The optimal choice of a relative performance indicator in product market competition

Hamamura, Jumpei and Hayakawa, Sho

Momoyama Gakuin University, Japan, University of Marketing and Distribution Sciences, Japan

16 May 2019
The optimal choice of a relative performance indicator in product market competition

Jumpei Hamamura
Momoyama Gakuin University
Faculty of Business Administration

Sho Hayakawa
University of Marketing and Distribution Sciences
Faculty of Commerce

This version: May, 2019

* Corresponding author: 1-1, Manabino, Izumi-city, Osaka 594-1198, Japan. Phone: +81 725 54 3131. Fax: +81 725 54 3202. Email id: jmhama@andrew.ac.jp

* Acknowledgements: Earlier versions of this research were presented in seminars at Hokuriku University, Kobe University and Keio University. Authors would like to thank participants in the conferences and seminars for their helpful suggestions. Needless to say, we are solely responsible for any remaining errors. This work is supported by the Ministry of Education, Culture, Sports, Science and Technology of Japan (MEXT) under Financial support of Grant-in-Aid for Young Scientists (18K12909).
The optimal choice of a relative performance indicator in product market competition

Abstract
In this research, we analytically explore what performance indicator is optimal in a market competition when the firm’s owner compensates the CEO based on the relative performance evaluation. The relative performance evaluation considered in previous studies compares the firm’s profit with the competitor’s profit. However, when the firm evaluates the CEO’s performance, another performance indicator is often adopted instead of profit. As a result, we show that given specific economic conditions, the owners adopt sales as a relative performance indicator to evaluate the CEO’s performance. This result has some important implications for the research on relative performance evaluation. First, it will affect future studies showing that there are different possible choices of relative performance indicators in management accounting assuming product market competition. Second, our study has an important implication for empirical research on relative performance evaluation in management accounting, in which a relative performance indicator is adopted as an independent variable.

Keywords: Non-cooperative game theory; CEO compensation; Relative performance evaluation; Performance indicator; Quantity competition
1. Introduction

In this paper, we explore what performance indicator is optimal in a market competition when the firm’s owner pays compensation to the CEO based on a relative performance evaluation. The relative performance evaluation is based on the comparison with the competitor’s performance, which is investigated in management accounting research. When the firm’s owner evaluates the CEO’s performance, the relative performance evaluation is adopted in practice. For example, Gong, Li, and Shin (2011) provide an important empirical contribution to the literature on relative performance evaluation by examining the relationship between the practical relative performance evaluation system and the CEO’s compensation. Specifically, Gong et al. (2011) examine S&P 1,500 firms and find that about 25% of them explicitly adopt the relative performance evaluation as a CEO’s reward system.

Considering the incentives in CEO’s contracts, it is important to take into account market competition, because the CEO’s performance is mainly assessed using competitors’ profit in relative performance evaluation. Hence, assuming product market competition, there are many analytical and empirical studies in management accounting (e.g., Albuquerque, 2009; Antle and Smith, 1986; Demeré, Krishnan, Sedatole, and Woods, 2016; Gibbons and Murphy, 1990; Hamamura, 2019; Hecht, Newman, and Tafkov, 2019; Holmström, 1982; Kramer et al., 2016; Matsumura and Shin, 2006; Murphy, 1999; Vrettos, 2013).

Assuming quantity or price competition in a product market, Aggarwal and Samwich (1999) and Fumas (1992) offer valuable contributions to the research on relative performance evaluation, which show that the firms assign a weight and positive or negative sign to competitors’ profit
considering the strategic effect of market competition¹. These studies, Aggarwal and Samwich (1999) or Fumas (1992), show that the firms choose to put a negative weight on a competitor’s profit to induce the CEO to commit to an aggressive strategy to obtain market share in a product market when they engage in strategic substitute competition in a product market (e.g., quantity competition)². When the firms engage in strategic complement competition, they choose instead to put a positive weight on a competitor’s profit to induce the CEO to commit to a collusive strategy based on soft competition and high price in a product market.

The empirical evidence collected in previous studies is consistent with analytical research (e.g., Antle and Smith, 1986; Albuquerque, 2009; Joh, 1999; Vrettos, 2013). The empirical literature on relative performance evaluation has mainly focused on whether the firms adopt the relative performance evaluation to establish the compensation of CEOs. The analysis of statistical data to examine relative performance evaluation has produced mixed results. For example, Antle and Smith (1986) investigate the relative performance evaluation of the CEO considering other firms in the same industry as peers. They demonstrate the existence of the relative performance evaluation of the CEO, showing that the ROA Return on Assets (ROA) of peer firms hurts the CEO’s compensation. Aggarwal and Samwich (1999) analyzes an optimal level and sign of weight put on competitors’ profit on CEO compensation contract with product market competitions which

¹ Prior studies, Aggarwal and Samwich (1999) or Fumas (1992), considers the weight put on competitors’ profit. This weight represents the impact of performance indicator on CEO compensation contract. Hence, when the weight is negative, CEO has to exceeds competitors’ profit. In addition, when this weight approaches zero, CEOs decide strategies by considering their own firm’s profit.

² Strategic substitute and complementally is defined as follows: When firms face strategic substitute competition in a product market, the firm decrease the strategy by increasing competitors’ strategy. In addition, firms face strategic complementally competition in a product market, the firm can increase the strategy by increasing competitors’ strategy.
are quantity and price competition. As a result, Aggarwal and Samwick (1999) show that the relative performance evaluation is adopted in firms that engage in intense competition in the industry. Also, Albuquerque (2009) investigates the relative performance evaluation from the perspective of a contract theory, assuming that the ROA of the same industry does not only have an impact on CEO’s compensation, but a similar size firm also impacts CEO’s compensation as a peer firm. Albuquerque (2009) considers the optimal peer group using empirical data. Showing by contract theory, the effectiveness of relative performance evaluation is eliminating the common peer group’s noise of performance and Albuquerque (2009) claims that similar size firm is optimal as a peer group from this analytically result. From this argue, Albuquerque (2009) investigates that ROA of same industry and similar size firm have an impact on CEO compensation instead of same industry firm’s ROA. As a result, Albuquerque (2009) demonstrates that ROA does not have an impact on the CEO’s compensation. While these studies use listed company and multi-industry data to consider relative performance evaluation of CEO, Vrettos (2013) explores the relative performance evaluation of CEOs using airline firms’ data. Vrettos (2013) considers the relationship between strategic interaction and relative performance evaluation, which is considered in industrial organizations, and the risk reduction effect is addressed by Holmström (1982). Vrettos (2013) observes both strategic interaction and risk reduction effect for the compensation of CEO in the airlines industry.

Prior empirical research used profit, for example, ROA, ROE, or EBIT, as dependent variables to investigate the relative performance evaluation of CEOs (e.g., Antle and Smith, 1986; Darrough, Shi, and Wang, 2017; Janakiraman, Lambert, and Larcker, 1992; Vrettos, 2013). Hence,
the relative performance evaluation which is considered by prior research is assumed to compare the firm’s profit with the competitor’s profit.

However, when the firm evaluates the CEO’s performance, other performance indicators are often adopted with the exception of profit. For example, a new firm needs to obtain market share to increase the consumers’ awareness. Hence, there are cases in which the firm’s CEO is evaluated using market share or sales in practice. In addition, it is found that indicators leading to profit, for example, customer’s satisfaction is adopted as a performance indicator in management accounting research (Ittner and Larcker, 1998). Therefore, other indicators can be used in the relative performance evaluation of CEOs rather than profit, which includes ROA.

This study focuses on sales as a performance indicator that is considered in management accounting research and practice because sales are often adopted as a performance indicator in practice. For example, Johnson & Johnson adopts sales growth as a performance indicator of the CEO’s compensation from a proxy statement\(^3\). In addition, it is easy to compare the firm’s sales with competitors using the available financial reports. Moreover, because it is essential to choose the performance indicator with respect to the corresponding strategy in management accounting, sales are chosen in the case in which the firm aims at obtaining market share by ignoring the marginal cost in a product market.

In this research, we analytically consider the choice of profit or sales as a relative performance indicator in quantity competition, applying the model of Aggarwal and Samwich

---

\(^3\) We obtain this information from Johnson & Johnson’s Proxy Statement for 2019, p.43: [http://www.investor.jnj.com/annual-meeting-materials/2019-proxy-statement](http://www.investor.jnj.com/annual-meeting-materials/2019-proxy-statement)
(1999) and Fumas (1992). As a result, we show that, in specific economic conditions, owners adopt sales as a relative performance indicator to evaluate the CEO’s performance. When weight put on competitor’s profit approaches zero and the degree of competitive intensity is enough large, owners have an incentive to threaten the competitor by other ways because the CEO does not have an incentive to emphasize the competitor’s performance. This means that weight put on competitor’s profit has the role of a commitment device to engage in softer competition in a product market. In this case, when marginal cost is small enough, the positive effect of threatening the competitor by informing it of excess supply exceeds the negative effect of excess supply for ignoring marginal cost by the CEO. In addition, if the owner does not choose this strategy, the competitor obtains market share because of the competitor’s threat of excess supply.

This result makes the following contributions to relative performance evaluation studies. First, given that this study concerns the choice of a relative performance indicator in management accounting assuming product market competition, its outcome will be useful for future research on this topic. Most studies that considered product market competition assumed that the owner evaluates the CEO using profit as a relative performance indicator (Aggarwal and Samwich, 1999; Fumas, 1992). However, our result will suggest that other relative performance indicators may be adopted in the CEO evaluation. Second, our study has an important implication for empirical research on relative performance evaluation in management accounting. In the empirical research on relative performance evaluation, previous studies used profit as a relative performance indicator (Joh, 1999; Vrettos, 2013). However, the outcome of this study will show that sales can also be adopted as a relative performance indicator in a specific economic environment.
2. Model

This section proposes an analytical model that describes the relative performance evaluation in firms, based on Aggarwal and Samwich (1999) or Fumas (1992). Let us assume that there are two firms, firm 1 and firm 2, in an industry that engages in differentiated quantity competition in a market. Each firm produces goods by marginal cost $c$ and sells them in a final goods market. Both firms have an owner and CEO who is delegated decision rights by owner.

Firm $i$’s owner maximizes following own firm’s profit

$$\pi_i = (p_i - c)q_i, \quad i = 1, 2,$$  \hspace{1cm} (1)

where $p_i$ is the market price and $q_i$ indicates the sales quantity of firm $i$ in a final goods market. Firm $i$’s owner chooses performance indicator to maximize Eq. (1).

In this research, we analyze the model that assumes the relative performance evaluation of CEOs. Fumas (1992) and Aggarwal and Samwich (1999) provide valuable insights on the relative performance evaluation when the firm engages in competition in a product market, and we apply these models in the current research. Also, we extend these models by considering the choice of an alternative relative performance indicator by the owner, namely, total sales. The owner can choose relative performance indicator by profit (denoted as Eq. (1)) or sales (denote as $p_i q_i$) to maximize Eq. (1).

---

4 Prior studies consider $n \geq 2$ firms which engage product market competition, while our model assumes 2 firms in a product market. In addition, Prior studies assume risk in a gross profit, while our model does not have a risk.
In addition, we assume that the owner compensates the CEO based on the performance. When the firm adopts the profit as a relative performance indicator, the CEO’s performance is defined by the following objective function:

\[ O_i^P = \pi_i + \alpha_i \pi_j, \]  

(2)

where superscript \( P \) denotes that the firm adopts profit as a relative performance indicator and \( \alpha_i < 0 \) is the constant of weight placed on the competitor’s profit in this study. Moreover, when the firm adopts firm’s sales as a relative performance indicator, the CEO’s performance is defined by the following objective function:

\[ O_i^S = p_i q_i + \alpha_i p_j q_j, \]  

(3)

where superscript \( S \) denotes that the firm adopts firm’s sales as a relative performance indicator and \( -1 \leq \alpha_i < 0 \) is the constant of weight placed on competitor’s sales in this research. In this study, we assume \( \alpha_1 = \alpha_2 = \alpha \) for simplicity, while we relax this assumption in additional analysis. This assumption means the level of weight put on competitor’s profit is same in performance evaluation on CEO compensation.

This study considers the choice of relative performance indicator when firms engage in quantity competition in a product market because Aggarwal and Samwich (1999) show that \( \alpha < 0 \) in quantity competition. When firms engage in quantity competition in a product market, CEOs decide sales quantity to maximize their performance considering the objective function. In this research, we assume the following demand function of firm \( i \)’s product:

\[ p_i = a - q_i - \theta q_j, \]  

(4)
where \( a \) is a positive constant greater than \( c \). In addition, \( \theta \) represents substitutability of products supplied by the two firms \((0 < \theta < 1) (1 - \theta \) is the degree of product differentiation). As \( \theta \) approaches 0, the market becomes a monopoly by a firm \( i \). Hereafter, \((i,j)\) represents \((1,2)\) or \((2,1)\), when two variables simultaneously appear in one equation. Table 1 represents notations in this study.

<table>
<thead>
<tr>
<th>Notations</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi )</td>
<td>Profit for a firm</td>
</tr>
<tr>
<td>( O )</td>
<td>RPE’s objective function of CEO</td>
</tr>
<tr>
<td>( i )</td>
<td>Subscript that indicates the firm</td>
</tr>
<tr>
<td>( j )</td>
<td>Subscript that indicates that the firm is different from firm ( i )</td>
</tr>
<tr>
<td>( P )</td>
<td>Superscript that indicates that the firm adopts profit as a relative performance indicator</td>
</tr>
<tr>
<td>( S )</td>
<td>Superscript that indicates that the firm adopts sales as a relative performance indicator</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Negative weight placed on competitor’s profit ((-1 \leq \alpha &lt; 0))</td>
</tr>
<tr>
<td>( p )</td>
<td>Market price</td>
</tr>
<tr>
<td>( q )</td>
<td>Quantity</td>
</tr>
<tr>
<td>( c )</td>
<td>Direct manufacturing cost per unit</td>
</tr>
<tr>
<td>( a )</td>
<td>Positive constant greater than ( c )</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Substitutability of products supplied by the two firms ((0 &lt; \theta &lt; 1) (1 - \theta ) is the degree of product differentiation)</td>
</tr>
</tbody>
</table>

The timeline of events proceeds as follows. At time 1, each owner chooses between profit and sales as a relative performance indicator to evaluate the CEO. At time 2, the CEOs decide the sales quantity in the product market. At time 3, profits and sales are realized. Finally, the owner compensates the CEO based on the CEO’s objective function. This is described in Figure 1. In addition, a competitor can observe all endogenous variables.
3. Analysis

3.1 Main model analysis

In this section, the subgame perfect Nash equilibrium is derived using backward induction. We consider the optimal strategies of CEOs under the given relative performance indicator. First, we consider that the owner evaluates each CEO using profit. In this case, CEOs are evaluated using Eq. (2). Because of the use of backward induction, we consider time 2 first and obtain the following strategy for firm $i$:

$$q^{(p,p)}_i = \frac{a - c}{2 + \theta + \alpha \theta}, \quad (5)$$

where superscript $(i,j) = (P,P), (P,S), (S,P), (S,S)$ represent combinations of performance indicators chosen by the owner. From Eq. (5), we obtain the following firm $i$’s profit:

$$\pi^{(p,p)}_i = \frac{(1 + \alpha \theta)(a - c)^2}{(2 + \theta + \alpha \theta)^2}. \quad (6)$$

Next, we consider that the owner evaluates each CEO using sales. In this case, CEOs are evaluated using Eq. (3) and we obtain the following strategy and profit for firm $i$. 

---

**Figure 1.** Timeline of the events

<table>
<thead>
<tr>
<th>Time 1</th>
<th>Time 2</th>
<th>Time 3</th>
<th>Time 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Each owner chooses a relative performance indicator</td>
<td>CEOs decide sales quantity</td>
<td>Profits and sales are realized</td>
<td>The owner rewards the CEO based on the CEO’s objective function</td>
</tr>
</tbody>
</table>
\[ q_i^{(S,S)} = \frac{a}{2 + \theta + \alpha \theta'} \]  
(7) 

\[ \pi_i^{(S,S)} = \frac{a((1 + \alpha \theta) a - (2 + \theta + \alpha \theta) c)}{(2 + \theta + \alpha \theta)^2}. \]  
(8) 

From this outcome, we obtain the following lemma.

**Lemma 1.** When the owner evaluates each CEO using sales, the profit of firm \( i \) is as follows:

\[ \pi_i^{(S,S)} = \frac{a((1 + \alpha \theta) a - c(2 + \theta + \alpha \theta))}{(2 + \theta + \alpha \theta)^2}. \]

Finally, we consider that firm \( i \) adopts profit and firm \( j \) adopts sales as a relative performance indicator to evaluate CEOs. In this case, optimal strategies which are chosen by firm \( i \) and \( j \) are as follows:

\[ q_i^{(P,S)} = \frac{(2 - \theta - \alpha \theta) a - 2 c}{4 - (1 + \alpha)^2 \theta^2}, \]  
(9) 

\[ q_j^{(P,S)} = \frac{(2 - \theta - \alpha \theta) a + (1 + \alpha) \theta c}{4 - (1 + \alpha)^2 \theta^2}. \]  
(10) 

The unique feature of this outcome is the quantity of firm \( j \) which chooses sales as a relative performance indicator. Generally, the quantity will decrease by increasing marginal cost, \( c \), in quantity competition in a product market, while the quantity of firm \( j \) increases by increasing marginal cost, \( c \). In this case, the marginal cost does not have a direct impact on firm \( j \)’s profit, because the sales that are used to evaluate firm \( j \)’s CEO do not reflect firm \( j \)’s marginal cost. The CEO of firm \( j \) will choose an aggressive strategy because the marginal cost does not affect the performance evaluation in this case. Especially, in this case, only a competitor considers marginal
costs because firm $i$’s CEO is evaluated using the firm’s profit. Hence, firm $j$’s CEO has a cost advantage in a product market competition compared to firm $i$. As a result, the increasing effect of marginal cost has a positive impact on firm $j$’s strategies and firm $j$ can choose a higher quantity in an engaging market. In addition, we obtain the following outcome by Eqs. (9) and (10):

$$
\pi_i^{(P,S)} = \frac{(Aa - 2c)((2 - \theta + a\theta(1 - (1 + \alpha)\theta)a - (2 - a(1 - a)\theta^2)c)}{(4 - (1 + a)^2\theta^2)^2},
$$

$$
\pi_j^{(P,S)} = \frac{(Aa + (1 + \alpha)\theta c)((2 - \theta + a\theta(1 - (1 + \alpha)\theta)a - (4 - \theta(1 - a + (1 + \alpha)^2\theta)c)}{(4 - (1 + a)^2\theta^2)^2}.
$$

where $A \equiv 2 - \theta - a\theta$. We represent this outcome as the following lemma.

**Lemma 2.** When firm $i$’s CEO is evaluated using profit and firm $j$’s CEO is evaluated using sales, quantities and profits in equilibrium are as follows:

$$
\pi_i^{(P,S)} = \frac{(Aa - 2c)((2 - \theta + a\theta(1 - (1 + \alpha)\theta)a - (2 - a(1 - a)\theta^2)c)}{(4 - (1 + a)^2\theta^2)^2},
$$

$$
\pi_j^{(P,S)} = \frac{(Aa + (1 + \alpha)\theta c)((2 - \theta + a\theta(1 - (1 + \alpha)\theta)a - (4 - \theta(1 - a + (1 + \alpha)^2\theta)c)}{(4 - (1 + a)^2\theta^2)^2}.
$$

From the outcome in equilibrium, we consider the best response strategy of firms in this study. First, when the competitor adopts profit as a relative performance indicator, we consider

$$
\pi_j^{(S,P)} - \pi_j^{(P,P)}
$$

to identify the best response strategy.

$$
\pi_j^{(S,P)} - \pi_j^{(P,P)} = \frac{(\theta(a^2\theta(2 + \theta) - (2 - \theta)\theta - 2a(2 - \theta^2))a(4 - \theta^3 - B)c}{A^2(2 + \theta + a\theta)^2},
$$

where we define $B \equiv a\theta(a\theta(2 + \theta) + 2(2 - \theta - \theta^2))$. When
\[
\frac{(2 - \theta^2)a - (2 - \theta - \theta^2)c - \sqrt{D}}{(a - c)(2 + \theta)} < \alpha < 0,
\]
(14)

holds (where \( \equiv 4(a - 3c)(a - c) + \theta^2(4a - 3c)c \)), Eq. (13) is negative. When Eq. (14) and conditions of other exogenous variables hold, each firm chooses sales as a relative performance indicator instead of profit, which is adopted as a relative performance indicator by a competitor.

From Eq. (14), we find that sales may be chosen as a relative performance indicator, when \(\alpha\) is close to zero. Because it is difficult to consider the economic intuition of this outcome from this result, we consider using a numerical example after identifying the equilibrium strategies of our model.

Next, calculating \(\pi_i^{(P,R)} - \pi_i^{(R,R)}\), we consider the best response strategy of firms, when a competitor adopts sales as a relative performance indicator.

\[
\pi_i^{(P,S)} - \pi_i^{(S,S)} = \frac{\left(\theta(\alpha^2\theta(2 + \theta) - \theta(2 - \theta) - 2\alpha(2 - \theta^2))a + 2(2 - \alpha\theta^2(1 + \alpha))c\right)c}{A^2(2 + \theta + \alpha\theta)^2},
\]
(15)

Eq. (15) is negative when

\[
\frac{(2 - \theta^2)a - (2 - \theta - \theta^2)c - \sqrt{D}}{(a - c)(2 + \theta)} < \alpha < 0,
\]
(16)

holds. Eq. (16) is equal to Eq. (14). From this outcome, when Eq. (16) holds, owners choose sales as a relative performance indicator in equilibrium. As a result, we obtain the following proposition:

**Proposition 1.** The owners choose sales as a relative performance indicator in equilibrium, when

\[
\frac{(2 - \theta^2)a - (2 - \theta - \theta^2)c - \sqrt{D}}{(a - c)(2 + \theta)} < \alpha < 0,
\]
holds, where \(D \equiv 4(a - 3c)(a - c) + \theta^2(4a - 3c)c\).
Now, we define \( \tilde{a} \equiv \frac{\left( (2 - \theta^2) a - (2 - \theta - \theta^2) c - \sqrt{D} \right)}{(a - c)(2 + \theta)} \) and consider \( \tilde{a} \).

First, we differentiate \( \tilde{a} \) by \( a \) to consider the impact of altering \( a \). As a result, we obtain

\[
\frac{\partial \tilde{a}}{\partial a} = -\frac{\left( (2 - \theta^2) a - (2 - \theta)(2 + \theta) c + \sqrt{D} \right) c}{(a - c)^2 (2 + \theta) \sqrt{4(a - 3c)(a - c)\theta^2 + (4a - 3c)c\theta^4}}. \tag{17}
\]

Hence, when \( a/c \geq \left( (2 - \theta)(2 + \theta) + \sqrt{(1 - \theta)(1 + \theta)(2 - \theta)(2 + \theta)} \right)/2 \) holds, the range of using sales as a relative performance indicator by firms become smaller by increasing \( a \). When \( a \) increases, the market becomes more attractive, and firms have an incentive to compete intensively in a product market. In this situation, the CEO has an incentive to obtain market share and excess supply because the CEO does not consider the marginal cost when the owner adopts sales as a relative performance indicator in our model. As a result, because increasing \( a \) decreases \( \tilde{a} \) and the range of Eq. (14) expands, the owner will choose the sales as a relative performance indicator. In addition, we consider the same analysis about marginal cost \( c \) and obtain the opposite outcome toward \( a \). This is because it has a negative effect on the firm’s profit by the decision of the CEO, if the CEO decides a strategy without considering marginal cost in the case of a higher \( c \).

Hence, despite increasing quantity and decreasing market price, because the firm cannot maintain enough profit with high marginal cost, increasing marginal cost, \( c \), narrows the range of Eq. (14). We summarize this outcome in the following proposition.
Proposition 2. Consider the impact of \( a \) on \( \tilde{a} \). Differentiating \( \tilde{a} \), we obtain the following outcome

\[
\frac{\partial \tilde{a}}{\partial a} = -\frac{(2(2 - \theta^2)a - (2 - \theta)(2 + \theta)c + \sqrt{D})c}{(a - c)^2(2 + \theta)\sqrt{4(a - 3c)(a - c)\theta^2 + (4a - 3c)c\theta^4}}.
\]

When this equation is negative, the range where firms choose sales as a relative performance indicator becomes narrow by increasing \( a \) and this equation is negative when \( a/c \geq \frac{(2 - \theta)(2 + \theta) + \sqrt{(1 - \theta)(1 + \theta)(2 - \theta)(2 + \theta)})}{2} \) holds.

Next, we consider the impact of \( \theta \) on \( \tilde{a} \), differentiating \( \tilde{a} \) on \( \theta \) as follows:

\[
\frac{\partial \tilde{a}}{\partial \theta} = \frac{\theta E - (2a(2 + 2\theta + \theta^2) + c(2 + \theta)(2 + \theta))\sqrt{D}}{(a - c)\theta^3(2 + \theta)^2\sqrt{D}},
\]

where \( E \equiv (8a^2(1 + \theta) - 4ac(8 + 8\theta - \theta^3) + 3c^2(2 + \theta)(4 + 2\theta - \theta^2)) \). From this outcome, we find that it is difficult to consider the sign of this expression.

We consider the numerical example to explain the intuition of choosing sales as a relative performance indicator in equilibrium. For example, we consider the case in which \( (a, c, \theta) = (1, 0.1, 1) \). In this case, \( \tilde{a} \) is

\[
\tilde{a} (1, 0.1, 1) = -0.259.
\]

where values have been rounded to the fifth decimal place or less. When \( \alpha \) exists in this range, the owners choose sales as a relative performance indicator.

Next, we fix \( a \) and \( \theta \) to \( a = 1 \) and \( \theta = 1 \) and consider the impact of altering \( c \) on \( \tilde{a} \). In addition, because altering \( a \) does not have a seriously impact to \( \tilde{a} \), we do not consider the
impact of $\alpha$ on $\tilde{\alpha}$ in this paper. For example, we consider the case in which $c = 0.05$, $c = 0.2$ and $c = 0.5$, and obtain the following result:

\[
\tilde{\alpha} (1, 0.05, 1) = -0.298, \quad (20)
\]
\[
\tilde{\alpha} (1, 0.2, 1) = -0.167, \quad (21)
\]
\[
\tilde{\alpha} (1, 0.5, 1) = 0.333. \quad (22)
\]

From this result, we find that the range of $\alpha$ expands when $c$ approaches zero. In addition, when $c$ is large enough, owners do not choose sales as a relative performance indicator in equilibrium. This result corresponds to differentiating $\tilde{\alpha}$ by $c$ and can be drawn at Figure 2.

**Figure 2.** Impact of altering $c$ on $\tilde{\alpha}$, when $a = 1$ and $\theta = 1$. 

Next, we fix $a = 1$ and $c = 0.2$ to consider the impact of altering $\theta$ on $\tilde{\alpha}$. For example, we consider the case in which $\theta = 0.1$, $\theta = 0.4$, $\theta = 0.7$ and $\theta = 1$ and obtain the following result:
\[ \alpha (1, 0.2, 0.1) = 1.124, \] (23)
\[ \alpha (1, 0.2, 0.4) = 0.104, \] (24)
\[ \alpha (1, 0.2, 0.7) = -0.126, \] (25)
\[ \alpha (1, 0.2, 1) = -0.259. \] (26)

From this result, we find that the range in which the owners choose sales as a relative performance indicator in equilibrium expands when \( \theta \) approaches to 1. This is because owners provide an incentive to CEOs to expand the market share by ignoring marginal cost when market competition is intense in this model. This is shown in Figure 3.

**Figure 3.** Impact of altering \( \theta \) on \( \alpha \), when \( a = 1 \) and \( c = 0.2 \).

From the above discussion, when \( \alpha \) approaches zero and \( \theta \) is enough large, owners have an incentive to threaten the competitor by other ways because the CEO does not have an incentive to emphasize the competitor’s performance. This means that \( \alpha \) has the role of a commitment device to engage in softer competition in a product market. In this case, when \( c \) is small enough,
the positive effect of threatening the competitor by informing it of excess supply exceeds the negative effect of excess supply for ignoring marginal cost by the CEO. In addition, if the owner does not choose this strategy, the competitor obtains market share because of the competitor’s threat of excess supply. Hence, in specific economic conditions, owners adopt sales as a relative performance indicator to evaluate the CEO’s performance.

3.2 Additional analysis

In this subsection, we relax the assumption of \( \alpha \) to \( \alpha_i \neq \alpha_j \) which is the general assumption of weight put on competitor’s profit. First, we consider the case in which the firms adopt profit as a relative performance indicator of the CEO. We obtain the following quantity and profit:

\[
q_{i}^{(p,p)} = \frac{(a - c)(2 - \theta(1 + \alpha_i))}{4 - \theta^2((1 + \alpha_i)(1 + \alpha_j + \alpha_i \alpha_j))},
\]  

(27)  

\[
\pi_{i}^{(p,p)} = \frac{(a - c)^2(2 - \theta - \theta \alpha_i)(2 - \theta(1 + \alpha_i(1 - \theta - \theta \alpha_j)))}{(4 - \theta^2((1 + \alpha_j + \alpha_i + \alpha_i \alpha_j)))^2}.
\]  

(28)  

Next, we consider the case in which the firms adopt sales as a relative performance indicator of the CEO. We obtain the following quantity and profit:

\[
q_{i}^{(s,s)} = \frac{a(2 - \theta(1 + \alpha_i))}{4 - \theta^2(1 + \alpha_i)(1 + \alpha_j)},
\]  

(29)  

\[
\pi_{i}^{(s,s)} = \frac{a(2 - \theta(1 + \alpha_i))(aF - ((2 - \theta)(2 + \theta) + \theta^2(\alpha_i + \alpha_j + \alpha_i \alpha_j))c)}{(4 - \theta^2((1 + \alpha_j + \alpha_i + \alpha_i \alpha_j)))^2},
\]  

(30)  

where \( F \equiv (2 - \theta) + \theta \alpha_i\left((1 - \theta) - \theta \alpha_j\right) \).
Finally, we consider the case in which firm $i$ adopts the profit and firm $j$ adopts sales as a relative performance indicator of the CEO. We obtain the following quantity and profit:

$$q_i^{(P)} = \frac{a((2 + \theta) - \theta \alpha_i) - 2c}{4 - \theta^2(1 + \alpha_i + \alpha_j + \alpha_i \alpha_j)}, \quad (31)$$

$$q_j^{(P,S)} = \frac{a(2 - \theta - \theta \alpha_j)(1 - \theta)c}{4 - \theta^2(1 + \alpha_i + \alpha_i + \alpha_i \alpha_j)}, \quad (32)$$

$$\pi_i^{(P,S)} = \frac{(a(2 - \theta(1 + \alpha_i)) - 2c)(aF - c(2 - \theta^2\alpha_i(1 + \alpha_j))}{(4 - \theta^2(1 + \alpha_j + \alpha_i + \alpha_i \alpha_j))^2}, \quad (33)$$

$$\pi_j^{(P,S)} = \frac{(a(2 - \theta(1 + \alpha_j)) + \theta c(1 + \alpha_j))}{(4 - \theta^2(1 + \alpha_i + \alpha_i + \alpha_i \alpha_j))^2} \times \left(\frac{a(2 - \theta + \theta \alpha_i(1 - \theta) - \theta \alpha_j)}{4 - \theta(1 + \alpha_j)\theta \alpha_j - (1 + \theta) - \theta^2 \alpha_i(1 + \alpha_j)}\right). \quad (34)$$

From the above outcome, we consider the equilibrium strategy of relative performance indicator that is chosen by the owners. First, we consider $\pi_i^{(P,P)} - \pi_i^{(S,P)}$ identifying the best response strategy, when the competitor adopts profit as a relative performance indicator.

$$\pi_i^{(P,P)} - \pi_i^{(S,P)} = -\frac{ac\theta\left((2 - \theta)\theta + (4 - 2\theta - \theta^2)\alpha_j + (2 - \theta - (2 + \theta)\alpha_j)\theta \alpha_i\right)}{(4 - \theta^2(1 + \alpha_j + \alpha_i + \alpha_i \alpha_j))^2}$$

$$+ \frac{c^2(4 - (1 + \alpha_i)\theta^3 + (4 - \theta(2 + \theta)(1 + \alpha_i)) \theta \alpha_j)}{(4 - \theta^2(1 + \alpha_j + \alpha_i + \alpha_i \alpha_j))^2}. \quad (35)$$

Eq. (35) is negative when

$$\frac{a}{c} > \frac{(4 - (1 + \alpha_i)\theta^3 + (4 - \theta(2 + \theta)(1 + \alpha_i)) \theta \alpha_j)}{\theta(2 - \theta)\theta + (4 - 2\theta - \theta^2)\alpha_j + \theta \alpha_i((2 - \theta - (2 + \theta)\alpha_j)}, \quad (36)$$

holds.
Next, we consider $\pi_i^{(p,S)} - \pi_i^{(s,S)}$ identifying the best response strategy, when the competitor adopts sales as a relative performance indicator.

$$\pi_i^{(p,S)} - \pi_i^{(s,S)} = -\frac{(2 - \theta)(1 + \alpha_j)\theta^2 + (4 - 2\theta - \theta^2)\theta\alpha_i - (2 + \theta)\theta^2\alpha_i\alpha_j)ac}{(4 - \theta^2(1 + \alpha_j + \alpha_i + \alpha_i\alpha_j))^2}$$

$$+ \frac{2(2 - (1 + \alpha_j)\theta^2\alpha_i)c^2}{(4 - \theta^2(1 + \alpha_j + \alpha_i + \alpha_i\alpha_j))^2}. \tag{37}$$

Eq. (37) is negative when

$$\frac{a}{c} > \frac{2(2 - (1 + \alpha_j)\theta^2\alpha_i)}{(2 - \theta)(1 + \alpha_j)\theta^2 + (4 - 2\theta - \theta^2)\theta\alpha_i - (2 + \theta)\theta^2\alpha_i\alpha_j)}, \tag{38}$$

holds. The overlap of Eqs. (36) and (38) is the range in which sales is chosen as a relative performance indicator in equilibrium. Since both of Eqs. (36) and (38) represent the lower limit of $a/c$, there is always a range in which both ranges overlap. Therefore, even if it is assumed that $\alpha_i \neq \alpha_j$, it can be understood that sales can be selected as a relative performance indicator in equilibrium. From the outcome of the model analysis, we obtain a unique result on the choice of a relative performance indicator, which has not been suggested by prior relative performance evaluation studies in management accounting.

4. Conclusions

We analyzed the optimal choice of profit or sales as a relative performance indicator, when the CEO, facing product market competition, is evaluated using relative performance evaluation by the owner. As a result, we showed that sales are the optimal choice as a relative performance indicator in a specific economic environment. In the literature on relative performance evaluation,
while previous studies assumed profit as a relative performance indicator of the CEO, the outcome of this research shows that the owner uses sales as a relative performance indicator given specific economic conditions. Hence, this outcome has the following important implication: the choice of a relative performance indicator is important when considering the performance of the CEO in future relative performance evaluation research.

This study has the following limitations. First, this study considers only quantity competition, because we consider the negative weight put on competitor’s profit. In addition, we do not consider the endogenous choice of $\alpha$, because it is difficult to consider the comparative statics and intuition of result for the complexity of the equilibrium outcome. Hence, it is necessary to consider the choice of relative performance indicator in endogenous $\alpha$ and price competition after this study. However, when the level of competitor’s weight placed on other firms’ profit, $\alpha$ is unobservable before decision-making, firms will adopt and report relative performance indicator by voluntary disclosure as a new commitment device in equilibrium. Hence, our assumption of exogenous $\alpha$ can be applied to analyze unobservable competitor’s weight of performance indicator. In addition, we consider only sales as a new relative performance indicator in this research. In management accounting, there are many performance indicators, and future research has to include them as relative performance indicators. While this research has these limitations, our model has an important impact on analytical and empirical research, because it addresses the issue of the choice of new relative performance evaluation research in product market competition.
References


