Parameter variation in the "log t" convergence test

Johnson, Paul A.

Vassar College

15 May 2019

Parameter variation in the "log *t*" convergence test

Paul A. Johnson
Department of Economics
Vassar College
Poughkeepsie, New York 12604
pajohnson@vassar.edu

May 15, 2019

This paper estimates a local linear version of the model used in Phillips and Sul's (2007, 2009) "log *t*" convergence test. It documents the economically and statistically significant within-sample variation in the estimated value of the key parameter of that test when applied to data for 18 OECD countries during the 20th century. This variation suggests the substantial waxing and waning of the forces driving convergence, possibly due to low-frequency shocks and changes in the level economic integration.

Keywords: Cross-country convergence, σ-convergence, transition

JEL Classification: O11, O47, F63

1. Introduction

Using their "log *t*" convergence test, Phillips and Sul (2009) find convergence in growth rates but not levels of per capita GDP for a group of 18 OECD countries over the 1870-2001 period. Their application of the test to various subsamples of their sample period reveals substantial across subsample variation in the estimated value of the parameter that indicates whether or not convergence is occurring. This note examines the parameter's stability by estimating a local linear regression version of the "log *t*" convergence regression that allows that parameter to vary over time. I find evidence of convergence in levels from 1950 to 2001 as well as both convergence in growth rates and divergence from in several periods between 1914 to 1950. This suggests that forces for convergence were weakened by low-frequency disturbances such as the Great Depression and WW2 and strengthened during periods of economic stability and increasing economic integration.

2. The "log *t*" convergence test

Letting $y_{i,t}$ denote the log of GDP per capita in country i at time *t*, Phillips and Sul (2007, 2009) write $y_{i,t} = b_{it}\mu_t$, where μ_t is the hypothesized steady-state growth path common to all countries and b_{it} describes the transition path of economy to the steady state growth path. They then define $h_{it} = ny_{it}/\sum_{j=1}^{n} y_{jt} = nb_{it}/\sum_{j=1}^{n} b_{jt}$ where *n* is the number of countries and say that convergence occurs if $\lim_{t\to\infty} h_{it} = 1$ for all *i*. To test for convergence so defined, Phillips and Sul propose the "log *t*" convergence test by specifying a model of the transition path, b_{it}, that yields a convergence test based on a conventional one-sided t-test of the hypothesis $\gamma = 0$ (no convergence) against the alternative $\gamma > 0$ (convergence) in the regression

$$\log\frac{H_1}{H_t} - 2\log(\log t) = a + \gamma \log t + u_t \qquad (1)$$

where $H_t = \frac{1}{n}\sum_{i=1}^{n}(h_{it} - 1)^2$ is the sample variance of h_{it}.[1] In addition to implying an absence of convergence, Phillips and Sul (2007) show that $\gamma < 0$ implies divergence, $0 < \gamma < 2$ implies convergence in growth rates, and $2 \leq \gamma$ implies convergence in levels.

Using data on a group of 18 OECD countries over the 1870-2001 period, Phillips and Sul (2009) report an estimate of γ of 1.71 with an estimated standard error of 0.07. As the estimate is statistically significantly different from both zero and two, they conclude that convergence in growth rates occurred for those countries over the sample period. To study the effects of transitions to the steady state they also apply their test to various subsamples of the data sea and find marked variation in the estimates of γ. For example, for the 1870-1929 period, the point estimate is – 0.42, with an estimated standard error of 0.04, and so convergence is rejected for this period. By contrast, for the 1940-2001 period, the point estimate is 1.14, with an estimated standard error of 0.01, implying convergence in growth rates for this period. This variation suggests that the forces driving convergence may have waxed and waned substantially over the sample period and warrants further investigation.

3. A local linear version of the "log *t*" convergence test

To study the possible variation in forces driving convergence, I propose use of a model that allows the γ parameter in Equation (1) to vary. Specifically, I propose replacing Equation (1) with

$$\log\frac{H_1}{H_t} - 2\log(\log t) = m(\log t) + u_t \tag{2}$$

[1] This test can be interpreted as a type of σ-convergence test. See Islam (2003), Durlauf, Johnson, and Temple (2005), and Johnson and Papageorgiou (2018) for surveys of the convergence literature and expositions of the different types of convergence tests.

where $m(x)$ is a smooth function of x. Letting $m'(x)$ denote the first derivative of $m(x)$, $m'(\log t) < 0$ implies that the rate of decline of H_t, the sample variance of h_{it}, at time t is consistent with long-run divergence while $0 < m'(\log t) < 2$ implies that it is consistent with long-run convergence in growth rates and $2 \leq m'(\log t)$ implies that it is consistent with long-run convergence in levels.

To estimate Equation (2), I use the local linear regression approach described in Loader (1999). Given a postulated relationship $z = m(x) + \epsilon$, this approach approximates $m(x)$ as $m(x) \approx a + b(x - x_0)$ for x in a neighborhood of x_0. A dataset of n pairs of observations $\{(x_i, z_i)\}_{i=1}^{n}$ can then be used to estimate a and b as the minimisers of $\sum_{i=1}^{n} w_i(x)\,(z_i - a - b(x_i - x))^2$ where $w_i(x) = K(x_i - x)/h(x)$, $K(s)$ is a kernel function, and $h(x)$ is the bandwidth so that only observations in the interval $(x - h(x), x + h(x))$ are used to estimate a and b with observations closer to x receiving greater weight. The resultant estimator of b is the weighted least squares estimator

$$\hat{b} = \frac{\sum_{i=1}^{n} w_i(x)(x_i - \bar{x}_w)z_i}{\sum_{i=1}^{n} w_i(x)(x_i - \bar{x}_w)^2}$$

where $\bar{x}_w = \sum_{i=1}^{n} w_i(x)x_i / \sum_{i=1}^{n} w_i(x)$. Loader (1999) argues that $\hat{b}$, the "local slope", is the appropriate estimator of the derivative $m'(x)$.

4. Data and results

Using the `locfit` package in R (Loader, 2013), I estimate the local linear regression version of Equation (2) using the same data used by Phillips and Sul (2009) for their analysis of convergence in 18 Western OECD countries.[2] Making use of portions of the R code written by Schnurbus, Haupt, and Meier (2017) for their replication of Phillips and Sul's (2009) results on convergence clubs, I follow Phillips and Sul (2009) and use the Hodrick-Prescott (1997) filter to remove the cyclical component of the data and then calculate the dependent variable $\log\frac{H_1}{H_t} - 2\log(\log t)$ as described above, before omitting the first third of the data points so that the effective sample period is 1914 to 2001. Estimation of Equation (1) using the data so constructed exactly reproduces Phillips and Sul's (2009) results for the 18 Western OECD countries.

To estimate the model in Equation (2) with this data, I use the `locfit` default tricube function for the kernel so $K(s) = (1 - |s|^3)^3$ and set the bandwidth so that the smoothing window contains 20% of the observations.[3] As the sample consists of 88 observations, I use a bootstrap method to construct confidence intervals for $m'(\log t)$. Specifically, motivated by the high persistence in the residuals which have a first-order autocorrelation coefficient of 0.96, I use a non-overlapping block bootstrap (Lahiri, 2003). To do so, I divide the residuals into 8 non-overlapping blocks of 11 sequential residuals and then sample the blocks with replacement to construct sets of 88 artificial errors. These are added to the fitted values from the original local linear regression to construct the artificial dependent variable and the model is estimated using this resampled data using exactly the same specification as for the original estimation. I perform 1000 such replications and then take the 5th-smallest and 5th-largest values of the local estimated slope at each data point as the lower and upper bounds respectively of the estimated 99% confidence interval for $m'(\log t)$.

---

[2] The countries are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, UK, and USA. The data are available at http://doi.org/10.3886/E108421V1.

[3] The results here are qualitatively robust to other choices of $K(s)$ available in `locfit` as well as to variations in the bandwidth so that the window contains between 5% and 50% of the observations.

Figure 1 below plots the estimated derivative of $m(\log t)$ computed as the local slope of the estimated local linear regression line and the estimated 99% confidence interval. Also shown are the regions where the values of γ in Equation (1) would correspond to long-run divergence and growth-rate and level convergence. The figure shows that the early 20th century is characterized by weak growth-rate convergence with the estimated derivative being slightly above zero until the turmoil of the Great Depression pushes it into the divergence region in 1930. The confidence interval includes only negative values of the derivative for much of the 1930s and early 1940s indicating statistically-significant divergence in this period. In the early 1940s the estimated derivative begins rising rapidly and crosses back into the growth-rate convergence region in 1948 before entering the level convergence region in 1950 with statistically-significant level convergence being evident after the mid-1950s. The estimated derivative continues to rise until it reaches about six in 1970 before falling to about three by about 1980, a value that it approximately maintains until the end of the sample period.

That the estimated derivative exceeds two, and often by a substantial amount, during much of the post-WW2 period, implies the presence of strong forces for level convergence in that period, especially during the 1960s and early 1970s. This contrasts with Phillips and Sul's (2009) finding of only growth-rate convergence when the entire sample period is considered. Some additional insight into the implications of the variations in the forces for convergence is provided by Figure 2 below which plots the sample variance of the raw (i.e. not Hodrick-Prescott filtered) log GDP per capita data for the same period as in Figure 1. During the post-WW2 period the variance declines rapidly implying σ-convergence among this group of countries during that period. [4] Also

[4] This paper is not intended to present tests of the convergence hypothesis per se so DeLong's (1988) well-taken point about the sample selection issues involved in using data a group of successful countries in this context doesn't detract from the conclusions.

evident is the σ-divergence caused by the Great Depression and WW2. That is, as Figure 1 suggests, the forces for convergence were weakened by those two low-frequency disturbances before being strengthened by the relative stability and increasing economic integration of the post-WW2 period.[5] By the end of the sample period, the rate of decline of the variance slows quite a bit suggesting the forces driving convergence are largely exhausted. This is consistent with the estimated derivative being close to two, the lower limit of the level-convergence region, and with the lower limit of the estimated 99% confidence interval falling below that value in the late 1990s, as shown in Figure 1.

5. Concluding remarks

The results presented here are not intended to question the findings of Phillips and Sul (2009) in any way. Rather, they document important changes in strength of the forces driving convergence during the 20th century. These changes are consistent with, for example, the results of Epstein Howlett, and Schulze (2003) who apply distribution dynamics methods (see Quah, 1997) to data from the OECD countries and find that, while the pre-1914 and 1914-1950 periods appear characterized by persistence within the cross-country distribution of per capita incomes, there is some mobility and some convergence in the post-1950 period but this weakens beginning in the early 1970s. The importance of the post-1950 convergence among the developed countries is underscored by Di Vaio and Enflo (2011) who cite the role of globalisation as a driver. Delong and Dowrick (2003) similarly argue that periods of increased globalisation such as those prior to WW1 and after WW2 tended to foster convergence among the developed countries.

[5] Repeating the calculations for Figure 1 with the raw data, so that both the trend and cyclical parts of the variation are included, yields a qualitatively similar result although, as might be expected, the estimated derivative is more variable, dipping into the growth-rate convergence zone during the early-1980s recession, and has a wider confidence interval.

Acknowledgements

I thank Steven Durlauf and Paul Ruud for comments on an earlier draft, Diana Henry for efficient research assistance, and Donggyu Sul for supplying the OECD dataset used in Phillips and Sul (2009). Financial support from the Lucy Maynard Salmon Research Fund is gratefully acknowledged. All errors are mine.

Declarations of interest: none.

References

De Long, B. 1988. "Productivity Growth, Convergence, and Welfare: Comment." *American Economic Review* 78 (5) : 1138-1154.

Di Vaio, G., and Enflo, K. 2011. "Did globalization drive convergence? Identifying cross-country growth regimes in the long run." *European Economic Review* 55 (6): 832-844.

Dowrick, S. and DeLong. J.B. 2003. "Globalization and Convergence" in Bordo M.D., A.M. Taylor and J.G. Williamson, eds (Eds.), *Globalization in Historical Perspective*, NBER Conference Report series. Chicago and London: University of Chicago Press.

Durlauf, S.N., Johnson, P. A. and Temple., J.R.W. 2005. "Growth Econometrics," in P. Aghion and S.N. Durlauf, eds., *Handbook of Economic Growth*, Amsterdam, North-Holland, 555-677.

Epstein, P., Howlett, P., and Schulze, M. 2003. "Distribution dynamics: Stratification, polarization, and convergence among OECD economies, 1870-1992." *Explorations in Economic History* 40 (1): 78-97.

Hodrick, R. J., and Prescott, E. C. 1997. "Postwar U.S. business cycles: An empirical investigation." *Journal of Money, Credit, and Banking* 29 (1): 1-16.

Islam, N. 2003. "What Have We Learnt from the Convergence Debate?" *Journal of Economic Surveys* 17 (3): 309-362

Johnson, P., and Papageorgiou, C. 2018. "What Remains of Cross-Country Convergence?", *Journal of Economic Literature*, forthcoming,

Lahiri , S. N. 2003. *Resampling Methods for Dependent Data*. New York: Springer-Verlag.

Loader, Catherine. 2013. locfit*: Local Regression, Likelihood and Density Estimation*, R package version 1.5-9.1. https://CRAN.R-project.org/package=locfit.

Loader, Clive. 1999. *Local Regression and Likelihood*. New York: Springer, New York.

Phillips, P. C. B., and Sul, D. 2007. "Transition modeling and econometric convergence tests." *Econometrica* 75 (6): 1771-1855.

Phillips, P. C. B., and Sul, D. 2009. "Economic transition and growth." *Journal of Applied Econometrics* 24 (7): 1153-1185.

Quah, Danny. .1997. "Empirics for growth and distribution: Stratification, polarization, and convergence clubs." *Journal of Economic Growth* 2 (1): 27–59.

R Core Team. 2016. R: *A language and environment for statistical computing*. Vienna: R Foundation for Statistical Computing. https://www.R-project.org/.

Schnurbus, J., Haupt, H., and Meier, V. 2017. "Economic transition and growth: A replication." *Journal of Applied Econometrics* 32 (5): 1039-1042.

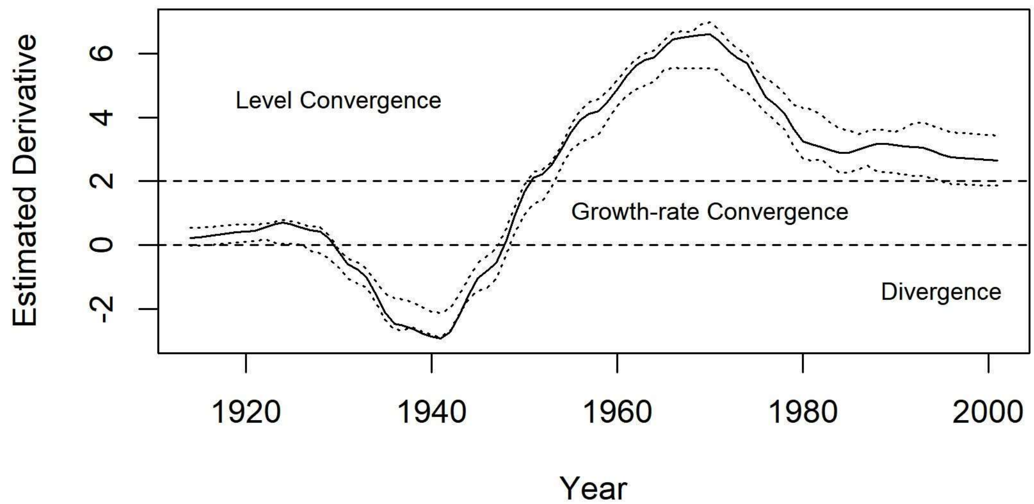

Figure 1: Estimated first derivative of $m(\log t)$ (solid line) and estimated 99% confidence intervals (dotted lines). Author's calculations using data from Phillips and Sul (2009)

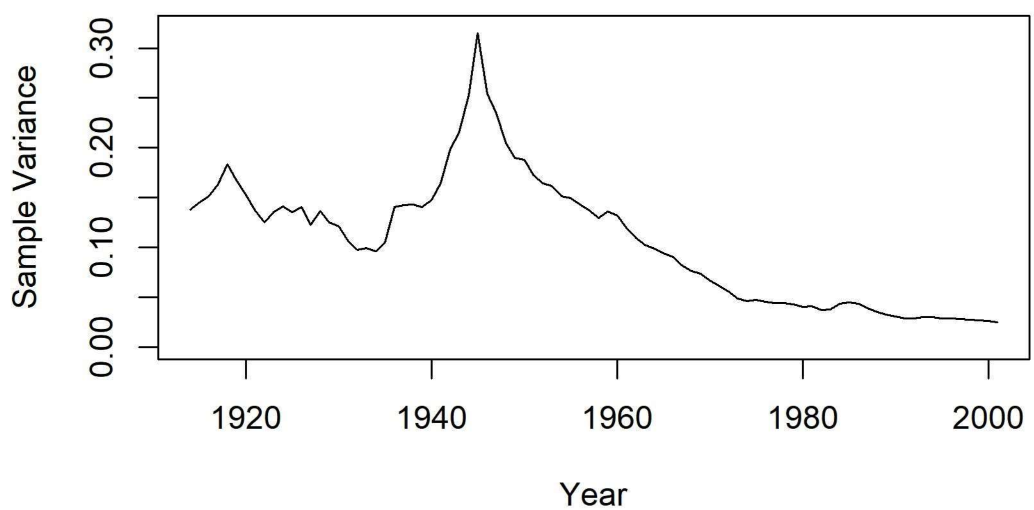

Figure 2: Cross-country variance of log GDP per capita for 18 Western OECD countries. Author's calculations using data from Phillips and Sul (2009)