivity analysis of the recovery time for a population under the impact of an environmental disturbance, *Natural Resource Modeling*, 32, 1-22.
2. Alley, K.D. 1992. On the banks of the Ganga, *Annals of Tourism Research*, 19, 125-127.
3. Antelman, G., and Savage, I.R. 1965. Surveillance problems: Wiener processes, *Naval Research Logistics Quarterly*, 12, 35-55.
4. Batabyal, A.A., and Beladi, H. 2017. Cleaning the Ganges in Varanasi to attract tourists, *Atlantic Economic Journal*, 45, 511-513.
5. Batabyal, A.A., and Beladi, H. 2019. Probabilistic Approaches to Cleaning the Ganges in Varanasi to Attract Tourists, *Natural Resource Modeling*, 32, 1-11.
6. Batabyal, A.A., and Nijkamp, P. 2005. Alternate strategies for managing resistance to antibiotics and pesticides, *Environmental Economics and Policy Studies*, 7, 39-51.
7. Beladi, H., Liu, L., and Oladi, R. 2013. On pollution permits and abatement, *Economics Letters*, 119, 302-305.
8. Chitravanshi, R. 2014. Tourist guides to hold annual meeting in Varanasi, devise clean Ganga strategy, *Business Standard*, June 28, http://www.business-standard.com/article/current-affairs/tourist-guides-to-hold-annual-meeting-in-varanasi-devise-clean-ganga-strategy-114062500741_1.html, Accessed on 7 December 2018.
9. Ciriacy-Wantrup, S.V. 1952. *Resource Conservation*. University of California Press, Berkeley, CA.

10. Das, P., and Tamminga, K.R. 2012. The Ganges and the GAP: An assessment of efforts to clean a sacred river, *Sustainability*, 4, 1647-1668.
11. Dhillon, A. 2014. Ganga management, Post Magazine, *South China Morning Post*, http://www.scmp.com/magazines/post-magazine/article/1589301/ganga-management, Accessed on 7 December 2018.
12. Gueant, O., Guesnerie, R., and Lasry, J. 2012. Ecological intuition versus economic "reason," *Journal of Public Economic Theory*, 14, 245-272.
13. Harris, R.L. 2000. *Information Graphics*. Oxford University Press, Oxford, UK.
14. Rinschede, G. 1992. Forms of religious tourism, *Annals of Tourism Research*, 19, 51-67.
15. Ross, S.M. 2014. *Introduction to Probability Models*, 11th edition. Academic Press, Amsterdam, The Netherlands.
16. Srinivas, R., Singh, A.P., and Sharma, R. 2017. A scenario based impact assessment of trace metals on ecosystem of river Ganges using multivariate analysis coupled with fuzzy decision-making approach, *Water Resources Management*, 31, 4165-4185.
17. Srinivas, R., Singh, A.P., Dhadse, K., Garg, C., and Deshmukh, A. 2018. Sustainable management of a river basin by integrating an improved fuzzy based hybridized SWOT model and geo-statistical weighted thematic overlay analysis, *Journal of Hydrology*, 563, 92-105.
18. Srivastava, P., Sreekrishnan, T.R., and Nema, A.K. 2017. Human health risk assessment and PAHs in a stretch of river Ganges near Kanpur, *Environmental Monitoring and Assessment*, 189, Article 445.
19. Tijms, H.C. 2003. *A First Course in Stochastic Models*. Wiley, Chichester, UK.