Incorporating fairness motives into the Impulse Balance Equilibrium concept: an application to experimental 2X2 games

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Abstract. Substantial evidence has accumulated in recent empirical works on the limited ability of the Nash equilibrium to rationalize observed behavior in many classes of games played by experimental subjects. This realization has led to several attempts aimed at finding tractable equilibrium concepts which perform better empirically, often by introducing a reference point to which players compare the available payoff allocations, as in impulse balance equilibrium and in the inequity aversion model. The purpose of this paper is to review these recent reference point literature and to advance two new, empirically sound, hybrid concepts.

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From efficiency to equality: the "distributive" reference point

In recent years experimental economists have accumulated considerable evidence that steadily contradicts the self-interest hypothesis embedded in equilibrium concepts traditionally studied in game theory, such as Nash's. The evidence suggests that restricting the focus of analysis to the strategic interactions among perfectly rational players (exhibiting equilibrium behavior) can be limiting, and that
considerations about fairness and reciprocity should be accounted for.
In fact, while models based on the assumption that people are exclusively motivated by their material self-interest perform well for competitive markets with standardized goods, misleading predictions arise when applied to non-competitive environments, for example those characterized by a small number of players (cf. FEHR & SCHMIDT, 2000) or other frictions. For example KAHNEMAN, KNETSCH & THALER (1986) find empirical results indicating that customers are extremely sensitive to the fairness of firms' short-run pricing decisions, which might explain the fact that some firms do not fully exploit their monopoly power.

One prolific strand of literature on equity issues focuses on relative measures, in the sense that subjects are concerned not only with the absolute amount of money they receive but also about their relative standing compared to others. BOLTON (1991), formalized the relative income hypothesis in the context of an experimental bargaining game between two players.
KIRCHSTEIGER (1994) followed a similar approach by postulating envious behavior. Both specify the utility function in such a way that agent i suffers if she gets less than player j, but she's indifferent with respect to j's payoff if she is better off herself. The downside of the latter specifications is that, while consistent with the behavior in bargaining games, they fall short of explaining observed behavior such as voluntary contributions in public good games.

A more general approach has been followed by FEHR & SCHMIDT (1999), who instead of assuming that utility is either monotonically increasing or decreasing in the well being of other player, model fairness as self-centered inequality aversion. Based on this interpretation, subjects resist inequitable outcomes, that is they are willing to give up some payoff in order to move in the direction of more equitable outcomes. More specifically, a player is altruistic towards other players if their material payoffs are below an equitable benchmark, but feels envy when the material payoffs of the other players exceed this level. To capture this idea, the authors consider a utility function which is linear in both inequality aversion and in the payoffs. Formally, for the two-player case ($i \neq j$):

$$\boldsymbol{\mathcal{U}_i = x_i - \alpha_i max\{x_j - x_i, 0\} - \beta_i max\{x_i - x_j, 0\}} \qquad (\mathbf{1})$$

Where x_i, x_j are player 1 and player 2's payoffs respectively and β_i, α_i are player i's inequality parameters satisfying the following conditions: $\beta_i \leq \alpha_i$ and $0 \leq \beta_i \leq 1$.
The second term in the equation is the utility loss from disadvantageous inequality, while the third term is the utility loss from advantageous inequality. Due to the above restrictions imposed on the parameters, for a given payoff x_i, player i's utility function is maximized at $x_i = x_j$, and the utility loss from disadvantageous inequality ($x_i < x_j$) is larger than the utility loss if player i is better off than player j ($x_i > x_j$).

Fehr and Schmidt show that the interaction of the distribution of types with the strategic environment explains why in some situations very unequal outcomes are obtained while in other situations very egalitarian outcomes prevail. In referring to the social aspects introduced by this utility function, one could think of inequality aversion in terms of an interactive framing effect (reference point dependence).

This payoff modification has proved successful in many applications, mainly in combination with the Nash equilibrium concept, and will therefore be employed in this study, although in conjunction with a different equilibrium type, as will be explained in the next section.

The "psychological" reference point

The predictive weakness of the Nash equilibrium is effectively pointed out by EREV & ROTH (1998), who study the robustness and predictive power of learning models in experiments involving at least 100 periods of games with a unique equilibrium in mixed strategies. They conclude that "…in some of the games the [Nash] equilibrium prediction does very badly" and that a simple learning model can be used to predict, as well as explain, observed behavior on a broad range of games, without fitting parameters to each game. A similar approach, based ex-post and ex-ante comparisons of the mean square deviations, will also be employed in this paper to assess to what extent the proposed hybrid model improves the fit of several games.

Based on the observation of the shortcomings of mixed Nash equilibrium in rationalizing observed behavior in many classes of games played by experimental subjects, an alternative tractable equilibrium has been suggested by SELTEN & CHMURA (forthcoming). IBE is based on learning direction theory (SELTEN & BUCHTA, 1999), which is applicable to the repeated choice of the same parameter in learning situations where the decision maker receives feedback not only about the payoff for the choice taken, but also for the payoffs connected to alternative actions. If a higher parameter would have brought a higher payoff, the player receives an upward impulse, while if a lower parameter would have yielded a higher payoff, a downward impulse is received. The decision maker is assumed to have a tendency to move in the direction of the impulse. IBE, a stationary concept which is based on transformed payoff matrices as explained in the next section, applies this mechanism to 2x2 games. The probability of choosing one of two strategies (for example Up) is treated as the parameter, which can be adjusted upward or downward. It is assumed that the second lowest payoff in the matrix is an aspiration level determining what is perceived as profit or loss. In impulse balance equilibrium expected upward and downward impulses are equal for each of both players simultaneously.

The main result of the paper by Selten and Chmura is that, for the games they consider, impulse balance theory has a greater predictive success than the other three stationary concepts they compare it to: Nash equilibrium, sample-7 equilibrium and quantal response equilibrium. While having the desirable feature of being a parameter-free concept as the Nash equilibrium, and of outperforming the latter, the aspiration level framework (to be described) expose the theory to a critique regarding the use of transformed payoffs in place of the original ones for the computation of the equilibrium.

The aspiration level can be thought of as a psychological reference point, as opposed to the social one considered when modeling inequality aversion: the idea behind the present work is that of utilizing IBE but replacing the aspiration level with inequity aversion (social) parameters. The motivation follows from the realization that in non-constant sum games (considered here) subjects' behavior also reflects considerations of equity. In fact, while finite repetition does little to enlarge the scope for cooperation or retaliation, non-constant sum games offer some cooperation opportunities, and it seems plausible that fairness motives will play an important role in repeated play of this class of games. A suitable consequence of replacing the aspiration level framework with the inequality aversion one is that the original payoffs can be utilized (and should, in order to avoid mixing social and psychological reference points).

Experimental setup: IBE

The table in Appendix A shows the 12 games, 6 constant sum games and 6 non-constant sum games on which Selten and Chmura have run experiments, which have taken place with 12 independent subject groups for each constant sum game and with 6 independent subject groups for each non-constant sum game. Each independent subject group consists of 4 players 1 and 4 players 2 interacting anonymously in fixed roles over 200 periods with random matching. In summary:

Players: *I={1,2}*
Action space: *{U,D}x{L,R}*
Probabilities in mixed strategy: $\{P_U,1\text{-}P_U\}$ and $\{Q_L,1\text{-}Q_L\}$
Sample size: (54 sessions) x (16 subjects) = 864
Time periods: T=200

In Appendix A, a non-constant sum game next to a constant sum game has the same best reply structure (characterized by the Nash equilibrium choice probabilities P_U, Q_L) and is derived from the paired constant sum game by adding the same constant to player 1's payoff in the column for *R* and 2's payoff in the row for *U*. Games identified by a smaller number have more extreme parameter values than games identifies by a higher number; for example, Game 1 and its paired non-constant sum Game 7 are near the border of the parameter space ($P_U \cong 0.1$ and $Q_L \cong 0.9$), while Game 6 and its paired non-constant sum Game 12 are near the middle of the parameter space ($P_U \cong 0.5$ and Q_L=0.6).

As pointed out, IBE involves a transition from the original game to the transformed game, in which losses with respect to the natural aspiration level get twice the weight as gains above this level. The impulse balance equilibrium depends on the best reply structure of this modified game, which is generally different from that of the original game, resulting therefore in different predictions for the games in a pair.
The present paper utilizes the data on the experiments involving 6 independent subject groups for each of the 6 non-constant sum games (games 7 through 12 in Appendix A). As anticipated above, this class of games is particularly conceptually suitable to the application of the inequality aversion

framework. Further, in completely mixed 2x2 games, mixed equilibrium is the unambiguous game theoretic prediction when they are played as non-cooperative one-shot games. Since non-constant sum games provide incentives for cooperation, such attempts to cooperation may have influenced the observed relative frequencies in Selten's experiment. Along these lines, it is particularly relevant to see whether inequality aversion payoff modifications can help improve the fit with respect to these frequencies.

The application of inequality aversion parameters to Impulse balance equilibrium provides an opportunity for testing Fehr & Schmidt's fairness model in conjunction with the IBE, which is itself a simple yet fascinating concept which has proven to be empirically successful in fitting the data in many categories of games and is nevertheless parsimonious due to the straight-forward formulation and parameter-free nature. By including a fairness dimension to it, the hope is to supply favorable empirical evidence and provide further stimulus to expand the types of games empirically tested.

Formally, this involves first modifying the payoff matrices of each game in order to account for the inequality parameters (β, α), than creating the impulse matrix based on which the probabilities are computed. In order to clarify the difference between the reference point utilized in Selten and Chmura (the aspiration level) and that utilized in this paper it is useful to start by summarizing the mechanics behind the computation of the IBE.

Let's consider the normal form game depicted in Figure 1 below,

Fig.1: structure of the 2x2 games (arrows point in the direction of best replies)

L (Q_L) →	R (1-Q_L)
$a_L + c_L$; b_U ↑	a_R ; $b_U + d_U$ ↓
a_L ; $b_D + d_D$	$a_R + c_R$; b_D

←

where $a_L, a_R, b_U, b_D \geq 0$ and $c_L, c_R, d_U, d_D > 0$

c_L and c_R are player 1's payoffs in favor of *U,D* while d_U, d_D are player 2's payoffs in favour of *L,R* respectively. Note that player 1 can secure the higher one of a_L, a_R by choosing one of his pure strategies, and player 2 can similarly secure the higher one of b_U, b_D. Therefore, the authors define the natural aspiration levels for the 2 players are given by:

s_i =max(a_L, a_R) for i=1 and s_i=max(b_U, b_D) for i=2

the transformed game (TG) is constructed by leaving player i's payoff unchanged if it is less or equal to s_i and by reducing the difference of payoffs greater than si by the factor ½. Algebraically, calling *x* the payoffs,

if $x \le s_i$ => $x' = x$

if $x > s_i$ => $x' = x - ½(x - s_i)$

If after the play, player i could have obtained a higher payoff with the other strategy, she receives an impulse in the direction of the other strategy, of the size of the foregone payoff in the TG.

Fig.2:Impulses in T.G. in the direction of unselected strategy

L (Q_L)	R (1-Q_L)
0 ; d_U*	c_R* ; 0
c_L* ; 0	0 ; d_D*

The concept of impulse balance equilibrium requires that player 1's expected impulse from *U* to *D* is equal to the expected impulse from *D* to *U*; likewise, pl.2's expected impulse from *L* to *R* must equal the impulse from R to L. Formally,

$P_U Q_R c_R$* =$P_D Q_L c_L$*

$P_U Q_L d_U$*=$P_D Q_R d_D$*

Which, after some manipulation, can be shown to lead to the following formulae for probabilities:

$$P_U = \frac{\sqrt{cl^*/cr^*}}{\sqrt{cl^*/cr^*}+\sqrt{du^*/dd^*}} \;;\; Q_L = \frac{1}{1+\sqrt{\frac{cl^*}{cr^*}\frac{du^*}{dd^*}}}$$

Experimental setup: equity-driven Impulse Balance Equilibrium

Replacing the aspiration level framework with the inequality aversion one doesn't require the computation of the TG based on aspiration level framing, as the original payoffs are now modified by including the inequality parameters (β, α). Formally, recalling that: $\mathcal{U}_i = x_i - \alpha_i max\{x_j -$ $xi,0-\beta imaxxi-xj,0$

Table 1: structure of the 2x2 games accounting for inequality aversion

L (Q_L)	R (1-Q_L)
$a_L + c_L - \alpha_i max\{b_U - a_L - c_L, 0\} - \beta_i max\{a_L + c_L - b_U, 0\}$; $b_U - \alpha_j max\{a_L + c_L - b_U, 0\} - \beta_j max\{b_U - a_L - c_L, 0\}$	$a_R - \alpha_i max\{b_U + d_U - a_R, 0\} - \beta_i max\{a_R - b_U - d_U, 0\}$; $b_U + d_U - \alpha_j max\{a_R - b_U - d_U, 0\} - \beta_j max\{b_U + d_U - a_R, 0\}$
$a_L - \alpha_i max\{b_D + d_D - a_L, 0\} - \beta_i max\{-b_D - d_D + a_L, 0\}$; $b_D + d_D - \alpha_j max\{a_L - b_D - d_{D_L}, 0\} - \beta_j max\{-b_D - d_D + a_L, 0\}$	$a_R + c_R - \alpha_i max\{b_D - a_R - c_R, 0\} - \beta_i max\{a_R + c_R - b_D, 0\}$; $b_D - \alpha_j max\{a_R + c_R - b_D, 0\} - \beta_j max\{b_D - a_R - c_{R,}, 0\}$

Based on these payoffs, the previous section's computations can be conducted in order to find the impulse balance mixed strategy equilibria corresponding to specific values of β and α.

Two measures of the relative performance of the I.A.-adjusted Impulse Balance concept: best fit and predictive power

Results in terms of Best fit

The preceding analysis served as an introduction to the more systemic method utilized in the next paragraphs to assess the descriptive and predictive success of the "pure" impulse balance equilibrium in comparison to the proposed Inequality Aversion hybrid.

Following a methodology which has been broadly utilized in the literature to measure the adaptive and predictive success of a point in a Euclidean space, the squared distance of observed and theoretical values is employed (cf. Erev & Roth, 1998 and Selten & Chmura). More precisely, the first part of the analysis consists, for each of the 6 non constant sum games, of a grid search with an MSD criterion on the (β, α) parameter space to estimate the best fitting parameters, i.e. those that minimize the distance between the model and the data.

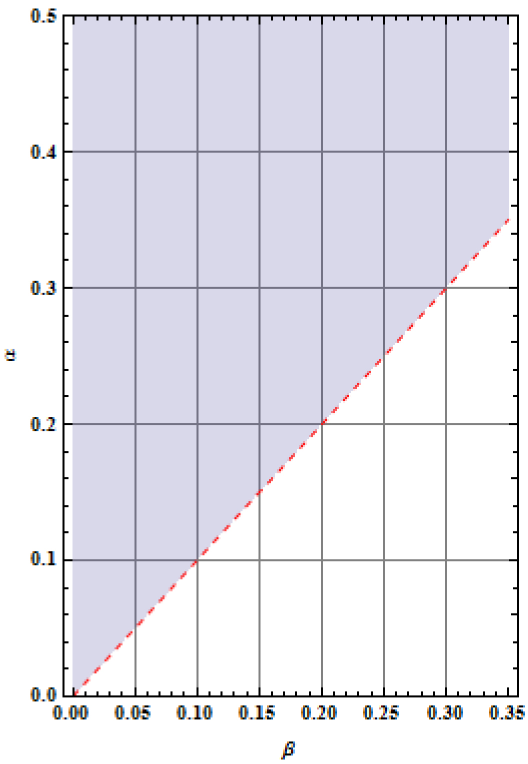
Algebraically, the mean over the 6 games in the best fit row will be given by:

$$\frac{1}{6}\sum_{i=7}^{12} MSD_i$$

where MSD_i is the mean of game i's squared distances $(\mathrm{f_{ui}} - \mathrm{P_u})^2 + (\mathrm{f_{li}} - \mathrm{Q_l})^2$

The inequality aversion parameters used in the hybrid model must satisfy the constraints $\beta i \leq \alpha i$ and $0 \leq \beta i \leq 1$. The relevant parameter space under investigation is then given, for each β ,by values $\alpha \in [\beta, 0.5]$. Graphically the parameter space can be represented as follows are as follows:

***Figure 3: The* correspondence between β and α**



$$\forall \beta \in [0,0.35], \alpha \in [\beta, 0.5]$$

In Table 1, a summary of the results of the explanatory power of the two models is presented for each non constant sum game, starting from the transformed or the original payoffs, respectively. The comparisons are made both within game class in column 5 (e.g. within transformed game *i*, *i*=7,...,12), and across game class in the last column (e.g. between original game *i* and transformed game *i*).

The reason of the two-fold comparison is that not only it is meaningful to assess whether the hybrid model can better approximate the observed frequencies than the I.B. concept, but it is especially important to answer the question: does the hybrid concept applied to the original payoffs of game *i* outperform the 'pure' I.B. applied to the transformed payoffs? In other words, since the inequality aversion concept overlaps to a certain extent to that of having impulses in the direction of the strategy not chosen, applying the inequality aversion adjustment to payoffs that have already been transformed to account for the aspiration level will result in "double counting". It is therefore more relevant to compare the best fit of hybrid equilibrium on O.G. (see rows highlighted in blue) to that obtained by applying impulse balance equilibrium to T.G.

Table 2: Ex-post (best fit) descriptive power of hybrid model vs I.B. equilibrium

	FREQUENCY $[f_u; f_l]$	**N.E. [Pu;Ql]**	**BEST FIT I.B.+I.A. [Pu;Ql]** (β, α)	**IBE [Pu;Ql]** (0;0)	**I.B.+I.A > IBE?**	**O.G.+I.B.+I.A. > T.G.+IBE?**
TG7	[.141;.564]		[.104;.634] (0;0)	[.104;.634]	NO	*n.a.*
OG7	[.141;.564]	[.091;.909]	[.099;.568] (.054;.055)	[.091;.500]	YES	YES
TG8	[.250;.586]		[.270;.586] (.043;.065)	[.258;.561]	YES	*n.a.*
OG8	[.250;.586]	[.182;.727]	[.257;.585] (.006;.468)	[.224;.435]	YES	YES
TG9	[.254;.827]		[.180;.827] (.07;.10)	[.188;.764]	YES	*n.a.*
OG9	[.254;.827]	[.273;.909]	[.232;.840] (.325;.327)	[.162;.659]	YES	YES
TG10	[.366;.699]		[.355;.759] (.089;.134)	[.304;.724]	YES	*n.a.*
OG10	[.366;.699]	[.364;.818]	[.348;.717] (.250;.254)	[.263;.616]	YES	YES
TG11	[.311;.652]		[.357;.652] (.012;.018)	[.354;.646]	YES	*n.a.*
OG11	[.311;.652]	[.364;.727]	[.344;.644] (.001;.425)	[.316;.552]	YES	YES
TG12	[.439;.604]		[.496;0.575] (0;0)	[.496;.575]	NO	*n.a.*
OG12	[.439;.604]	[.455;.636]	[.439;.604] (.022;.393)	[.408;.547]	YES	YES

Inspection of Table 1 suggests a strong positive answer to the following two relevant questions regarding the ability of the proposed concept to fit the observed frequencies of play: within the same class of payoffs (TG or OG), is the descriptive power of the hybrid concept superior to that of

the IBE? And, perhaps more importantly, is this still true when the two concepts are applied to their natural payoffs, namely the original and the transformed respectively?
The last two columns of Table 1 contain the answers to the two questions, based on a comparison of the mean squared deviations of the predicted probabilities from the observed frequencies under the two methods.

Results in terms of Predictive power

The next step in evaluating the performance of the inequality aversion-adjusted impulse balance equilibrium concept is studying its ex ante predictive power. This is done by partitioning the data into subsets, and simulating each experiment using parameters estimated from the other experiments. By generating the MSD statistic repeatedly on the data set leaving one data value out each time, a mean estimate is found making it possible to evaluate the predictive power of the model. In other words, the behavior in each of the 6 non-constant sum games is predicted without using that game's data, but using the data of the other 5 games to estimate the probabilities of playing up and down. By this cross-prediction technique (known as jackknifing), one can evaluate the stability of the parameter estimates, which shouldn't be substantially affected by the removal of any one game from the sample. Erev & Roth (1998) based their conclusions on the predictive success and stability of their learning models by means of this procedure, and it has therefore been employed in this work.

Table 2, above, shows summary MSD scores (100*Mean-squared Deviation) organized as follows: each of the first 6 columns represents one non-constant sum game, while the last column gives the average MSD over all games, which is a summary statistic by which the models can be roughly compared. The first three rows present the MSDs of the Nash equilibrium and of the I.B. equilibrium predictions (for $\beta=0=\alpha$) on the transformed and original payoffs respectively. The remaining three rows display MSDs of the I.A.+I.B. model on the original payoffs: in the fourth row, the parameters are separately estimated for each game (12 parameters in total); in the fifth row, the estimated 2 parameters that best fit the data over all 6 games (and over all but Game 7) are employed (the same two β, α that minimize the average score over all games are used to compute the MSDs for each game); in the last row the accuracy of the prediction of the hybrid model is showed when behavior in each of the 6 games is predicted based on the 2 parameters that best fit the other 5 games (and excluding Game 7).

Table 3: MSD scores of the IBE and of the proposed equilibrium concept

Model	G 7	G 8	G 9	G 10	G 11	G 12	Mean
Nash equilibrium, O.G. 0 parameters (0;0) All games G8-12	6.076	1.225	.354	.708	.422	.064	1.475 **.555**
I.B. equilibrium, T.G. 0 parameters (0;0)All games G8-12	.315	.035	.416	.224	.094	.205	.215 **.195**
I.B. equilibrium, O.G. 0 parameters(0;0) All games G8-12	.330	1.174	1.825	.878	.497	.209	.819 **.917**
Hybrid by game, O.G. 12 parameters All games G8-12	.090	.003	.031	.033	.056	.000	.035 **.025**
Hybrid best fit, O.G. 2 parameters All games (.157,.160) G8-12 (.252,.257)	.746 -	.178 **.042**	.428 **.098**	.152 **.033**	.140 **.173**	.030 **.034**	.279 **.076**
Hybrid predict, O.G. 2 parameters All games Without G 7	2.220 -	.238 **.044**	.585 **.149**	.186 **.033**	.141 **.189**	.031 **.035**	.567 **.09**

Table 3 summarizes further evidence in favor of the newly developed equity-driven impulse balance equilibrium. One can see from the third row that if the parameters of inequality aversion are allowed to be fit separately in each game, the improvements in terms of reduction of MSD are significant, both with respect to the Nash and impulse balance equilibrium.

Moreover, even when restricting the number of parameters to 2 (common to all games, cf. row 5 “best fit”), the mean MSD is still more than five times smaller than Nash’s. If one doesn’t include the extremely high MSD reported in both cases for Game 7 (for reasons discussed below), the gap actually increases, as the hybrid concept’s MSD becomes more than seven times smaller than Nash’s. With respect to the overall MSD mean of the IBE, when considering all games the hybrid has a higher MSD, although the same order of magnitude (.279 and .215 respectively). If one focuses only on games 8-12, again we have a marked superiority of the hybrid model over the IBE, as the MSD of the latter is more than twice that of the new concept.

A similar pattern is appears in the last row of the table, concerning the predictive capability: if Game 7 is excluded, the values are in line with the ones obtained in the fifth row, indicating stability of the parameters who survive the cross-validation test. One comforting consideration regarding the appropriateness of the exclusion of Game 7 comes from the widespread anomalous high level of its MSD score in all rows of the table, which for both Nash and Hybrid predict is about four times the corresponding mean level obtained over the six games. It is plausible that this evidence is related to the location of Game 7 in the parameter space. It is in fact located at near the border, as previously pointed out, and therefore may be subject to the overvaluation of extreme probabilities by the subjects due to overweighting of small probabilities. An addition to the present work, which is currently in progress, considers incorporating fairness motives in the quantal response equilibrium notion, one that has recently attracted considerable attention thanks to its ability to rationalize behavior observed in experimental games. In addition to providing an interesting case for comparison, it should also allow to shed light on the suspected anomalous nature of Game 7.

Quantal Response Equilibrium and Inequity Aversion

The former analysis has also been conducted utilizing the quantal response equilibrium concept (henceforth QRE) in conjunction with preferences that are again allowed to be affected by the counterparty's fate, via the inequity aversion parameters. Before showing the results, which are given in Table 4 and Table 5 and show an even better overall performance of this concept compared to the one examined in the previous sections, let's briefly describe the QRE. The concept, introduced by (Mckelvey, Palfrey and Thomas, 1995), models games with noisy players: these probabilistic choice models are based on quantal best responses to the behavior of the other parties, so that deviations from optimal decisions are negatively correlated with the associated costs. That is to say, individuals are more likely to select better choices than worse choices, but do not necessarily succeed in selecting the very best choice. In the exponential form of quantal response equilibrium, considered here, the probabilities are proportional to an exponential with the expected payoff multiplied by the logit precision parameter (λ) in the exponent: as λ increases, the response functions become more responsive to payoff differences. Formally,

$$\boldsymbol{P_{ij}} = \frac{e^{\lambda\pi_{ij}(P_{-i})}}{e^{\lambda\pi_{ij}(P_{-i})}+e^{\lambda\pi_{ik}(P_{-i})}} \qquad \textbf{(2)}$$

Where *i,j*=1,2 are the players ($k \neq j$), P_{ij} is the probability of player *i* choosing strategy *j* and π_{ij} is player *i*'s expected payoff when choosing strategy *j* given the other player is playing according to the probability distribution P_{-i}.

Two measures of the relative performance of the I.A.-adjusted Quantal Response Equilibrium: best fit and predictive power. Results in terms of Best fit

The following is a companion table to Table 2, as it reports the results of comparisons between the new hybrid model and the IBE concept, the former always outperforming the one employing the 'pure' IBE on the transformed games. Note that the penultimate column now compares the performance of the two proposed concepts, showing that the one employing QRE outperforms the in five of the six games[1].

Table 4: Ex-post (best fit) descriptive power of QRE with inequity aversion

	FREQUENCY [f_u; f_l]	**N.E. [Pu;Ql]**	**BEST FIT QRE+I.A. [Pu;Ql] (β, α) λ**	**IBE [Pu;Ql] (0;0)**	**QRE+I.A > IBE+I.A?**	**O.G.+QRE+IA > T.G.+IBE?**
TG7	[.141;.564]		[.] (;)	[.104;.634]		*n.a.*
OG7	[.141;.564]	[.091;.909]	[.141;.564] (.105;.209) $\lambda=0.335$	[.091;.500]	YES	YES
TG8	[.250;.586]		[.] (.)	[.258;.561]		*n.a.*
OG8	[.250;.586]	[.182;.727]	[.250;.586] (.097;.386) $\lambda=0.335$	[.224;.435]	YES	YES
TG9	[.254;.827]		[] (.)	[.188;.764]		*n.a.*
OG9	[.254;.827]	[.273;.909]	[.254;.827] (.083;.316) $\lambda=0.6$	[.162;.659]	YES	YES
TG10	[.366;.699]		[.] (.)	[.304;.724]		*n.a.*
OG10	[.366;.699]	[.364;.818]	[.366;.699] (.250;.254) $\lambda=0.31$	[.263;.616]	YES	YES
TG11	[.311;.652]		[] ()	[.354;.646]		*n.a.*
OG11	[.311;.652]	[.364;.727]	[.311;.652] (.003;.02) $\lambda=0.91$	[.316;.552]	YES	YES
TG12	[.439;.604]		[] ()	[.496;.575]		*n.a.*
OG12	[.439;.604]	[.455;.636]	[.439;.604](.042;.137) $\lambda=0.55$	[.408;.547]	same	YES

[1] in game 12 they achieve a substantially equal equilibrium prediction.

As before, in order to assess the performance of the concepts over multiple games, the parameters are restricted to be the same over all the games, as shown in the penultimate row in Table 5: the QRE+IA concept displays a better fit than the IBE+IA (smaller mean square deviation) in all but game 11, achieving a mean MSD of .147 as opposed to .279 for the latter. As for the predictive power, measured through jackknifing (cross-predicting), when all games are considered the mean MSD is substantially lower for the QRE-based concept incorporating fairness motives, averaging .219 vs. a score of .567 for the IBE-based one.

Table 5: MSD scores of the proposed equilibrium concepts

Model	G 7	G 8	G 9	G 10	G 11	G 12	Mean
Nash equilibrium, O.G. 0 parameters (0;0) All games	6.076	1.225	.354	.708	.422	.064	**1.475**
I.B. equilibrium, O.G. 0 parameters(0;0) All games	.330	1.174	1.825	.878	.497	.209	**.819**
I.B. equilibrium, T.G. 0 parameters (0;0)All games	.315	.035	.416	.224	.094	.205	**.215**
Hybrid QRE by game, O.G. 18 parameters	5.5* 10^-6	2.4* 10^-7	7.5* 10^-6	6.4* 10^-7	7.4* 10^-8	5.7* 10^-6	**3.3*10^-6**
Hybrid best fit, O.G. parameters (β, α, λ) 2 par. IBE+IA (.157,.160) 3 par.QRE+IA (.147,.243,.43)	.746 .251	.178 .012	.428 .397	.152 .036	.140 .163	.030 .027	**.279** **.147**
Hybrid predict, O.G. 2 par. IBE+IA 3 par. QRE+IA	2.220 .415	.238 .016	.585 .640	.186 .038	.141 .177	.031 .029	**.567** **.219**

Two important considerations should be remarked at this point. Firstly, for what concerns the overall fit, even without excluding the potentially problematic game 7, the QRE+IA concept outperforms the traditional impulse balance equilibrium applied to the transformed games (MSD scores are .147 and .215, respectively); this is noteworthy, since it wasn't the case for the other hybrid concept[2]. Secondly, the above considerations are confirmed by the predictions obtained with

[2] In fact, the 'pure' impulse balance equilibrium obtains dramatically higher MSD scores when the original games are employed in place of the transformed ones, with an almost four-fold increase. The intuition behind this is, loosely speaking, that the IBE is not as parameter-free as it looks: that is, by utilizing transformed payoffs for each game

the jackknifing technique: for the QRE+IA specification the mean MSD score based on cross-predictions is not substantially higher than the one calculated when the parameters that best fit all games are employed (.219 and .147, respectively). This doesn't hold for the IBE+IA concept, whose score roughly doubles from .279 to .567[3].

Based on the above comparisons, the inequity aversion generalization of the quantal response equilibrium concept appears to emerge as the best performing in terms of goodness of fit among the considered stationary concepts. Based on this realization and following the behavioral stationary concept interpretation of mixed equilibrium[4], one may conclude that the proposed other-regarding generalization of the QRE is the behavioral stationary concept that best models the probability of choosing one of two strategies in various non constant-sum games spanning a wide parameter space. More specifically, even when restricting the degrees of freedom of the parametric models and comparing the goodness of fit utilizing the same parameters (β, α, λ if any) for all six games, the other-regarding QRE outperforms all of the other stationary concepts considered here. The order, starting with the most successful with the goodness of fit decreasing progressively, is the following (see the grey highlighted rows in Table 5): QRE+IA, IBE on the transformed games, IBE+IA and Nash equilibrium.

Of course, the previous comparison is biased against the more parsimonious concepts, in particular the parameter-free Nash equilibrium and IBE concepts (see footnote 1 regarding the latter). In order to trade off the predictive parsimony of a theory against its descriptive power, one can employ Selten's Measure of Predictive Success (Selten, 1991). This is currently ongoing work.

(although based on common definition of aspiration level), it effectively allows for game-specific adjustments similar to those obtained by adding a parameter which can take different values in each game.

[3] Note also that the QRE+IA mean of the MSD when cross-predicting is approximately equal to the mean score for the 'pure' IBE on all transformed games, further confirming the stability of the parameters in the other-regarding version of QRE.

[4] that sees it as the result of evolutionary (or learning) processes in a situation of frequently repeated play with two populations of randomly matched opponents.

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Appendix A: Games utilized in Selten & Chmura; in the present paper only games 7 to 12 (non-constant sum games) are investigated.

Constant Sum Games

Game 1

10 8	0 18
9 9	10 8

Game 2

9 4	0 13
6 7	8 5

Game 3

8 6	0 14
7 7	10 4

Game 4

7 4	0 11
5 6	9 2

Game 5

7 2	0 9
4 5	8 1

Game 6

7 1	1 7
3 5	8 0

Non-Constant Sum Games

Game 7

10 12	4 22
9 9	14 8

Game 8

9 7	3 16
6 7	11 5

Game 9

8 9	3 17
7 7	13 4

Game 10

7 6	2 13
5 6	11 2

Game 11

7 4	2 11
4 5	10 1

Game 12

7 3	3 9
3 5	10 0

L: left R: right
U: up D: down

Player 1's payoff is shown in the upper left corner
Player 2's payoff is shown in the lower right corner