Inequality and education choice

Uchida, Yuki and Ono, Tetsuo

Seikei University, Osaka University

13 February 2019

Inequality and Education Choice*

Yuki Uchida† Tetsuo Ono‡
Seikei University Osaka University

Abstract

This study presents a two-class successive generations model with human capital accumulation and the choice to opt out of public education. The model demonstrates the mutual interaction between inequality and education choice and shows that this interaction leads to two locally stable steady-state equilibria. The existence of multiple stable equilibria implies a negative association between inequality and public education enrollment, which is consistent with evidence from Organisation for Economic Co-operation and Development (OECD) countries. This study also presents a welfare analysis using data from OECD countries and shows that introducing a compulsory public education system leaves the first generation worse off, although it realizes an equal society and improves welfare for future generations of lower-class individuals.

- JEL Classification Numbers: D70, H52, I24
  Keywords: Public education, opting out, inequality

*The financial support from the Grant-in-Aid for Early-Career Scientists from the Japan Society for the Promotion of Science (No. 18K12802, Uchida) and the Grant-in-Aid for Scientific Research (C) from the Japan Society for the Promotion of Science (No. 18K01650, Ono) is gratefully acknowledged.

†Corresponding author. Yuki Uchida: Faculty of Economics, Seikei University, 3-3-1 Kichijoji-Kitamachi, Musashino, Tokyo 180-8633, Japan. E-mail: yuchida@econ.seikei-u.ac.jp.

‡Graduate School of Economics, Osaka University, 1-7, Machikaneyama, Toyonaka, Osaka 560-0043, Japan. E-mail: tono@econ.osaka-u.ac.jp

1 Introduction

Compulsory school attendance laws, enforced in nearly all developed countries, require parents to have their children attend public or private school for a designated period. Public schools are entirely funded by local and state taxes, whereas private schools tend to obtain funding by charging their students tuition fees. Parents can choose either option depending on their income and preferences. Because public schooling is a kind of government intervention, higher-income parents who benefit less from it are more likely to choose private schooling. Therefore, we expect an association between higher inequality and higher enrollment (or larger spending) in private education institutions, as observed in data from the Organisation for Economic Co-operation and Development (OECD) countries depicted in Figure 1.[1]

[Figure 1 is here.]

de la Croix and Doepke (2009) develop a political economy theory that attempts to explain the above-mentioned association. They assume an exogenous income distribution and focus on the extent to which expanding inequality affects education choice. However, the reverse effect, namely the effect of education choice on inequality, is abstracted from their analysis.[2] Indeed, studies suggest that the reverse effect is important (Saint-Paul and Verdier, 1993; Zhang, 1996) and that it exists across and within countries over time (De Gregorio and Lee, 2002; Teulings and Van Rens, 2008).

The presence of this reverse effect implies a dynamic interaction between inequality and education choice over time: inequality affects adults' education choices and subsequently governmental policy, and this in turn determines the degree of inequality in the next generation. Cardak (2004a, 2004b) attempts to demonstrate this dynamic education choice–inequality interaction in a two-period overlapping generations model. In particular, Cardak (2004a) focuses on two structural parameters, namely preferences for the education of children and heritability of human capital from parents to children, and shows that these are key factors in explaining the variations in inequality and education choice.

The present study instead focuses on the initial conditions of inequality and aims to demonstrate that two countries sharing the same structural parameters show different

[1] Exceptions are Belgium and South Korea. Belgium shows high enrollment and low private spending because private schools are heavily subsidized by public funding (OECD, 2017). South Korea shows low enrollment and high private spending because parents tend to spend much more on supplementary private education due to excessive competition for entrance into schools and colleges (OECD, 2016).

[2] The reverse effect is briefly analyzed in their discussion paper version (de la Croix and Doepke, 2003). However, wages are assumed to be constant within and across generations. This point is further discussed in Section 1.1.

paths of inequality and education choice in the short and long run depending on the initial inequality conditions. In addition, we investigate the welfare implications of the differences from the political economy viewpoint. For this purpose, we follow the simple two-class successive generations model with human capital accumulation presented by Gradstein and Justman (1996) and de la Croix and Doepke (2004). We extend their frameworks by introducing the choice to opt out of public education, as in Cardak (2004a, 2004b). In particular, the model used in this study has two types of family dynasties classified according to their level of human capital (i.e., a low type and a high type). Agents from either type of family enter adulthood with a stock of human capital invested by their parents, earn after-tax income, and obtain utility from consumption and their children's human capital. Agents compare the maximized utility under each type of education and choose the one with the highest value.

Every adult agent votes on the public education expenditure in each period. This study assumes that the low type are the majority in the population. We compute the low type's preferred public education expenditure and analyze the corresponding education choice by adult agents. We show that low-type adults always choose public education because they pay less than they receive from public education. However, the high type's decision depends on income inequality. As inequality increases, the income discrepancy between the two types increases and so does the high type's tax burden. Therefore, high-type adults opt out of public education when inequality is high, while they choose public education when inequality is low. Hence, two education choices exist depending on the level of inequality in society.

As noted earlier, education choice and expenditure influence human capital formation, which in turn determines inequality in the next generation. We demonstrate this dynamic interaction between inequality and education choice across generations and show that the interaction leads to two locally stable steady-state equilibria. One steady state shows a polarized income distribution with high-type agents opting out of public education; the other steady state has perfect equality and full public education enrollment. The findings of this study are therefore novel in that they show the existence of multiple locally stable steady-state equilibria, which were not fully demonstrated in Cardak (2004a, 2004b). In addition, the finding of multiple equilibria implies a positive association between inequality and private education choice, and this may help clarify the differences in inequality and education choice among countries sharing similar economic backgrounds as observed in Figure 1.

To investigate the welfare implications of the presented model, we compare the utility of the two steady states by considering an economic environment in which the equilibrium converges to the higher-inequality steady state. We then introduce an alternative education system into this environment, namely a compulsory public school system that prohibits students from opting out of public school. This system forces the economy into the lower-inequality steady state. Therefore, we can evaluate the multiple stable steady-state equilibria by comparing the higher-inequality steady state in the mixed education system with the steady state in the compulsory public school system.

We show by simulation that almost every generation of the high type is worse off by the introduction of compulsory public schooling since expenditure on education departs from its optimal level.[3] However, the new system has a mixed effect on low-type agents. The first generation is worse off since the negative effect of the tax burden increase outweighs the positive effect of the per capita public education expenditure increase. From the second generation onward, there is an additional positive effect via the human capital formation generated by the compulsory public school system. This effect together with the positive effect of public education expenditure may outweigh the negative effect of the tax burden. The result suggests an intergenerational trade-off and that the two equilibria are not Pareto-ranked. It also suggests that the shift from a mixed education system to a compulsory public school system that aims to improve equality is not Pareto-improving.

The remainder of this paper is organized as follows. Next, we review the related literature, followed by a description of the model in Section 2. Section 3 considers agents' voting behavior and Section 4 describes the political equilibrium in each period. Section 5 shows the existence and stability of a steady-state equilibrium and clarifies the role of the structural parameter values in the determination of inequality, individual education choice, and policy. Section 6 presents a welfare analysis of the political equilibria in addition to considering the welfare implications of a compulsory public school system as an alternative. Section 7 offers some extensions to the basic model. Section 8 provides concluding remarks. All proofs are provided in the appendices.

1.1 Related Literature

Apart from the studies mentioned above, this study is related to the following three strands of the literature. The first is the static analyses of public and private education choices (e.g., Stiglitz, 1974; Epple and Romano, 1996; Glomm and Ravikumar, 1998; Hoyt and Lee, 1998; Bearse, Glomm and Patterson, 2005; de la Croix and Doepke, 2009; Arcalean and Schiopu, 2015). In particular, the present study is closely related to de la Croix and Doepke (2003). They first introduce a static model that focuses on the effect of inequality

[3]The introduction of compulsory public schooling increases the average human capital level. This in turn increases tax revenue and per capita public education expenditure, and thus may benefit the future type H. In fact, our simulation shows that the type-H is better off by the introduction of compulsory public schooling roughly from generation 800 onward. Since this benefit occurs in the far distant future, we ignore such an improvement and hence conclude that almost every generation of the type H is worse off.

on education choice. Within this framework, they present a multiplicity of equilibria displaying a short-run positive association between inequality and enrollment in private education institutions. Then, they extend their model by including the reverse effect and show an example of a period-2 cycle. However, the long-run association observed in Panel (a) of Figure 1 is not addressed. In addition, they assume constant wages within and across generations, so a change in the population distribution is the only cause of inequality. The present study advances the work of de la Croix and Doepke (2003) by demonstrating endogenous wage determination via human capital accumulation. Within this extended framework, we calibrate the model to a set of OECD member countries and present a long-run multiplicity of equilibria under empirically plausible parameter values, which is not fully addressed in de la Croix and Doepke (2003).

The second is the dynamic inequality analyses in public or private education regimes (e.g., Glomm and Ravikumar, 1992; Saint-Paul and Verdier, 1993; Gradstein and Justman, 1997; Benabou, 2000; de la Croix and Doepke, 2004; Galor, Moav, and Vollrath, 2009). However, this study departs from prior work by allowing for an endogenous education choice accompanied by voting on education policy. While Gradstein and Justman (1996) and Ono (2016) conduct similar analyses, they focus on private education as a supplement to public education. The present study instead focuses on the ability to opt out of public education, which leads to novel implications for the multiplicity and efficiency of the equilibria.[4]

The third strand relates to political economy analyses of redistribution and private education (Hassler, Rodriguez Mora, Storesletten, and Zilibotti, 2003; Hassler, Storesletten, and Zilibotti, 2007; Arawatari and Ono, 2009, 2013). In earlier frameworks, the multiple self-fulfilling expectations of agents on future in-cash redistribution policies were found to create two types of equilibria: one characterized by low inequality and high redistribution and the other characterized by high inequality and low redistribution. This multiple equilibria story implies a negative correlation between inequality and redistribution. While this is relevant to our study, these earlier works consider private education and in-cash transfers, while our study instead focuses on in-kind public education provision and allows for private education as an alternative choice.

[4] Tournemaine and Tsoukis (2015) also focus on the ability to opt out of public education. They employ a model with infinitely lived households and assume that such households choose education (i.e., private or public schooling) at the beginning of the economy and commit to their decisions in the following periods. The present study instead employs a model with overlapping generations and assumes that each generation chooses education from the viewpoint of its utility maximization. This assumption enables us to demonstrate changes in education choices across time and generations.

2 Model

We consider a discrete time successive generations economy beginning at time 1. The economy is populated by individuals who live in two periods (youth and adulthood) and belong to one of two types of family dynasties indexed by $i \in \{L, H\}$. This assumption simplifies the real economy, but it enables us to demonstrate the dynamic motion of inequality in a tractable way.

A type-i adult in period 1 is endowed with h_1^i units of human capital, where $0 < h_1^L < h_1^H$. Thus, type-L and type-H individuals in period-1 have low and high human capital, respectively. As demonstrated below, members of type H endogenously choose more education for their children than members of type L, meaning that inequality always exists within the two types. However, the extent of this inequality is determined endogenously through individuals' choices.

Each adult produces one child; thus, the population remains constant from generation to generation. The proportion of type-L individuals within each generation is ϕ, leaving $1-\phi$ as the proportion of type-H individuals, where ϕ is constant across generations and satisfies $0.5 < \phi < 1$. Therefore, type-L individuals are the majority in the economy in every period, which reflects the real-world right-skewed income distribution.

2.1 Preferences and Budget Constraints

Upon entering adulthood at time t, a type-i individual has a stock of human capital h_t^i that defines his or her effective labor capacity. He or she then inelastically supplies his or her human capital to firms to receive wages. We assume that wages are normalized to one in each period, implying that labor income is equal to the human capital level.

A type-i adult of generation t derives utility from his or her current consumption, c_t^i, and from his or her child's anticipated future income, h_{t+1}^i. Consequently, we can express the agent's preferences with the following utility function:

$$u_t^i = \ln c_t^i + \gamma \ln h_{t+1}^i,$$

where $\gamma(>0)$ is a common parameter that reflects the bequest motive. We employ this logarithmic utility function to make our analysis more manageable.

Adults have a choice between public and private education for their children, which they choose to maximize utility. However, regardless of their choice, they must pay income taxes to finance public education. Therefore, the budget constraint of a type-i adult in period t is

$$c_t^i + e_t^i \leq (1-\tau_t)h_t^i,$$

where $e_t^i (\geq 0)$ denotes type-i's private education expenditure in period t and τ_t is the period-t income tax rate.[5]

Let $q_t^i \in \{0, 1\}$ denote a binary variable representing type i's education choice: $q_t^i = 0$ when choosing private education and $q_t^i = 1$ when choosing public education. The child's level of education, h_{t+1}^i, is determined by his or her parents' human capital, h_t^i, and the parents' choice of schooling, either x_t or e_t^i, where x_t is per capita public education. In particular, we assume $h_{t+1}^i = D\left(h_t^i\right)^{1-\eta}\left(q_t^i x_t + \left(1 - q_t^i\right) e_t^i\right)^{\eta}$, where $D(> 0)$ is the total factor productivity of human capital and $\eta \in (0, 1)$ is the elasticity of schooling. We assume the following with respect to γ and η.

Assumption 1. $\gamma\eta \in (0, 1)$.

Assumption 1 is satisfied as long as $\gamma \in (0, 1)$. In Section 6, we estimate γ based on data from OECD countries and find that $\gamma = 0.138$. This estimate fits well with Cardak's (2004a) estimate of 0.13 and de la Croix and Doepke's (2004) estimate of 0.169.

2.2 Education Choice

Given the tax rate, public education, and his or her human capital, each adult chooses consumption and education to maximize his or her utility subject to the budget constraint. In particular, he or she compares the maximum utility of each education choice and chooses the option with the highest value.

Suppose that a type-i adult chooses private education, $q_t^i = 0$. He or she solves the utility maximization problem by allocating disposable income between private education and consumption as follows:

$$e_t^i = \frac{\gamma\eta}{1 + \gamma\eta}(1 - \tau_t)h_t^i,$$
$$c_t^i = \frac{1}{1 + \gamma\eta}(1 - \tau_t)h_t^i.$$

The type-i adult's utility from providing private education for his or her child, denoted by $V_{e,t}^i$, is

$$V_{e,t}^i = (1 + \gamma\eta)\ln(1 - \tau_t)h_t^i + \gamma \ln D\left(h_t^i\right)^{1-\eta} + \gamma\eta \ln \mu, \tag{1}$$

where

$$\mu \equiv \frac{\gamma\eta}{(1 + \gamma\eta)^{(1+\gamma\eta)/\gamma\eta}}.$$

[5]Private education in the present framework is assumed to be fully funded by the tuition fees paid by students' parents. However, purely private education is limited, and many private schools in OECD countries receive some government subsidies to bridge the revenue gap. The present model should be understood as a simplification of real private schooling.

Alternatively, suppose that the type-i adult chooses public education, $q_t^i = 1$. He or she chooses $e_t^i = 0$, and thus consumes all disposable income. In this case, the type-i adult's utility from choosing public education for his or her child, denoted by $V_{x,t}^i$, is

$$V_{x,t}^i = \ln(1-\tau_t)h_t^i + \gamma \ln D\left(h_t^i\right)^{1-\eta} + \gamma\eta \ln x_t. \tag{2}$$

Given the set of policies, (x_t, τ_t), each adult chooses between these education alternatives for his or her child to maximize utility. We assume that each adult chooses private education when the two alternatives are indifferent. Therefore, the type-i adult's education choice is

$$q_t^i = \begin{cases} 1 \text{ (public education)} & \text{if } \; V_{x,t}^i > V_{e,t}^i \Leftrightarrow \mu(1-\tau_t)h_t^i < x_t, \\ 0 \text{ (private education)} & \text{if } \; V_{x,t}^i \leq V_{e,t}^i \Leftrightarrow \mu(1-\tau_t)h_t^i \geq x_t. \end{cases} \tag{3}$$

The timing of events in period t is as follows. First, adult agents vote on public education, x_t. Given the voting outcome, the tax rate τ_t is set to satisfy the government's budget constraint. Second, given x_t and τ_t, each agent chooses either public or private education to maximize his or her utility. In choosing private education, agents decide how to divide their disposable income between consumption and private education subject to their budget constraints. We follow the backward induction approach to solve this multi-stage game. In particular, we first solve the second-stage problem in Section 3 and then solve the first-stage problem in Section 4.

3 Period-t Economic Equilibrium

We define the period-t economic equilibrium in the present model as follows.

Definition 1. Given h_t^i $(i = L, H)$ and x_t, the *period-t economic equilibrium* is a set of an allocation, $\left\{q_t^i, c_t^i, e_t^i, h_{t+1}^i\right\}_{i=L,H}$ and a tax rate, τ_t, such that the following conditions hold:

(i) Given h_t^i, x_t, and τ_t, a type-i agent chooses q_t^i and the corresponding c_t^i and e_t^i to maximize his or her utility;

(ii) Given h_t^i, x_t, and q_t^i, τ_t is set to satisfy the government's budget constraint, $\left\{q_t^L\phi + q_t^H(1-\phi)\right\} x_t = \tau_t h_t$;

(iii) Given h_t^i, x_t, and (q_t^i, e_t^i), which satisfy conditions (i) and (ii), h_{t+1}^i is determined by $h_{t+1}^i = D\left(h_t^i\right)^{1-\eta}\left(q_t^i x_t + \left(1-q_t^i\right)e_t^i\right)^\eta$.

To find the period-t economic equilibrium solution, we introduce an inequality index ρ_t. Let h_t denote the average human capital in period t, $h_t \equiv \phi h_t^L + (1-\phi)h_t^H$, and let ρ_t

denote the ratio of h_t^L to h_t,

$$\rho_t \equiv \frac{h_t^L}{h_t} \in (0,1].$$

The index ρ_t suggests that a larger (smaller) ρ_t implies lower (higher) income inequality between the high-type and low-type groups, and thus a more equal (unequal) society. By using this inequality index and the definition of average human capital, we can rewrite the ratio h_t^H/h_t as

$$\frac{h_t^H}{h_t} = \frac{1-\phi\rho_t}{1-\phi}.$$

Therefore, we replace the two state variables h_t^L and h_t^H with h_t and ρ_t in the following analysis.

By using the definitions of h_t and ρ_t, we can reformulate the condition in (3) and obtain the corresponding pair of education choices in the period-t economic equilibrium. First, suppose that both types of adults choose private education, $\left(q_t^L, q_t^H\right) = (0,0)$. Condition (ii) in Definition 1 implies that the government's budget constraint is reduced to $\tau_t = 0$ because no agent will choose public education. By substituting $\tau_t = 0$ into (3) and rearranging the terms, we obtain

$$\left(q_t^L, q_t^H\right) = (0,0) \text{ if } x_t \leq x_t^{00} \equiv \mu\rho_t h_t, \tag{4}$$

where the superscript "00" of x_t^{00} implies $\left(q_t^L, q_t^H\right) = (0,0)$.

Second, suppose that only type-L adults choose public education, $\left(q_t^L, q_t^H\right) = (1,0)$. The government's budget constraint is $\phi x_t = \tau_t h_t$. By substituting this into (3) and rearranging the terms, we obtain

$$\begin{aligned} q_t^L &= 1 \text{ if } x_t^{10} \equiv \frac{\mu\rho_t}{1+\mu\phi\rho_t} h_t < x_t, \\ q_t^H &= 0 \text{ if } x_t \leq \bar{x}_t \equiv \frac{\mu\left(1-\phi\rho_t\right)/(1-\phi)}{1+\mu\phi\left(1-\phi\rho_t\right)/(1-\phi)} h_t, \end{aligned}$$

where the superscript "10" of x_t^{10} means $\left(q_t^L, q_t^H\right) = (1,0)$ and the bar of $\bar{x}_t$ indicates the upper limit of x_t, which induces type-H adults to choose private education. Therefore, we obtain

$$\left(q_t^L, q_t^H\right) = (1,0) \text{ if } x_t^{10} < x_t \leq \bar{x}_t. \tag{5}$$

Third, suppose that both types of adults choose public education, $\left(q_t^L, q_t^H\right) = (1,1)$. The government's budget constraint is then $x_t = \tau_t h_t$. Following the same procedure above, we obtain

$$\left(q_t^L, q_t^H\right) = (1,1) \text{ if } x_t^{11} \equiv \frac{\mu\left(1-\phi\rho_t\right)/(1-\phi)}{1+\mu\left(1-\phi\rho_t\right)/(1-\phi)} h_t < x_t, \tag{6}$$

where the superscript "11" of x_t^{11} indicates $\left(q_t^L, q_t^H\right) = (1, 1)$. Finally, the case $\left(q_t^L, q_t^H\right) = (0, 1)$ is not feasible in the present framework because of the income distribution assumption.

The analysis thus far suggests that education choice is affected by the four threshold values of x_t, denoted by x_t^{00}, x_t^{10}, $\bar{x}_t$, and x_t^{11}. The order of these values depends on the inequality ρ_t. In particular, there are three critical values of ρ_t, denoted by ρ^l, ρ^m, and ρ^h, where $0 < \rho^l < \rho^m < \rho^h < 1$, such that

$$\begin{cases} x_t^{00} \lesseqgtr x_t^{11} \Leftrightarrow \rho_t \lesseqgtr \rho^l, \\ x_t^{00} \lesseqgtr \bar{x}_t \Leftrightarrow \rho_t \lesseqgtr \rho^m, \\ x_t^{10} \lesseqgtr x_t^{11} \Leftrightarrow \rho_t \lesseqgtr \rho^h. \end{cases} \tag{7}$$

We provide the proof of (7) in Appendix A.1. Figure 2 illustrates the four cases of ρ_t that classify the ordering of the four threshold values of x_t and the corresponding education choice by each type of adult. Figure 3 summarizes the four cases, which are precisely stated in the following proposition.

[Figure 2 here.]

[Figure 3 here.]

Proposition 1. *There is a unique period-t economic equilibrium if any of the following three conditions hold: (i)* $x_t \leq \min\{x_t^{10}, x_t^{11}\}$, *(ii)* $\max\{\bar{x}_t, x_t^{00}\} < x_t$, *or (iii)* $\rho_t \in \left(0, \rho^l\right)$ *and* $x_t \in [x_t^{00}, x_t^{11}]$. *Otherwise, there are multiple period-t economic equilibria.*

Proof. See Appendix A.2.

The result in Proposition 1 suggests that the economic equilibrium is unique if the level of public education is low or high. If not, there are multiple equilibria or a unique equilibrium. To understand this result, let us first consider the case of a low x such that $x_t \leq \min\{x_t^{10}, x_t^{11}\}$. Because of the low level of public education expenditure, the tax rate could be reduced to satisfy the government's budget constraint, regardless of education choice. This fact implies a low tax burden, making private education more affordable even for type-L adults. Therefore, there is a unique economic equilibrium at which both types of adults choose private education if $x_t \leq \min\{x_t^{10}, x_t^{11}\}$.

Second, consider the case of a high x such that $\max\{\bar{x}_t, x_t^{00}\} < x_t$. The government is required to set a high tax rate to satisfy its budget constraint. This creates a negative income effect, which in turn makes private education less affordable, even for type-H adults. Thus, there is a unique economic equilibrium at which both types of adults choose public education if $\max\{\bar{x}_t, x_t^{00}\} < x_t$.

Finally, for the intermediate case such that $\min\{x_t^{10}, x_t^{11}\} < x_t \leq \max\{\bar{x}_t, x_t^{00}\}$, the uniqueness or multiplicity of the equilibria depend on the tax rate that satisfies the government's budget constraint. For example, consider a low inequality case such that $\rho_t \in \left(0, \rho^l\right)$ holds, as illustrated in Panel (a) of Figure 2, and focus on the public education expenditure level with $x_t \in (x_t^{11}, \bar{x}_t]$. If the tax rate is sufficiently low that $\tau_t = \phi x_t/h_t$, then type-H adults can afford to invest in private education, and this choice is consistent with the condition of $\tau_t = \phi x_t/h_t$. However, if the tax rate is sufficiently high that $\tau_t = x_t/h_t$, then they find it optimal to choose public rather than private education from the viewpoint of utility maximization. This choice is consistent with $\tau_t = x_t/h_t$.

4 Period-t Political Equilibrium

Based on the characterization of the period-t economic equilibrium in Section 3, we demonstrate voting on education policy. We assume that adults vote sincerely since every agent has zero mass and thus no individual vote can change the outcome. In addition, in each period t, adult agents determine public education through a political process of majority voting. Assuming $\phi > 0.5$, type-L adults constitute the majority. Therefore, the political objective function in period t, denoted by Ω_t, is the indirect utility function of adult type-L agents.

Definition 2. Given ρ_t, a *period-t political equilibrium* is a level of public education expenditure, x_t, such that x_t maximizes type-L adults' utility subject to each type's education choice as well as the corresponding consumption functions and government's budget constraints.

We write the period-t political objective function according to the pair of education choices, $\left(q_t^L, q_t^H\right)$ demonstrated in (4), (5), and (6). Recall that the government's budget constraint is

$$\tau_t h_t = \left\{ \begin{array}{ccc} \phi x_t & \text{if} & \left(q_t^L, q_t^H\right) = (1,0), \\ x_t & \text{if} & \left(q_t^L, q_t^H\right) = (1,1), \end{array} \right\}$$

and substitute this into the indirect utility function for type-L adults. Then, the political objective function becomes

$$\Omega_t = \left\{ \begin{array}{lll} \Omega_{00,t} \equiv V_{e,t}^L\big|_{\tau_t=0} = (1+\gamma\eta)\ln h_t\rho_t + \gamma \ln D\left(h_t\rho_t\right)^{1-\eta} + \gamma\eta\ln\mu & \text{if} & \left(q_t^L, q_t^H\right) = (0,0), \\ \Omega_{10,t} \equiv V_{x,t}^L\big|_{\tau_t=\phi x_t/h_t} = \ln\left(h_t - \phi x_t\right)\rho_t + \gamma \ln D\left(h_t\rho_t\right)^{1-\eta} + \gamma\eta\ln x_t & \text{if} & \left(q_t^L, q_t^H\right) = (1,0), \\ \Omega_{11,t} \equiv V_{x,t}^L\big|_{\tau_t=x_t/h_t} = \ln\left(h_t - x_t\right)\rho_t + \gamma \ln D\left(h_t\rho_t\right)^{1-\eta} + \gamma\eta\ln x_t & \text{if} & \left(q_t^L, q_t^H\right) = (1,1). \end{array} \right. \tag{8}$$

The functions $\Omega_{00,t}$, $\Omega_{10,t}$, $\Omega_{11,t}$ have the following properties: $\Omega_{00,t}$ is independent of x_t because both types opt out of public education, whereas $\Omega_{10,t}$ and $\Omega_{11,t}$ depend on x_t because either or both types choose public education. In particular, the solutions that maximize $\Omega_{10,t}$ and $\Omega_{11,t}$ are, respectively

$$\left\{ \begin{array}{c} \arg\max \Omega_{10,t} = x_t^* \equiv \frac{\gamma\eta}{\phi(1+\gamma\eta)} h_t, \\ \arg\max \Omega_{11,t} = x_t^{**} \equiv \frac{\gamma\eta}{1+\gamma\eta} h_t \left(< x_t^*\right). \end{array} \right. \tag{9}$$

In addition, the following conditions hold:

$$\begin{cases} \Omega_{11,t} < \Omega_{10,t} \ \forall x_t > 0, \\ x_t^{00} < x_t^{**} = \arg\max \Omega_{11,t}, \\ \Omega_{10,t}|_{x_t = x_t^{00}} < \Omega_{00,t}, \\ \Omega_{00,t} < \Omega_{11,t}|_{x_t = x_t^{**}} < \Omega_{10,t}|_{x_t = x_t^{*}}, \end{cases} \tag{10}$$

where the proof is provided in Appendix A.3.

Given h_t and ρ_t, the period-t political equilibrium solution is $x_t = \arg\max \Omega_t$ and the corresponding education choices are in (4), (5), and (6). The tax rate is set to satisfy the government's budget constraint. In the following, we consider two cases: a low-inequality state, $\rho_t \in [\rho^m, 1]$, where $\bar{x}_t \leq x_t^{00}$ holds, and a high-inequality state, $\rho_t \in (0, \rho^m)$, where $x_t^{00} < \bar{x}_t$ holds.

4.1 A Low-inequality State: $\rho_t \in [\rho^m, 1]$

The education choice when $[\rho^m, 1]$ is depicted in Panels (c) and (d) of Figure 2. Figure 4 shows the corresponding political objective function. For illustration purposes, we use the properties of Ω_t in (9) and (10).

[Figure 4 here.]

When x_t is below (above) the critical value x_t^{00}, the government expects both types of adults to choose private (public) education, and the adults actually make that choice. When x_t is below x_t^{00}, there are at most three economic equilibria. However, the government finds it optimal to expect that both types of adults choose private education because this choice attains the highest utility for $x_t \leq x_t^{00}$. Thus, the political objective function is

$$\Omega_t = \begin{cases} \Omega_{00,t} & \text{if} \quad 0 < x_t \leq x_t^{00}, \\ \Omega_{11,t} & \text{if} \quad x_t^{00} < x_t < h_t. \end{cases}$$

The solution that maximizes Ω_t is $x_t^{**} = \arg\max \Omega_{11,t}$ because $\Omega_{11,t}|_{x_t = x_t^{**}} > \Omega_{00,t}$ holds, as shown in (10).

Lemma 1. *For $\rho_t \in [\rho^m, 1]$, the period-t* the *voting solution is $x_t^{**} = \arg\max \Omega_{11,t}$.*

Type-L adults pay less than they receive from public education and thus prefer public education to private education. As decisive voters, they choose the per capita public education expenditure given their expectations of type-H voters' choices. Type-H adults may prefer private education to public education because they pay more than they receive from the latter. However, their costs of providing public education in terms of their utility decrease as ρ_t increases (i.e., as their income level relative to the average, $h_t^H / h_t = (1 - \phi\rho_t) / (1 - \phi)$, decreases). In particular, if ρ_t is above ρ^m, the benefits in terms of utility outweigh the costs of public education to type-H adults, and thus these adults find it optimal to choose public education. Therefore, when inequality is sufficiently low that $\rho^m \leq \rho_t \leq 1$, it is optimal for type-L adults to choose a per capita public education expenditure of $x_t^{**} = \arg\max \Omega_{11,t}$, given the expectation that type-H adults also choose public education.

4.2 A High-inequality State: $\rho_t \in (0, \rho^m)$

Panels (a) and (b) of Figure 2 show the education choice when $\rho_t \in (0, \rho^m)$. Figure 5 illustrates the corresponding political objective function. Because the government finds it optimal to expect the education choice that attains the highest utility, the political objective function when $\rho_t \in (0, \rho^m)$ is

$$\Omega_t = \begin{cases} \Omega_{00,t} & \text{if} \quad 0 < x_t \leq x_t^{00}, \\ \Omega_{10,t} & \text{if} \quad x_t^{00} < x_t \leq \bar{x}_t, \\ \Omega_{11,t} & \text{if} \quad \bar{x}_t < x_t < h_t. \end{cases}$$

The main difference from the previous case is that the political objective might be maximized at $x_t \in (x_t^{00}, \bar{x}_t]$, where type-$H$ adults opt out of public education, while type-L adults do not.

[Figure 5 here.]

To find a political equilibrium solution, consider the following two cases: $x_t^* \leq \bar{x}_t$ as illustrated in Panel (a) of Figure 5 and $x_t^* > \bar{x}_t$ as illustrated in Panel (b) of Figure 5. Consider first the case when $x_t^* \leq \bar{x}_t \Leftrightarrow \rho_t \leq \rho^* \equiv (1 - \gamma\eta(1-\phi)/\mu\phi)/\phi (< \rho^m)$. As the figure shows, Ω_t is maximized at $x_t^* = \arg\max \Omega_{10,t}$. At this public education level, type-H adults opt out of public education, while type-L adults do not. This case arises as a political equilibrium outcome when inequality is sufficiently high that $\rho_t \in (0, \rho^*)$. This set is non-empty if and only if $\phi > \gamma\eta/(\mu + \gamma\eta)$. Therefore, there is a period-t political equilibrium with $(q_t^L, q_t^H) = (1, 0)$ and $x_t = x_t^*$ if $\rho_t \in (0, \rho^*)$ and $\phi \in (\gamma\eta/(\mu + \gamma\eta), 1)$.

Next, consider the case when $x_t^* > \bar{x}_t \Leftrightarrow \rho_t > \rho^*$ as in Panel (b) of Figure 5. As the figure shows, there are two candidates for the period-t voting solution: one is $x_t = \bar{x}_t$, where type-H adults opt out of public education, and the other is $x_t = \arg\max \Omega_{11,t} = x_t^{**}$, where both types of adults choose public education. Type-L adults, as decisive voters, choose either education type to maximize utility. Appendix A.4 shows that there is a critical value of ρ_t, denoted by $\rho^{**} \in (\rho^*, \rho^m)$, such that $\Omega_{10,t}|_{x_t=\bar{x}_t} \lesseqgtr \Omega_{11,t}|_{x_t=x_t^{**}} \Leftrightarrow \rho_t \gtreqless \rho^{**}$. The following lemma summarizes the results thus far.

Lemma 2. *Assume $\rho_t \in (0, \rho^m)$. Given ρ_t and h_t, the period-t voting solution is*

$$\arg\max \Omega_t = \begin{cases} x_t^{**} = \arg\max \Omega_{11,t} & \textit{if} \quad \rho_t \in (\rho^{**}, \rho^m), \\ x_t^* = \arg\max \Omega_{10,t} & \textit{if} \quad \rho_t \in (0, \rho^*] \text{ and } \phi \in \left(\frac{\gamma\eta}{\mu+\gamma\eta}, 1\right), \\ \bar{x}_t & \textit{if} \quad \rho_t \in (\max(0, \rho^*), \rho^{**}]. \end{cases}$$

Consider first a situation where $\rho_t \in (0, \rho^*]$ and $\phi \in \left(\frac{\gamma\eta}{\mu+\gamma\eta}, 1\right)$ hold: type-H adults are endowed with a sufficiently high income level but they have a low share of the population in their generation. They thus choose private education because they benefit less from public education because of its lower per capita expenditure. Given this choice by type-H adults, $\Omega_{10,t}$ is the indirect utility function of type-L adults. As decisive voters, they choose the per capita public education expenditure x_t that maximizes $\Omega_{10,t}$, $x_t^* = \arg\max \Omega_{10,t}$. As Panel (a) of Figure 5 shows, this choice is feasible if $x_t^* \leq \bar{x}_t$. In the current situation, the condition $x_t^* \leq \bar{x}_t$ actually holds because the low share of type-H individuals in the generation is equivalent to a high share of those of type L, which thus implies a low per capita level of public education expenditure.

Next, consider a situation where $\rho_t \in (\max(0, \rho^*), \rho^{**}]$ holds: inequality is high but less severe than that observed in the first case. Type-H adults still prefer private to public education, but type-L adults cannot choose an "interior" solution, $x_t^* = \arg\max \Omega_{10,t}$. Their choice is constrained by the upper limit, $\bar{x}_t$. We hereafter refer to $\bar{x}_t$ as a "corner" solution. Finally, if inequality is sufficiently low that $\rho_t \in (\rho^{**}, \rho^m)$, type-$H$ adults choose public education and the political objective is maximized at $x_t^{**} = \arg\max \Omega_{11,t}$.

The corner solution arises when the proportion of type-L adults is sufficiently low that $\phi \in (1/2, \gamma\eta/(\mu + \gamma\eta)]$. A low ϕ implies a small tax burden for each agent for the given level of public education expenditure x. This lowers the marginal cost of public education, thereby inducing type-L adults to prefer higher public education expenditure. However, type-H adults will opt out when public education expenditure is below $\bar{x}_t$. If such expenditure is above $\bar{x}_t$, type-H adults prefer public to private education. Therefore, the upper limit, $\bar{x}_t$, constrains type-L adults' choice of public education as long as type-H adults opt out of public education.

4.3 Voting Outcome and Education Choice

Summarizing the results in Lemmas 1 and 2, we obtain the voting solution in period t and the corresponding education choice.

Proposition 2. *Given the inequality index ρ_t, the period-t voting solution, (x_t, τ_t), is*

$$(x_t, \tau_t) = \begin{cases} \left(x_t^{**}, \frac{\gamma\eta}{1+\gamma\eta}\right) & \text{if} \quad \rho_t \in (\rho^{**}, 1], \\ \left(x_t^{*}, \frac{\gamma\eta}{1+\gamma\eta}\right) & \text{if} \quad \rho_t \in (0, \rho^*] \text{ and } \phi \in \left(\frac{\gamma\eta}{\mu+\gamma\eta}, 1\right), \\ \left(\bar{x}_t, \frac{\phi\mu(1-\phi\rho_t)}{(1-\phi)+\phi\mu(1-\phi\rho_t)}\right) & \text{otherwise.} \end{cases}$$

The corresponding education choice is

$$\left(q^L, q^H\right) = \begin{cases} (1,1) & \text{if} \quad \rho_t \in (\rho^{**}, 1], \\ (1,0) & \text{if} \quad \rho_t \in (0, \rho^{**}]. \end{cases}$$

Proposition 2 states that type-H adults choose public education if inequality is sufficiently low that $\rho_t \in (\rho^{**}, 1]$; otherwise, they choose private education. A small ρ_t implies a high income disparity between the two types of adults, meaning that type-H adults could owe a large tax burden. In particular, if $\rho_t \leq \rho^{**}$, the negative tax burden effect dominates the positive effect of public education. This fact incentivizes type-H adults to opt out of public education. However, if inequality is sufficiently low that $\rho_t > \rho^{**}$, the positive public education effect outweighs the negative tax burden effect and type-H adults choose public education.

To consider the implications of the result in Proposition 2 more in detail, we compare a low-inequality economy with $\rho_t > \rho^{**}$ and a high-inequality economy with $\rho_t \leq \rho^{**}$. In particular, we consider two possible cases of the high-inequality economy. The first case is that it is distinguished by $x_t = x_t^*$ as observed in Panel (a) of Figure 5. In this case, the two economies differ with respect to the enrollment rates in public schooling, but they are identical in terms of tax burdens. The high-inequality economy experiences a lower enrollment rate in public schooling than the low-inequality economy. This fact may imply a lower tax burden for individuals in the high-inequality economy. However, this positive tax burden effect is offset by the higher per capita public education expenditure. Thus, the aggregate public education expenditure and corresponding tax rates are the same between the two economies.

The second case is that the high-inequality economy is distinguished by $x_t = \bar{x}_t$, as observed in Panel (b) of Figure 5. The per capita public education expenditure in this case, $x_t = \bar{x}_t$, is lower than that in the former case. This fact implies that the positive tax burden effect outweighs the negative per capita public education expenditure effect in the high-inequality economy. Thus, the high-inequality economy experiences a lower tax burden and a lower enrollment rate in public schooling compared with the low-inequality economy. The United States and the United Kingdom are examples of high-inequality economies, whereas Nordic countries are examples of low-inequality economies.

5 Steady-state Equilibrium

The analysis in the previous section demonstrated that public education expenditure can serve as a political outcome for a given inequality index, ρ_t. Public education expenditure influences human capital formation, which in turn determines inequality in the next generation, ρ_{t+1}. To consider the dynamic interaction between inequality and public education, we demonstrate the movement in the inequality index across periods as well as the existence and stability of a *steady-state equilibrium* in which $\rho_{t+1} = \rho_t$ holds along the equilibrium path.

Given the initial condition, $\rho_1 (> 0)$, the political equilibrium sequence $\{\rho_t\}$ is characterized by the first-order difference equation, $\rho_{t+1} = P(\rho_t)$, where

$$P(\rho_t) = \begin{cases} P_{11}(\rho_t) \equiv \left[\phi + (1-\phi)^{\eta}\left(\frac{1}{\rho_t} - \phi\right)^{1-\eta}\right]^{-1} & \text{if} \quad \rho_t \in (\rho^{**}, 1], \\ P_{10}(\rho_t) \equiv \left[\phi + \left(\frac{\phi}{1+\gamma\eta}\right)^{\eta}(1-\phi\rho_t)\left(\frac{1}{\rho_t}\right)^{1-\eta}\right]^{-1} & \text{if} \quad \rho_t \in (0, \rho^{*}] \text{ and } \phi \in \left(\frac{\gamma\eta}{\mu+\gamma\eta}, 1\right), \\ \bar{P}_{10}(\rho_t) \equiv \left[\phi + (1-\phi)(1+\gamma\eta)^{1/\gamma}\left(\frac{1-\phi\rho_t}{(1-\phi)\rho_t}\right)^{1-\eta}\right]^{-1} & \text{otherwise.} \end{cases}$$

where the subscripts "11" and "10" in $P(\cdot)$ imply $\left(q^L, q^H\right) = (1,1)$ and $(1,0)$, respectively. The three cases correspond to those in Proposition 2. Appendix A.5 provides the derivations of $P_{11}(\rho_t)$, $P_{10}(\rho_t)$ and $\bar{P}_{10}(\rho_t)$.

A closer analysis of $P(\cdot)$ reveals that the function has the following properties (see Appendix A.6 for the formal proof of the following statement). First, $P_{11}(\cdot)$, $P_{10}(\cdot)$ and $\bar{P}_{10}(\cdot)$ are strictly increasing in ρ_t. Second, $P_{10}(\cdot) \gtreqless \bar{P}_{10}(\cdot)$ if and only if $\rho_t \gtreqless \rho^*$. Third, $\bar{P}_{10}(\cdot) < P_{11}(\cdot)\ \forall \rho_t \in (0,1]$. Fourth, $P_{11}(\cdot)$ satisfies $P_{11}(1) = 1$ and $P'_{11}(1) = 1-\eta \in (0,1)$. Fifth, $\bar{P}_{10}(\cdot)$ satisfies $\bar{P}_{10}(0) = 0$ and $\lim_{\rho\to 0}\left(\partial \bar{P}_{10}(\cdot)/\partial \rho_t\right) = \infty$; $P_{10}(\cdot)$ satisfies $P_{10}(0) = 0$, and $\lim_{\rho\to 0}\left(\partial P_{10}(\cdot)/\partial \rho_t\right) = \infty$. These properties imply that (i) there is a locally stable steady-state equilibrium with $\rho = 1$ and (ii) $P(\cdot)$ is strictly increasing in ρ_t but discontinuous at $\rho_t = \rho^{**}$. Figure 6 illustrates the possible patterns of $P(\cdot)$ when $\phi \in (\gamma\eta/(\mu+\gamma\eta), 1)$. The $\phi \in (1/2, \gamma\eta/(\mu+\gamma\eta)]$ case is qualitatively similar, but the threshold value ρ^* is negative and thus irrelevant. From the figure, we obtain the following proposition.

[Figure 6 here.]

Proposition 3. *If $\bar{P}_{10}(\rho^{**}) > \rho^{**}$, there is a unique stable steady-state equilibrium with $\rho = 1$; if $\bar{P}_{10}(\rho^{**}) \leq \rho^{**}$, there are two locally stable steady-state equilibria, one with $\rho \in (0, \rho^{**}]$ and the other with $\rho = 1$.*

The unique stable steady-state equilibrium is distinguished by perfect equality between the two types of agents and 100% enrollment in public schooling: $\rho = 1$ and $\left(q^L, q^H\right) = (1,1)$ (see Panel (a) of Figure 6). However, another type of equilibrium exists when multiple stable steady states are realized (see Panels (b) and (c) of Figure 6), distinguished by the presence of income inequality and type-H agents opting out of public education: $\rho < 1$ and $\left(q^L, q^H\right) = (1,0)$. Thus, the figure shows low inequality and low (i.e., no) enrollment in private education institutions in the former steady-state equilibrium and high inequality and high enrollment in the latter. These multiple steady states that are unique to the present model imply that higher inequality is associated with lower public education enrollment. The model implications are thus consistent with the empirical pattern observed in Panel (a) of Figure 1.

To consider the movement of inequality across periods more in detail, let us first consider the low-inequality case where $\rho_t > \rho^{**}$. In this case, both types of agents choose public education for their children, $\left(q^L, q^H\right) = (1,1)$, and this choice reduces inequality in the next generation. Because of this positive effect on equality, the economy eventually converges to the perfect equality state with $\rho = 1$ and $\left(q^L, q^H\right) = (1,1)$.

Next, consider the high-inequality case where $\rho_t \leq \rho^{**}$. In this case, type-H adults opt out of public schooling and instead choose private education for their children: $q^H = 0$. This choice works to expand inequality in the next generation, whereas type-L adults' choice of public education, $q^L = 1$, works in the opposite direction. Which effect outweighs the other depends on the inequality level. When inequality is high (low), the positive effect on equality produced by the choice of type-L adults is larger (smaller) than the negative effect caused by the choice by type-H adults. This fact implies that the inequality level converges to a steady state where only type-L adults choose public education (i.e., $\left(q^L, q^H\right) = (1,0)$), with its decreasing (increasing) trend across generations.

For an intuitive interpretation of the condition $\bar{P}_{10}\left(\rho^{**}\right) \gtrless \rho^{**}$ in Proposition 3, we reformulate it as

$$\bar{P}_{10}\left(\rho^{**}\right) \gtrless \rho^{**} \Leftrightarrow \frac{1-\phi\rho^{**}}{1-\phi} \cdot \frac{1}{\rho^{**}} \gtrless (1+\gamma\eta)^{1/\gamma\eta}, \tag{11}$$

where ρ^{**}, defined in Subsection 4.2, satisfies $\Omega_{10,t}\big|_{x_t=\bar{x}_t} = \Omega_{11,t}\big|_{x_t=x_t^{**}}$, or

$$\left(\frac{1-\phi\rho^{**}}{1-\phi}\right)^{\gamma\eta} = 1 + \mu\phi \cdot \frac{1-\phi\rho^{**}}{1-\phi}, \;\; \mu \equiv \frac{\gamma\eta}{(1+\gamma\eta)^{(1+\gamma\eta)/\gamma\eta}}. \tag{12}$$

Eq. (12) indicates that ρ^{**} is a function of ϕ and $\gamma\eta$, $\rho^{**} = \rho^{**}(\phi, \gamma\eta)$. Thus, we can illustrate the condition in (11) in a $\phi - \gamma\eta$ space, as Figure 7 shows. The figure suggests that the model is more likely to produce multiple steady-state equilibria if ϕ and $\gamma\eta$ are lower.

[Figure 7 here.]

To explain this argument, we first consider the effect of $\gamma\eta$. A low γ means a low weight attached to the utility of children's human capital, while a low η means low elasticity in human capital with respect to public education expenditure. These factors imply that type-L agents, as decisive voters, attach less weight to the utility of public education, meaning that they prefer lower public education expenditure. This in turn means that type-H agents gain more from opting out of public education. In addition, their tax burden could be reduced or unchanged by choosing private education. Because of these positive effects, a low $\gamma\eta$ encourages type-H adults to opt out of public education and to attain an equilibrium with $\rho < 1$.

Next, we consider the role of ϕ in the steady-state equilibria outcome. Recall the definition of $\rho_{t+1} \equiv h_{t+1}^L / h_{t+1}$, or,

$$\rho_{t+1} \equiv \frac{h_{t+1}^L}{h_{t+1}} = \frac{h_{t+1}^L}{\phi h_{t+1}^L + (1-\phi) h_{t+1}^H}.$$

This expression indicates that the parameter ϕ has two effects in the determination of ρ_{t+1}. First, given h_{t+1}^H, a lower ϕ implies a larger proportion of type-H agents. This fact leads to higher average human capital, h_{t+1}, and thus a lower $\rho_{t+1} \equiv h_{t+1}^L / h_{t+1}$ for a given h_{t+1}^L. Second, a lower ϕ implies lower aggregate public education expenditure and thus a lower tax burden on type-H agents. This produces a positive income effect on private education expenditure by type-H agents, in turn increasing the human capital of type-H adults, h_{t+1}^H, and average human capital, h_{t+1}; hence, this decreases ρ_{t+1}. Because of these two negative effects, the model produces an equilibrium with $\rho_{t+1} < 1$ if ϕ is low.

However, an economy with a low $\gamma\eta$ and ϕ also has an equilibrium with $\rho = 1$ if the initial condition of ρ is high. A higher ρ implies a lower income gap and thus lower income for type-H agents. Because of this negative income effect, type-H agents find it optimal to choose public education over private education. Therefore, an equilibrium with $\rho = 1$ for low values of $\gamma\eta$ and ϕ also exists.

Thus far, we have assumed that human capital productivity, represented by D, is common between the two types of agents. However, D may represent a durable productive asset such as generic ability, technology transfer, or business succession that children inherit from parents. Based on this view, we can alternatively assume that the distribution of D is positively correlated with human capital, $D^H > D^L$, where D^i $(i = H, L)$ is the human capital productivity of type-i agents (Gradstein and Justman, 1996). This assumption implies that, on average, children born to higher-income families are endowed with greater human capital productivity (Behrman and Taubman, 1989).

Under this alternative assumption, the law of the motion of human capital when $(q^L, q^H) = (1, 1)$ is reformulated as

$$\rho_{t+1} = \left[\phi + \frac{D^H}{D^L} (1-\phi)^{\eta} \left(\frac{1}{\rho_t} - \phi \right)^{1-\eta} \right]^{-1}.$$

This equation implies a stable steady-state equilibrium with $\rho < 1$, which seems more realistic than the equilibrium with $\rho = 1$, which assumes $D^H = D^L = D$. However, the qualitative results remain unchanged. Therefore, for analytical tractability, we retain the assumption of $D^H = D^L = D$ in the following analysis.

6 Welfare Analysis

We use simulations to investigate the model's welfare implications. In the analysis, we set the parameters ϕ, η, γ, and D as in the following. First, recall that $1-\phi$ is the proportion of type-H agents and only type-H agents opt out of public schooling. The proportion $1-\phi$ therefore represents enrollment in private primary institutions as a percentage. We set $1-\phi$ at 0.124 because the average enrollment rate in the high-inequality group in Figure 1 was 12.4% during 2000–2013. Appendix A.7 describes the data source and the classification of high- and low-inequality groups.

Second, for $\gamma\eta$, we focus on the public education expenditure–GDP ratio in the steady-state equilibrium distinguished by $\rho = 1$. The ratio in this equilibrium is $x/h = \gamma\eta/(1+\gamma\eta)$. We can estimate $\gamma\eta$ by using the average ratio of expenditure on primary-level public education to GDP observed in the low-inequality group in Figure 1. The average ratio in that group was 0.0150 during 2000–2013, allowing us to determine $\gamma\eta$ by solving $0.015 = \gamma\eta/(1+\gamma\eta)$ for $\gamma\eta$: $\gamma\eta \simeq 0.0152$.

Third, the estimate in Card and Krueger (1992) implies an elasticity of school quality of 0.12. In addition, simulation studies suggest that η is in the range from 0.1 to 0.3 (Cardak, 2004a) and from 0.05 to 0.15 (Glomm and Ravikumar, 1998). Following these results, we set $\eta = 0.11$. Thus, with $\gamma\eta = 0.0152$, we have $\gamma = 0.138$. This estimate fits well with that of 0.13 by Cardak (2004a) and that of 0.169 by de la Croix and Doepke (2004). Finally, we normalize D to $D = 1$. The values of γ and η obtained here satisfy the conditions that realize the multiple equilibria depicted in Figure 7. Thus the analysis shows multiple stable steady-state equilibria under empirically plausible parameters.

The existence of multiple equilibria indicates that a country with high initial inequality falls into the higher inequality state, whereas a country with low initial inequality converges to the lower inequality state. The former country is thus inferior to the latter in terms of equality. One way to resolve this problem is to introduce a compulsory public school system as an alternative education regime. This limited education choice forces the economy into a steady state with perfect equality, which is identical to the steady state with $\rho = 1$ in the mixed education system analyzed thus far.

The introduction of compulsory public schooling improves inequality, but another question is raised: does compulsory public schooling benefit individuals in terms of utility? To answer this question, we compare the two systems in the following way. We set the initial conditions h_1^L and h_1^H to attain an equilibrium path that converges to the unequal steady state with $\rho < 1$ in the mixed education system. Then, as illustrated in Figure 8, we take the ratios of mixed education systems to compulsory public school systems in terms of per capita public education expenditure (Panel (a)), type-L agents' utility (Panel (b)), type-H agents' utility (Panel (c)), and social welfare defined by a weighted average utility of the two types, $\phi V^L + (1-\phi)V^H$ (Panel (d)). We plot these from generation 1 to generation 80 for the three initial inequality scenarios.[6]

[Figure 8 here.]

In Panel (a), a ratio of more than one implies that expenditure in the compulsory public school system is higher than that in the mixed education system. In Panels (b)–(d), a ratio of less than one implies that utility in the compulsory public school system is higher than that in the mixed education system because the logarithmic utility function takes negative values in the numerical analysis. For example, the ratio in Panel (b) is $0.99(<1)$ if $V^L = -1.0$ (-1.01) in the compulsory public school (mixed education) system. Table 1 summarizes the numerical results from generation 1 to generation 4.

[Table 1 here.]

To interpret the results in Figure 8 and Table 1, we first note the indirect utility of type-L agents:

$$V^L_{mix,t} = \ln\left(h_t - \phi\bar{x}_t\right)\frac{h^L_t}{h_t} + \gamma\ln D\left(h^L_t\right)^{1-\eta} + \gamma\eta\ln\bar{x}_t,$$
$$V^L_{comp,t} = \ln\left(h_t - x^{**}_t\right)\frac{h^L_t}{h_t} + \gamma\ln D\left(h^L_t\right)^{1-\eta} + \gamma\eta\ln x^{**}_t,$$

where $V^L_{mix,t}$ and $V^L_{comp,t}$ are the indirect utility in the mixed education system and compulsory public school system, respectively.

These expressions show that introducing a compulsory public school system has opposing effects on the utility of type-L agents. First, the tax burden increases from $\phi\bar{x}_t$ to x^{**}_t. Second, per capita public education expenditure may increase from $\bar{x}_t$ to x^{**}_t. The numerical result in Panel (a) of Figure 8 shows that expenditure increases for the two lower initial inequality scenarios. However, for the highest initial inequality scenario (represented by the dashed curve), expenditure on compulsory public schooling outweighs that in the mixed education system from about generation 64 onward. The numerical result in Panel (b) of Figure 8 shows that the negative effect of the tax burden increase outweighs the positive effect of the public education expenditure increase, suggesting that introducing a compulsory public school system makes type-L agents in the first generation worse off.

From the second generation onward, there is an additional positive effect via the human capital formation generated by the compulsory public school system. The terms h^L_t/h_t and $D\left(h^L_t\right)^{1-\eta}$ in the above expressions represent this effect. The compulsory public

[6] In Figure 8, the corner solution, $\bar{x}$, is realized in the mixed education system.

school system encourages human capital formation by type-L agents. This benefit rises as the initial human capital level increases (i.e., the lower is initial inequality). In addition, this effect amplifies the positive effect of public education. Therefore, for the baseline and low initial inequality cases, introducing a compulsory public school system makes all the generations from the second one better off. However, for the high initial inequality case, it takes a long time to realize this welfare improvement because the negative effect remains strong as initial inequality increases. Indeed, the welfare of type-L agents improves only from generation 77 onward.[7]

Panel (c) plots the ratio of type-H agents from generations 1 to 80. In this setting, they choose private education in the mixed education regime. Thus, their indirect utility is

$$V^H_{mix,t} = (1+\gamma\eta)\ln\left(h_t - \phi\bar{x}_t\right)\frac{h^H_t}{h_t} + \gamma \ln D\left(h^H_t\right)^{1-\eta} + \gamma\eta\ln\mu,$$
$$V^H_{comp,t} = \ln\left(h_t - x^{**}_t\right)\frac{h^H_t}{h_t} + \gamma \ln D\left(h^H_t\right)^{1-\eta} + \gamma\eta\ln x^{**}_t,$$

where $V^H_{mix,t}$ and $V^H_{comp,t}$ are the indirect utility in the mixed education system and compulsory public school system, respectively.

Introducing a compulsory public school system has two negative effects on the utility of type-H agents in the initial period: the tax burden increases from $\phi\bar{x}_t$ to x^{**}_t and expenditure on human capital formation decreases from e^H_t to x^{**}_t. Thus, the change makes type-H agents in the first generation worse off. From the second generation onward, these agents are also worse off because of the additional negative effect caused by the delay in human capital formation generated by the compulsory public school system. The terms h^H_t/h_t and $D\left(h^H_t\right)^{1-\eta}$ in the above expressions illustrate this effect. The numerical result suggests that the shift from the mixed education system to the compulsory public school system is not Pareto-improving.

Finally, we investigate the effect of the compulsory public school system on social welfare defined by the weighted average utility of the two types, $\phi V^L + (1-\phi)V^H$. Its introduction decreases social welfare in the first generation because both types of agents are worse off. However, the effect on welfare from the second generation onward depends on the initial inequality level: welfare improves earlier as initial inequality decreases, as illustrated in Panel (d). This result suggests that the social welfare ranking of the multiple equilibria depends on the initial inequality condition.

[7]In other words, when private education is available, type-L agents are more likely to be worse off as initial inequality increases.

7 Extensions and Further Analysis

Thus far, the following issues have been abstracted from the analysis: (i) fertility decisions; (ii) intergenerational income mobility; (iii) the policy preferences of type-H agents; and (iv) cases including three or more types of agents. This section briefly considers how the analysis and results would change when any of these issues are included in the analysis. A supplementary explanation of the results presented in this section is provided in Appendix B.

7.1 Fertility Decisions

In this subsection, we briefly introduce the fertility decisions of adults into the model. In particular, we follow de la Croix and Doepke (2004) and assume that adults care about both family size and education level. The utility maximization of a type-i adult is as follows:

$$
\begin{aligned}
&\max_{\{c_t^i, n_{t+1}^i, e_t^i, q_t^i\}} \ln c_t^i + \gamma \ln \left(1 + n_{t+1}^i\right) h_{t+1}^i \\
&\text{s.t. } c_t^i + \left(1 + n_{t+1}^i\right) e_t^i \leq (1 - \tau_t)\, h_t^i \left(1 - \delta \left(1 + n_{t+1}^i\right)\right), \\
&h_{t+1}^i = D \left(h_t^i\right)^{1-\eta} \left\{q_t^i x_t + \left(1 - q_t^i\right) e_t^i\right\}^\eta \\
&\text{given } h_t^i,\ \tau_t,\ \text{and } x_t,
\end{aligned}
$$

where $n_{t+1}^i (> 0)$ is the fertility rate, and thus $1 + n_{t+1}^i$ is the number of children per type-i adult, and $\delta \in (0,1)$ is an adult's time spent raising one child.

We solve the utility maximization problem for the cases of $q_t^i = 1$ and 0, and obtain

$$
1 + n_{t+1}^i\big|_{q_t^i=0} = \frac{\gamma (1-\eta)}{\delta (1+\gamma)} < 1 + n_{t+1}^i\big|_{q_t^i=1} = \frac{\gamma}{\delta (1+\gamma)}.
$$

This expression indicates that family size when choosing private education ($q_t^i = 0$) is smaller than that when choosing public education ($q_t^i = 1$). Following the same procedure as in Section 3, we obtain the indirect utility functions, $V_{x,t}^i$ and $V_{e,t}^i$, as well as the following condition of education choice:

$$
q_t^i = \begin{cases} 1 \text{ (public education)} & \text{if } \ V_{x,t}^i > V_{e,t}^i \Leftrightarrow \tilde{\mu}(1-\tau_t) h_t^i < x_t, \\ 0 \text{ (private education)} & \text{if } \ V_{x,t}^i \leq V_{e,t}^i \Leftrightarrow \tilde{\mu}(1-\tau_t) h_t^i \geq x_t, \end{cases}
$$

where $\tilde{\mu}$ is defined as $\tilde{\mu} \equiv (1-\eta)^{(1-\eta)/\eta} \eta\delta$. This condition implies that a higher fertility cost (i.e., a larger δ) incentivizes adults to substitute family size with education level (i.e., having fewer children with a higher standard of education) by choosing private schooling.

Given the condition above, we can write the education choices of both types of agents in the period-t economic equilibrium as follows:

$$\left(q_t^L, q_t^H\right) = \begin{cases} (0,0) & \text{if} \quad x_t \leq \tilde{x}_t^{00}, \\ (1,0) & \text{if} \quad \tilde{x}_t^{10} < x_t \leq \tilde{x}_t, \\ (1,1) & \text{if} \quad \tilde{x}_t^{11} < x_t, \end{cases}$$

where $\tilde{x}_t^{00}$, $\tilde{x}_t^{10}$, $\tilde{x}_t^{11}$, and $\tilde{x}$, as defined in Appendix B, correspond to x_t^{00}, x_t^{10}, x_t^{11}, and $\bar{x}_t$ in the main analysis, respectively. The expression suggests that the education choice in the period-t economic equilibrium remains qualitatively unchanged when fertility choice is introduced into the analysis. Thus, the policy preferences of the type-L majority and associated political equilibrium characterization would also remain qualitatively unchanged.

The long-run consequences of inequality, however, are affected by the introduction of fertility decisions. The steady state with $\rho < 1$, which is observed in the absence of fertility decisions, is infeasible in the long run. That is, a unique steady state with $\rho = 1$ always exists in the presence of fertility decisions. To show this, let us denote by ϕ_t the share of type-L agents in generation t and by N_t^L $\left(N_t^H\right)$ the type-L (type-H) population in generation t. When $\left(q_t^L, q_t^H\right) = (1,1)$, both types of agents choose public education and thus the population growth rates for each are identical. Thus, $\phi_{t+1} = \phi_t$ when $\left(q_t^L, q_t^H\right) = (1,1)$.

When $\left(q_t^L, q_t^H\right) = (1,0)$, the fertility rates of type-L and type-H agents are

$$1 + n_{t+1}^L = \frac{\gamma}{\delta(1+\gamma)} \text{ and } 1 + n_{t+1}^H = \frac{\gamma(1-\eta)}{\delta(1+\gamma)},$$

respectively. The share of type-L agents in generation $t+1$, ϕ_{t+1}, is thus

$$\phi_{t+1} = \frac{\left(1+n_{t+1}^L\right) \cdot N_t^L}{\left(1+n_{t+1}^L\right) \cdot N_t^L + \left(1+n_{t+1}^H\right) \cdot N_t^H} = \frac{N_t^L}{N_t^L + (1-\eta)N_t^H} > \frac{N_t^L}{N_t^L + N_t^H} = \phi_t.$$

This expression indicates that the share of type-L agents increases as long as the education choice remains at $\left(q_t^L, q_t^H\right) = (1,0)$. Hence, in some future period, the economy changes so that both types of agents choose public education. Therefore, the steady state with $\rho < 1$ is infeasible in the long run in the presence of fertility decisions.

7.2 Intergenerational Income Mobility

The analysis in Subsection 7.1 showed that the economy reaches a unique steady state with perfect equality and no enrollment in private schooling in the long run when adults control fertility from the viewpoint of their utility maximization. However, this model prediction is not consistent with the evidence in OECD countries. To address this issue, we add intergenerational income mobility into the framework, following Bernasconi and Profeta (2012) and Uchida (2017).

First, children either inherit or not their parents' ability, represented by human capital, with some probability. In particular, children whose parents are type L are endowed with h_t^L with probability $1-u$, but with h_t^H with probability $u \in (0,1)$. On the contrary, children whose parents are type H are endowed with h_t^H with probability $1-d$, but with h_t^L with probability $d \in (0,1)$. Thus, u and d represent the probabilities of upward and downward intergenerational mobility for type-L and type-H dynasties, respectively. Second, parents do not recognize their children's ability (i.e., inherited human capital) when they make their schooling decisions. This fact implies that parents choose education to maximize their expected utility.

In this setting, the expected utility functions of type-L and type-H agents are

$$U_t^L = \ln c_t^L + \gamma \ln\left(1+n_{t+1}^L\right)\left[(1-u)D\left(h_t^L\right)^{1-\eta} + uD\left(h_t^H\right)^{1-\eta}\right]\left\{\left(1-q_t^L\right)e_t^L + q_t^L x_t\right\}^\eta,$$
$$U_t^H = \ln c_t^H + \gamma \ln\left(1+n_{t+1}^H\right)\left[dD\left(h_t^L\right)^{1-\eta} + (1-d)D\left(h_t^H\right)^{1-\eta}\right]\left\{\left(1-q_t^H\right)e_t^H + q_t^H x_t\right\}^\eta,$$

respectively. These expressions suggest that the terms including mobility do not affect individual decisions on fertility, consumption, and education because of the assumption of a logarithmic utility function. This in turn implies that they do not also affect the policy preferences of type-L adults. However, mobility does affect the motion of the share of type-L agents, ϕ_t, as we demonstrate below.

Suppose that in some period t, $\left(q_t^L, q_t^H\right) = (1,0)$ holds in the period-t political equilibrium. Then, ϕ_{t+1} is given by

$$\phi_{t+1} = \frac{(1-u)N_{t+1}^L + dN_{t+1}^H}{N_{t+1}^L + N_{t+1}^H},$$

where $(1-u)N_{t+1}^L$ is the number of type-L children whose parents are also of type L, dN_{t+1}^H is the number of type-L children whose parents are type H, and $N_{t+1}^L + N_{t+1}^H$ is the population in generation $t+1$. By using $N_{t+1}^L = \frac{\gamma}{\delta(1+\gamma)}N_t^L$ and $N_{t+1}^H = \frac{\gamma(1-\eta)}{\delta(1+\gamma)}N_t^H$, the above expression is reformulated as

$$\phi_{t+1} = \frac{(1-u)\phi_t + d(1-\eta)(1-\phi_t)}{\phi_t + (1-\eta)(1-\phi_t)} = (1-u) - (1-\eta)(1-u-d)\frac{1}{\frac{\phi_t}{1-\phi_t} + (1-\eta)}.$$

Under the assumption of $1-u-d>0$, a unique $\phi \in (0,1)$ satisfies $\phi_t = \phi_{t+1}$. This fact suggests that the steady state distinguished by $\left(q^L, q^H\right) = (1,0)$ and $\rho < 1$ is feasible in the long run as long as the pair (u,d) is chosen to satisfy $\phi_t = \phi_{t+1}$.

7.3 Type-H Majority

Thus far, we have conducted the analysis by assuming that type-L agents are in the majority, which reflects the right-skewed income distribution in the real economy. However, recent studies report that richer and better educated citizens are more likely to vote (see Hodler, Luechinger, and Stutzer, 2015 and the references therein). Thus, how would the result change if type-H agents constituted the majority?

To consider the type-H majority case, we assume either that $\phi < 1/2$ holds or that $\phi \geq 1/2$ but the voting propensity of type-L agents is low. Under this assumption, the political objective function is the indirect utility function of type-H adults as follows:

$$\Omega_t = \begin{cases} \Omega_{00,t} \equiv V^H_{e,t}\big|_{\tau_t=0} = (1+\gamma\eta)\ln h^H_t + \gamma \ln D\left(h^H_t\right)^{1-\eta} + \gamma\eta \ln \mu & \text{if} \quad (q^L_t, q^H_t) = (0,0), \\ \Omega_{10,t} \equiv V^H_{e,t}\big|_{\tau_t=\phi x_t/h_t} = (1+\gamma\eta)\ln\left(1-\phi x_t/h_t\right) h^H_t & \\ \qquad\qquad +\gamma \ln D\left(h^H_t\right)^{1-\eta} + \gamma\eta \ln \mu & \text{if} \quad (q^L_t, q^H_t) = (1,0), \\ \Omega_{11,t} \equiv V^H_{x,t}\big|_{\tau_t=x_t/h_t} = \ln\left(1-x_t/h_t\right) h^H_t + \gamma \ln D\left(h^H_t\right)^{1-\eta} + \gamma\eta \ln x_t & \text{if} \quad (q^L_t, q^H_t) = (1,1). \end{cases}$$

The objective function has the following properties (see Appendix B for the proof):

$$\Omega_{00,t} \geq \Omega_{10,t}; \ = \text{ holds if } x_t = 0, \text{ and}$$
$$\Omega_{00,t} > \Omega_{11,t}.$$

These conditions state that type-H agents, as the majority, prefer no provision of public education to both types of agents. This is because when they choose the provision of public education either to type-L agents or to both types of agents, they pay more than they receive from such a provision. Therefore, a unique period-t political equilibrium is distinguished by $(x,\tau) = (0,0)$ when type-H agents are the majority. This result together with that in the main analysis suggests that when we consider voting such that the political objective function is the weighted sum of the utility functions of both types of agents, public education is more likely to be provided in the political equilibrium as the political weight attached to type-L agents increases.

7.4 Three or More Types of Agents

To check the robustness of the results for cases of three or more types of agents, we finally consider the three types of family dynasties indexed by $i \in \{L, M, H\}$. The proportion of type-i individuals in each generation is ϕ^i with $\sum_i \phi^i = 1$. Education choice, q^i_t, in (3) applies to the present case, and the government's budget constraint is given by $\sum_i q^i_t \phi^i x_t = \tau_t h_t$, where average human capital is now redefined as $h_t \equiv \sum_i \phi^i h^i_t$.

The education choices of these three types of agents in the period-t economic equilibrium are summarized as follows (see Appendix B for the derivation):

$$\left(q^L_t, q^M_t, q^H_t\right) = \begin{cases} (0,0,0) & \text{if} \quad x_t \leq \mu h^L_t, \\ (1,0,0) & \text{if} \quad \mu\left(1-\frac{\phi^L x_t}{h_t}\right) h^L_t < x_t \leq \mu\left(1-\frac{\phi^L x_t}{h_t}\right) h^M_t, \\ (1,1,0) & \text{if} \quad \mu\left(1-\frac{(\phi^L+\phi^M)x_t}{h_t}\right) h^M_t < x_t \leq \mu\left(1-\frac{(\phi^L+\phi^M)x_t}{h_t}\right) h^H_t, \\ (1,1,1) & \text{if} \quad \mu\left(1-\frac{x_t}{h_t}\right) h^H_t < x_t. \end{cases}$$

This expression shows that the choices are qualitatively similar to those of the two-type case in the sense that lower-income agents are more likely to choose public education. This result, accompanied with the finding reported in the previous subsection, suggests that public education is more widely provided as the political weight to lower-income agents increases.

8 Conclusion

This study presents a political economy theory to explain why countries with higher inequality are associated with lower public education enrollment. We base the theory on a two-class (high and low) successive generations model with human capital accumulation and the choice to opt out of public education accompanied by voting on education policy. This condition creates multiple locally stable steady-state equilibria: one with low inequality and high public education enrollment and the other with high inequality and low public education enrollment. This study is novel in that it shows the negative correlation observed in OECD countries in the mutual interaction of inequality and education.

From an equity viewpoint, it is desirable to attain a low-inequality steady state. One path to this steady state involves introducing compulsory public schooling. We use a simulation to investigate the welfare implications of introducing this reform and find that it makes high-income families worse off, while improving the outcomes of future generations of low-income families at the expense of the current generation. These results suggest that the multiple equilibria are not Pareto-ranked and that the shift from the existing mixed education system to a compulsory public school system is not Pareto-improving.

As a caveat to the analysis, note that we base our analysis on the assumption that the tax rate is adjusted to satisfy the government budget constraint. This implies that per capita expenditure on public education may decrease as the number of opting-out students increases. However, in the real world, fewer students in public education may result in a higher quality of education for these students. To consider this possibility, we alternatively assume that the tax rate is fixed. Under this assumption, the government budget constraint becomes

$$\tau h_t = \begin{cases} \phi x_t & \text{if } \left(q_t^L, q_t^H\right) = (1,0), \\ x_t & \text{if } \left(q_t^L, q_t^H\right) = (1,1), \end{cases}$$

where $\tau \in (0,1)$ is fixed. This constraint indicates that students in public education can benefit from a higher per capita expenditure-to-GDP ratio, x_t/h_t, as the number of opting-out students increases. However, there is no voting on spending since x_t is adjusted to satisfy the government budget constraint given τ and h_t.

Further, we base our analysis on the assumption of fixed class sizes. This assumption makes the analysis tractable and yields clear intuitions. In particular, the assumption enables us to obtain a closed-form solution and demonstrate the evolution of human capital across generations. In addition, from an empirical point of view, the assumption is reasonable—at least for some class-structured societies. However, the result would change if we assumed intergenerational class mobility. Section 7.2 attempts to include mobility into the framework, but the analysis is limited in the sense that mobility is exogenous. Relaxing this assumption would be interesting to explore and is left for future work.

A Appendices

A.1 Proof of (7)

Recall the definition of x_t^{00}, x_t^{10}, $\bar{x}_t$, and x_t^{11} in the text. We compare these as follows:

$$\begin{aligned}
x_t^{00} \gtrless x_t^{11} &\Leftrightarrow 0 \gtrless f(\rho_t) \equiv \mu\phi(\rho_t)^2 - (1+\mu)\rho_t + 1, \\
x_t^{00} \gtrless \bar{x}_t &\Leftrightarrow 0 \gtrless g(\rho_t) \equiv \mu\phi^2(\rho_t)^2 - (1+\mu\phi)\rho_t + 1, \\
x_t^{10} \gtrless x_t^{11} &\Leftrightarrow 0 \gtrless h(\rho_t) \equiv \mu\phi(1-\phi)(\rho_t)^2 - (1+\mu(1-\phi))\rho_t + 1,
\end{aligned}$$

where (i) $f(0) = g(0) = h(0) > 0$, (ii) $f(\cdot) < g(\cdot) < h(\cdot)$ for any $\rho_t \in (0,1]$, and (iii) $f'(\cdot) < 0, g'(\cdot) < 0$, and $h'(\cdot) < 0$ for any $\rho_t \in (0,1)$. As illustrated in Figure A.1, there are three critical values of ρ_t, denoted by ρ^l, ρ^m, and ρ^h, where $0 < \rho^l < \rho^m < \rho^h < 1$, such that $f(\rho^l) = 0$, $g(\rho^m) = 0$, and $h(\rho^h) = 0$. From Figure A.1, we obtain (7).

[Figure A.1 here.]

■

A.2 Proof of Proposition 1

Suppose that $\rho_t \in (0, \rho^l)$. Figure A.1 shows that in this case, $x_t^{00} < x_t^{11}$, $x_t^{00} < \bar{x}_t$, and $x_t^{10} < x_t^{11}$ hold. In addition, direct calculation leads to

$$x_t^{10} < x_t^{00} \text{ and } x_t^{10}, x_t^{11} < \bar{x}_t.$$

Thus, we obtain $x_t^{10} < x_t^{00} < x_t^{11} < \bar{x}_t$, as illustrated in Panel (a) of Figure 2. This figure shows that there is a unique economic equilibrium if $x_t \in (0, x_t^{10})$, $(x_t^{00}, x_t^{11}]$, or $(\bar{x}_t, h_t)$; otherwise, there are multiple economic equilibria.

Following the same procedure, we can show the uniqueness or multiplicity of the economic equilibria for the remaining three cases: $\rho_t \in [\rho^l, \rho^m)$, $[\rho^m, \rho^h)$, and $[\rho^h, 1]$. There is a unique economic equilibrium if any of the following three conditions hold: (i) $\rho_t \in [\rho^l, \rho^m)$ and $x_t \in (0, x_t^{10}]$ or $(\bar{x}_t, h_t)$, as illustrated in Panel (b) of Figure 2; (ii) $\rho_t \in [\rho^m, \rho^h)$ and $x_t \in (0, x_t^{10}]$ or (x_t^{00}, h_t), as illustrated in Panel (c) of Figure 2; and (iii) $\rho_t \in [\rho^h, 1]$ and $x_t \in (0, x_t^{11}]$ or (x_t^{00}, h_t), as illustrated in Panel (d) of Figure 2. Proposition 1 summarizes the results established thus far.

■

A.3 Proof of (10)

The first condition, $\Omega_{11,t} < \Omega_{10,t}\ \forall x_t > 0$, is immediate from the definitions of $\Omega_{11,t}$ and $\Omega_{10,t}$. We show the second condition, $x_t^{00} < x_t^{**}$, with a direct comparison:

$$x_t^{00} < x_t^{**} \Leftrightarrow \mu\rho_t h_t < \frac{\gamma\eta}{1+\gamma\eta} h_t \Leftrightarrow \rho_t < (1+\gamma\eta)^{1/\gamma\eta},$$

which holds for any $\rho_t < 1$ and $\gamma\eta \in (0,1)$.

To show the third condition, $\Omega_{10,t}|_{x_t=x_t^{00}} < \Omega_{00,t}$, we compare $\Omega_{10,t}|_{x_t=x_t^{00}}$ with $\Omega_{00,t}$, and obtain

$$\begin{aligned}\Omega_{10,t}|_{x_t=x_t^{00}} < \Omega_{00,t} &\Leftrightarrow \ln(1-\phi\mu\rho_t)\,\rho_t h_t + \gamma\eta \ln \mu\rho_t h_t < (1+\gamma\eta)\ln \rho_t h_t + \gamma\eta \ln \mu \\ &\Leftrightarrow \ln(1-\phi\mu\rho_t) < 0.\end{aligned}$$

The last inequality holds since $\ln(1-\phi\mu\rho_t) < \ln 1 = 0$.

To show the fourth condition, we first compare $\Omega_{00,t}$ with $\Omega_{11,t}|_{x_t=x_t^{**}}$, and obtain

$$\begin{aligned}\Omega_{00,t} < \Omega_{11,t}|_{x_t=x_t^{**}} &\Leftrightarrow (1+\gamma\eta)\ln \rho_t h_t + \gamma\eta \ln \mu < \ln\left(1-\frac{\gamma\eta}{1+\gamma\eta}\right)\rho_t h_t + \gamma\eta \ln \frac{\gamma\eta}{1+\gamma\eta} h_t \\ &\Leftrightarrow \gamma\eta \ln \rho_t < 0,\end{aligned}$$

where the last inequality holds since $\ln \rho_t < \ln 1 = 0$. The inequality $\Omega_{11,t}|_{x_t=x_t^{**}} < \Omega_{10,t}|_{x_t=x_t^{*}}$ is immediate since $\Omega_{11,t}|_{x_t=x_t^{**}} < \Omega_{10,t}|_{x_t=x_t^{**}} < \Omega_{10,t}|_{x_t=x_t^{*}}$.

■

A.4 Proof of Lemma 2

The text provides the following statement:

$$\arg\max \Omega_t = x_t^* \text{ if } \rho_t \in (0,\rho^*] \text{ and } \phi \in \left(\frac{\gamma\eta}{\mu+\gamma\eta}, 1\right).$$

The remaining task is to show that there is $\rho^{**} \in (\rho^*, \rho^m)$ such that

$$\Omega_{10,t}|_{x_t=\bar{x}_t} \lesseqgtr \Omega_{11,t}|_{x_t=x_t^{**}} \Leftrightarrow \rho_t \gtreqless \rho^{**}, \tag{13}$$

where

$$\begin{aligned}\Omega_{10,t}|_{x_t=\bar{x}_t} &= \ln(h_t - \phi\bar{x}_t)\,\rho_t + \gamma \ln D\,(h_t\rho_t)^{1-\eta} + \gamma\eta \ln \bar{x}_t, \\ \Omega_{11,t}|_{x_t=x_t^{**}} &= \ln(h_t - x_t^{**})\,\rho_t + \gamma \ln D\,(h_t\rho_t)^{1-\eta} + \gamma\eta \ln x_t^{**}.\end{aligned}$$

A direct comparison of $\Omega_{10,t}|_{x_t=\bar{x}_t}$ with $\Omega_{11,t}|_{x_t=x_t^{**}}$ leads to

$$\begin{aligned}
\Omega_{10,t}|_{x_t=\bar{x}_t} &\lesseqgtr \Omega_{11,t}|_{x_t=x_t^{**}} \\
&\Leftrightarrow \ln\left(h_t - \phi\frac{\mu^{\frac{1-\phi\rho_t}{1-\phi}}}{1+\mu\phi\frac{1-\phi\rho_t}{1-\phi}}h_t\right) + \gamma\eta\ln\frac{\mu^{\frac{1-\phi\rho_t}{1-\phi}}}{1+\mu\phi\frac{1-\phi\rho_t}{1-\phi}}h_t \\
&\lesseqgtr \ln\left(h_t - \frac{\gamma\eta}{1+\gamma\eta}h_t\right) + \gamma\eta\ln\frac{\gamma\eta}{1+\gamma\eta}h_t \\
&\Leftrightarrow \ln\frac{\left(\mu\frac{1-\phi\rho_t}{1-\phi}\right)^{\gamma\eta}}{\left(1+\mu\phi\frac{1-\phi\rho_t}{1-\phi}\right)^{1+\gamma\eta}} \lesseqgtr \ln\frac{(\gamma\eta)^{\gamma\eta}}{(1+\gamma\eta)^{1+\gamma\eta}} \\
&\Leftrightarrow \underbrace{\left[\frac{1-\phi\rho_t}{1-\phi}\right]^{\gamma\eta/(1+\gamma\eta)}}_{LHS} \lesseqgtr \underbrace{1+\mu\phi\frac{1-\phi\rho_t}{1-\phi}}_{RHS}, \qquad (14)
\end{aligned}$$

where the LHS and RHS in (14) are increasing in ρ_t.

At $\rho_t = \rho^* \equiv (1-\gamma\eta(1-\phi)/\mu\phi)/\phi$,

$$LHS|_{\rho_t=\rho^*} > RHS|_{\rho_t=\rho^*} \Leftrightarrow \left(\frac{1}{\phi}\right)^{\gamma\eta/(1+\gamma\eta)} > 1,$$

which holds for any $\phi \in (0,1)$ and $\gamma\eta \in (0,1)$. It also holds that

$$\lim_{\rho\to\rho^m} LHS < \lim_{\rho\to\rho^m} RHS \Leftrightarrow \Omega_{10,t}|_{x_t=\bar{x}_t,\rho=\rho^m} < \Omega_{11,t}|_{x_t=x_t^{**},\rho=\rho^m},$$

where the second inequality condition holds, as shown in Lemma 1. Therefore, a unique ρ_t, denoted by $\rho^{**} \in (\rho^*, \rho^m)$, satisfies (14) with an equality.

To summarize, the results thus are

$$\arg\max\Omega_t = \begin{cases} x_t^{**} = \arg\max\Omega_{11,t} & \textit{if} \quad \rho^{**} < \rho_t < \rho^m, \\ x_t^{*} = \arg\max\Omega_{10,t} & \textit{if} \quad \rho_t \in (0,\rho^*] \text{ and } \phi \in \left(\frac{\gamma\eta}{\mu+\gamma\eta}, 1\right), \\ \bar{x}_t & \textit{if} \quad \max(0,\rho^*) < \rho_t \leq \rho^{**}, \end{cases}$$

where

$$\max(0,\rho^*) = \begin{cases} 0 & \text{if} \quad \phi \in \left(\frac{1}{2}, \frac{\gamma\eta}{\mu+\gamma\eta}\right], \\ \rho^* & \text{if} \quad \phi \in \left(\frac{\gamma\eta}{\mu+\gamma\eta}, 1\right), \end{cases}$$

because $\rho^* \gtreqless 0 \Leftrightarrow \phi \gtreqless \frac{\gamma\eta}{\mu+\gamma\eta}$.

■

A.5 Derivation of $P_{11}(\cdot)$, $\bar{P}_{10}(\cdot)$, and $P_{10}(\cdot)$

First, assume $\rho_t \in (\rho^{**}, 1]$: both types of agents choose public education, $(q^L, q^H) = (1,1)$. The average human capital in period $t+1$ is

$$\begin{aligned}
h_{t+1} &= \phi h_{t+1}^L + (1-\phi)h_{t+1}^H \\
&= \phi D\left(h_t^L\right)^{1-\eta}(x_t)^\eta + (1-\phi)D\left(h_t^H\right)^{1-\eta}(x_t)^\eta.
\end{aligned}$$

By using this expression, we can reformulate $\rho_{t+1} = h^L_{t+1}/h_{t+1}$ as

$$\begin{aligned}
\rho_{t+1} &= \frac{D\left(h_t^L\right)^{1-\eta}(x_t)^\eta}{\phi D\left(h_t^L\right)^{1-\eta}(x_t)^\eta + (1-\phi)D\left(h_t^H\right)^{1-\eta}(x_t)^\eta} \\
&= \left[\phi + (1-\phi)\left(\frac{h_t^H}{h_t^L}\right)^{1-\eta}\right]^{-1} \\
&= \left[\phi + (1-\phi)^\eta\left(\frac{1}{\rho_t} - \phi\right)^{1-\eta}\right]^{-1},
\end{aligned}$$

where the equality on the third line comes from $h_t^H/h_t^L = (1/\rho_t - \phi)/(1-\phi)$.

Next, assume $\rho_t \in (0, \rho^{**}]$: type-$L$ agents choose public education and type-H agents choose private education. The human capital equation of type-H agents is

$$\begin{aligned}
h_{t+1}^H &= D\left(h_t^H\right)^{1-\eta}\left(\frac{\gamma\eta}{1+\gamma\eta}(1-\tau_t)h_t^H\right)^\eta \\
&= Dh_t^H\left(\frac{\gamma\eta}{1+\gamma\eta}\right)^\eta\left(1-\frac{\phi x_t}{h_t}\right)^\eta,
\end{aligned}$$

where the first equality comes from the private education function, $e_t^H = \gamma\eta(1-\tau_t)h_t^H/(1+\gamma\eta)$, and the second equality comes from the government's budget constraint, $\phi x_t = \tau_t h_t$. With $h_{t+1}^L = D\left(h_t^L\right)^{1-\eta}(x_t)^\eta$, the period $t+1$ inequality index, ρ_{t+1}, becomes

$$\begin{aligned}
\rho_{t+1} &= \frac{D\left(h_t^L\right)^{1-\eta}(x_t)^\eta}{\phi D\left(h_t^L\right)^{1-\eta}(x_t)^\eta + (1-\phi)Dh_t^H\left(\frac{\gamma\eta}{1+\gamma\eta}\right)^\eta\left(1-\frac{\phi x_t}{h_t}\right)^\eta} \\
&= \left[\phi + (1-\phi)\frac{h_t^H\left(\frac{\gamma\eta}{1+\gamma\eta}\right)^\eta\left(1-\frac{\phi x_t}{h_t}\right)^\eta}{\left(h_t^L\right)^{1-\eta}(x_t)^\eta}\right]^{-1}. \qquad (15)
\end{aligned}$$

Assume the corner solution,

$$x_t = \bar{x}_t \equiv \frac{\mu\frac{1-\phi\rho_t}{1-\phi}}{1+\mu\phi\frac{1-\phi\rho_t}{1-\phi}}h_t.$$

By substituting this into (15) and rearranging the terms, we obtain $\bar{P}_{10}(\cdot)$, as in the text. Alternatively, assume the interior solution, $x_t = x_t^* = \gamma\eta h_t/\phi(1+\gamma\eta)$. By substituting this into (15) and rearranging the terms, we obtain $P_{10}(\cdot)$, as in the text.

■

A.6 Properties of $P_{11}(\cdot)$, $\bar{P}_{10}(\cdot)$, and $P_{10}(\cdot)$

(i) Claim 1: $P_{11}(\cdot)$, $\bar{P}_{10}(\cdot)$, and $P_{10}(\cdot)$ are strictly increasing inρ_t.

This claim is immediate from the expressions of $P_{11}(\cdot)$, $\bar{P}_{10}(\cdot)$, and $P_{10}(\cdot)$ in the text.

(ii) Claim 2: $P_{10}(\cdot) \gtreqless \bar{P}_{10}(\cdot)$ if and only if $\rho_t \gtreqless \rho^*$.

We directly compare $P_{10}(\cdot)$ with $\bar{P}_{10}(\cdot)$ and obtain

$$P_{10}(\cdot) \gtreqless \bar{P}_{10}(\cdot) \Leftrightarrow (1-\phi)(1+\gamma\eta)^{1/\gamma}\left(\frac{1-\phi\rho_t}{(1-\phi)\rho_t}\right)^{1-\eta} \gtreqless \left(\frac{\phi\rho_t}{1+\gamma\eta}\right)^{\eta}\left(\frac{1-\phi\rho_t}{\rho_t}\right)$$
$$\Leftrightarrow \rho_t \gtreqless \rho^* \equiv \frac{1}{\phi}\cdot\left[1-\frac{1-\phi}{\phi}\cdot\frac{\gamma\eta}{\mu}\right].$$

(iii) Claim 3: $\bar{P}_{10}(\cdot) < P_{11}(\cdot)\ \forall\rho_t \in (0,1]$.

We directly compare $\bar{P}_{10}(\cdot)$ with $P_{11}(\cdot)$ and obtain

$$\begin{aligned}&\bar{P}_{10}(\cdot) < P_{11}(\cdot)\\ &\quad\Leftrightarrow (1-\phi)^{\eta}\left(\frac{1}{\rho_t}-\phi\right)^{1-\eta} < (1-\phi)(1+\gamma\eta)^{1/\gamma}\left(\frac{1-\phi\rho_t}{(1-\phi)\rho_t}\right)^{1-\eta}\\ &\quad\Leftrightarrow 1 < (1+\gamma\eta)^{1/\gamma},\end{aligned}$$

which holds for any $\gamma\eta \in (0,1)$.

(iv) Claim 4: $P_{11}(\cdot)$ satisfies $P_{11}(1) = 1$ and $P_{11}'(1) = 1-\eta \in (0,1)$.

$P_{11}(1) = 1$ is immediate from the definition of $P_{11}(\cdot)$ in the text. The first differentiation of $P_{11}(\cdot)$ with respect to ρ is

$$P_{11}'(\rho_t) = \left[\phi + (1-\phi)^{\eta}\left(\frac{1}{\rho_t}-\phi\right)^{1-\eta}\right]^{-2}(1-\phi)^{\eta}(1-\eta)\left(\frac{1}{\rho_t}-\phi\right)^{-\eta}\frac{1}{(\rho_t)^2}.$$

We evaluate this at $\rho_t = 1$ to obtain $P_{11}'(1) = 1-\eta \in (0,1)$.

(v) Claim 5: $\bar{P}_{10}(\cdot)$ satisfies $\bar{P}_{10}(0) = 0$ and $\lim_{\rho\to 0}\left(\partial\bar{P}_{10}(\cdot)/\partial\rho_t\right) = \infty$; $P_{10}(\cdot)$ satisfies $P_{10}(0) = 0$ and $\lim_{\rho\to 0}\left(\partial P_{10}(\cdot)/\partial\rho_t\right) = \infty$.

We obtain $\bar{P}_{10}(0) = 0$ and $P_{10}(0) = 0$ by directly substituting $\rho_t = 0$ into $\bar{P}_{10}(\cdot)$ and $P_{10}(\cdot)$. To show $\lim_{\rho\to 0}\left(\partial\bar{P}_{10}(\cdot)/\partial\rho_t\right) = \infty$, we differentiate $\bar{P}_{10}(\cdot)$ with respect to ρ_t. After rearranging the terms, we obtain

$$\frac{\partial\bar{P}_{10}(\cdot)}{\partial\rho_t} = \frac{(1-\phi)^{\eta}(1+\gamma\eta)^{1/\gamma}(1-\eta)}{\left[\phi+(1-\phi)(1+\gamma\eta)^{1/\gamma}\left(\frac{1-\phi\rho_t}{(1-\phi)\rho_t}\right)^{1-\eta}\right]^2\cdot(\rho_t)^2\cdot\left(\frac{1-\phi\rho_t}{\rho_t}\right)^{\eta}},$$

or

$$\begin{aligned}\frac{\partial\bar{P}_{10}(\cdot)}{\partial\rho_t} &= (1-\phi)^{\eta}(1+\gamma\eta)^{1/\gamma}(1-\eta)\times\Big[(\phi)^2(\rho_t)^{2-\eta}(1-\phi\rho_t)^{\eta}\\ &\quad+2\phi(1-\phi)(1+\gamma\eta)^{1/\gamma}\frac{\rho_t(1-\phi\rho_t)}{1-\phi}+\left\{(1-\phi)(1+\gamma\eta)^{1/\gamma}\right\}^2\left(\frac{1}{1-\phi}\right)^{2(1-\eta)}(\rho_t)^{\eta}\Big]^{-1}.\end{aligned}$$

By evaluating this at $\rho_t = 0$, we obtain $\lim_{\rho_t\to 0}\bar{P}_{10}'(\cdot) = +\infty$.

To show $\lim_{\rho \to 0} \left(\partial P_{10}\left(\cdot\right) / \partial \rho_t \right) = \infty$, we follow the same procedure described above. Differentiating $P_{10}\left(\cdot\right)$ with respect to ρ_t yields

$$\begin{aligned}\frac{\partial P_{10}\left(\cdot\right)}{\partial \rho_t} &= \left(\frac{\phi}{1+\gamma\eta}\right)^{\eta} \left[\phi\rho_t + \left(1-\eta\right)\left(1-\phi\rho_t\right)\right] \\ &\times \left[\left(\phi\right)^2 \left(\rho_t\right)^{2-\eta} + 2\phi \left(\frac{\phi}{1+\gamma\eta}\right)^{\eta} \left(1-\phi\rho_t\right)\rho_t + \left(\frac{\phi}{1+\gamma\eta}\right)^{2\eta} \left(1-\phi\rho_t\right)^2 \left(\rho_t\right)^{\eta}\right]^{-1}.\end{aligned}$$

We evaluate this at $\rho_t = 0$ and obtain

$$\lim_{\rho_t \to 0} \frac{\partial P_{10}\left(\cdot\right)}{\partial \rho_t} = \left(\frac{\phi}{1+\gamma\eta}\right)^{\eta} \left[0 + \left(1-\eta\right)\left(1-0\right)\right] \times \left(0\right)^{-1} = +\infty.$$

■

A.7 Data Description

The sources of data used in Panel (a) of Figure 1 and the simulation analysis in Section 6 are as follows. We use data on 33 OECD member countries because our concern is about the association between inequality and education choice in developed economies. We take the average of the Gini coefficients (market income, before taxes and transfers) during 2000–2013 for each country (Source: OECD.Stat (http://stats.oecd.org/), accessed on June 29, 2017). We classify the countries into two groups: a high-inequality group including countries above the OECD average and a low-inequality group including countries below the OECD average.

The high-inequality group includes Austria, Belgium, Chile, Estonia, Finland, France, Germany, Greece, Ireland, Israel, Italy, Japan, Latvia, Luxemburg, Poland, Portugal, Spain, Turkey, the United Kingdom, and the United States. The low-inequality group includes Australia, Canada, Czech Republic, Denmark, Iceland, Korea, the Netherlands, New Zealand, Norway, Slovak Republic, Slovenia, Sweden, and Switzerland. Hungary and Mexico are not included in either group because of the lack of data.

Based on the classification above, we also compute the average percentage of enrollment in primary education in private institutions for each group (Source: UNESCO Institute for Statistics (http://uis.unesco.org/), accessed on June 29, 2017). According to the data source, private enrollment refers to "pupils or students enrolled in institutions that are not operated by a public authority but controlled and managed, whether for profit or not, by a private body such as a nongovernmental organization, religious body, special interest group, foundation or business enterprise." We also take the data on government expenditure on primary education as a percentage of GDP for each country from the same source.

The sources of Panel (b) of Figure 1 are as follows. The data of Gini coefficients are collected from OECD.Stat (http://stats.oecd.org/) (accessed on February 7, 2019). We compute the ratio of private to total spending on primary education using the following data: OECD, 2019, Education spending (indicator). doi: 10.1787/ca274bac-en (Accessed on 07 February 2019), and OECD, 2019, Private spending on education (indicator). doi: 10.1787/6e70bede-en (Accessed on 07 February 2019). According to the data source, private spending on education refers to "expenditure funded by private sources which are households and other private entities."

■

B Supplementary Materials (Not for Publication)

B.1 Supplementary Explanation for Subsection 7.1

Here, we derive the four critical values of x_t, $\tilde{x}_t^{00}$, $\tilde{x}_t^{10}$, $\tilde{x}_t^{11}$, and $\tilde{x}_t$. We first consider education choice by a type-i adult. Suppose that he or she chooses private education, $q_t^i = 0$. Solving his or her utility maximization problem by assuming $q_t^i = 0$ leads to the following indirect utility function:

$$V_{e,t}^i = \ln \frac{1}{1+\gamma} (1-\tau_t) h_t^i + \gamma \ln \frac{\gamma(1-\eta)}{\delta(1+\gamma)} D \left(h_t^i\right)^{1-\eta} \left(\frac{\eta\delta}{1-\eta} (1-\tau_t) h_t^i\right)^{\eta}.$$

Alternatively, suppose that $q_t^i = 1$ holds. Then, the indirect utility function becomes

$$V_{x,t}^i = \ln \frac{1}{1+\gamma} (1-\tau_t) h_t^i + \gamma \ln \frac{\gamma}{\delta(1+\gamma)} D \left(h_t^i\right)^{1-\eta} (x_t)^{\eta}.$$

Thus, type-i's education choice is

$$q_t^i = \begin{cases} 1 \text{ (public education)} & \text{if } V_{x,t}^i > V_{e,t}^i \Leftrightarrow \tilde{\mu}(1-\tau_t)h_t^i < x_t, \\ 0 \text{ (private education)} & \text{if } V_{x,t}^i \leq V_{e,t}^i \Leftrightarrow \tilde{\mu}(1-\tau_t)h_t^i \geq x_t, \end{cases} \tag{16}$$

where $\tilde{\mu}$ is defined as $\tilde{\mu} \equiv (1-\eta)^{(1-\eta)/\eta} \eta\delta$.

Next, consider education choices in the period-t economic equilibrium. Within the framework in Subsection 7.1, the government's budget constraint in period t is

$$\left\{q_t^L \left(1+n_{t+1}^L\right) \phi_t + q_t^H \left(1+n_{t+1}^H\right) (1-\phi_t)\right\} x_t = \tau_t h_t.$$

When $\left(q_t^L, q_t^H\right) = (0,0)$, the constraint is reduced to $\tau_t = 0$ because both types of agents opt out of public education. The substitution of $\tau_t = 0$ into (16) leads to

$$q_t^L = 0 \text{ if } \tilde{\mu} h_t^L \geq x_t; \; q_t^H = 0 \text{ if } \tilde{\mu} h_t^H \geq x_t.$$

Thus, we obtain $\left(q_t^L, q_t^H\right) = (0,0)$ if $x_t \leq \tilde{x}_t^{00} \equiv \tilde{\mu}\rho_t h_t$.

When $\left(q_t^L, q_t^H\right) = (1,0)$, the period-$t$ government's budget constraint is $\left(1+n_{t+1}^L\right) \phi_t x_t = \tau_t h_t$, or

$$\tau_t = \frac{\phi_t \gamma}{\delta(1+\gamma)} \cdot \frac{x_t}{h_t}.$$

We substitute this into (16) and obtain

$$q_t^L = 1 \text{ if } \tilde{x}_t^{10} \equiv \frac{\tilde{\mu}\rho_t}{1 + \frac{\phi_t\gamma}{\delta(1+\gamma)}\tilde{\mu}\rho_t} h_t < x_t,$$

$$q_t^H = 0 \text{ if } x_t \leq \tilde{x}_t \equiv \frac{\tilde{\mu}\frac{1-\phi_t\rho_t}{1-\phi_t}}{1 + \tilde{\mu}\frac{1-\phi_t\rho_t}{1-\phi_t}\frac{\phi_t\gamma}{\delta(1+\gamma)}} h_t.$$

Finally, when $\left(q_t^L, q_t^H\right) = (1,1)$, the period-$t$ government's budget constraint is

$$\left\{\left(1+n_{t+1}^L\right)\phi_t + \left(1+n_{t+1}^H\right)(1-\phi_t)\right\} x_t = \tau_t h_t,$$

or

$$\tau_t = \frac{\gamma}{\delta(1+\gamma)} \cdot \frac{x_t}{h_t}.$$

With (16), we obtain

$$\begin{aligned}
q_t^L &= 1 \text{ if } \tilde{\mu}\left(1 - \frac{\gamma}{\delta(1+\gamma)} \cdot \frac{x_t}{h_t}\right) h_t^L < x_t, \\
q_t^H &= 1 \text{ if } \tilde{\mu}\left(1 - \frac{\gamma}{\delta(1+\gamma)} \cdot \frac{x_t}{h_t}\right) h_t^H < x_t.
\end{aligned}$$

Therefore, we have $\left(q_t^L, q_t^H\right) = (1,1)$ if

$$\tilde{x}_t^{11} \equiv \tilde{\mu}\left(1 - \frac{\gamma}{\delta(1+\gamma)} \cdot \frac{x_t}{h_t}\right) h_t^H < x_t.$$

B.2 Supplementary Explanation for Subsection 7.2

First, we compare $\Omega_{00,t} \geq \Omega_{10,t}$ and obtain

$$\begin{aligned}
\Omega_{00,t} \geq \Omega_{10,t} &\Leftrightarrow (1+\gamma\eta)\ln h_t^H \geq (1+\gamma\eta)\ln(1-\phi x_t/h_t)\, h_t^H \\
&\Leftrightarrow 0 \geq (1+\gamma\eta)\ln(1-\phi x_t/h_t),
\end{aligned}$$

where the right-hand side of the second line is negative if $x_t > 0$ and zero if $x_t = 0$.

Let $\tilde{x}_t^{**}$ denote $\arg\max \Omega_{11,t}$: $\tilde{x}_t^{**} \equiv \gamma\eta h_t/(1+\gamma\eta)$. Direct calculation leads to

$$\Omega_{00,t} > \Omega_{11,t}\big|_{x_t=\tilde{x}_t^{**}} \Leftrightarrow \ln h_t^H > \ln h_t,$$

which holds for any h_t^H and h_t.

B.3 Supplementary Explanation for Subsection 7.4

Suppose first that $\left(q_t^L, q_t^M, q_t^H\right) = (0,0,0)$ holds. The government's budget constraint is reduced to $\tau_t = 0$. The substitution of $\tau_t = 0$ into (3) leads to

$$q_t^L = 0 \text{ if } x_t \leq \mu h_t^L;\ q_t^M = 0 \text{ if } x_t \leq \mu h_t^M;\ \text{and } q_t^H = 0 \text{ if } x_t \leq \mu h_t^H.$$

Thus, we obtain $\left(q_t^L, q_t^M, q_t^H\right) = (0,0,0)$ if $x_t \leq \mu h_t^L$.

Second, suppose that $\left(q_t^L, q_t^M, q_t^H\right) = (1,0,0)$ holds. The government's budget constraint is rewritten as $\phi^L x_t = \tau_t h_t$. By substituting this into (3) and rearranging the terms, we obtain

$$q_t^L = 1 \text{ if } \mu\left(1 - \frac{\phi^L x_t}{h_t}\right) h_t^L < x_t;$$
$$q_t^M = 0 \text{ if } x_t \le \mu\left(1 - \frac{\phi^L x_t}{h_t}\right) h_t^M;$$
$$q_t^H = 0 \text{ if } x_t \le \mu\left(1 - \frac{\phi^L x_t}{h_t}\right) h_t^H.$$

Thus, $\left(q_t^L, q_t^M, q_t^H\right) = (1,0,0)$ holds if

$$\mu\left(1 - \frac{\phi^L x_t}{h_t}\right) h_t^L < x_t \le \mu\left(1 - \frac{\phi^L x_t}{h_t}\right) h_t^M.$$

Third, suppose that $\left(q_t^L, q_t^M, q_t^H\right) = (1,1,0)$ holds. The government's budget constraint is rewritten as $\left(\phi^L + \phi^M\right) x_t = \tau_t h_t$. We substitute this into (3) and rearrange the terms to obtain

$$q_t^L = 1 \text{ if } \mu\left(1 - \frac{\left(\phi^L + \phi^M\right) x_t}{h_t}\right) h_t^L < x_t;$$
$$q_t^M = 0 \text{ if } \mu\left(1 - \frac{\left(\phi^L + \phi^M\right) x_t}{h_t}\right) h_t^M < x_t;$$
$$q_t^H = 0 \text{ if } x_t \le \mu\left(1 - \frac{\left(\phi^L + \phi^M\right) x_t}{h_t}\right) h_t^H.$$

Thus, we have $\left(q_t^L, q_t^M, q_t^H\right) = (1,1,0)$ if

$$\mu\left(1 - \frac{\left(\phi^L + \phi^M\right) x_t}{h_t}\right) h_t^M < x_t \le \mu\left(1 - \frac{\left(\phi^L + \phi^M\right) x_t}{h_t}\right) h_t^H.$$

Finally, suppose that $\left(q_t^L, q_t^M, q_t^H\right) = (1,1,1)$ holds. The government's budget constraint is rewritten as $x_t = \tau_t h_t$. We substitute this into (3) and rearrange the terms to obtain

$$q_t^i = 1 \text{ if } \mu\left(1 - \frac{x_t}{h_t}\right) h_t^i < x_t,\ i = L, M, H.$$

Thus, we have $\left(q_t^L, q_t^M, q_t^H\right) = (1,1,1)$ if

$$\mu\left(1 - \frac{x_t}{h_t}\right) h_t^H < x_t.$$

References

[1] Arawatari, R., and Ono, T., 2009. A second chance at success: A political economy perspective. *Journal of Economic Theory* 144(3), 1249–1277.

[2] Arawatari, R., and Ono, T., 2013. Inequality, mobility and redistributive politics. *Journal of Economic Theory* 148(1), 353–375.

[3] Arcalean, C., and Schiopu, I., 2015. Inequality, opting-out and public education funding. *Social Choice and Welfare* 46(4), 811–837.

[4] Bearse, P., Glomm, G., and Patterson, D.M., 2005. Endogenous public expenditures on education. *Journal of Public Economic Theory* 7(4), 561–577.

[5] Behrman, J.R., and Taubman, P., 1989. Is schooling 'mostly in the genes'? Nature-nurture decomposition using data on relatives. *Journal of Political Economy* 97, 125–146.

[6] Benabou, R., 2000. Unequal societies: Income distribution and the social contract. *American Economic Review* 90, 96–129.

[7] Bernasconi, M., and Profeta, P., 2012. Public education and redistribution when talents are mismatched. *European Economic Review* 56, 84–96.

[8] Card, D., and Krueger, A.B., 1992. Does school quality matter? Returns to education and the characteristics of public schools in the United States. *Journal of Political Economy* 100, 1–40.

[9] Cardak, B.A., 2004a. Education choice, endogenous growth and income distribution. *Economica* 71(281), 57–81.

[10] Cardak, B.A., 2004b. Education choice, neoclassical growth, and class structure. *Oxford Economic Papers* 56(4), 643–666.

[11] de la Croix, D., and Doepke, M., 2003. To segregate or to integrate: Education politics and democracy. http://papers.ccpr.ucla.edu/index.php/pwp/article/view/PWP-CCPR-2003-031 (accessed on February 16, 2018).

[12] de la Croix, D., and Doepke, M., 2004. Public versus private education when differential fertility matters. *Journal of Development Economics* 73(2), 607–629.

[13] de la Croix, D., and Doepke, M., 2009. To segregate or to integrate: Education politics and democracy. *Review of Economic Studies* 76(2), 597–628.

[14] De Gregorio, J., and Lee, J-W., 2002. Education and income inequality: New evidence from cross-country data. *Review of Income and Wealth* 48(3), 395–416.

[15] Epple, D., and Romano, E.R., 1996. Public provision of private goods. *Journal of Political Economy* 104, 57–84.

[16] Galor, O., Moav, O., and Vollrath, D., 2009. Inequality in land ownership, the emergence of human-capital promoting institutions, and the great divergence. *Review of Economic Studies* 76(1), 143–179.

[17] Glomm, G., and Ravikumar, B., 1992. Public versus private investment in human capital: Endogenous growth and income inequality. *Journal of Political Economy* 100, 818–834.

[18] Glomm, G., and Ravikumar, B., 1998. Opting out of publicly provided services: A majority voting result. *Social Choice and Welfare* 15(2), 187–199.

[19] Gradstein, M., and Justman, M., 1996. The political economy of mixed public and private schooling: A dynamic analysis. *International Tax and Public Finance* 3(3), 297–310.

[20] Gradstein, M., and Justman, M., 1997. Democratic choice of an education system: Implications for growth and income distribution. *Journal of Economic Growth* 2(2), 169–183.

[21] Hassler, J., Rodriguez Mora, J.V., Storesletten, K., and Zilibotti, A., 2003. The survival of the welfare state. *American Economic Review* 93(1), 87–112.

[22] Hassler, J., Storesletten, K., and Zilibotti, F., 2007. Democratic public good provision. *Journal of Economic Theory* 133(1), 127–151.

[23] Hodler, R., Luechinger, S., and Stutzer, A., 2015. The effects of voting costs on the democratic process and public finances. *American Economic Journal: Economic Policy* 7, 141–171.

[24] Hoyt, W.H., and Lee, K., 1998. Educational vouchers, welfare effects, and voting. *Journal of Public Economics* 69(2), 211–228.

[25] Organisation for Economic Co-operation and Development(OECD), 2016. *Education Policy Outlook: Korea*, available at www.oecd.org/education/policyoutlook.htm (accessed on February 7, 2019).

[26] Organisation for Economic Co-operation and Development(OECD), 2017. *Education Policy Outlook: Belgium*, available at www.oecd.org/edu/profiles.htm (accessed on February 7, 2019).

[27] Ono, T., 2016. Inequality and the politics of redistribution. *International Tax and Public Finance* 23(2), 191–217.

[28] Saint-Paul, G., and Verdier, T., 1993. Education, democracy and growth. *Journal of Development Economics* 42(2), 399–407.

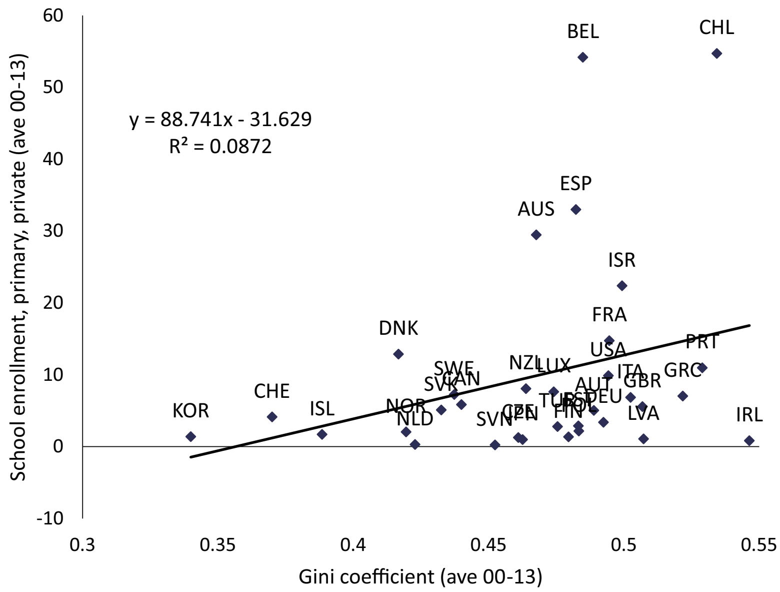
[29] Stiglitz, J.E., 1974. The demand for education in public and private school systems. *Journal of Public Economics* 3(4), 349–385.

[30] Teulings, C., and Van Rens, T., 2008. Education, growth, and income inequality. *Review of Economics and Statistics* 90(1), 89–104.

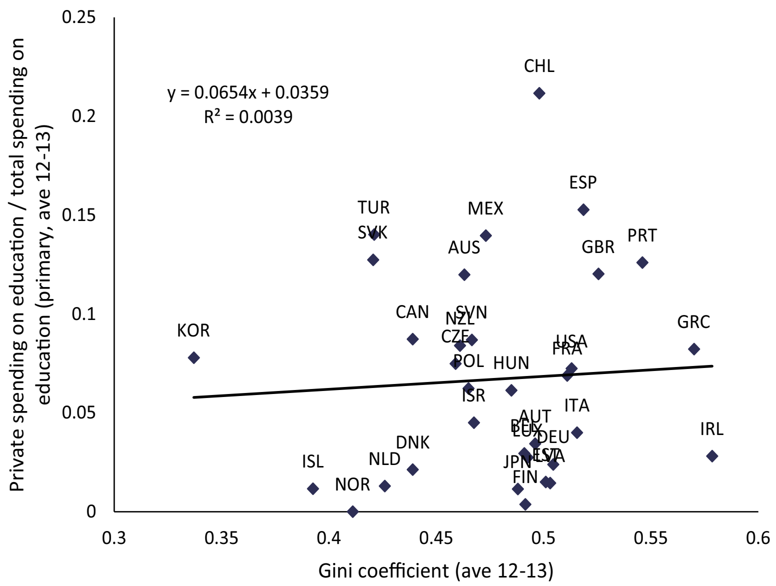
[31] Tournemaine, F., and Tsoukis, C., 2015. Public expenditures, growth, and distribution in a mixed regime of education with a status motive. *Journal of Public Economic Theory* 17, 673–701.

[32] Uchida, Y., 2017. Education, social mobility, and the mismatch of talents. *Economic Theory* https://doi.org/10.1007/s00199-016-1027-7

[33] Zhang, J., 1996. Optimal investments in education and endogenous growth. *Scandinavian Journal of Economics* 98(3), 387–404.



(a)



(b)

Figure 1: Average Gini coefficient and the average percentage of enrollment in primary private education institutions for OECD countries during 2000–2013 (Panel (a)); average Gini coefficient and the average ratio of private to total spending on primary education during 2012–2013 (Panel (b)). Sources of the data are described in Appendix A.7.
Note: Sample periods are different for each panel because of different data availability.

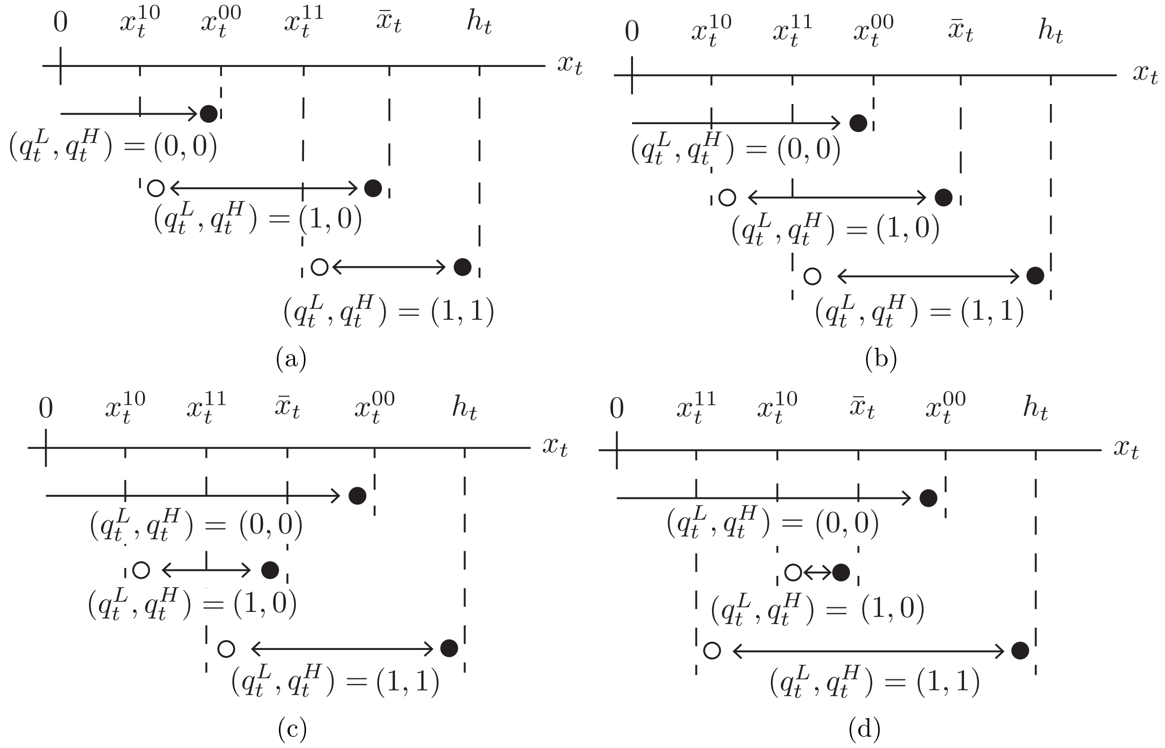


Figure 2: $\rho_t \in \left(0, \rho^l\right)$ case (Panel (a)); $\rho_t \in \left[\rho^l, \rho^m\right)$ case (Panel (b)); $\rho_t \in \left[\rho^m, \rho^h\right)$ case (Panel (c)); $\rho_t \in \left[\rho^h, 1\right]$ case (Panel (d)).
Note: The horizontal arrows below the x_t line show the ranges of x_t realizing the education choice $\left(q_t^L, q_t^H\right)$ in the economic equilibrium. For example, in Panel (a), the education choice of $\left(q_t^L, q_t^H\right) = (0, 0)$ occurs when x_t is set within the range $(0, x_t^{00})$.

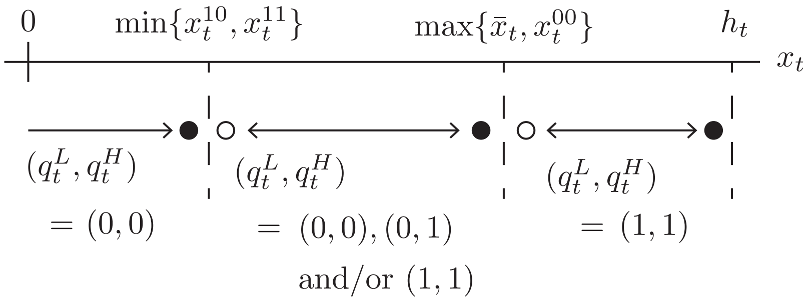


Figure 3: Education choice $\left(q_t^L, q_t^H\right)$ classified according to x_t.
Note: See the note in Figure 2 for an explanation of the horizontal arrows below the x_t line.

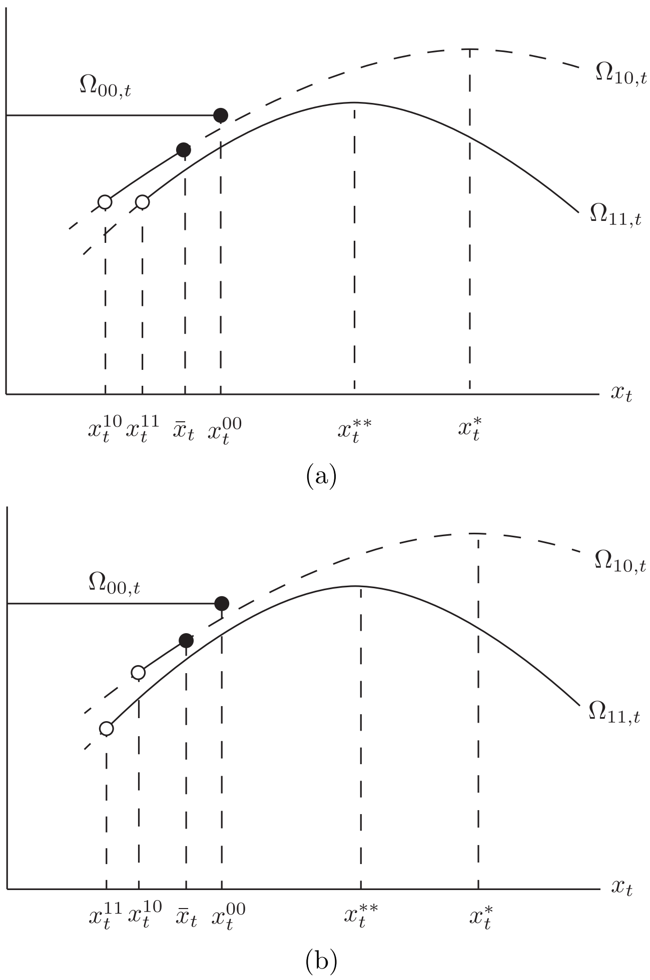


Figure 4: Illustration of the political objective function for the $\rho_t \in \left[\rho^m, \rho^h\right)$ case (Panel (a)) and the $\rho_t \in \left[\rho^h, 1\right]$ case (Panel (b)).

Note: The solid (dotted) curves illustrate the feasible (infeasible) political objective function. For example, in Panel (a), the education choice $\left(q_t^L, q_t^H\right) = (1, 0)$ arises in the economic equilibrium for the range of $x_t \in (x_t^{10}, \bar{x}_t]$. The corresponding political objective function, $\Omega_{10,t}$, is illustrated by the solid curve within the range $(x_t^{10}, \bar{x}_t]$. Outside that range, the function is illustrated by the dotted curve, implying that the choice of $\left(q_t^L, q_t^H\right) = (1, 0)$ is infeasible.

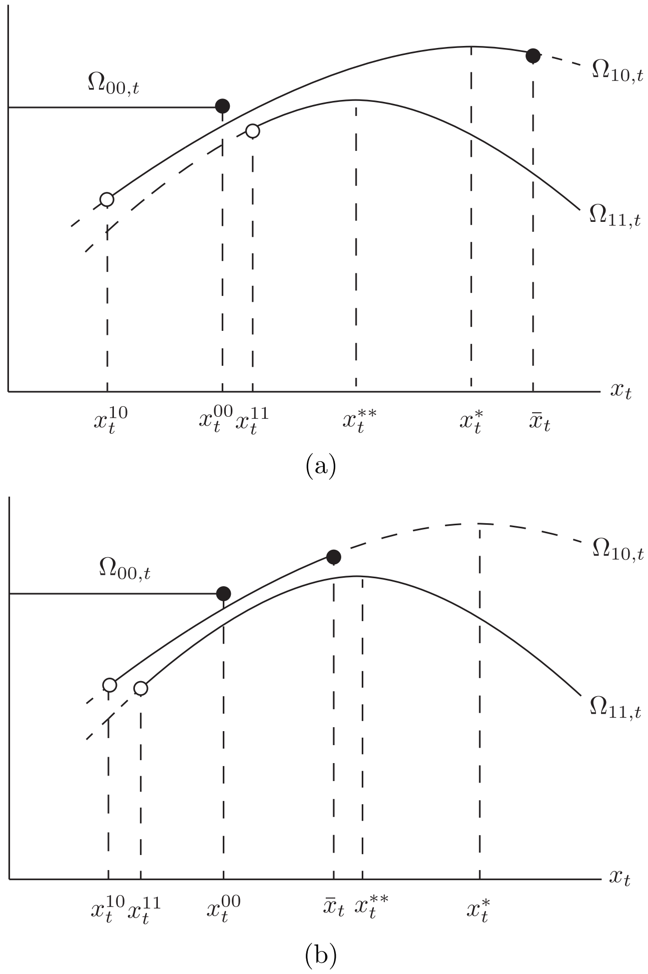


Figure 5: Illustration of the political objective function for the case of $\rho_t \in \left(0, \rho^l\right)$ and $x_t^* \leq \bar{x}_t$ (Panel (a)) and the case of $\rho_t \in \left[\rho^l, \rho^m\right)$ and $x_t^* > \bar{x}_t$ (Panel (b)).
Note: See the note in Figure 4 for an explanation of the solid and dotted curves.

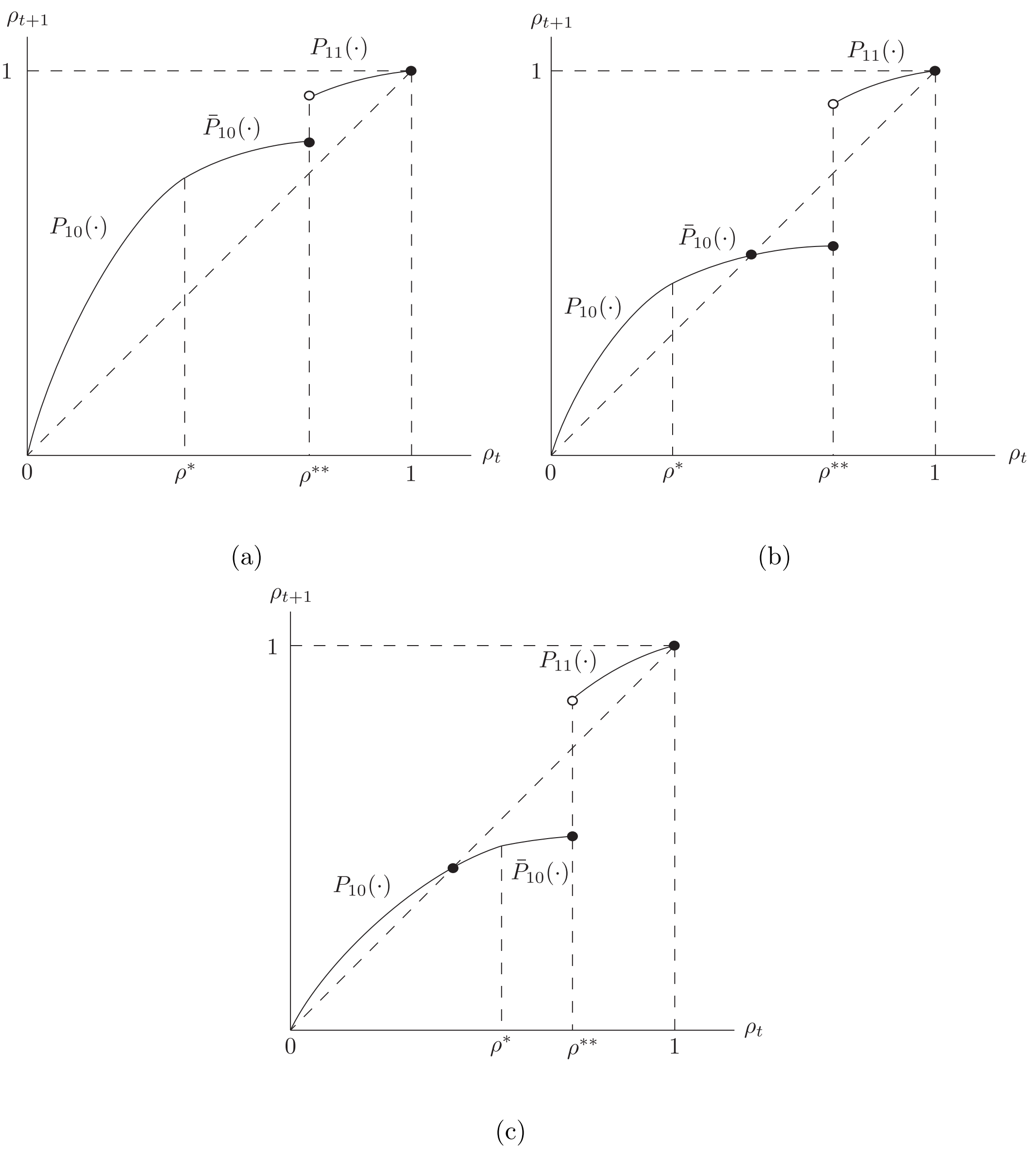


Figure 6: $\bar{P}_{10}(\rho^{**}) > \rho^{**}$ case (Panel (a)) and $\bar{P}_{10}(\rho^{**}) \leq \rho^{**}$ case (Panels (b) and (c)).
Note: The curves denoted by $P_{10}(\cdot)$, $\bar{P}_{10}(\cdot)$, and $P_{11}(\cdot)$ illustrate the motions of ρ_t represented by $\rho_{t+1} = P_{10}(\rho_t)$, $\bar{P}_{10}(\rho_t)$, and $P_{11}(\rho_t)$, respectively.

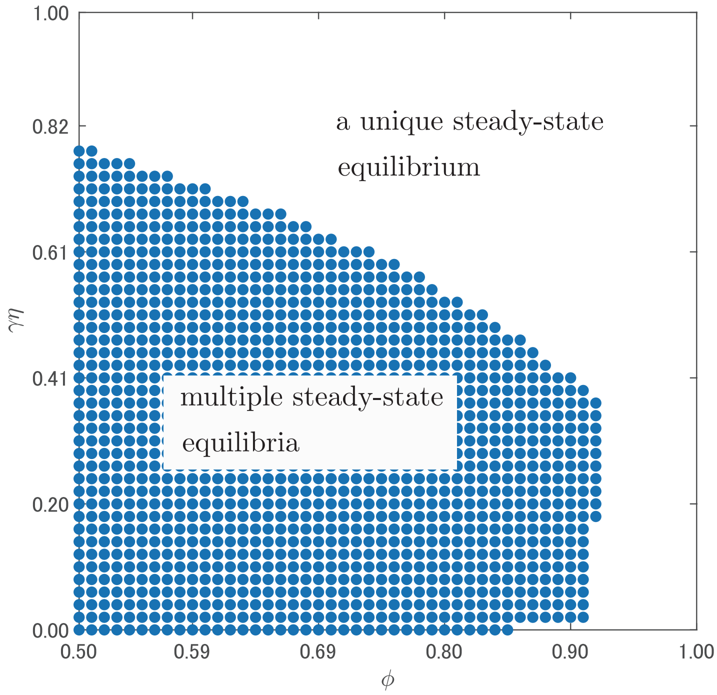


Figure 7: The horizontal axis takes ϕ, and the vertical axis takes $\gamma\eta$. Multiple steady-state equilibria for the shaded area; a unique steady-state equilibrium for the non-shaded area.

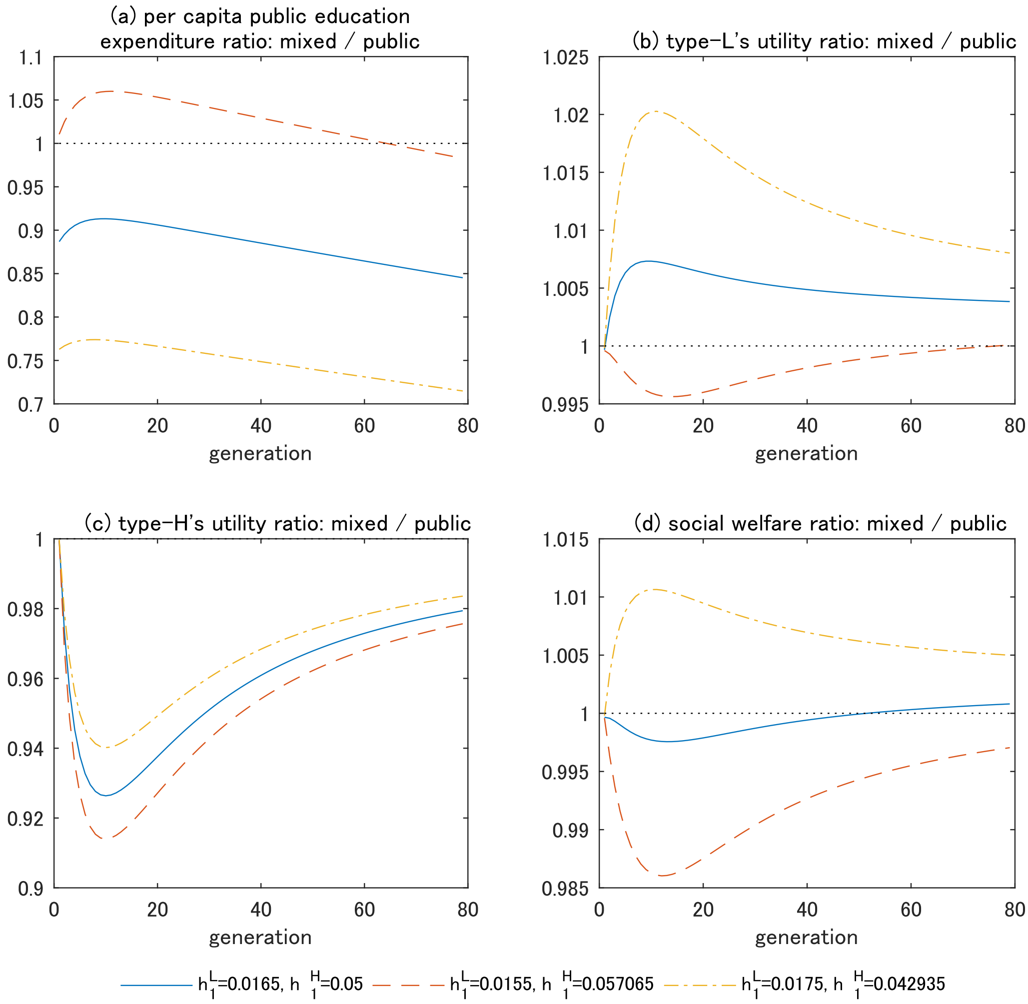


Figure 8: Ratios of mixed education systems to compulsory public school systems in terms of per capita public education expenditure (Panel (a)), utility of type-L agents (Panel (b)), utility of type-H agents (Panel (c)), and social welfare (Panel (d)) for the three initial inequality scenarios. The solid curves, dashed curves, and dot-dashed curves correspond to the baseline case, high initial inequality case, and low initial inequality case, respectively.

(a) Type-L's utility

	Baseline	High inequality	Low inequality
Generation 1	0.99967	0.99961	0.9998
Generation 2	1.0025	0.9993	1.0063
Generation 3	1.0043	0.99879	1.0108
Generation 4	1.0055	0.99822	1.014

(b) Type-H's utility

	Baseline	High inequality	Low inequality
Generation 1	0.99955	0.99943	0.99975
Generation 2	0.97352	0.96847	0.979
Generation 3	0.95657	0.94861	0.96528
Generation 4	0.94529	0.93553	0.95604

(c) Social welfare

	Baseline	High inequality	Low inequality
Generation 1	0.99966	0.99959	0.9998
Generation 2	0.99957	0.99633	1.0034
Generation 3	0.99932	0.99369	1.0059
Generation 4	0.999	0.99159	1.0076

Table 1: Ratios of mixed education systems to compulsory public school systems in terms of the utility of type-L agents (Panel (a)), the utility of type-H agents (Panel (b)), and social welfare (Panel (c)) for the three initial inequality scenarios from generation 1 to generation 4.

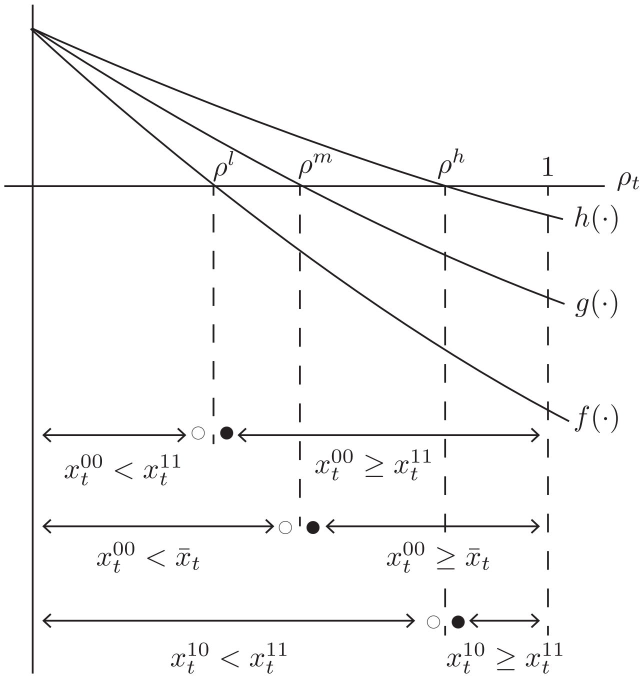


Figure A.1: Illustration for Condition (7).