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Abstract: The paper develops a four-sector general equilibrium model where the fair wage hypothesis is valid and there is agricultural dualism for analyzing the consequence of an inflow of foreign capital on the skilled-unskilled wage inequality and the unemployment of skilled labour in a developing economy. The unskilled workers are fully employed but there is imperfection in the market for unskilled labour. On the contrary, the skilled wage is set by the firms by minimizing the unit cost of skilled labour and their efficiency depends on the relative income distribution and the unemployment rate. The analysis finds that an inflow of foreign capital worsens the relative wage inequality but lowers the unemployment of skilled labour. It provides an alternative theoretical foundation to the empirical finding that inflows of foreign capital might have produced unfavourable effect on the wage inequality in the developing countries during the liberalized regime by increasing the relative demand for skilled labour.

JEL classification: F13; J41; O15

Keywords: Fair wage hypothesis; agricultural dualism; skilled labour; unskilled labour; relative wage inequality; foreign capital; unemployment.
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1. Introduction

The deteriorating skilled-unskilled wage inequality is a matter of deep concern in the developing nations during the liberalized regime although as per the celebrated Stolper-Samuelson theorem it should have improved owing to economic liberalization. The theoretical literature explaining the deteriorating wage inequality in the developing countries includes works of Feenstra and Hanson (1996), Marjit and Acharya (2003), Marjit and Kar (2005), Yabuuchi and Chaudhuri (2007), Marjit, Beladi and Chakrabarti (2004) and Chaudhuri and Yabuuchi (2007). They have shown how trade liberalization, international migration of labour and inflows of foreign capital might produce unfavourable effects on the wage inequality in the South given the specific structural characteristics of the less developed countries, such as features of labour markets, structures of production, nature of capital mobility etc.

As per the empirical literature, growth in foreign direct investment which is positively correlated with the relative demand for skilled labour has been one of the prime factors\(^1\) responsible for the growing incidence of wage inequality in the South. The paper of Feenstra and Hanson (1996) is based on the famous Dornbusch-Fischer-Samuelson continuum-of-goods framework. According to them, inflows of foreign capital induced greater production of skilled-intensive commodities in Mexico, thereby leading to a relative decrease in the demand for unskilled labour. Marjit, Beladi and Chakrabarti (2004) have analyzed how diverse trade pattern and market fragmentation in world trade can adversely affect the skilled-unskilled wage inequality in the developing countries. They have also studied the consequences of an improvement of terms of trade and inflows of foreign capital on wage inequality with or without trade fragmentation. Their paper shows that without trade fragmentation improvements in terms of trade and/or inflows of foreign capital may worsen wage inequality if the vertically integrated skilled export sector is more capital-intensive vis-à-vis the import-competitive sector. But, with trade fragmentation the consequences on the

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\(^1\) See Harrison and Hanson (1999), Hanson and Harrison (1999), Curie and Harrison (1997), and Beyer, Rojas and Vergara (1999) in this context.
relative wage inequality might be different. However, all these papers have considered full-employment framework and hence have ignored the problem of unemployment which is a salient feature of these economies. But how to explain unemployment as a general equilibrium phenomenon depends on which type of labour we are considering. Harris-Todaro (1970) type of model is one way to explain unemployment in a general equilibrium setup where the efficiency of each worker is considered to be exogenously given and equal to unity. However, in such a model unemployment is specific to the urban sector only and is suitable to explain unemployment of unskilled labour only. There is a paper in the literature by Beladi, Chaudhuri and Yabuuchi (2008) that has used a 2×3 Harris-Todaro setup to examine the consequences of international mobility of different factors of production on the relative wage inequality. But it does not account for the unemployment of skilled labour which is a disquieting problem in a developing economy where skilled labour is scarce. Apart from this none of above works takes into consideration agricultural dualism which is another important feature of the developing economies.

Agricultural dualism is a common symptom of the developing countries. The distinction between advanced and backward agriculture can be made on the basis of inputs used, economies of scale, efficiency and elasticity of substitution. Many of the farmers in the agricultural sector of a developing economy stick to old and unscientific methods of cultivation although in other parts of the economy the introduction of the so called ‘Green Revolution’ technology brought about revolutionary changes with respect to production technologies and modern inputs use and the increase in factor productivity. However, the improved technology was designed for the best areas (irrigation, high soil fertility) with chemical intensive technology. Although, Green Revolution has modernized agricultural technology, it is limited only to a few parts of a developing economy and only rich (large) farmers have been benefited from it. The small and marginal farmers continue to depend on rain-fed backward agricultural technique. Therefore, the adoption of the Green Revolution technology has led to an increase in the extent of agricultural dualism in a developing economy.

The present paper seeks to examine the consequence of an inflow of foreign capital on the skilled-unskilled wage inequality in the presence of agricultural dualism and when there exists

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2 An inflow of foreign capital may worsen the wage inequality even with trade fragmentation if the traded intermediate good sector is capital-intensive relative to the import-competing sector.
unemployment of skilled labour which is explained by the ‘fair wage hypothesis’ (FWH). Following Agell and Lundborg (1992, 1995) treatment of the FWH, we assume that the efficiency of a skilled worker is sensitive to the functional distribution of income. Consequently, the return to capital and the wage rates of different types of labour appear as arguments in the efficiency function. We consider a four-sector general equilibrium framework for the purpose of analysis where there are two agricultural sectors, one low-skill manufacturing sector and one high-skill sector. The agricultural sectors use unskilled labour and land-capital as the two inputs. Sector 1 is the advanced agricultural sector while sector 2 is the backward agricultural sector. The input ‘land-capital’ is broadly conceived to include durable capital equipments of all kinds. It is sensible to assume that sector 1 (sector 2) is more land-capital-intensive (unskilled labour-intensive) than sector 2 (sector 1). Sector 3 produces a low-skill manufacturing commodity using unskilled labour and capital. On the other hand, sector 4 produces a high-skill commodity with the help of skilled labour and capital. Wages are set according to the FWH in high-skill sector while in the other three sectors where unskilled labour is used competitive forces or trade union activities determine the wages. Unskilled workers in the low-skill manufacturing sector are organized and through collective bargaining they can ensure a higher unionized wage, $W^*$, than what their counterparts earn in the two agricultural sectors, $W$. Using such a framework we show that an inflow of foreign capital in the two manufacturing sectors worsens the skilled-unskilled wage inequality and lowers the unemployment of skilled labour. Thus this paper provides an alternative theoretical foundation to the empirical finding that foreign capital inflows might have produced an adverse effect on the relative wage inequality in the liberalized regime.

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3 The existence of unemployment of skilled labour can be explained using the efficiency wage theories. One version of efficiency wage theory is based upon the work of Shapiro and Stiglitz (1984) where the work-effort of a worker is positively related to both the wage rate and unemployment rate. However, it should be kept in mind that the Shapiro and Stiglitz (1984) type of unemployment is relevant only where there is ‘hire and fire’ recruitment policy of labour. A more generalized version of efficiency wage theory is the ‘fair wage hypothesis’ (FWH). Agell and Lundborg (1992, 1995), Feher (1991), Akerlof and Yellen (1990), etc. have explained unemployment as a general equilibrium phenomenon using the FWH.

4 See Bardhan (1972) and Chaudhuri (2007) in this context.
2. The model

We consider a four-sector economy where all the sectors operate at close vicinity. There are two types of labour: unskilled and skilled. Sector 1 is the advanced agricultural sector that uses unskilled labour and land-capital as inputs. Sector 2 is the backward agricultural sector which also uses the same two inputs. Sector 1 (sector 2) uses land-capital (unskilled labour) more intensively than sector 2 (sector 1). These two are the unorganized sectors where unskilled workers receive a competitive wage, $W$. Sector 3 produces a low-skill manufacturing commodity with the help of unskilled labour and capital. Unskilled workers in this sector are organized. They successfully bargain with their employers to secure a higher unionized wage, $W^*$, than their counterparts in the other two sectors. Sector 4 produces a high-skill commodity using skilled labour and capital. So capital is perfectly mobile between sectors 3 and 4 while land-capital is perfectly mobile between the first two sectors of the economy. Skilled labour is specific to sector 4 while unskilled labour is imperfectly mobile between the first three sectors although it is perfectly mobile between sector 1 and sector 2. The efficiency of each unskilled worker is assumed to be exogenously given and equal to unity. We assume that the fair wage hypothesis (FWH) is valid and is applicable to skilled workers only. This gives rise to unemployment of skilled labour. On the contrary, there is no unemployment of unskilled labour. The unskilled workers first try to get employment in the higher paid manufacturing sector (sector 3) and those who are unable to get employment in that sector are automatically absorbed in the other two sectors where the wage rate is completely flexible. All the commodities are internationally traded and hence their prices are given internationally. The production functions exhibit constant returns to scale with positive but diminishing marginal productivity to each factor. All markets excepting the unskilled labour market in sector 3 and the skilled labour market are perfectly competitive. The capital stock of the economy consists of both domestic capital and foreign capital which are perfect substitutes. Finally, commodity 2 is chosen as the numeraire.

The following symbols will be used for formal presentation of the model.

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5 In reality, there exists open unemployment of unskilled labour which can be explained either by the Harris-Todaro (1970) migration theory or by the ‘consumption efficiency hypothesis’ of Leibenstein (1957) and Bliss and Stern (1978). As we like to emphasize on the FWH and the resultant skilled unemployment we ignore unemployment of unskilled labour. However, the basic results of the paper remain unaltered even if one accommodates unskilled unemployment following either of the two above-mentioned ways.
Given the perfectly competitive commodity markets the three price-unit cost equality conditions relating to the four industries are as follows.

\[ Wa_{L1} + Ra_{N1} = P_1 \]  \hspace{1cm} (1)
\[ Wa_{L2} + Ra_{N2} = 1 \]  \hspace{1cm} (2)
\[ W^*a_{L3} + ra_{K3} = P_3 \]  \hspace{1cm} (3)
\[ \frac{W_s}{E} a_{S4} + ra_{K4} = P_4 \]  

(4)

As the unskilled workers in this model earn two different wage rates in the different sectors the average unskilled wage is given by\(^6\)

\[ W_A \equiv W(\lambda_{z1} + \lambda_{z2}) + W^* \lambda_{z3} \equiv W + \lambda_{z3}(W^* - W) \]  

(5)

Following Agell and Lundborg (1992, 1995) we assume the efficiency of each skilled worker to be positively related to the skilled wage relative to the average unskilled wage, skilled wage relative to the return to land-capital, skilled wage relative to capital and the skilled unemployment rate.\(^7\) Thus we have

\[ E = E(W_s, W_s, W_s, R, r, v); \]

with \( E_1, E_2, E_3, E_4 > 0; E_{11}, E_{22}, E_{33} < 0; E_{13} = E_{12} = E_{14} = E_{23} = E_{24} = E_{34} = 0 \) \(^8\)

(6)

The unit cost of skilled labour, \( \sigma \), is given by

\[ \sigma = \left( \frac{W_s}{E()} \right) \]  

(7)

Apart from skilled labour, capital is used in production in sector 3. Assuming capital to be perfectly mobile intersectorally and its uniform return be \( r \) economy-wide, each firm in sector 3 minimizes its unit cost of skilled labour as given by (7). The first-order condition of minimization is

\[ E = \frac{W_s}{W_A} E_1 + \frac{W_s}{R} E_2 + \frac{W_s}{r} E_3 \]  

(8)

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\(^6\) Note that \( \lambda_{z3} = 1 - (\lambda_{z1} + \lambda_{z2}) \).

\(^7\) The micro foundation of such an efficiency function is available in Agell and Lundborg (1992, 1995).

\(^8\) The cross-effects have been assumed to zero which is a simplifying assumption. However, Agell and Lundborg (1992, 1995) in some cases have also made this assumption.
where: \(E_1 = \left(\frac{\partial E}{\partial W_A} \right) > 0; E_2 = \left(\frac{\partial E}{\partial W_S} \right) > 0; \) and, \(E_3 = \left(\frac{\partial E}{\partial W_R} \right) > 0\) are the partial derivatives of the efficiency function with respect to \(\frac{W_A}{W}; \frac{W_S}{W}; \frac{W_R}{r}\), respectively.

Full-employment conditions for unskilled labour, land-capital and capital are given by the following three equations, respectively.

\[ a_{l1}X_1 + a_{l2}X_2 + a_{l3}X_3 = L \]  
\[ a_{N1}X_1 + a_{N2}X_2 = N \]  
\[ a_{K3}X_3 + a_{K4}X_4 = K \]

Sectors 1 and 2 use the same two inputs: unskilled labour and land-capital and hence can be classified in terms factor intensities. It is natural to assume that sector 1 (sector 2) is more land-capital (unskilled labour) intensive than sector 2 (sector 1).

There is unemployment of skilled labour in the economy and the rate of unemployment is \(v\). The skilled labour endowment equation is, therefore, given by

\[ a_{S4}X_4 = E(1-v)S \]

The general framework consists of equations (1) – (4), (6) and (8) – (12). There are ten independent equations and the same number of endogenous variables; namely, \(W, R, r, W_S, E, v, X_1, X_2, X_3\) and \(X_4\).

Using (12), equation (11) can be rewritten as

\[ a_{K3}X_3 + \left(\frac{a_{K4}}{a_{S4}}\right)E(1-v)S = K \] (11.1)

The two agricultural sectors; namely, sectors 1 and 2 together form a Heckscher-Ohlin-subsystem (HOSS) as they use the same two inputs. \(W\) and \(R\) are determined from equations (1) and (2). Given \(W^*, r\) is determined from equation (3). The equilibrium values of \(W_S, E, v, X_1, X_2\) and \(X_3\) are obtained by solving equations (4), (6), (8) – (10) and (11.1). Finally, plugging the values of \(W_S, r, E\) and \(v\) into (12), \(X_4\) is found.
Unskilled workers in this system earn two different wages – either the unionized wage, $W^*$, in sector 3 or a lower competitive wage, $W$, in sector 1 and sector 2. The average wage for unskilled labour is given by equation (5). The skilled–unskilled wage gap in the present case improves (worsens) in absolute terms if the gap between $W_S$ and $W_A$ falls (rises). On the other hand, the wage inequality improves (deteriorates) both in absolute and relative terms if $(\hat{W}_S - \hat{W}_A) < (> 0$.

3. Comparative Statics

In this section of the paper we will examine the consequence of an inflow of foreign capital and on the relative wage inequality. The effect of liberalized investment policy on the skilled unemployment will also be analyzed.

As $r$ is obtained from equation (3), it follows from (4) that $\frac{W_S}{E}$ is constant. So the skilled wage and the efficiency of skilled labour must move in the same direction and in the same proportion.

Differentiating equations (4) and (5) we get, respectively

$$\hat{W}_S = \hat{E} \tag{13}$$

$$\hat{W}_A = \frac{\lambda_A (W^* - W)}{W_A} \hat{X}_3 \tag{14}$$

Totally differentiating equation (6) one gets

$$\hat{E} = \varepsilon_1 (\hat{W}_S - \hat{W}_A) + \varepsilon_2 \hat{W}_s + \varepsilon_3 \hat{W}_S + \varepsilon_4 \hat{v} \tag{15}$$

where $\varepsilon_1, \varepsilon_2, \varepsilon_3$ and $\varepsilon_4 > 0$ are the elasticities of $E(\cdot)$ with respect to $\frac{W_S}{W_A}, \frac{W_S}{R}, \frac{W_S}{r}$ and $v$, respectively.

Dividing both sides of (8) by $E$ one obtains

$$1 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \tag{16}$$

This is the famous modified Solow condition as obtained in Agell and Lundborg (1992, 1995). Substituting $(\varepsilon_2 + \varepsilon_3) = (1 - \varepsilon_1)$ into (15), using (13) and simplifying we find

$$\varepsilon_1 \hat{W}_A = \varepsilon_4 \hat{v} \tag{17}$$
Equation (17) implies that the average unskilled wage, $W_A$, and the unemployment rate of skilled labour, $v$, are positively related. This establishes the following corollary.

**Corollary 1:** The average unskilled wage, $W_A$, and the rate of skilled unemployment, $v$, are proportionately related.

Totally differentiating equations (6), (8), (9), (10) and (11.1), using (13) and (14) and solving by Cramer’s rule the following results can be derived,

When $\hat{N} > 0$, $\hat{X}_1 > 0$; $\hat{X}_2 < 0$ if $\lambda_{A}^2\varepsilon_4 \leq A_1 A_2$; $\hat{X}_3$, $\hat{X}_4 = 0$ and, (18.1)

when $\hat{K} > 0$, $\hat{X}_1 < 0$; and, $\hat{X}_2 > 0$; $\hat{X}_3 < 0$; and, $\hat{X}_4 > 0$ if $\lambda_{A}^2\varepsilon_4 \leq A_1 A_2$ (18.2)

where: $A_1 = \left(\frac{\lambda_{A}^2\varepsilon_4}{1 - \varepsilon_4}\right) > 0$; and, $A_2 = \left(\frac{\lambda_{A}^2\varepsilon_4}{W_A}\right) > 0$. (18.3)

As mentioned earlier sectors 1 and 2 together form a HOSS displaying Heckscher-Ohlin properties and sector 1 is land-capital-intensive while sector 2 is unskilled labour-intensive with respect to the other factor of production. So if the endowment of land-capital, $N$, rises sector 1 must expand while sector 2 should contract following the Rybczynski effect. Our analysis suggests that the output levels in the HOSS respond normally (i.e. as per the relative factor intensities) to changes in factor endowments under the sufficient condition that $\lambda_{A}^2\varepsilon_4 \leq A_1 A_2$.

We assume that this condition holds. We should note that any changes in the endowment of land-capital cannot affect the output levels of sectors 3 and 4.

Our analysis also suggests that sectors 1 and 3 contract while sectors 2 and 4 expand following an increase in the capital endowment of the economy under the sufficient condition that $\lambda_{A}^2\varepsilon_4 \leq A_1 A_2$.

Again differentiating equations (6), (8), (9), (10) and (11.1), using (13) and (14) and solving the following expressions are obtained.

$$\hat{W}_S = (\lambda_{A}^1 A_1 - \lambda_{A}^2 A_2) [(\varepsilon_1 A_2 E + A_4)\varepsilon_4] \left(\frac{\hat{K}}{\Delta}\right)$$

(19)

$$\hat{W}_A = \left(\frac{\lambda_{A}^1 A_1 - \lambda_{A}^2 A_2}{\Delta}\right) \varepsilon_1 A_3 A_4 \hat{K}$$

(20)

and,
\[ \hat{v} = \frac{\lambda_{N1} \lambda_{Z2} - \lambda_{N2} \lambda_{Z1}}{\Delta} A_2 A_4 \hat{K} < 0 \]  
(21)

where: \[ A_3 = \left[ \left( \frac{W_s}{W_A} \right)^2 E_{11} + \left( \frac{W_s}{R} \right)^2 E_{22} + \left( \frac{W_s}{r} \right)^2 E_{33} \right] < 0; \text{ and,} \]

\[ A_4 = \left[ \frac{\lambda_{Z3} (W_s^* - W_s)}{W_A} \right] < 0. \]

\[ \Delta = \lambda_{N1} \lambda_{Z2} - \lambda_{N2} \lambda_{Z1} \left[ (\lambda_{K3} e_4 - A_4 e_1 A_2) A_3 + \lambda_{K4} (e_1 A_2 E + A_4) e_4 \right] \]

Using (18.3) and (22) it can be shown that
\[ \Delta > 0 \text{ if } \lambda_{K3} e_4 \leq A_4 e_1 A_2 \]  
(23)

Subtracting (20) from (19) the following result can be easily derived.
\[ \frac{(\hat{W}_S - \hat{W}_A)}{\hat{K}} < 0 \text{ if } \lambda_{K3} e_4 \leq A_4 e_1 A_2 \]  
(24)

From (21) the following proposition follows immediately.

**Proposition 1:** The skilled-unskilled wage inequality deteriorates following an inflow of foreign capital if \[ \lambda_{K3} e_4 \leq A_4 e_1 A_2. \]

Proposition 1 can intuitively be explained as follows. We note that \( W, R, r \) and \( \frac{W_s}{E} \) are determined from the price system consisting of equations (1) – (4) and hence are independent of factor endowments. An inflow of foreign capital leads to a contraction of sector 3 and an

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9 This is shown in Appendix I.

10 This has been derived in Appendix II.

11 The skilled wage per efficiency unit, \( \frac{W_s}{E} \), is determined from equation (4) once \( r \) is obtained from (3). However, the individual determination of \( W_s \) and \( E \) requires the use of equations of the output system. Thus, these variables depend on factor endowments although their ratio does not.
expansion of sector 4. The demand for skilled labour rises in sector 4 that raises the skilled wage. On the other hand, a contraction of sector 3 releases unskilled labour which in turn increases the availability of unskilled labour in the HOSS. This produces a Rybczynski type effect leading to an expansion of sector 2 and a contraction of sector 1 as sector 2 is unskilled labour-intensive. As the higher unskilled wage-paying sector (sector 3) contracts both in terms of output and employment the aggregate unskilled wage income and hence the average unskilled wage must fall. Consequently, the skilled-unskilled wage inequality worsens.

Now let us examine the consequence of the liberalized investment policy on the unemployment of skilled labour, denoted, $U$, and is given by

$$U = vS$$  \hspace{1cm} (25)

Differentiating (25) and using (18) and (21) the following result can be obtained.

$$\frac{\dot{U}}{K} = \left(\frac{\dot{v}}{K}\right) = \left(\frac{\lambda_{KL_2} - \lambda_{KL_1}}{\Delta}\right)\epsilon_i A_2A_3 < 0$$  \hspace{1cm} (26)

Hence the following proposition can now be established.

**Proposition 2:** An inflow of foreign capital lowers the level of skilled unemployment if $\lambda_{KL_3}\epsilon_i A_3 \leq A_1\epsilon_i A_2$

From corollary 1 we have seen that the average unskilled wage, $W_A$, and the skilled unemployment rate, $v$, are positively related. As sector 2 expands and sector 3 contracts following an inflow of foreign capital, $W_A$, falls which in turn raises, $v$. The aggregate skilled unemployment, $vS$, rises as $S$ has not changed.

4. **Concluding remarks**

The paper has developed a four-sector general equilibrium model where the fair wage hypothesis (FWH) is valid and there is agricultural dualism so as to analyze the effects of inflows of foreign capital on the skilled-unskilled wage inequality and the unemployment of skilled labour. There are two types of labour in the model: unskilled and skilled. The unskilled workers are fully

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12 These results have been proved in Appendix I.
employed but there is imperfection in the unskilled labour market. On the contrary, the skilled wage is set by the firms by minimizing the unit cost of skilled labour and their efficiency depends on the relative income distribution and the unemployment rate. Besides, there is agricultural dualism where advanced agriculture and backward agriculture coexist and these sectors together form a Heckscher-Ohlin-subsystem. This theoretical analysis deserves special attention because no attempt has earlier been made to use the efficiency wage theory, especially the FWH version of the theory, for analyzing the consequences of liberalized economic policies on the skilled-unskilled wage inequality in a developing economy. This exercise has found that an inflow of foreign capital worsens the relative wage inequality and lowers the unemployment of skilled labour. Thus the paper provides an alternative theoretical foundation to the empirical finding that inflows of foreign capital might have produced unfavourable effect on the wage inequality in the developing countries by increasing the relative demand for skilled labour.

References:


Appendix I:

Differentiating (9), (10), (11.1), (6) and (8) and using (13), (14) and the modified-Solow condition as given by (16) we obtain, respectively

\[ \lambda_{N1} \dot{X}_1 + \lambda_{N2} \dot{X}_2 = \dot{N} \]  
(A.1)

\[ \lambda_{L1} \dot{X}_1 + \lambda_{L2} \dot{X}_2 + \lambda_{L3} \dot{X}_3 = 0 \]  
(A.2)

\[ \lambda_{K3} \dot{X}_3 + \lambda_{K4} \dot{W}_s - A_4 \ddot{v} = \dot{K} \]  
(A.3)

\[ \varepsilon_1 A_2 \dot{X}_3 - \varepsilon_4 \ddot{v} = 0 \]  
(A.4)

\[ A_3 \dot{W}_s - A_4 \dot{X}_3 - E \varepsilon_4 \ddot{v} = 0 \]  
(A.5)

where:

\[ A_1 = (\frac{\lambda_{K4} v}{1 - v}) > 0; \quad A_2 = (\frac{\lambda_{K3} (W^* - W)}{W_A}) > 0; \]
\[ A_3 = [(\frac{W_s}{W_A})^2 E_{11} + (\frac{W_s}{R})^2 E_{22} + (\frac{W_s}{r})^2 E_{33}] < 0; \quad \text{and,} \]
\[ A_4 = [(\frac{W_s}{W_A})^2 E_{11} \frac{\lambda_{K3} (W^* - W)}{W_A}] < 0. \]  
(A.6)

Writing equations (A.1) – (A.5) in a matrix notation one gets

\[
\begin{bmatrix}
\lambda_{N1} & \lambda_{N2} & 0 & 0 & 0 \\
\lambda_{L1} & \lambda_{L2} & \lambda_{L3} & 0 & 0 \\
0 & 0 & \lambda_{K3} & \lambda_{K4} & -A_1 \\
0 & 0 & \varepsilon_1 A_2 & 0 & -\varepsilon_4 \\
0 & 0 & -A_4 & A_3 & -E \varepsilon_4
\end{bmatrix}
\begin{bmatrix}
\dot{X}_1 \\
\dot{X}_2 \\
\dot{X}_3 \\
\dot{W}_s \\
\ddot{v}
\end{bmatrix}
=
\begin{bmatrix}
\dot{N} \\
0 \\
\dot{K} \\
0 \\
0
\end{bmatrix}
\]  
(A.7)

Here the determinant of the coefficient matrix is

\[ \Delta = (\lambda_{N1} \lambda_{L2} - \lambda_{N2} \lambda_{L1})[(\lambda_{K3} \varepsilon_4 - A_1 \varepsilon_1) A_3 + \lambda_{K4} (\varepsilon_1 A_2 E + A_4) \varepsilon_4] \]  
(A.8)

\[ (\varepsilon_1 A_2 E + A_4) = [(\frac{\lambda_{K3} (W^* - W)}{(W_A)^2}) E_{11} + (\frac{W_s}{W_A}) E_{11}] > 0 \quad \text{(as E}_1 > 0; \quad \text{and, E}_{11} < 0) \]  
(A.9)

Using (A.9) from (A.8) it follows that
\[ \Delta > 0 \text{ if } \lambda_{K3} e_4 \leq A_1 e_1 A_2 \]  

(23)

[Note that \((\lambda_{N1} A_{L2} - \lambda_{N2} A_{L1}) > 0 \) as sector 1 is land-capital-intensive relative to sector 2.]

Solving (A.7) by Cramer’s rule we can derive the following expressions.

\[
\hat{X}_1 = \left( \frac{\lambda_{L2} \hat{N}}{\Delta} \right) \left[ (\lambda_{K3} e_4 - A_1 e_1 A_2) A_3 + \lambda_{K4} e_4 (e_1 A_2 E + A_4) \right] + \left( \frac{\lambda_{N2} \lambda_{L3} A_1 e_4}{\Delta} \right) \hat{K} \]  

(A.10)

\[
\hat{X}_2 = -\left( \frac{\lambda_{L2} \hat{N}}{\Delta} \right) \left[ (\lambda_{K3} e_4 - A_1 e_1 A_2) A_3 + \lambda_{K4} e_4 (e_1 A_2 E + A_4) \right] - \left( \frac{\lambda_{N1} \lambda_{L3} A_1 e_4}{\Delta} \right) \hat{K} \]  

(A.11)

\[
\hat{X}_3 = \left( \frac{\lambda_{N1} A_{L2} - \lambda_{N2} A_{L1}}{\Delta} \right) A_1 e_4 \hat{K} \]  

(A.12)

Using (18.3), (22) and (A.9) from (A.10) and (A.11) one can find that

When \( \hat{N} > 0 \), \( \hat{X}_1 > 0 \); \( \hat{X}_2 < 0 \); and, \( \hat{X}_3 = 0 \) if \( \lambda_{K3} e_4 \leq A_1 e_1 A_2 \);  

(18.1)

When \( \hat{K} > 0 \), \( \hat{X}_1 < 0 \); and, \( \hat{X}_2 > 0 \); and, \( \hat{X}_3 < 0 \) if \( \lambda_{K3} e_4 \leq A_1 e_1 A_2 \)  

(18.2)

Again, differentiating (12) and using (13) one obtains

\[
\hat{X}_4 = \hat{W}_S - \left( \frac{v}{1-v} \right) \hat{\nu} \]  

(A.13)

Using (19) and (21) from (A.13) one gets

\[
\hat{X}_4 = (\lambda_{N1} A_{L2} - \lambda_{N2} A_{L1}) [(e_1 A_2 E + A_4) e_4 - \frac{v e_1 A_2 A_4}{1-v}] \hat{K} \]  

(A.14)

Using (A.9) and (23) from (A.14) it follows that

\[
\left( \frac{\hat{X}_4}{\hat{K}} \right) > 0 \text{ if } \lambda_{K3} e_4 \leq A_1 e_1 A_2 \]  

(A.15)

Appendix II:

Solving (A.7) we can also derive the following expressions.

\[
\hat{W}_S = \left( \frac{\lambda_{N1} A_{L2} - \lambda_{N2} A_{L1}}{\Delta} \right) [(e_1 A_2 E + A_4) e_4] \hat{K} \]  

(19)

\[
\hat{\nu} = \left[ \frac{\lambda_{N1} A_{L2} - \lambda_{N2} A_{L1}}{\Delta} (e_1 A_2 A_3) \right] \hat{K} \]  

(21)
Again using (A.12) from (14) one can derive
\[
\hat{W}_A = \frac{(\lambda_{N2}A_{L2} - \lambda_{N2}A_{L1})\varepsilon_4}{\Delta} A_2 A_4 \hat{K}
\]  
(20)

Subtracting (20) from (19) we find
\[
(\hat{W}_S - \hat{W}_A) = \frac{(\lambda_{N1}A_{L2} - \lambda_{N2}A_{L1})\varepsilon_4}{\Delta} [(\varepsilon_1 A_2 E + A_4) - A_2 A_3] \hat{K}
\]  
(A.16)

Using (18.3), (22) and (A.9) from (A.16) one finds that
\[
\frac{(\hat{W}_S - \hat{W}_A)}{\hat{K}} > 0 \text{ if } \lambda_{K3} \varepsilon_4 \leq A_4 \varepsilon_1 A_2 .
\]

Skilled unemployment is,
\[
U = vS
\]  
(25)

Differentiating (25) one gets
\[
\hat{U} = \hat{v} + \hat{S}
\]  
(A.17)

Putting \(\hat{S} = 0\) and using (18) the following result can be obtained.
\[
\frac{\hat{U}}{\hat{K}} = \frac{\hat{v}}{\hat{K}}
\]

Using (21) we get
\[
\frac{\hat{U}}{\hat{K}} = \frac{\hat{v}}{\hat{K}} = \frac{(\lambda_{N1}A_{L2} - \lambda_{N2}A_{L1})\varepsilon_1}{\Delta} A_2 A_3
\]  
(26)

From (26) it follows that
\[
\frac{\hat{U}}{\hat{K}} < 0 \text{ if } \lambda_{K3} \varepsilon_4 \leq A_4 \varepsilon_1 A_2
\]

[Note that \((\lambda_{N1}A_{L2} - \lambda_{N2}A_{L1}) > 0 ; \varepsilon_1, A_4 > 0; A_2 < 0; \text{ and, } \Delta > 0 \text{ if } \lambda_{K3} \varepsilon_4 \leq A_4 \varepsilon_1 A_2 .\]

So, the unemployment of skilled labour decreases following an inflow of foreign capital.