Minimum Wages and Housing Rents: Theory and Evidence from Two Countries

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Abstract

Whether the minimum wage is an effective redistributive policy is still controversial. I investigate this issue from a new perspective by focusing on the effect of minimum wage hikes on housing rents. It is informative for two reasons. First, if minimum wage hikes increase housing rents, some of the benefits accidentally fall on homeowners rather than workers. Second, housing rents serve as an indicator as to whether and how much minimum wages are beneficial for workers, which I show by developing a spatial equilibrium model. I empirically analyze the causal impact of the minimum wage increase on housing rents in the United States and Japan. In both countries, minimum wages hikes increase housing rents in urban areas: 10% minimum wage increase induces 1%-2% increase in the United States and 2.5%-5% increase in Japan. While the unintended incidence on homeowners is arguably moderate, it is non-negligible. Moreover, it may be more salient if minimum wages induce unemployment. I also suggest the importance of heterogeneous welfare impacts on different groups of minimum wage workers.

JEL classification: J21, J38, J61, R23, R38

Keywords: Minimum Wages, Housing Rents, Incidence, Capitalization, Heterogeneous Workers

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1 Introduction

In spite of extensive research, whether minimum wage is an effective redistributive policy tool is still a controversial issue (e.g., Card and Krueger, 1994, 2000; Neumark and Wascher, 2000; Dube, Lester, and Reich, 2010, 2016; Neumark, Salas, and Wascher., 2014; Meer and West, 2016; Harasztosi and Lindner, 2018; Neumark, 2019). In evaluating the desirability of minimum wages, most studies focus on the adverse effect of minimum wages on employment. However, minimum wages may have an impact on other dimensions of the economy, such as hours worked, price levels, and consumption schedules. Assessing minimum wages becomes a more complicated task once these effects are also taken into account.

In this article, I analyze the effectiveness of minimum wage as a redistributive policy from a new perspective: Its effect on housing rents. Although this is a simple question, to the best of my knowledge, this is the first study to formally analyze this issue.\(^1\)

The results yield at least two important implications on the desirability of minimum wages.\(^2\) First, if minimum wage hikes increase housing costs, it increases living costs for low-income households while unintentionally benefiting homeowners. Since housing costs takes up a large share of budgets, which is particularly the case for low-income households, revealing this unintentional incidence on homeowners is indispensable in evaluating minimum wage as a redistributive policy.\(^3\)

Second, housing rents serve as an index (i.e., a sufficient statistic) capturing the inclusive cost and benefits of minimum wages. I develop a spatial equilibrium model à la Rosen (1979) and Roback (1982) to consider the effect of minimum wages on housing rents.\(^4\) I show that changes in housing rents adequately reveal quantitative information on whether and how much they increase the utility of minimum wage workers. The basic idea is that if the minimum wage hike in a region is attractive, this region experiences increased housing demand and the housing cost becomes higher.\(^5\) Notably, the cost

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\(^1\)I should note here that Tidemann (2018) independently and almost simultaneously gives a formal analysis to this issue. See the latter part of this introduction for the key differences and advantages of this paper. At the journalistic level, the impact of minimum wages on housing rents has attracted attention and been discussed in informal ways (e.g., Greenblatt, 2015; Ozimek, 2015). In this paper, I first formalize and extend the theoretical arguments and then reveal the causal relationship between the two using data from the United States and Japan.

\(^2\)To be precise, if minimum wages are increased in every local jurisdiction so that workers remain indifferent about their residence. Thus, my argument is about the ‘local’ minimum wage hikes. In many countries, minimum wage rates vary at the local level and it is likely to be informative.

\(^3\)In general, redistributive policies yield unintended consequences and might lead to unexpected incidence. For example, Rothstein (2010) finds that the Earned Income Tax Credit (EITC) may benefit employers because it reduces wage rates through increasing labor supply. While this channel is not present in the context of minimum wage because the wage rate is fixed, it underscores the importance of considering general equilibrium effects in evaluating redistributive policies.

\(^4\)More specifically, my model extends a spatial equilibrium model to evaluate the efficiency of public goods provision (e.g., Brueckner, 1982; Cellini, Ferreira, and Rothstein, 2010; Coate and Ma, 2017).

\(^5\)Numerous studies show that migration and commuting decisions are affected by minimum wages (e.g., Cadena, 2014; Giulietti, 2014; McKinnish, 2017; Zhang, 2017; Monras, 2018; Pérez, 2018).
and benefit captured by this approach is inclusive in the sense that the effects on employment and the consumption re-scheduling (e.g., labor-leisure choice) are incorporated. Thus, focusing on the changes of housing rents may be quite informative in assessing the minimum wages.

My model also yields several other interesting implications. First, when minimum wage workers are heterogeneous, I show that the change in rents can be expressed as the weighted average of the willingness-to-pay (WTP) of heterogeneous workers for the minimum wage increase. An important consequence of this result is that a minimum wage increase harms some workers while benefiting others. As a redistributive policy, benefiting some workers at the cost of other workers might be hard to justify. It also provides the basis for uncovering group-specific willingness-to-pay for a particular policy. Second, migration responses, which are used to evaluate minimum wage policies in previous works (e.g., Cadena, 2014; Giulietti, 2014; Monras, 2018), might be more complicated than previously deemed in the literature. This result reinforces the rationale for focusing on the effect of minimum wages on housing rents.\footnote{It does not mean that the migration response is not an adequate welfare indicator, and I view that housing rents and migration responses are complementary. See the discussions in Section 2 and Appendix A.2.}

In light of these theoretical results, I investigate the effect of minimum wage hikes on housing rents using two distinct data: one from the United States and the other from Japan. Exploiting local minimum wage changes, I adopt a difference-in-difference (DD) identification strategy and several variants of it to plausibly estimate the causal effect and its heterogeneity.

Interestingly, the two distinct settings lead to similar conclusions on the impact of minimum wage hikes on housing rents. In the United States, although non-urban areas may see no impact, urban areas experience 1%-2% increase in response to 10% increase in minimum wage rates. In Japan, where minimum wage increase was concentrated in urban areas in my sample period, the rents of low-quality apartments experience around a 2.5-5% increase in response to 10% minimum wage increase. Assuming that 10% increase of minimum wages leads to 10% increase of earnings and households spend about a third their budget on housing, around 3%-6% of the increased earnings go to the pocket of homeowners in the urban areas of the United States. In Japan, this figure is 7.5%-15%. While the exact magnitude of the positive impact may depend on the particular housing markets I focus on, both cases exhibit arguably moderate but non-negligible benefits on homeowners. Moreover, the fraction of such unintended incidence is larger when the benefit is smaller because, for instance, minimum wage hikes damage workers through reduced employment opportunities or increased commodity prices. Thus, such an unintentional incidence on homeowners may be an important counter-argument for local minimum wages.

The positive effect on rents implies that minimum wage hikes improve workers’ welfare. Thus, the
detrimental effects of minimum wage increases are not so large that they lower the utility of workers. However, this result requires qualifications once minimum wage workers’ heterogeneity is taken into account. I highlight the potential importance of heterogeneity by showing the possibility that in the non-urban areas of the United States, whites do not benefit nor suffer from the minimum wage increase while blacks greatly benefit from it. Heterogeneity is also important when housing rents increase but some groups of workers face unemployment: In this case, some workers face both the risk of unemployment and increased living costs.

This paper is related with several strands of literature beyond that of minimum wages and unemployment. First, it contributes to the growing literature of evaluating the desirability of minimum wages in a spatial framework (e.g., Cadena, 2014; Giulietti, 2014; McKinnish, 2017; Zhang, 2017; Monras, 2018; Pérez, 2018). Unlike these studies, I focus on the role of the housing market in assessing minimum wages. I theoretically argue that identifying the effect of minimum wage on rents is very appealing. Moreover, housing rents are also informative in detecting the unintended incidence on homeowners, which provides additional information on the efficiency of redistribution through minimum wages. Some of the ideas presented in my paper is also given by the independent work of Tidemann (2018), who finds a moderately negative effect of minimum wage increase on housing rents in the United States. Compared with his study, my theoretical framework more explicitly links the welfare implications to the housing rents, allowing for endogenous behavioral adjustments and heterogeneity among minimum wage workers. Empirically, I use both the data on the United States and Japan, and reveal that minimum wages hikes dynamically increase housing rents in urban areas. I also present a method for uncovering the heterogeneous welfare implications among different groups of workers and point out the racial difference in the United States.

This study also relates to the literature on the effect of minimum wages on output prices (e.g., Lemos, 2008; MaCurdy, 2015; Harasztosi and Lindner, 2018). In general, these studies tend to focus on the impact of minimum wages on supplier’s cost in investigating the impact on commodity prices. In contrast, my focus is on how the minimum wage hikes affect the housing market by changing the migration decision of workers. This question is of great practical importance because housing represents one of the largest spending categories, and thus investigating its price is indispensable. Moreover, focusing on housing allows me to infer information on preferences by using a spatial equilibrium model.

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7 Since Tidemann (2018) and I use the same data source for US housing rents, it is interesting to see why I find positive impacts while he gets negative estimates. In Appendix C.1.3, I use an empirical specification similar to his and argue that his estimates may become statistically insignificant once clustering of standard errors are done at the state level rather than the county level. Moreover, he does not incorporate dynamic impacts of minimum wages, which might hide the positive impact on rents.

8 Moreover, housing supply crucially depends on the stock, and the amount of supply does not immediately respond to the increase in production costs of housing.
Finally, this paper relates to the literature of evaluating public policy using housing rents (e.g., Brueckner, 1982; Cellini, Ferreira, and Rothstein, 2010; Kuminoff, Smith, and Timmins, 2013; Coate and Ma, 2017). I apply this approach to the novel setting of evaluating minimum wages. Moreover, my theoretical results also apply to policies other than minimum wages. In particular, the comparison between housing rents and migration responses in assessing the desirability of policies may be useful in choosing appropriate outcome variables. I also present a heuristic but empirically implementable method to relate the gradient of the rent function to welfare implications of heterogeneous groups of people.

This paper is organized as follows. In Section 2, I present the model and derive several propositions to better understand the empirical results. In Section 3, I empirically investigate the dynamic relationship between minimum wages and housing rents. I also present a method for investigating heterogeneous welfare impacts and analyze the racial difference. Section 4 investigates the same issue by using Japanese data and finds that the results in Section 3 are also valid in a completely different context. Section 5 concludes.

2 Theoretical Framework

2.1 The Model

I show that housing rents of a region increase if and only if the minimum wage increase improves the welfare of residents by extending the capitalization model used to evaluate the efficiency of public goods provision (e.g., Brueckner, 1982; Cellini, Ferreira, and Rothstein, 2010; Coate and Ma, 2017). This model shows that housing rents become higher when public goods are more efficiently provided because it makes the location more attractive. I apply this idea to the context of the labor market and show that the housing rents are increased if and only if the minimum wage increase makes the labor market more attractive to workers. Note that I am implicitly assuming that the change in the minimum wage is local, and so it applies to local minimum wage hikes such as state-level ones. It implies that the effect of minimum wages on housing rents discussed in the empirical analysis may be more conservative than the theory in Section 2 suggests because other local governments experience some increase in minimum wages and it affects the outside utility \( u \). Appendix C.2.3 informally address this issue by exploiting Japanese slow and almost piecewise linear minimum wage increase.

I present a static model to motivate my empirical analysis in the simplest manner. Consider a small open region \( i \). The amount of available housing service in region \( i \) is denoted by \( H(R_i) \), where \( R_i \) is the rental price per unit of apartment quality. I assume \( H' \geq 0 \) so that the supply function is upward-sloping. When \( H' = 0 \) for all \( R_i \geq 0 \), the housing supply is perfectly inelastic. This assumption is widely adopted in the literature because housing supply is mainly driven by stocks, but I do not impose

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9 It implies that the effect of minimum wages on housing rents discussed in the empirical analysis may be more conservative than the theory in Section 2 suggests because other local governments experience some increase in minimum wages and it affects the outside utility \( u \). Appendix C.2.3 informally address this issue by exploiting Japanese slow and almost piecewise linear minimum wage increase.
it from the outset because some supply responses to housing rents may be expected in reality.\textsuperscript{10}

I begin with the model of homogeneous minimum wage workers. Their utility is given by $u(x, l)$, where $x$ is the numeraire (non-housing) consumption vector and $l$ is the amount of leisure. There are many workers (i.e. their effect on the market is assumed to be infinitesimally small), and each consumes one unit of housing. I assume that $u_x, u_l > 0$ and that $u$ is quasi-concave. They are also endowed with $M$ units of the numeraire.

The minimum wage rate is given by $w_i$, but workers face the risk of unemployment. Noting that the minimum wage rate may affect the unemployment, I assume that with probability $p(w_i) \in [0, 1]$, minimum wage worker can work at the wage rate $w_i$. On the other hand, with probability $1 - p(w_i)$, workers cannot find a job and thus earn nothing.\textsuperscript{11} If employed, she can choose her preferred working hours. When a worker can find a job, the budget constraint is $\psi x_i + R_i + w_i l_i = w_i T + M$, where $\psi$ is the price of the national good $x$ in region $i$, $l_i$ is the amount of leisure time in region $i$, $R_i$ is the unit cost of apartments in region $i$, and $T$ is the endowment of time. I assume that $\psi$ is determined at the national level, and unaffected by the minimum wage rate of region $i$.\textsuperscript{12} If she is unemployed, the budget constraint is $\psi x_i + R_i = M$. Note that $l_i = T$ in this case. Let $V^e(w_i, \psi, I)$ and $V^u(\psi, I)$ be the indirect utility function when a worker is employed and unemployed, respectively. The results remain intact if there are $n > 1$ goods, but I assume that $n = 1$ for the ease of exposition.

Assuming that workers are expected utility maximizers, the expected utility can be written as

$$
P V^e(w_i, \psi, w_i T + M - R_i) + (1 - p) V^u(\psi, M - R_i).$$

(1)

Each worker maximizes (1) by choosing the location $i$. Then, in the equilibrium, the following migration equilibrium condition must hold for those living in region $i$:

$$
P V^e(w_i, \psi, w_i T + M - R_i) + (1 - p) V^u(\psi, M - R_i) + \theta \geq \bar{u},$$

(2)

where $\theta$ is the degree of the attachment to region $i$. Here, $\theta$ varies at the individual level and is distributed according to $G(\theta)$, with $dG/d\theta \equiv g \geq 0$.\textsuperscript{13} Although $\theta$ can be interpreted in various ways, I prefer to interpret it as the degree of friction in geographical mobility. Suppose that the utility $p V^e(w_i, \psi, w_i T + M - R_i) + (1 - p) V^u(\psi, M - R_i)$ marginally increases. In a small open economy without migration costs, in the absence of adjustments in the economy, infinitely many people flow

\textsuperscript{10}This is particularly relevant in my context because I allow for dynamic impacts of minimum wages in the empirical analysis.

\textsuperscript{11}Introducing non-negative unemployment benefits does not alter the main implications.

\textsuperscript{12}This assumption is plausible under my assumption of small open economy. It is also widely adopted in spatial equilibrium models (e.g., Diamond, 2016). In my empirical specification, I take the log of rents and minimum wage rates and control for flexible year effects, implying that the relevant variables can be interpreted as real rather than nominal.

\textsuperscript{13}For notational simplicity, I do not explicitly write that $g$ depends on $\theta$. 

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into region $i$. This corresponds to the case of $g \simeq \infty$. On the other hand, in a closed economy, no one moves into or out of region $i$, which corresponds to $g \simeq 0$.

When (2) holds with equality, it defines the marginal worker who is indifferent between region $i$ and other regions. I denote such a worker’s $\theta$ by $\overline{\theta}$. The population level in region $i$ is now written as $N_i \equiv \lim_{\theta \to \infty} G(\theta) - G(\overline{\theta})$, where I assume that the first term is finite.

In equilibrium, the following housing market clearing condition should also hold:

$$N_i = H(R_i).$$  \hspace{1cm} (3)

The equilibrium conditions (2) and (3) determine two endogenous variables $(R_i, \overline{\theta})$. Differentiation of (3) with respect to $w_i$ yields

$$-g \left[ \frac{\partial \overline{\theta}}{\partial w_i} + \frac{\partial \overline{\theta}}{\partial R_i} \frac{\partial R_i}{\partial w_i} \right] = H' \frac{\partial R_i}{\partial w_i}. \hspace{1cm} (4)$$

Combined with Roy’s identity, (2) gives

$$\frac{\partial \overline{\theta}}{\partial w_i} = - \left[ p'(V^e - V^u) + p \frac{\partial V^e}{\partial T} (T - l_i) \right], \quad \frac{\partial \overline{\theta}}{\partial R_i} = p \frac{\partial V^e}{\partial I} + (1 - p) \frac{\partial V^u}{\partial I}. \hspace{1cm} (5)$$

Then, after rearrangement, (4) can be written as

$$\frac{\partial R_i}{\partial w_i} = \frac{p'(V^e - V^u) + p \frac{\partial V^e}{\partial I} (T - l_i)}{p \frac{\partial V^e}{\partial I} + (1 - p) \frac{\partial V^u}{\partial I} + \frac{H'}{g}} \geq 0. \hspace{1cm} (6)$$

Note that (6) shows that the desirability of minimum wages can be inferred from the gradient of the rent function. The numerator of (6) measures the expected utility change due to a minimum wage increase. On the other hand, the denominator of (6) is the sum of the expected marginal value of income and the product of $H'$ and $1/g$. For simplicity, suppose first that $H'/g = 0$ so that the denominator becomes the expected marginal value of income. Then, (6) quantifies the money-metric utility impact of the minimum wage increase. Thus, (6) can reveal both qualitative and quantitative implications for the desirability of minimum wages.

The numerator of (6) is ambiguous in sign, while the denominator of (6) is positive. Thus, the minimum wage increase may either benefit or harm workers. If $p' \geq 0$, so that minimum wage increases do not increase the unemployment rate, the first term in the numerator is positive because of the increased employment.\(^{14}\) When $p' < 0$, the sign of (6) can be negative. If minimum wage hikes significantly exacerbate the labor market conditions by reducing the number of jobs, the region is no longer attractive to workers, even though the wage rate has increased. Workers relocate away from the region, and this

\(^{14}\) $V^e - V^u \geq 0$ follows from revealed preference because employed workers can choose not to work.
imposes downward pressure on housing rents. The second term represents the effect that the currently employed workers experience the wage increase, which is always desirable for workers. The gradient of the rent function is positive if and only if these effects are, on net, positive.

Although qualitative implications remain the same, quantitative interpretations require qualifications when \( g < \infty \) or \( H' > 0 \). When \( g < \infty \), so that the economy is not perfectly open, some utility differences relative to the outside is allowed for, and the migration responses are relatively weak. It mitigates the response of apartment rents as long as \( H' > 0 \).\(^{15}\) When \( H' > 0 \), increases in rents induce an increase in housing supply, and it again mitigates the increase in rents.

(6) is no longer money-metric when \( H'/g > 0 \), but it still provides bounds on the money-metric values of minimum wages. When minimum wage is positively valued (i.e., \( \frac{dR_i}{dw_i} > 0 \)), it provides a lower-bound estimate for the money-metric utility gain of the minimum wage increase. On the other hand, it serves as an upper-bound for the utility loss of minimum wage increase when it is negatively valued (i.e., \( \frac{dR_i}{dw_i} < 0 \)). Thus, it is conservative in terms of assessing benefits and optimistic in evaluating damages.

Summarizing the arguments so far, I obtain the following proposition:

**Proposition 1.**

(a) Suppose that either the assumption of free mobility or an inelastic housing supply holds. Then, the gradient of rents is equal to money-metric utility changes of workers caused by the minimum wage increase. Minimum wage increase may decrease apartment rents if they cause significant unemployment.

(b) Suppose that both the assumptions of free mobility and an inelastic housing supply are violated. Even in this case, the gradient of rents provides correct qualitative implications on the desirability of minimum wages. Moreover, it is a lower-bound estimate for the money-metric utility gain and an upper-bound estimate for the money-metric utility loss in response to changes in minimum wage rates.

The cost and benefit captured in this approach include unemployment and re-optimization in response to the new wage rate.\(^{16}\) Suppose, for instance, that workers change their working hours in response to minimum wage hikes. It may mask the effect of minimum wage rates on total earnings.

\(^{15}\)When \( H' = 0 \), the population \( N_i \) is virtually fixed by the housing market, which is why \( H' > 0 \) is needed for this result.\(^{16}\)Note, however, that Proposition 1 is about marginal minimum wage changes, just like many other models belonging to the sufficient statistics approach (Chetty, 2009). Thus, some additional assumptions are needed to evaluate non-marginal ones. See Chetty (2009) for methods and qualifications about obtaining welfare impacts of non-marginal policies from the assessment of marginal policy changes.
However, increased leisure time is valuable to workers. This is potentially very important in the context of minimum wage policies, as many of minimum wage workers are either homemakers and students. A large fraction of their leisure time is devoted to housework or human capital accumulation, which is arguably very valuable for these people. The housing rents capitalize such benefits, and Proposition 1 provides a method to measure such inclusive benefits of minimum wages.

One may also be interested in whether migration responses, i.e., the population level of $i$, can also serve as an indicator about the desirability of a minimum wage increase. Indeed, recent studies such as Cadena (2014), Giulietti (2014), and Monras (2018) use migration responses to infer the desirability of minimum wage policies. They suppose that if a minimum wage increase is desirable for workers, the region attracts workers, and immigration inflow is promoted. Their analysis, however, does not consider the role of housing market.

The housing market equilibrium condition (3) immediately reveals that the population level yields the same qualitative implications. Indeed, from (3) and (6), I have

$$\frac{\partial N_i}{\partial w_i} = H' \frac{\partial R_i}{\partial w_i} = H' \frac{p'(V^e - V^u) + p \frac{\partial V^e}{\partial I} (T - l_i)}{p \frac{\partial V^e}{\partial I} + (1 - p) \frac{\partial V^u}{\partial I} + \frac{H'}{g}} \geq 0. \quad (7)$$

(7) clearly shows that the signs of $\frac{\partial N_i}{\partial w_i}$ and $p'(V^e - V^u) + p \frac{\partial V^e}{\partial I} (T - l_i)$ match, implying that migration responses also yield qualitatively correct implications on the desirability of minimum wages.

However, there are several notable differences between housing rents and migration responses. First, changes in rents reveal the incidence of minimum wages on homeowners, while migration responses do not. Second, migration responses do not provide money-metric information on minimum wages. In contrast to rents, quantitative welfare implications cannot be obtained by focusing on migration responses. Third, migration responses do not vanish when $H' \simeq \infty$, while rents do not respond in this case. The reason is simply that housing prices are determined completely by the supply side, and any utility differentials are absorbed by migration responses. Thus, in this case, focusing on migration responses is valuable at the cost of quantitative implications. However, in real housing markets, stocks are very important and a rapid adjustment in supply may be hard to expect. Finally, in the presence of worker heterogeneity, migration responses may exhibit a complex pattern while rents are affected in a simple and interpretable way. I discuss this point in the next subsection. Based on these arguments, although migration responses are informative, I focus on apartment rents in the empirical analysis.  

\[17\] These activities may involve externalities, especially to their family members. Externalities within a family are incorporated as long as relocation and labor-leisure choices are made at the family level, but broader externalities are not capitalized in the apartment rents.

\[18\] I also theoretically argue in Appendix A.2 that migration responses may be particularly useful in the presence of local inflation.


2.2 Heterogeneity in Minimum Wage Workers

Now, I discuss the case that minimum wage workers may be heterogeneous. Minimum wage workers are composed of workers differing in various dimensions, such as race, marriage status, full-time education, and immigration status. The welfare evaluation for a minimum wage increase may significantly differ across these different groups, and it is interesting to analyze what we can learn from housing rents under such heterogeneity. I assume that housing supply is fixed (i.e., $H' = 0$) to focus on the role of heterogeneity.\(^{19}\)

When workers are homogeneous, Proposition 1 implies that changes in rents are sufficient for inferring the WTP to live in the region (Chetty, 2009). However, without further assumptions, which I impose in a later section, it is impossible to uncover the WTP for all types of workers since the rent level is a single index and the information is insufficient. I show that the changes in rents can be written as the weighted average of WTP of different groups. A larger weight is placed on the group with large population share and on the group with higher mobility.

Assume that there are $k(= 1, 2, \ldots, K)$ types of minimum wage workers that differ in utility function, employment probability, and endowment denoted by $u^k(x, l)$, $p^k_i(w_i)$, and $M^k$, respectively.\(^{20}\) On the other hand, they face the same housing rent and the minimum wage. To simplify the analysis, I assume in this section that the utility function is quasi-linear so that $u^k(x, l) = x + u^k(l)$, where $x$ is a scalar and denotes the amount of numeraire consumption (see Appendix A.3 for a generalization). Each resident has attachment $\theta^k$ to region $i$, which is distributed according to $G^k_i(\theta^k)$ with $dG^k_i/\theta^k \equiv g^k_i \geq 0$. Here, $G^k_i$ gives the number of workers whose attachment level is less than $\theta^k$, and $g^k_i$ is the marginal change in the number of workers in response to changes in $\theta^k$.\(^ {21}\) One naturally expects that $g^k_i$ is higher when (i) the size of the group $k$ is larger and (ii) the mobility of group $k$ is higher.

I assume that $\theta$ is measured in monetary units and enters the utility in an additive way. Then, the utility function of type $k$ is written as

$$WTP^k_i(w_i, R_i) = p^k_i(w_i, R_i) \equiv p^k_i(w_iT + M^k - R_i - w_i l^*_i + u^k(l^*_i)) + (1 - p^k_i)(M^k - R_i + u^k(T)) + \theta^k,$$  

where the leisure $l^*_i$ is optimally chosen. The effects on the first and second terms can be thought as the WTP of type $k$ for living in region $i$. I define $WTP^k_i(w_i, R_i) \equiv p^k_i(w_iT + M^k - R_i - w_i l^*_i + u^k(l^*_i)) + (1 - p^k_i)(M^k - R_i + u^k(T))$.

The migration condition is now written as

$$WTP^k_i + \theta^k = \bar{u}^k,$$  

\(^{19}\)In Appendix A.3, I generalize the model to allow for the elastic housing supply and general utility functions.\n
\(^{20}\)One can also make the endowment of time heterogeneous without any changes to results.\n
\(^{21}\)As in the previous subsection, I simplify the notation by not explicitly stating the dependence of $g^k_i$ on $\theta$.\n
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where $\bar{u}^k$ is the outside utility for type $k$, which is assumed to be exogenous. The marginal resident has the attachment level $\bar{\theta}^k$.

In the absence of changes in rents in response to a marginal minimum wage increase, $(9)$ tells us that $\frac{\partial \bar{\theta}^k}{\partial w^i} = -\frac{\partial WTP^k_i}{\partial w^i}$, which translates into the migration of $g^k_i \frac{\partial WTP^k_i}{\partial w^i}$ type $k$ workers.

However, changes in rents also induce population changes. Let $R^i(w^i)$ be the equilibrium rent level under the minimum wage rate $w^i$.

Since $\forall k$, $\frac{\partial WTP^k_i}{\partial R^i} = -1$, the marginal population change of type $k$ in response to a marginal rent increase is written as $-g^k_i$. The population is fixed at $H$, so one must have

$$\frac{\partial R^i}{\partial w^i} \sum_k g^k_i = \sum_k (g^k_i \frac{\partial WTP^k_i}{\partial w^i}).$$  

(10)

It can be rewritten as

$$\frac{\partial R^i}{\partial w^i} = \sum_k (z^k_i \frac{\partial WTP^k_i}{\partial w^i}),$$  

(11)

where $z^k_i = \frac{g^k_i}{\sum_k g^k_i}$ and $\sum_k z^k_i = 1$. Noting that $g^k_i$ reflects both the population size and the mobility of type $k$, I obtain the following proposition.

**Proposition 2.** In response to marginal a minimum wage increase $w^i$, the changes in rents $\frac{\partial R^i}{\partial w^i}$ can be written as $\sum_k (z^k_i \frac{\partial WTP^k_i}{\partial w^i})$. Thus, $\frac{\partial R^i}{\partial w^i}$ is a weighted average of changes in WTP of each group. The weight for group $k$ is larger when it has larger share of population and is more mobile.

Proposition 2 shows that the gradient of the rent function still provides information about the impact of minimum wage increase on the WTP even when minimum wage workers are heterogeneous. It tends to reflect large and mobile groups’ WTP. This result can be intuitively understood in the following way. Suppose there are two groups, $k_1$ and $k_2$, and the minimum wage increase improves the welfare of group $k_1$, but it harms the welfare of group $k_2$ because unemployment is more detrimental to group $k_2$. Suppose also that group $k_1$’s mobility cost is much lower than group $k_2$’s. Then, the minimum wage increase induces massive inward migration of people of group $k_1$, while little migration response is induced for group $k_2$. Since a fixed housing supply implies constant population in the region, the inward migration of people of group $k_1$ should be offset by the increase in rents. That is, the amount of the increase should be determined so that it offset the increase of $WTP^k_{i1}$, implying that it reflects the WTP of group 1.

---

22Rent $R^i$ is determined as a function of $w^i$ that clears the housing market.

23Recently, Wong (2018) shows that the gradient of the rent function in a standard discrete choice model is equal to a weighted average of marginal WTP of heterogeneous individuals. She also shows that a larger weight is given to an individual whose choice is uncertain. Although there are several formal similarities in terms of the mechanism, my result implies that the size of marginal migration responses is critical in determining the weight, rather than featuring individual choice uncertainty in discrete choice models.
Note that in this example, group $k_2$’s WTP is decreased after the minimum wage increase because the rent is increased significantly. Generalizing this observation leads to the following corollary of Proposition 2

**Corollary 1.** The group with the lowest $\partial WTP_k^i / \partial w_i$ experiences the decrease in $WTP_k^i$ due to the adjustment of rents. As a result, the population of type $k$ decreases.

Corollary 1 implies that under our assumption of a perfectly inelastic housing supply, some group must be adversely affected by the increase in the minimum wage. For example, it is possible that housing rents are significantly increased because it is beneficial for university students, but it makes it difficult for truly poor minimum wage workers to find a job. In this case, they face both worsened labor market conditions and increased housing costs. As long as one views minimum wage policies as a redistributive policy, it is difficult to justify this situation because university students can often seek financial support from their parents or siblings and their expected lifetime earnings is relatively high. Thus, in testing the effectiveness of minimum wage policies, it is useful to analyze the heterogeneity of minimum wage workers.

Corollary 1 also has an implication on the interpretation of migration responses to minimum wage changes. It suggests that minimum wage hikes yield both “winners” and “losers.” Those who suffer from the minimum wage increase move out of the region, while those who benefit move in. Thus, even when the net migration response is zero, minimum wage hikes promote both inward and outward migration. The presence of heterogeneity in minimum wage workers can be seen when both directions of migration are active while the net migration response is close to zero. At the same time, the complexity of migration responses is likely to make it difficult to empirically recover information about minimum wages from them. In particular, the heterogeneity may lead to qualitatively wrong implications when one focuses on the migration responses of specific groups of workers.\[24\]

In the presence of heterogeneity, exactly identifying WTP of all groups requires quite detailed information, but some progress can still be made by making some approximations. In particular, the share of population is observable in the data and it is useful in obtaining an idea about the weights $z_k$. As long as the mobility of a group does not differ significantly across regions, the weight is larger for a group with a large population share. In Section 3.3 I develop this idea to roughly estimate heterogeneous welfare impacts by race in the United States.

In Appendix A, I provide several theoretical extensions of the model. First, I explicitly consider

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\[24\] Indeed, emigration in the heterogeneous case provides an alternative explanation to the results in the literature. For example, Cadena (2014) focuses on the migration responses of foreign immigrant workers. Corollary 1 implies that when they value labor market conditions differently from native workers, emigration may occur even when the minimum wage increase is positively valued by immigrant workers.
housing quality and how the estimated results may need modifications when some housing qualities are unobservable. It may either increase the relatively high-quality apartments' rents. Second, I consider the effect of local inflation. If it is significant and the data on local price levels is weak, migration responses may be a preferable outcome variable. Third, I generalize the model with heterogeneous workers and derive the generalized formula for the rent gradients.

3 Evidence from the United States

To empirically evaluate the effect of minimum wage hikes on housing rents, I utilize the experiences of two distinct counties: the United States and Japan. I start with the analysis of the United States.

3.1 Data

I use the county-level data from 1984 to 2018. To obtain the monthly rent data for each county, I use the fair market rent (FMR) series from the Department of Housing and Urban Development (HUD), which are widely utilized as a measure of housing rents for low-income households (e.g., Quingley, Raphael, Smolensky, 2001; Saiz, 2003, 2007; Tidemann, 2018; Sharpe, 2019). FMRs are published for implementing Section 8 housing voucher program and calculated by utilizing various data sources such as the Decennial Census, American Community Survey, and random digit dialing telephone surveys. FMRs include the shelter rent plus basic utility costs. The FMRs capture the 40th percentile rent of the distribution of monthly rents of all units occupied by recent movers. In calculating the distribution of rents, non-market rental housing as well as units less than two years old are excluded. Thus, the FMRs can be interpreted as the price for a rental unit of moderate quality available in markets (Saiz, 2003, 2007).

I use the FMRs for two-bedrooms units since it is regarded as most fundamental by HUD. FMRs for studio, one-bedroom, three-bedrooms, and four-bedrooms apartments are also published. They are calculated by greatly relying on FMRs for two-bedroom apartments, and using these alternative FMRs lead to similar conclusions.

25 Since the FMRs for a fiscal year \( t \) (i.e., from the October of year \( t \) to the September of year \( t + 1 \)) are determined using the information in the previous fiscal year (i.e., from the October of year \( t - 1 \) to the September of year \( t \)), I use the fair market rent of fiscal year \( 1985 \leq t \leq 2019 \) as the rent of year \( t - 1 \). This approach follows Saiz (2003, 2007).

26 Since FMRs are related to housing assistance, they in turn have an effect on housing rents. See, for example, Collinson and Ganong (2018) and McMillen and Singh (2018). My estimates may include such feedback effect, but I believe such an inclusive estimate is also of interest. In Japan, on the other hand, housing assistance program is generally of small scale and such problem is likely to small.

27 Prior to 1995, the 45th percentile was used. In the main analysis, I assume that year-fixed effect adequately captures the effect of this change in the evaluation point. Reassuringly, I obtain qualitatively similar results when I drop observations prior to 1995, although they are somewhat more imprecise. It should also be noted that from 2001, some large cities use the evaluation point of the 50th percentile. While it is likely to have only minor impact on the conclusion since such cities are not many, the direction of the bias is unknown, but excluding samples using the 50th percentile has only minor impacts on results. See https://www.huduser.gov/portal/datasets/fmr.html#history for further details the FMR data.

28 FMRs for studio, one-bedroom, three-bedrooms, and four-bedrooms apartments are also published. They are calculated by greatly relying on FMRs for two-bedroom apartments, and using these alternative FMRs lead to similar conclusions.
are calculated at the level of the FMR area, which usually corresponds the county but it is often estimated at a coarser level in metropolitan areas. When necessary, I use as weights the population level from 2000 census to construct the county-level FMRs. Summary statistics are available in Table C.1.

Here, I point out several drawbacks of the FMR data one should keep in mind in interpreting the results. First, while some efforts are made to make housing units comparable, housing characteristics are not available in the FMR data. This is not the case in the Japanese data, as the Japanese dataset and a unique housing market characteristic allow me to largely control for observed and unobserved housing quality. Thus, I indirectly check the seriousness of this problem using a different context and reassuringly, I obtain similar results. Second, FMRs may “jump” because of the availability of reliable data sources such as the decennial census (Collison and Ganong, 2018). There are several reasons why I suppose that this problem is not crucial. First, my empirical specifications flexibly allow for lagged impacts. Thus, even when FMRs take some time to reflect the true housing market conditions, it would be captured by the lagged terms. Second, densely populated counties, which I particularly focus on, are less likely to suffer from this issue since American Housing Survey, which is conducted more frequently than the decennial census and includes detailed housing information, has been available for large metropolitan areas and HUD utilizes this information. Third, although results become more imprecise due to the reduced number of observations, qualitative implications are similar if I repeat the analysis using samples only from year \( t \geq 2007 \), from which HUD started utilizing annual American Community Survey.

I also take the density of each county from 2000 census and regard dense counties as urban counties with relatively tight housing supply. The threshold density for qualifying as an urban county is 200 people per square mile of land area. It selects about 15% most densely populated counties. Below, I report results for both full samples and densely populated (urban) samples. I have also worked on samples consisting of counties with density less than 200, but the results are both qualitatively and quantitatively similar to that in the full sample. Henceforth, the results in the full sample can also be interpreted as the results from “rural” samples with the density less than 200.

I assign to each county the state minimum wage rate. Since I use lags and leads of them in some specifications, I collect the annual data of state minimum wage rates from 1979 to 2019 to maximize the number of available samples. I use the minimum wage rates in January of year \( t \) as the minimum wage.

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29I use the same weight in every year to reduce the concern about endogeneity. While the population level in 2000 might contain some endogeneity, the bias is likely to be very small since in calculating weights, I only use the level, not the change of the population.

30The qualitative implications are insensitive to the choice of the threshold. The results are also robust if I split the sample based on the level of population rather than the density. Note also that, as can be seen in Table C.1, the distribution of density is quite fat-tailed.

31I acknowledge that the data on state minimum wages largely come from Minimum Wage/Living Wage Data Set
wage rate of year $t$.\textsuperscript{32} Throughout the analysis, I use the log of the nominal minimum wage rates. However, since I include year-fixed effects in all specifications, the results are completely unchanged if the minimum wage rates are converted into real terms by using the federal-level price index. Estimations are done by the OLS. Since the variable of interest, the minimum wage, is a state-level variable, I cluster standard errors at the state level in all specifications (Bertrand, Duflo, and Mullainathan, 2004).

### 3.2 Empirical Specifications and Results

Housing markets are expected to adjust only gradually because the demand for housing is expected to change only gradually. In testing the effect of minimum wages on employment, Meer and West (2016) point out this issue and suggest using empirical specifications explicitly considering the dynamic impact of minimum wages on employment. Since the housing demand of each region depends on the labor market conditions, the gradual labor market responses are also likely to induce gradual adjustments in the housing market. Moreover, search frictions in the housing market (e.g., Wheaton, 1990) is also likely to prevent immediate responses of the housing demand. Since the model in Section 2 deals with the long-run migration equilibrium, allowing for dynamic impacts of minimum wages in empirical specifications seems important. As described below, I follow the empirical strategy of Meer and West (2016) to investigate the dynamic impact on housing rents.

The first specification is what Meer and West (2016) call “long-differences specifications” given by the following:

$$
\Delta_r \ln (FMR)_{it} = \text{year}_t + \gamma_i \cdot r + \beta_0 \Delta_r \ln (MW)_{it} + \epsilon_{it},
$$

where $\Delta_r$ is the $r$-period differencing operator (i.e., $\Delta_r x_t \equiv x_t - x_{t-r}$), $i$ is the index of each county, and $t$ is the year.$^{33}$ Since this involves the differencing, the second term in (12) corresponds to the county-specific linear time trends in a regression equation without differencing. Except for county and year fixed effects and county-specific linear trends, I do not include other control variables because I am interested in the effect of minimum wages through any channels. For example, suppose I include the county population as a control. However, minimum wages may affect the housing rents through affecting population levels, inducing the over-controlling problem. Since how minimum wages affect housing rents is unclear a priori, I omit county-year level control variables. I estimate (12) with and without county-specific linear time trends to see the robustness of the results.

\textsuperscript{32}In the United States, state minimum wage changes usually take place in January.

\textsuperscript{33}In Appendix C.1.3, I estimate a standard DD model in levels and show that similar implications are obtained.
In Table 1 and 2, I report the results using the full sample and ones using only densely populated counties, respectively. The index of each column corresponds to the size of differencing, i.e., $r$.

It turns out the results are sharply different between full samples and urban samples. When full samples are used, Table 1 shows no indication that minimum wages affect housing rents in any way. All estimates are close to zero and statistically insignificant, implying there is no immediate nor gradual impact on housing rents.

However, the picture is completely different if I focus on the counties with populous counties. Table 2 shows sizable and statistically significant positive impacts of minimum wage increase on housing rents. The estimates show that 10\% minimum wage increase induces around 1\% of the rent increase. Thus, for urban residents, minimum wages are likely to improve their utility and counties with higher minimum wages are attractive places. Recall that the benefit of minimum wages captured by the housing rents is inclusive of unemployment and various behavioral adjustments, such as labor-leisure choice. Thus, it provides a novel evidence that minimum wage improve workers welfare in urban areas, even after taking into account the impact of minimum wages through many channels.

Regarding the incidence of minimum wages, the positive impact shows that some benefit of minimum wages unintentionally goes to homeowners. While this unintentional incidence is arguably not so large, it should be considered in evaluating minimum wages as a redistributive policy tool. I use a back-of-the-envelop calculation to see the size of this incidence after estimating another specification.

Table 2 also exhibits a clear pattern that the coefficient becomes larger as the time span $r$ expands more. This pattern is consistent with the premise that minimum wage increase gradually affects the housing rents, implying that the positive impact is likely to be driven by minimum wages, not by pre-existing trends. It is also consistent with the result of Dube, Lester, and Reich (2016) and Meer and West (2016) showing that minimum wage hikes do not immediately reduce employment but it does gradually. If there is some inertia in the response of local labor market conditions, it is natural that the housing demand for that area is also adjusted gradually.

Given that minimum wages may have gradual effect on housing rents, it is informative to estimate a model incorporating the more direct and flexible dynamic relationship. I estimate the following equation:

$$
\Delta_1 \ln(FMR)_{it} = \text{year}_t + \gamma_i + \sum_{r=-k}^{s} \beta_r \Delta_1 \ln(MW)_{it-r} + \epsilon_{it}.
$$

(13)

The model includes the lags and the leads of the minimum wage rates. The number of lagged terms are denoted by $s$ and that of the leads by $k$. If minimum wages have a dynamic impact, including lagged terms is necessary. Lead terms are meant to control for pre-existing trends. If the minimum wage
### Table 1: Long-differences specifications: Full samples

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Standard errors in parentheses are clustered at the state level.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

### Table 2: Long-differences specifications: Densely populated samples

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Standard errors in parentheses are clustered at the state level.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$
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Standard errors in parentheses are clustered at the state level.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 3: Distributed lag models

increase is unrelated with the trend, the coefficients should be zero because future minimum wage rates are not likely to affect housing rents.\textsuperscript{34}

The results are reported in Table 3. “Sum of MW effects” indicate the sum of coefficients of contemporaneous and lagged terms. Again, the results from densely populated samples are in start contrast to that from the full samples. In columns 1-2, I include only lagged minimum wage rates up to three years. Column 1 uses the full sample while column 2 uses densely populated samples. The estimates are positive in both cases, although it is statistically significant only in urban samples.

\textsuperscript{34}It is not entirely impossible that future minimum wage rates are somewhat capitalized into housing rents. However, the focus is on the housing for rent and this issue is likely to be minor as long as rental markets are sufficiently liquid.
Column 3 and 4 adds one lead term, and the results are almost the same. Note also that the coefficients of the 5th lag is closer to zero than other lagged and contemporaneous variables, implying that the full effect of minimum wages appears within five years.

The results are sharply different between the two cases in columns 5-6, where I include two lead terms.\textsuperscript{35} The full sample case in column 5 has much lower coefficient than columns 1 and 3. On the other hand, in the densely populated sample in column 6, the effect is increased up to 0.196 with statistical significance at 5\% level. Since the inclusion of leads are likely to control for the presence of pre-trends, the positive coefficient for the full sample case in column 1 may be driven by the trends rather than the true policy impact. On the other hand, the positive impact in urban counties are strengthened by the same modification to the model, implying that it indeed captures the true impact of minimum wages. Overall, the distributed lag model predicts that the rents are 1.4\%-2\% increased in urban areas after 10\% increase in minimum wage rates.

Overall, I conclude that in urban areas, minimum wage hikes have a gradual dynamic impact on housing rents. In total, 10\% increase in the minimum wage induces around 1-2\% increase in housing rents. In light of Proposition 1 and 2, the positive impact on rents implies that at least some minimum wage workers benefit from the minimum wage increase. Thus, even considering the potential detrimental effect on employment, minimum wage hikes in region $i$ increase the WTP of some workers to live in region $i$. However, the rent increase may harm some group of workers in urban areas, implying that one should be careful about the housing affordability of low-income households in urban areas.

In Appendix C.1.4, I estimate the model without log-transformation to obtain estimates of the monetary marginal WTP about 1\$ increase in real minimum wages. While specifications without log-transformation may be vulnerable to extreme values in OLS estimation, results resemble the patterns in specifications with log-transformation. In the long-difference specification with five-year time span, the estimated marginal WTP is $30. To assess its magnitude, consider a full-time minimum wage worker with the labor supply of 160 hours per month. Then, without any behavioral response and induced unemployment, it increases the WTP by 160, which is the increase in the monthly earnings. Thus, for such a worker, $(160\$ - 30\$)/160\$ \approx 81\%$ of the benefits are lost for some reason, such as job destruction effects of minimum wages. This fraction is likely to be an upper-bound of the lost benefit. Note first that the estimated benefit of 30\$ is a lower-bound estimate of the true benefit due to housing supply increase and migration costs (Proposition 1). Moreover, the total benefit might be less than 160\$. For example, if minimum wage hikes induce unemployment, it would be less than 160\$. Another possibility is that minimum wage workers tend to work less than the full-time, since many of

\textsuperscript{35} Including three lead terms hardly changes results.
them are secondary wage earners such as students. Thus, even in a quite conservative estimate, the side-effects of minimum wages are not large enough to reduce workers’ welfare and keep around 20% of the total benefits.

The results also show that some benefits of minimum wage hikes unintentionally goes to owners of urban residential properties. If minimum wage workers in urban areas spend a third of their budget on housing, around 2%-6% of increased earnings go to homeowners. The size of this incidence may be arguably not very large, but still a non-negligible amount. Moreover, it is even larger if households spend more fraction of earnings on housing or they face job-destruction effect of unemployment. Note that positive total dynamic impacts on rents indicate that the housing supply is not perfectly elastic even in the medium run. Policies to expand housing supply, such as de-regulation on housing markets and public housing provision, may curb rent increases and prevent the windfall gains on homeowners.

On the other hand, we find no impact on housing rents in rural areas. In light of the theory in Section 2, there are several ways to interpret this result. First, the negative impact of minimum wages may be more salient in rural areas than in urban areas. For example, suppose that jobs in cities are more productive but workers are paid below their marginal product for some reason. Since minimum wage hikes are likely to destruct jobs with low productivity, rural areas with low productivity may be disproportionately destroyed. In such a case, rural areas experience smaller impacts on housing rents. Second, housing supply may be more elastic in rural areas since there are utilized lands. Such elastic supply pushes the impact of minimum wages toward zero regardless of how minimum wages affect workers. Third, migration may be more inactive in rural areas due to high pecuniary or psychological migration costs. For instance, residents in rural areas are more likely to be born in these areas and feel strong attachments to that region. If this is the case, the marginal worker who is indifferent between the particular area and other areas is insensitive to the difference in minimum wage rates, leading to the small capitalization of minimum wages. While these explanations are not mutually exclusive, the welfare implication is quite different. More specifically, the first case implies that rural residents suffer more from minimum wage hikes, while the second and third cases do not imply it. In the next subsection, I provide some evidence that the first explanation may be more relevant.

Before going to the next section, I comment on the possible concern that the positive impact on housing rents may seem inconsistent with the findings of Cadena (2014) and Monras (2018), who find that low-skilled workers “vote in one’s feet” and move out of the states experiencing minimum wage

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36 On the other hand, the estimated benefit here does not include the benefit of behavioral adjustments. For instance, if workers attach high value to their leisure, minimum wage hikes may allow them to reduce the hours worked and greatly improve the welfare.

37 In this back-of-the-envelope calculation, I assume no adjustment in labor supply and no unemployment.

38 This case can be interpreted as high g in the model of Section 2.
hikes. Indeed, these results imply that a minimum wage increase is harmful by significantly destroying jobs. The difference may come from details of empirical specifications, as other studies document positive impact on the number of immigrants (e.g., Giulietti, 2014). That said, the positive response of rents may still seem somewhat contradictory to their results.

There are several possible ways to reconcile the positive impact on housing rents and the acceleration of the out-migration. First, as clarified in Corollary 1, the migration may exhibit a complex pattern in the presence of minimum wage workers’ heterogeneity. If some disadvantaged group of workers suffers from unemployment caused by a minimum wage increase, they are harmed in two ways: reduced employment rate and increased housing costs. In such a case, these group of workers may move out to find a job and affordable housing. Emigration of some groups of workers may be driven by these responses of housing markets, not only by those of labor markets.\footnote{In Appendix B, I present suggestive evidence indicating workers’ heterogeneity in the Japanese context.}

Second, out-migration might be driven by additional factors not included in the canonical model. For example, in contrast to my model and the models of Cadena (2014) and Monras (2018), workers may have a chance to migrate after seeing whether they are employed or not. In particular, the unemployed people may be more likely to move to seek a job (Greenwood, 1985). In such a case, the out-migration is likely to be more stimulated than these models predict.\footnote{To describe such a situation in my model, assume that the unemployed people can choose to leave the region to obtain a job in a rural area rather than staying in the region and earning nothing. In this case, housing cost increase promotes out-migration because it increases the cost of staying in the region without a job.}

Third, the increase in renters might be driven by workers who keep residing in the same prefecture but start renting a house. For example, students might choose to commute from their parents’ house if the minimum wage rate is low, while they may live apart from their parents when it is high. In such a case, housing demand increases without cross-state migration responses observable in the data.

### 3.3 Heterogeneous Impacts by Race

Minimum wage workers can be divided into various groups by observable characteristics, such as gender and race. In this subsection, I present a heuristic but empirically implementable method to uncover group-specific welfare impacts of minimum wages on workers by using the results from Section 2.2. I then apply the method to reveal the heterogeneity with respect to race. The main advantage of this approach is the lax data requirement: Some aggregate-level housing rents data and characteristics of the housing markets (at least including the population level of each group) are sufficient. It is more heuristic compared with the attempts to uncover preferences by estimating discrete choice models using household-level information (e.g., Bayer, Ferreira, McMillan, 2007; Kuminoff, Smith, Timmins, 2007; Kuminoff, Smith, Timmins, 2007).
2013), but still provide some information even when such a method is not applicable.

I start with the formula in Proposition 2 relating the marginal welfare impact and the impact on housing rents:

\[
\frac{\partial R_i}{\partial w_i} = \sum_k (z_i^k \frac{\partial WTP_i^k}{\partial w_i}).
\]

Multiplying \( w_i/R_i \) to this,

\[
\frac{\partial \ln R_i}{\partial \ln w_i} = \frac{\partial R_i}{\partial w_i} \frac{w_i}{R_i} = \sum_k \left[ z_i^k \left( \frac{w_i}{R_i} \right) \frac{\partial WTP_i^k}{\partial w_i} \right].
\]

Define \( \lambda_i^k \) as the ratio between the rent and the willingness-to-pay of group \( k \), so that \( R_i \equiv \lambda_i^k WTP_i^k \).

I also assume that \( WTP_i^k \) is positive without loss of generality.\(^{41}\) The above expression is now written as

\[
\frac{\partial \ln R_i}{\partial \ln w_i} = \sum_k \left[ \left( \frac{\partial WTP_i^k}{\partial w_i} \frac{w_i}{WTP_i^k} \right) z_i^k \lambda_i^k \right].
\] (14)

(14) suggests the way to uncover the elasticity of the willingness-to-pay in a linear regression framework. More specifically, if one defines explanatory variables \( \frac{z_i^k}{\lambda_i^k} \ln w_i \) for each \( k \), then the associated coefficient corresponds to the elasticity of the WTP with respect to minimum wage rates \( \frac{\partial WTP_i^k}{\partial w_i} \frac{w_i}{WTP_i^k} \). In practice, the elasticity \( \frac{\partial WTP_i^k}{\partial w_i} \frac{w_i}{WTP_i^k} \) may be heterogeneous across regions \( i \). While it is an important point, to focus on the heterogeneity across different groups of workers, I simply assume away such heterogeneity.

The next problem is about how to obtain information on \( z_i^k \). While exactly knowing it is difficult, \( z_i^k \equiv g_i^k / \sum_k g_i^k \) is closely related with the population share of group \( k \) in that region since \( g \) is the mass of marginal workers indifferent between region \( i \) and other areas. One heuristic way to construct the explanatory variables is to define \( z^k \) as the population share of group \( k \) by assuming that the mobility cost is the same for every group. Further assuming that \( \lambda_i^k \) does not differ much across regions and groups, about which I have no available dataset, the estimated coefficients are directly comparable across groups to determine which group of workers benefited or suffered. Since this approach is ad-hoc in nature and rely on strong assumptions, it is important to investigate the sensitivity of results to alternative views on \( g \) and \( \lambda \).\(^{42}\)

Note also that this approach estimates the size of the welfare impact relative to that of another group. As long as the magnitude of \( \lambda \) is unknown, the absolute size of the benefit is not identified.

\(^{41}\)From (9), \( WTP_i^k = \bar{u} - \bar{\theta}_i^k \). Note that \( WTP_i^k \) is the benefit of residing in region \( i \) net of the housing costs.

\(^{42}\)In my context, blacks may be more likely to be a minimum wage worker. They may also have lower migration costs (e.g., Diamond, 2016). These considerations increase \( z_i^{black} \). On the other hand, it can also be argued that blacks are generally poorer and have higher expenditure share for housing, implying high \( \lambda^k \). As a robustness check, I multiply the population of whites or blacks by 1.5 and construct weights using the same procedure as the baseline case. It yields similar conclusions.
\[ \Delta \ln FMR = \gamma_t + \gamma \cdot r + \beta_{\text{whites}} \Delta \ln MW + \beta_{\text{blacks}} \Delta \ln MW + \varepsilon_t. \]

\( \beta_{\text{white}} \) is \( \lambda_{\text{white}} \frac{\partial WTP_{\text{white}}}{\partial w_i} \frac{w_i}{WTP_{\text{white}}} \) and the analogous expression holds for \( \beta_{\text{black}} \). As long as \( \lambda \) is similar, the estimates indicate the heterogeneity in the welfare impact of minimum wage hikes.

I report the results in Table 4 with various time spans \( r \). Panel A shows the substantial heterogeneity between whites and blacks. While whites do not seem to experience welfare gains, blacks benefited substantially. Note that we have no statistically significant impact in Table 1 assuming homogeneous minimum wage workers. Thus, the previous finding is driven by the null effect on white people, which

\[ z_{i}^{\text{whites}} = \frac{\#\text{whites}}{\#\text{whites} + \#\text{blacks}} \] for each county \( i \), and \( z_{i}^{\text{blacks}} = 1 - z_{i}^{\text{whites}} \). Table C.1 provides the summary statistics.

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**Table 4: Long-differences specifications: Heterogeneity by Race**

<table>
<thead>
<tr>
<th></th>
<th>(1) ( \Delta_1 \ln FMR )</th>
<th>(2) ( \Delta_2 \ln FMR )</th>
<th>(3) ( \Delta_3 \ln FMR )</th>
<th>(4) ( \Delta_4 \ln FMR )</th>
<th>(5) ( \Delta_5 \ln FMR )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Full samples</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Whites ( (\beta_{\text{whites}}) )</td>
<td>0.0087 (0.01753)</td>
<td>0.0018 (0.0206)</td>
<td>-0.0194 (0.0264)</td>
<td>-0.0241 (0.0279)</td>
<td>-0.0111 (0.0331)</td>
</tr>
<tr>
<td>Blacks ( (\beta_{\text{blacks}}) )</td>
<td>0.0659 (0.0569)</td>
<td>0.1298 (0.0438)</td>
<td>0.2120 (0.0469)</td>
<td>0.2796 (0.0538)</td>
<td>0.4095 (0.0775)</td>
</tr>
<tr>
<td>Observations</td>
<td>104788</td>
<td>101706</td>
<td>98624</td>
<td>95542</td>
<td>92460</td>
</tr>
<tr>
<td><strong>Panel B: Densely populated areas</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Whites ( (\beta_{\text{whites}}) )</td>
<td>0.0351 (0.0301)</td>
<td>0.0425 (0.0382)</td>
<td>0.0674 (0.0406)</td>
<td>0.0925 (0.0499)</td>
<td>0.1209 (0.0607)</td>
</tr>
<tr>
<td>Blacks ( (\beta_{\text{blacks}}) )</td>
<td>-0.0130 (0.0390)</td>
<td>0.0288 (0.0504)</td>
<td>0.0611 (0.0723)</td>
<td>0.0730 (0.0892)</td>
<td>0.1021 (0.1080)</td>
</tr>
<tr>
<td>Observations</td>
<td>15674</td>
<td>15213</td>
<td>14752</td>
<td>14291</td>
<td>13830</td>
</tr>
</tbody>
</table>

Standard errors in parentheses are clustered at the state level.

* \( p < 0.1 \), ** \( p < 0.05 \), *** \( p < 0.01 \)
constitute the majority. Even though blacks benefit from the minimum wage increase their small population share masks its impact on rents. However, in counties with large black population share, the rent increase is observed.

The positive impact on black minimum wage workers in the full sample implies that the null impact found in Table 1 is unlikely to be driven by elastic housing supply or high migration costs in rural areas. As long as these elements affect both whites and blacks in the similar way, one expects to see no impact on housing rents since they negate the capitalization itself. However, Table 4 indicates the benefit received by blacks are indeed capitalized into housing rents, rejecting that these factors are important. Thus, the null impact in the full sample does imply the null impact on welfare, not the failure of the capitalization mechanism.

Panel B confines the sample to densely populated counties. Here, although the estimates on blacks are imprecise, both whites and blacks seem to have benefited almost equally. This pattern suggests the considerable heterogeneity of local labor markets between urban and rural areas (c.f., Moretti, 2011). For example, this pattern is consistent with the premise that black people face worse labor market conditions and engage in minimum wage jobs in rural areas than in urban areas. It is also consistent that whites are more likely to lose jobs in rural areas than in urban areas. While my analysis does not reveal the mechanism behind the heterogeneity, it underscores the importance of local labor market conditions in assessing the desirability of minimum wages. It also implies that in urban areas, blacks may be harmed by the minimum wage increase because they suffer from the increased housing cost. In urban areas, minimum wage increase attracts whites and housing rents increase. However, without large improvement for blacks’ labor market conditions, this effect might outweigh the benefit for blacks.

In Table 4, positive impacts become more salient as the time span expands. This pattern reinforces the premise that minimum wages have a dynamic impact on the housing market. Even if one focuses on racial subgroups, this dynamic pattern is robustly observed. Such a dynamic pattern also implies that the heterogeneity in the estimated coefficients is driven by the welfare impact of minimum wages, not by other reasons such as the housing expenditure share relative to the WTP \( \lambda_k \). Since \( \lambda^k \) is not affected by the size of the time span, the heterogeneity in \( \lambda^k \) alone does not explain such a dynamic pattern.

In Appendix C.1.4, I show that the results are similar without applying log-transformation and present the estimates of monetary WTPs.

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\[ \lambda^k \]

There may also exist heterogeneity in the housing market. For example, Early, Carrillo, and Olsen (2019) show that black households pay higher rents for identical housing than white ones.
4 Evidence from Japan

I have seen that at least in urban areas of the United States, minimum wage increase has a positive impact on housing rents. I investigate the external validity of this result using Japanese data. As we see later, Japan has exogenously increased minimum wage rates in many urban areas. Thus, if the positive impact on housing rents is observed by exploiting this variation, it could support the positive impact of minimum wages in urban areas. Confirming this pattern in completely different contexts and data reinforces the external validity of the previous findings. Moreover, Japanese institutional setting and features of the housing market provide several unique advantages, as I describe below.

4.1 Data

4.1.1 Housing Rents

I first describe my data and some features of Japanese housing market. I focus on Japanese rental apartments market from October 1, 2002 to September 30, 2013.\footnote{This is because Japanese minimum wage in year $t$ is implemented from around October and is effective until September of year $t + 1$. Thus, I call an observation during October 1 of year $t$ and September 30 of year $t + 1$ as an observation in year $t$. Strictly speaking, there is a slight timing difference in the implementation across prefectures, but I do not consider such difference because minimum wage rates have already been determined on October 1. Moreover, since the data is about posted advertisement, the actual transaction must occur with some time lag. Thus, it is highly likely that advertisers consider the market conditions at the time of the transaction in posting advertisements. In this paper, I avoid unnecessary complications and use minimum wage rate in year $t$ to explain rents during October 1, year $t$ and September 30, year $t + 1$. See Appendix C.2.1 for the rationale of this time period.} Thus, $t = 2002, \ldots, 2012$.

The Japanese rental apartment market is mainly for non-rich people and suitable for detecting the effect of minimum wages on housing markets. Although some units are aimed at families, Japanese rental markets are primarily for singles and couples, because non-poor families with children typically own a house or an apartment unit. Moreover, Japanese market-based housing assistance program is much less extensive than the United States and the supply of public housing is limited, implying that many minimum wage workers have to rely on the market without any subsidies.\footnote{Only the unemployed are applicable to the housing assistance program, called \textit{juutaku kakaho kyu-fu kin}. This program is not very well-known and likely to be underutilized. Moreover, the length of the program is usually three months and cannot be extended for more than nine months.} Thus, the rental apartment market is likely to reflect the true effect of redistributive policies.

The data contains all apartments posted on At Home, which is one of the most popular on-line real estate search engines in Japan. At Home covers all prefectures in Japan and deals with a wide variety of rental apartments. However, since the dataset is not a random sample of all apartments for rent in Japan, the representativeness may not be guaranteed. Although At Home covers various apartments and this problem is unlikely to be very serious, this point should be kept in mind in interpreting the
results. For each apartment unit, the dataset records the posted monthly rent, the prefecture in which the apartment is located, the floor on which it is located, construction year, square footage, and the material with which the apartment is made. Unlike FMRs, utility costs are not included in the rents. The unit of the observation is the posted advertisement and so the rent may not necessarily equal to the transaction price. However, the posted price should play a very important role in determining the transaction price, implying that it is useful in investigating whether minimum wages affect rents. To mitigate the effect of mis-recorded values, I delete certain outliers and eliminate duplicate observations in the raw data to the extent possible. Summary statistics on apartments are provided in Table C.5.

Unfortunately, I do not have data on renters’ information because the observational unit is each apartment advertisement posted on the on-line real estate search engine. Thus, I cannot directly observe which apartment rooms are occupied by minimum wage earners and are more likely to be affected by the minimum wage rates without making further assumptions. However, a unique feature of Japanese apartments is quite helpful in determining which apartments are likely to be rented by minimum wage workers. Specifically, I exploit the fact that Japanese apartments depreciate very fast compared with those of other countries. Here, I discuss the significance of the depreciation and its background.

Generally speaking, Japanese residential structures depreciate very fast. Yoshida (2016) quantitatively finds that residential properties annually depreciate at around 7% in Japan, in contrast to 1% in the United States. There are several possible explanations for the difference, but one of the most important reasons might be the construction material. In Japan, most apartments are made of woods and light-gauge steel. In my data, more than 85% of the apartments in my sample are made of these materials. They are cheap but not very durable. Indeed, they depreciate very fast: legally, they are completely depreciated within 20–40 years, depending on the structure of the buildings. Such buildings are not likely to go through major repairs and renovations, and they are typically demolished when they are sufficiently damaged. The absence of major renovations implies that most unobserved quality of the apartment, such as the quality of the interior, the exterior, infrastructure, fire and earthquake resistance, and sanitation, are quite accurately predicted by the age of the apartment. This feature is helpful in my identification strategy because the observable age of the building is a strong predictor of

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47 It may slightly inflate the estimated elasticity compared to the the case of FMRs as long as utility costs are unrelated with housing demand.
48 See Appendix C.2.1 for the details.
49 In Japan, more durable, high-rise buildings for residence are called “mansions” and they are differentiated from apartments. I focus on apartments throughout this paper.
50 It is partly due to the frequent revision of Japanese construction standards. Since apartments built under old standards are less valued, it leads to frequent destruction. The frequent destruction may also come from people’s strong preference toward newer housing due to changes in lifestyles and other reasons.
51 I exclude apartments made of reinforced-concrete, which are rare except in Okinawa prefecture, because they are more durable than typical apartments in Japan.
the unobserved quality of the apartment unit.\footnote{Note that I do not have information on housing quality in FMRs data. Japanese data allows me to explicitly control for housing quality measured by apartments’ age and other observable characteristics.}

To confirm this point in my data, I conduct a simple regression analysis to predict housing rents.\footnote{The regression equation is 
\[ \ln\left(\text{rent}_{ijt}\right) = \delta\text{age}_{i} + \beta\text{X}_{i} + \text{pref}_{j} + \text{year}_{t} + \epsilon_{ijt}, \]
where \(\text{X}_{i}\) are apartment-level control variables, \(\text{pref}_{j}\) are prefecture fixed effects, and \(\text{year}_{t}\) are time fixed effects. \(\text{X}_{i}\) includes the square footage of apartment \(i\) and its square, which floor the apartment unit is on, and the material of which the apartment is made.}
The result shows that, even controlling for observable characteristics, rent decreases by around 1% as the building becomes one year older. A thirty-year-old apartment room, which is around the 95th percentile of my sample, is approximately 30% cheaper than a new apartment. This result confirms the conjecture that in the Japanese rental housing market, the age of the apartments is a very strong predictor for the housing quality.

Given the large price difference by apartments’ age, it is highly likely that minimum wage workers who are affected by the minimum wage increase sort into old apartments while non-minimum wage workers sort into new apartments. Housing quality, which includes cleanliness, safety, and the appearance of the building, are generally deemed as normal goods. Given the significant price difference, such a sorting mechanism seems to be working strongly. In the empirical analysis, after controlling for observable characteristics, I use apartment age as a proxy for the overall housing quality and regard it as the intensity of the treatment in testing the effect of the minimum wage increase. Then, I allow for a heterogeneous impact of minimum wages on new apartments and old apartments.

4.1.2 Minimum Wages in Japan

In Japan, there are forty-seven prefectures, and each of them has a different minimum wage rate. There is no difference in the minimum wage rate within a prefecture.\footnote{Strictly speaking, different minimum wage rates are applied for some specified types of jobs. In this paper, I focus on the effect of the generally applied minimum wage rates.}

I first explain how minimum wage rates are determined in Japan and then discuss how it is useful in identification by significantly mitigating policy endogeneity.\footnote{See Kambayashi, Kawaguchi, and Yamada (2013), Aoyagi, Genelli, and Tawk (2016), and Hara (2017) for more details about the centralized nature of minimum wage settings in Japan.} In particular, one may expect that a local government is likely to raise its minimum wage rate when the housing cost is increasing to support the poor, which is likely to upwardly bias the effect of the minimum wage increase on apartment rents. Such policy endogeneity was not likely to be the case in Japan during the relevant period.

The Japanese minimum wage setting is highly centralized and unresponsive to trends in local housing markets. Japanese prefectural minimum wages are determined by the following process. First, the
Figure 1: **Japanese Prefectural Minimum Wage Rates: 2002–2012.**

*Notes:* The figure shows the trends of nominal minimum wage rates for each of 47 Japanese prefectures during 2002-2012.

The central government classifies prefectures into four categories, and it assigns the targeted amount of minimum wage increase to each category. The categorization is reviewed only once in five years and changes in the classification are rare, implying that the classification is not sensitive to prefectural trends. Given the targeted rate, local governments can make fine-adjustments to the minimum wage increase. A concern for the endogeneity is that local governments may take into account trends in housing markets. However, the targeted rate explains almost all variations of the actual minimum wage increase, and local economic factors, such as local shocks to housing market, are not well-considered (Tamada, 2011). In this sense, although there is local variation of minimum wage rates across prefectures, the Japanese minimum wage setting is similar to a centralized system and almost unaffected by the trends of local housing markets.

Moreover, from 2007 to 2012, a new consideration took the primary role in setting minimum wages due to the national policy change, which further reduces the concern about the endogeneity. As explained in detail in Kawaguchi and Mori (2013) and Hara (2017), during the period, the primary consideration in setting the minimum wage rate became closing the gap between the quality of life of minimum wage workers and people relying on public assistance (*seikatsu-hogo*). As a result, since 2007, prefectures with large earnings gaps between minimum wage workers and public assistance recipients experienced larger increase in minimum wage rates to ensure that minimum wage workers
Figure 2: Relative increase of prefectural minimum wage.

Notes: The map shows the minimum wage rates in 2012 divided by those in 2002. A prefecture is colored if it experiences more than a 9% percentage-points or larger relative minimum wage increase than the prefecture with least relative minimum wage increase. The map is created by the command ‘mptile’ on Stata. I thank Chigusa Okamoto for sharing the code to plot the Japanese prefectural map.
are wealthier than those receiving public assistance. Since the gap was generally large in urban areas, the policy results in a plausibly exogenous minimum wage increase in urban prefectures. Such a policy is likely to weaken the connection in policymaking between the minimum wage increase and the trend in housing markets, which in turn mitigates the endogeneity problem. Figure 1 shows the evolution of minimum wage rates by prefectures. Minimum wage rates were kept almost constant before 2007, but they started increasing since 2007 due to the policy change. The heterogeneous speed of the minimum wage increase across prefectures creates large variations in minimum wage rates.

In Figure 2, I show the ratio of the minimum wage rate in 2002 to that of 2012 by prefectures. The colored areas are mainly urban areas, including the three largest urban employment areas (UEAs), namely Tokyo, Osaka, and Nagoya. However, since they are geographically distant UEAs, each of them faces significantly different economic situations and has an independent housing market, which again mitigates the concern for policy endogeneity.

Note that Japanese minimum wages were increased only gradually. Since I do not use a panel data in Japanese analysis, I cannot use the same panel data method of Meer and West (2016) in uncovering dynamic effects. However, the slow minimum wage increase implies that simply regressing housing rents on minimum wage levels resembles the final impact since significant time has already passed to reach the current minimum wage level. Thus, as seen in the next subsection, I do not use differencing techniques in this context.

### 4.2 Empirical Strategies

I combine several empirical specifications to test the effect of minimum wage hikes on apartment rents. The first specification is the following simple DD regression:

\[
\ln(rent)_{ijt} = \alpha + \beta_0X_i + \beta_1pref_j + \beta_2year_t + \beta_3Y_{jt} + \delta_1\ln(MW)_{jt} + \epsilon_{ijt},
\]

where \(i\) is each apartment observed in prefecture \(j\) in year \(t\). \(X_i\) is the vector of characteristics of each apartment unit. \(Y_{jt}\) is the prefecture-year specific covariates such as the average wage of male

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56 Kawaguchi and Mori (2013) confirm that the policy change is likely to be an exogenous variation in analyzing the effect on unemployment.

57 Since minimum wages are increased steadily over time, controlling for state-specific linear time trends, which is often done in state-panel DD analysis, is likely to mask the effect of minimum wages. In the main analysis, I choose not to control for parametric state-specific trends. However, linear trends may alleviate problems about the presence of trends and the simultaneous minimum wage increase. I report the results with such linear trends in Appendix C.2.3. Overall, I find larger positive impacts on housing rents.

58 Since the minimum wage rate in year \(t\) is effective from October of year \(t\) to September of year \(t + 1\), I refer to observations during this period as those in year \(t\).

59 \(X_i\) includes the apartment’s age and its square, the square footage of the apartment \(i\) and its square, the floor the apartment unit is located on, and the material with which the apartment is made.
or female workers.\textsuperscript{60} $MW_{jt}$ denotes the minimum wage rate of prefecture $j$ in year $t$. The parameter of interest is $\delta_1$, which is the elasticity of the housing rents with respect to the minimum wage rate.

I have argued that minimum wage hikes are likely to have a larger impact on lower-quality housing, but (16) assumes the homogeneous effect among all apartments. To allow for the heterogeneity, I estimate the following equation:

$$
\ln(\text{rent})_{ijt} = \alpha + \beta_0 X_i + \beta_1 \text{pref}_j + \beta_2 \text{year}_{it} + \beta_3 \text{class}_i \times \text{pref}_j \\
+ \beta_4 \text{class}_i \times \text{year}_{it} + \beta_5 Y_{jt} + \delta_1 \ln(MW)_{jt} + \delta_2 \ln(MW)_{jt} \times age_i + \epsilon_{ijt}.
$$

(17)

As discussed, $age_i$ can be regarded as the intensity of treatment. Hence, I include $age_i$ and the interaction of it with the minimum wage rate.\textsuperscript{61} Variables $\delta_1$ and $\delta_2$ are parameters of interest. I call $\delta_1$ the “primary effect,” which indicates the effect for brand-new apartments. On the other hand, I refer to $\delta_2$ as the “differential effect,” meaning the heterogeneity of the effect of depending on apartment age. The total impact on an $x$-year-old apartment’s rent in response to 1% minimum wage increase is $\delta_1 + \delta_2 \times x\%$. The signs of $\delta_1$ and $\delta_2$ are theoretically ambiguous, as discussed in the previous section.

The equation (17) controls for prefecture-specific and year-specific effects of apartments’ age to reduce concerns about endogeneity.\textsuperscript{62} To control for the interaction effects of apartments’ age without imposing functional-form assumptions, I define an indicator variable $\text{class}_i$ that can take three values, namely old, medium, and new. I classify apartment $i$ with $0 \leq age_i \leq 11$ as new, $12 \leq age_i \leq 24$ as medium, $age_i \geq 25$ as old.\textsuperscript{63} However, the implications are similar even if I replace the variable $\text{class}_i$ with $age_i$.

I also tried a specification controlling for (the log of) the male and female average wage rates in prefecture $j$ at time $t$ in $Y_{jt}$.\textsuperscript{64} While controlling for the average wages allows me to better incorporate the relative significance of minimum wage rates and local economic conditions, it may introduce the issue of “over-control” because average wages are also affected by minimum wages to some extent. This issue is more serious for the inclusion of the female average wage because women are more likely

\textsuperscript{60}In this paper, however, I do not report results in specification (16) with controls $Y_{jt}$. The inclusion of the average wages leads to similar results.

\textsuperscript{61}Note that $X_i$ includes $age_i$ and $age_i^2$.

\textsuperscript{62}In particular, the term $\text{class} \times \text{pref}$ is important because it mitigates the concern that old apartments in some prefectures may be systematically expensive or cheap.

\textsuperscript{63}These classifications are chosen given that wooden apartments and light-gauge steel framed apartments, which are the most common type of apartments in my sample, completely depreciate in around 25 years. Other classifications also lead to similar results.

\textsuperscript{64}The data are taken from Basic Survey on Wage Structure, which reports the average total earnings and working hours in every June. I calculate the average wage rate from these variables and use year $t$’s information to explain observations from October of year $t$ to September of year $t+1$. The specification in (17) is more flexible than regressing the log of the Kaitz index on housing rents, which Card and Krueger (1995) argue is more convincing.
to be minimum wage workers. Thus, I report the results without controls, controlling for only the male average wage, and controlling for both male and female average wages.

Note that the regression equation (17) omits prefecture-year dummies. The omission is because I am interested in identifying the primary effect $\delta_1$. If I include prefecture-year dummies, they absorb the effect of $\ln(MW)$, and the primary effect cannot be identified. In the sense that I utilize prefecture-year level variations, the regression is based on a DD identification strategy.

Including the dummies renders (17) a DDD-type regression because it relies on comparing low-quality apartments and high-quality apartments within a given prefecture and a year. To better identify the differential effect, I also implement such a DDD-type regression. The regression equation follows:

$$\ln(\text{rent})_{ijt} = \alpha + \beta_0X_i + \beta_1\text{pref}_j + \beta_2\text{year}_t + \beta_3\text{class}_i \times \text{pref}_j$$

$$+ \beta_4\text{class}_i \times \text{year}_t + \beta_5\text{pref}_j \times \text{year}_t + \delta_2\ln(MW)_{jt} \times \text{age}_t + \epsilon_{ijt},$$

(18)

where $i$ is each apartment observed in prefecture $j$ in year $t$, and $X_i$ are the characteristics of each apartment unit. Note that $Y_{jt}$ is included in the prefecture-year dummies.

The parameter of interest in (18) is $\delta_2$, which is the differential effect between low-quality and high-quality apartments. The sign and the magnitude of $\delta_2$ is informative about the heterogeneity in the treatment effect. Unless the primary effect is assumed to be zero, this effect cannot be identified with the total effect. Still, the estimation of (18) is expected to better identify the differential effect because prefecture-year dummies control for local unobserved factors in a very flexible manner.

Estimations are done by OLS. In every specification, I calculate the standard errors by clustering the error at the prefecture level since minimum wage, the policy of interest, varies at this level (Bertrand, Duflo, and Mullainathan, 2004).

4.3 Estimation Results

The regression results are reported in Table 5. In column 1, I report the result from the simple DD specification (16). It shows a statistically significant result that the rent increases by 2.6% in response to 10% minimum wage increase. Thus, even when I do not include the heterogeneity of the treatment effect, the specification (16) detects the positive impact on rents.

To further investigate the heterogeneity of the impact, I estimate the specification (17). In column 2, I estimate (17) without controlling for prefecture-year level controls $Y_{jt}$. The primary effect $\delta_1$ is around 0.16, implying that a 10% minimum wage hike reduces the rent of new apartments by 1.6%. The estimate is somewhat imprecise and indistinguishable from zero. The differential effect $\delta_2$, on the

---

65The number of prefectures in Japan is forty-seven, which is within the “safe zone” in using clustered standard errors (Angrist and Pischke, 2009).
other hand, is positive and significantly different from zero at the 10% level. The coefficient is 0.0072, implying that $x$-years-old apartments experience a 0.0072$x$% larger increase than the new apartments in response to 10% minimum wage increase.

I report the marginal effect of minimum wage hikes on old apartments (25 years old) and very old apartments (35 years old). An old apartment experiences a rent increase of around 3.3% when the minimum wage increases by 10%, which is statistically significant at the 1% level. A very old apartment experiences an increase of around 4%, which is also significant at 1% level. Overall, the result reveals the larger impact on the rents of low-quality apartments.

In column 3 and 4, I add the average wage rates. Including them in the regression equation leads to better controls of the relative significance of minimum wages and local economic conditions while potentially introducing over-controlling problems. In column 3, I estimate (17) while controlling for the log of the male average income. I further add the log of the female average income in column 4. In both cases, the results are consistent with column 1. Moreover, as hypothesized, minimum wage hikes have larger effect on lower-quality apartments. Taking both effects into consideration, old apartments experience around 3.7% increase in rents, and very old apartments experience an increase of around 4.4%.

To estimate the heterogeneity of the impact while more flexibly control for local confounding factors, I estimate (18), which controls for prefecture-year dummies. Note that this strategy does not allow me to estimate the primary effect. The result in column 5 of Table 5 suggests that the differential

---

### Table 5: The Effect of Minimum Wage on Apartment Rents

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(MW)</td>
<td>0.2624***</td>
<td>0.1555</td>
<td>0.1878*</td>
<td>0.1909*</td>
<td>0.1990*</td>
</tr>
<tr>
<td>($\delta_1$)</td>
<td>(0.0970)</td>
<td>(0.1144)</td>
<td>(0.1094)</td>
<td>(0.1095)</td>
<td></td>
</tr>
<tr>
<td>ln(MW) × age</td>
<td>0.0072*</td>
<td>0.0072*</td>
<td>0.0072*</td>
<td>0.0088**</td>
<td></td>
</tr>
<tr>
<td>($\delta_2$)</td>
<td>(0.0038)</td>
<td>(0.0038)</td>
<td>(0.0038)</td>
<td>(0.0038)</td>
<td></td>
</tr>
<tr>
<td>Marginal effect on old apartments</td>
<td>0.3343***</td>
<td>0.3680***</td>
<td>0.3705***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>($\delta_1 + 25\delta_2$)</td>
<td>(0.09686)</td>
<td>(0.09146)</td>
<td>(0.09265)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Marginal effect on very old apartments</td>
<td>0.4058***</td>
<td>0.4401***</td>
<td>0.4424***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>($\delta_1 + 35\delta_2$)</td>
<td>(0.1138)</td>
<td>(0.1093)</td>
<td>(0.1106)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prefecture-year level controls</td>
<td>No</td>
<td>No</td>
<td>M average wage</td>
<td>M&amp;F average wages</td>
<td>Dummies</td>
</tr>
<tr>
<td>Observations</td>
<td>5696368</td>
<td>5696368</td>
<td>5696368</td>
<td>5696368</td>
<td>5696368</td>
</tr>
</tbody>
</table>

Standard errors in parentheses are clustered at the prefecture level.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$
effect increases slightly: it is now 0.0088 instead of 0.0072 in specification (17). Overall, controlling for prefecture-year dummies does not substantially alter the results, which is reassuring because the significance of confounding factors seems limited.

All of my specifications suggest that rents of low-quality apartments are increased in response to minimum wage hikes. Given that the minimum wage increase is concentrated in urban areas, I conclude that the positive impact on housing rents in urban areas is also confirmed in Japan as well. From Proposition 1 and 2, the positive impact on rents implies the positive welfare impact for some minimum wage workers. By the same back-of-the-envelope calculation as the case of the United States, again assuming that housing rents consume around 33% of earnings, around 2.5%-5% of the increase in rents imply that around 7.5%-15% of the additional earnings flows into homeowners’ pocket. This number is larger than the United States, but both indicate the arguably small but non-negligible incidence for homeowners. One should note, however, that the different magnitude of the elasticity in the United States and Japan may not necessary come from cross-country difference, but from the different source the data comes from. For example, if Japanese data is assembled in a similar way to the FMR, the results may more closely resemble those in the United States.

4.4 Placebo Tests

I conduct placebo tests to determine whether my results in Table 5 can be interpreted as describing causal relationships. The most important concern for DD or DDD identification strategies is that the effect of minimum wages may erroneously reflect prefecture-specific trends rather than the true effect of minimum wages. In particular, if prefectures experiencing rapid minimum wage hikes have a particular type of the trend, the estimation results in Table 5 are likely to be affected by it.

In conducting the placebo tests, I exploit the fact that during 2002–2006, minimum wage rates were kept almost constant (see Figure 1). I assume that minimum wage rates were increased in this period just as they were increased during 2008–2012, during which minimum wages were significantly increased due to the policy change. Then, I repeat the same analysis as in the previous subsection, replacing the actual minimum wage rates with hypothetical ones and limiting the sample to 2002–2006. The estimated coefficient in this placebo test is expected to pick up the trends of prefectures with large minimum wage hikes, but not the effect of minimum wages since they were almost constant. Thus, if prefectures experiencing rapid minimum wage hikes have a particular pre-trend, it would be detected

67The sample size in this period is relatively small because on-line real estate search engines were less popular during this period. As a result, some prefectures do not have many observations. The results are similar, however, if I drop samples from prefectures with few observations.

68More formally, for \( t \in \{2002, 2003, 2004, 2005, 2006\} \), I replace \( \ln(MW)_t \) with \( \ln(MW)_{t+6} \).
by this placebo test.

The results presented in Table 6 confirm that my results are unlikely to be driven by the pre-trends. In column 1, I estimate (16) using the fictitious minimum wage increase without controlling for $Y_{jt}$. The estimated marginal effect is reassuringly non-positive, and more strikingly, it is significantly negative at the 5% level. The negative trend is also observed in the specification (17) reported in columns 2-4, with some statistically significant negative coefficients. It suggests that my results are not driven by the pre-trend, and it might be somewhat larger if one takes the negative trend into account.

Although the main conclusion that the increase in rents of low-quality apartments is unlikely to be driven by pre-trends, from a conservative perspective, the placebo tests imply that the composition of the marginal effects might be affected by the presence of pre-trends. First, the primary effects in columns 2-4 of Table 6 are lower than those in Table 5. Although they are not statistically significant at the conventional level, it suggests the possibility that the primary effect may actually be larger than the estimate of Table 5 due to the presence of the negative pre-trend. Second, the differential effect in column 2-5 is close to those estimated in Table 5. The estimates are quite imprecise, and I do not view this result as convincing evidence that the differential effect is zero. Still, it might raise a concern that the differential effect is actually closer to zero than estimated in Table 5. Taken together, due to the negative pre-trends of the primary effect, new (high-quality) apartments may also have experienced an increase in rents, in contrast to the estimates in Table 5.69

In sum, the placebo tests generally reinforce my main finding of the positive impact on the rents of

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69Given that the apartments in my sample are not very expensive, it may be possible that the sorting mechanism is incomplete and minimum wages have an effect on high-quality apartments. See Section 2.1 for more discussion as to how high-quality apartments can be affected by minimum wages.
low-quality apartments. Given that the placebo test also suggests that high-quality apartments might also have experienced some increase in rents, pre-trends tend to reinforce the positive impact on apartment rents.

4.5 Additional Results and Further Robustness Checks

In Appendix B, I investigate the impact of minimum wage increase on housing supply and migration, as these outcome variables are suggested to be important by the theoretical framework developed in Section 2. In sum, I find positive impact on housing supply but somewhat complex effects on migration.

In addition to the placebo test in Section 4.4, as a robustness check, I re-estimate my DD specifications with controlling for linear time trends in Appendix C.2.3. Although allowing for parametric trends is becoming more and more popular in minimum wage studies (e.g., Cadena, 2014), I do not include them in my main analysis because Japanese minimum wage increase was gradual. In such a case, time trends are likely to erroneously capture the effect of minimum wages and the interpretation of the results are difficult. Still, it might reduce the concern about pre-trends while utilizing the full-sample. I show that by controlling for the linear trend, the positive impact of the minimum wage hikes on rents is magnified, which is consistent with the result of the negative pre-trend in Section 4.4.

5 Concluding Remarks

I contribute to the debate over the effectiveness of minimum wage as a redistributive policy from a new perspective: its effect on housing rents. It is valuable mainly for two reasons. First, since housing costs account for a large share of budgets, minimum wage workers are significantly harmed if housing costs increase in response to minimum wage hikes. Second, changes in housing rents are informative about whether minimum wage hikes improve workers’ utility. I show this point by using a spatial equilibrium model, allowing for minimum wage workers’ heterogeneity.

Based on the theoretical framework, I empirically investigate whether and how minimum wages affect housing rents in the United States and in Japan. In the United States, 10% increase in minimum wage induces around a 1-2% increase in urban areas. The corresponding figure is 2.5%-5% in Japanese apartments for rent. These results imply that at least in urban areas, minimum wage hikes improve workers’ welfare even when unemployment and other behavioral responses to minimum wages such as labor-leisure choice are incorporated. The positive impacts on housing rents appear gradually, suggesting the dynamic impacts of housing rents are important not only in the labor market (e.g., Dube, Lester, and Reich, 2016; Meer and West, 2016) but also in the housing market. I also emphasize the
potential importance of heterogeneity in minimum wage workers and local labor markets, both from theoretical and empirical perspectives.

The positive impact on housing rents also imply that some benefits of minimum wage hikes unintentionally go to homeowners. Back-of-the-envelope calculations suggest that without any job destruction and other behavioral responses, 3%-6% (7.5%-15%) of the benefits go to homeowners in the United States (Japan). The figure is arguably moderate but non-negligible. Moreover, the ratio of such unintentional incidence might be larger if minimum wages harm workers through other channels, such as increased unemployment. Such an unintended consequence might be avoided if minimum wages are increased in all local jurisdictions at the same time. As long as it keeps the spatial equilibrium condition unaltered, the housing rent increase would not occur. However, the heterogeneity in the local labor market requires different minimum wages across regions. This seems to be an important trade-off in choosing between national-level or local-level minimum wages.

My analysis is the first step in relating housing rents and minimum wages, and it can be extended in various ways. Perhaps the most fruitful way is to investigate the effect of minimum wages on housing rents in other contexts in addition to the United States and Japan. Another important issue is the role of heterogeneity among minimum wage workers, to which I present a canonical methodology in Section 3.3. Refining the approach and investigating the heterogeneity in other dimensions are important: Especially, heterogeneity of local housing markets may be important for quantitative implications (Saiz, 2010; Glaeser and Gyourko, 2018). Since my approach is agnostic about the cause of heterogeneity among different minimum wage workers, it is also interesting to analyze the mechanism behind the cause of heterogeneous impacts, such as the structure of local labor markets (Moretti, 2011) from the viewpoint of minimum wages. These issues remain for future research.

References


Appendix (Not for Publication)

A Theoretical Extensions

In this Appendix, I extend the theoretical framework in Section 2 in several ways. I also discuss how such extensions might modify the interpretation of the empirical results.

A.1 Housing Quality

Throughout Section 2, I have assumed that housing quality is homogeneous. This assumption can be relaxed in several ways. For simplicity, I focus on the case of homogeneous workers and the fixed housing supply ($H' = 0$).

First, assume that each housing unit is endowed with the vector $y$. $y$ may include both observed (e.g., square footage, age) and unobserved (e.g., the attractiveness of appearance) factors related with housing quality. Accordingly, the utility function now depends on $y$: $U(x, y, l)$. Assuming that the hedonic approach can be applied, let $\xi$ be the price vector corresponding to each component of $y$. The budget constraint of each consumer is rewritten as $\psi x_i + R_i + \xi y_i = w_i T + M$ if employed and $\psi x_i + R_i + \xi y_i = M$ if unemployed. The indirect utility functions are also written as $V^e(w_i; \psi, \xi, I)$ and $V^u(\psi, \xi, I)$.

If the cost $\xi y_i$ is directly paid by workers, the model is the same as the case in which $y$ is included in $x$. However, at least part of the cost $\xi y_i$ is usually included in rents. If $\xi y_i$ is included in rents, the observed housing rents is now rewritten as $R_{tot}^i = R_i + \xi y_i$. Since the change in the minimum wage rate $w_i$ affects $\xi y_i$, $\partial R_{tot}^i / \partial w_i \neq \partial R_i / \partial w_i$ in general.

To see how my result should be modified, consider first the simplest case in which housing quality $y$ is supplied by the perfectly competitive sector with cost function $c(y)$, which I assume to be linear in each element to ensure that the hedonic approach is valid. Perfect competition implies that $\xi$ is equal to the marginal cost of providing each component of $y$. Since all workers are identical ex ante, all of the employed workers choose $y^*_e$ and the unemployed chooses $y^*_u$. Note that $\partial y^*_e / \partial w_i = 0$ because the minimum wage rate does not matter for the unemployed people’s choice, while $\partial y^*_u / \partial w_i \neq 0$ in general. Under these assumptions, the same derivation of Proposition 1 shows that $\partial R_i / \partial w_i$ reveals the willingness to pay. However, $R_i$ is not directly observable in data, and the question becomes whether and how the observed rents are informative about the desirability of minimum wages.

\footnote{Note that the housing quality $y$ is not a stock variable and adjusted smoothly, which may be plausible if $y$ mainly consists of the maintenance quality. However, the implications are similar as long as the hedonic approach is approximately valid.}
Now that the employed and the unemployed choose the different housing quality, there are two types of housing in the market, that is, that of high quality and low quality. The observed rent of high-quality housing is \( R_i + \xi y_e^* \) while that of the low-quality one is \( R_i + \xi y_u^* \). The fraction of the high-quality housing equals the number of the employed \( p \). The rest housing units \( 1 - p \) are of low-quality.

Let \( R_{hi}^* = R_i + \xi y_e^* \) and \( R_{lo}^* = R_i + \xi y_u^* \) be the observed rent of the high-quality and low-quality apartments, respectively.

In this context, it seems natural to assume \( \xi y_e^* > \xi y_u^* \) so that the employed spend more on housing quality.\(^{71}\) Moreover, it is also natural to suppose \( \xi \partial y_e^*/\partial w_i > 0 \) because workers with higher wage are likely to enjoy better housing. Under these assumptions, I obtain the following proposition

**Proposition 3.** Suppose that housing units are vertically differentiated. Then, compared with the case of no differentiation, high-quality housing experiences the increase in rents in response to minimum wage hikes (i.e., \( \partial R_{hi}^*/\partial w_i > \partial R_i/\partial w_i \)). While the response of the observed rents low-quality housing is the same as before (i.e., \( \partial R_{lo}^*/\partial w_i = \partial R_i/\partial w_i \)), the share of low-quality housing increases if and only if \( p' < 0 \), implying that some housing units experience the discontinuous drop in the observed rents.

When minimum wage are marginally increased, two forces are at work in Proposition 3. First, it raises the wage of the employed workers. As long as the housing quality is increasing in the wage rate, it increases the observed rent of high-quality housing because workers demand better housing quality. Second, some people may get unemployed due to the job destruction effect of minimum wages. They start demanding housing units of lower quality. As a result, the quality of some housing units becomes lower, leading to the discontinuous decrease in the observed housing rents.

Proposition 3 shows that the qualitative welfare implications might become ambiguous when apartments are vertically differentiated. \( \partial R_{hi}^*/\partial w_i > \partial R_i/\partial w_i \) implies that the observed marginal rent increase may overstate the true welfare improvement of the employed workers because it also includes the marginal increase of spending on housing. However, the envelope theorem implies that such re-scheduling of consumption has no first-order effect on welfare. On the other hand, the unemployment leads to the large drop in the observed rents of some apartments. However, workers expect the welfare impact of unemployment and the impact of \( p' \) is captured in \( R_i \). Thus, the quality adjustment makes the observed rents underestimate the true benefit. These two effects act in the opposite direction and the welfare implication of the observed rents is not clear-cut as before.\(^{72}\)

\(^{71}\)This situation naturally occurs when the marginal utility of private consumption is high for the unemployed. Note that the restriction is only on the total amount of spending and I do not require that the employed enjoy the better housing quality in every dimension.

\(^{72}\)These mechanisms might serve as an explanation for the potential rents increase of high-quality apartments discussed in Section 4.4. If there is some unobserved quality of housing, the rents of high-quality apartments may rise in response to minimum wage hikes.
This analysis underscores the importance of controlling for the observed housing quality. Indeed, if \( y \) consists only of the observed factors, \( R \) can be, in principle, identified. However, in practice, observing all elements of \( y \) might be infeasible. In such a case, some bias may remain about the welfare implication judged from the rents data. This point emphasizes the merit of using Japanese data, as many unobserved characteristics are approximated by the apartments’ age.

The discussion in this section is also illuminating in understanding the sorting mechanism. I have argued in the Japanese context that old apartments are more likely to be occupied by minimum wage workers, and so likely to be affected more intensively. To see this point, suppose there is some workers who earn more than minimum wage (\( w > w_i \)). Then, since the wage is higher, this type consumes better housing units. As a result, the sorting equilibrium in which non-minimum wage workers reside in better apartments realizes. As long as \( y \) includes apartments’ age, the sorting according to the apartments’ age is likely to emerge.

### A.2 Local Commodity Prices

In the main analysis, the commodity price are determined at the national level and local minimum wages have no effect on it. Although it is plausible in some sense, minimum wages may affect the price of local goods through increasing the supply cost.

In this Section, I explore the case where minimum wages affect the local price of commodities. Let \( x_i \) be the consumption vector of local goods, and \( \psi_i(w_i) \) be the associated price vector. Then, (6) is modified as follows:

\[
\frac{\partial R_i}{\partial w_i} = p'(V^e - V'^u) + p\frac{\partial V^e}{\partial l_i} + \left( p\frac{\partial V^e}{\partial \psi_i} + (1 - p)\frac{\partial V'^u}{\partial \psi_i} \right) \frac{\partial \psi_i}{\partial x_i} \geq 0.
\]

Since (19) is about the nominal rent, it can be positive without welfare improvement if minimum wage hikes cause inflation. This can be seen most clearly if \( p = 1 \) for all \( w_i \), \( H' = 0 \), \( M = 0 \), and the derivative of \( \psi_i \) with respect to \( w_i \) is 1. Suppose that \( w_i \) increases by \( \varepsilon > 0 \%. \) In this case, the Roy’s identity implies that the numerator of (19) is \( \varepsilon \frac{\partial V^e}{\partial l_i}(w_i(T - l_i) - \psi_i x_i) \), and \( \frac{\partial R_i}{\partial w_i} = \varepsilon w_i(T - l_i) - \varepsilon \psi_i x_i > 0 \). However, it implies that the revenue and the expenditure increases by the same amount in response to the minimum wage hike without any behavioral adjustment, and the utility remains constant.

The response of the nominal rents is informative especially when one is interested in the incidence of the minimum wage increase. However, if prices adjust locally, the welfare implications from the nominal rents are contaminated by the inflation. If it is the effect of interest, rents should be converted into real terms. The simplest solution is to divide the rents and the minimum wage rates by the local
price index (excluding housing). It uncovers the effect of rents unmediated by the inflation of other commodity goods.\(^7\)

It is instructive to compare housing rents and migration responses as a welfare measure in the presence of local commodity inflation. As long as people make their migration decisions based only on real values, the migration responses yield qualitatively correct implications. Indeed, in the above example, the nominal rent goes up, but people do not move since the utility is kept constant. However, it assumes that migration decisions are made with full access to the information on local commodity prices, which seems a little too strong. Moreover, as I discuss in the main text, migration responses may exhibit complex patterns if workers are heterogeneous. Overall, I conclude that both measures are complementary.

Recently, it is argued that the effect of prices on commodity prices may be heterogeneous across industries (e.g., MaCurdy, 2015; Harasztosi and Lindner, 2018). It complicates the adjustment of the inflation because it may affect consumers’ welfare through distorting the relative price. Inappropriate adjustment for inflation might mask such effects capitalized into apartment rents. On the other hand, the inflation should be adjusted to obtain real welfare implications. Although some measures can be done, completely disentangling these two effects of inflation seems a very difficult task.

### A.3 Generalizations of the Heterogeneous Case

I introduce the elastic housing supply as well as the general utility function in the heterogeneous case. I drop the assumption of the quasi-linear utility function and use the general utility functions \(u^k(x, l)\) for all \(k\). For each \(k\), the utility maximization problem leads to the indirect utility functions \(V^{ek}(w_i, \psi_i, l)\) if employed and \(V^{uk}(\psi_i, l)\) if unemployed. The slope of the housing supply function \(H_i\) is also allowed to be positive and heterogeneous across regions. Finally the amount of the endowment \(M_k\) is also heterogeneous.

Assuming that \(\theta\) enters the utility in an additive way, the expected utility of type \(k\) is written as

\[
p^k_i V^{ek}(w_i, \psi_i, M^k + w_i T - R_i) + (1 - p^k_i) V^{uk}(\psi_i, M^k - R_i) + \theta^k_i.
\]

I define \(EU^k_i \equiv p^k_i V^{ek}(w_i, \psi_i, M^k + w_i T - R_i) + (1 - p^k_i) V^{uk}(\psi_i, M^k - R_i)\), which is the expected utility of residing in region \(i\). Proposition 2 can be generalized in the following way:

**Proposition 4.** In response to marginal a minimum wage increase \(w_i\), the changes in rents \(\partial R_i/\partial w_i\) can be written as

\[
\sum_k (\omega^k_i \partial EU^k_i / \partial w_i), \quad \text{where} \quad \omega^k_i \equiv g^k_i / (H'_i - \sum_k (g^k_i \partial EU^k_i / \partial R_i)).
\]

However, the local inflation lowers the real value of the endowment \(M\). To make sure that such an adjustment more accurately, the endowment should be compensated for the depreciation of the real value. It is, however, difficult to implement empirically. In practice, minimum wage workers do not own the substantial amount of the endowment and this effect might be of little concern.
Proof. For each $k$, The migration condition is rewritten as

$$EU^k_i + \theta^k_i = u^k,$$  \hspace{1cm} (21)

where $u^k$ is the outside utility for type $k$, which is assumed to be exogenous. The marginal resident has the attachment level $\theta^k_i$.

The housing market equilibrium condition is

$$N_i = H(R_i).$$  \hspace{1cm} (22)

(21) and (22) define the equilibrium.

In the absence of changes in rents in response to a marginal minimum wage increase, (21) shows that $\partial \theta^k_i / \partial w_i = -\partial EU^k_i / \partial w_i$, which translates into the population change $g^k_i \partial EU^k_i / \partial w_i$ of type $k$ workers. On the other hand, $\partial \theta^k_i / \partial R_i = -\partial EU^k_i / \partial R_i$, implying that the population change of type $k$ in response to the marginal increase of rents is $g^k_i \partial EU^k_i / \partial R_i$. Note that the population level $N_i \equiv \sum_k \left( \lim_{\theta \to \infty} G^k(\theta) - G^k(\theta^k) \right)$

The differentiation of (22) with respect to $w_i$ yields

$$\frac{\partial R_i}{\partial w_i} \left( \sum_k (g^k_i \partial EU^k_i / \partial R_i) \right) + \sum_k (g^k_i \partial EU^k_i / \partial w_i) = \left( \frac{\partial R_i}{\partial w_i} \right) H',$$  \hspace{1cm} (23)

which can be rewritten as

$$\frac{\partial R_i}{\partial w_i} = \sum_k (\omega^k_i \partial EU^k_i / \partial w_i),$$  \hspace{1cm} (24)

where $\omega^k_i \equiv g^k_i / (H' - \sum_k (g^k_i \partial EU^k_i / \partial R_i))$. \hfill \blacksquare

Proposition 4 shows that the gradient of the rent function can be expressed in the similar way as Proposition 2, but the weight becomes more complex and in general, no longer sums up to one.\footnote{Proposition 2 is the special case where $H' = 0$ and $\partial EU^k_i / \partial R_i = -1$.} To ease the interpretation, suppose first that the housing supply is inelastic ($H'_i = 0$). The weights $\omega^k_i$ is now $g^k_i / \sum_k (-g^k_i \partial EU^k_i / \partial R_i)$. The denominator is now a weighted sum of the marginal utility of the private consumption.

The first important implication is that, as long as housing supply is inelastic, the minimum wage hike must create the winner and the loser just as in Proposition 2. More specifically, the group $k$ with the lowest $\frac{\partial EU^k_i / \partial w_i}{-\partial EU^k_i / \partial R_i}$ must be damaged while that with the highest one must benefit.\footnote{It can be seen by investigating the sign of

$$-\frac{\partial EU^k_i / \partial w_i}{-\partial EU^k_i / \partial R_i} - \frac{\sum (g^k_i \partial EU^k_i / \partial w_i)}{\sum (g^k_i \partial EU^k_i / \partial R_i)}. \hspace{1cm} (25)$$}
The next natural question is when a particular group is likely to be a winner or a loser of minimum wage hikes. To understand this point, suppose there are two groups: the rich group and the poor group. Other things being equal, if the numerator is positive, $\frac{\partial EU_k^i}{\partial w_i} / \partial R_i$, large when the denominator is small, i.e., the marginal utility from private consumption is small. As the inequality expands, $-\partial EU_k^i / \partial R_i$ becomes much smaller for the rich group. In such a case, the poor group is likely to have the smaller $\frac{\partial EU_k^i}{\partial w_i} - \frac{\partial EU_k^i}{\partial R_i}$, and get harmed by the minimum wage increase. On the other hand, if $\partial EU_k^i / \partial w_i$ is negative, the larger inequality makes the poor group more likely to benefit. The argument is summarized in the following Corollary:

**Corollary 2.** If the housing supply is inelastic, some group must experience the welfare decrease while others experience the increase. Suppose the case of the two groups: the rich group and the poor group. Then, as the inequality (measured by the ratio of the marginal utility of private consumption) expands, the poor group is more likely to be damaged by the minimum wage increase if the rich positively values the minimum wage increase. On the other hand, the poor group is more likely to benefit from it if the rich negatively values the minimum wage increase.

To see an implication of Corollary 2 in an example, suppose that the “rich” minimum wage workers are university students, while the poor workers are truly poor. If the students are less likely to suffer from the unemployment because they may be able obtain financial support from other sources. Thus, even when both groups positively value minimum wage increase, the relative valuation differs. As long as housing supply is inelastic, the minimum wage hike must create the winner and the loser just as in Proposition 2. The poor workers are likely to suffer when the students are wealthier than them.

The key intuition behind Corollary 2 is that when $-\partial EU_k^i / \partial R_i$ is small (i.e., the marginal utility from additional consumption is small), group 2 must experience the large absolute change in rents to retain the indifference condition. When it positively values the minimum wage hike, it can accept the large increase in rents, which is harmful to group 1. On the other hand, when the minimum wage increase is harmful to group 2, it must be compensated by the large decrease in rents, which is beneficial to group 1.

Now, let me suppose $H_i' > 0$. It is straightforward to observe that $\omega_i \rightarrow 0$ as $H_i' \rightarrow \infty$. This result is expected: when the housing supply is elastic, the rent is determined by the supply side and the increase in demands does not matter. As a result, the rent gradient approaches zero. The elastic supply
creates the possibility the possibility of Pareto improvement. At the same time, if minimum wage is detrimental, it might harm every group of workers.

**B Housing Supply and Migration in Japan**

In this section, I report evidence about the effect of minimum wages on these outcomes. Such additional information allows for a deeper and more accurate understanding of how minimum wage hikes affect the economy.

I begin with reviewing the theoretical predictions about housing supply and migration responses. As for housing supply, it is likely to be positively related to housing rents because the supply function is upward-sloping. Given the evidence in the previous Section that apartment rents increase due to minimum wage increases, the supply of rental housing is likely to be increased.\(^{76}\)

The impacts on migration responses are more complicated. In the case of homogeneous workers, migration responses also reflect the qualitative desirability of minimum wages. However, the theoretical portion in the heterogeneous case yields more complex patterns of migration responses. Proposition 2 suggests that the change in rents is the weighted average of heterogeneous workers. Thus, in response to the minimum wage increase, those who receive the largest benefit move into the region while those who receive the smallest benefit move out. It implies that both migration inflow and outflow occur after large changes in minimum wage rates. Thus, even when the total population remains constant, minimum wage hikes would promote migration in both directions.

I use prefectural panel data to investigate the effect of minimum wages on rental housing supply and migration responses. Unfortunately, detailed information on these outcomes is not available on an annual basis, and I use somewhat rough measure of them. I obtain the annual number of newly constructed rental housing from the Survey of Building Construction Work Started, conducted by the Ministry of Land, Infrastructure, Transport, and Tourism. The data are available separately for private and public rental housing. Investigating the supply response of public housing is informative on how governments respond to the housing demand shifts induced by minimum wage increases. The annual level of inward and outward migration is obtained from the Basic Resident Register, collected by Ministry of Internal Affairs and Communications.

Since the responses of both migration and construction requires some time, I allow for time lags in the estimation. More specifically, I use the minimum wage rate from October of year \(t - 1\) to September of year \(t\) to explain the number of newly constructed rental apartments during April of year \(t\) and March

\(^{76}\)A minimum wage increase may induce the shift in the supply function itself by increasing construction and maintenance costs. It causes the decrease in \(H'\), and the positive impact on the supply of the apartments may be mitigated.
Similarly, I use such minimum wage rates to explain the level of migration during January $t$ and December $t + 1$. Accordingly, I use the average wage rates in year $t − 1$ (at the time of minimum wage settings) as control variables. The results are similar if we adopt alternative leads or lags. I collect the relevant data during 2003–2013 because the outcomes during this period are explained by minimum wage rates during 2002–2012.

My data does not allow me to identify the groups of rental housing or people most likely to be affected by minimum wages, which prevents me from investigating the heterogeneity of treatment effects. I instead estimate the following simple DD regression equation:

$$\ln Z_{jt} = \delta \ln (MW)_{jt} + \beta Y_{jt} + \text{pref}_j + \text{year}_t + \epsilon_{jt}, \quad (26)$$

where $Z_{jt}$ is the explained variable in prefecture $j$ in year $t$ and $Y_{jt}$ is a vector of control variables. In this Section, I control for the log of male average wage rates. As in the previous Section, I take a lag of the control variable to capture the significance of the minimum wage increase at the time of the hike.\footnote{The results are similar if I drop this control variable or add the log of the female average wage rate.}

To test the significance of the pre-trends, I also report the results of placebo tests. As explained in Section 3.2, Japanese minimum wage increases are determined without referring to the error term in (26), but some effects might be driven by trends rather than the true effect of minimum wages. Standard errors are clustered at the prefectural level (Bertrand, Duflo, and Mullainathan, 2004).

Note that, as seen below, the pre-trends are identified only imprecisely; hence, one should be careful about quantitative implications in this Section. That said, even if the large fraction of effects is driven by trends other than changes in minimum wages, the correlations between minimum wages and the outcome variables are informative in interpreting my main results.

The results are reported in Table B.1. Column 1 shows that a minimum wage increase induces the increase of newly constructed private rental housing. Of course, the share of new housing to the stock of all housing is small, and the supply adjustment takes time. Still, the result indicates that housing

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ln (new \ private)$</td>
<td>4.100***</td>
<td>11.069***</td>
<td>0.351</td>
<td>0.893***</td>
</tr>
<tr>
<td>$\ln (MW)$</td>
<td>(1.024)</td>
<td>(2.976)</td>
<td>(0.213)</td>
<td>(0.234)</td>
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<td>517</td>
<td>505</td>
<td>517</td>
<td>517</td>
</tr>
</tbody>
</table>

Standard errors in parentheses are clustered at the prefecture level.

\* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table B.1: New Construction of Rental Housing, and Migration Responses
supply is increased, which is consistent with the theoretical prediction.

Column 2 shows the supply response of public rental housing. It shows a much stronger positive response than in private rental markets. Japanese public apartments are primarily targeted toward poor households who are likely to be affected by a minimum wage increase. The strong increase in supply might imply the possibility that governments might try to mitigate the increase in rents by supplying more public housing. Thus, it may be the case that governments implicitly consider redistributive policies and housing market responses as intertwined, although more direct evidence is needed to fully justify this interpretation.

The result that the housing supply is increased reinforces the result that minimum wage hikes increase the WTP of some workers. I explain this point using the model introduced in Section 2.1. If the housing supply is responsive, $H'$ is expected to be large. Then, (6) implies that the positive impact on housing rents becomes quite small, although housing supply responses $H' \frac{\partial R}{\partial w}$ is large since $H'$ is large. Thus, even when the positive impact on rents is too small to detect using econometric techniques, the housing supply may respond significantly. Since $\text{sgn}(H' \frac{\partial R}{\partial w}) = \text{sgn}(\frac{\partial R}{\partial w})$, the positive supply response implies an increase in WTP. Relatedly, according to Proposition 1, the positive response of the housing supply indicates that the increase in rents is a lower-bound estimate for the money-metric benefits of minimum wage hikes. Thus, the aggregate benefit might be somewhat larger than estimated in the previous section.

Columns 3 and 4 report the effect on inward and outward migration, respectively. The results show that both inward and outward migration are promoted by minimum wage hikes, although the effect on the inward migration is marginally insignificant at the 10% level ($p \approx 0.11$). This is consistent with the prediction of my theoretical results in the heterogeneous case. A minimum wage increase must create “winners” and “losers” among minimum wage workers, which induces migration in both directions. The coefficient in column 4 is somewhat larger than the coefficient in column 3, and the net effect on migration might be negative, although this result may be due to the lack of detailed data on migration. As discussed at the end of Section 4.3, this pattern might call for modifying the timing of migration decisions in my model and other canonical models in the literature. In particular, it raises the possibility that housing cost increase, rather than labor market conditions, may enhance out-migration. It underscores the importance of including housing markets in studying migration responses to minimum wages.

To determine whether the results are driven by the common trend rather than the effects posited in this paper, I conduct the placebo test in the analogous way as Section 5.1. I confine the sample to the pre-treatment period (2003–2006) and use minimum wage rates during 2009–2012 to explain the
If prefectures experiencing rapid minimum wage increase have a particular pre-trend, the placebo test is likely to capture it.

The results in Table B.2 confirm that there are no statistically significant pre-trends. The point estimates are closer to zero than those in Table B.1. As to the estimates for construction, both private and public construction exhibit positive but much smaller point estimates. However, the large standard errors might imply that the causal effects revealed in Table B.1 might be smaller. As for migration responses, the main effect on immigration cannot be statistically distinguished from the roughly estimated pre-trend. Although the statistical significance is limited, the result suggests that emigration might be relatively more strongly affected by minimum wage hikes.

Overall, the analysis on housing supply and migration responses reinforces my main findings. The increased housing supply implies that the WTP of some minimum wage workers are enhanced. The result that both inward and outward migration are increased suggests that worker heterogeneity plays a key role. Thus, it seems indispensable to investigate whether minimum wage hikes really help the targeted group of people, although my data are not informative on this issue. Due to the data availability, the analysis cannot focus on the specific group most likely to be affected by minimum wages, and thus, results should be interpreted with caution. Still, the results in this Section reveal interesting relationships between minimum wages, housing supply, and migration responses.

C Details and Further Discussions on the Empirics

C.1 The United States

C.1.1 Sample Selection Procedure

The sample period 1984-2018 are chosen to utilize all available FMRs data in consecutive years. This period also include numerous minimum wage changes and provide sufficient variation, many of which

\[^{78}\text{That is, I use the minimum wage rate in year } t + 6 \text{ to explain outcomes in year } t \ (t = \{2003, 2004, 2005, 2006\}).\]
are caused by the federal minimum wage increase (see, e.g., Cadena, 2014; Monras, 2018). The sample consists of counties (or their equivalent) in 48 continental US states and District of Columbia. Although almost all counties are included in the sample, some observations are excluded due to the following reasons: (i) 2000 census does not provide the population density, (ii) within a given county-year pair, FMRs are calculated using a different percentile point in different areas. I also drop counties where FMRs are not complete for all sample periods to create a balanced panel data. This procedure leaves me 3082 counties.

### C.1.2 Summary Statistics

I provide summary statistics on the US data.

#### C.1.3 DD specification in levels.

In the main text, the empirical specifications for the United States rely on differencing. In this Appendix, I illustrate that similar conclusions appear without differencing operations.

I estimate the following standard DD regression equation:

$$\ln(FMR)_{it} = \gamma_t + \beta_0 \ln(MW)_{it} + \varepsilon_{it}. \quad (27)$$

The results are reported in Table C.2. Column 1 uses the full sample, showing the negative but insignificant effect. Note that while the negative estimate also appears in Tidemann (2018), it is insignificant in my results while he argues it is significantly negative. The likely reason behind this discrepancy is that I cluster standard errors at the state level, rather than at the county level as in Tidemann (2018). Thus, Tidemann’s conclusion may not be robust to the presence of state-year specific unobserved effects.

Column 2 presents the estimate for densely populated samples. The estimate is positive and significant at the 10% level. The magnitude is similar to what I find by using distributed lag models in Table 3. Thus, it confirms the robustness of my arguments in the main text.

In the main text, I do not report results for non-densely populated samples (i.e., samples not belonging to densely populated samples) because full sample estimates turn out to be very close in most

<table>
<thead>
<tr>
<th></th>
<th>Observations</th>
<th>mean</th>
<th>SD</th>
<th>p25</th>
<th>Median</th>
<th>p75</th>
<th>p90</th>
</tr>
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<td>FMRs</td>
<td>107870</td>
<td>535.3397</td>
<td>204.9815</td>
<td>383</td>
<td>497</td>
<td>641</td>
<td>779</td>
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<td>Density</td>
<td>3082</td>
<td>238.7314</td>
<td>1680.387</td>
<td>17.4</td>
<td>42.65</td>
<td>102.1</td>
<td>314.5</td>
</tr>
<tr>
<td>Share of Whites</td>
<td>3082</td>
<td>.9083684</td>
<td>.1490095</td>
<td>.8911884</td>
<td>.981169</td>
<td>.9967864</td>
<td>.998657</td>
</tr>
<tr>
<td>Share of Blacks</td>
<td>3082</td>
<td>.0916316</td>
<td>.1490095</td>
<td>.0032136</td>
<td>.018831</td>
<td>.1088115</td>
<td>.3163947</td>
</tr>
</tbody>
</table>

Table C.1: **Summary Statistics: The United States**

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51
Table C.2: Standard DD models

specifications. However, in the specification (27), there is a notable difference and I report it in column 3. Here, the estimate is significantly negative. While it may indicate the negative impact on housing rents in rural areas, I view it as capturing the pre-trends rather than true impacts. Indeed, in Table 3, I find that the lead terms of minimum wages are negative in the full sample case. I deem that this pre-trend is captured by the standard DD specification (27). On the other hand, Table 3 shows positive pre-trends for urban samples. Thus, the positive impact might be larger than what Table C.2 suggests.

C.1.4 Estimations and Welfare Implications without Log-transformation

Throughout the paper, I apply log-transformations to housing rents and minimum wage rates. This has several advantages. First, it enables me to interpret the estimated coefficients as elasticity, which is scale-free and facilitate comparisons across specifications and countries. Moreover, log-transformation mitigates the effect of extreme values, which is particularly reasonable given the high rent of several large cities (Collinson and Ganong, 2018). However, benefits come at a cost. First, while the theory predicts that without log-transformation, the monetary WTP can be estimated. Log-transformation complicates the interpretation of the estimated coefficients as the measure of welfare improvement. Second, in analyzing the heterogeneous welfare impact, I have to introduce an unknown variable $\lambda^k_i$ without log-transformation. Directly estimating the formula in 6 and Proposition 2 is appealing in circumventing these disadvantages. Note that without log transformation, year fixed effects no longer absorbs the effect of inflation. I convert FMRs and minimum wage rates into 2018 US dollars using CPI-U.

The regression equation is the following:

$$\Delta FMR_{it} = \text{year}_i + \gamma_i \cdot r + \beta_{\text{whites}}(\Delta rMW_{it} \cdot z_{i}^{\text{whites}}) + \beta_{\text{blacks}}(\Delta rMW_{it} \cdot z_{i}^{\text{blacks}}) + \varepsilon_{it}, \quad (28)$$

where $\beta_{\text{white}} = \partial WTP^{\text{whites}} / \partial w$ and $\beta_{\text{blacks}} = \partial WTP^{\text{black}} / \partial w$. I do not differentiate whites and blacks in the case without heterogeneity.
### Table C.3: Long-differences specifications without log transformation

<table>
<thead>
<tr>
<th></th>
<th>(1) $\Delta_1 FMR$</th>
<th>(2) $\Delta_2 FMR$</th>
<th>(3) $\Delta_3 FMR$</th>
<th>(4) $\Delta_4 FMR$</th>
<th>(5) $\Delta_5 FMR$</th>
<th>Panel A: Full samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta MW$</td>
<td>2.3597 (1.6696)</td>
<td>3.2818 (2.2789)</td>
<td>3.4094 (3.1154)</td>
<td>4.9914 (4.0721)</td>
<td>7.6709 (5.1671)</td>
<td>Observations 104788</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>104788</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>15674</td>
</tr>
</tbody>
</table>

Standard errors in parentheses are clustered at the state level.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

### Table C.4: Long-differences specifications: Heterogeneity by race without log transformation

<table>
<thead>
<tr>
<th></th>
<th>(1) $\Delta_1 FMR$</th>
<th>(2) $\Delta_2 FMR$</th>
<th>(3) $\Delta_3 FMR$</th>
<th>(4) $\Delta_4 FMR$</th>
<th>(5) $\Delta_5 FMR$</th>
<th>Panel B: Densely populated areas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whites</td>
<td>1.8692 (1.6528)</td>
<td>2.4893 (3.1246)</td>
<td>1.9747 (4.0515)</td>
<td>3.1164 (5.1189)</td>
<td>5.0622 (4.2676)</td>
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<td>14291</td>
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</table>

Standard errors in parentheses are clustered at the state level.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$
The case without heterogeneity is reported in Table C.3. Note first that the basic pattern found in the main text is preserved. Namely, urban counties see the rent increase while rural ones may not. Moreover, the coefficients become larger as the time span \( r \) gets larger, suggesting the dynamic effect of minimum wages. The estimated WTP for urban residents is around 30$ using the time span of five years, meaning that 1$ real minimum wage increase induces (monthly) 30$ increase in the WTP.

Turning to the heterogeneous case, Table C.4 reports the results. The basic pattern found in the main text is confirmed: blacks benefit in rural areas while whites benefit in urban areas. Moreover, the coefficient becomes larger as the time span expands, which is consistent with the dynamic effect of minimum wages. Interestingly, the benefit of blacks in rural areas and that of whites in urban areas are very similar. Note that in this case, the weight of blacks \( z_{blacks} \) may be underestimated because blacks may have lower mobility cost (Diamond, 2016) and are more likely to be a minimum wage worker. In such a case, the WTP of blacks might be overestimated and that of whites is underestimated.\(^{79}\) If this is the case, the positive impact on blacks in a rural area may be overestimated, while that in urban areas is underestimated. If this effect is sufficiently strong, the racial heterogeneity within a region might disappear. However, it does not explain why the results are very different in urban and rural areas. Other factors, such as local labor market conditions and housing supply elasticity, are necessary to understand this difference.

C.2 Japan

C.2.1 Sample Selection Procedure

I focus on the data from October, 2002 to September, 2013, implying that \( t = 2002, ..., 2012 \). This time period includes five years before and after the policy change (\( t = 2007 \)). I do not use the data on \( t \geq 2013 \) for several reasons. First, the policy change was intended to end at \( t = 2012 \) and the source of the minimum wage increase thereafter is unclear (Kawaguchi and Mori, 2013). Second, two important national policy changes may affect the market. The first change is the increase of the VAT in 2014. Since rental apartments are exempted from VAT, it changes the relative price between housing and other goods. The second change is major revision on the amount of housing assistance to livelihood protection recipients in 2015.\(^{80}\) Third, beginning from 2013, minimum wages are increased relatively parallelly across prefectures. Such a parallel increase is not likely to affect the cross-sectional spatial arbitrage condition and bias the estimate toward zero. I do not use data between January, 1999 and September, 2002 because there was some minimum wage increase during this period, which makes

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\(^{79}\)Suppose that the true weight is \( z^* \) and the true coefficient is \( WTP^* \). Then, the estimated coefficient identifies \( WTP^*(z/\hat{z}) \), where \( \hat{z} \) is the used weight. It shows that \( WTP^* \) is overestimated if and only if the true weight is larger than the used weight.

\(^{80}\)On the other hand, during my sample period, the level of the housing assistance was held fixed in most jurisdictions.
Turning to the sample selection, I delete observations with either (i) floor area smaller than $5m^2$ or larger than $80m^2$, (ii) monthly rents higher than 150,000 yen, (iii) negative apartment’s age, or (iv) missing values on the located floor or the construction material. I also drop apartments made of reinforced concrete (RC) or steal reinforced concrete (SRC) to make sure that the apartments in the sample are not very durable. I also delete apartments older than 45 to ensure that my results are not driven by potentially mis-recorded extreme values. Since my dataset on rental apartments comes from raw posted data, some observations are re-posted multiple times when they are vacant for a long time. Since my dataset records all postings, a re-posted observation is counted as a new observation even though the same observation is already included in the dataset. To mitigate this concern, I randomly keep only one observation out of all repeated observations of the same unit within one year. Qualitative implications remain the same even when I work with the data without the above-mentioned data cleaning processes.

### C.2.2 Summary Statistics

Summary statistics on the At Home data is provided in Table C.5.

### C.2.3 Prefecture-Specific Linear Trends

In this section, I re-estimate the results in Table 5 while including the prefecture-specific trends ($\text{pref}_j \times t$). As seen in Figure 1, Japanese minimum wages are adjusted gradually. Thus, including the linear trend may unintentionally absorb the true effect of the minimum wage hikes. Still, it is likely to significantly alleviate the difference in trends, which is one of the most important concerns for DD analysis. Moreover, it may also alleviate the problem that all prefectures experience some increase in minimum wages. My model in Section 2 assumes that the outside utility is fixed, but it may actually be endoge-
<table>
<thead>
<tr>
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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(MW)</td>
<td>0.7624***</td>
<td>0.6604***</td>
<td>0.6446***</td>
<td>0.6324***</td>
</tr>
<tr>
<td></td>
<td>(0.2155)</td>
<td>(0.2038)</td>
<td>(0.2052)</td>
<td>(0.2079)</td>
</tr>
<tr>
<td>ln(MW) × age</td>
<td>0.0076*</td>
<td>0.0076*</td>
<td>0.0076*</td>
<td>0.0076*</td>
</tr>
<tr>
<td></td>
<td>(0.0038)</td>
<td>(0.0038)</td>
<td>(0.0038)</td>
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</tr>
<tr>
<td>Prefecture-year level controls</td>
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<td>No</td>
<td>M average wage</td>
<td>M&amp;F average wages</td>
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<td>Observations</td>
<td>5696368</td>
<td>5696368</td>
<td>5696368</td>
<td>5696368</td>
</tr>
</tbody>
</table>

Standard errors in parentheses are clustered at the prefecture level.

* \( p < 0.1 \), ** \( p < 0.05 \), *** \( p < 0.01 \)

Table C.6: **Re-estimation of Table 5 with prefecture-specific linear trends.**

nous and causes underestimation about the true positive impact. By controlling for the linear trend, the effects of minimum wage increase with small minimum wage adjustments are likely to be absorbed and the estimated effect may better respects the assumptions in Section 2.\(^{81}\)

The results are reported in Table C.6.\(^{82}\) Compared with Table 5, the primary effect is much larger. According to column 1, 10% minimum wage increase raises the apartment rents by 7.6%. This result is consistent with the negative pre-trend discussed in Section 4.4. Indeed, the difference between the coefficient in the column 1 of Table 5 and that in the column 1 of Table 6 is approximately 0.6, which is close to the coefficient in Table C.6.\(^{83}\) The differential effect is slightly larger, but similar to the estimates in Table 5. As discussed in Section 4.4, taking into account the presence of the trends seems to significantly increase the positive impact on housing rents.

\(^{81}\)Note that the minimum wage increase is closer to the linear way in prefectures with small increase (See Figure 1).
\(^{82}\)The results on the DDD analysis is omitted because the linear trend is completely absorbed by the prefecture-year specific dummies.
\(^{83}\)The remaining difference by around 0.16 might be partially attributed to the underestimation due to the simultaneous gradual minimum wage increase.