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Bank of Israel

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The views expressed in this paper are those of the authors only, and do not necessarily represent those of the Bank of Israel.

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Abstract

We formulate and estimate a small New Keynesian model for the Israeli economy. Our goal is to construct a small but still realistic model that can be used to support the inflation targeting process. The model contains three structural equations: An open economy Phillips curve for CPI inflation (excluding the housing component), an aggregate demand curve for the output gap and an interest parity condition for the nominal exchange rate. The model is closed with an interest rate reaction function (Taylor-type rule) and an ad hoc equation for the housing component of the CPI, which is dominated by exchange rate changes. In the specification of the model we had to pay special attention to the crucial role of the exchange rate in the transmission of monetary policy in Israel, which has a direct effect on almost 60 percent of the CPI. The model is estimated by the GMM method, using quarterly data for the period 1992:I to 2005:IV. In the estimation of the structural equations we tried to remain as close as possible to the theoretical formulation by restricting the dynamics to one lag at most. We use the model to characterize an "optimal" simple interest rate rule. We find that the monetary authority should respond to an hybrid backward-forward looking rate of inflation and does not benefit from direct reaction to exchange rate measures.

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The model of the monetary policy of Israel is presented in the context of the Keynesian model, with its implementation in the Israeli economy. The model, which we have developed, helps to design the economic policies of the central bank to control inflation and achieve the required interest rate. The model, between the banks and the changing inflation, is the core of the model. It uses the real change of the central bank's interest rate and the inflation rate. The Taylor-type rule (Taylor type rule) is used to determine the central bank's interest rate, which is influenced by the inflation rate and the real change of the central bank's interest rate. In this way, the model presents a detailed description of the economy, including production and consumption functions, and the relationship between inflation and interest rate. The model uses the first-order approximation of the central bank's policy, which is then tested against historical data. The model is tested using the GMM method, and the results are compared with the empirical and theoretical models.
1. Introduction

In this paper we formulate and estimate a small New Keynesian model for the Israeli economy. Our goal is to construct a small but realistic model that can be used for forecasting, policy analysis, risk assessment and supporting the inflation targeting process. The model contains three structural equations: A Phillips curve for consumer price index inflation (excluding the housing component), an aggregate demand curve for the output gap (in the business sector) and an interest parity condition for the nominal exchange rate. The model is closed with an estimated reaction function (Taylor-type rule) for the Bank of Israel's interest rate.

The model developed here is the smallest one which can be used to describe the main ingredient of the transmission mechanism in a small open economy. An important advantage of a small model is its simplicity and clarity, which help to enhance the communication among the various groups that are related to the monetary policy and its outcomes.

The theoretical framework and part of the empirical specification of the model is based on the works of Svensson (2000), Adolfson (2001), Linde et al. (2004) and Monacelli (2005). Our aim at this paper is to implement this framework in the context of the characteristics of the Israeli economy and to arrive at a workable model that could support the inflation targeting process.

The New Keynesian model in a closed economy contains (at least) three basic equations: one for inflation, one for the output gap and one for the nominal interest rate.1 In a small open economy the exchange rate has an important role in the transmission mechanism of monetary policy. As described in Svensson (2000), the exchange rate affects both the aggregate demand (output gap equation) and supply (inflation equation) sides. On the demand side the exchange rate affects the relative price of both imports and exports of goods and services. On the supply side it affects consumer price directly, through the price of imported consumer goods which are part of the consumption basket, and indirectly through the price of imported raw materials.

In the Israeli economy there is another important and unique channel through which the exchange rate affect prices – by affecting the housing component of the consumer price index (CPI). In the next section we shall describe this unique relation. However, we note that this component constitutes about 20 percent of the CPI and

that the pass-through from the exchange rate to this non-traded component is almost immediate and complete.

In the analysis of inflation in an open economy one has to distinguish between the prices of locally produced goods, which are mainly affected by aggregate demand, and the prices of imported goods which are influenced directly by the exchange rate. If one assumes a gradual pass-through from the exchange rate to import prices, then one has to specify and estimate separate equations for the locally produced goods (home goods) and for the imported goods (see, for example, Monacelli, 2005). In most, if not all, empirical works, the New Keynesian Phillips Curve is estimated in terms of the GDP deflator (for the home goods inflation equation) and the national account's total import price deflator (for the imported goods inflation equation). In this paper we concentrate on specifying and estimating an inflation equation in terms of the CPI. The difficulty that one has to face is that the two components of the CPI (i.e., locally produced goods and imported goods inflation) are unobservable. We try to overcome this difficulty by specifying the imported goods inflation equation as a distributed lag on the world import price inflation adjusted to exchange rate changes, and augmenting this equation in the CPI inflation Phillips curve. Outcomes of such an exercise are estimates of the price inflation of the two components, which could serve as indicators in the analysis of the CPI inflation developments.

Another issue to which we pay attention in the specification and estimation of the inflation equation is the distinction between the price of imported raw material (which affects production costs) and the price of imported consumption goods (which affects aggregate demand and consumption costs). This distinction, which is usually neglected in most of the empirical works (Battini et al., 2005, is an exception), enables us to assess the effect of a shock to the world's relative price of raw material.

The main equations of the model are based on micro structure. As is well known such an emphasis strengthens the theoretical consistency of the model but it may weaken empirical aspects. In the estimation stage we tried to keep the

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2 For examples, see Litemo (2006a), Batini et al. (2005)
3 There are several reasons that a CPI-oriented model will be more appropriate to the support of inflation targeting. At first the inflation target is in terms of CPI. Moreover the CPI is published with a relatively short lag and is not revised.
4 Such as a rise in the price of oil, which is an important (imported) raw material for the Israeli economy.
5 As will be detailed below, the output gap and inflation of locally produced goods are derived with DSGE formulation. The specification of the interest rate rule, the imported goods inflation equation and the housing price inflation equation are ad hoc.
specification of the estimated equations as close as possible to the DSGE formulation, by starting with the DSGE specification and then allowing some additional dynamics, whenever this seemed necessary, but restricting it to only one lag of the various variables, which was found to strengthen the robustness of the estimated equations.

In the next section we present a short background of the economy. In Section 3 we develop the specification of the model's equations. In Section 4 we present and discuss the estimation results. In Section 5 we describe the choice of the monetary policy rule – based on simple optimization methods. Section 6 describes and discuss the characteristics of the model and Section 7 offers conclusions.

2. A short background to the economy

During the estimation period (1992 to 2005) the Israeli economy's real and nominal sides experienced several major changes. A short description of those developments will highlight several aspects with regard to the concrete implementation of the theoretical framework. Specifically, we refer to the crucial role of the exchange rate in the transmission process and the specific breakdown of the CPI inflation.

On the real side we can name the large immigration from the former Soviet Union that resulted in an average yearly population growth of 3.6 percent during the years 1990 to 1999. Another important development was the prolonged structural change in the industrial sector: the high-tech industry grew rapidly while traditional industries were stagnating.

An important development in the nominal side was the declaration of inflation targets in 1992, and the implementation of the inflation targeting regime since then. During the years 1992 to 2000 inflation declined from a yearly rate of 9.4 percent to 0.0 percent. From 2001 to 2006 the average inflation rate was 1.6 percent, in the lower part of inflation target band (1 to 3 percent). Another major development was the transition to a flexible exchange rate regime that took place in the middle of 1997. That development increased the sensitivity of the exchange rate to external shocks. As a result the volatility of the exchange rate and (thereby) prices increased. On the other hand, the transition increased the sensitivity of the exchange rate to the interest rate

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6 In the ad hoc specification of the import price inflation we allowed two lags.
7 For example, see Justman (2002).
and thus enhanced the effectiveness of the interest rate as an instrument of stabilizing the inflation and the output gap.⁸

**Figure 1: Annual CPI inflation and the inflation target band, 1986-2006**

As we shall see in section 4, despite the major changes that characterized the Israeli economy during the estimation period, the rather standard New Keynesian model fits the data rather well. This conclusion is based on the following: (1) the sign and magnitude of most estimated parameters is in the range of the results obtained in similar models for other economies. (2) A dynamic within sample simulation that replicates fairly well the paths of the endogenous variables (except for the exchange rate path). (3) Cross correlations produced from stochastic simulations of the model are in line with the observed data.⁹

The Israeli economy is a very open economy¹⁰ and as can be expected the exchange rate plays a major role in the transmission of monetary policy. As we shall see in section 4, the pass-through from the exchange rate to import prices, and through it to the CPI, is very high but still gradual, that is, part of the (direct) effect of the

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⁸ For a survey on the exchange rate regimes during 1986 to 2005 see Elkayam (2003).
⁹ The dynamic simulation and cross-correlation comparison are presented in Argov et al. (2007). They are derived using a similar, though not identical, version of the model designated for practical use at the central bank.
¹⁰ In 2006 the ratio of export and import to GDP is 45 and 44 percents, respectively.
exchange rate changes comes with a lag. As we shall see, assuming immediate pass-through results in biased estimates of the inflation equation.

Figure 2: Quarterly Percent Change in the CPI Housing Component and the Shekel/Dollar Exchange Rate, 1999-2006

In the Israeli economy, the exchange rate also has a direct effect on part of the locally produced goods and services (even non-traded ones). That effect is a result of practices that developed during the high inflation era (1974 to 1985), with inflation reaching a yearly rate of 400 percent in 1984-85. One of the ways to avoid the consequences of the high inflation was to link prices of goods and services to the exchange rate of the shekel against the US Dollar.\textsuperscript{11} A relatively large market in which that practice still exists is real estate (house prices and rents are nominated in US Dollar and are linked to it, at least in the short- and medium-terms). The housing component constitutes 20 percent of the CPI and since 1999 most of it is based on the price of rented dwellings.\textsuperscript{12} In figure 2 we can see the high (and almost perfect) correlation between the change in the housing component and the exchange rate

\textsuperscript{11} In 1985 a stabilization program took place and inflation declined to 16-21 percent in the years 1986 to 1992. Since 1999 the average yearly inflation rate has been 1.4 percent.

\textsuperscript{12} The CPI housing component is made of owner-occupied housing (77%), rental housing (20%) and other related expenditures (3%). The price of owner-occupied housing is measured by the rental equivalent approach (which is based on the rental market). As a result 97% of the CPI housing component is based on the price of rentals.
changes. This unique relation, which complicates the inflation targeting process in Israel, led us to specify and estimate a separate equation for the inflation of the housing component (since 1999). The theory of the New Keynesian Phillips Curve was applied to the CPI excluding housing.\textsuperscript{13}

3. The theoretical model

The model described below is largely based on Svensson (2000), Adolfson (2001) and Linde et al. (2004). At several stages we deviate from other formulations, in order to adjust the model to the special characteristics of the Israeli economy. For the sake of completeness, and in order to highlight the meaning of the structural parameters that we try to estimate, we shall present the main stages of the model's development, even at the cost of some repetition of previous papers. For more background on the formulations the reader is referred to the papers above.

3.1 Households (output demand)

The domestic economy is populated by a continuum of infinitely-lived households indexed by \( j \), who consume Dixit-Stiglitz bundles of domestic and imported goods, denoted \( C^h_i \) and \( C^f_i \) respectively. Domestic goods are produced by a continuum of firms producing differentiated goods marked by index \( i \). Let \( C(i)^h \) denote the quantity consumed of good \( i \), and define:

\[
C^h_i = \left[ \int_0^1 C(i)^{\frac{\eta_h \eta_f}{\eta_h - 1} - 1} \, di \right]^{\frac{\eta_h - 1}{\eta_f}},
\]

where \( \eta_h > 1 \) is the elasticity of substitution between domestic goods.\textsuperscript{14} Cost minimization, subject to a given level of domestic consumption \( (C^h_i) \) leads to the following demand function for good \( i \):

\[
C(i)^h = \left( \frac{P(i)^h}{P^h} \right)^{-\frac{\eta_f}{\eta_h}} C^h_i,
\]

\textsuperscript{13} We also excluded the fruit and vegetables component (whose weight in the CPI is 3.4 percent) as it is noisy and unpredictable.

\textsuperscript{14} A similar aggregating equation holds for the imported consumption bundle.
where \( P(i)^h \) is the price of good \( i \) and the corresponding cost minimizing price aggregator of locally produced goods is given by:
\[
P_t^h = \frac{1}{\eta} \left[ \int_0^1 P(i)^{1-h} \, di \right]^{1-\eta}.
\]

The composite consumption index is defined by:
\[
C_i = \left( 1 - w_f^j \right) \left( C_i^h \right)^{\frac{\eta}{\eta-1}} + \left( w_f^j \right) \left( C_f^i \right)^{\frac{\eta}{\eta-1}}
\]

where \( w_f^j \) is the long-run share of imports in consumption, and \( \eta \) is the elasticity of substitution between imported and domestic goods. The corresponding minimum cost price aggregator (consumer price index) is:
\[
P_t^c = \left[ \left( 1 - w_f^j \right) (P_t^h)^{1-\eta} + \left( w_f^j \right) (P_t^f)^{1-\eta} \right]^{\frac{1}{\eta}},
\]

where \( P_t^h \) and \( P_t^f \) are the price aggregators of domestically produced and imported goods, all in local currency units.

Household \( j \)'s contemporaneous utility depends on its own consumption relative to lagged aggregate consumption \( C_{t-1} \) according to
\[
u(C(j)_i) = \left( C(j)_i - h C_{t-1} \right)^{\frac{\eta}{1-\eta}},
\]

where the parameter \( h \in (0,1) \) represents the extent of habit formation,\(^{15}\) and \( \sigma^{-1} \) is the inter-temporal elasticity of substitution. Household \( j \) chooses a sequence of consumption, domestic bond holdings and foreign bond holdings to maximize utility:
\[
\max_{C(j), B(j), B(\cdot)^f} \mathbb{E} \sum_{k=0}^{\infty} \beta^k u(C(j)_{t+k}),
\]
subject to the flow budget constraint:
\[
C(j)_t + \frac{B(j)_t}{(1+i^j)P^c} + \frac{\varepsilon_i B(j)^f_t}{(1+i^j)^{\Phi} P^c_t} = \frac{B(j)_{t-1}}{P^c_t} + \frac{\varepsilon^f_i B(j)^f_{t-1}}{P^c_t} + X(j)_t,
\]

\(^{15}\) See Smets and Wouters (2003) and Christiano et al. (2005).
where \( B_{i(j)_l} \) is household \( j \)'s holdings of one-period nominal bonds denominated in the domestic currency; \( B_{i(j)_l} \) is its foreign counterpart denominated in the foreign currency (dollar); \( \Phi \) is a risk premium paid on foreign assets; \( \varepsilon \) is the nominal exchange rate (the price of foreign currency in terms of the domestic currency); \( X_{i(j)_l} \) is household \( j \)'s share of aggregate real profits in the domestic economy; \( i \) and \( i^* \) are domestic and foreign nominal risk-free interest rates, respectively (i.e., the domestic bonds yield the gross return of \((1 + i)\) shekels and the foreign bonds yield the gross return of \((1 + i^*)\) dollars); \( \beta \) is the quarterly discount rate; and \( E \) is the expectation operator.

After aggregating the log-linearized first order conditions of the maximization problem, one obtains the following equations (the Euler equation, the uncovered interest parity condition, the optimal intra-temporal allocation across domestic and imported goods bundles and the aggregate price level). Lower case letters denote log deviation from steady state.

\[
\begin{align*}
(7) & \quad c_t = \frac{1}{1 + h} E_t c_{t+1} + \frac{h}{1 + h} c_{t-1} - \frac{1}{h} (i_t - E_t \pi^*_t) \\
(8) & \quad \varepsilon_t = E_t e_{t+1} + i^*_t - i_t + \phi_t \\
(9) & \quad c^b_t = c_t - \eta (p^b_t - p^*_t) \\
(10) & \quad p^*_t = (1 - w^*_t) p^b_t + w^*_t p^f_t
\end{align*}
\]

where \( \varepsilon_t \) is the log of the nominal exchange rate; and \( \pi^*_t = p^*_t - p^c_{t-1} \) is the CPI inflation rate.\(^{16}\) Notice that in reality we do not have observations on the CPI components, \( p^b_t \) and \( p^f_t \); only \( p^*_t \) is measurable. In the empirical part we shall try to overcome this problem by assuming a specific structure of the imported inflation equation (we shall return to that issue in the next subsection).

\(^{16}\) As discussed in the previous section \( \pi^c \) is the CPI excluding housing, fruit and vegetables. Below, describing monetary policy, we will mark the overall CPI by \( \pi^{cpi} \).
Assuming the demand for world trade is characterized by the same intra-temporal elasticity of substitution, $\eta$, the demand for the local economy's exports, $x^h_t$, is given by:\(^{17}\)

\[
x^h_t = y^*_i - \eta(p^*_i - e_i, p^*_i)
\]

where $y^*_i$ is the world's trade and $p^*_i$ is the price of consumer goods in the foreign economies. Following Monacelli (2005), we assume that in the export sector, prices are flexible and follow the law of one price (L.O.P.). Therefore $p^*_i - e_i$ is the export price in foreign economies' currency (dollars).

Let us define the real exchange rate as:

\[
g_t = p^*_i + e_i - p^*_i.
\]

Following Adolfson (2001) and Munacelli (2005), we assume that the pass-through from the exchange rate and the relevant world prices to the import price at the local market is gradual. Let us define $\psi^*_i$ as a temporary deviation from the law of one price (L.O.P. gap), that is:

\[
\psi^*_i = p^*_i - (p^*_i + e_i).
\]

Log-linearization of the national account identity yields:

\[
y_i = \gamma c^*_i + \gamma g^*_i + \gamma x^h_t + (1 - \gamma_e - \gamma g - \gamma s)inv^h_i,
\]

where $y_i$ is the output gap, $g^*_i$ and $inv^h_i$ are the log deviations from equilibrium of public consumption and investment (both in value added terms), and $\gamma_i$ is the long-run share in output of component $i$.

Using equations (7)-(14) we derive the output gap equation:

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\(^{17}\) $x^h_t$ is also a continuum of differentiated goods with constant elasticity of substitution $\eta_h$, similar to equation (1).
Equation (15) can be written more compactly as:

(15)  \[ y_t = \frac{1}{1+h} E_y y_{t+1} + \frac{1}{1+h} y_{t+1} - (1-h) y_{t+1} (i_t - E_i, \sigma_t) \]

where for any variable \( x_t \) we define

\[ \tilde{x}_t = x_t - \frac{1}{1+h} E_x x_{t+1} - \frac{1}{1+h} x_{t+1}. \]

and:

\[ a_i = \frac{(1-h) y_{t+1}}{(1+h) \sigma_t}; \quad a_y = \frac{\eta(\gamma', w_{t+1} + \gamma_{t+1})}{(1-w_{t+1})}; \quad a_{g_t} = \gamma_{t+1}; \quad a_{\sigma_t} = (1-\gamma - \gamma_{t+1} - \gamma_\sigma); \quad a_y = \gamma_\sigma. \]

Equation (15a) illustrates the basic variables determining the output gap: the real interest rate, the real exchange rate, the temporary deviation from the L.O.P, government spending, investments and world trade, all in deviation form. Notice that we still have to specify how the L.O.P. gap, \( \psi_t \), is measured.

3.2 Domestic producers (inflation equation)

Following Rotemberg (1982) domestic firms face menu costs. They choose a price sequence to minimize the cost of price changes and the cost of deviating from their flexible price. Formally all firms face the following problem:

(16)  \[ \min_{\{p(i)_{t+1}, \ldots, p(i)_{t+n}\} \in \mathbb{R}} \sum_{t=0}^{n} \delta^t \left[ (p(i)_{t+1} - \hat{p}(i)_{t+1})^2 + c(p(i)_{t+1} - p(i)_{t+1})^2 \right], \]

where \( p(i)_{t+1} \) is the price set by the domestic firm \( i \); \( \hat{p}(i)_{t+1} \) is the optimal flexible price (i.e., the price that would have been chosen in the absence of adjustment costs); and \( \delta \) is a discount factor.

The first order condition for the firm's minimization problem is given by:
\[
\hat{p}(i)^h = p(i)^h + \frac{\theta}{1-\theta} y^h_i,
\]

The RHS side of (21) is the log deviation of nominal marginal costs, where the cost minimizing input price aggregator is given by:

\[
p(i)^h = (1-w^f_i) p_i^{sh} + w^f_i p_i^{sf},
\]

where \(p_i^{sh}\) and \(p_i^{sf}\) are the prices of domestic and imported inputs to production (in domestic currency).

Following Svensson (2000) we assume that the local price of inputs is similar to it's consumer counterpart:

\[
p_i^{sh} = p_i^h.
\]

Averaging (17) across all firms, and plugging in (21)-(23) we arrive at the following equation for the inflation in the domestically produced consumption goods:
This equation is distinct from the basic closed-economy New Keynesian model in that the real price of imported inputs affects inflation as well as the output gap. This is a source of influence of the exchange rate on domestic prices.

Now, following Gali and Gertler (1999), we assume that only a fraction $\lambda$ of the firms set their price accordingly, while a fraction $(1-\lambda)$ use a simple rule of adjusting their price according to last period's domestic inflation. We also assume $\delta=1$, and get:

\[
\pi_t^h = \lambda E_t \pi_{t+1}^h + (1-\lambda) \pi_{t+1}^h + \lambda \frac{1}{c} \theta y_t + \lambda \frac{w_f^r}{c} (p_t^f - p_t^h).
\]

Recall that neither $zf_t$ nor $h_t$ are observable. In the following we shall write $(p_t^f - p_t^h)$ as a sum of three components: the first is exogenous and measurable $p_t^{\ast \ast} = (p_t^{\ast} - p_t^h)$. The second is endogenous and measurable, $\frac{1}{1-w_f^r} q_t$, and the third, $\psi_t^r + \frac{w_f^r}{1-w_f^r} \psi_t^r$, is not measurable, and we shall have to use a proxy for it.\(^{18}\)

Applying this separation on equation (25) we get:

\[
\pi_t^h = \lambda E_t \pi_{t+1}^h + (1-\lambda) \pi_{t+1}^h + \lambda \frac{1}{c} \theta y_t + \lambda \frac{w_f^r}{c} \left[ p_t^{\ast \ast} + \frac{1}{1-w_f^r} q_t + \left( \psi_t^r + \frac{w_f^r}{1-w_f^r} \psi_t^r \right) \right],
\]

where $p_t^{\ast \ast} = p_t^{\ast} - p_t^h$ represents a temporary deviation (from trend) in the world's relative price of inputs. $\psi_t^r$ is the L.O.P. gap in the inputs sector and defined by:

\[
\psi_t^r = p_t^f - (p_t^{\ast} + e_t).
\]

Equation (26) is the Phillips curve for the inflation in locally produced goods. As mentioned earlier inflation components are unobservable and therefore it is impossible to directly estimate equation (26). We will use equation (10) in first difference to eliminate the local inflation in favor of total inflation ($\pi^r$) and imported inflation ($\pi^f$):

\[^{18}\text{For derivation of equation (26) see appendix A.}\]
Now equation (13), in first difference, can be used to replace the (unobserved) imported inflation by its determinants – the change in world import prices of consumer goods \((\Delta p^{*}_t)\), nominal depreciation \((\Delta e)\) and the change in the L.O.P gap \((\Delta \psi^*_t)\):

\[
\begin{align*}
\pi^*_{t} &= \lambda E_t \pi^*_t + (1-\lambda) \pi^*_{t-1} + (1-w_f^t)\frac{\theta}{c} 1_\Theta v_t + (1-w_f^t)\lambda \frac{w_f^t}{c} \left[p^{*\alpha} + \frac{1}{(1-w_f^t)} q_t + \left(\psi^*_t + \frac{w_f^t}{1-w_f^t}\psi_t^* \right)\right] \\
&\quad + w_f^t \left[\pi^*_t - \lambda E_t \pi^*_{t-1} - (1-\lambda) \pi^*_{t-1}\right].
\end{align*}
\]

where \(dec_t = \Delta p^*_t + \Delta e_t\).

Our next step is to specify an equation for the local prices of imported consumer goods \((p^*_t)\) which will define the L.O.P. gap \((\psi^*_t)\). One possibility is to assume immediate (and complete) pass-through, that is: \(p^*_t = e_t + p^*_t\) (as in Svensson, 2000, for example). In that case \(\psi^*_t = 0\) and equation (29) is fully measurable. A more reasonable possibility is to follow Adolfson (2001) and Munacelli (2005), who assumed price stickiness in imported goods as well as in the domestic goods. Consequently a dynamic equation arises, linking the imported goods inflation, \(\pi^*_t\), to its real marginal cost \(\psi^*_t\). Direct estimation of such an equation is possible only if there exist data on both the local and imported goods in the CPI. However, typically there are no statistics on this separation. One approach, taken in Leitemo (2006a) and Linde et al. (2004), is to use the national accounts deflators. For \(\pi^*_t\) they used the GDP price deflator and for \(\pi^*_t\) they used the import price deflator. However, this kind of approach is not free of problems. First, the pricing theory above relates to market prices, i.e., either consumer prices or producer prices, so the components of the CPI or the WPI are more relevant than the national account deflators. Second, at least in Israel, the quarterly data of the national accounts are noisy and relatively unreliable in comparison with CPI series. Thirdly, the inflation target is in terms of the CPI. For these reasons we choose to estimate an equation in terms of the CPI alone.
The solution we choose is in the spirit of Adoldson (2001) and Munacelli (2005), but rather ad hoc. We assume that \( p_t^{f} \) and \( p_t^{zf} \) evolve according to the following distributed lag process, through which we can calculate \( \psi_t^{f} \) and \( \psi_t^{zf} \):

\[
(30) \quad p_t^{f} = \alpha_t(E_t p_{t-1}^{f} + E_t e_t) + \alpha_2(p_t^{f} + e_t) + \alpha_3(p_{t-1}^{f} + e_{t-1}) + (1-\alpha_1 - \alpha_2 - \alpha_3)(p_{t-2}^{f} + e_{t-2}),
\]

\[
(31) \quad p_t^{zf} = \alpha_t(E_t p_{t-1}^{zf} + E_t e_t) + \alpha_2(p_t^{zf} + e_t) + \alpha_3(p_{t-1}^{zf} + e_{t-1}) + (1-\alpha_1 - \alpha_2 - \alpha_3)(p_{t-2}^{zf} + e_{t-2}).
\]

Namely, we assume the existence of some rigidities in the price setting of imported goods so that the final price (\( p_t^{f} \) or \( p_t^{zf} \)) is influenced partly by the expected relevant world price (adjusted for exchange rate currency), partly by the contemporaneous level and partly by the first two lags. In the following we shall estimate the \( \alpha_1 \), \( \alpha_2 \) and \( \alpha_3 \). Of course, we can add more leads or lags to this equation and test their significance. The choice of the first two lags is based on empirical results.

Using these assumptions we can characterize the L.O.P. gaps in the imported consumption goods and imported intermediate inputs as follows:

\[
(32) \quad \psi_t^{f} = \alpha_t E_t dec_{t-1} - (1-\alpha_1 - \alpha_2) dec_t - (1-\alpha_1 - \alpha_2 - \alpha_3) dec_{t-1},
\]

\[
(33) \quad \psi_t^{zf} = \alpha_t E_t dez_{t-1} - (1-\alpha_1 - \alpha_2) dez_t - (1-\alpha_1 - \alpha_2 - \alpha_3) dez_{t-1},
\]

where \( dec_t = \Delta p_t^{f} + \Delta e_t \) and \( dez_t = \Delta p_t^{zf} + \Delta e_t \).

Plugging (32) and (33) in equation (29) we derive a fully observable equation for the CPI (excluding housing, fruit and vegetables) inflation. In the equation below all the parameters can be identified and estimated:

\[
(34) \quad \pi_t^{f} = \lambda E_t \pi_{t-1}^{f} + (1-\lambda) \pi_{t-1}^{zf} + (1 - w_f^{*}) \lambda b_f y_t + (1 - w_f^{*}) \lambda b_q q_t + \left[ \alpha_t E_t d\bar{e}zc_{t-1} - (1-\alpha_1 - \alpha_2) d\bar{e}zc_t - (1-\alpha_1 - \alpha_2 - \alpha_3) d\bar{e}zc_{t-1} \right] + w_f^{*} \left[ \alpha_1 d\bar{e}c_{t-1} + \alpha_2 d\bar{e}c_t + \alpha_3 d\bar{e}c_{t-1} + (1-\alpha_1 - \alpha_2 - \alpha_3) d\bar{e}c_{t-2} \right],
\]

where \( d\bar{e}zc_t = dez_t + \frac{w_f^{*}}{1- w_f^{*}} dec_t \)

\( d\bar{e}c_t = E_t dec_t - \lambda E_t dec_{t-1} - (1- \lambda) E_{t-1} dec_{t-1}. \)

The non-structural parameters are:

\[
b_f = \frac{\theta}{c(1-\theta)} \quad b_q = \frac{w_f^{*}}{c}.
\]
3.3 Foreign currency market (the exchange rate equation)

We start with the UIP condition derived from the households' first order conditions (equation 8). Accordingly, the spot nominal exchange rate is affected by future rate expectations, $e^\text{exp}_{t+1}$, and the adjusted interest rate differential:

$$(8)' \quad e_t = e^\text{exp}_{t+1} + i_t - i^* + \phi_t .$$

Preliminary experiments showed that lags of the exchange rate, and foreign and local interest rates also contribute to the explanation of the spot exchange rate. Accordingly, we assume that households' expectations with respect to the exchange rate are partly rational and partly adaptive, where $\omega$ measures the degree of rationality:19

$$(35) \quad e^\text{exp}_{t+1} = (1 - \omega) e^\text{exp}_{t+1} + \omega E_t e_{t+1} .$$

Combining (8)' with (35) yields the following equation for the exchange rate:

$$(36) \quad e_t = \omega E_t e_{t+1} + (1 - \omega) e_{t-1} + (i_t^* - i_t) - (1 - \omega) (i_{t-1}^* - i_{t-1}) + \phi_t - (1 - \omega) \phi_{t-1} .$$

3.4 Monetary policy (interest rate rule)

We assume that the central bank follows an inflation forecast based rule of the form:

$$(37) \quad i_t = (1 - \kappa_\theta) [r_t + \pi^\text{target}_t + \kappa_z E_i (\pi^\text{exp}_{t+\theta} - \pi^\text{target}_t) + \kappa_i y_t + \kappa_z z_t] + \kappa_i i_{t-1} ,$$

where $\pi^\text{exp}_{t+\theta} = p^\text{exp}_{t+\theta} - p^\text{exp}_{t+\theta-4}$ is year-on-year overall inflation rate at $t+\theta$, $\pi^\text{target}_t$ is the inflation target, $r_t$ is the natural real interest rate and $z_t$ represents additional variables that might enter the rule. The inflation forecast horizon is $\theta$ quarters. Notice that while the behavioral model related only to the CPI excluding housing, fruits and vegetables, the forecast relates to the overall CPI (excluding only fruit and vegetables). The motivation is clear: the inflation target is defined on overall inflation and therefore monetary policy is likely to react to overall inflation. In order to close the model a simple, ad hoc, equation will be specified and estimated for the CPI housing component. In the next sections we will devote detailed discussion on the estimated rule's parameters, the possibility of including the exchange rate in the rule and the choice of $\theta$.

19 A similar approach was taken by Leitemo and Soderstrom (2005b).
Table 1: The Model Equations

The (non-housing) inflation equation (33):

\[
\begin{align*}
\pi'_{t} &= \lambda E,\pi'_{t+1} + (1-\lambda)(1-w')_{t}\lambda b_{t}y_{t},
\end{align*}
\]

\[
\begin{align*}
&+ (1-w')_{t}\lambda b_{t} \left[ p_{t}^{\infty} \right] \left[ q_{t} + \left[ \alpha_{1}E,\delta e_{c_{1}t} - (1-\alpha_{1} - \alpha_{2})\delta e_{c_{1}t} - (1-\alpha_{1} - \alpha_{2} - \delta e_{c_{1}t}) \right] \right] \\
&+ w'_{t}\left[ \alpha_{1}\delta e_{c_{1}t} + \alpha_{2}\delta e_{c_{1}t} + \alpha_{3}\delta e_{c_{1}t} + (1-\alpha_{1} - \alpha_{2} - \delta e_{c_{1}t}) \right],
\end{align*}
\]

where $\delta e_{c_{1}t} = \delta e_{c_{1}t} + \frac{w'_{t}}{(1-w'_{t})} \delta e_{c_{1}t}$.

\[
d\delta e_{c_{1}t} = E,\delta e_{c_{1}t} - \lambda E,\delta e_{c_{1}t} - (1-\lambda)E_{t-1,\delta e_{c_{1}t}}.
\]

The housing price inflation equation:

\[
\begin{align*}
\pi'_{t}^{\text{house}} &= b_{0} + b_{1}E,\Delta e_{c_{1}t} + b_{2}\Delta e_{t} + b_{3}\Delta e_{t-1},
\end{align*}
\]

The CPI identity:

\[
\pi_{t}^{\text{CPI}} = 0.2 \cdot \pi_{t}^{\text{house}} + 0.8 \cdot \pi_{t}^{e}.
\]

The output gap equation:

\[
\begin{align*}
\gamma_{t} &= a_{1}(i_{t} - E,\pi_{t+1}) + a_{2}q_{t} + a_{3}\tilde{q}_{t} + a_{4}\tilde{\gamma}_{t} + a_{5}\tilde{\gamma}_{t} + a_{6}\tilde{\gamma}_{t}^{*},
\end{align*}
\]

where for any variable $x_{t} (y_{t}, q_{t}, \tilde{q}_{t}, \tilde{\gamma}_{t})$ we define $\tilde{x}_{t} = x_{t} - \frac{h}{1+h} E, x_{t+1} - \frac{1}{1+h} x_{t-1},$

and the estimated parameters are:

\[
\begin{align*}
a_{1} &= \frac{(1-h)y_{t}}{(1+h)e_{t}}; \quad a_{2} = \frac{\eta(y_{t}w_{t} + \gamma_{t})}{(1-w'_{t})}; \quad a_{3} = \gamma_{t}; \quad a_{4} = (1-\gamma_{t} - \gamma_{t} - \gamma_{t}); \quad a_{5} = \gamma_{t}; \quad a_{6} = \gamma_{t}.
\end{align*}
\]

The nominal exchange rate:

\[
\begin{align*}
e_{t} - E, e_{t+1} &= \omega \phi + (1-\omega)(e_{t-1} - E, e_{t+1}) + [(i_{t} - i_{t}) - (1-\omega)(i_{t-1} - i_{t})].
\end{align*}
\]

The interest rate equation:

\[
\begin{align*}
i_{t} = (1-\kappa_{t}) \left[ r_{t} + \pi'_{t} + \kappa_{x} E, (\pi'_{t+1} - \pi'_{t}) + \kappa_{y} (y_{t} + y_{t}) + \kappa_{q} (q_{t} + q_{t}) + \kappa_{\gamma} (\Delta e_{c_{1}t} + \Delta e_{c_{1}t}) \right] + \kappa_{i} i_{t-1},
\end{align*}
\]

where: $\pi'_{t+1} = \sum_{i=0}^{\delta} \pi'_{t+i}.$
4. Estimation of the model

The estimation is based on quarterly data between 1992:I and 2005:IV. The variables that were used for the local economy, Israel, are: the consumer price index (excluding housing, fruit and vegetables), the CPI housing component, the shekel/dollar nominal exchange rate, the effective Bank of Israel nominal interest rate, business sector product, gross investment, government purchases and the forward 5-to 10-year real yield on government indexed bonds (which serves as a proxy for the time varying natural rate of interest). For the foreign economy we used: the unit value of imported consumer goods and imported inputs to production, the one-month LIBID dollar interest rate and the industrial country's volume of imports (to approximate world demand). In addition, the unit value of industrialized country's imports was used as an instrumental variable. In order to construct the gap variables the Hodrick-Prescott filter was employed. More information on the data, their source and constructing the final variables is available in Appendix B.

For convenience we summarized the operative equations of the model in Table 1. Each equation was estimated separately by the GMM method. In the estimation procedure, we assume rational expectations, meaning all variables indexed \( t+1 \) or \( t+2 \) are replaced with the actual variable in that quarter.

4.1 Estimation of the inflation equation

The inflation equation was derived in Section 3 above and can be viewed in table 1 equation (F.1). The estimated equation is related to CPI excluding housing, fruit and vegetables. To the estimated equation we added a constant (which turned out insignificant) and seasonal dummy variables. The estimation results seem more robust when replacing the leveled variables with moving averages. Therefore, the output gap term \( y_t \) and the real price of imported inputs (all terms in curly brackets) were replaced with a two period moving average; for example, instead of \( y_t \) we used \( 0.5 \cdot (y_t + y_{t-1}) \). All variables were multiplied by 4, so the dependent variable is the annualized quarterly inflation.
### Table 2: Estimation of the Inflation Equation (F.1)

Estimation for the period 1992:1-2005:IV (56 observations)*

<table>
<thead>
<tr>
<th></th>
<th>Immediate pass-through F.1a</th>
<th>Gradual pass-through F.1b</th>
<th>Gradual pass-through excluding 1998:III F.1c</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0.580 (17.5)</td>
<td>0.591 (12.6)</td>
<td>0.565 (14.8)</td>
</tr>
<tr>
<td>$w_f^c$</td>
<td>0.091 (12.3)</td>
<td>0.464 (4.7)</td>
<td>0.556 (5.4)</td>
</tr>
<tr>
<td>$b_y$</td>
<td>0.018 (1.6)</td>
<td>0.086 (2.2)</td>
<td>0.102 (2.0)</td>
</tr>
<tr>
<td>$b_q$</td>
<td>0.122 (6.12)</td>
<td>0.039 (1.3)</td>
<td>0.030 (1.1)</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.0 (2.3)</td>
<td>0.097 (2.3)</td>
<td>0.123 (3.7)</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>1.0 (13.6)</td>
<td>0.424 (15.6)</td>
<td>0.416 (15.6)</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>0.0 (16.4)</td>
<td>0.337 (19.2)</td>
<td>0.328 (19.2)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.792</td>
<td>0.822</td>
<td>0.849</td>
</tr>
<tr>
<td>S.E.</td>
<td>2.42</td>
<td>2.31</td>
<td>2.15</td>
</tr>
<tr>
<td>DW</td>
<td>2.65</td>
<td>3.08</td>
<td>2.38</td>
</tr>
<tr>
<td>Jstat</td>
<td>0.252 (0.996)**</td>
<td>0.231 (0.953)**</td>
<td>0.237 (0.951)**</td>
</tr>
</tbody>
</table>

* The numbers in parenthesis are t-statistics.

** The number in parenthesis is the p-value for the test where the null hypothesis is that the over identifying restrictions are satisfied. The statistic for the test, $T \cdot J$, is asymptotically $\chi^2$ with degrees of freedom equal to the number of over identifying restrictions (number of instruments less estimated parameters).

As mentioned above, in the estimation procedure, we replaced the variables that represent expectations with their actual realizations. As a result, the unexpected factor of each such realization is included in the error term of the estimated equation, and these elements are potentially correlated with any variable that appears at the same date (including the exogenous variables). For example, we replaced $E_t dec_{t+1}$ and
$E_{t+1}dE_{t+2}$ with $dec_{t+1}$ and $dec_{t+2}$, respectively. As a result, any variable dated $t+1$ or $t+2$ cannot serve as an instrument. Notice that the above equation includes $E_{t},dec_{t}$ which is replaced by $dec$. As a result, only lags of the various variables (including the exogenous variables) can be incorporated in the set of the instrumental variables. Throughout, the sets of instrumental variables for each estimation are documented in Appendix C.

We begin by assuming immediate (perfect) pass-through from the exchange rate and world prices to the domestic import price, i.e., we assume $\psi'_{n} = \psi'_{c} = 0$. In equation (F.1) this means $\alpha_{1}, \alpha_{3} = 0$ and $\alpha_{2} = 1$.

The estimates of the parameters of equation (F.1a) under the immediate pass-through assumption are presented in the first column of table 2. As is evident from the table, the equation has a good explanatory power, all the estimates have the right sign, and apart from $b_{y}$, all are significant (at the 5 percent significance level). However, for the weight of imports in consumption goods ($w'_{c}$) we obtained an estimate of 0.09, which appears too small. We would expect a value closer to 0.4. Earlier trials with several alternative specifications suggested that the bias in this estimate may be related to the presumably false assumption of immediate (perfect) pass-through. To account for this, we proceeded by assuming that the pass-through in the prices of imported goods (consumer goods and inputs to production) is gradual and of the form of equations (30) and (31), which means that the L.O.P. gaps in imported consumer goods and imported inputs are in the form of equations (32) and (33). Hence we will estimate the $\alpha$ parameters.

The estimates of equation (F.1b) when assuming gradual pass-through are presented in the second column of table 2. As we can see, the parameters $\alpha_{1}, \alpha_{2}$ and $\alpha_{3}$ are significant, supporting the hypothesis of gradual pass-through. Furthermore, both the weight of imports in consumption and the coefficient of the output gap are larger, and of a magnitude closer to what we would expect. The weight of imports we obtain is 0.46, which is of the order of magnitude we would expect. However this estimation seems to deliver some extent of serial correlation. It is found that omitting the third

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20 Under the assumption of complete pass-through, $E_{t-1}dE_{t}$ is not included in the equation and thus we can include exogenous variables dated at $t$ in the instrumental variables set.

21 This can be inferred from the high value (close to 3) of the Durbin-Watson statistic, although this statistic does not provide a formal test in our case.
quarter of 1998 (equation (F.1c) in the third column of Table 2) relieves much of the serial correlation without changing the magnitude of the estimated parameters.

The reason that 1998:III is problematic lies in the unexpected 15% depreciation of the currency in the last quarter of that year (following the LTCM crisis). That depreciation was behind the 5% increase in the CPI in that quarter. Under the assumption of rational expectations, we use the actual inflation of 1998:IV as an unbiased proxy for the inflation expectations generated in 1998:III. Obviously, it is a poor proxy in that quarter and as a consequence caused the undesired serial correlation.

A possible means for assessing the reasonability (of the order of magnitude) of the estimated parameters is to compare to other similar works. However, the comparison is not trivial since the specification of the equations is not exactly the same in all papers. Furthermore, we compare different economies in different time periods. Nevertheless, we choose three works in which the parameters were estimated by classical methods: Lopez (2003) estimated a model for Colombia, Caputo (2004) estimated a model for Chile, and Leitemo (2006a) estimated a model for the UK. In the following comparison we shall use the estimates of equation (F.1b) in Table 2. Table 3 summarizes the comparison.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation F.1b</th>
<th>Lopez Colombia CPI</th>
<th>Caputo Chile CPI</th>
<th>Leitemo England GDP deflator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient of expected inflation</td>
<td>0.591</td>
<td>0.352</td>
<td>0.562</td>
<td>0.580</td>
</tr>
<tr>
<td>output gap affect on the CPI</td>
<td>0.086*0.5 = 0.051</td>
<td>0.027</td>
<td>0.067</td>
<td></td>
</tr>
<tr>
<td>output gap affect on the domestic prices by</td>
<td>0.086</td>
<td></td>
<td>0.035</td>
<td>0.07</td>
</tr>
<tr>
<td>Coefficient of real exchange rate bi</td>
<td>0.039</td>
<td>-</td>
<td>0.035</td>
<td>-</td>
</tr>
</tbody>
</table>
First we shall compare the coefficient of expected inflation ($\lambda$ in equation F.1). As can be seen the estimate here (0.591) is quite similar to that of Caputo (0.562) and Leitemo (0.580) and larger than that obtained by Lopez (0.352). Regarding the coefficient of the output gap, our estimate with regard to CPI inflation is similar to the one obtained by Caputo and larger than the one obtained by Lopez. Litemo estimated equation using the GDP deflator and got a value of 0.067. The analogous value here is ($b_y$) 0.086.

As for the real exchange rate coefficient, it can only be compared with Caputo (2004), which estimated 0.035 in comparison to 0.039 here.

### 4.2 Estimation of the CPI housing component equation

The above equation relates to the inflation of the CPI excluding the housing and fruit and vegetables components. Since the inflation target is in terms of the overall CPI we need to specify and estimate an equation for the housing component as well. As is evident from Figure 2 (Section 2), this component is influenced mainly by the (shekel/dollar) exchange rate developments. Based on this we estimated equation (F.2a) where the housing component inflation ($\pi_{h}^{\text{prev}}$) is explained by the (current, lead and lagged) exchange rate depreciations ($\Delta e$), a constant, and seasonal dummy variables (not reported). All variables were multiplied by 4, so that the dependent variable is the annualized quarterly CPI housing component inflation. The equation was estimated by GMM. The resulted estimates are shown in Table 4.

For purposes of the model's long run properties and convergence, in the simulations we will use an homogeneous version of the equation (exchange rate coefficients sum to one) in which we omit the lead of the exchange rate. This ensures that monetary policy will not have a permanent effect on the relative price of housing. The estimation results of the restricted equation (F.2b) are presented in the second column of Table 2.
Table 4: Estimation of CPI Housing Component Inflation Equation (F.2)

Estimation for the period 2000:I-2006:III (27 observations) *

<table>
<thead>
<tr>
<th></th>
<th>With lead homogeneous restriction</th>
<th>Without lead homogeneous restriction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F.2a</td>
<td>F.2b</td>
</tr>
<tr>
<td>$b_0$</td>
<td>-0.979 (-4.17)</td>
<td>-0.833 (-2.4)</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.166 (1.76)</td>
<td>0.0</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.774 (18.14)</td>
<td>0.864 (54.85)</td>
</tr>
<tr>
<td>$b_3$</td>
<td>0.148 (3.77)</td>
<td>1 - $b_2$</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.923</td>
<td>0.936</td>
</tr>
<tr>
<td>S.E.</td>
<td>3.243</td>
<td>2.834</td>
</tr>
<tr>
<td>DW</td>
<td>1.53</td>
<td>1.91</td>
</tr>
<tr>
<td>$J_{stat}$</td>
<td>0.312 (0.209)**</td>
<td>0.315 (0.383)**</td>
</tr>
</tbody>
</table>

* The numbers in parenthesis are t-statistics.
** The number in parenthesis is the p-value for the test where the null hypothesis is that the over-identifying restrictions are satisfied. The statistic for the test, $T \cdot J$, is asymptotically $\chi^2$ with degrees of freedom equal to the number of over-identifying restrictions (number of instruments less estimated parameters).

4.3. Estimation of the output gap equation (F.4)

Estimation of the output gap equation, allowing for gradual pass-through of the form described in equation (32), did not yield reasonable results. This need not necessarily indicate the lack of gradual pass-through, but rather signals weak identifying power when it appears only in levels terms (in the output gap equation). Hence, we shall proceed to estimate the equation under the assumption of immediate pass-through. For the sake of simplicity we shall write the final output gap equation (15) in a compact form (F.4), where all parameters can be identified and estimated. The equation is further simplified by defining, for any variable $x_t$ ($y_t, q_t, g^*_t, inv_t, y^*_t$),
\[ \hat{x}_t = x_t - \frac{h}{1+h} E_t x_{t+1} - \frac{1}{1+h} x_{t-1}, \] i.e., \( \hat{x}_t \) is the deviation of \( x_t \) from a weighted average of its lead and lag.

In the following, we added a constant (which turned out insignificant in the first three versions) and seasonal dummy variables to the various estimated equations. In the estimation, the annualized real interest rate gap was divided by 4, so as to express all variables in quarterly term. The equation was estimated by GMM method.

The estimation results of equation (F.4a) are presented in the first column of Table 5. The results show an intermediate value of "habit persistence" (0.542). We used the estimated value of \( w_x \) from the inflation equation (F.1b) (in Table 2) in order to derive estimates of the structural parameters (\( \sigma, \eta \) and \( \gamma_c \)).

Comparing the estimated parameters that express-long run shares in GDP (\( \gamma_x, \gamma_g, \gamma_{inv}, \) and \( \gamma_c \)) with the actual average share we find that the consumption share is estimated accurately. However, the estimated share of exports seems too low (estimated 0.156 compared to 0.3 actual) and the share of government purchases seems too high (estimated 0.225 compared to 0.06 actual). We should note that what we call 'actual' is not necessarily the correct parameter. The long-run shares relate to the components of the GDP in value added terms, for which actual data are not available. The assessment regarding the value of the 'actual' is based on each component's share in total uses. In the second column of Table 5 (equation F.4b), we present estimates of the equation when imposing that assessment with regard to the actual shares. As can be seen, the estimated values of the other parameters remain quite similar to those in equation (F.4a).

In equations (F.4c) and (F.4d) we present estimates analogous to (F.4a) and (F.4b) when eliminating habit persistence. i.e., under the constraint that \( h=0 \). As can be seen, under this restriction the explanatory power of the equation is reduced, the coefficient of the interest rate (\( a_i \)) increases, and that of the real exchange rate (\( b_q \)) decreases.

In Table 6 we compare the estimated parameters of (F.4a) to those obtained by Lopez (2004), Caputo (2004) and Leitemo (2006a). As can be seen, the coefficients of expected output gap, and of the real exchange rate are larger, here, than those obtained in the other papers.
Table 5: Estimation of the Output Gap Equation (F.4)

Estimation for the period 1992:1-2005:IV (56 observations) *

<table>
<thead>
<tr>
<th></th>
<th>No output weight restrictions, with ( h ) (F.4a)</th>
<th>With output weight restrictions, with ( h ) (F.4b)</th>
<th>No output weight restrictions, ( h = 0 ) (F.4c)</th>
<th>With output weight restrictions, ( h = 0 ) (F.4d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h )</td>
<td>0.542 (5.9)</td>
<td>0.542 (5.9)</td>
<td>0.0 (-)</td>
<td>0.0 (-)</td>
</tr>
<tr>
<td>( a_i )</td>
<td>-0.424 (-3.5)</td>
<td>-0.399 (-4.07)</td>
<td>-0.819 (-6.2)</td>
<td>-0.703 (-7.4)</td>
</tr>
<tr>
<td>( a_q )</td>
<td>0.268 (5.6)</td>
<td>0.173 (4.2)</td>
<td>0.125 (2.6)</td>
<td>0.060 (1.5)</td>
</tr>
<tr>
<td>( a_c )</td>
<td>0.156 (1.5)</td>
<td>0.30 (-)</td>
<td>0.392 (7.1)</td>
<td>0.30 (-)</td>
</tr>
<tr>
<td>( a_g )</td>
<td>0.225 (5.4)</td>
<td>0.06 (-)</td>
<td>0.222 (6.3)</td>
<td>0.06 (-)</td>
</tr>
<tr>
<td>( a_{inv} )</td>
<td>0.139 (7.7)</td>
<td>0.16 (-)</td>
<td>0.167 (7.4)</td>
<td>0.16 (-)</td>
</tr>
</tbody>
</table>

Solving for deep parameters by plugging in \( w_f^* = 0.464 \).

<table>
<thead>
<tr>
<th></th>
<th>( \sigma )</th>
<th>( \eta )</th>
<th>( \gamma_c )</th>
<th>( \sigma )</th>
<th>( \eta )</th>
<th>( \gamma_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma )</td>
<td>0.336 (2.8)</td>
<td>0.336 (3.5)</td>
<td>0.266 (2.5)</td>
<td>0.683 (7.4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \eta )</td>
<td>0.379 (3.8)</td>
<td>0.275 (5.6)</td>
<td>0.136 (2.7)</td>
<td>0.062 (1.5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \gamma_c )</td>
<td>0.480 (4.5)</td>
<td>0.48 (-)</td>
<td>0.218 (3.3)</td>
<td>0.48 (-)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>( R^2 )</th>
<th>S.E.</th>
<th>DW</th>
<th>Jstat</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^2 )</td>
<td>0.712</td>
<td>2.05</td>
<td>2.90</td>
<td>0.221 (0.964)**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.E.</td>
<td>0.699</td>
<td>2.04</td>
<td>3.01</td>
<td>0.221 (0.988)**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DW</td>
<td>0.503</td>
<td>2.67</td>
<td>2.59</td>
<td>0.257 (0.937)**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jstat</td>
<td>0.536</td>
<td>2.50</td>
<td>2.66</td>
<td>0.261 (0.974)**</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* The numbers in parenthesis are t-statistics.
** The number in parenthesis is the p-value for the test where the null hypothesis is that the over identifying restrictions are satisfied. The statistic for the test, \( T \cdot J \), is asymptotically \( \chi^2 \) with degrees of freedom equal to the number of over-identifying restrictions (number of instruments less estimated parameters).
4.5. Estimation of the exchange rate equation

Since the middle of 1997 the Bank of Israel has not intervened in the foreign exchange market, so that the exchange rate has been determined by market forces. The final exchange rate equation (35) was modified assuming that the unobserved risk premium is a fixed parameter (see equation F.5). The equation was estimated by GMM method for the period 1997:III to 2005:IV. The annualized interest rate differentials were divided by 4. The results are summarized in table 7.

**Table 6: Cross Country Comparison of the Output Gap Equation**

<table>
<thead>
<tr>
<th>parameter</th>
<th>Equation (F.4a)</th>
<th>Lopez Columbia</th>
<th>Caputo Chile</th>
<th>Leitemo England</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{1+h} )</td>
<td>0.649</td>
<td>0.109</td>
<td>0.453</td>
<td>0.53</td>
</tr>
<tr>
<td>( a_t )</td>
<td>-0.424</td>
<td>-0.668</td>
<td>-</td>
<td>-0.28</td>
</tr>
<tr>
<td>( a_q )</td>
<td>0.268</td>
<td>0.002</td>
<td>0.016</td>
<td>0.11</td>
</tr>
<tr>
<td>( a_x )</td>
<td>0.156</td>
<td>0.092</td>
<td>0.026</td>
<td>0.25</td>
</tr>
</tbody>
</table>

**Table 7: Estimation of the Exchange Rate changes Equation (F.5)**

Estimation for the period 1997:III-2005:IV (34 observations) *

<table>
<thead>
<tr>
<th>( \omega )</th>
<th>0.569</th>
<th>( \phi )</th>
<th>3.383</th>
</tr>
</thead>
<tbody>
<tr>
<td>(9.53)</td>
<td>(2.43)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>R²</th>
<th>0.428</th>
<th>S.E.</th>
<th>2.52</th>
<th>DW</th>
<th>2.78</th>
<th>Jstat</th>
<th>( 0.114 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.693)**</td>
</tr>
</tbody>
</table>

* The numbers in parenthesis are t-statistics.
** The number in parenthesis is the p-value for the test where the null hypothesis is that the over identifying restrictions are satisfied. The statistic for the test, \( T \cdot J \), is asymptotically \( \chi^2 \) with degrees of freedom equal to the number of over-identifying restrictions (number of instruments less estimated parameters).

The parameter of the future exchange rate is estimated to be 0.569, and the estimate of the average annualized risk premium is 3.383 percent. Additional

---

\( e_t \) is 100*log(Nominal Exchange Rate).
experiments hint at the possibility that the average risk premium declined towards the end of the estimation period, concurrent with the decline in the inflation rate.

4.6. Estimation of the interest rate rule

We considered equation (F.6) where in addition to the inflation gap, \( E_t(\pi_{i,t+\theta}^m - \pi_t^T) \), the monetary authority may react directly to the output gap, the real exchange rate gap and the nominal depreciation (all in terms of two quarters moving average). The equation was estimated by GMM, and the instrumental variables are listed in Appendix C. The equation was estimated with various \( \theta \) ranging from 0 to 4.\(^{23}\) Only for \( \theta \) equal to 0 and 1 did we receive reasonable and stable results. A possible explanation of that result is that the actual yearly inflation for three and four quarters ahead is a "bad" estimate of expected inflation, due to the large fluctuations of the inflation rate during the estimation period. When we replaced actual ex-post yearly inflation with one-year-ahead inflation expectations which are derived from the capital market (CM), we obtained better results. As can be seen in Table 8 the estimated equations using CM expectations is not "worse" (in terms of fit) even than those that use yearly inflation with \( \theta \) equal to 0 and 1.

In the first three columns of Table 8 we present the results of the estimated equations under the restrictions: \( \kappa_q = 0, \kappa_{\Delta e} = 0 \), that is, without the real exchange rate gap or exchange rate changes. As can be seen, for the outcome based rule (i.e, for \( \theta=0 \)) we get for the inflation gap a coefficient greater than one (1.39), a positive and significant coefficient for the output gap (0.233), and a coefficient of 0.772 for the lagged interest rate. When we move yearly inflation one quarter ahead (i.e, for \( \theta=1 \)) the coefficients of the inflation and output gaps increase but the inertia also increases. When we use CM expectations instead of actual inflation (which can be interpreted as moving to \( \theta=4 \)) the coefficients of the inflation and output gaps increase further but the inertia reduces to a similar rate of that in the case of \( \theta=1 \).

In the last three columns of Table 8 we present the results when we add the real exchange rate gap and the exchange rate changes to the equations. For the estimates of the parameters \( \kappa_i, \kappa_\pi, \kappa_y \) the results are analogous to those in the first three columns. The effect of the exchange rate gap is significant in the three equations and the exchange rate changes is significant in F.6d and F.6f.

\(^{23}\) The use of a smoothed measure of inflation and the restriction of \( \theta \) to at most 4 quarters ahead is based on the conclusions of the paper by Levine et al. (2003).
According to the results above it seems that the Bank of Israel reacted to the exchange rate, in addition to the inflation and output gaps. The natural question is why? Is it because exchange rate fluctuations appear as a factor in its loss function? Alternatively, is it because reducing the exchange rate fluctuations helps to reduce the fluctuations of the inflation and or the output gap? In the next section we shall use the model to answer that question. However the main challenge of the next section is to choose the value of the parameters of a forecast-base rule for the model.

Table 8: Estimation of the Interest Rate Equation (F.6)

<table>
<thead>
<tr>
<th></th>
<th>( \theta = 0 )</th>
<th>( \theta = 1 )</th>
<th>CM</th>
<th>( \theta = 0 )</th>
<th>( \theta = 1 )</th>
<th>CM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \kappa_q = \kappa_{de} = 0 ) (F.6a)</td>
<td>( \kappa_q = \kappa_{de} = 0 ) (F.6b)</td>
<td>( \kappa_q = \kappa_{de} = 0 ) (F.6c)</td>
<td>( \kappa_q = \kappa_{de} = 0 ) (F.6d)</td>
<td>( \kappa_q = \kappa_{de} = 0 ) (F.6e)</td>
<td>( \kappa_q = \kappa_{de} = 0 ) (F.6f)</td>
</tr>
<tr>
<td>( \kappa_l )</td>
<td>0.772 (40.3)</td>
<td>0.876 (29.6)</td>
<td>0.807 (29.7)</td>
<td>0.804 (35.8)</td>
<td>0.873 (35.4)</td>
<td>0.806 (33.0)</td>
</tr>
<tr>
<td>( \kappa_r )</td>
<td>1.390 (9.97)</td>
<td>2.526 (4.13)</td>
<td>3.391 (7.23)</td>
<td>1.255 (9.91)</td>
<td>1.619 (3.37)</td>
<td>2.841 (6.55)</td>
</tr>
<tr>
<td>( \kappa_y )</td>
<td>0.233 (2.80)</td>
<td>0.330 (1.83)</td>
<td>0.397 (3.55)</td>
<td>0.448 (3.60)</td>
<td>0.764 (3.06)</td>
<td>0.625 (4.02)</td>
</tr>
<tr>
<td>( \kappa_q )</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>0.422 (2.83)</td>
<td>1.120 (2.90)</td>
<td>0.452 (2.37)</td>
</tr>
<tr>
<td>( \kappa_{de} )</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>0.083 (2.12)</td>
<td>0.121 (1.47)</td>
<td>0.193 (5.02)</td>
</tr>
<tr>
<td>R^2</td>
<td>0.945</td>
<td>0.927</td>
<td>0.943</td>
<td>0.944</td>
<td>0.932</td>
<td>0.944</td>
</tr>
<tr>
<td>S.E.</td>
<td>1.00</td>
<td>1.15</td>
<td>1.02</td>
<td>1.02</td>
<td>1.13</td>
<td>1.03</td>
</tr>
<tr>
<td>DW</td>
<td>1.69</td>
<td>1.85</td>
<td>1.74</td>
<td>1.57</td>
<td>1.54</td>
<td>1.73</td>
</tr>
<tr>
<td>Jstat</td>
<td>0.203 (0.925)**</td>
<td>0.228 (0.872)**</td>
<td>0.229 (0.903)**</td>
<td>0.166 (0.941)**</td>
<td>0.224 (0.794)**</td>
<td>0.181 (0.939)**</td>
</tr>
</tbody>
</table>

* The numbers in parenthesis are t-statistics.
** The number in parenthesis is the p-value for the test where the null hypothesis is that the over identifying restrictions are satisfied. The statistic for the test, \( T \cdot J \), is asymptotically \( \chi^2 \) with degrees of freedom equal to the number of over-identifying restrictions (number of instruments less estimated parameters).
5. Deriving an Optimal Simple Monetary Policy Rule

In this section we will search for an optimal-simple monetary policy rule, simple in the sense that the rule takes the form of equation (F.6), optimal in the sense that it will be based on a central bank loss function (to be defined below). Specifically, we would like to choose the inflation forecast horizon (defined by the $\theta$ parameter) and test whether the model justifies direct reaction to the exchange rate.

The simulations held for this purpose consists of the following equations:

a. The inflation equation (F.1b) which allows for gradual pass-through.
b. The housing component equation (F.2b), adjusted for long run convergence to the general inflation target. This insures that the model converges – relative prices are constant in the long run, and that monetary policy can not affect the long run relative price of housing.
c. The output gap equation (F.4a) which allows for habit formation in consumption and does not impose restrictions on the weights of the GDP components.
d. The exchange rate equation, modified in two manners. First, we found that allowing the forward weight ($\omega$) to be greater than 0.5 embodies potential determinacy problems. Therefore we reduced the weight to 0.45.\textsuperscript{24} Second, we assume an exogenous varying risk premium rather than a constant term.\textsuperscript{25}
e. An auto regressive equation for all exogenous variables to account for their dynamics. In general the equations were estimated by OLS. For details on these equations see Argov et al. (2007).\textsuperscript{26}
f. In order to perform stochastic simulations, standard deviations must be defined for each shock (equation residual). These were based on the estimated equations. In all simulations we assume absence of monetary policy shocks and a constant inflation target. Details on the standard deviations are presented in Appendix D.

The objective of monetary policy is to minimize the variation in macroeconomic variables.\textsuperscript{27} To get a first taste, Table 9 presents the standard deviation of key model variables under various values of $\theta$, i.e., the inflation forecast

\textsuperscript{24} In Argov et al. (2007) $\omega$ was estimated to be 0.45 in a similar specification. In their estimation a larger set of instrumental variables was used.
\textsuperscript{25} The risk premium variable was taken from Hecht and Pompushko (2006).
\textsuperscript{26} Section 7.
\textsuperscript{27} To be even more exact – to minimize variation of macroeconomic variables around their flexible price level.
horizon. In these simulations we used the first three parameter estimates from equation F.6f ($\kappa_i = 0.81, \kappa_\pi = 2.84, \kappa_y = 0.63$); we assumed no direct reaction to the real exchange rate nor the nominal depreciation ($\kappa_q, \kappa_Ae = 0$). The last row of Table 9 is the following weighted sum of variance:

$$L = 1.0 \cdot Var(\pi^{\text{pi}}) + 0.5 \cdot Var(y) + 4.0 \cdot Var(\Delta i).$$

This weighted sum ($L$) will later serve as the monetary policy's loss function. The weights in $L$ are somewhat non-standard; in particular, the interest rate weight is usually chosen to be less than unity. In our case, the weights were chosen so that the standard deviations under the simple-optimal rules fall in the neighborhood of those under the empirical (estimated) rules. Specifically, only interest rate weights far greater than 1 generate interest rate variations close to those observed in the data.

Table 9: Standard Deviations of Main Variables Using Estimated Parameters Under Various Forecast Horizons

$k_i = 0.81, \ k_\pi = 2.84, \ k_y = 0.63$

<table>
<thead>
<tr>
<th>S.D. of variable</th>
<th>$\theta = 0$</th>
<th>$\theta = 1$</th>
<th>$\theta = 2$</th>
<th>$\theta = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^{\text{pi}}$</td>
<td>3.75</td>
<td>3.72</td>
<td>3.86</td>
<td>4.23</td>
</tr>
<tr>
<td>$\pi^c$</td>
<td>3.92</td>
<td>3.94</td>
<td>4.13</td>
<td>4.52</td>
</tr>
<tr>
<td>$y$</td>
<td>4.35</td>
<td>4.15</td>
<td>3.95</td>
<td>3.77</td>
</tr>
<tr>
<td>$\Delta i$</td>
<td>1.00</td>
<td>1.02</td>
<td>1.04</td>
<td>1.02</td>
</tr>
<tr>
<td>$q$</td>
<td>4.15</td>
<td>4.02</td>
<td>3.90</td>
<td>3.80</td>
</tr>
<tr>
<td>$\Delta e$</td>
<td>9.09</td>
<td>8.86</td>
<td>8.75</td>
<td>8.75</td>
</tr>
</tbody>
</table>

| Weighted Sum of Variance | 27.52 | 26.61 | 27.02 | 29.16 |

1. Inflation and depreciation rates are annualized, $\Delta i$ is quarterly change in annualized interest rate.
2. $1.0 \cdot Var(\pi^{\text{pi}}) + 0.5 \cdot Var(y) + 4.0 \cdot Var(\Delta i)$.

$^28 \theta = 0$ is a backward-looking rule, $\theta = 1,2$ are hybrid backward-forward-looking rules and $\theta = 3$ is a forward-looking rule. Current inflation is included in each rule. For details see equation (F.6).

$^29$ For example Svensson (2000) uses a value of 0.01, Leitemo and Soderstrom (2005b) use 0.1.
It is evident from the table that the greater the forecast horizon, the smaller are the variance in the output gap, the real exchange rate and nominal depreciation (up to $\theta = 2$). In contrast, the standard deviation of CPI inflation is lowest with a one-period-ahead forecast horizon ($\theta = 1$). The standard deviation of interest rate changes is not very sensitive to the forecast horizon. To a large extent, these are the result of the fast and strong pass-through from exchange rate shocks to CPI prices. Following such shocks, inflation rises sharply for one quarter. The peak in year-on-year inflation is in the second quarter. Therefore, the longer the forecast horizon the quicker the interest rate starts re-adjusting to its pre-shock level, and a smaller output sacrifice is generated. Immediate and direct reaction to the peak of year-on-year inflation ($\theta = 1$) minimizes the variance in quarterly inflation.

It is seen that the loss function $L$ is minimized with $\theta = 1$, i.e., using a hybrid backward-forward-looking inflation measure of one-period-ahead inflation expectations, current inflation and two lags of inflation realizations. $\theta = 1$ being superior to $\theta = 0$ and $\theta = 3$ is rather robust to the loss function choice (it holds as long as the weight on the output gap is smaller than 1.75 and the weight on the interest rate is smaller than 33%). However it is somewhat harder to distinguish between $\theta = 1$ and $\theta = 2$. If the loss function weight on the output is 0.8 then $\theta = 2$ is superior.

Table 10 presents standard deviations of key variables (and a weighted sum) using $\theta = 1$ and the parameters of estimated equation (F.6f) when gradually allowing for direct reaction to the nominal and real exchange rates.

It is seen that direct reaction to the level of the real exchange rate ($\kappa_q > 0$) reduces the variance in inflation while raising the variance in output. In part, this is due to the dominance of shocks that generate a negative correlation between the real-exchange-rate and the output gap, for instance, nominal exchange rate and foreign interest rate shocks. Therefore it is found by the weighted sum to be sub-optimal (as long as the output weight in $L$ is bigger than 0.25). Direct reaction to the nominal depreciation ($\kappa_{\Delta e} > 0$) slightly improves the variance in inflation while generating moderately larger interest rate and output standard deviations. In sum, it does not seem to contribute much to central bank performance. The results outlined here for $\theta = 1$ hold for other $\theta$ values as well.

---

30 Due to the dollarized housing price component and a rather short import price pass-through lag structure (see equation 30).
Table 10: Standard Deviations of Main Variables Using Estimated Parameters and Implications of Direct Reaction to the Exchange Rate,

\[ \kappa_i = 0.81, \quad \kappa_x = 2.84, \quad \kappa_y = 0.63, \quad \theta = 1 \]

<table>
<thead>
<tr>
<th>S.D. of variable</th>
<th>( \kappa_q = 0.00 )</th>
<th>( \kappa_q = 0.45 )</th>
<th>( \kappa_q = 0.00 )</th>
<th>( \kappa_q = 0.45 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^c_{p} )</td>
<td>3.72</td>
<td>3.59</td>
<td>3.64</td>
<td>3.51</td>
</tr>
<tr>
<td>( \pi^c )</td>
<td>3.94</td>
<td>3.80</td>
<td>3.94</td>
<td>3.81</td>
</tr>
<tr>
<td>( y )</td>
<td>4.15</td>
<td>4.49</td>
<td>4.18</td>
<td>4.48</td>
</tr>
<tr>
<td>( \Delta i )</td>
<td>1.02</td>
<td>1.05</td>
<td>1.06</td>
<td>1.10</td>
</tr>
<tr>
<td>( q )</td>
<td>4.02</td>
<td>4.35</td>
<td>3.96</td>
<td>4.24</td>
</tr>
<tr>
<td>( \Delta e )</td>
<td>8.86</td>
<td>8.85</td>
<td>8.56</td>
<td>8.55</td>
</tr>
</tbody>
</table>

1. Inflation and depreciation rates are annualized, \( \Delta i \) is quarterly change in annualized interest rate.
2. \( 1.0 \times \text{Var}(\pi^c_{p}) + 0.5 \times \text{Var}(y) + 4.0 \times \text{Var}(\Delta i) \).

Having the results of Tables 9 and 10 in mind we turn to formally derive an optimal-simple rule, based on minimizing the loss function \( L \). We take the following steps:

a. We find the inflation and output reaction measures (\( \kappa_{\pi}, \kappa_{y} \)) that minimize the loss function while restricting the smoothing parameter (\( \kappa_{i} \)) to 0.831 and the exchange rate reaction parameters (\( \kappa_{q}, \kappa_{\Delta e} \)) to zero.

b. Taking the optimal parameters found in step (a), we minimize the loss function by means of the exchange rate reaction parameters. In this we follow the approach taken by Leitemo and Soderstrom (2005b).

c. For comparison purposes we optimize on all four parameters (\( \kappa_{\pi}, \kappa_{y}, \kappa_{q}, \kappa_{\Delta e} \)).

d. Steps (a)-(c) are repeated for each forecast horizon (\( \theta = 0, 1, 2, 3 \)).

31 Optimizing on all three parameters drives the optimal parameters to unreasonable areas in which the reaction and smoothing parameters are very high, especially when the forecast horizon grows. Since it was found in this study, like others, that a smoothing parameter value of 0.8 reflects central banks' actual conduct, we chose to fix it to this value.
Tables 11-14 report the optimized parameters, the resulting standard deviations and loss function values. For comparison, we found that using the globally optimal interest rate rule (assuming commitment), the loss function value is 24.10.

Similar to what we found with the estimated rule, the hybrid backward- and one-period-ahead forward-looking rule ($\theta = 1$) seems to deliver the lowest (optimized) loss function values. In this case we found the optimal inflation reaction parameter ($\kappa_\pi$) is 2.93, larger than what was found in the estimation assuming a short forecast horizon. Similar to the estimation, we learn from the tables that as the forecast horizon increases, the optimal inflation reaction parameter grows.

The optimal output gap reaction parameter ($\kappa_y$) is 0.9, again higher than the estimated values. This result might have a simple operational reason: the model assumes perfect knowledge of the current state of the output gap; of course, in reality, not only does the central bank not know the true measure of the gap, it does not even have a precise real-time HP filter estimate due to lags in data publication and well documented end-of-sample filtering problems.

For all forward looking rules ($\theta > 0$) we find that direct reaction to exchange rate measures does not contribute in reducing the loss function. In some cases the optimized parameters are negative! We conclude that the forecast of CPI inflation summarizes the relevant information, even in a very open economy, even when some inflation components are eccentrically linked to the exchange rate. In contrast, we find some benefit in nominal exchange rate reaction in backward-looking rules ($\theta = 0$).

Based on these results we will employ a simple optimal rule with $\theta = 1$, $\kappa_i = 0.8$, $\kappa_\pi = 2.93$, $\kappa_y = 0.5$ and no direct reaction to exchange rate measures. The output gap reaction parameter was reduced from the one found in the optimization procedure due to the operational considerations mentioned above. This rule generates a loss function value of 26.8, only 1.4% higher than the best of the optimal simple rules found above and 11.1% higher than the globally optimal (commitment) rule.
Table 11: Optimal Forecast-Based Rules, Standard Deviations of Main Variables and Loss Function\(^2,3\)

\[ \theta = 0 \]

<table>
<thead>
<tr>
<th></th>
<th>Optimizing on: ( \kappa_i, \kappa_y, \kappa_q )</th>
<th>Optimizing on: ( \kappa_q )</th>
<th>Optimizing on: ( \kappa_{ie} )</th>
<th>Optimizing on: ( \kappa_{ie}, \kappa_i, \kappa_y, \kappa_q, \kappa_{ie} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa_i )</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>( \kappa_\pi )</td>
<td>2.81</td>
<td>2.81</td>
<td>2.81</td>
<td>2.54</td>
</tr>
<tr>
<td>( \kappa_y )</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>1.03</td>
</tr>
<tr>
<td>( \kappa_q )</td>
<td>--</td>
<td>0.16</td>
<td>--</td>
<td>0.18</td>
</tr>
<tr>
<td>( \kappa_{ie} )</td>
<td>--</td>
<td>--</td>
<td>0.24</td>
<td>0.24</td>
</tr>
<tr>
<td>S.D. of variable(^1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \pi^{cpi} )</td>
<td>3.83</td>
<td>3.78</td>
<td>3.73</td>
<td>3.77</td>
</tr>
<tr>
<td>( \pi^c )</td>
<td>4.00</td>
<td>3.95</td>
<td>3.99</td>
<td>4.03</td>
</tr>
<tr>
<td>( y )</td>
<td>4.04</td>
<td>4.11</td>
<td>4.08</td>
<td>4.04</td>
</tr>
<tr>
<td>( \Delta i )</td>
<td>1.04</td>
<td>1.05</td>
<td>1.07</td>
<td>1.05</td>
</tr>
<tr>
<td>( q )</td>
<td>4.01</td>
<td>4.07</td>
<td>3.94</td>
<td>3.95</td>
</tr>
<tr>
<td>( \Delta e )</td>
<td>9.10</td>
<td>9.07</td>
<td>4.68</td>
<td>8.69</td>
</tr>
<tr>
<td>Loss Function Value(^2)</td>
<td>27.21</td>
<td>27.16</td>
<td>26.83</td>
<td>26.78</td>
</tr>
</tbody>
</table>

1. Inflation rates are annualized, \( \Delta i \) is quarterly change in annualized interest rate.
2. We find the \( \kappa \) parameters that minimize the objective function:
   \[ 1.0*\text{Var}(\pi^{cpi}) + 0.5*\text{Var}(y) + 4.0*\text{Var}(
   \Delta i) \]
3. For all simulations, \( \kappa_i \) is fixed on 0.8.
Table 12: Optimal Forecast-Based Rules, Standard Deviations of Main Variables and Loss Function\(^{2,3}\)

\(\theta = 1\)

<table>
<thead>
<tr>
<th></th>
<th>Optimizing on: (\kappa_i, \kappa_{\pi}, \kappa_y)</th>
<th>Optimizing on: (\kappa_q)</th>
<th>Optimizing on: (\kappa_{\Delta e})</th>
<th>Optimizing on: (\kappa_i, \kappa_{\pi}, \kappa_y, \kappa_q, \kappa_{\Delta e})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\kappa_i)</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>(\kappa_{\pi})</td>
<td>2.93</td>
<td>2.93</td>
<td>2.93</td>
<td>2.81</td>
</tr>
<tr>
<td>(\kappa_y)</td>
<td>0.90</td>
<td>0.90</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>(\kappa_q)</td>
<td>--</td>
<td>0.01</td>
<td>--</td>
<td>0.02</td>
</tr>
<tr>
<td>(\kappa_{\Delta e})</td>
<td>--</td>
<td>--</td>
<td>0.12</td>
<td>0.12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>S.D. of variable(^1)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi^{\text{cpi}})</td>
<td>3.73</td>
<td>3.72</td>
<td>3.68</td>
</tr>
<tr>
<td>(\pi^c)</td>
<td>3.96</td>
<td>3.95</td>
<td>3.96</td>
</tr>
<tr>
<td>(y)</td>
<td>3.95</td>
<td>3.96</td>
<td>3.98</td>
</tr>
<tr>
<td>(\Delta i)</td>
<td>1.09</td>
<td>1.09</td>
<td>1.11</td>
</tr>
<tr>
<td>(q)</td>
<td>3.93</td>
<td>3.94</td>
<td>3.90</td>
</tr>
<tr>
<td>(\Delta e)</td>
<td>8.84</td>
<td>8.84</td>
<td>8.66</td>
</tr>
</tbody>
</table>

| Loss Function Value\(^2\) | 26.48          | 26.48          | 26.41          | 26.40          |

1. Inflation rates are annualized, \(\Delta i\) is quarterly change in annualized interest rate.
2. We find the \(\kappa\) parameters that minimize the objective function:
   \[
   1.0 \cdot \text{Var}(\pi^{\text{cpi}}) + 0.5 \cdot \text{Var}(y) + 4.0 \cdot \text{Var}(\Delta i)
   \]
3. For all simulations, \(\kappa_i\) is fixed on 0.8.
Table 13: Optimal Forecast-Based Rules, Standard Deviations of Main Variables
and Loss Function$^{2,3}$

$\theta = 2$

<table>
<thead>
<tr>
<th></th>
<th>Optimizing on: $\kappa_i, \kappa_\pi, \kappa_y$</th>
<th>Optimizing on: $\kappa_q$</th>
<th>Optimizing on: $\kappa_{\Delta e}$</th>
<th>Optimizing on: $\kappa_\pi, \kappa_y, \kappa_q, \kappa_{\Delta e}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_i$</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>$\kappa_\pi$</td>
<td>3.12</td>
<td>3.12</td>
<td>3.12</td>
<td>3.38</td>
</tr>
<tr>
<td>$\kappa_y$</td>
<td>0.81</td>
<td>0.81</td>
<td>0.81</td>
<td>0.80</td>
</tr>
<tr>
<td>$\kappa_q$</td>
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<td>-0.05</td>
<td>--</td>
<td>-0.13</td>
</tr>
<tr>
<td>$\kappa_{\Delta e}$</td>
<td>--</td>
<td>--</td>
<td>-0.06</td>
<td>-0.09</td>
</tr>
<tr>
<td>S.D. of variable$^1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi^{\text{cpi}}$</td>
<td>3.78</td>
<td>3.79</td>
<td>3.81</td>
<td>3.76</td>
</tr>
<tr>
<td>$\pi^c$</td>
<td>4.06</td>
<td>4.08</td>
<td>4.07</td>
<td>4.04</td>
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<tr>
<td>$y$</td>
<td>3.87</td>
<td>3.85</td>
<td>3.86</td>
<td>3.87</td>
</tr>
<tr>
<td>$\Delta i$</td>
<td>1.14</td>
<td>1.14</td>
<td>1.12</td>
<td>1.14</td>
</tr>
<tr>
<td>$q$</td>
<td>3.87</td>
<td>3.84</td>
<td>3.89</td>
<td>3.87</td>
</tr>
<tr>
<td>$\Delta e$</td>
<td>8.68</td>
<td>8.68</td>
<td>8.77</td>
<td>8.77</td>
</tr>
</tbody>
</table>

1. Inflation rates are annualized, $\Delta i$ is quarterly change in annualized interest rate.
2. We find the $\kappa$ parameters that minimize the objective function:
   \[1.0\text{Var}(\pi^{\text{cpi}}) + 0.5\text{Var}(y) + 4.0\text{Var}(\Delta i)\]
3. For all simulations, $\kappa_i$ is fixed on 0.8.
Table 14: Optimal Forecast-Based Rules, Standard Deviations of Main Variables and Loss Function$^{2,3}$

$\theta = 3$

<table>
<thead>
<tr>
<th>Optimizing on: $\kappa_i, \kappa_\pi, \kappa_y$</th>
<th>Optimizing on: $\kappa_q$</th>
<th>Optimizing on: $\kappa_{i\pi}, \kappa_q, \kappa_{i\pi}$</th>
<th>Optimizing on: $\kappa_\pi, \kappa_y, \kappa_q, \kappa_{i\pi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_i$</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>$\kappa_\pi$</td>
<td>3.46</td>
<td>3.46</td>
<td>3.46</td>
</tr>
<tr>
<td>$\kappa_y$</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>$\kappa_q$</td>
<td>--</td>
<td>0.12</td>
<td>--</td>
</tr>
<tr>
<td>$\kappa_{i\pi}$</td>
<td>--</td>
<td>--</td>
<td>0.04</td>
</tr>
</tbody>
</table>

S.D. of variable$^1$

| $\pi^{cpi}$ | 4.05 | 4.02 | 4.03 | 4.05 |
| $\pi^c$ | 4.36 | 4.33 | 4.35 | 4.36 |
| $y$ | 3.68 | 3.67 | 3.64 | 3.69 |
| $\Delta i$ | 1.19 | 1.22 | 1.22 | 1.18 |
| $q$ | 3.78 | 3.82 | 3.75 | 3.81 |
| $\Delta e$ | 8.61 | 8.62 | 8.56 | 8.58 |

Loss Function Value$^2$

| 28.85 | 28.84 | 28.85 | 28.81 |

1. Inflation rates are annualized, $\Delta i$ is quarterly change in annualized interest rate.
2. We find the $\kappa$ parameters that minimize the objective function:
   \[1.0 \times \text{Var}(\pi^{cpi}) + 0.5 \times \text{Var}(y) + 4.0 \times \text{Var}(\Delta i)\]
3. For all simulations, $\kappa_i$ is fixed on 0.8.
6. The model's impulse response functions

In this section we present dynamic elasticities (impulse responses) of the endogenous variables, with respect to each of the following shocks: monetary policy, nominal exchange rate and the output gap (demand shock). The simulations are based on equations (F.1b), (F.2b), (F.4a) and (F.5) as described in Section 5. To close the model we used the "optimal" simple rule of Section 5. (The parameters of the rule are: \( \theta = 1, \kappa_i = 0.8, \kappa_x = 2.93, \kappa_y = 0.5, \kappa_q = 0.0 \) and \( \kappa_{\Delta e} = 0.0 \).) In addition, we will present the same impulse responses when monetary policy is operated according to estimated equation (F.6a). This alternative employs a backward-looking rule (\( \theta = 0 \)) and less aggressiveness in reaction to inflation and the output gap. In the following we shall refer briefly to some interesting characteristics of the model, that can be inferred from the impulses.

Figure 6.1 presents impulse responses with respect to one percentage point (p.p.) shock to the nominal interest rate. Notice that all the endogenous variables are immediately affected by the shock and that the immediate response is the largest. Usually in estimated models one would expect to find a hump-shaped response of inflation and the output gap to the interest rate shock. The large and quick response of the endogenous variables in this model is partly due to the existence of the exchange rate channel and the large immediate exchange rate pass-through. The unexpected increase in the interest rate causes an immediate decline of the exchange rate which immediately affects inflation and the real exchange rate, and through it – the output gap. Another reason for the quick response is the relatively large coefficients of expectations in the inflation, output gap and exchange rate equations (that is, the large degree of which the model is forward-looking).

In figure 6.2 we present the impulse responses with respect to an unexpected 10-percentage-points (p.p.) shock to the exchange rate. Notice that here as well (as in figure 6.1), the immediate response is the largest. Due to the concurrent positive response of the interest rate, the 10 p.p. shock ends in only a 7.5 p.p. devaluation in the first quarter. Concurrently, CPI inflation increases by 2.5 p.p., indicating immediate pass-through of 1/3 with respect to the CPI inflation. The nominal interest rate increases immediately by 0.5 p.p. and peaks at 0.9 p.p. above the steady state (in the baseline simulation). The real rate temporarily drops in the first quarter; however, as the nominal rate continues to rise and inflation re-converges, the real rate peaks at
1.2 p.p. two quarters following the shock. As a consequence, output falls for two years, where the peak is two quarters after the shock.

In the alternative simulation with the empirical monetary policy rule, the interest rate path is 0.2 p.p. lower for three periods. As a result, inflation and output are higher. In addition, we notice that the interest rate starts reverting down to the steady state one period after the baseline case. This is due to the backward-looking nature of the policy rule. The year-on-year inflation measure used in the rule drops four quarters after the shock – the time in which the interest rate reverts to steady state. Ceteris paribus, this causes a deeper output gap reaction.

The above simulations highlight the role of the exchange rate in the transmission mechanism of monetary policy in an open economy. The high exchange rate pass-through means high sensitivity of the inflation rate to shocks to the exchange rate. On the other hand, the high exchange rate pass-through also enhances the effectiveness of the nominal interest rate as a tool for stabilizing inflation.

In figure 6.3 we show the impulse responses with respect to a shock in the output gap equation. In the first quarter the nominal interest rate increases in the intention to stabilize output. This causes an immediate decrease of the exchange rate, which sets off the inflationary effect of the demand shock so that year-on-year inflation is soon below the steady state. This open-economy feature demonstrates the policy tradeoff between stabilization of output and inflation in light of demand-side shocks. This calls for a careful choice of the output gap reaction parameter. As can be seen from the alternative simulation, empirically, interest rates reacted less to output gaps, leaving some more short-run inflationary pressures.
Figure 6.1: Impulse Response Function to 1 p.p. Interest Rate Shock
With optimal-simple rule (baseline) Vs. estimated rule (alternative)

\(i\) – Nominal interest rate
\(\pi^{(p)}\) – Quarterly CPI inflation (annualized)
\(y\) – output gap
\(\Delta e\) – Quarterly depreciation rate (annualized)

_base – Baseline simple-optimal rule
_alt – Alternative estimated rule
Figure 6.2: Impulse Response Function to 10 p.p. Exchange Rate Shock

With optimal-simple rule (baseline) Vs. estimated rule (alternative)

- $i$ – Nominal interest rate
- $r$ – Real interest rate
- $\pi^cpi$ – Quarterly CPI inflation (annualized)
- $\pi^{4cpi}$ – Year on year CPI inflation
- $y$ – output gap
- $q$ – real exchange rate gap
- $\Delta e$ – Quarterly depreciation rate (annualized)
- $e$ – (log) Nominal exchange rate

_base – Baseline simple-optimal rule
_alt – Alternative estimated rule
Figure 6.3: Impulse Response Function to 1 p.p. Output Gap Shock

With optimal-simple rule (baseline) Vs. estimated rule (alternative)

\begin{itemize}
  \item $i$ – Nominal interest rate
  \item $\pi^{q}$ – Quarterly CPI inflation (annualized)
  \item $y$ – output gap
  \item $\Delta e$ – Quarterly depreciation rate (annualized)
  \item $e$ – (log) Nominal exchange rate
  \item $\pi^{4}$ – Year on year CPI inflation
  \item $q$ – real exchange rate gap
  \item _base – Baseline simple-optimal rule
  \item _alt – Alternative estimated rule
\end{itemize}
7. Conclusions

In this paper, we specified and estimated a New Keynesian model for the Israeli economy. The specification relies on standard New Keynesian theory, with some adjustments to Israel's unique features. Classical estimation of such models is often unsuccessful. In our case, classical estimation techniques did produce fairly good results that are consistent with theory and with results obtained for other economies.

One of the main findings is the importance of the exchange rate in the Israeli economy’s transmission mechanism. The exchange rate affects CPI inflation directly through import prices and housing prices, and indirectly through its influence on the output gap. In addition, the direct influence of the exchange rate on inflation, and its effect on the output gap, are stronger than in other economies. The sensitivity of inflation and the output gap to the exchange rate and to the exogenous shocks that characterize it, is manifested in their relatively large fluctuations, and as a result also in the fluctuations of the nominal interest rate.

On the other hand, the intensity of the exchange rate channel increases the influence of monetary policy on inflation. Thus, changes in the interest rate rapidly affect prices via the exchange rate.

The estimation of the inflation equation showed that there is a rapid pass-through from prices abroad and from the exchange rate to import prices in the domestic economy. About 10 percent of a one-quarter-ahead expected depreciation is transmitted immediately into higher import prices. Once a depreciation has occurred, some 40 percent of it is transmitted into higher import prices in the same quarter, while the remaining 50 percent is transmitted in the following two quarters.

The estimation showed that the weight of imported goods and services in overall CPI is about 0.37. This estimate is higher than the one obtained for other open economies (where it is about 0.3). In addition, the exchange rate directly affects the housing component of the CPI (which accounts for about 20 percent of the CPI). The pass-through in this component is rather fast – some 85 percent is transmitted immediately and the rest in the following quarter. This instance, of the price of a component that is not imported (and is not even tradable) being almost completely linked to the shekel/dollar exchange rate, is, to the best of our knowledge, unique to the Israeli economy – a legacy from the era of hyper-inflation that prevailed during the years 1978 to 1985. In order to better understand and forecast inflation and to
reliably estimate the exchange rate /CPI relation, we found it worthwhile to separate the housing component from the CPI and to estimate a separate equation for it.

Also, in the case of the output gap equation it was found that the exchange rate (in this case, the real exchange rate) has a larger effect than that found in similar studies of other economies. This finding is further evidence of the Israeli economy’s degree of openness and its sensitivity to exogenous global shocks.

The estimation of the inflation and output gap equations yielded a high coefficient for the expectations term (0.6 and 0.65 respectively). This indicates a low degree of inertia in both variables.

In most economies, a relatively long lag is found in the influence of the interest rate on inflation. In contrast, our study found that the transmission mechanism from the interest rate to inflation in the Israeli economy is rapid, even in comparison to other small open economies. This finding is a result of two factors: 1) the immediate effect of the interest rate on the exchange rate and the strong and rapid effect of a depreciation on inflation; 2) the high coefficients obtained for the expectation variables in the equations for inflation and the output gap. The strong influence of expectations in these two equations is consistent with the New Keynesian theory, which emphasizes the importance of the public’s expectations in the transmission mechanism.

In the search for a simple optimal monetary policy rule we found two policy implications: (1) regardless of the strong and fast exchange rate channel described above, the central bank does not gain much by directly reacting to its (nominal or real) innovations. The CPI inflation provides enough information for conduct of monetary policy. (2) Due to the fast transmission from shocks and interest rates to inflation, and the low persistence characterizing it, monetary policy should be based on a hybrid backward-forward-looking measure of inflation.
Appendix A – Deriving Equation (26)

Equation (25) is a hybrid local prices Phillips Curve in which inflation dynamics are determined by the output gap and the real price of imported inputs.

\[
\pi_t^h = \lambda E_t \pi_{t+1}^h (1-\lambda) \pi_{t-1}^h + \lambda \frac{\theta}{c} y_t + \lambda \frac{w_{t}}{c} (p_{t}^{\phi} - p_{t}^{h}).
\]

For practical purposes (since \( p_t^h \) is unobservable) we shall re-write the last term. First let us add and subtract \( p_t^c \), the local price of imported consumption goods:

\[
(A.1) \quad (p_t^{e} - p_t^{h}) = (p_t^{e} - p_t^{c}) + (p_t^{c} - p_t^{h}).
\]

Next we shall use the definition of \( p_t^c \) given in equation (10), in order to eliminate \( p_t^c \):

\[
(A.2) \quad (p_t^{e} - p_t^{h}) = (p_t^{e} - p_t^{c}) + \frac{1}{(1-w_{t}^{e})} (p_t^{c} - p_t^{h}).
\]

Next we can use the general definition of incomplete pass-through given in equations (13) and (27):

\[
(A.3) \quad (p_t^{e} - p_t^{h}) = (p_t^{e} - p_t^{c}) + \frac{1}{(1-w_{t}^{e})} (p_t^{c} + e_t - p_t^{h}) + \left( \psi_t^{e} + \frac{w_{t}^{e}}{1-w_{t}^{e}} \psi_t^{h} \right).
\]

Finally we define the relative price of imported production inputs \( p_t^{r} = (p_t^{r} - p_t^{h}) \) and use the definition of the real exchange rate (12):

\[
(A.4) \quad (p_t^{e} - p_t^{h}) = p_t^{r} + \frac{1}{(1-w_{t}^{e})} q_t + \left( \psi_t^{e} + \frac{w_{t}^{e}}{1-w_{t}^{e}} \psi_t^{h} \right).
\]

In this form \((p_t^{e} - p_t^{h})\) is a sum of three components: the first, \( p_t^{r} \), is exogenous and measurable, The second, \( \frac{1}{(1-w_{t}^{e})} q_t \), is endogenous and measurable, and the third, \( \psi_t^{e} + \frac{w_{t}^{e}}{1-w_{t}^{e}} \psi_t^{h} \), is not measurable and for it we shall have to use a proxy. Plugging (A.4) in equation (25) we derive equation (26):

\[
\pi_t^h = \lambda E_t \pi_{t+1}^h (1-\lambda) \pi_{t-1}^h + \lambda \frac{\theta}{c} y_t + \lambda \frac{w_{t}}{c} \left[ p_t^{r} + \frac{1}{(1-w_{t}^{e})} q_t + \left( \psi_t^{e} + \frac{w_{t}^{e}}{1-w_{t}^{e}} \psi_t^{h} \right) \right].
\]
Appendix B – Description of the Data

The estimation of the model used three types of data: gaps, rates of change and interest rates. This appendix will describe each type and its source. In general, quarterly rates of change and interest rates are annualized while the gaps are expressed in conventional quarterly terms. However, in the estimation of each equation we normalize all variables to the same term, thus allowing the direct interpretation of the estimated parameters.

1. Gaps

All the gaps were calculated using the HP filter. In what follows, we denote the smoothing of series $X$ using the HP filter as $\text{HP}(X)$. The gap of the original series $Z$ (for example, business sector real output) is denoted by $z$ (the output gap) and is calculated as follows:

$$
(B.1) \quad z = \left[\log(Z) - \text{HP}(\log(Z))\right] \times 100.
$$

The term in square brackets is the deviation rate of $Z$ from its trend. Multiplying by 100 expresses the gap in percent.

Following is a description of the gaps included in the model:

$y$ – the output gap which is based on the business sector real GDP ($Y$). The trend was calculated using data starting from 1968:I. Source: Bank of Israel (BoI) database.

$g^h$ – the public consumption gap which is based on the real purchases component of the public sector ($G^h$). The trend was calculated using data starting from 1964:I. Source: BoI database.

$\text{inv}^h$ – the investment gap which is based on real gross investment ($\text{INV}^h$). The trend was calculated using data starting from 1964:I. Source: BoI database.

$y^*$ – the world trade gap which is based on the real imports of the industrialized countries$^{32}$ ($Y^*$). The trend was calculated on the basis of data starting in 1960:I. Source: the IFS database. The IFS database series contains a break in 1996:I due to a definition change. Therefore the IFS yearbook and older versions of the series were used to chain data before 1996 to the reliable database series following 1996.

$^{32}$ Volume of imports divided by unit value.
\( q \) – the real exchange rate gap is based on the real exchange rate \( (Q) \) which is calculated as follows:

\[
(B.2) \quad Q_t = \frac{P_t^\ast \cdot E_t}{P_t^r},
\]

where \( P_t^r \) is the CPI excluding housing, fruit and vegetables, \( E_t \) is the nominal exchange rate and \( P_t^\ast \) is Fisher's price index for imported consumer goods (in dollar terms). The trend was calculated using data starting from 1991:I. Source of all variables: BoI database.

\( p^{i\ast} \) – the relative world price of imported inputs gap, based on the price ratio calculated as follows:

\[
(B.3) \quad P_t^{i\ast} = \frac{P_t^{i\ast\ast}}{P_t^r},
\]

where \( P_t^{i\ast\ast} \) is a weighted average of Fisher's price index for imported inputs to production and investment goods (in dollar terms). The weights are based on the relative weights of imported inputs and investment goods. (The weight of inputs to production is 0.75.) \( P_t^r \) is Fisher's price index for imported consumer goods (in dollar terms). The trend of the ratio was calculated using data starting from 1991:I. Source of all variables: BoI database.

2. Rates of change

The percentage rate of change in variable \( X \), denoted by \( \Delta x \) (or \( \pi^x \)), is calculated as follows:

\[
(B.4) \quad \Delta x = (x - x_{-1}) \ast 100 \ast 4,
\]

where

\[
(D.5) \quad x = \log(X).
\]

Multiplying by 100 transforms the rate of change into percent and multiplying by 4 annualizes it.

Following is a description of the change variables included in the model:

\( \pi^{opi} \) – inflation in the consumer price index (CPI). Source: BoI database.
\( \pi^{c} \) – inflation in the CPI excluding the housing, fruit and vegetables components. The fruit and vegetables data are very noisy, with irregular seasonality. The prices of owner-occupied housing services in Israel are measured by the imputed rent approach. However rents account for only 15% of the housing market and, due to old habits, are nominated in US dollars. Therefore, in the short run the price index of housing services is mainly determined by the evolution of the exchange rate despite being a non-traded good. We choose not to deal with this complexity in our behavioral model (Phillips curve and output gap equation), and therefore omitted it from the general price index. In total, we exclude 25% of the CPI. Source: BoI database.

\( \pi^{\text{house}} \) – inflation in the CPI housing component. Source: BoI database.

\( \Delta e \) – depreciation in the nominal exchange rate based on the shekel/dollar exchange rate. Source: BoI database.

\( \Delta p^{c} \) – percent change in the world price of imported consumption goods based on Fisher's price index for imported consumption goods (in dollar terms). Source: BoI database.

\( \Delta p^{z} \) – percent change in the world price of inputs. Based on a weighted average Fisher's price index for imported inputs to consumption and investment goods (in dollar terms). Source: BoI database.

\( \Delta p^{\text{**}} \) - percent change in the price of world trade (serves only as an instrumental variable) based on the unit value of industrialized countries' imports. Source: the IFS database.

The following additional change-related variables are included in the model estimation:

\( \pi^{T} \) – the announced year-ahead inflation target (in terms of total CPI). Source: BoI database.

\( \pi^{CM} \) – one-year-ahead inflation expectations derived from the capital markets (in terms of total CPI). The expectations are derived from the difference between the yields on nominal and CPI-indexed government bonds. Source: BoI database.

3. Interest rates
The interest rates are annualized. \( i \) is the effective Bank of Israel interest rate and \( i^{*} \) is the one-month LIBID dollar interest rate. The average forward rate of interest for five
to ten years, denoted in the equations as \( r^n \), is a proxy for the natural real interest rate. It is derived from the trade in government fixed rate CPI linked bonds (“Galil”). The source of all interest rates: BoI database.

The expectations variables also require explanation. Whenever an expectation variable appears in the estimated equations (for instance, \( E_{xt+1} \), \( E_{xt+2} \) or \( E_{t-1} xt \)) we followed the rational expectations assumption and used the actual value (\( x_{t+1} \), \( x_{t+2} \) and \( x_t \) respectively) as an unbiased proxy.

Appendix C – Instrumental Variables
C.1 - The (non-housing, fruit and vegetables) inflation equation:
Equation (F.1a) includes the following instrumental variables:
- The constant and seasonal dummies.
- **Current value and first three lags of** the percent change in world prices of imported consumer goods and production inputs, the percent change of world trade prices, the relative world price of imported inputs (deviation from trend) and the dollar interest rate.
- **The first three lags of** the quarterly CPI inflation (excluding housing, fruit and vegetables), the nominal exchange rate depreciation, the output gap, the real exchange rate gap and the effective BoI interest rate.

Total of 39 instrumental variables.

Equations (F.1b) and (F.1c) include the following instrumental variables:
- The constant and seasonal dummies.
- **The first three lags of** the quarterly CPI inflation (excluding housing, fruit and vegetables), the percent change in world prices of imported consumer goods and production inputs, the percent change of world trade prices, the nominal exchange rate depreciation, the output gap, the relative world price of imported inputs (deviation from trend), the real exchange rate gap, the effective BoI interest rate and the dollar interest rate.

Total of 34 instrumental variables.
C.2 - The CPI housing component inflation equation:
Equation (F.2) includes the following instrumental variables:
- The constant and seasonal dummies.
- **Current value and first two lags of** the rate of the dollar interest rate.
- **The first two lags of** the CPI housing component inflation, the nominal exchange rate depreciation and the effective BoI interest rate.

Total of 13 instrumental variables.

C.3 - The output gap equation:
Equation (F.4) includes the following instrumental variables:
- The constant and seasonal dummies.
- **Current value and first two lags of** the public consumption gap, the dollar interest rate and the percent change in world prices of imported consumer goods and production inputs.
- **The first two lags of** the output gap, the investment gap, the effective BoI interest rate, the natural real interest rate, the quarterly CPI inflation (excluding housing, fruits and vegetables), the nominal exchange rate depreciation and the real exchange rate gap.

Total of 33 instrumental variables.

C.4 - The exchange rate equation:
Equation (F.5) includes the following instrumental variables:
- The constant.
- **Current value and first two lags of** the dollar interest rate.
- **The first two lags of** the output gap, the investment gap, the effective BoI interest rate, the natural real interest rate and the exchange rate.

Total of 8 instrumental variables.

C.5 - Interest rate equation:
Equations (F.6a), (F.6b), (F.6d), and (F.6e) include the following instrumental variables:
- The constant.
- Current value of the inflation target.
- Current value and first two lags of the dollar interest rate and percent change in world prices of imported consumer goods.
- The first two lags of the output gap, the effective BoI interest rate, the natural real interest rate, the real exchange rate gap and the nominal exchange rate depreciation.
- The first four lags of the quarterly CPI inflation (excluding housing, fruit and vegetables).

Total of 22 instrumental variables.

Equations (F.6c) and (F.6f) includes the following instrumental variables:

- The constant.
- Current value of the inflation target.
- Current value and first two lags of the dollar interest rate and percent change in world prices of imported consumer goods.
- The first lag of the one year ahead inflation expectations derived from the capital markets.
- The first two lags of the output gap, the effective BoI interest rate, the natural real interest rate, the real exchange rate gap and the nominal exchange rate depreciation.
- The first four lags of the quarterly CPI inflation (excluding housing, fruit and vegetables).

Total of 23 instrumental variables.
Appendix D – Standard deviations for stochastic simulations

The shocks' standard deviations and equations structure are presented in table D-1.

Table D-1: Standard Deviations in Stochastic Simulations

<table>
<thead>
<tr>
<th>Variable the shock relates to</th>
<th>Equation</th>
<th>Standard deviation</th>
<th>Variable the shock relates to</th>
<th>Equation</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ( \pi^c )</td>
<td>(F.1b)</td>
<td>2.30</td>
<td>9 ( g^h )</td>
<td>AR(2)</td>
<td>2.87</td>
</tr>
<tr>
<td>2 ( \pi^{house} )</td>
<td>(F.2b)</td>
<td>2.80</td>
<td>10 ( inv^n )</td>
<td>AR(1)</td>
<td>9.08</td>
</tr>
<tr>
<td>3 ( y )</td>
<td>(F.4a)</td>
<td>2.00</td>
<td>11 ( r^p )</td>
<td>AR(1)</td>
<td>0.36</td>
</tr>
<tr>
<td>4 ( \Delta e )</td>
<td>(F.5)</td>
<td>10.0</td>
<td>12 ( \phi )</td>
<td>AR(1)</td>
<td>0.58</td>
</tr>
<tr>
<td>5 ( i )</td>
<td>(F.6)</td>
<td>0.00</td>
<td>13 ( \Delta p^c )</td>
<td>AR(0)</td>
<td>7.88</td>
</tr>
<tr>
<td>6 ( p^{\pi*} )</td>
<td>AR(2)</td>
<td>1.54</td>
<td>14 ( q )</td>
<td>(D.1)</td>
<td>0.86</td>
</tr>
<tr>
<td>7 ( i^* )</td>
<td>AR(2)</td>
<td>0.32</td>
<td>15 ( \Delta p^{\pi*} )</td>
<td>(D.2)</td>
<td>0.93</td>
</tr>
<tr>
<td>8 ( y^* )</td>
<td>AR(2)</td>
<td>1.58</td>
<td>16 ( \pi^t )</td>
<td>AR(1)</td>
<td>0.00</td>
</tr>
</tbody>
</table>

a. Price changes and interest rates are expressed in annualized terms. Gaps are in regular quarterly terms. See Appendix B for details.
b. The exchange rate equation (F.5) was adjusted so that \( \omega = 0.45 \).
c. The dynamics in the real exchange rate can be defined by the following identity:
   \[
   (D.1) \quad q_t = q_{t-1} + 0.25 \cdot (\Delta p_t^{\pi*} + \Delta e_t - \pi_t^c) - \varepsilon_t^e,
   \]
   where \( \varepsilon_t^e \) represents changes in the natural real exchange rate. We assume \( \varepsilon_t^e \)
   is a white noise shock with standard deviation of 0.86. Similarly, the dynamics in the quarterly change in input prices \( (\Delta p_t^{\pi*}) \), can be defined through the following identity:
   \[
   (D.2) \quad \Delta p_t^{\pi*} = 4 \cdot (p_t^{\pi*} - p_{t-1}^{\pi*}) + \Delta p_t^{\pi*} + 4 \cdot \varepsilon_t^{\pi*},
   \]
   where \( \varepsilon_t^{\pi*} \) represents changes in the natural imported consumption-input price ratio. We assume \( \varepsilon_t^{\pi*} \) is a white noise shock with standard deviation of 0.93.
d. (1)-(4) are based on estimated equations reported in this paper. It should be clear that these standard errors are only approximations since the estimated residuals include exceptional errors.
e. (6)-(12) are based on the auto-regressive estimations reported in Argov et al. (2007).\textsuperscript{33}

f. (13)-(15) were estimated using full information maximum likelihood.\textsuperscript{34} For purpose of this estimation we used the estimated monetary policy rule (F.6a).

\textsuperscript{33} Section 7.

\textsuperscript{34} To be clear only those standard deviations were estimated. The rest, as well as the models coefficients, were based on the limited information estimations.
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