World Equivalent Factor Endowments Determine Local Factor Rewards When Countries Have Different Productivities

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Abstract – This study derived the solution of general trade equilibrium by the Trefler Hicks-Neutral HOV Model (Trefler model), which reflects productivities different across countries. The study found that the Trefler model is with a single diversification cone of commodity price[2], although it is with two diversification cones of factor endowments. This feature provides a chance to attain the general trade equilibrium and non-equalized factor price. The study uses a geometric approach on a generalized Integrated World Equilibrium (IWE) diagram, which presents the equivalent factor endowments defined by Trefler (1993). The non-equalized factor price at the equilibrium by the Trefler model is with two useful properties. The first one is that the equilibrium prices are the functions of world equivalent factor endowments so that it remains the same when the allocation of equivalent factor endowments changes. The second property is that the localized factor prices ensure gains from trade for countries participating in trade. A new logic explored is that the world equivalent factor endowments determine world commodity price and local factor rewards of all countries.

Keywords:
Factor content of trade; factor price non-equalization; General equilibrium of trade; Integrated World Equilibrium; IWE

1. Introduction

Giving factor endowments of two countries in an open economic system, same or not same technologies across countries, under assumptions of identical consumption preference and constant return, how are world price and local factor rewards determined? This is a central question for international economics.

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[2] Fisher (2011) provided this insight concept. He named it as "goods price diversification cone". It is the counterpart of the diversification cone of factor endowments. It tells what makes sure for the positive factor prices.

The world price (local factor prices and commodity price) determination and general equilibrium are the same issues by different descriptions. Fewer studies focused on this issue directly.

The Leontief paradox inspired many researchers to make similar investigations on the trade pattern of various other countries. It also inspired the numerous HOV studies to incorporate different technologies across countries by assorted approaches.

Vanek (1968)'s HOV model provided a powerful vehicle for the analyses of factor contents of trade, which are flexible both for the same technologies and for different technologies. The share of GNP in the HOV model connected prices with trade and consumption.

The Integrated World Equilibrium (Dixit and Norman, 1980) is remarkable to illustrate equalized factor price by factor content of trade. It provided a practical view of price-trade equilibrium. It identified the feature of equalized factor price with mobile factors. Helpman and Krugman (1985) normalize the assumption of integrated equilibrium, which presented equilibrium analyses in a simple way. Deardroff (1994) derived the conditions of the FPE for many goods, many factors, and many countries by using the IWE approach. He discussed the FPE for all possible allocations of factor endowments.

Many studies, like Gale and Nikaido (1965), Chipman (1969), Krugman (2000), Fisher(2011), Leamer (1998, 2000), and Rassekh and Thompson(1993) had argued the need of factor price non-equalization when considering different technologies across countries.

Deardroff (1979) proposed a two-cone approach to present productions with different technologies. He identified the Heckscher-Ohlin chain of the rank of comparative advantage for the case of two factors.

Trefler (1993) extended Leontief (1953) idea of the productivity-equivalent unit to introduce productivity parameters for each factor of each country in HOV studies. He provided an effective and simple way to measure factor endowments by equivalent productivity while remaining technology the same for all countries. He provided an artful model to present relative factor prices across countries. Trefler (1995) turned to another method to introduce technology matrix differences by a uniform argument parameter across countries.

Fisher and Marshall (2008) and Fisher (2011) introduced another insight approach to characterized different technologies. They measure the factor endowments with different technologies by virtual factor endowments. Fisher (2011) also proposed another two important terms: the goods price diversification cone and the intersection of goods price cones, which are very helpful to understand price properties in equilibriums. Feenstra and Taylor (2012, p.102) provided the concept of effective factors to interpret factor endowments with different productivities.

Guo (2015) derived a price-trade equilibrium for the Heckscher-Ohlin model and demonstrated that the equalized factor price and common commodity price at the equilibrium depended directly on world factor endowments. He demonstrated that equalized factor price makes sure of gains from trade for the countries participating in trade. This study borrows Guo (2015) approach and derived the price-trade equilibrium of the 2x2x2 Hicks-Neutral HOV model (we call it the Trefler model). The study found that the Trefler model has only one cone for commodity price, although it has two cones of factor diversification. It is more available to get full relationship among factor prices, commodity price, and trade volumes. The study attained the first result of the price-trade equilibrium and factor price non-equalization when countries have different productivities.

This paper is divided into three sections. Section 2 introduces the price-trade equilibrium of the Trefler Hicks-Neutral HOV Model by a geometric method. Section 3 illustrates gains from trade by the equilibrium. Section 4 examines the equilibrium conditions of many commodities and many factors. The last one is the conclusion.

2. The General Trade Equilibrium

2.1 Trefler Model

We first denote a "standard" 2x2x2 Trefler model based on Trefler (1993).

Suppose that country home is the USA, which serves as the benchmark country. Its technological matrix is

$$A^H = \begin{bmatrix} a_{1K}^H & a_{2K}^H \\ a_{1L}^H & a_{2L}^H \end{bmatrix} \tag{2-1}$$

where A^H is the 2x2 technology matrix. Its element $a_{ik}^H(w/r)$ is minimizing input requirement of factor k needed to product one unit of output i, i=1,2, k=L(labor), K(capital).

The foreign country's technological matrix is

$$A^F = \Pi^{-1}A^H = \begin{bmatrix} 1/\pi_K & 0 \\ 0 & 1/\pi_L \end{bmatrix} A^H = \begin{bmatrix} a_{1K}^H/\pi_K & a_{2K}^H/\pi_K \\ a_{1L}^H/\pi_L & a_{2L}^H/\pi_L \end{bmatrix} \tag{2-2}$$

or

$$A^H = \Pi A^F = \begin{bmatrix} \pi_K & 0 \\ 0 & \pi_L \end{bmatrix} A^F$$

where Π is a 2x2 diagonal productivity-argument matrix, its element π_K is capital productivity-argument parameter, π_L is labor productivity-argument parameter. When $\pi_K = \pi_L = \delta$, it is the single factor-augmenting parameter model defined in Trefler (1995) as

$$A^F = \delta A^H \tag{2-3}$$

When $\delta < 1$, the home country productivities are higher in both factors than the productivities of the foreign country.

We denote the production constraints and cost functions of two countries by using the technology of the home country as

$$A^H X^H = V^H \tag{2-4}$$

$$(A^H)' W^H = P^H \tag{2-5}$$

$$\Pi^{-1} A^H X^F = V^F \tag{2-6}$$

$$(\Pi^{-1} A^H)' W^F = P^F \tag{2-7}$$

where V^h is the 2 x 1 vector of factor endowments with elements *K* as capital and *L* as labor; X^h is the 2 x 1 vector of output; W^h is the 2 x 1 vector of factor prices with elements r as rental and w as wage; P^h is a 2 x 1 vector of commodity prices with elements p_1^h and p_2^h; $h = H, F$ for countries.

We call the four equations above as the Trefler model.

The Trefler model is with two diversification cones of factor endowments. For the home country, we express it in algebra as

$$\frac{a_{K1}^H}{a_{L1}^H} > \frac{K^H}{L^H} > \frac{a_{K1}^H}{a_{L2}^H} \tag{2-8}$$

For the foreign country, it is

$$\frac{a_{K1}^H \pi_L}{a_{L1}^H \pi_K} > \frac{K^F}{L^F} > \frac{a_{K1}^H \pi_L}{a_{L2}^H \pi_K} \tag{2-9}$$

A unique feature for the Trefler model is that it is with a single cone of commodity price, although technologies are different. Its cost ratio ranks, which show the rays of commodity price cone in algebra, are

$$\frac{a_{K1}^{H}}{a_{K2}^{H}} = \frac{a_{K1}^{H}\pi_k}{a_{K2}^{H}\pi_k} > \frac{P_1^*}{P_2^*} > \frac{a_{L1}^{H}}{a_{L2}^{H}} = \frac{a_{L1}^{H}\pi_L}{a_{L2}^{H}\pi_L} \tag{2-10}$$

Most cases of technology difference across countries are with two commodity price cones. In addition, the world commodity price must lie in the interception of the two price cones. The feature of the single price cone reduces the difficultness of analyses of price-trade equilibrium. It provides a chance to get an equilibrium as Guo (2019) did for the Heckscher-Ohlin model.

To incorporate productivities more efficiency, Trefler introduced the measurement of factor endowments by the productivity-equivalent unit. We call the factor endowments measured by the equivalent productivity unit as "equivalent factor endowments" briefly.

Express the equivalent factor endowments of the foreign country as

$$V^{EF} = \Pi V^F = \begin{bmatrix} \pi_K K^F \\ \pi_L L^F \end{bmatrix} \tag{2-11}$$

where V^{EF} is the 2x1 vector of equivalent factor endowments in country foreign.
Denote the equivalent reward of the foreign country as

$$W^{EF} = \Pi^{-1} W^E \tag{2-12}$$

where W^{EF} is the 2x2 vector of the equivalent reward of the foreign country.
We rewrite the Trefler model (2-4) through (2-7) as

$$A^H X^H = V^H \tag{2-13}$$

$$(A^H)' W^H = P^H \tag{2-14}$$

$$A^H X^F = V^{EF} \tag{2-15}$$

$$(A^H)' W^{EF} = P^F \tag{2-16}$$

This is the equivalent factor version of the Trefler model. Trefler described it as "Factor price equalization hypothesis and HOV theorem hold".

Let denote world equivalent factor endowments for the Trefler model as

$$V^{EW} = \begin{bmatrix} K^{EW} \\ L^{EW} \end{bmatrix} = \begin{bmatrix} K^H + \pi_K K^F \\ L^H + \pi_L L^F \end{bmatrix} \tag{2-17}$$

where K^{EW} is the world equivalent capital endowments; L^{EW} is the world equivalent labor endowments. The home country as a benchmark country, its factor endowments are its equivalent factor endowments.

2.2 General Equilibrium of Trade by Equivalent Integrated World Equilibrium

The trade vector of the home is

$$T^H = X^H - C^H = (1-s)X^H - sX^F \tag{2-18}$$

where $T^H$is the 2x1 trade vector; C^H is the 2x1 consumption vector; s is the home country's share of GNP to world GNP.

Denote the factor content of trade as follows,

$$F^H = A^H T^H = (1-s)V^H - s\Pi V^F = V^H - sV^{EW} \tag{2-19}$$

We now present the equivalent factor endowments of the model in Figure 1. It is a generalized Integrated World Equilibrium (IWE) diagram with the equivalent factor endowments. We call it the equivalent-IWE or E-IWE diagram.

The dimensions of the figure represent world equivalent factor endowments. The origin for country home is the lower left corner, for country foreign is the right upper corner. ON and OM are the rays of the cone of diversification. Any point within the parallelogram formed by $ONO'M$ is an available allocation of factor endowments of two countries. Suppose that an allocation of the factor endowments is at point E.

The first thing we mentioned is that factor endowments of the home country and the equivalent factor endowments of the foreign country are under the same cone. They used the same technology matrix. We can see that from (2-14) and (2-15).

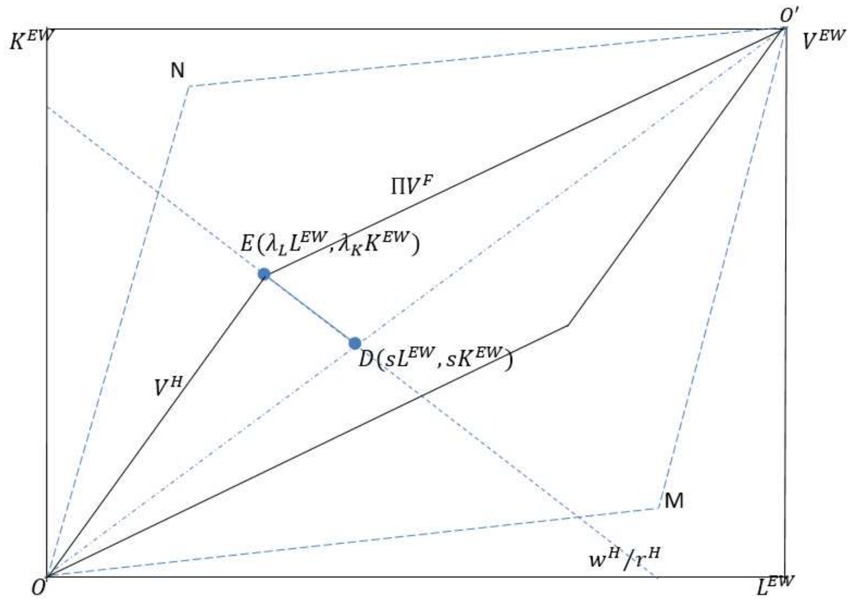

Figure 1 Equivalent-IWE Diagram

We introduce two share parameters of the home factor endowments to their world equivalent factor endowments, respectively,

$$\lambda_L = \frac{L^H}{L^{EW}} \tag{2-20}$$

$$\lambda_K = \frac{K^H}{K^{EW}} \tag{2-21}$$

We denote the home factor endowments as

$$L^H = \lambda_L L^{EW} \tag{2-22}$$

$$K^H = \lambda_K K^{EW} \tag{2-23}$$

The allocation of point E is $E(\lambda_L L^w, \lambda_K K^w)$.

Substituting (2-22) and (2-23) into (2-19) yields

$$\begin{bmatrix} F_K^H \\ F_L^H \end{bmatrix} = \begin{bmatrix} \lambda_K K^{EW} - sK^{EW} \\ \lambda_L L^{EW} - sL^{EW} \end{bmatrix} \tag{2-24}$$

Using trade balance of factor contents of trade yields

$$\frac{r^H}{w^H} = -\frac{\lambda_L L^{EW} - sL^{EW}}{\lambda_K K^{EW} - sK^{EW}} = \frac{(s-\lambda_L)L^{EW}}{(\lambda_K - s)K^{EW}} \tag{2-25}$$

In the IWE analysis for the Heckscher-Ohlin model with the same technology assumption, Dixit and Norman (1980) had shown that the equalized factor price remains the same when the allocation of factor endowments changes (we call it the Dixit-Norman price principle). Is the principle valid for the equivalent-IWE? We thought that it is yes. Woodland (2013, p70) made a forceful argument for the property of the

Dixit-Norman principle. He stated, “The answer to this question is ‘yes’, since this allocation of world factor endowments between countries leaves the world supply of goods and, hence, incomes unchanged and so supplies will still match the unchanged world demands”. For the E-IWE, the allocation of world equivalent factor endowment between countries also leaves the world supply of goods and income unchanged and so supplies will still match the unchanged world demands.

The mobility of the equivalent factor may not be so realistic. However, a quantitative relationship in the E-IWE is similar to the one in the IWE. Imagine an allocation of commodity outputs changes when world total commodity outputs remain the same. It is very like the situation of product outsourcing nowadays. If the world production capacity (world fully-employed equivalent factor endowments) remains the same, the world outputs will remain the same, so as, world income, world consumption, and prices and local factor rewards.

We take the result or the assumption of the fixed price in the E-IWE. Introduce a constant C as

$$C = \frac{(s-\lambda_L)}{(\lambda_K - s)} \tag{2-26}$$

Substituting it into (2-25) yields

$$\frac{r^H}{w^H} = C\frac{L^{EW}}{K^{EW}} \tag{2-27}$$

The factor price ratio (r^H/w^H) is unchanged or fixed within the parallelogram by $ONO'M$ on the E-IWE diagram. Therefore, C should be a constant. Equation (2-27) illustrates that the rent/wage ratio of the home country is the function of the world equivalent factor endowments. This is why the prices are constant when the allocation of equivalent factor endowments changes within the parallelogram by $ONO'M$ in the E-IWE diagram.

We have interesting to know what value C takes. We denote an allocation of factor endowment at D, which is a point at the diagonal line of the E-IEW box. At that point $D(sL^{EW}, sK^{EW})$, its two parameters of fator endowment ratios to world factor endowments are $\lambda_{Ld} = s$ and $\lambda_{Kd} = s$, where s is country home' share of GNP for the allocation. There is no trade at this point.

We now suppose that allocation E is nearby to allocation D or imagine point E moves to close to its equilibrium point D.

If the allocation E is above the diagonal line OO', there are always $s - \lambda_L > 0$ and $\lambda_K - s > 0$.

Taking $\lambda_L \to s$ and $\lambda_k \to s$ yields

$$\lim_{\substack{\lambda_L \to s \\ \lambda_k \to s}} \frac{(s-\lambda_L)}{(\lambda_K - s)} = 1 = C \tag{2-28}$$

We see that constant C equals 1. From (2-26), we have the share of GNP for equilibrium as

$$s = \frac{1}{2}(\lambda_L + \lambda_K) = \frac{1}{2}\left(\frac{K^H}{K^{EW}} + \frac{L^H}{L^{EW}}\right) \tag{2-29}$$

In addition, equation (2-27) is reduced as

$$\frac{r^H}{w^H} = \frac{L^{EW}}{K^{EW}} \tag{2-30}$$

This is true for every allocation of factor endowments within the parallelogram formed by $ONO'M$.

With the equilibrium share of GNP (2-29) and the rent/wage ratio (2-30), we now obtain the whole equilibrium solution of the model as

$$W^{H*} = \begin{bmatrix} \frac{L^{VW}}{K^{VW}} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{L^H + \pi_L L^F}{K^H + \pi_K K^F} \\ 1 \end{bmatrix} \tag{2-31}$$

$$P^* = (A^H)' \begin{bmatrix} \frac{L^{VW}}{K^{VW}} \\ 1 \end{bmatrix} \tag{2-32}$$

$$W^{F*} = \Pi \begin{bmatrix} \frac{L^{VW}}{K^{VW}} \\ 1 \end{bmatrix} \tag{2-33}$$

$$F_K^h = \frac{1}{2}\frac{K^h L^{EW} - K^{EW} L^h}{K^{EW}}, \qquad F_L^h = -\frac{1}{2}\frac{K^h L^{EW} - K^{EW} L^h}{L^{EW}}, \quad (h = H, F) \tag{2-34}$$

$$T_1^h = x_1^h - s x_1^w, \qquad T_2^h = x_2^h - s x_2^w \;, \quad (h = H, F) \tag{2-35}$$

$$s = \frac{1}{2}\left(\frac{K^H}{K^{EW}} + \frac{L^H}{L^{EW}}\right) \tag{2-36}$$

Walras' equilibrium allows dropping one market clear condition. We take $w^{H*} = 1$. It just serves as benchmark price referred both by all of the other factors in domestic or in international and by world common commodity prices.

We notice that the relative factor prices of two countries after factor price localization are under the following relationship,

$$r^F = \pi_K r^H \tag{2-37}$$

$$w^F = \pi_L w^H \tag{2-38}$$

This is just derived by Trefler (1993).

The factor price of the foreign country can be rewritten as

$$W^{F*} = \Pi \begin{bmatrix} \frac{L^{VW}}{K^{VW}} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{L^H + \pi_L L^F}{K^H/\pi_K + K^F} \\ \pi_L \end{bmatrix} \tag{2-39}$$

If we assume the $w^F = 1$ as the base price of the system, the price above will be

$$W^{F*} = \begin{bmatrix} \frac{L^H/\pi_L + L^F}{K^H/\pi_K + K^F} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{L^{VW*}}{K^{VW*}} \\ 1 \end{bmatrix} \tag{3-40}$$

where L^{VW*} and K^{VW*} are the world equivalent factor endowments measured by the technologies of the foreign country.

The world equilibrium prices (one set of commodity price and two sets of local factor prices) are the function of the world equivalent factor endowments. It demonstrates that when the allocations of equivalent factor endowments of the two countries changes, the equilibrium prices remain the same. The equilibrium prices themselves proof that the assumption of the fixed price in E-IWE is right.

From the factor content of trade (2-34), we see that when $\frac{K^H}{L^H} > \frac{K^{EW}}{L^{EW}}$, then $F_K^H > 0$. This is just the content of the "equivalent-productivity" Heckscher-Ohlin theorem.

The localized factor price (2-30) displays that the relative factor price (rent/wage) in the home country, in reversely, is proportional to their world equivalent factor endowments. It does not relate to benchmark technologies. Moreover, it does not relate to commodity prices. It is endogenously determined by the exogenous equivalent factor endowments.

The changes of allocations of the equivalent factor endowments within parallelogram $ONO'M$ in the E-IWE box arise the changes of shares of GNP and the changes of trade volumes of two countries. This does not affect world commodity price and localized factor prices.

The price solution above illustrates that the price in E-IWE box more stable. The technology matrix A^h keeps unchanging since $A^h = A^h(w/r) = A^h(K^{VW}/L^{VW})$, where $(h = H, F)$.

2.3 Trade Box and Trade Competition

We now view the trade equilibrium from the trade box identified by the cone of commodity price.

Trades redistribute national welfares, which are measured by GNP. This is a major trade consequence.

The boundaries of the share of GNP corresponding the cone of commodity price (2-10) are

$$s_b^H(p) = s(p\left(\frac{a_{K1}^H}{a_{K2}^H}, 1\right)) = \frac{a_{K1}^H x_1^H + a_{K2}^H x_2^H}{a_{K1}^H x_1^w + a_{K2}^H x_2^w} = \frac{K^H}{K^{EW}} \tag{2-39}$$

$$s_a^H(p) = s\left(p\left(\frac{a_{L1}^H}{a_{L1}^H}, 1\right)\right) = \frac{a_{L1}^H x_1^H + a_{L2}^H x_2^H}{a_{L1}^H x_1^w + a_{L2}^H x_2^w} = \frac{L^H}{L^{EW}} \tag{2-40}$$

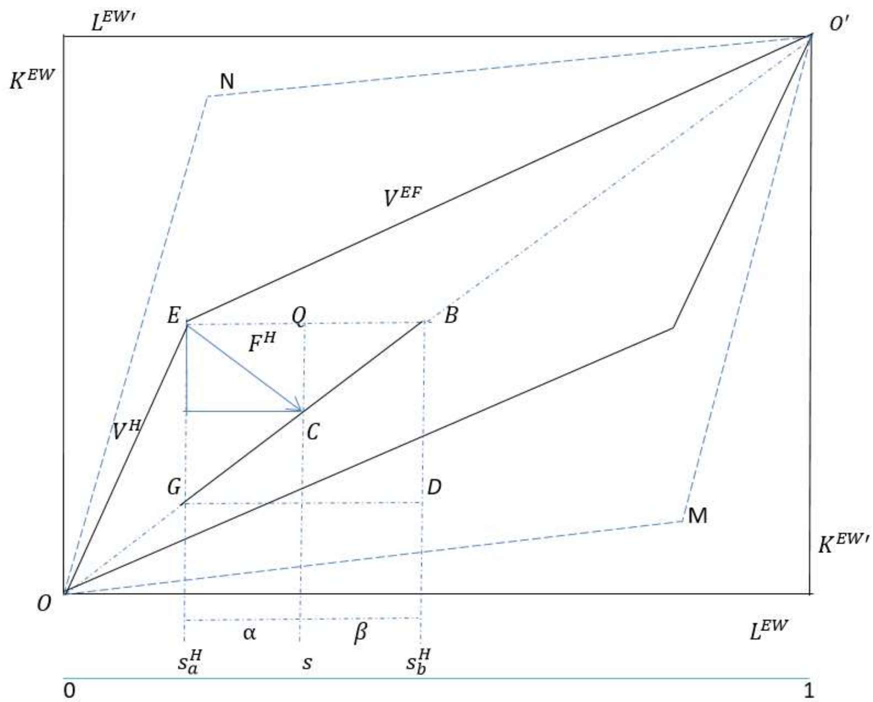

Figure 2 Equivalent-IWE with Trade Box

They identify the trade box $EBDG$ in Figure 2. If a commodity price lies in the cone of commodity price (2-10), the share of GNP will lie in the trade box. The trade vector F^H must end at a point of diagonal line GB of the trade box. Otherwise, it is either violating the assumption of the same consumption taste or yielding a negative reward of factor.

The share of GNP of the home country s divides the trade box into two parts in Figure 2. Their lengths are α and β, respectively, at horizontal direction as

$$\alpha = (s - \lambda_L), \qquad \beta = (\lambda_K - s) \tag{2-41}$$

When α increases, the share of GNP of the home country increases, the share of GNP of the foreign country decreases. On the contrary, when β increases, the share of GNP of the foreign country increases, the share of GNP of the home country decreases. In trade competitions, the both countries need to reach their maximum GNP share through free trade.

We notice that the trade box not only is the trade area but also is the redistributable area of the share of GNP for the two countries. Outside the box, they are not redistributable by trade (the trade outside of the trade box will course a factor payment being negative). Therefore, α is redistributable part of the share of GNP for the home country; β is redistributable part of the share of GNP for the foreign country. Under free trade, each country needs to maximue their redistributable share of GNP.

Helpman and Krugman (1985, pp23) introduced the term "volume of trade" as

$$VT = 2p_1\left(x_1^h - sx_1^W\right) = 2p_2\left(x_2^h - sx_2^W\right) \qquad (h = H, F) \tag{2-42}$$

Based on their concept, we introduce the volume of factor content of trade as

$$VF = 2r^H\left(K^h - sK^{EW}\right) = 2w^H\left(L^h - sL^{EW}\right) \tag{2-43}$$

We now demonstrate that α is the home country's share of GNP by VF.

The home country exports EG as capital service and imports GC as labor service. The GC indicates the share of GNP of capital service EG plus labor service GC. GC is the share of GNP measured at the diagonal OO' direction. Its size equals to α numerically.

We see $\alpha = \beta$ when trade reaches its equilibrium. they both are the share of GNP of F^Has

$$\alpha = \beta = \frac{VF}{World\ GNP} = \frac{2w^{H*}\left(L^H - sL^{EW}\right)}{w^{H*}L^{EW} + r^*K^{EW}} \tag{2-44}$$

The share of world trade volume of factor content is

$$WVF = 2 \times VF = \alpha + \beta = (\lambda_k - \lambda_K) = \left(\frac{K^H}{K^{EW}} - \frac{L^H}{L^{EW}}\right) \tag{2-45}$$

It implies that the size of world trade equals to the size of the trade box identified the cone of commodity price.

3. Autarky Price and Comparative Advantage

> "Proofs of the static gains from trade fall into the unrefutable category yet these are some of the most important results in all of economics". (Leamer and Levinsohn, 1995, p.1342)

Guo (2015) provided a computable autarky price by the logic that "autarky" factor endowments determined its "autarky" price. It sourced from the logic that world factor endowments determine world price in the Heckscher-Ohlin model. He also provided a mathematical proof for autarky price by using the IWE diagram.

The autarky prices of two countries before trade can be expressed

$$r^{ha} = \frac{L^h}{K^h} \qquad (h = H, F) \tag{4-2}$$

$$w^{ha} = 1 \qquad (h = H, F) \tag{4-3}$$

$$p_1^{ha} = a_{k1}\frac{L^h}{K^h} + a_{L1} \qquad (h = H, F) \tag{4-4}$$

$$p_2^{ha} = a_{k2}\frac{L^h}{K^h} + a_{L2} \qquad (h = H, F) \tag{4-5}$$

where superscript ha indicates the autarky price of country h.

Gains from trade are measured by

$$-W^{ha\prime}F^h > 0 \qquad (h = H, F) \tag{4-8}$$

$$-P^{ha\prime}T^h > 0 \qquad (h = H, F) \tag{4-9}$$

We add the negative sign in inequalities above since we expressed trade by net export, T^h . In most other literatures, they express trade by net import.

Appendix A is proof of gains from trade by inequality (4-8) for the price solution of this paper. It implies that localized-equalized factor prices make sure that countries participating in the trade in the Trefler model gains from trade.

The result of gains from trade is another good side effect of the equilibrium of trade. It is one important property of the equilibrium and the factor price non-equalization.

When $\pi_K < 1$ and $\pi_L < 1$, the home country is with Adam Smith's absolute advantage in the productions of both commodities. This study demonstrates that even when one country is with absolute disadvantages in technologies in productions of both commodities, the two countries still have benefits to do trade by the factor endowment differences. In addition, both countries gain from trade.

In addition, the localized factor prices satisfy the factor price restriction on the factor content of trade (Helpman 1984) as the follows

$$\left(w^j - w^i\right)' F^{ij} > 0 \tag{4-10}$$

where w^l is the 2x2 factor price vector for country l, $l = i, j$, $F^{ij}$is the 2x2 vector of factor content of trade exporting from country i to country j.

We presented a numerical example in Appendix B to display the gains from trade for a situation of absolute advantage in technologies and for displaying Helpman factor price criterion.

We now summarize the result of this study to a theorem.

The theorem – Factor Price Non-Equalization Theorem

Suppose that two countries are engaged in free trade, having an identical homothetic taste but different technologies and different factor endowments by the Trefler model. When the common commodities price formulated, factor prices (w^H, r^H) and (w^F, r^F) of the two countries are localized. The total amounts of world equivalent factor endowments determine the common commodity price and the localized factor prices, which make sure that the two countries gain from trade.

Proof

For a giving E-IWE box, the price solution of one set of common commodity price and two sets of localized factors is unique. We have provided the proof the equilibrium price as equations (2-31) through (2-33). Appendix A presented proof of the gains from trade by the equilibrium price. The prices at the equilibrium are functions of world equivalent factor endowments. Besides the mathematical proof of the equilibrium solution, the theorem is supported by three principles of international economics. The first one is that it makes sure gains from trade. This is a necessary requirement of international trade theory for the solution. The second is that the world price and local factor price remain the same when the allocation of the equivalent factor endowments of two countries changes. This is both economic principle and the "dynamic check" for the solution in mathematics. The third is that the size of trade box identified by the price cone equals the size of the world volume of factor contents of trade. It does imply the important price-trade relation inside models. This is not a coincident result.

End Proof

The equilibrium shows the unification of the trade direction, the None-FPE theorem, and gains from trade. They confirmed each other.

4. Many Commodities and Factors

For higher dimension (many-factor, many-commodity, and may-country model) equilibrium, Guo (2018) provided a solution for the Heckscher-Ohlin model. It demonstrates that general equilibriums are available for the cases of un-even technology matrix. A higher dimension Trefler model can be converted into the model like equations (2-13) through (2-16), which is the Heckscher-Ohlin model mathematically. The general equilibrium in the higher dimension Heckscher-Ohlin model can be generalized to the higher dimension Trefler model without difficulties. We concern if the Trefler model still is one commodity price cone in the cases of many commodity and many factors. We use a three-factor and three-commodity model to illustrate that it is still at one price cone.

Suppose that country home's technological matrix is

$$A^H = \begin{bmatrix} a_{11}^H & a_{12}^H & a_{13}^H \\ a_{21}^H & a_{22}^H & a_{23}^H \\ a_{31}^H & a_{32}^H & a_{33}^H \end{bmatrix} \tag{4-1}$$

The foreign country's technological matrix is

$$A^F = \Pi^{-1}A^H = \begin{bmatrix} 1/\pi_1 & 0 & 0 \\ 0 & 1/\pi_2 & 0 \\ 0 & 0 & 1/\pi_3 \end{bmatrix} A^H = \begin{bmatrix} a_{11}^H/\pi_1 & a_{12}^H/\pi_1 & a_{13}^H/\pi_1 \\ a_{21}^H/\pi_2 & a_{22}^H/\pi_2 & a_{23}^H/\pi_2 \\ a_{31}^H/\pi_3 & a_{32}^H/\pi_3 & a_{33}^H/\pi_3 \end{bmatrix} \tag{4-2}$$

For the 3 x 3 x 2 model, the cone of commodity prices is on a tetrahedron shape. The cone of diversification of factor endowments also is a shape of the tetrahedron. Figure 3 shows the tetrahedron of commodity prices. A commodity price vectors lie in the tetrahedron will ensure positive factor prices.

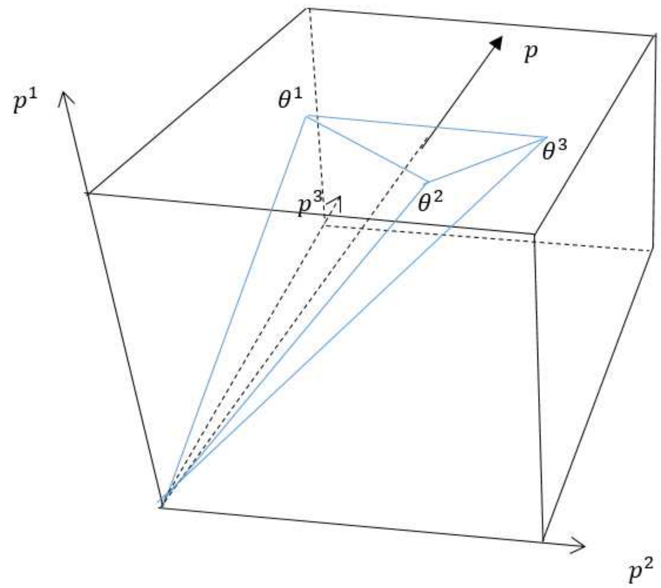

Figure 3 The Cone of Commodity Price by 3 Commodities and 3 Factors

We rewrite the unit cost function of the foreign country, $A^{F\prime}W = P$, as

$$\begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \end{bmatrix} w_1 \ /\pi_1 + \begin{bmatrix} a_{21} \\ a_{22} \\ a_{23} \end{bmatrix} w_2 \ /\pi_2 + \begin{bmatrix} a_{31} \\ a_{32} \\ a_{33} \end{bmatrix} w_3 \ /\pi_3 = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} \tag{4-3}$$

Each column of $A^{F\prime}(W)$ represnets the optimal unit coefficients from a single factor. Denote

$$\theta^{F1} = 1/\pi_1 \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \end{bmatrix}, \theta^{F2} = 1/\pi_2 \begin{bmatrix} a_{21} \\ a_{22} \\ a_{23} \end{bmatrix}, \ \theta^{F3} = 1/\pi_3 \begin{bmatrix} a_{31} \\ a_{32} \\ a_{33} \end{bmatrix} \tag{4-4}$$

Those three vectors are the three rays or ridges that compose the price tetrahedron in Figure 3.

For the home country, each column of $A^{H\prime}(W)$ for a single factor can be expressed respectively as

$$\theta^{H1} = \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \end{bmatrix}, \theta^{H2} = \begin{bmatrix} a_{21} \\ a_{22} \\ a_{23} \end{bmatrix}, \ \theta^{H3} = \begin{bmatrix} a_{31} \\ a_{32} \\ a_{33} \end{bmatrix} \tag{4-5}$$

The cone by (3-5) and the cone by (3-6) are the same since θ^{Fi} and θ^{Fi} are at the same line. Generally, the trade equilibrium of many commodity and many factors are available.

Conclusion

The approach for the equilibrium is the geometric method on the E-IWE diagram. It assumes that price is constant in the E-IWE. The geometric equilibrium solution returned to approve the assumption.

The solution is a Walrasian equilibrium. It reached budge balance: the total demand for each commodity just equals the aggregate endowment.

This is the first analytical try to reach the factor price non-equalization by world trade equilibrium. It attained for the 2x 2x2 Trefler model. The study explored a principle that world equivalent factor endowments determine world prices (one set of commodity price and two sets (or many sets) of localized factor prices) when countries have different productivities. It is a generalized Dixit-Norman price property.

The factor price equalization by same productivities is a special case of the factor price non-equalization by different productivities.

Appendix A

We express the gains from trade for country H as

$$-W^{Ha\prime}F^{H} > 0 \tag{A-1}$$

Adding trade balance condition $W^{*\prime}F^{H} = 0$ on (A-1) yields

$$-(W^{Ha\prime} - W^{*\prime})F^{H} > 0 \tag{A-2}$$

The factor content of trade of the home country in (A-2) is

$$F^{H} = \begin{bmatrix} \frac{1}{2}\frac{K^{H}L^{EW} - L^{EW}L^{H}}{K^{EW}} \\ -\frac{1}{2}\frac{K^{h}L^{w} - K^{w}L^{h}}{L^{EW}} \end{bmatrix} \tag{A-3}$$

The equilibrium factor price is

$$W^{*} = \begin{bmatrix} \frac{L^{EW}}{K^{EW}} \\ 1 \end{bmatrix} \tag{A-4}$$

The autarky price is

$$W^{Ha} = \begin{bmatrix} \frac{L^{H}}{K^{H}} \\ 1 \end{bmatrix} \tag{A-5}$$

Substituting (A-3) through (A-5) into (A-2) yields

$$-\begin{bmatrix} \frac{L^{H}}{K^{H}} - \frac{L^{EW}}{K^{EW}} & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2}\frac{K^{H}L^{EW} - L^{EW}L^{H}}{K^{EW}} \\ -\frac{1}{2}\frac{K^{h}L^{w} - K^{w}L^{h}}{L^{EW}} \end{bmatrix} > 0 \tag{A-6}$$

Reduced it to

$$-(\frac{L^H}{K^H}-\frac{L^{EW}}{K^{EW}})\times\frac{1}{2}\frac{K^H L^{EW}-L^{EW}L^H}{K^{EW}}>0 \tag{A-7}$$

Rewrite it as

$$-\left(\frac{L^H}{K^H}-\frac{L^{EW}}{K^{EW}}\right)\times\frac{1}{2}\frac{\frac{L^{EW}}{K^{EW}}-\frac{L^H}{K^H}}{K^{EW}}(K^{EW})K^H>0 \tag{A-8}$$

We have

$$\left(\frac{L^H}{K^H}-\frac{L^{EW}}{K^{EW}}\right)^2\times\frac{1}{2}K^H>0 \tag{A-9}$$

It is true. Therefore, equation (A-1) is valid.

We can prove the gains from trade for the foreign country as $-W^{Fa\prime}F^F>0$ by the similarly way. We will not repeat it.

Appendix B A numerical example for the Trefler Model

Consider two countries, home and foreign, two commodities, 1 and 2, two factors, capital, and labor. The technological matrix for the home country is

$$A^H=\begin{bmatrix}a_{K1} & a_{K2}\\ a_{L1} & a_{L2}\end{bmatrix}=\begin{bmatrix}3.0 & 1.1\\ 1.5 & 1.4\end{bmatrix}$$

The technological matrix for the foreign country is

$$A^F=\begin{bmatrix}1.0/0.9 & 0\\ 0 & 1.0/0.7\end{bmatrix}A^H=\begin{bmatrix}3.3333 & 1.1111\\ 2.1428 & 2.0\end{bmatrix}$$

The factor endowments for the two countries are

$$\begin{bmatrix}K^H\\ L^H\end{bmatrix}=\begin{bmatrix}5500.0\\ 3280.0\end{bmatrix},\qquad \begin{bmatrix}K^F\\ L^F\end{bmatrix}=\begin{bmatrix}2444.4\\ 2857.1\end{bmatrix}$$

The outputs of the two countries by full employment of factor endowments are

$$\begin{bmatrix}x_1^H\\ x_2^H\end{bmatrix}=\begin{bmatrix}1600.\\ 700\end{bmatrix},\qquad \begin{bmatrix}x_1^F\\ x_2^F\end{bmatrix}=\begin{bmatrix}400\\ 1000\end{bmatrix}$$

The share of GNP of the home country is calculated as 0.6712. The trade and prices, under free trade, reach the following equilibrium:

$$\begin{bmatrix}T_1^H\\ T_2^H\end{bmatrix}=\begin{bmatrix}257.46\\ -441.15\end{bmatrix},\qquad \begin{bmatrix}T_1^F\\ T_2^F\end{bmatrix}=\begin{bmatrix}-257.46\\ 441.15\end{bmatrix}$$

$$\begin{bmatrix}F_K^H\\ F_L^H\end{bmatrix}=\begin{bmatrix}331.22\\ -231.42\end{bmatrix},\qquad \begin{bmatrix}F_K^F\\ F_L^F\end{bmatrix}=\begin{bmatrix}-368.03\\ 330.61\end{bmatrix}$$

$$\begin{bmatrix} p_1^* \\ p_2^* \end{bmatrix} = \begin{bmatrix} 3.5961 \\ 2.0987 \end{bmatrix}, \qquad \begin{bmatrix} r^H \\ w^H \end{bmatrix} = \begin{bmatrix} 0.6987 \\ 1.000 \end{bmatrix}, \qquad \begin{bmatrix} r^F \\ w^F \end{bmatrix} = \begin{bmatrix} 0.6288 \\ 0.7 \end{bmatrix}$$

The localized factor prices are under the following relations

$$r^F = 0.9r^H$$

$$w^F = 0.7w^H$$

The autarky prices of two countries are estimated as

$$\begin{bmatrix} p_1^{Ha} \\ p_2^{Ha} \end{bmatrix} = \begin{bmatrix} 3.3436 \\ 2.0145 \end{bmatrix}, \quad \begin{bmatrix} r^{Ha} \\ w^{Ha} \end{bmatrix} = \begin{bmatrix} 0.6145 \\ 1.0000 \end{bmatrix}$$

$$\begin{bmatrix} p_1^{Fa} \\ p_2^{Fa} \end{bmatrix} = \begin{bmatrix} 6.0389 \\ 3.2987 \end{bmatrix}, \quad \begin{bmatrix} r^{Fa} \\ w^{Fa} \end{bmatrix} = \begin{bmatrix} 1.1688 \\ 1.0000 \end{bmatrix}$$

The gains from trade for the two countries by commodity price are

$$-\begin{bmatrix} p_1^{Ha} & p_2^{Ha} \end{bmatrix} \begin{bmatrix} T_1^H \\ T_2^H \end{bmatrix} = 27.87$$

$$-\begin{bmatrix} p_1^{Fa} & p_2^{Fa} \end{bmatrix} \begin{bmatrix} T_1^F \\ T_2^F \end{bmatrix} = 99.52$$

The gains from trade for the two countries by localized factor prices are

$$-\begin{bmatrix} r^{Ha} & w^{Ha} \end{bmatrix} \begin{bmatrix} F_K^H \\ F_L^H \end{bmatrix} = 27.87$$

$$-\begin{bmatrix} r^{Fa} & w^{Fa} \end{bmatrix} \begin{bmatrix} F_K^F \\ F_L^F \end{bmatrix} = 99.55$$

We add a negative sign since we use trade volume as exports. The example illustrates that a country even with absolute disadvantages in production still has comparative advantages in production by the difference of factor endowments. Both countries gain from trade.

The factor price restriction on the factor content of trade by Helpman (1984) are

$$\left(w^j - w^i\right)' F^{ij} = \left(\begin{bmatrix} r^F \\ w^F \end{bmatrix} - \begin{bmatrix} r^H \\ w^H \end{bmatrix}\right)' \begin{bmatrix} F_K^H \\ F_L^H \end{bmatrix} = 22.05 > 0$$

$$\left(\begin{bmatrix} r^H \\ w^H \end{bmatrix} - \begin{bmatrix} r^F \\ w^F \end{bmatrix}\right)' \begin{bmatrix} F_K^F \\ F_L^F \end{bmatrix} = 30.00 > 0$$

The world volume of factor contents of trade is

$$\frac{4w^{H*}\left(L^H - sL^{EW}\right)}{w^{H*}L^{EW} + r^* K^{EW}} = 0.0146$$

The size of the trade box is

$$\frac{K^H}{K^{EW}} - \frac{L^H}{L^{EW}} = 0.6428 - 0.6282 = 0.0146$$

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