The calculation of Solvency Capital Requirement using Copulas

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Abstract

Our aim is to present an alternative methodology to the standard formula imposed to the insurance regulation (the European directive knows as Solvency II) for the calculus of the capital requirements. We want to demonstrate how this formula is now obsolete and how is possible to obtain lower capital requirement through the theory of the copulas, function that are gaining increasing importance in various economic areas. A lower capital requirement involves the advantage for the various insurance companies not to have unproductive capital that can therefore be used for the production of further profits. Indeed the standard formula is adequate only with some particular assumptions, otherwise it can overestimate the capital requirements that are actually needed as the standard formula underestimates the effect of diversification.

Keywords. Solvency II, Solvency Capital Requirement, Standard Formula, Value-at-Risk, Copula.
JEL classification: C13, C15, C18, C61.

I. Introduction

Insurance regulation requires companies to hold adequate capital to cope with the various risks. The insurance companies must therefore calculate the "Solvency Capital Requirement" (SCR), that is the level of funds that companies must hold by law to be able to operate on the market. The SCR, which is based on the concept of Value-at-Risk (VaR), is calculated through the standard formula defined directly by the insurance regulation which, in general, can overestimate capital requirements. In this regard, we provide an application assuming, for simplicity, the independence between two risks taken into consideration. This application highlights the inadequacy of the standard formula for the calculation of capital requirements. In case of asymmetric distribution the standard formula imposed by the supervisory authority can effectively overestimate the capital requirement. In the event that the hypothesis of independence for the distributions is violated the theory of copulas is actually suitable for the construction of an internal model to be used for the calculation of capital requirements for the various insurance companies.

The first use of copulas in insurance was introduced by Wang (1992), who used them as a tool to analyze the dependency structure of the risks that made up insurance portfolios. Edward Frees, Carriere and Valdez (1995) used the copulas to evaluate the joint distribution; Blum, Dias and Embrechts (2002) discuss the use of copulas to study the dependence in alternative risk transfer products. Thanks to their ability to analyze the dependency structure between the functions of marginal distributions, the copulas are an excellent way to calculate the SCR through an alternative methodology to the standard formula.

II. Solvency II

Banking and insurance legislation and regulation have always played a key role, due to the constantly increasing risks (especially after the latest financial crises) for both savers and companies. To contain these risks an effective monitoring system must be relied upon. A failure of a systemic bank or a large insurance company would cause serious damage to the whole financial system and consequently to individual savers. Banks and insurers therefore need minimum requirements, called capital requirements, in order to have sufficient capital to cover any unexpected losses and adequate to cope with various risks (such as credit risk, market risk...) deriving from the activities carried out.

As a starting point we must present the definition of solvency understood as the capacity of a debtor (be it a business, a financial intermediary, a sovereign state, a private citizen) to repay his debts at maturity. Solvency II is precisely based on the definition of solvency and it defines the provisions for calculating the Solvency Capital Requirements (as well as Minimum Capital Requirements, the MCR). As previously mentioned the SCR currently used is based on the concept of Value-at-Risk (VaR) with a confidence level of 99.5%, with an annual time horizon.

Definition 1. *(Value-at-Risk).* The VaR represents the maximum potential loss hypothesizable, on a given time horizon and with a certain level of probability. In formulas we have:

$$\mathrm{VaR}_\alpha(X) = \inf\{x \in \mathbf{R} : \Pr(X > x) \leq 1 - \alpha\} \tag{1}$$

where α (with $\alpha \in (0, 1)$) indicates the confidence level and the random variable X the loss.

In any case the SCR can be calculated according to two methods:

- The *standard formula*;
- an internal model (complete or partial), which must be validated by the supervisory authority.

Regarding the standard formula we have to consider the following risks:

- market risk;
- health risk;
- default risk;
- life risk;
- non-life risk;
- intangibles risk;
- operational risk.

We are in presence of a "modular" structure because the *Basic SCR* (BSCR) is obtained by aggregating the various submodules (Life, Non-Life, Health, Market, Default, Intangible Asset) through a correlation matrix provided directly by the EIOPA Delegated Acts. To obtain the final SCR we must subtract the adjustment (Adj), which express the capacity to absorb the losses, and add the SCR_{op}, which represents the requirement of capital for operational risk, to the BSCR. So, we have:

$$\mathrm{SCR} = \text{BSCR-Adj+SCR}_{op}$$

where

$$\mathrm{BSCR} = \sqrt{\sum_{ij} Corr_{ij} \cdot SCR_i \cdot SCR_j} + SCR_{intangibles} \tag{2}$$

where i and j represent the various modules and $Corr_{ij}$ represents the correlation coefficient between them.

Now suppose to consider only two risks, denoted by X and Y (with $Z = X + Y$), for which, according to (2), we have

$$\mathrm{SCR}_Z = \sqrt{SCR_X^2 + SCR_Y^2 + 2 \cdot \rho_{X,Y} \cdot SCR_X \cdot SCR_Y}. \tag{3}$$

The SCR can be rewritten as the difference between the value of the VaR and the expected value of the loss:

$$\mathrm{SCR}_Z = \mathrm{VaR}_{99.5\%}(Z) - \mathrm{E}(Z) \tag{4}$$

where $\mathrm{E}(Z)$ represents the expected value of the loss, placed in reserve by the companies. Consider now the Value-at-Risk of the two individual risks, for which we obtain

$$\begin{aligned} \mathrm{VaR}_{99.5\%}(X) - \mathrm{E}(X) = k_X \cdot \sigma(X) \\ \mathrm{VaR}_{99.5\%}(Y) - \mathrm{E}(Y) = k_Y \cdot \sigma(Y) \end{aligned} \tag{5}$$

and, therefore, for the sum of the two risks we have

$$\mathrm{VaR}_{99.5\%}(Z) - \mathrm{E}(Z) = k \cdot \sigma(Z) \tag{6}$$

where the standard deviation $\sigma(Z)$ is

$$\sigma(Z) = \sqrt{\sigma_X^2 + \sigma_Y^2 + 2 \cdot \rho_{X,Y} \cdot \sigma_X \cdot \sigma_Y}. \tag{7}$$

By replacing we have

$$\mathrm{VaR}_{99.5\%}(Z) - \mathrm{E}(Z) = k \cdot \sqrt{\sigma_X^2 + \sigma_Y^2 + 2 \cdot \rho_{X,Y} \cdot \sigma_X \cdot \sigma_Y}. \tag{8}$$

The standard formula (according to (3)) proposed by Solvency II is based on volatilities and, in presence of the two risks X and Y, we have:

$$\begin{aligned} &\mathrm{VaR}_{99.5\%}(Z) - \mathrm{E}(Z) = \\ &= \sqrt{k_X^2 \cdot \sigma_X^2 + k_Y^2 \cdot \sigma_Y^2 + 2 \cdot \rho_{X,Y} \cdot k_X \cdot k_Y \cdot \sigma_X \cdot \sigma_Y}. \end{aligned} \tag{9}$$

The problem arises because (8) and (9) are the same only if we have $k_X = k_Y = k$ and this happens only if we are in presence of distributions that belong to the same "location scale family" (such as in the case of normal distributions). If the distribution is actually normal then we have $k = 2.58$, while if the distribution is not symmetrical we have k variable based on asymmetry. We can illustrate this problem of the standard formula analyzing both the case of having, for the two risks, normal distributions (symmetric distribution) or the gamma distributions (asymmetric distribution). If we suppose both distributions (normal and gamma) independent, the correlation results equal to 0 and, therefore, it isn't yet necessary to apply the theory of the copulas.
To estimate the parameters k_X, k_Y, k we must:

- create the two independent distributions;

- obtain a distribution as the sum of the two previous ones;
- calculate the expected value and the VaR for the three distributions;
- obtain, according to the values obtained, the values of k_X, k_Y, k.

Let's start now with the case of the two normal distributions using one million simulations, $\mu_1 = 3$ and $\sigma_1 = 2$, $\mu_2 = 3$ and $\sigma_2 = 2$ and $p_x = 0.995$, where μ represents the expect value, σ represents the standard deviation and p_x represents the value of the percentile. We obtaion the figure (1).

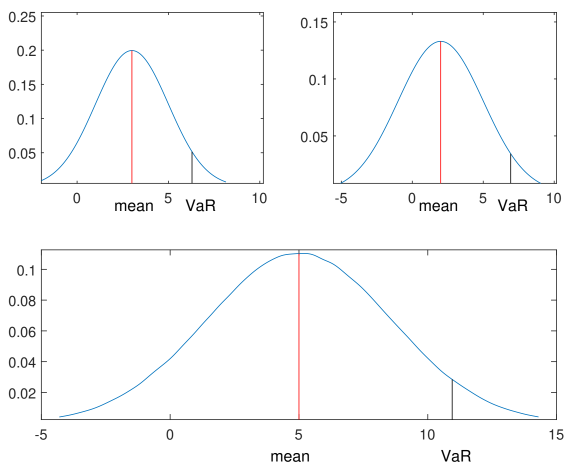

Figure 1: VaR and expected value for normal distributions.

We are now in position to estimate the parameters k_X, k_Y, k. From the formula (6) we obtain k equal to

$$k = \frac{\mathrm{VaR}_{99.5\%}(Z) - \mathrm{E}(Z)}{\sigma(Z)}$$

and we also have

$$k_X = \frac{\mathrm{VaR}_{99.5\%}(X) - \mathrm{E}(X)}{\sigma(X)},$$

$$k_Y = \frac{\mathrm{VaR}_{99.5\%}(Y) - \mathrm{E}(Y)}{\sigma(Y)}.$$

We obtain $k_X = k_Y = k = 2.58$. So, actually, for normal distributions we have the same values of the parameters.

We now analyze the case of asymmetric distributions, using two gamma distributions. In this case we using one million simulations, $\alpha_1 = 3$ and $\beta_1 = 2$, $\alpha_2 = 2$ and $\beta_2 = 3$ and $p_x = 0.995$, where α is the shape parameter and β represent the rate parameter. We calculate the parameters and we get $k_X = 3.62$, $k_Y = 3.84$ e $k = 3.45$. The three values are different from each other and, moreover, we have $k < k_X$ e $k < k_Y$, for which the capital requirement is overestimated. Therefore, in general, if we are in presence of asymmetric distributions the standard formula overestimates the capital requirement as it underestimates the effect of diversification. The problem of overestimating the capital requirements by the standard formula can be solved through the use of the copulas.

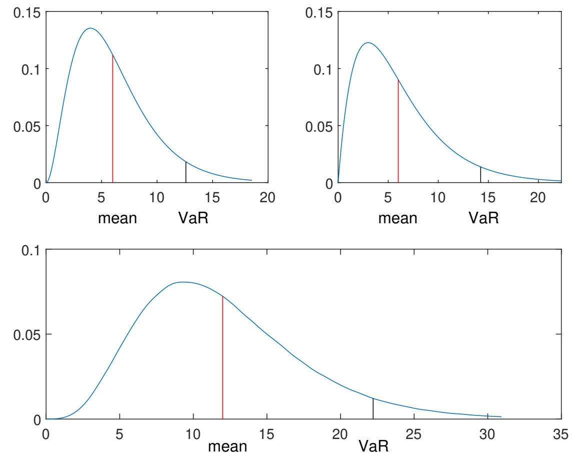

Figure 2: VaR and expected value for gamma distributions.

III. Copulas

Thanks to the copulas we can study the interaction between the random variables described by a joint distribution function. Thanks to Sklar's Theorem, this interaction can be easily expressed in terms of marginal distribution functions and a copula function. The Sklar's Theorem represents the main theorem concerning the copulas: it explains the link between the marginal distribution functions and the multivariate distribution function.

Theorem 1. ***Sklar's Theorem.*** *Let H be a joint distribution function with margins $F(x)$ and $G(y)$. Then there exists a copula $\mathcal{C}$ such that for all $x, y \in \bar{\mathbf{R}}$:*

$$H(x, y) = \mathcal{C}(F(x), G(y)). \tag{10}$$

If F and G are continuous, then $\mathcal{C}$ otherwise, $\mathcal{C}$ is uniquely determined on $RanF \times RanG$. Conversely, if $\mathcal{C}$ is a copula and F and G are distribution functions, then the function H defined by (10)*is a joint distribution function with margins F e G.*

Thanks to the Theorem we show how it is possible to obtain, with the marginal distribution functions and the copula function, the joint distribution function. Moreover, if the marginal distribution functions are continuous, we have:

$$H(F^{-1}(u), G^{-1}(v)) = \mathcal{C}(u, v).$$

In our study we use Archimedean copulas. The construction of this class depends on a function ϕ which is denoted by the name of *generator* (obtainable from the Laplace transform). They are the most widely used family of copulas as they have very useful advantages, such as the ease with which they can be built, the large number of copulas belonging to this family and they have various useful properties that characterize them (such as symmetry).
In terms of joint distribution function H and marginal distribution functions F and G, we have:

$$\phi(H(x, y)) = \phi(F(x)) + \phi(G(y)) \tag{11}$$

and, specifically, for the copulas:

$$\phi(\mathcal{C}(u, v)) = \phi(u) + \phi(v) \tag{12}$$

from which, finally, we obtain:

$$\mathcal{C}(u, v) = \phi^{[-1]}(\phi(u) + \phi(v)). \tag{13}$$

The function obtained in this way takes the name of "Archimedean copulas". Of notable importance is the parameter θ (we will show how to estimate this parameter in the next section) which measures the dependence between u and v. A negative value of the parameter indicates a negative dependency between u and v, on the contrary a positive value of the parameter indicates a positive dependency between u and v. In figures (3) and (4) we use Frank copula to show the difference between a positive or negative θ.
The copulas are useful for our study since they provide complete information on the dependence between the individual risk factors; the joint distribution allows, without assumptions and particular hypotheses, to analize them in an appropriate way. These functions allow, in fact, to analyze risks individually while studying the dependencies between them that can impact the SCR. Moreover, if the marginal distribution functions are continuous, the joint distribution (or multivariate) can still be obtained.

IV. Calculation of capital requirements

As seen previously, the standard formula of Solvency II is only valid in a particular case (normal distribution) and, for this reason, it doesn't always appear to be adequate

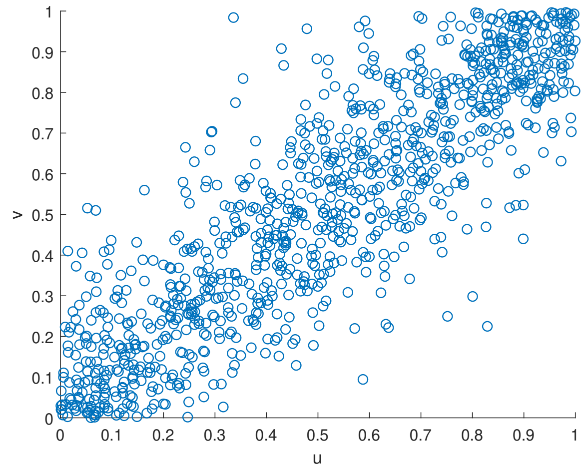

Figure 3: Frank copula with $\theta = 10$.

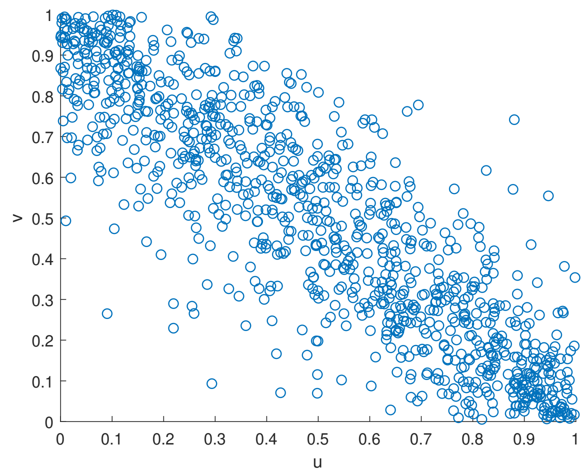

Figure 4: Frank copula with $\theta = -10$.

for the calculation of the capital requirements. Our aim is to develop an alternative algorithm based on the theory of copulas (recall that the copulas provide complete information on the dependence between the various risks) or calculate the capital requirements of an insurance company. We therefore want to illustrate the actual usefulness of the copulas in the insurance field. Hence we want to calculate the capital requirements both with the theory of the copulas and with the standard formula of Solvency II and then we will compare the results. We want to show practically that the use in the standard formula of a distribution different from the normal one actually provokes an overestimation of the capital requirements due to the underestimation of the effect of diversification (recall that if we use two asymmetric distributions, such as two gamma distributions, we violate the hypothesis at the base of the standard formula, i.e.

Table I: Some typologies of Archimedean copulas

Type of Copula	$\mathcal{C}(u,v)$	$\phi_\theta(t)$	$\theta \in$
Clayton	$[\max(u^{-\theta}+v^{-\theta}-1,0)]^{-1/\theta}$	$\frac{1}{\theta}(t^{-\theta}-1)$	$[-1,\infty)\backslash\{0\}$
Frank	$-\frac{1}{\theta}\ln(1+\frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{e^{-\theta}-1})$	$-\ln(\frac{e^{-\theta t}-1}{e^{-\theta}-1})$	$(-\infty,\infty)\backslash\{0\}$
Ali-Mikhail-Haq	$\frac{uv}{1-\theta(1-u)(1-v)}$	$\ln(\frac{1-\theta(1-t)}{t})$	$[-1,1)$
Gumbel-Hougaard	$\exp(-[(-\ln\ u)^\theta+(-\ln\ v)^\theta)]^{1\backslash\theta})$	$(-\ln\ t)^\theta$	$[1,\infty)$

$k_X = k_Y = k$).

In this section we assume that the reader is familiar with the methods of generating uniform independent variables and with the algorithms to obtain samples from a given univariate distribution.

We calculate the SCR only for two risks, but this procedure can be extended to all risks. The calculation of capital requirements is applied to the "Premium and Reserve risk" submodule of the "non-life" risk-module. The "non-life" underwriting risk-module consists of the following sub-risks:

- premium risk: the risk that the premiums of new contracts plus the reserve of initial premiums will be insufficient to pay claims;
- reserve risk: the risk that the available reserves are insufficient with an annual time horizon;
- lapse risk: the risk of losses due to the exercise of options by policyholders (for example the option to terminate the contract before the agreed deadline);
- cat risk: the risk of losses deriving from catastrophic events (for example an earthquake or even man-made disasters).

The "Premium and Reserve Risk" sub-module consists of the lines of businesses (LoB) of the table (II), while the correlation between the various LoBs is illustrated in the figure (5). We focus only on two LoB (because the process can also be extended to more LoBs with a similar procedure) and with a correlation of 0.5, in order to analyze a case where the LoB interact with each other in deep way. In our study we don't use a specific insurance dataset, but we resort to simulations. Infact we use two gamma distributions for the LoBs because, indeed, we have evidence that the random variable loss is adequately described by a gamma distribution and, again for the same reason not to use a specific dataset, the parameters of the gamma distributions have been hypothesized (in any case, even with different parameters of the distributions, the final result, that is to show the overestimation of the capital requirements by the standard formula, doesn't change). Based on the correlation of 0.5 we decided to calculate the capital requirement for the following LoBs:

- “Motor, third-party liability”: insurance on civil liability resulting from the circulation of motor vehicles. These insurances concern the resulting responsibilities from the use of land vehicles;
- “Marine, aviation, transport”: maritime, aeronautical and transport insurance. They relate to damage suffered by maritime, lake, river and air vehicles, including damage suffered by goods transported by such vehicles.

Table II: Lines of Business

LoB	LoB description
1	Motor, third-party liability
2	Motor, other classes
3	Marine, aviation, transport (MAT)
4	Fire and other property damage
5	Third-party liability
6	Credit and suretyship
7	Legal expenses
8	Assistance
9	Miscellaneous
10	Non-proportional reinsurance property
11	Non-proportional reinsurance casualty
12	Non-proportional reinsurance - MAT

A. The calculation of SCR with the standard formula

We are now in position to estimate the capital requirement based on the use of the standard formula, therefore we must calculate the Value-at-Risk, for the two branches chosen, with a confidence level of 99.5%. So we must:

CorrLob	1	2	3	4	5	6	7	8	9	10	11	12
1: M (3rd party)	1											
2: M (other)	0.5	1										
3: MAT	0.5	0.25	1									
4: Fire	0.25	0.25	0.25	1								
5: 3rd party liab	0.5	0.25	0.25	0.25	1							
6: credit	0.25	0.25	0.25	0.25	0.5	1						
7: legal exp.	0.5	0.5	0.25	0.25	0.5	0.5	1					
8: assistance	0.25	0.5	0.5	0.5	0.25	0.25	0.25	1				
9: misc.	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	1			
10: reins. (prop)	0.25	0.25	0.25	0.5	0.25	0.25	0.25	0.5	0.25	1		
11: reins. (cas)	0.25	0.25	0.25	0.25	0.5	0.5	0.5	0.25	0.25	0.25	1	
12: reins. (MAT)	0.25	0.25	0.5	0.5	0.25	0.25	0.25	0.25	0.5	0.25	0.25	1

Figure 5: Correlation matrix LoB.

- calculate the quantiles (VaR) at 99.5% for the two distributions,
- calculate the two expected values of the two functions of marginal distributions,
- calculate for each branch, according to formula (5), the difference between the quantile and the expected value,
- finally, calculate the SCR for the two branches according to the formula (2).

As a starting point, we set the values of the distributions using $\alpha_1 = 2, \beta_1 = 3$, $\alpha_2 = 3, \beta_2 = 2$ and $\rho = 0.5$. We calculate the inverse of marginal gamma distributions and we get a quantile equal to 22.29 for the first distribution, while the second is equal to 18.55. We must now only calculate the expected value of the two distributions and calculate the difference between the value of the quantile and we have that the expected value for both distributions is equal to 6. Therefore the differences between the quantiles and the expected values of the LoBs are, respectively, 16.290 and 12.548. We apply the formula (2) and we get that capital requirement, based on the standard Solvency II formula, is equal to 25.04. Note that in this case the capital requirements of each LoB were calculated independently and only at the end we obtained the overall SCR based on the correlation provided by the regulation.

B. The calculation of SCR with the theory of the copulas

We show now the approach for calculating capital requirements through the theory of copulas. So we must:

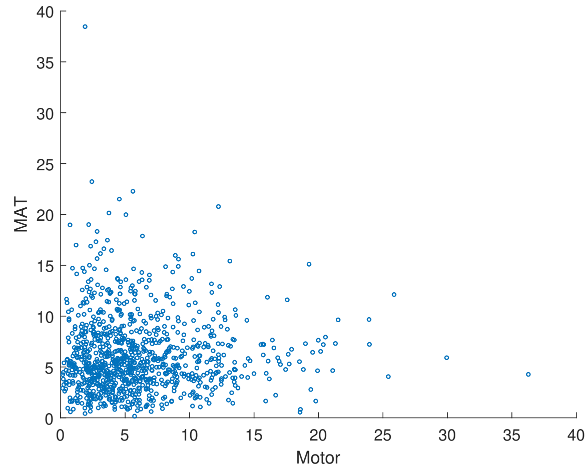

Figure 6: LoBs distribution.

- set the correlation between the distribution functions equal to that of Solvency II,
- estimate the dependency parameter θ assuming a specific copula,
- generate (based on the θ obtained, the selected Archimedean copula and the number of observations) the values (u, v) of the copula,
- obtain, through the copula, the values of the two gamma distributions,
- calculate the quantile at 99.5% and the expected value of the random variable sum.
- calculate the capital requirement as the difference between the quantile and the expected value.

We want to have a correlation between the distributions equal to 0.5, in order to have the same used by Solvency II and to obtain a comparison between the final results as consistent as possible. To do this we specifically build a cost function that estimates the dependency parameter θ which makes the empirical correlation of the gamma distributions equal to 0.5 and we, finally, obtain $\theta = 1.77$.

We generate random scenarios using the Clayton copula, the estimated dependency parameter and, as in the calculation of capital requirements based on the standard formula, the scenarios generated are 1,000,000. We get the values u and v of the copula (figure (5)) and to obtain the values of the distributions of the potential losses of the two LoBs (figure (8)) we calculate again the inverse. If we verify that the empirical correlation is the same as that of Solvency II we will obtain that the correlation is 0.501, therefore perfectly in line with the desired one.

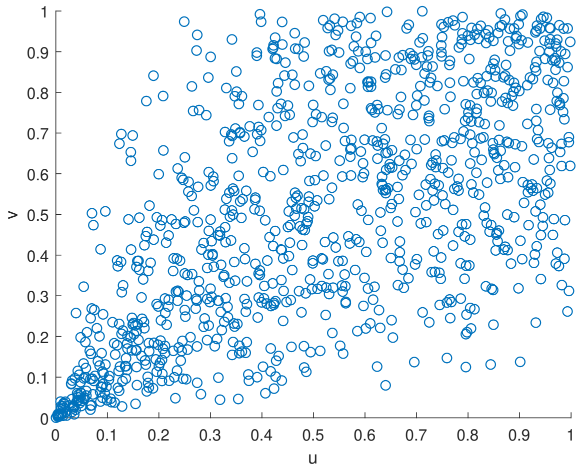

Figure 7: Clayton copula according to the generated θ.

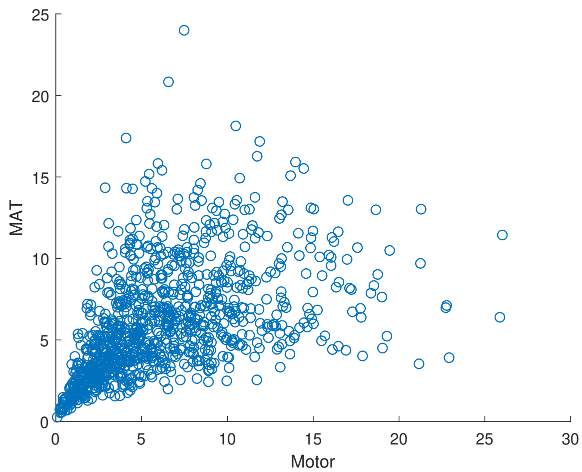

Figure 8: Distribution of LoBs obtained from the Clayton copula.

In figures (9) and (10) we show the contour diagram and the joint distribution function for the Lobs.

At this point we have to sum the values of the two distributions and, again, calculate the quantile at 99.5% and the expected value of the random variable sum and we obtain an expected value equal to 11.99 and a quantile equal to 33.39. The difference between quantile and expected value represents the value of the SCR and we obtain that the SCR calculated with the copulas is 21.39. Note that the capital requirement is about 20% lower than the previous SCR (25.04) so, actually, our analysis showed that the standard formula provided by the regulation overestimates the capital requirement necessary for insurance companies. This is a considerable problem for the various companies as more capital needs to be set aside than what is actually needed.

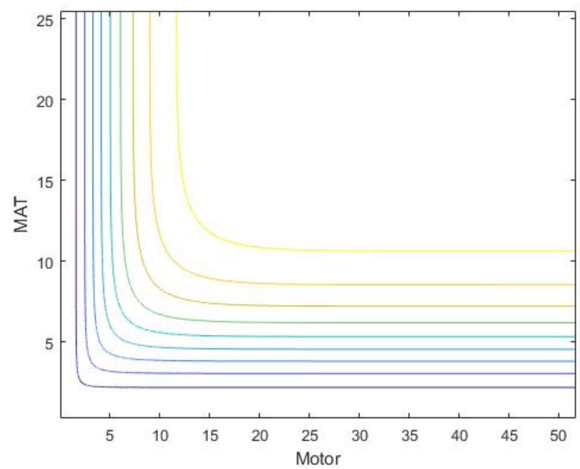

Figure 9: Contour diagram for the two LoBs.

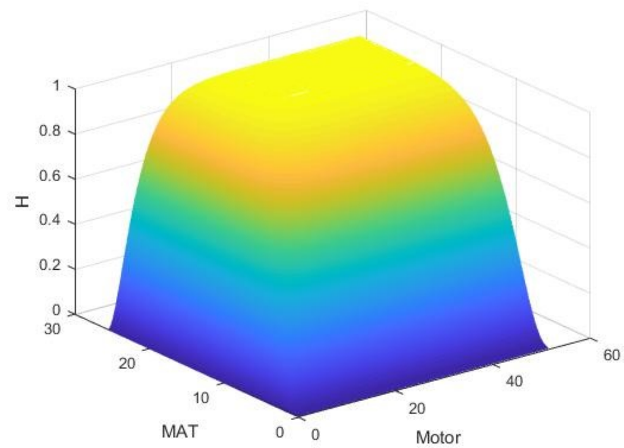

Figure 10: Joint distribution for the two LoBs.

The excess capital set aside is unproductive (it could be used to produce potential profits). So the copulas are a useful tool to calculate the SCR thanks to their ability to analyze the dependency structure between the marginal distribution functions. Therefore we have shown the limits of the standard formula provided by the regulation. The analysis carried out for the two LoBs shows a significant difference between the values obtained with the two methodologies: the use of internal models based on the theory of copulas is an appropriate alternative methodology, despite the cost to implement this method.

V. Conclusion

In our study we have shown the problem of the standard formula for the calculation of capital requirements (Solvency Capital Requirement, SCR) provided by the supervisory authority (due to the European Union Solvency II directive) based on the concept of Value-at-Risk (VaR) with a confidence level of 99.5%.

The formula, unless specifical assumptions, can lead to an overestimation of capital requirements. The provision of a greater amount of capital than necessary implies the presence of unproductive capital that could be invested in other activities to produce potential profits. The copulas, instead, thanks to their properties can solve the problem deriving from the use of the standard formula. The literature analysis shows that there are no studies that provide a concrete application concerning the theory of copulas relating to the Solvency II directive for the calculation of capital requirements; the main studies in this field concern purely theoretical aspects. So we proceed to the calculation of the capital requirements both with the standard formula method and through the implementation of an alternative methodology based on the theory of the copulas.

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