Social wealth and optimal care

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Abstract

Many accidents result in losses that cannot be perfectly compensated by a monetary payment. Moreover, often injurers control the magnitude rather than the probability of accidents. We study the characteristics of optimal levels of care and distribution of risk under these circumstances and show that care depends on the aggregate wealth of society but does not depend on wealth distribution. We then examine whether ordinary liability rules, regulation, insurance, taxes and subsidies can be used to implement the first-best outcome. Finally, our results are discussed in the light of fairness considerations (second best).

JEL codes: K13

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1 Introduction

Often accidents result in bodily injury, death or other noncompensable losses—in addition to pecuniary damages—to large numbers of passive victims. Moreover, such accidents at times involve precaution technologies that reduce the likely magnitude of the harm rather than affecting its probability. We study the optimal design of liability rules for accidents having these characteristics, considering both the allocation of risk and the level of care.\(^1\) We focus on the question whether the level of care taken by injurers should depend on individuals’ risk aversion and wealth. With respect to wealth, we look at both the aggregate wealth of society and its distribution among individuals. These issues bear, for instance, on the setting of safety standards in developing countries and on the question whether they should be similar to those set in richer countries.

Our approach departs from previous literature in two respects. First, we consider noncompensable harm, that is, harm to a nonreplaceable commodity such as life or health. There is evidence that such injuries change the individuals’ marginal utility of money (Viscusi and Evans, 1990). The literature on tort liability has mainly analyzed accidents where the harm is pecuniary and, thus, can be perfectly compensated, at least in principle.\(^2\) There is consensus on the fact that, if harm is pecuniary, the first best solution consists of separating the incentive problem from the risk-sharing and the wealth-distribution problems. Consequently, liability rules should provide incentives for injurers to take the level of care that minimizes the total accident costs, insurance should provide for an optimal allocation of risk, and the income tax system should provide for redistribution.\(^3\)

When insurance is not available or insurers cannot cope with the moral hazard problem effectively, then perfectly separating risk-sharing from incentives is not possible and the socially optimal level of care depends on the parties’ risk aversion (Shavell, 1982 and 1987). Likewise, when redistribution by independent means is not possible, the socially optimal level of care depends on the distribution of wealth among individuals, but it is still independent from the aggregate wealth of society (Arlen 1992; Miceli and Segerson, 1995).

In order to bring noncompensable harm into the analysis, we employ a state-dependent expected utility representation for the individuals’ utilities\(^4\) (SDEU) and compare our results with those above. This is to take account of the fact that, after an accident, victims may find themselves in an inferior 'state’ (being on a wheelchair or having lost a child), in which their utility is less than it was before the accident. Thus, damage compensation is not enough to bring victims back to their initial utility, irrespective of the magnitude of the compensation.

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\(^1\) Calabresi (1970) indicated both incentives and risk-sharing as two important goals for the liability system (the third being the minimization of the administrative costs of the system). The subsequent economic literature first tackled the incentive problem, starting from Brown (1973), and later focused also on the risk-sharing problem, beginning with Shavell (1982). See also Landes and Posner (1987) and Shavell (1987).

\(^2\) For a recent review see Arlen (2000).

\(^3\) See also Shavell (1981) and Kaplow and Shavell (1994 and 2000).

\(^4\) Arrow (1964); Hirschleifer (1970); Keeney and Raiffa (1976); Karni (1985).
Injurers, in turn, might also be in an inferior state after the accident. They might suffer from bad reputation, loss of job opportunities, or remorse as a result of being involved in an accident in addition to having to pay damages (Nielsen and Winter, 1997). The second departure from existing literature consists of bringing into consideration the distinction between precautions that affect the magnitude of the harm (self-insurance) and precautions that affect the probability of accidents (self-protection). The distinction between self-insurance and self-protection has been developed in the insurance literature (Ehrlich and Becker, 1967; Sweeney and Beard, 1992), which deals with a party who takes precautions in order to prevent harm to himself. In contrast, our model involves an injurer creating risks for passive victims. The tort literature has exploited this distinction with reference to the incentive problem (Boyd and Ingberman, 1994; Dari-Mattiacci and De Geest, 2005), while papers dealing with incentives and risk focus on self-protection only. The present analysis bridges this gap in the literature by focusing on accidents of the self-insurance type, where the probability of occurrence is exogenous, but the injurer can reduce the magnitude of the harm by taking care. Safety measures such as evacuation plans, pollution-containment procedures, timely alarms, helmets, or safety harnesses reduce the magnitude of the harm but not the probability of the accident. Our model applies to those situations and focuses both on risk and on incentives.

In the analysis, we first describe a care-free world—in which the only problem is to allocate risk optimally—and a risk-free world—in which the only problem is induce an optimal level of care. In relation to these two benchmark cases, we study the properties of the first-best allocation of risk and the first-best level of care in a world in which both aspects are important. In particular, we focus on four characteristics of the first best and demonstrate the following results:

1. Mutuality: The costs of accidents should be borne in some proportion by both parties. As a result, an accident results in a reduction of both wealth and utility for both parties, which runs against principles of full compensation of innocent victims.

2. Care and risk: Compared to accidents involving only pecuniary losses, accidents involving noncompensable losses do not necessarily require higher levels of care. If the marginal utility of money is greater after an accident, a higher level of care is socially optimal; however, the opposite might be true if marginal utility of money is less after an accident. This result also obtains when parties are risk-neutral as long as at least one of them suffers a noncompensable loss.

3. Care and society’s wealth: The socially optimal level of care depends on society’s initial wealth. In contrast, the distribution of the social wealth among individuals is irrelevant for the level of care. Whether care increases or decreases in society’s wealth depends in turn on the individuals’ tolerance to risk.

4. Care and probability of accidents: The socially optimal level of care increases when the probability of accidents increases.

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5 In the analysis we show that our main results apply both to firms and to individuals.

6 See also Jullien, Salanié and Salanié (1999) focusing on the relationship between risk-aversion and level of care.
It is shown that these characteristics apply to the first best—where the allocation of risk is optimal—but are also (weakly) persistent in the second best—where fairness concerns suggest that one party (usually the victim) should be fully insured against accidents. The analysis is then extended to a setting with varying numbers of victims holding total harm fixed. The presence of multiple victims allows for a natural spreading of risk and is shown to have the same consequences as an increase of social wealth.

After describing the characteristics of the first and second best policies concerning care and the risk-sharing, we investigate what instruments can be used to implement these policies. A mix of regulation of safety, taxes and subsidies is enough to achieve the first best. This should be no surprise. In fact, the policy maker has three instruments to reach three goals: enforce the desired level of care and redistribute wealth in the good and the bad state of the world. We further examine whether ordinary liability rules such as strict liability and simple negligence, possibly paired with some forms of redistributive taxation or private insurance can reach the first or the second best. In general, contrary to the literature dealing with compensable losses (Shavell, 1982; Miceli and Segerson, 1995) we find that the second best is attainable while the first best requires additional instruments. Our results are different because we consider instances in which risk is not perfectly diversifiable.

Our analysis suggests that the level of care that should be enforced by means of regulation or liability rules depends on the aggregate wealth of society but not on the distribution of such a wealth among individuals. The reason is intuitive: any distributional issue can be corrected by redistributing wealth, while the total wealth of society is an insurmountable constraint. The paper is structured as follows: Section 2 presents the properties of the first- and second-best levels of care and distribution of risk. Section 3 discusses the implementation of the first and second best by means of ordinary liability rules, regulation, insurance, taxes and subsidies. Section 4 concludes.

2 Analysis

2.1 The basic framework

We consider a simple society with two different groups of identical individuals, injurers and victims, who are initially endowed with wealth \( w = w_0 \) and \( y = y_0 \), respectively, and are strangers to each other. Note that \( W_0 = w_0 + y_0 \) represents society’s initial aggregate wealth—henceforth simply society’s wealth.

An injurer’s activity may result in an accident with an exogenous probability \( p > 0 \); if an accident occurs, a victim suffers a pecuniary loss \( h(x) \), which depends on the injurer’s pecuniary investment in care \( x \), with \( h'(x) < 0, h''(x) > 0, h(0) = H > 0, \) and \( h'(-\infty) \to 0 \). We assume that it is always profitable both for the injurer and for society that the injurer undertakes such an activity. Thus, our analysis does not address questions concerning the optimal level of activity.

The loss also implies the destruction of an irrereplaceable asset for both par-
ties. To capture this aspect, we employ a SDEU representation of individuals’ utilities: \( u_i(w) \) and \( v_i(y) \) denote the injurer’s and the victim’s utility, which are functions of their wealth, with \( u_i'(w), v_i'(y) > 0 \) and \( u_i''(w), v_i''(y) \leq 0; i = b \) in the accident state (the ”bad” state) and \( i = g \) in the no-accident state (the ”good” state). We also require the following assumption to hold:

**A1: The good state is better than the bad state.** An individual never prefers the bad state over the good state, whatever his wealth; \( \forall w : u_g(w) \geq u_b(w) \) and \( \forall y : v_g(y) \geq v_b(y) \).

We will first consider the case in which a benevolent social planner can command a certain level of care \( x \) and a certain allocation of risk \((w_b, w_g; y_b, y_g)\). The planner’s objective is to maximize social welfare, defined as follows:

\[
SW = p[u_b(w_b) + v_b(y_b)] + (1 - p)[u_g(w_g) + v_g(y_g)]
\]  
subject to the resource constraints \( w_b + y_b = w_0 + y_0 - h(x) - x \), in the bad state, and \( w_g + y_g = w_0 + y_0 - x \), in the good state.

**2.2 Benchmark cases**

We will analyze the social planner’s problem against two standard benchmark cases: one in which the planner only deals with risk-spreading and another in which the planner only deals with incentives.

**B1: The care-free world.** The first scenario concerns a world without safety technology, in which the injurer is not able to reduce the pecuniary loss \( H \) to the victim in case of accident and hence the planner only optimizes the spreading of risk. We refer to this scenario as the care-free world. Standard results show that a Pareto-efficient allocation of risk implies that both the injurer’s and the victim’s wealth are larger in the good state than in the bad one: \( y_b < y_g \) and \( w_b < w_g \). This allocation of risk is said to be comonotonic in the sense that, the richer the society as a whole in a state, the richer its members individually.\(^7\)

**B2: The risk-free world.** In the second scenario, there is a safety technology but both individuals are risk-neutral and have state-independent utility functions—that is, the harm is entirely pecuniary: A risk-free world. In this case, any feasible allocation of risk is Pareto efficient, and the first-best level of care satisfies:

\[
ph'(\hat{x}) + 1 = 0
\]  
The socially optimal, risk-free level of care \( \hat{x} \) increases in \( p \) but depends neither on society’s aggregate wealth, nor on the distribution of wealth among individuals.

\(^7\)See Borch (1962) and Raviv (1979) for Expected Utility models; extensions to Non Expected Utility models are provided by Landsberger and Meilijson (1994) and Chateauneuf, Dana and Tallon (2000).
2.3 First-best allocation of risk and level of care

Our model encompasses situations in which both risk and care are of importance, thus the first best is characterized by a certain level of care \( x \) and a certain sharing of the aggregate risk in each state \((w_b, w_g; y_b, y_g)\) which maximize (1). In this section, we examine the characteristics of the first best and relate them to the benchmark cases discussed above.

2.3.1 Mutuality

Accidents reduce the aggregate wealth of society. However, this loss can in principle be allocated in many different ways between the parties involved. The following proposition puts some restrictions on such feasible allocations of risk.

**Proposition 1** The first-best allocation of risk is comonotonic, \( w_b \leq w_g \) and \( y_b \leq y_g \) (mutuality principle), and satisfies Borch’s conditions, \( u'_b(w_b) = v'_b(y_b) \) and \( u'_g(w_g) = v'_g(y_g) \).

**Proof.** See appendix. ■

According to the Mutuality Principle developed in the insurance literature (Borch, 1962), since the aggregate social wealth in the good state is always larger than in the bad state, it is efficient to give both individuals a level of wealth in the good state that is at least as large as in the bad state. Proposition 1 extends such a principle to accident cases in which also the cost of care is considered. Note, however, that not all comonotonic allocations are efficient but only those comonotonic allocations that satisfy Borch’s conditions.\(^8\) Those conditions yield that an efficient allocation of risk is reached when, in each state, the aggregate social wealth is shared so that the injurer’s marginal utility of wealth equals the victim’s marginal utility of wealth.

It is important to remark that the optimal allocation of risk implies that, in relation to the parties’ wealth, neither the injurer nor the victim obtains full insurance, that is, neither of them obtains the same wealth in the bad state as in the good state.\(^9\) This implies that in relation to the parties’ utility, neither the injurer nor the victim is protected against adverse changes in his utility, that is, their utility is necessarily less in the bad state than in the good state. This result matches what already observed about the allocation of risk in a care-free world.

2.3.2 Care and risk

In order to characterize the first-best level of care and compare this general scenario with the benchmark case of a risk-free world, we need to impose some

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\(^8\) Note that we arrive at a unique solution in that we have implicitly assigned the same weight to the victim and the injurer in the calculation of social welfare. More generally, given different weights one can arrive at different Pareto optimal allocations.

\(^9\) Unless some restrictive conditions are satisfied, the relations in proposition 1 are strict inequalities.
structure on the way in which the accident affects the marginal utility of wealth for the parties. We consider two alternative situations in turn:

A2a: Money is more useful in the good state. The parties’ marginal utility is never greater in the bad state than in the good state, at any level of wealth; $\forall w : u'_g(w) \geq u'_b(w)$ and $\forall y : v'_g(y) \geq v'_b(y)$.

A2b: Money is more useful in the bad state. Under this alternative assumption, the marginal utility of money is never less in the bad state than in the good state, at each level of the parties’ wealth; $\forall w : u'_g(w) \leq u'_b(w)$ and $\forall y : v'_g(y) \leq v'_b(y)$.

The next proposition addresses the question how the general case compares with the risk-free benchmark. In particular, is the level of care greater or less than in a risk-free world?

**Proposition 2** For any first-best allocation of risk:

i) Under assumption A2a, the first-best level of care may be greater or less than the risk-free level of care;

ii) Under assumption A2b, the first-best level of care is greater than the risk-free level of care.

**Proof.** In the appendix, it is shown that given a first-best allocation of risk, the first-best level of care $x^*$ satisfies the following condition:

$$ph'(x^*) + 1 = (1 - p) \left(1 - \frac{v'_g(y_g)}{v'_b(y_b)}\right)$$

Moreover, according to Borch’s conditions, the allocation of risk satisfies:

$$\frac{v'_g(y_g)}{v'_b(y_b)} = \frac{u'_g(w_g)}{u'_b(w_b)}$$

The condition in (3a), implies that the optimal level of care $x^*$ does not necessarily minimize the expected cost of accidents as it is the case in an risk-free world. In contrast, the optimal level of care is adjusted to respond to risk. Consequently, the first-best level of care $x^*$ is greater than the first-best level of care $\hat{x}$ in a risk-free world if the right-hand side of (3a) is positive, and is less otherwise.

Recall that, according to proposition 1, we must have $y_g \geq y_b$ and, by concavity, $v'_g(y_g) \leq v'_g(y_b)$. Therefore: i) Under assumption A2a we have

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10The situations we are about to describe have been extensively discussed in the literature about irreplaceable commodities (Cook and Graham, 1977) or about self-protection expenditures and the willingness to pay for safety, health and life (Dehez and Drèze, 1987; Jones-Lee, 1974).

11Note that the condition in the text does not guarantee the uniqueness of the result, since the RHS in (3a) is not monotonic in $x$. 

\( v'(y_b) \geq v'(y_b) \), such that it may be the case that \( \frac{v'(y_b)}{v'(y_b)} \geq 1 \) and, thus, also \( x^* \geq \hat{x} \). ii) Under assumption A2b we have \( v'(y_b) \leq v'(y_b) \), which implies \( \frac{v'(y_b)}{v'(y_b)} \leq 1 \) and \( x^* \geq \hat{x} \). 

These results imply that the first-best level of care is generally different from the level of care that minimizes the sum of accident loss and cost of care, and it depends on the difference in marginal utilities between states. Our findings can be explained as follows: Care expenditures are a cost both in the good state and in the bad state, while reducing the magnitude of the loss in the bad state only. More precisely, socially optimal investment in care reduces social wealth in the good state—a self-evident observation—while increasing social wealth in the bad state. It is as if care realized an implicit transfer of wealth from the good state to the bad state, with the effect of reducing the difference in wealth between states.

When the marginal utility of money is larger in the bad state (A2b), it is clear that increasing care expenditures above the risk-free level is advantageous. When, instead, the marginal utility of money is larger in the good state, two contrasting forces govern the result. On the one hand, transferring money from the good state to the bad state is less desirable and might result in first-best care being less than in the risk-free world. On the other hand, risk-aversion might impose higher care expenditures, reversing the result. The following corollary clarifies the role of risk-aversion in determining the first-best level of care.

**Corollary 3** For any first-best allocation of risk, if either or both parties are risk-neutral and at least one of the parties has state-dependent preferences:

i) Under assumption A2a, the first-best level of care may be greater or less than the risk-free level of care;

ii) Under assumption A2b, the first-best level of care is greater than the risk-free level of care.

**Proof.** If at least one of the parties is risk-neutral, his marginal utility is constant, which implies according to Borch’s conditions: \( \frac{v'(y_b)}{v'(y_b)} = \frac{v'(y_b)}{v'(y_b)} = k \). Thus, it is easy to see that the relation between \( x^* \) and \( \hat{x} \) depends on the sign of \( 1 - k \), which is turn is determined by whether A2a or A2b applies. Note that this result also obtains when one of the parties has state-independent preferences as long as the other party has state-dependent preferences.

The logic of this result is that the risk-neutral party can indemnify the risk-averse party, counteracting the effect of risk aversion. Thus, the only relevant factor determining departures from the risk-free level of care is the marginal utility of money. The case in which only one party has state-dependent preferences may be a realistic description of accidents between a firm and an individual. These types of accidents may cause major or even fatal physical injuries, which are neither fully reparable nor fully compensable.

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12 Too see why this is the case, write \( (3) \) as \( h'(x^*) + 1 = \frac{1-p}{p} v'(y_b) \). Given that the RHS is negative, we have \( h'(x^*) + 1 < 0 \), which implies that the total accident cost in the bad state (victim’s loss plus cost of care) is decreasing at the socially optimal level of care.
2.3.3 Care and society’s wealth

An often debated question is whether the first-best level of care increases or decreases when society’s wealth increases. That is, are rich societies better off by exerting greater or less care than poor ones? The next proposition shows that the relationship between care and society’s wealth depends on society’s tolerance to risk in the good state relative to that in the bad state. Society’s tolerance to risk in a given state is defined as the sum of the individual indexes of tolerance to risk in that state. In turn, an individual’s index of risk tolerance is simply the inverse of his index of absolute risk aversion. That is, an individual who is only slightly risk averse is said to have a high tolerance to risk. Formally, let

\[ t_v^g = \frac{-u'(y_0)}{u''(y_0)}, \quad t_v^b = \frac{-u'(w_0)}{u''(w_0)}, \quad t_u^g = \frac{-u'(w_0)}{u''(w_0)}, \quad \text{and} \quad t_u^b = \frac{-u'(w_0)}{u''(w_0)} \]

be the indexes of absolute tolerance to risk, respectively for the victim and the injurer in the good and bad states.\(^{13}\)

**Proposition 4** For any first-best allocation of risk:

i) If society’s risk tolerance is larger in the good state, then the first-best level of care decreases in society’s wealth.

ii) If society’s risk tolerance is larger in the bad state, then the first-best level of care increases in society’s wealth.

**Proof.** Society’s initial wealth is \( W_0 = w_0 + y_0 \). In the appendix, it is shown that:

\[
\frac{\partial x^*}{\partial W_0} = \frac{t_u^g + t_v^b}{t_u^g + t_v^b} - 1
\]

(5)

Given that according to the second order condition for the first best,\(^{14}\) the denominator has a negative sign, we have:

\[
\text{sign} \frac{\partial x^*}{\partial W_0} = \text{sign} \left( 1 - \frac{t_u^g + t_v^b}{t_u^g + t_v^b} \right)
\]

yielding the results. \(\blacksquare\)

Four important implications follow from this proposition. First, the first-best level of care depends on society’s wealth. However, the direction of this relation in turn depends on society’s tolerance to risk. Thus, for richer societies it might be optimal to take either more or less care than for poorer societies, depending on a further condition.

Second, this condition states that society should take more care when richer than when poorer if its risk tolerance is higher in the bad state and, vice versa, society should take less care when richer than when poorer if the opposite applies. As we have previously observed, the fact that the harm can be reduced

\(^{13}\)Note that generally speaking \( t_v^g \neq t_v^b \) and \( t_u^g \neq t_u^b \). Note also that, although at the individual level, risk aversion and risk tolerance are inversely related, at the aggregate level, things are less clear. This implies that for a society to display more risk tolerance in a state, a sufficient (but not necessary) condition is that all individuals be less risk averse in that state.

\(^{14}\)See the proof of proposition 2 in the appendix.
by taking care implements an implicit transfer of wealth from the good state to
the bad state, thereby reducing the spread between the states. Thus, as society
becomes reacher, the issue is to decide in which state to benefit from this in-
crease in wealth. According to proposition 4, more wealth should be available
in the state where society is less sensitive to the dispersion in the distribution
of wealth between the states.

Third, the first-best level of care only depends on society’s initial wealth
and not on the distribution of wealth among individuals. This means that
two equally rich countries differing only with regard to the distribution of
wealth should invest the same amount to resources in care. Purely redis-
tributive changes in individuals’ initial wealth that keep the initial aggregate
wealth constant do not affect care.

Finally, due to the mutuality principle, as a society becomes reacher both
decidings become reacher in both states. However, which party most benefits
from this improvement depends on risk tolerance. From Borch’s conditions we can
derive the following relationships (see appendix).

\[
\begin{align*}
\frac{\partial y_b}{\partial W_0} &= \frac{t^v_b}{t^v_b} \frac{\partial w_b}{\partial W_0} \\
\frac{\partial y_g}{\partial W_0} &= \frac{t^v_g}{t^v_g} \frac{\partial w_g}{\partial W_0}
\end{align*}
\]

These conditions imply that the party exhibiting larger tolerance to risk in
a state will capture a larger portion of the increase in society’s wealth in that
state. This in turn implies that if the injurer’s risk tolerance is larger in both
states, the injurer should pay a decreasing portion of the total cost of accidents
as society becomes richer. This result can lead to a decrease of the injurer’s
liability towards the victim. Vice versa, if the victim’s risk tolerance is larger
in both states, the injurer should pay an increasing portion of the total cost of
accidents as society becomes richer, yielding the opposite result.

2.3.4 Care and probability of accidents

Proposition 5 An increase in the probability of accidents leads to an increase
in the first-best level of care, an increase in both individuals’ wealth in the bad
state, and a decrease in both individuals’ wealth in the good state.

Proof. See appendix. ■

An increase in \( p \) implies that the bad state becomes relatively more probable
than the good state. Hence, all else equal, society moves to a more risky (in
the sense of the first order stochastic dominance) distribution of wealth between
states, which deteriorates its welfare. An increase in care implicitly transfers
some wealth from the good state to the bad state, as previously explained. This
transfer is advantageous because it realizes a less spread distribution of wealth
between states (in the sense of the second order stochastic dominance), reducing

\(^{15}\)Consider, for instance, the countries \((w_0, y_0)\) and \((w'_0, y'_0)\) with \(w_0 + y_0 = W_0 = w'_0 + y'_0\).
society’s exposure to risk. As care increases and wealth is implicitly transferred from the good state to the bad state, the mutuality principle guarantees that both individuals become richer in the bad state and, consequently, poorer in the good state.

2.4 Fairness and second best: One party is fully insured

In this section, we focus on situations in which one party obtains full insurance against accidents. Full insurance implies that a party receives the same wealth, irrespective of the state of the world that materializes. Fully insuring one party realizes a second best (Pareto-constrained) outcome, since we have seen that in the first best both parties should bear some risk, even when the other one is risk neutral.

Accident prevention policies might be constrained by several considerations. Fairness, for instance, might require that innocent victims do not suffer any reduction in their wealth as a consequence of accidents that they were not in the position to avoid. We refer to this outcome as second best. Note, however, that the same reasoning could in theory apply to injurers involved in activities that are perceived as socially valuable, such as doctors or rescue teams in relation to harm arising from unsuccessful rescue operations. In this case we speak of dual second best; although the results below are derived with respect to the second best, they also apply to the dual second best.

Consider the case in which the victim is fully insured so that his wealth is constant across states: \( y_b = y_g = \bar{y} \). A specific case of full insurance obtains when \( \bar{y} = y_0 \), that is, the victim is guaranteed its initial level of wealth in all states. In this section, we treat this problem as a special case of the more general framework in which \( \bar{y} > 0 \) is arbitrarily set. This framework encompasses both the case where \( \bar{y} > y_0 \) and the case where \( \bar{y} < y_0 \), that is, the victim’s wealth is constant across states but might be either greater or less than his initial wealth.

We obtain more specifically the following results regarding the properties of the second best:

**Proposition 6** When fairness is considered:

i) The second-best allocation of risk is comonotonic;

ii) Under assumption A2a, the second-best level of care may be greater or less than the risk-free level of care; under assumption A2b, the second-best level of care is greater than the risk-free level of care;

iii) If the injurer’s risk tolerance is larger in the good state, then the second-best level of care decreases in society’s wealth; if the injurer’s risk tolerance is larger in the bad state, then the second-best level of care increases in society’s wealth;

iv) The second-best level of care increases with the probability of accidents.

**Proof.** Claim i) is self-evident. Concerning claims ii) to iv), given that the victim’s wealth is constant across states, the social problem in (1) simplifies to

\[
SW_{\bar{y}} = p_0 (w_0 + y_0 - \bar{y} - h(x) - x) + (1 - p)u_0 (w_0 + y_0 - \bar{y} - x)
\]  

(6)
where the only independent variable is the level of care $x$. The solution, denoted as $x\_y$, satisfies the following first order condition:

$$ph'(x\_y) + 1 = (1 - p) \left(1 - \frac{u'_g(w_0 + y_0 - \hat{y} - x\_y)}{u'_b(w_0 + y_0 - \hat{y} - h(x\_y) - x\_y)}\right)$$

The rest of the proof is analogous to the proofs of propositions 1 to 5. ■

Accordingly to this Pareto-constrained solution, the injurer obtains $w_b = w_0 + y_0 - \hat{y} - x - h(x)$, in the bad state, and $w_g = w_0 + y_0 - \hat{y} - x$, in the good state. It is easy to see that this allocation of risk does not generally correspond to the first best, since there is no guarantee that Borch’s conditions in proposition 1 are satisfied. Nevertheless, the resulting second-best allocation of risk and level of care have most of the characteristics found for the first best.

### 2.5 The harm is spread among $N$ victims

Where an accident involves $N$ victims, we can compare situations in which the harm resulting from an accident is spread among a varying number of victims. The first best is found by maximizing:

$$SW = pu_b(w_b) + (1 - p)u_g(w_g) + \sum_{i=1}^{N} (pu'_b(y^i_b) + (1 - p)v'_g(y^i_g))$$

subject to:

$$w_b + \sum_{i=1}^{N} y^i_b = W_0 - h(x) - x$$
$$w_g + \sum_{i=1}^{N} y^i_g = W_0 - x$$
$$w_0 + \sum_{i=1}^{N} y^i_0 = W_0$$

with $h(x)$ denoting the aggregate harm to all victims. Conditions (3) still hold and Borch’s conditions now are:

$$u'_b(w_b) = v'_b(y^i_b) \forall i = 1, ..., N$$
$$u'_g(w_g) = v'_g(y^i_g) \forall i = 1, ..., N$$

such that (5) becomes:

$$\frac{\partial x^*}{\partial W_0} = \frac{T_g}{1 + h'(x)} \left(\frac{T_g}{T_b} - 1\right)$$
where: \( T_g = t_g^u + \sum_{i=1}^{N} t_i^v \) and \( T_b = t_b^u + \sum_{i=1}^{N} t_i^v \). It is easy to see that propositions 1 to 5 apply.

In the specific case where all victims have the same initial wealth \( y_0 \) and the same preferences, we have \( W_0 = w_0 + N y_0 \), \( T_g = t_g^u + N t_g^v \) and \( T_b = t_b^u + N t_b^v \), and it can be shown that an increase in the number of victims affects care in the same way as an increase in society’s wealth:

\[
\frac{\partial x^*}{\partial N} = y_0 \frac{\partial x^*}{\partial W_0}
\]

where the right-hand side obeys proposition 4 also apply. These results can be summarized by the following proposition:

**Proposition 7** For any first-best allocation of risk:

i) As the number of victims increases, if the injurer’s wealth in one state increases (decreases) then the victims’ wealth must also increase (decrease). More precisely:

ii) If society’s risk tolerance is larger in the good state and if \( y_0 \in [y_b, y_g] \), then both individuals’ wealth increases in the bad state but decreases in the good state as \( N \) increases;

iii) If society’s risk tolerance is larger in the bad state and if \( y_0 < y_b \), then both individuals’ wealth decreases in the bad state as \( N \) increases, while the effect is ambiguous in the good state;

iv) If society’s risk tolerance is larger in the bad state and if \( y_0 > y_g \), then individuals’ wealth increase in the good state as \( N \) increases, while the effect is ambiguous in the bad state;

**Proof.** See the appendix.

If the harm is spread among many victims—each of them bearing a smaller share in the harm—and risk tolerance is larger in the good state, the first- (and second-)best level of care decreases with the number of victims. *Vice versa*, if risk tolerance is larger in the bad state, care increases with the number of victims. Increasing the number of victims has an effect that is qualitatively the same as the one associated to an increase in the victims’s initial wealth. As shown in the proposition, the way a change in the number of victims affects the results depends on the distribution of wealth between injurer and victims prior to the change. Moreover, given that \( \frac{\partial y_b}{\partial N} = \frac{t_b^u}{t_b^u} \frac{\partial w_b}{\partial N} \) and \( \frac{\partial y_g}{\partial N} = \frac{t_g^u}{t_g^u} \frac{\partial w_g}{\partial N} \), the way in which a change in the number of victims affects the final allocation of risk depends on the relative risk tolerance of injurers and victims. Thus, if the injurer’s risk tolerance is less than the victims’ risk tolerance in both states, an increase in the number of victims makes it efficient to allocate more risk to them. Obviously, the reverse may also hold.

Nell and Richter (2003) also present a model with \( N \) victims but focus on the effect that risk spreading has on the desirability of different liability rules, in the case of a pure pecuniary loss. They show that an increase in the number of victims makes it desirable to allocate more risk to the victims due to better risk-spreading.
3 Implications for the design of liability rules

So far, we have studied care and risk-sharing policies by a benevolent planner, who can directly implement both of them. In the following, we extend the analysis to consider whether these two objectives can be reached by means of ordinary policy instruments, such as regulation, taxes and subsidies, insurance, and tort liability.

3.1 Regulation with taxes and subsidies

As shown in propositions 1, 4 and 5, the efficient allocation of wealth is not linear, in the sense that an increase in society’s wealth does not result in a proportional change in the individuals’ wealth. The specific shape of this relationship depends on the curvature of individual utility functions and on the safety technology. However, for practical purposes, effective insurance policies, regulation or liability ought to be simple enough to be implementable. In this section, we show that it is possible to reach the first best by using simple linear rules governing the level of care and how the harm and the cost of care are shared between the parties.

Consider a simple sharing rule for both the harm and the cost of care, such that the injurer bears an amount $\beta h(x)$ of the harm and an amount $\alpha x$ of the cost of care. The social problem consists of finding $(\alpha^*, \beta^*, X)$ that maximize:

$$SW = pu_b(w_0 - \beta h(x) - \alpha x) + (1 - p)u_g(w_0 - \alpha x) + pv_b(y_0 - (1 - \beta)h(x) - (1 - \alpha)x) + (1 - p)v_g(y_0 - (1 - \alpha)x)$$

Ignoring the arguments of the utility functions, the first-order conditions yield:

$$u'_b = v'_b$$  \hspace{1cm} (7)

$$pu'_b + (1 - p)u'_g = pv'_b + (1 - p)v'_g$$  \hspace{1cm} (8)

$$ph'(X) + 1 = (1 - p) \left(1 - \frac{v'_g}{v'_b}\right)$$  \hspace{1cm} (9)

Note that (7) requires that marginal utilities of victim and injurer in the bad state be equal; combining (7) and (8) we have also that the marginal utilities of victim and injurer in the good state should be equal: $u'_g = v'_g$, which satisfies Borch’s conditions, yielding in turn that the allocation of risk is the same as in the first-best. Likewise, (9) is the same as (3a), yielding that the level of care is also the same as in the first best, $X = x^*$. This result should not be surprising, since it is due to the fact that there are as many independent instruments as variables to control: $X$ controls care, while $\alpha^*$ and $\beta^*$ control the allocation of risk in the good and the bad state.

The solution just described may also be implemented through a mix of regulation and lump-sum transfers. Regulation sets and enforces the required level

\[14\]
of care, while taxes and subsidies realize the desired transfers of wealth between the parties in each state. To illustrate, assume that the injurer pays both the cost of care and the accident loss, while the victim pays two different lump-sum taxes $\tau_b$ in the bad state and $\tau_g$ in the good state, which are used to subsidize the injurer. Regulation has the task to enforce the level of care $X = x^*$. Assuming that this goal is achieved by means of fines or other forms of punishment for violators, then the efficient choice of $(\tau_b, \tau_g)$ by the planner maximizes:

$$SW = pw_b(w_0 - h(x^*) - x^* + \tau_b) + (1 - p)u_g(w_0 - x^* + \tau_g) + pv_b(y_0 - \tau_b) + (1 - p)v_g(y_0 - \tau_g)$$

and thus, the optimal values of $(\tau_b^*, \tau_g^*)$ satisfy:

$$u'_b(w_0 - h(x) - x^* + \tau_b^*) = v'_b(y_0 - \tau_b^*)$$

$$u'_g(w_0 - x + \tau_g^*) = v'_g(y_0 - \tau_g^*)$$

which have the same form as Borch’s conditions.

### 3.2 Strict liability

We now turn to liability rules, which are alternative tools to reallocate wealth and give incentives to take care. An important result of the previous analysis is that the first best requires enough instruments to reallocate wealth across states. However, liability rules allow transfers between the injurer and the victim in the bad state—in the form of damages payments—while ruling out any payment in the good state. Thus, it may be expected that liability falls short of controlling all of the three variables pertaining to risk-sharing and care and hence will not be enough to implement the first best. In the following we also compare the performance of liability rules to the second best.

Consider first strict liability: the injurer pays damages equal to $\lambda h(x)$ whenever an accident occurs, where $\lambda > 0$. With $\lambda = 1$, the injurer pays perfectly compensatory damages to the victim—the victims obtains full compensation for his pecuniary losses and, thus, has a constant wealth across states $y_b = y_g = y_0$. However, strict liability can also be designed as to allow for supracompensatory damages ($\lambda > 1$, such as punitive damages) or infracompensatory ($\lambda < 1$) damages, in which cases the victim receives a state-dependent wealth which is $y_0 + (\lambda - 1)h(x)$ in the bad state, and $y_0$ in the good state.

**Proposition 8** Under strict liability with perfect compensatory damages $\lambda = 1$, the injurer chooses a second-best level of care. The associated allocation of risk is also second best. If damages are infru- or supracompensatory, the outcome is neither a first best nor a second best.

**Proof.** Assume that the liability rule is strict liability $\lambda$. Under this liability rule, the injurer will take care as to maximize:

$$pw_b(w_0 - \lambda h(x) - x) + (1 - p)u_g(w_0 - x)$$
Let $x_\lambda$ denote the injurer’s level of care, which satisfies the following first order condition:

$$p\lambda h'(x_\lambda) + 1 = (1 - p) \left( 1 - \frac{u'_g(w_0 - x_\lambda)}{u'_g(w_0 - \lambda h(x_\lambda) - x_\lambda)} \right)$$

Thus, when $\lambda = 1$ the injurer’s choice of care is the same as in the second best in (6). The wealth of the victim is thus the constant allocation $\bar{y} = y_0$ which provides him with full insurance. Hence, $\lambda = 1$ allows to implement the second best both in terms of care and of risk.

When $\lambda \neq 1$, $x_\lambda$ does not meet the condition for a second best level of care. Moreover, such a liability rule generally does reach neither a first best nor a second best allocation of risk. With supracompensatory damages, the victim receives a greater wealth in the bad state than in the good state: $y_0 + (\lambda - 1)h(x) - y_0 = h_w$, implying that the associated allocation of risk is not comonotonic, hence it cannot be first-best efficient. In contrast, with infracompensatory damages we have $y_0 + (\lambda - 1)h(x) < y_0$, then, the allocation is comonotonic but it will be only by chance that Borch’s conditions are met; moreover care is not set at the first-best level.

From this proposition it emerges that increasing or decreasing the damage amount affects both the level of care and the sharing of the risk, bringing the outcome away from the second best (but possibly improving over it), without being able to reach the first best.

One of the sources of inefficiency for the strict liability rule is that it only allows for a transfer in the bad state. This shortcoming could be corrected by adding a tax $\tau$ in the good state, while $\lambda$ takes care of the transfer in the bad state. The planner’s problem is to maximize:

$$SW = pu_b(w_0 - \lambda h(x) - x) + (1 - p)u_g(w_0 - x + \tau)$$
$$+ pv_b(y_0 + (\lambda - 1)h(x)) + (1 - p)v_g(y_0 - \tau)$$

Which gives a result similar to Borch’s conditions:

$$u'_b(w_0 - \lambda h(x) - x) = v'_b(y_0 + \lambda h(x))$$
$$u'_g(w_0 - x + \tau) = v'_g(y_0 - \tau)$$

However, the injurer chooses $x$ as to maximize

$$pu_b(w_0 - \lambda h(x) - x) + (1 - p)u_g(w_0 - x + \tau)$$

and hence we have:

$$p\lambda h'(x) + 1 = (1 - p) \left( 1 - \frac{u'_g(w_0 - x + \tau)}{u'_g(w_0 - \lambda h(x) - x)} \right)$$

It is easy to see that in general $\lambda = 1$ does not satisfy Borch’s conditions; however, the only way to obtain first best care is to set $\lambda = 1$, which in turn would be in contrast with the first-best allocation of risk. Thus, a simple tax in addition to strict liability cannot reach the first best as the same instrument $\lambda$ is used to allocate wealth in the bad state and to regulate caretaking.
3.3 Negligence

Under the negligence rule, the injurer pays damages only if negligent, that is if his level of care is below $X$. Here the only policy instrument is the due care level $X$. In fact, if the injurer abides by the standard of care, he does not pay damages to the victim, thus $\lambda$ becomes irrelevant as concerns the allocation of risk.

However, the parameter $\lambda$ is important in respect of the question of incentive compatibility. When the standard of care is set at the level $X$, the utility level of the injurer is defined as:

$$U(w_0, x) = \begin{cases} pu_b(w_0 - x) + (1 - p)u_g(w_0 - x), & \text{if } x \geq X \\ pu_b(w_0 - \lambda h(x) - x) + (1 - p)u_g(w_0 - x), & \text{otherwise} \end{cases} \quad (10)$$

As a result, under the negligence rule, the injurer obtains a sure outcome $(w_0 - X)$ if he adheres to the due care standard, and a risky outcome $(p, w_0 - \lambda h(x) - x; 1 - p, w_0 - x)$ if he does not. Note that according to the first line of (10), the injurer has no incentives to choose $x > X$. According to the second line of (10), when he does not comply with $X$, the injurer chooses the same level of care as under strict liability; $x_\lambda$ denotes this level of care.

Thus, negligence raises two issues: Will the injurer comply with the due care? Assuming he does, how does the outcome compare with the first and second best?

**Proposition 9** Under the negligence rule with a due care standard $X$:

i) If $X \leq x_\lambda$, then the injurer complies with the due care standard;

ii) If $X > x_\lambda$, then the injurer complies with the due care standard only if the following condition is satisfied:

$$pu_b(w_0 - \lambda h(x_\lambda) - x_\lambda) + (1 - p)u_g(w_0 - x_\lambda) \leq pu_b(w_0 - X) + (1 - p)u_g(w_0 - X) \quad (11)$$

iii) The allocation of risk is generally not first best. If the injurer complies, the allocation of risk is dual second best.

**Proof.**

i) If $X \leq x_\lambda$, then: $u_b(w_0 - \lambda h(x) - x_\lambda) \leq u_b(w_0 - x_\lambda) \leq u_b(w_0 - X)$

which implies in turn:

$$pu_b(w_0 - \lambda h(x_\lambda) - x_\lambda) + (1 - p)u_g(w_0 - x_\lambda) \leq pu_b(w_0 - X) + (1 - p)u_g(w_0 - X)$$

Thus, the injurer complies with due care.

ii) if $X > x_\lambda$, then:

$$pu_b(w_0 - x_\lambda) + (1 - p)u_g(w_0 - x_\lambda) \geq pu_b(w_0 - X) + (1 - p)u_g(w_0 - X)$$
but condition (11) is not always satisfied. In several cases, the injurer may prefer to be found liable and bear the loss rather than comply with the due care standard.

iii) When the injurer complies, the victim is not compensated for his loss. The injurer only bears the cost of care and does not face any risk. This outcome is the dual of the second best described above, where the victim did not face any risk.

Note that the standard may be $X = x^*$. However, given the costs allocation associated to the negligence, the outcome in terms of risk sharing can never be neither first best. But it is easy to see that whatever the standard $X$, the injurer complies as far as it entails a risk reduction as compared to not complying. This requirement is obviously met once we have $X \leq x_\lambda$; hence, the first best in term of prevention may be obtained if $X = x^* \leq x_\lambda$. Moreover, by setting $X = x_\lambda$ and $\lambda = 1$ the planner can reach for sure the second-best level of care. Concerning the allocation of risk, note the negligence rule implements a second-best allocation of risk where the injurer, rather than the victim, is fully insured. Finally, the level of care that is second best when the injurer is fully insured can be reached provided that the incentive-compatibility conditions set in the proposition above are satisfied.

3.4 Liability and insurance

We now consider the issue of liability and insurance combined. Assume that the injurer has the opportunity to buy third-party liability insurance in a competitive insurance market and that the insurer can cheaply control the injurer’s level of care. Under strict liability, the amount of insurance coverage and the level of care chosen by the injurer are the solution to the maximization of:

$$pu_b(w_0 - h(x) - q - x - m) + (1 - p)u_g(w_0 - x - m)$$

In a competitive insurance market, insurers charge a pure premium which allow them to break even: $m = pq$. The first order condition that characterizes the equilibrium insurance purchase $\hat{q}$ is thus:

$$u'_b(w_0 - h(x) - x + \hat{q} - \hat{m}) = u'_g(w_0 - x - \hat{m})$$

with $\hat{m} = pq$.

Condition (12) implies that the injurer’s choice of insurance coverage is designed to equalize his marginal utility of money between states: in a sense, he is perfectly insured against the variability of his marginal utility between the states. However, he does not use insurance coverage to be perfectly hedged against the variations of his wealth between the states since money does not have the same value in all states (see assumptions A2). Hence, although the insurance market is competitive, the injurer does not buy full insurance, $\hat{q} \neq h(x)$. He will overinsure, $\hat{q} > h(x)$, if money is more useful in the good state (A2a) and underinsure, $\hat{q} < h(x)$, if money is more useful in the bad state (A2b).  

See Cook and Graham (1977) for the first statement of this result in a pure individual decision set up.
On the other hand, when he buys pure competitive liability insurance, the level of care that the injurer chooses is set at the level that would be optimal in a risk-free world in (2), since conditions (9) and (12) imply condition (2).

**Proposition 10** If liability coverage is offered in a competitive insurance market, then strict liability does generally yield neither the first-best level of care nor the first best allocation of risk. The level of care is also not second best.

**Proof.** See text above.  

The reason is that insurance markets allow the injurer to individually reallocate his wealth among different states of the nature, in such a way that his marginal utility of wealth be equal between states. Nevertheless, this does not represent a Pareto efficient allocation, since according to Borch’s conditions, the first best allocation of risk is only obtained through transfers between individuals, state by state, in order that individual marginal utilities equate in each separate state.

Consider now the simple negligence when the standard of care is set at the level \(X\). If the injurer has the opportunity to buy liability insurance on purely competitive markets, the utility level of the injurer is defined as:

\[
U(w, x) = \begin{cases} 
pu_b(w - x) + (1 - p)u_g(w - x), & \text{if } x \geq X \\
pu_b(w - h(x) - x + q - pq) + (1 - p)u_g(w - x - pq), & \text{otherwise}
\end{cases}
\]

The result is straightforward:

**Proposition 11** If liability coverage is offered in a competitive insurance market, then under the negligence rule with a standard of care \(X\):

i) If \(X \leq \hat{x}\), then the injurer complies with the standard of care.

ii) If \(X > \hat{x}\), then the injurer complies with the standard of care only when

\[
pu_b(w - h(\hat{x}) - \hat{x} + q - m) + (1 - p)u_g(w - \hat{x} - m) \leq pu_b(w - X) + (1 - p)u_g(w - X)
\]

(13)

iii) The allocation of risk is generally not first best efficient.

**Proof.** The injurer needs insurance coverage only when he expects to be found liable. Thus, the proof of i) and ii) are the same as in proposition 9.

iii) Either the injurer is not liable, and the allocation of wealth is such that the injurer bears the full cost of care while the victim bears the harm loss in the case of an accident, which is not a first-best allocation of risk; or the injurer is liable, and hence he bears the cost of care and the harm, which is not a first-best outcome.  

Remark that the introduction of a purely competitive insurance market for first-party insurance—where victims purchase coverage for the losses they could suffer in case of an accident—would not attain the efficient sharing of risk either. To see this, consider a situation where the injurer complies with the standard of care. If there exists competitive markets for first-party insurance, the victim is
allowed to buy a contract \((Q, M)\), where \(Q\) is the coverage and \(M\) the premium, such that \(M = pQ\) and:

\[
v'_0(y - h(X) + Q - pQ) = v'_0(y - pQ)
\]

Once more, with a competitive first-party insurance market, the victim has the opportunity to reallocate his wealth between states in order to hedge himself against the variability of his individual marginal utilities. But this is only individually and not socially optimal.

4 Conclusions

This paper focuses on accidents having two characteristics: the harm has a noncompensable component and care reduces the magnitude of the harm but not the probability of accidents. Our framework departs from previous literature, which considers either perfectly compensable losses and/or contexts in which care reduces the probability of accidents (Nell and Richter, 2003; Zivin, Just and Zilberman, 2006).

Our finding shed new light on the question whether and to what extent safety standards should depend on wealth (Arlen 1992; Miceli and Segerson, 1995; Shavell, 1982). We find that both in the first best and in the second best—when either the victim or the injurer is fully insured—care does depend on society’s wealth, but it does not depend on the distribution of such wealth among individuals. Moreover, the effect of an increase in wealth on the socially desirable level of precaution is not straightforward, as it is not always desirable to increases precaution when society becomes rich.

Since taking care creates a cost in the good state (when there is no accident) and a benefit in the bad state (when an accident occurs), we have described the problem of deciding whether to increase care when society becomes richer as a problem of allocating such additional wealth between the good state and the bad state. As we have shown, more wealth should be allocated to the state where society has more tolerance to risk. This implies that care increases with wealth if society is more tolerant to risk in the bad state and, vice versa, care decreases with wealth if society is more tolerant to risk in the good state.

Extending the one-agent framework introduced in the insurance literature (Ehrlich and Becker, 1967; Jones-Lee, 1974) to a two-agent framework, we have tackled a second basic question, whether a risk-averse society should take more or less care than a risk neutral society. Also in this case, the answer is not unambiguous and depends on whether the marginal utility of wealth is larger in the good or in the bad state. This yields that accident that cause temporary impairment, such as a reversible medial condition, which trigger some monetary expenditures might make it worthwhile for a risk-averse society to spend more on precaution than a risk-neutral society. In contrast, if the accident causes a permanent disability which reduces the individual’s capacity to derive utility from money, risk aversion reduces the socially desirable level of care instead of increasing it.
In relation to the literature on the optimal allocation of risk (Borch, 1962; Arrow, 1964), we have shown that the mutuality principle—yielding that both parties should bear some risk in equilibrium—extends to situations in which the magnitude of the accident is endogenous and depends on care choices by one of the parties. Moreover, we have shown that the optimal allocation of risk depends on the (exogenous) probability of accidents, a result that obtains even though the parties share the same information and believes on the probability of the different states of the world.

After having tackled the problem of optimally allocating risk and choosing care at an abstract level, we examine different policy measures to implement the desired levels of risk and care. Besides first-best solutions we also consider the implementability and characteristics of second-best outcomes where one party is fully insured against the accident loss. Although the optimal risk-sharing rules are complex and non-linear, we show that they can be mimicked by implementing simple linear rules according to which the parties have to share both the cost of care and the accident loss.

The issue whether strict liability or negligence is the superior liability rule has been long discussed in the literature. Nell and Richter (2003) show that the negligence rule has the advantage of spreading risk among several parties rather than concentrating it on the injurer and hence might induce a better allocation of risk. However, if harm is non-monetary, the common wisdom (Shavell, 2004, ch. 11; Arlen, 1992, note 3) is that the negligence rule is preferable because it allows a separation between the incentive problem and the risk-allocation problem. We show that the superiority on either rule is not established: the negligence rule might produce better incentives to take care but strict liability induces an allocation of risk that is preferable when fairness consideration impose the compensation of innocent victims.

Many relevant variables have been left out of the analysis. Of importance for the implementation of our recommendations could be issues concerning activity levels and potential insolvency, which are analyzed in previous literature (Shavell, 1980; Ganuza and Gomez, 2008).

References


APPENDIX

Proof of proposition 1: See Landsberger and Meilijson (1994), and Chateauneuf, Dana and Tallon (2000), for economies without transaction costs (cost of care, or expenditures needed for the sharing of aggregate resources of society).

We assume that there exists some values of $x > 0$ such that $h(x) + x < H$. Assume now that the feasible allocation $[(w_b, w_g); (y_b, y_g)]$ with $w_b \leq w_g$ and simultaneously $y_b \geq y_g$, associated to a care level $x$, is Pareto optimal. Now for the same level of care, define an alternative feasible allocation $[(\hat{w}_b, \hat{w}_g); (\hat{y}_b, \hat{y}_g)]$ where $\hat{w}_b \leq \hat{w}_g$ and simultaneously $\hat{y}_b = \hat{y}_g$, such that:

\[
\begin{align*}
\hat{w}_b &= w_b + (1-p)(y_b - y_g) \\
\hat{w}_g &= w_g - p(y_b - y_g) \\
\hat{y}_b &= py_b + (1-p)y_g = \hat{y}_g \\
\hat{w}_b + \hat{y}_b &= w_b + y_b \\
\hat{w}_g + \hat{y}_g &= w_g + y_g
\end{align*}
\]

By definition, both individuals obtain the same expected individual wealth irrespective of the allocation we choose, since: $p\hat{y}_b + (1-p)\hat{y}_g = py_b + (1-p)y_g$ for the victim and $p\hat{w}_b + (1-p)\hat{w}_g = pw_b + (1-p)w_g$ for the injurer.

However, $(\hat{w}_b, \hat{w}_g)$ is less spread than $(w_b, w_g)$ in the sense of the second stochastic dominance order, since given that for the same probabilities $(p, 1-p)$ we have the following order for injurer’s wealth in the different states: $w_b < \hat{w}_b \leq \hat{w}_g < w_g$. The allocation $(\hat{y}_b, \hat{y}_g)$ is also less spread than $(y_b, y_g)$ since we have: $y_b > \hat{y}_b = \hat{y}_g > y_g$. Recall that, by assumption, both individuals are risk averse to second-order dominance shifts in risk. Thus $[(\hat{w}_b, \hat{w}_g); (\hat{y}_b, \hat{y}_g)]$ Pareto dominates $[(w_b, w_g); (y_b, y_g)]$; hence a contradiction.

Define two real numbers $\mu_b$ and $\mu_g$ as the shadow prices of the aggregate resource constraints of society. If an interior solution exists for the problem of the social planner, then it corresponds to a vector $(x, w_b, w_g, y_b, y_g)$ which satisfies to the following conditions:

\[
\begin{align*}
(A) : & -\mu_b h'(x) - (\mu_g + \mu_b) = 0 \\
(B) : & pu'_b(w_b) - \mu_b = 0 \\
(C) : & (1-p)u'_g(w_g) - \mu_g = 0 \\
(D) : & pv'_b(y_b) - \mu_g = 0 \\
(E) : & (1-p)v'_g(y_g) - \mu_g = 0
\end{align*}
\]

Conditions (B) to (E) define the rule that should be used by the planner to implement a first best allocation of risk. Using (B) and (D) together, and (C) and (E) together, we obtain:

\[
\begin{align*}
u'_b(w_b) &= v'_b(y_b), \text{ with } w_b + y_b = w_0 + y_0 - x - h(x) \\
u'_g(w_g) &= v'_g(y_g), \text{ with } w_g + y_g = w_0 + y_0 - x
\end{align*}
\]
which are known to be Borch’s conditions.

**Proof of proposition 2:** Using Borch’s conditions, conditions (B) to (E) together yield:

\[ \mu_b + \mu_g = pv'_b(y_b) + (1 - p)v'_g(y_g) \]

Substituting in (A) and rearranging, we have that the first-best level of care satisfies the condition:

\[ (F) : -h'(x) = 1 + \frac{1 - p}{p} \frac{v'_g(y_g)}{v'_b(y_b)} \]

(F) implies that at optimum: \(-h'(x) - 1 > 0\). Second order conditions require the following inequality to hold:

\[
\frac{h''(x^*)}{1 + h'(x^*)} (t^u + t^v) + \frac{t^u + t^v}{t^b + t^g} (1 + h'(x^*)) - 1 < 0
\]

where: \(t^v = \frac{v'_b(y_b)}{v'_g(y_g)}\), \(t^u = \frac{v'_g(y_b)}{v'_b(y_g)}\), \(t^b = -\frac{u'_b(y_b)}{u'_g(y_g)}\) are the indexes of absolute tolerance to risk for the victim and the injurer in each state; given the various restrictions imposed on preferences and safety technology, this inequality is satisfied.

Finally, after some tedious manipulations of (F), it is easy to obtain condition (3) in the text.

**Proof of proposition 4:** Recall that the aggregate wealth is \(W_0 = y_0 + w_0\).

Then, given that \(\frac{\partial y_b}{\partial W_0} + \frac{\partial w_b}{\partial W_0} = 1 - (1 + h'(x)) \frac{\partial x}{\partial W_0}\) and \(\frac{\partial y_g}{\partial W_0} + \frac{\partial w_g}{\partial W_0} = 1 - \frac{\partial x}{\partial W_0}\), the impact of an increase in \(W_0\) on the risk sharing rules may be obtained by first totally differentiating Borch’s conditions:

\[
u'_b(w_b) \frac{\partial w_b}{\partial W_0} = v'_b(y_b) \frac{\partial y_b}{\partial W_0} \Rightarrow \frac{\partial y_b}{\partial W_0} = \frac{t^v_b}{t^v_b + \frac{t^u_b}{t^b + t^g}} \left(1 - (1 + h'(x^*)) \frac{\partial x^*}{\partial W_0}\right)\]

and substituting the constraints we obtain:

\[ (G) : \frac{\partial y_b}{\partial W_0} = \frac{t^v_b}{t^v_b + \frac{t^u_b}{t^b + t^g}} \left(1 - \frac{\partial x^*}{\partial W_0}\right) \]

\[ (H) : \frac{\partial y_g}{\partial W_0} = \frac{t^v_g}{t^v_g + \frac{t^u_g}{t^b + t^g}} \left(1 - \frac{\partial x^*}{\partial W_0}\right) \]

Then, differentiating condition (F) and rearranging, we obtain:

\[ (I) : \frac{h''(x^*)}{1 + h'(x^*)} \frac{\partial x}{\partial W_0} = \frac{1}{t^u_b t^v_b} \frac{\partial y_b}{\partial W_0} - \frac{1}{t^u_g t^v_g} \frac{\partial y_g}{\partial W_0}\]

After substituting (G) and (H) into (I), we have Exp. (5).
Note that, as one might expect from the Mutuality Principle, it is easy to verify that: $1 > (1 + h'(x^*)) \frac{\partial x^*}{\partial W_0} \Rightarrow \frac{\partial y_b}{\partial W_0} > 0$ and $1 > \frac{\partial x^*}{\partial W_0} \Rightarrow \frac{\partial y_g}{\partial W_0} > 0$. 

**Proof of proposition 5:** Given that $\frac{\partial y_b}{\partial p} + \frac{\partial w_b}{\partial p} = -(1 + h'(x^*)) \frac{\partial x^*}{\partial p}$ and $\frac{\partial y_g}{\partial p} + \frac{\partial w_g}{\partial p} = -\frac{\partial x^*}{\partial p}$, the impact on the risk sharing rules of any increase in $p$ can be obtained by first totally differentiating Borch’s conditions to have:

\[
\begin{align*}
&u''_b(w_b) \frac{\partial w_b}{\partial p} = v''_b(y_b) \frac{\partial y_b}{\partial p} \Rightarrow \frac{\partial y_b}{\partial p} = \frac{\partial w_b}{\partial p} \\
&u''_g(w_g) \frac{\partial w_g}{\partial p} = v''_g(y_g) \frac{\partial y_g}{\partial p} \Rightarrow \frac{\partial y_g}{\partial p} = \frac{\partial w_g}{\partial p}
\end{align*}
\]

Substituting the constraints we obtain:

\[
\begin{align*}
(J) : & \quad \frac{\partial y_b}{\partial p} = -\frac{t_v^b}{t_v^b + t_v^b} (1 + h'(x^*)) \frac{\partial x^*}{\partial p} \\
(K) : & \quad \frac{\partial y_g}{\partial p} = -\frac{t_v^g}{t_v^g + t_v^g} \frac{\partial x^*}{\partial p}
\end{align*}
\]

Then, differentiating condition (3a) and rearranging, we obtain:

\[
(L) : \quad \frac{h''(x^*)}{1 + h'(x^*)} \frac{\partial x^*}{\partial p} - \frac{1}{t_v^b} \frac{\partial y_b}{\partial p} + \frac{1}{t_v^g} \frac{\partial y_g}{\partial p} = -1 \frac{p}{p(1-p)}
\]

Substituting (J) and (K) in (L) gives:

\[
\frac{\partial x^*}{\partial p} = \frac{-\frac{t_v^b + t_v^g}{1 + h'(x^*)} \frac{t_v^b}{t_v^b + t_v^b} (1 + h'(x^*)) - 1}{p(1-p)} > 0
\]

Coming back to (J) and (K), and remembering that at optimum we must have $-(1 + h'(x^*)) > 0$, it follows that $\frac{\partial y_b}{\partial p} > 0$ and $\frac{\partial y_g}{\partial p} < 0$. 

**Proof of proposition 7:** From Borch’s conditions, we have:

\[
\begin{align*}
\frac{\partial y_b}{\partial N} &= \frac{t_v^b}{\partial w_b} \frac{\partial w_b}{\partial N} \Rightarrow \text{sign} \frac{\partial y_b}{\partial N} = \text{sign} \frac{\partial w_b}{\partial N} \\
\frac{\partial y_g}{\partial N} &= \frac{t_v^g}{\partial w_g} \frac{\partial w_g}{\partial N} \Rightarrow \text{sign} \frac{\partial y_g}{\partial N} = \text{sign} \frac{\partial w_g}{\partial N}
\end{align*}
\]

which proves i). Now, using $\frac{\partial x^*}{\partial N} = y_0 \frac{\partial x^*}{\partial W_0}$, the aggregate wealth constraints
lead to:

\[
\frac{\partial w_b}{\partial N} + N \frac{\partial y_b}{\partial N} = (y_0 - y_b) - (h'(x^*) + 1) \frac{\partial x^*}{\partial N}
\]

\[
= y_0 \left( 1 - (h'(x^*) + 1) \right) \frac{\partial x^*}{\partial W_0} - y_b
\]

\[
\frac{\partial w_g}{\partial N} + N \frac{\partial y_g}{\partial N} = (y_0 - y_g) - \frac{\partial x^*}{\partial N}
\]

\[
= y_0 \left( 1 - \frac{\partial x^*}{\partial W_0} \right) - y_g
\]

such that the sign of the RHS in each expression gives the sign of the LHS. Using that \(1 > (1 + h'(x^*)) \frac{\partial x^*}{\partial W_0}\) and \(1 > \frac{\partial x^*}{\partial W_0}\), the results \(ii)\) to \(iv)\) of the proposition are proven. \(\blacksquare\)