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20 May 2019

Online at <https://mpra.ub.uni-muenchen.de/94050/>

MPRA Paper No. 94050, posted 21 May 2019 15:45 UTC

## Child Care Policy and Capital Mobility<sup>†</sup>

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### Abstract

Earlier reports have described effects of child care policy on fertility and education investment in an endogenous fertility model. Nevertheless, these studies examine closed economies in which capital accumulation is achieved by saving or small open economies in which capital accumulation is not considered. We can regard a capital mobility model as another model for which capital accumulation in one country affects capital accumulation in another country. Our paper presents consideration of capital mobility and examines how child care policy in one country affects another country. Results show that child allowances and education subsidies positively or negatively affect human capital accumulation in the foreign country even if fertility and human capital accumulation can be raised in the country in which child care policies are provided.

**JEL Classification:** J13

**Keywords:** Capital Mobility, Child Allowance, Education Subsidy, Endogenous Fertility, Human Capital

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<sup>†</sup> Research conducted for this paper was supported financially by JSPS KAKENHI No.17K03746 and No.17K03791. Nevertheless, any remaining error is the authors' responsibility.

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## 1. Introduction

Many studies have examined child care policy in an endogenous fertility model. Van Groezen, Leers and Meijdam (2003) derive that a child allowance can raise fertility in a small open economy. Zhang (1997) sets the model of quality and quantity of children with human capital and derives that a child allowance raises fertility and reduces human capital accumulation. However, the subsidy for education investment facilitates human capital accumulation and reduces fertility.

Effects of child care policy are examined in the model of a small open economy or a closed economy as described in reports of the related literature. In a small open economy, physical capital accumulation is not considered. The interest rate and wage rate are constant over time. However, in a closed economy, physical capital accumulation is considered. The interest rate and wage rate depend on the level of physical capital stock. If child care policies change the level of the physical capital stock, then this change affects fertility and human capital accumulation.

Van Groezen and Meijdam (2008), Fanti and Gori (2009), and others derive the effects of a child allowance on fertility in the closed economy model. Fanti and Gori (2009) show the negative effect of a child allowance on fertility attributable to a decrease in physical capital accumulation. Shintani and Yasuoka (2019) derives the effect of education subsidy on human capital accumulation in a closed economy as set by de la Croix and Doepke (2003). Compared with the case of a small open economy, in the case of closed economy, the positive effect of an education subsidy on human capital accumulation is weaker.

This paper sets a two-country model with capital mobility and without labor mobility. Because of capital mobility, if the level of the physical capital stock changes in one country, then that in the other country changes because of arbitrage transactions. This is not considered in a small open economy or a closed economy. The study described in this paper examines derivation of the effects of child care policy on fertility and human capital accumulation in a two-country model. Results derived from the analyses described herein are presented below. If a child allowance is provided in one country, then fertility rises and human capital accumulation is prevented in this country at that time. However, human capital accumulation is facilitated over time because of an increase in physical capital accumulation. In foreign countries, human capital accumulation is positively affected. If a subsidy for education investment is provided in one country, then fertility decreases in this country. Human capital accumulation is facilitated in this country at that time. However, because of the negative effects of

physical capital accumulation, the positive effect on human capital accumulation is weakened in this country over time. In the other country, human capital accumulation is prevented. Results are not obtained for a closed economy or for a small open economy.

The remainder of this paper is presented as follows. Section 2 sets the model. Section 3 derives the equilibrium. Section 4 presents examination of the effect of child allowance and subsidies for education investment in a two-country model. Final section concludes this paper.

## 2. Model

This model economy includes agents of three types: households, firms, and government. The individuals in households live in two periods: young and old. During the young period, they work inelastically to obtain labor income. Labor income is allocated to expenditures for child care, education investment for children, and savings for consumption during the old period. Then, the budget constraint in the young period can be shown as presented below.

$$(1 - \tau)w_t h_t = (z_t - q_t)n_t + (1 - x)e_t n_t + s_t \quad (1)$$

Therein,  $w_t$  and  $h_t$  respectively denote the wage rate per effective labor and human capital stock.  $z_t$  represents the child care cost per capita.  $n_t$  denotes the number of children (fertility).  $e_t$  stands for the education investment for a child.  $s_t$  represents the saving.  $q_t$  and  $x$  respectively signify the subsidy for the number of children and education investment. These subsidies are financed by the income tax. The tax rate is  $\tau$ .  $t$  represents the period.

During the old period, consumption is paid for by savings, as shown below.

$$(1 + r_{t+1})s_t = c_{t+1} \quad (2)$$

Substituting (2) into (1), the lifetime budget constraint is obtainable as

$$\frac{c_{t+1}}{1 + r_{t+1}} + (z_t - q_t)n_t + (1 - x)e_t n_t = (1 - \tau)w_t h_t. \quad (3)$$

The utility function is assumed as the following equation,<sup>1</sup>

$$u_t = \alpha \ln n_t h_{t+1} + (1 - \alpha) \ln c_{t+1}, 0 < \alpha < 1. \quad (4)$$

In that equation,  $h_{t+1}$  denotes the human capital in  $t + 1$  period. Also,  $h_{t+1}$  represents the human

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<sup>1</sup> Some utility forms of fertility and education exist. The utility function assumed for the analyses in this paper is the same as that of De la Croix and Doepke (2003). Zhang (1997) sets the dynasty model. The education investment raises the offspring utility.

capital of children.

Human capital accumulation is assumed as

$$h_{t+1} = e_t^\varepsilon h_t^{1-\varepsilon}, 0 < \varepsilon < 1. \quad (5)$$

Then, the optimal allocations to maximize (4) subject to (3) and (5) can be derived as follows.

$$c_{t+1} = (1 + r_{t+1})(1 - \tau)w_t h_t \quad (6)$$

$$e_t = \frac{\varepsilon(\bar{z} - \bar{q})w_t h_t}{(1 - \varepsilon)(1 - x)} \quad (7)$$

$$1 + g_t = \frac{h_{t+1}}{h_t} = \left( \frac{\varepsilon(z - q)w_t}{(1 - \varepsilon)(1 - x)} \right)^\varepsilon \quad (8)$$

$$n_t = \frac{\alpha(1 - \varepsilon)(1 - \tau)}{\bar{z} - \bar{q}} \quad (9)$$

We assume  $z_t = \bar{z}w_t h_t$  and  $q_t = \bar{q}w_t h_t$ , respectively.<sup>2</sup>  $g_t$  denotes human capital growth rate, that is, income growth rate.

In this model economy, the product function is assumed as

$$Y_t = K_t^\theta L_t^{1-\theta}, 0 < \theta < 1. \quad (10)$$

Therein,  $Y_t$ ,  $K_t$ , and  $L_t$  respectively denote the output, capital stock and effective labor. Effective labor is given as  $L_t = N_t h_t$ .  $N_t$  represents the population size of younger people. With a competitive market, the wage rate and interest rate are given as equal to marginal productivity.

$$w_t = (1 - \theta)k_t^\theta \quad (11)$$

$$1 + r_t = \theta k_t^{\theta-1} \quad (12)$$

The capital stock is assumed to be fully depreciated in one period.

Government subsidies are provided for the number of children and the education investment for children. With a balanced budget, the government budget constraint is given as shown below.

$$\tau w_t h_t = \bar{q} w_t h_t n_t + x e_t n_t \rightarrow \tau = \bar{q} n_t + x \frac{e_t n_t}{w_t h_t} \quad (13)$$

### 3. Equilibrium in an open economy

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<sup>2</sup> If the child care cost  $z_t$  is constant over time in the growth model, then the child care cost is small. The effect of cost vanishes. Therefore, we set the child care cost as the cost of the proportional wage rate. This child care cost, which is proportional to the wage rate, can be derived by considering the child care service sector as a two-sector model. However, for simplicity, we do not consider the child care service sector.

This model economy has two countries, A and B. In A country, child care policies are provided. However, B country has no child care policy. Between the two countries, capital mobility exists. However, labor supply can not move between the two countries. Defining  $y_t^i$  as the variable  $y_t$  in  $i$  country, the capital stock between the two countries is given as shown below.

$$r_t^A = r_t^B \rightarrow k_t^A = k_t^B = k_t \quad (14)$$

The saving at  $t$  period is presented below.

$$(1 - \alpha)((1 - \tau)N_t^A h_t^A + N_t^B h_t^B)(1 - \theta)k_t^\theta \quad (15)$$

Defining  $\mu$  as the share of the investment for capital stock between the two countries,  $k_t^i$  is shown as

$$k_{t+1}^A = \frac{K_{t+1}}{N_{t+1}^A h_{t+1}^A} = \frac{\mu(1 - \alpha)((1 - \tau)N_t^A h_t^A + N_t^B h_t^B)(1 - \theta)k_t^\theta}{n^A N_t^A (1 + g_t)^A h_t^A}, \quad (16)$$

$$k_{t+1}^B = \frac{K_{t+1}}{N_{t+1}^B h_{t+1}^B} = \frac{(1 - \mu)(1 - \alpha)((1 - \tau)N_t^A h_t^A + N_t^B h_t^B)(1 - \theta)k_t^\theta}{n^B N_t^B (1 + g_t)^B h_t^B}, \quad (17)$$

where

$$n^A = \frac{\alpha(1 - \varepsilon)(1 - \tau)}{\bar{z} - \bar{q}}, \quad (18)$$

$$n^B = \frac{\alpha(1 - \varepsilon)}{\bar{z}}, \quad (19)$$

$$(1 + g_t)^A = \left( \frac{\varepsilon(\bar{z} - \bar{q})w_t}{(1 - \varepsilon)(1 - x)} \right)^\varepsilon, \quad (20)$$

$$(1 + g_t)^B = \left( \frac{\varepsilon \bar{z} w_t}{1 - \varepsilon} \right)^\varepsilon. \quad (21)$$

We assume that  $h_t^A = h_t^B$  and  $N_t^A = N_t^B$  in  $t$  period.

Because of  $k_{t+1}^A = k_{t+1}^B$ ,  $\mu$  can be derived as shown below.

$$\mu = \frac{1}{\frac{n^B (1 + g_t)^B}{n^A (1 + g_t)^A} + 1} \quad (22)$$

Given  $k_t, h_t^A, h_t^B$ , we can obtain  $k_{t+1}^A = k_{t+1}^B = k_{t+1}$  from (14) and (16)–(22). Defining  $k_t^A = k_t^B = k$  as the physical capital stock in the steady state.  $k$  is given as (14) and (16)–(22).

#### 4. Policy Analysis

This paper presents an examination of how child care policies such as child allowances and education subsidies affect fertility and human capital accumulation in the two countries. Being different from a closed economy and a small open economy, this model economy demonstrates that the child care policies in one country affect the capital stock in another country. This paper presents consideration of country A as the country in which the child care policies are provided. Country B has no child care policies.

#### 4.1 Child Allowance

If the child allowance is provided in country A, then the government budget constraint (13) is shown as

$$\tau = \bar{q}n_A. \quad (23)$$

Substituting (23) into (18), one can obtain the fertility in country A as

$$n^A = \frac{\alpha(1-\varepsilon)}{\bar{z} - (1-\alpha(1-\varepsilon))\bar{q}}. \quad (24)$$

An increase in  $\bar{q}$  raises fertility in A country at  $t$  period. However, as shown by (20),  $(1+g_t)^A$  decreases at  $t$  period. These results are the same as those in the case of a small open economy, as derived by Zhang (1997) and others.

Before the effect in  $t+1$  period, we calculate  $\frac{dn^A(1+g_t)^A}{d\bar{q}}$  at the approximation of  $\bar{q} = 0$  as

$$\frac{dn^A(1+g_t)^A}{d\bar{q}} = -\frac{\alpha^2(1-\varepsilon)^2}{\bar{z}^2} \left( \frac{\varepsilon\bar{z}(1-\theta)k_t^\theta}{1-\varepsilon} \right)^\varepsilon < 0. \quad (25)$$

Therefore, we can obtain  $\frac{d\mu}{d\bar{q}} < 0$  because of (22). The share of total investment for capital stock in country A decreases.<sup>3</sup> Total differentiation of (16) with respect to  $k_{t+1}^A$ ,  $n^A$ ,  $(1+g_t)^A$ ,  $\mu$ ,  $\tau$ ,  $\bar{q}$  at the approximation of  $\bar{q}$ , one can obtain  $\frac{dk_{t+1}^A}{d\tau}$  as

$$\frac{dk_{t+1}^A}{d\bar{q}} = \frac{k_{t+1}^A}{\mu} \frac{d\mu}{d\bar{q}} - \frac{\mu(1-\alpha)(1-\theta)k_t^\theta}{n^A(1+g_t)^A} \frac{d\tau}{d\bar{q}} - \frac{k_{t+1}^A}{n^A(1+g_t)^A} \frac{dn^A(1+g_t)^A}{d\bar{q}}. \quad (26)$$

The sign of  $\frac{dk_{t+1}^A}{d\bar{q}}$  is ambiguous because  $\frac{k_{t+1}^A}{\mu} \frac{d\mu}{d\bar{q}} < 0$ ,  $\frac{\mu(1-\alpha)(1-\theta)k_t^\theta}{n^A(1+g_t)^A} \frac{d\tau}{d\bar{q}} > 0$  and  $\frac{k_{t+1}^A}{n^A(1+g_t)^A} \frac{dn^A(1+g_t)^A}{d\bar{q}} < 0$ . However, as shown by (17) and (22), we can obtain  $\frac{dk_{t+1}^B}{d\bar{q}} > 0$  if the

<sup>3</sup> Once the government provides a child allowance at country A, one can obtain  $k_{t+1}^A > k_{t+1}^B$  temporally. Then, the capital stock inflow occurs in country B. That is,  $\mu$  decreases.

negative effect of  $\tau$  on capital accumulation is small. Therefore, because of the arbitrage condition (14), it is consistent with  $\frac{dk_{t+1}^A}{d\bar{q}} > 0$ . As shown by (17), the income growth rate in country B at  $t + 1$  period  $(1 + g_t)^B$  increases. Because of  $\frac{dk_{t+1}^A}{d\bar{q}} > 0$ , the income growth rate in country A at  $t + 1$  period  $(1 + g_t)^A$  increases, too. Then, the following proposition can be established.

**Proposition 1**

Child allowance in country A raises the fertility and reduces income growth rate in country A. The income growth rate in country B increases in  $t + 1$  period if the negative effect of taxation on capital accumulation is small.

When considering a closed economy or a small open economy, then no effect occurs in country B. However, because of the capital stock flow, the child allowance can affect income growth in country B.

**4.2 Education Subsidy**

If the education subsidy is provided in country A, the government budget constraint (13) is

$$\tau = x \frac{e_t^A n^A}{w_t h_t^A} = \frac{x\alpha\varepsilon(1-\tau)}{1-x}. \quad (27)$$

In this case, the education subsidy reduces fertility and raises education investment in country A at  $t$  period.<sup>4</sup> These results are the same with the case of small open economy as derived by Zhang (1997) and others.

By substituting (27) into  $n^A(1 + g_t)^A$ , one can obtain the following equation:

$$n^A(1 + g_t)^A = \frac{\alpha(1-\varepsilon)(1-\alpha\varepsilon x)}{\bar{z}} \left( \frac{\varepsilon\bar{z}w_t}{(1-\varepsilon)(1-x)} \right)^\varepsilon. \quad (28)$$

Before the effect in  $t + 1$  period, we calculate  $\frac{dn^A(1+g_t)^A}{d\bar{q}}$  at the approximation of  $x = 0$  as

$$\frac{dn^A(1 + g_t)^A}{dx} = \varepsilon(1-\alpha)n^A(1 + g)^A > 0. \quad (29)$$

Therefore, we can obtain  $\frac{d\mu}{dx} > 0$  because of (22).  $k_{t+1}^A < k_{t+1}^B$  can be obtained temporally.

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<sup>4</sup> In the case of  $\bar{q} = 0, \tau > 0, x > 0$ , fertility and education investment are  $n = \frac{\alpha(1-\varepsilon)(1-\tau)}{\bar{z}}$  and  $e_t = \frac{\varepsilon\bar{z}w_t h_t}{(1-\varepsilon)(1-x)}$ .



Capital stock flow from country B to country A occurs. The share of total investment for capital stock in country A increases. However, the capital stock in  $t + 1$  period does not always increase. Total differentiation of (16) with respect to  $k_{t+1}^A$ ,  $n^A$ ,  $(1 + g_t)^A$ ,  $\mu$ ,  $\tau$ , and  $x$ , at the approximation of  $x = 0$ ,  $\frac{dk_{t+1}^A}{d\tau}$  can be obtained as

$$\frac{dk_{t+1}^A}{dx} = \frac{k_{t+1}^A}{\mu} \frac{d\mu}{dx} - \frac{\mu(1-\alpha)(1-\theta)k_t^\theta}{n^A(1+g_t)^A} \frac{d\tau}{dx} - \frac{k_{t+1}^A}{n^A(1+g_t)^A} \frac{dn^A(1+g_t)^A}{dx}. \quad (30)$$

The sign of  $\frac{dk_{t+1}^A}{dx}$  is ambiguous because  $\frac{k_{t+1}^A}{\mu} \frac{d\mu}{dx} > 0$ ,  $\frac{\mu(1-\alpha)(1-\theta)k_t^\theta}{n^A(1+g_t)^A} \frac{d\tau}{dx} > 0$ ,  $\frac{k_{t+1}^A}{n^A(1+g_t)^A} \frac{dn^A(1+g_t)^A}{dx} > 0$ . An increase in education subsidy reduces the capital stock per capita in  $t + 1$  period because of the tax burden and dilution effect, even if the investment for capital stock share  $\mu$  increases and generally  $\frac{dk_{t+1}^A}{dx} < 0$  is obtainable. Additionally,  $\frac{dk_{t+1}^B}{dx} < 0$  can be obtained because of (22). Then, considering (14), it is consistent with  $\frac{dk_{t+1}^A}{dx} < 0$ . Therefore, the income growth rate in A country at  $t + 1$  period  $(1 + g_t)^A$  decreases. As shown by (17), the income growth rate in country B at  $t + 1$  period  $(1 + g_t)^B$  decreases. Then, the following proposition can be established.

### Proposition 2

Education subsidies in country A reduces fertility in country A. The income growth rate in country A rises instantaneously. However, the income growth rate in country A decreases over time because of a decrease in physical capital stock. The income growth rate in country B decreases in  $t + 1$  period.

An education subsidy in country A reduces income growth in country B because of capital flow from country B to country A. This result is not derived at the closed economy model and small open economy model.

## 5. Conclusions

This paper presents an examination of how the child care policy affects fertility and human capital accumulation. The related literature includes examination of the child care policy effects in the closed economy or a small open economy. No effect of policy from one country affects another country in a small open economy or a closed economy. However, considering capital mobility, the setting presented by this paper should be examined. Results demonstrate that the child care policy in one country

negatively or positively affects human capital accumulation in the other country.

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