A Pareto Criterion on Systemic Risk

Weijia Wang and Shaoan Huang

Central University of Finance and Economics, The Center for Economic Research, Shandong University

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Abstract

Perfect risk sharing is not an optimal design for the financial system because it can increase systemic risk by facilitating risk contagion among financial institutions. However, risk sharing dominates betting according to most Pareto efficiency criteria. One reason for this might be that those Pareto criteria consider individual risk rather than systemic risk and neglect that betting may reduce systemic risk by segmenting the financial system and preventing financial contagion. Refining Pareto criterion to cover systemic risk, I propose the systemic Pareto criterion which has two features: 1) satisfying facts that betting dominates risk sharing when systemic risk is considered. 2) being applicable to scenarios with the constant aggregate endowment to which current criteria cannot provide compelling suggestions. One implication from this paper is that betting can act as the stabilizer of the economy and prohibiting betting is not always helpful for financial stability.

Keywords: Risk Sharing, Heterogenous Beliefs, Pareto Efficiency, Systemic Risk

JEL: D61, D62, G18

139th Xueyuan South Road, Central University of Finance and Economics, Beijing, China. Email: wangwj06@gmail.com
1. Introduction

The view that perfect risk sharing is the optimal allocation has been widely held by economists for a long time. However, since the 2007/08 U.S. financial crisis, a growing body of literature has pointed out that perfect risk sharing can undermine the financial stability by allowing risk propagating among financial network and increasing systemic risk. For example, Stiglitz (2010a,b) questions if full integration or full risk sharing is always desirable for the financial system and concludes that risk sharing is not the optimal financial architecture due to the high probability of system-wide failure. Ace-moglu et al. (2015) argues that whether a more complete financial network is better or not depends on the magnitude and the number of negative shocks. The similar view is also held by Dasgupta (2004), Allen et al. (2012), Battiston et al. (2012) and Castiglionesi et al. (2017).

Traditionally, risking sharing is encouraged because it can diversify individual risk. However, the incentive of diversification also makes the financial institutions more interconnected or homogenous and increases the systemic risk. This tension between the two roles is also discussed by Beale et al. (2011) and Haldane and May (2011).

Another reason that risk sharing is welcomed may come from the equivalence of Pareto optima and risk sharing, which is discussed by Back (2010). If the agents are risk averse and homogenous in beliefs about the uncertainty, risk sharing naturally arises from the Pareto optimal allocation. Although the standard Pareto optimality criterion faces difficulties under heterogeneous belief conditions and has been modified to various forms, the risk sharing allocation is still optimal according to those criteria. This is the
problem I attempt to solve, thus I propose a Pareto criterion on systemic risk under which risk sharing can be dominated by betting. The intuition behind the ranking is that risk sharing increases systemic risk while betting reduces it.

In addition to the ”risk sharing always wins” issue, another trouble that standard Pareto criterion encounters is mentioned by Hammond (1981) and Nielsen (2003, 2018). Since the ex-ante Pareto efficient allocation may not be ex-post optimal due to the overconfidence or information misperception of agents, those studies suggest that substituting the standard (ex-ante) Pareto criterion with ex-post welfare optimality. However, as Mongin (2016) criticizes, the distinctions between ex-ante and ex-post Pareto optimality are ”between differing and identical subjective probabilities rather than between two temporal stages of analysis” and ”the ex-post Pareto principle is intended to serve at the ex-ante stage”. In this paper, I have no intention of going further into those arguments but respect the traditions that making the welfare comparison at the ex-ante stage.

From ex-ante perspective, Gilboa et al. (2014) and Brunnermeier et al. (2014) refine the standard Pareto criterion and propose their versions of Pareto efficiency— no-betting Pareto and belief-neutral Pareto criteria. They both favor risk sharing which is more efficient in their views and disapprove betting which could cause ex-post inefficiency or negative externality. Their conclusions contradict with mine because they do not take systemic risk into account. For example, the derivatives backed by the housing values were shared by the markets in the United States in 2007 and most people agreed on that housing prices will rise in the future (see Foote et al. (2012)).
risk-sharing allocation based on similar beliefs seems fine from the above criteria, but in fact, it contained great systemic risk and betting on housing prices could reduce this risk.

In addition, I follow this strand of research and assume that the social planner does not know the objective (true) probability and take no stand when the agents hold heterogenous beliefs. The reasons why heterogenous beliefs arise are various, such as irrationality of agents, different background or experience and distortions in updating (see Brunnermeier et al. (2014)). Whatever the reasons may be, the planner cannot use a ”presentative” belief to make welfare comparisons. This plausible assumption makes our analysis necessary, or else the planner is able to use the only correct probability to measure social welfare. Since my work is closely related to theirs, I will leave the further discussion for Section 4.

Another strand of literature related to mine is the utilitarian aggregation of individual preference. Harsanyi (1955) shows that a standard Pareto condition holds if and only if the utility function of society can be presented by a convex average of individuals’ utility function when they hold homogenous beliefs about uncertainty. Gilboa et al. (2004) extends this axiom under Savage (1972)’s framework where all agents are Subjective Expected Utility (SEU) maximizer but restrict the Pareto condition to the alternatives on which agents hold identical beliefs. Furthermore, Alon and Gayer (2016) and Danan et al. (2016) also aggregate preferences of individuals under various restricted Pareto conditions which, however, cannot accommodate the heterogeneity in belief and utility simultaneously.

In this paper, I follow the tradition of Gilboa et al. (2014) and focus
on decentralized decisions making under heterogenous beliefs without the intention of providing complete social ranking by aggregating preferences of individuals.

2. Example

In this section, I will give a dilemma that cannot be perfectly solved by recent Pareto criteria proposed by Brunnermeier et al. (2014) and Gilboa et al. (2014). And then I will show how the systemic Pareto criterion I propose solves it.

*The Kings dilemma.*

Two kingdoms stand next to each other. A bad king rules one of them and has a daughter. To assure the princess will marry to the strongest man, he announces that he will marry his daughter to the winner of the contest. A good king rules another kingdom and he has two sons, Aaron and Ben. They can choose to attend the contest or not and then there would be three potential outcomes for each of them: (no attendance, attendance and success, attendance and defeat). Suppose for each prince, the payoffs over the outcomes are the same: (0, 1, −1). If Aaron and Ben are very confident (but no one knows the exact probability of the outcomes) to triumph in the contest, then they would like to try their luck provided the expected payoffs are high enough. Assume the good king is Paretian and should he let his sons attend the contest?

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2This example is adapted from Nielsen (2018).
If he decides to respect the will of Aaron and Ben and let them go (to bet), he will have to accept the "one win and one lose" consequence which violates the Pareto principle at the ex-post stage. This scenario named "spurious unanimity" has been studied by [Gilboa et al. (2014)](https://doi.org/10.1093/certa/mct001) and [Mongin (2016)](https://doi.org/10.1093/certa/mct001). Otherwise, he will hurt the feelings of his sons and reject the potential Pareto improvement in the ex-ante stage.

No-betting and belief-neutral Pareto criteria are two recent accomplishments in Pareto efficiency refinement under heterogenous beliefs. Next, I will show that they cannot provide compelling answers for the good king.

### 2.1. No-betting Pareto criterion

For two allocations $f$ and $g$, no-betting Pareto reckons that $f$ dominates $g$ if 1) all agents involved prefer $f$ to $g$ under their subjective beliefs and 2) there exists a hypothetical probability which can rationalize the preference.

Suppose the princes are risk neutral and maximizing expected utilities with respect to their subjective probabilities and a hypothetical probability over (Aaron wins, Ben wins) is $(p, 1 - p)$. It is easy to show that no such hypothetical belief can rationalize their options of attending the contest. Because Aaron’s decision to go requires that $p \times 1 - (1 - p) \times 1 > 0$ while Ben’s decision to go requires that $(1 - p) \times 1 - p \times 1 > 0$. No-betting Pareto criterion excludes the option of letting them go owing to no common beliefs shared by the princes and suggests that the good king should not support their decisions of betting.

The underlying assumption for this suggestion is that ex-post efficiency is more important than ex-ante efficiency. However, compared with the cost of neglecting ex-ante efficiency and ensuring ex-post efficiency (which is hard
to measure), it might be less costly to ensure ex-ante efficiency and remedy the ex-post inefficiency by proper transfer plans (i.e., the good king ask the winning son give his fortune to the losing son). In such sense, no-betting Pareto criterion does not offer a compelling suggestion for the good king.

2.2. Belief neutral Pareto criterion

For two allocations $f$ and $g$, belief neutral Pareto views that $f$ dominates $g$ if for every belief $p$ in the convex hull of beliefs of all agents, every agent weakly prefers $f$ to $g$ and at least one agent strictly prefers $f$ to $g$.

Assume that Aaron and Ben are both 90% certain of winning in the contest and $(\lambda, 1 - \lambda)$, where $0 < \lambda < 1$, are weights of beliefs assigned to Aaron and Ben. Then the linear combination of their beliefs over (Aaron wins, Ben wins) is $(0.8\lambda + 0.1, 0.9 - 0.8\lambda)$. For every belief in the belief set, the expected utilities of betting for Aaron and Ben are $(0.8\lambda + 0.1) * 1 - (0.9 - 0.8\lambda) = 1.6\lambda - 0.8$ and $(0.9 - 0.8\lambda) * 1 - (0.8\lambda + 0.1) = 0.8 - 1.6\lambda$, respectively. Apparently, they cannot be positive or negative at the same time, which means that the allocation of betting is neither dominating nor being dominated by the allocation of no betting. Therefore, belief-neutral Pareto criterion cannot offer advice for the good king.

2.3. Systemic Pareto criterion

Here I will show how the systemic Pareto criterion solves this dilemma by adding extra disturbance. The King’s dilemma may remind us of game theory where multiple Nash equilibriums arise and we only need the most

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3 This is the incompleteness of belief-neutral Pareto criterion and more details can be found in [Brunnermeier et al. (2014)]
reasonable one—the perfect equilibrium. Next, I apply the spirit of trembling hand perfect equilibrium\(^4\) to solve the dilemma. Assume the irritable bad king may lose his temper with a small probability \(\varepsilon > 0\). When he gets angry, he could aggress upon the good king if none of his sons attend the contest or break his promise even if the princes complete the contest, with the results that Aaron and Ben both receive the worst payoff \(-c \leq -1\). Now the set of potential outcomes is (no attendance and no invasion, no attendance and invasion, attendance and Aaron wins, attendance and Ben wins, attendance and no one wins). And the corresponding payoffs for Aaron and Ben are \((0, -c, 1, -1, -c)\) and \((0, -c, -1, 1, -c)\), respectively. The expected payoff of not going for them is

\[
(1 - \varepsilon) \times 0 + \varepsilon \times (-c) = -c \times \varepsilon
\]

According to the persistent belief (see, for example, Battigalli and Bonanno (1997) and Ko and Huang (2012)), people are reluctant to adjust their beliefs when new information is unfavorable to their priors. Back to this example, the possibility of the bad king going mad changes from 0 to \(\varepsilon\) and this shock should have a negligible effect on the beliefs of winning for Aaron and Ben (since they are quite sure of their victory). Thus Aaron’s belief on (attendance and Aaron wins, attendance and Ben wins, attendance and no one wins) would be \((0.9, 0.1 - \varepsilon, \varepsilon)\) and the same adjustment is for Ben. The expected payoff of betting for them is

\[
0.9 \times 1 - (0.1 - \varepsilon) - \varepsilon \times c = 0.8 + \varepsilon - c \times \varepsilon
\]

\(^4\)Since the bad king is not a player, this is not the exact trembling hand refinement in game theory.
Table 1: Aaron’s (Ben’s) strategy

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Expected payoffs</th>
<th>Loss caused by shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beting</td>
<td>$0.8 + \varepsilon - c \varepsilon$</td>
<td>$c \varepsilon - \varepsilon$</td>
</tr>
<tr>
<td>No betting</td>
<td>$-c \varepsilon$</td>
<td>$c \varepsilon$</td>
</tr>
<tr>
<td>Comparison</td>
<td>$0.8 + \varepsilon - c \varepsilon &gt; -c \varepsilon$</td>
<td>$c \varepsilon - \varepsilon &lt; c \varepsilon$</td>
</tr>
</tbody>
</table>

Compared with the original story, the chance of the bad king going mad reduces the payoffs of Aaron and Ben regardless of their choices, but the extents to which the payoffs are reduced are different. If the good king prevents his sons from attending the contest, the payoffs reduced for each prince is $0 + c \varepsilon = c \varepsilon$. If not, the payoffs reduced for each prince is $0.8 - (0.8 + \varepsilon - c \varepsilon) = c \varepsilon - \varepsilon$. Clearly, payoffs decrease greater if the princes do not bet and the difference of reduced payoffs can help the good king make the decision. The analysis is summarized in Table 1.

In addition to the welfare of his sons, the good king also cares about the changes caused by the "trembling hands". Intuitively, the society wants the potential negative shock has the small, rather than large, effect on people’s expectation. For example, although unpredictable earthquake or disease outbreaks may happen with a very small probability tomorrow, it should be better for everyone to neglect it and live normally. In the same sense, the good king should allow his sons to attend the contest because they would suffer less expected utility loss in that situation.

This is how I incorporate systemic risk into the Pareto principle. From utilitarian perspectives, the social planner only cares about the level of utility of the agents. However, the variance of utilities of the agents also matters
when the "King’s dilemma" arises. The variance represents the risk of the whole system because the smaller the variances are for every agent, the more resilient the system is.

Here I interpret this example: Aaron and Ben represent two parties who want to bet; The good king is the social planner or government who cares about the welfare of each party and the systemic risk; The bad king signifies the economy or nature. Nature is hardly predictable so that we can never reject the possibility that a disaster happens and we get the worst results (i.e., Aaron and Ben get $-c$ when the bad king goes mad). The negligible possibility serves as the trembling hands in Nash equilibriums and helps the planner find the efficient allocation for each one without comparing the ex-ante and ex-post efficiencies. Betting dominates risk sharing because betting reduces systemic risk, which can be seen from the example—Aaron and Ben are less affected by the idea of the crazy king when they bet.

3. Model

After the informal illustration, I will present the systemic Pareto criterion formally in this section.

3.1. Setup

The environment setup in this paper is similar to that of Gilboa et al. (2014). There is a measurable state space $(S, \Sigma)$ and an outcomes set $X$. The mappings from $S$ to $X$ are contained in an acts set $\mathcal{F}$. The set $N = \{1, 2, ..., n\}$ contains all the agents in the society and each agent $i \in N$ has a preference relation $\succsim_i$ over $\mathcal{F}$. The preference $\succsim_i$ is characterized by maximizing $\int_S u_i(f(s))dp_i$, where $u_i$ is a utility functions mapping $X$ to $\mathbb{R}$.
and $p_i$ is a probability measure on $(S, \Sigma)$. In addition, there is no aggregate uncertainty in society, which means the total wealth of society is constant until the "trembling hands" reduces the total wealth.

### 3.2. Pareto criterion

Now I will give the formal definition that appeared in the previous example. First, I define belief persistence as follows:

**Definition 1.** An agent is belief persistent if the probability assigned to a new state rise from 0 to $\varepsilon > 0$ and the probability of the favored state which he thought was most likely to happen would not change.

For example, in King’s Dilemma, Aaron and Ben are belief persistent because the probability of bad king going crazy increases from 0 to $\varepsilon$ but they still believe that they will win with the probability 0.9. In other words, the king going crazy and losing the contest seem the same for them so those two events share the probability assigned previously to losing the contest. That agents are belief persistent is a strong assumption but can be relaxed by allowing the shock has a larger effect on the probability of unfavored states. What follows is the definition of another essential concept—systemic risk.

**Definition 2.** Given all agents are belief persistent, for two allocations $f$ and $g$, $f$ involves less systemic risk than $g$ if the inclusion of a new state, the probability of which increase from 0 to $\varepsilon > 0$, reduces less expected utility loss of each agent in $f$ than in $g$. That is, $\forall i$,

$$\Delta U^f_i \leq \Delta U^g_i$$
with the inequality holding strictly for at least one agent, where $\Delta U_i^a, a \in (f, g)$ is the reduction of expected utility of agent $i$ in allocation $f$ or $g$.

Now the systemic Pareto criterion can be defined as follows:

**Definition 3.** Given the agents are belief persistent, for two alternatives $f$ and $g$, $f$ is more systemic Pareto (SP) efficient than $g$ if:

1. every agent in the society prefers $f$ than $g$, that is $\forall i$,

\[
\int_S u_i(f)dp_i \geq \int_S u_i(g)dp_i,
\]

with the inequality holding strictly for at least one agent.

2. $f$ involves less systemic risk than $g$.

Condition (1) guarantees the systemic Pareto criterion is a refinement of the standard Pareto criterion and Condition (2) gives the reason why an allocation dominates another one when they are both standard Pareto efficient.

**Proposition 1.** Given the agents are belief persistent and the aggregate endowment is constant, betting is more SP efficient than risk sharing.

**Proof.** For any allocation resulting from voluntary trade, condition (1) is always satisfied and thus not crucial for justifying that betting is more efficient. Betting dominates risk sharing because betting is less systemic risky and next I will prove it.

Consider an allocation of betting $f$. For an agent, the subset $S_{max}$ contains all the states he think is most likely to happen, that is $S_{max} = \arg\max_{s \in S} p_i(s)$. And $S_{max}^c$ contains all the other states which are not included
in $S_{max}$. The expected utility of him can be expressed as $\int_{S_{max}} u_i(f)dp_i + \int_{S_{cmax}} u_i(f)dp_i$. And for $s_1 \in S_{max}$ and $s_2 \in S_{cmax}^c$, $u_i(f(s_1)) \geq u_i(f(s_2))$ holds for betting. Suppose the probability of a state $s_0$ increases from 0 to $\varepsilon$ and $u_i(f(s_0)) \leq u_i(f(s)), \forall s \neq s_0$. Now the expected utility is $\int_{S_{max}} u_i(f)dp_i + \int_{S_{cmax}} u_i(f)dp_i + p_i(s_0) * u_i(f(s_0))$. Because the agents are belief persistent, $\int_{S_{max}} u_i(f)dp_i$ is not affected but $\int_{S_{cmax}} u_i(f)dp_i$ is changed to $\int_{S_{cmax}} u_i(f)dp_i$ due to the inclusion of the new state. Hence, the reduction of expected utility is $\int_{S_{max}} u_i(f)dp_i - \int_{S_{cmax}} u_i(f)dp_i - p_i(s_0) * u_i(f(s_0)) = \varepsilon * u_i(f(s_2^*)) - \varepsilon * u_i(f(s_0))$ for some $s_2^* \in S_{max}$.

In an allocation of risk sharing $g$, the constant endowment guarantees that $\forall s_3 \in S$, $u_i(f(s_1)) \geq u_i(g(s_3)) \geq u_i(f(s_2))$ with inequalities holding strictly for some $i$. And his expected utility is $\int_S u_i(g)dp_i$ and reduction of expected utility is $\int_S u_i(g)dp_i - \int_{S_{max}} u_i(g)dp_i - p_i(s_0) * u_i(g(s_0)) = \varepsilon * u_i(g(s_3^*)) - \varepsilon * u_i(f(s_0))$ for some $s_3^* \in S$. With $u_i(f(s_0)) = u_i(g(s_0))$ and $u_i(f(s_2^*)) \leq u_i(g(s_3^*))$, it is obvious that $\varepsilon * u_i(f(s_2^*)) - \varepsilon * u_i(f(s_0)) \leq \varepsilon * u_i(g(s_3^*)) - \varepsilon * u_i(f(s_0))$ with inequality holding strictly for some $i$. This implies that $f$ is less systemic risky than $g$ by definition.

4. Discussion

In this section, I will compare the systemic Pareto criterion with no betting and belief neutral Pareto criteria. Here I offer three angles to distinguish mine from those two criteria.

4.1. Constant endowment

One seemingly strong assumption applied in this paper is that no aggregate uncertainty in the model. Specifically, the total wealth of society is
constant before including a new state.

This is contrary to the setting in \textcite{Brunnermeier et al. 2014} who assigns a positive or negative externality to trade before evaluating its efficiency. Belief-neutral Pareto criterion works when the externality is uniformly positive or negative under every reasonable belief in the combinations of beliefs of all agents. For example, given negative externality caused by betting reducing social wealth, belief-neutral Pareto criterion is able to identify the negative externality and claims the betting is inefficient.

However, its ability is compromised when the negative externality presumption is abandoned. For instance, in the example \textit{Speculative Motive} of \textcite{Brunnermeier et al. 2014}, authors assign no clear negative externality to betting but claim that betting can cause agents to take excessive risk and impose negative effects on society. This is questionable because a riskier individual allocation may involve less systemic risk and does not necessarily mean that it is inefficient for society. Overall, the rationale behind belief-neutral Pareto criterion depends on the presumption of externality.

I am not denying that in some cases we can identify its externality of a trade before it is completed, but this may not be true for most cases. Usually, we are uncertain of the results caused by a deal and cannot know its externality in advance. Therefore, I assume there is no aggregate uncertainty in the model and this helps us to assess social welfare in those cases.

4.2. Parentalism

The belief-neutral Pareto criterion is paternalistic because it ignores individual preferences when evaluating social welfare. One trouble caused by Parentalism is that an inefficient allocation $f$ can be undominated by any
other allocation $g$. Consider an economy with two agents—Aaron and Ben, using Aaron’s belief, Aaron is indifferent between $f$ and $g$ while Ben prefers $g$ to $f$. In addition, using Ben’s belief, Ben is indifferent between $f$ and $g$, but Aaron also thinks that $g$ is better than $f$. As a result, each one is indifferent between $f$ and $g$ using his own belief but the society reckons $f$ as inefficient.

To the contrary, the systemic Pareto criterion does not require planner consider preferences generated by maximizing ones’ expected utilities using other’s beliefs. The condition (1) in the Proposition ensures that the systemic Pareto criterion coincides with the notion of standard Pareto principle.

4.3. Robustness

No betting Pareto criterion is not robust to the inclusion of a new state. For example, $f$ did not dominate $g$ according to no betting Pareto criterion before a new state was included. However, if a new state which increases the wealth of every agent arises, then $f$ dominates $g$ after the inclusion of the new state.

The systemic Pareto criterion also considers the inclusion of a new state but it is different from the above case. The shock included in this paper which reduces the wealth of all agents and acts like a ”trembling hand”. Then if $f$ dominates $g$ according to standard Pareto and $f$ involves less systemic risk than $g$, we can conclude that $f$ is systemic Pareto efficient than $g$.

Furthermore, adding the model a new state which increases the wealth of every agent is beyond the scope of belief persistence. It is difficult to argue that such a state is against or for the priors. For example, if the bad king will give Aaron and Ben a castle when a new state realized. This is hard to say whether Aaron and Ben are less certain of their defeat or victory when
they take such a possibility into consideration.

5. Conclusion

This paper proposes a Pareto criterion refinement which takes the risk of the financial system into account. It shows that under certain assumptions, perfect risk sharing is not the optimal allocation and can be dominated by betting, which echoes with the research of Stiglitz (2010b), Beale et al. (2011) and Haldane and May (2011).

It is worthy of noting that I am not denying the positive effect of risk sharing in normal situations. When the global economy is stable, for example, more trade liberalization and financial integration can bring more benefit for participating countries. What I am suggesting is that when the economy is ambiguous and there is a small but positive probability that we all suffer, it is not reasonable to pursue perfect risk sharing and forbid betting. Furthermore, betting may reduce systemic risk and act as a stabilizer of the economy.

Reference


