# A family of rules to share the revenues from broadcasting sport events

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# A family of rules to share the revenues from broadcasting sport events*

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**Abstract**

We consider the problem of sharing the revenues from broadcasting sport league events, introduced by Bergantiños and Moreno-Ternero (2019). We characterize a family of rules compromising between two focal and somewhat polar rules: the *equal-split* rule and *concede-and-divide*. The characterization only makes use of three basic axioms: *equal treatment of equals*, *additivity* and *maximum aspirations*. We also show further interesting features of the family: (i) if we allow teams to vote for any rule within the family, then a majority voting equilibrium exists; (ii) the rules within the family yield outcomes that are fully ranked according to the Lorenz dominance criterion; (iii) the family provides rationale for existing schemes in real-life situations.

***JEL numbers****: D63, C71, Z20.*

***Keywords****: resource allocation, broadcasting, sport events, concede-and-divide, equal-split.*

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# 1 Introduction

The sale of broadcasting and media rights is currently the biggest source of revenue for most sports organizations. This sale is often carried out through some sort of collective bargaining involving all participating organizations (teams) in a given competition on the one hand, and broadcasting companies on the other hand. Thus, an ensuing key problem arises in which the revenues collected from the sale have to be shared among the teams. This is, by no means, a straightforward problem, mostly because the individual contribution to the revenues is not known. Furthermore, the revenue is sizable, which renders the solution of the problem crucial for the management of most sports organizations.

In a recent paper (Bergantiños and Moreno-Ternero, 2019), we introduce a formal model to analyze this problem. Therein, two focal and somewhat polar rules stand out. On the one hand, the so-called *equal-split* rule which splits the audience of each game equally among the two teams.[1] On the other hand, the so-called *concede-and-divide*, which concedes each team the audience coming from its fan base (the loyal viewers watching all games played by that team) and divides equally the residual. The two rules have distinguishing merits, but they treat fans in two opposite and somewhat extreme ways. More precisely, the *equal-split* rule essentially ignores the existence of fan bases as it considers, de facto, that both teams participating in a game contributed equally to the revenues collected from broadcasting that game. On the other hand, *concede-and-divide* essentially ignores the existence of casual viewers as it considers, de facto, that viewers watching a game are either fans of one participating team, or compulsive viewers, who watch all games in the season. Reality seems to be somewhat in between and compromising between both rules, which provide meaningful lower and upper bounds (depending on whether the team has a weak or strong fan base), seems to be a natural move.

We take in this paper the axiomatic approach and consider three basic and intuitive axioms for allocation rules, satisfied by the *equal-split* rule and *concede-and-divide*: *equal treatment of equals*, *additivity* and *maximum aspirations*. The first one says that if two teams have the same audiences, then they should receive equal amounts. The second one says that revenues should

[1] As we shall argue later, revenues can be reduced to audiences provided one assumes a constant pay-per-view fee for each game.

be additive on audiences.[2] The third axiom says that no team can receive more than its *claim*, i.e., the total revenue obtained from all the games in which the team was involved. We show that these three axioms, which seem to be innocuous independently, have a strong bite when combined as they actually characterize a family of rules that offer a compromise between the previous two rules. This is the main result of our paper.

Each rule in the family we characterize is simply defined by a certain convex combination of the *equal-split* rule and *concede-and-divide*. More precisely, for a given parameter $\lambda \in [0,1]$, the rule $R^{\lambda}$ selects, for each problem, the convex combination of the solutions suggested by the *equal-split* rule and *concede-and-divide* for that problem, with weights $\lambda$ and $1-\lambda$, respectively. Note that, when the set of options is equipped with a convex structure (as in this case), averaging between different positions that people may take concerning the best way of approaching problems is an appealing way of finding some common ground between them.[3] What is remarkable in our setting is that this position is normatively supported by three simple and intuitive principles, as our characterization shows.

We then explore the family so derived and discover further interesting features of it.

First, we show that, if we allow teams to vote for any rule within the family, then a majority voting equilibrium exists, i.e., a rule that cannot be overturned by any other rule within the family through majority rule. This is a consequence of the fact that the rules within the family satisfy the so-called *single-crossing property*, which allows one to separate those teams who benefit from the application of one rule or the other, depending on the rank of their claims.[4]

Second, we show that the rules within the family yield outcomes that are fully ranked according to the Lorenz dominance criterion, the most fundamental principle for the evaluation of inequality (e.g., Dasgupta, Sen and Starret, 1973). More precisely, for each problem, and each pair of rules within the family, the outcome suggested by the rule associated with a higher parameter dominates (in the sense of Lorenz) the outcome suggested by the other rule, which is equivalent to saying that the former will be more egalitarian than the latter. Thus, the

[2] An interpretation is that the aggregation of the revenue sharing in two seasons (involving the same teams) is equivalent to the revenue sharing in the hypothetical combined season aggregating the audiences of the corresponding games in both seasons.

[3] The idea of averaging as a means of compromising is a recurrent theme in game theory and the theory of resource allocation (e.g., Thomson, 2018).

[4] It is well known that a sufficient condition for the existence of a majority voting equilibrium is that voters exhibit *intermediate* preferences over the set of alternatives (e.g., Gans and Smart, 1996).

parameter describing the family allows for the control of the relative equality of the outcomes, for any problem.

One-parameter families such as the one we derive in this paper have been frequently singled out in the literature. For instance, in the literature on the measurement of income inequality, Atkinson (1970) famously introduced a family of inequality measures, characterized by a weighting parameter measuring aversion to inequality. Somewhat related, Donalson and Weymark (1980) generalized the social-evaluation function corresponding to the focal Gini inequality index to derive the well-known (one-parameter) family of generalized Gini inequality indices.[5] In a context more similar to ours, Moulin (1987) characterized a family compromising between the equal and proportional surplus sharing methods. As a matter of fact, his family is the convex combination of those two methods and one of the axioms used for its characterization is precisely *additivity*. Thus, the parallelism with our result is strong.[6] Compromises between the proportional and *constrained* equal-award rules (thus, satisfying the standard non-negativity condition for claims problems) have also been considered by Thomson (2015a,b). And alternative compromises between the proportional rule and the so-called Talmud rule (e.g., Aumann and Maschler, 1985) have been explored by Moreno-Ternero and Thomson (2017). Finally, Moreno-Ternero and Villar (2006) introduced a one-parameter family of rules for claims problems generalizing the Talmud rule and encompassing (as extreme cases) the polar constrained equal-awards and constrained equal-losses rules. The rules within such a family also happen to satisfy the single-crossing property and be fully ranked according to the Lorenz dominance criterion (e.g., Moreno-Ternero, 2011).

The rest of the paper is organized as follows. We introduce the model in Section 2. We present the axiomatic characterization leading to the family in Section 3. Section 4 is devoted to explore additional properties of the rules within the family. In Section 5, we apply the family to the case of the Spanish Football League and apply the rules of our family to explore several allocation schemes therein. We also contrast them with the current scheme being implemented by the Spanish National Professional Football League Association. We conclude in Section 6.

[5] See also Weymark (1981), Donalson and Weymark (1983), and Bossert (1990).

[6] Something similar happens in minimum cost spanning tree problems, where Trudeau (2014) characterizes the convex combination of the folk rule (e.g., Bergantiños and Vidal-Puga, 2007) and the so-called cycle-complete rule (e.g., Trudeau, 2002), also making use of *additivity*.

# 2 The model

We consider the model introduced by Bergantiños and Moreno-Ternero (2019). Let $N$ describe a finite set of teams. Its cardinality is denoted by $n$. We assume $n \geq 3$. For each pair of teams $i, j \in N$, we denote by $a_{ij}$ the broadcasting audience (number of viewers) for the game played by $i$ and $j$ at $i$'s stadium. We use the notational convention that $a_{ii} = 0$, for each $i \in N$. Let $A \in \mathcal{A}_{n \times n}$ denote the resulting matrix of broadcasting audiences generated in the whole tournament involving the teams within $N$.[7]

Let $\alpha_i(N, A)$ denote the total audience achieved by team $i$, i.e.,

$$\alpha_i(N, A) = \sum_{j \in N} (a_{ij} + a_{ji}).$$

Without loss of generality, we normalize the revenue generated from each viewer to 1 (to be interpreted as the "pay per view" fee). Thus, we sometimes refer to $\alpha_i(N, A)$ by the *claim* of team $i$. When no confusion arises, we write $\alpha_i$ or $\alpha_i(A)$ instead of $\alpha_i(N, A)$.

For each $(N, A) \in \mathcal{P}$, we define $\overline{\alpha}$ as the average audience of all teams. Namely,

$$\overline{\alpha} = \frac{\sum_{i \in N} \alpha_i}{n}.$$

For each $A \in \mathcal{A}_{n \times n}$, let $||A||$ denote the total audience of the tournament. Namely,

$$||A|| = \sum_{i,j \in N} a_{ij} = \frac{1}{2} \sum_{i \in N} \alpha_i = \frac{n\overline{\alpha}}{2}.$$

A (broadcasting) **problem** is a pair $(N, A)$, where $A \in \mathcal{A}_{n \times n}$ is defined as above. The family of all the problems is denoted by $\mathcal{P}$.

A (sharing) **rule** is a mapping that associates with each problem the list of the amounts the teams get from the total revenue. Thus, formally, $R : \mathcal{P} \to \mathbb{R}^n$ is such that, for each $(N, A) \in \mathcal{P}$,

$$\sum_{i \in N} R_i(N, A) = ||A||.$$

[7] We are therefore assuming a tournament in which each team plays each other team twice: once home, another away. Our model could be extended to tournaments in which some teams play other teams a different number of times. In such a case, $a_{ij}$ would denote the broadcasting audience in all games played by $i$ and $j$ at $i$'s stadium.

Two rules stand out as focal for this problem (e.g., Bergantiños and Moreno-Ternero, 2019). The *equal-split rule*, which splits equally the audience of each game (among the two teams), and *concede-and-divide*, which takes into account the number of fans of each team. They are defined in a similar way. First, each team $i$ tentatively receives its claim ($\alpha_i$). Second, they each subtract from it an amount associated to the remaining $n-1$ teams. In the case of the *equal-split* rule, an equal share of half of the team's total audience ($\beta_i = \frac{\alpha_i/2}{n-1}$); in the case of *concede-and-divide*, the average audience per game that the remaining teams played ($\gamma_i = \frac{\sum_{j,k\in N\setminus\{i\}}(a_{jk}+a_{kj})}{(n-2)(n-1)}$).[8] Formally,

**Equal-split rule**, $ES$: for each $(N, A) \in \mathcal{P}$, and each $i \in N$,

$$ES_i(N, A) = \alpha_i - (n-1)\beta_i = \frac{\alpha_i}{2}.$$

**Concede-and-divide**, $CD$: for each $(N, A) \in \mathcal{P}$, and each $i \in N$,

$$CD_i(N, A) = \alpha_i - (n-1)\gamma_i = \frac{(n-1)\,\alpha_i - ||A||}{n-2}.$$

We now consider a family of rules that offer a compromise between the *equal-split rule* and *concede-and-divide*. They are defined as convex combinations of the two rules. Formally,

$EC$**-family**, $\left\{EC^\lambda\right\}_{\lambda\in[0,1]}$: for each $\lambda \in [0,1]$, each $(N, A) \in \mathcal{P}$, and each $i \in N$,

$$EC_i^\lambda(N, A) = \lambda ES_i(N, A) + (1-\lambda)CD_i(N, A).$$

At the risk of stressing the obvious, note that when $\lambda = 0$ then $EC^\lambda$ coincides with *concede-and-divide*, whereas when $\lambda = 1$ then $EC^\lambda$ coincides with the *equal-split* rule. That is, $EC^0 \equiv CD$ and $EC^1 \equiv ES$.

[8] The term *concede-and-divide*, which was coined by Thomson (2013) in a different setting, is justified here by an intuitive procedure, based on a form of statistical estimation aiming to capture the loyal viewers of each team, which leads to this rule (see Bergantiños and Moreno-Ternero (2019) for further details).

Note also that, for each $(N, A) \in \mathcal{P}$, each $i \in N$, and each $\lambda \in [0,1]$,

$$\begin{aligned} EC_i^{\lambda}(N,A) &= \lambda\frac{\alpha_i}{2} + (1-\lambda)\frac{(n-1)\,\alpha_i - ||A||}{n-2} \\ &= \lambda\frac{\alpha_i}{2} + (1-\lambda)\frac{(n-1)\,\alpha_i - \frac{n}{2}\overline{\alpha}}{n-2} \\ &= \lambda\frac{\alpha_i}{2} + (1-\lambda)\frac{2n\alpha_i - 2\alpha_i - n\overline{\alpha}}{2\,(n-2)} \\ &= \lambda\frac{\alpha_i}{2} + (1-\lambda)\frac{n\,(\alpha_i - \overline{\alpha})}{2\,(n-2)} + (1-\lambda)\frac{(n-2)\,\alpha_i}{2\,(n-2)} \\ &= \frac{\alpha_i}{2} + \frac{n(1-\lambda)}{2\,(n-2)}\,(\alpha_i - \overline{\alpha})\,. \end{aligned} \tag{1}$$

## 3 A characterization

We now introduce three natural axioms for rules.

The first axiom is a minimal requirement of *impartiality*, a basic requirement of justice (e.g., Moreno-Ternero and Roemer, 2006). It says that if two teams have equal audience, then they should receive equal amounts.

**Equal treatment of equals**: For each $(N, A) \in \mathcal{P}$, and each pair $i, j \in N$ such that $a_{ik} = a_{jk}$, and $a_{ki} = a_{kj}$, for each $k \in N \setminus \{i, j\}$,

$$R_i(N, A) = R_j(N, A).$$

The second axiom says that revenues should be additive on $A$. This is an axiom with a long tradition in axiomatic work (e.g., Shapley, 1953). In our setting, among other things, it precludes the allocation of revenue $a_{ij}$ to depend on any other information contained in the matrix $A$. Formally,

**Additivity**: For each pair $(N, A)$ and $(N, A') \in \mathcal{P}$,

$$R\,(N, A + A') = R\,(N, A) + R\,(N, A')\,.$$

The next axiom says that each team should receive, at most, the total audience of the games played by the team. It therefore formalizes a natural upper bound, akin to the standard requirement of claims boundedness for the problem of adjudicating conflicting claims (e.g., O'Neill, 1982; Thomson, 2018).

**Maximum aspirations**: For each $(N, A) \in \mathcal{P}$ and each $i \in N$,

$$R_i(N, A) \leq \alpha_i.$$

Our next result says that just the three previous axioms together characterize the EC family of rules. This is remarkable as the three axioms are intuitive and basic and none of them seem to convey strong implications or have a flavor reminiscent of the rules.

**Theorem 1** *A rule satisfies additivity, equal treatment of equals, and maximum aspirations if and only if it is an EC rule.*

**Proof.** In Bergantiños and Moreno-Ternero (2019), we prove that the rules $ES$ and $CD$ satisfy the three axioms. As, for each $\lambda \in [0, 1]$, $EC^{\lambda} = \lambda ES + (1 - \lambda) CD$, it is straightforward to see that each rule within the $EC$-family also satisfies the three axioms.

Conversely, let $R$ be a rule satisfying the three axioms. Let $(N, A) \in \mathcal{P}$. For each pair $i, j \in N$, with $i \neq j$, let $1^{ij}$ denote the matrix with the following entries:

$$1^{ij}_{kl} = \begin{cases} 1 & \text{if } (k, l) = (i, j) \\ 0 & \quad \text{otherwise.} \end{cases}$$

Notice that $1^{ij}_{ji}$ is the *zero matrix*, i.e., the matrix with only zero entries.

Let $k \in N$. By *additivity*,

$$R_k(N, A) = \sum_{i,j \in N: i \neq j} a_{ij} R_k\left(N, 1^{ij}\right). \tag{2}$$

By *equal treatment of equals*, for each pair $k, l \in N \setminus \{i, j\}$ we have that $R_i(N, 1^{ij}) = R_j(N, 1^{ij}) = x^{ij}$, and $R_k(N, 1^{ij}) = R_l(N, 1^{ij}) = z^{ij}$. As $\sum_{k \in N} R_j(N, 1^{ij}) = ||1^{ij}|| = 1$ we deduce that

$$z^{ij} = \frac{1 - 2x^{ij}}{n - 2}.$$

Let $k \in N \setminus \{i, j\}$. By *additivity*, $R_j\left(N, 1^{ij} + 1^{ik}\right) = x^{ij} + z^{ik}$, and $R_k\left(N, 1^{ij} + 1^{ik}\right) = z^{ij} + x^{ik}$. By *equal treatment of equals*, $R_j\left(N, 1^{ij} + 1^{ik}\right) = R_k\left(N, 1^{ij} + 1^{ik}\right)$. Thus,

$$\begin{aligned} x^{ij} + \frac{1 - 2x^{ik}}{n - 2} &= x^{ik} + \frac{1 - 2x^{ij}}{n - 2} \Leftrightarrow \\ (n - 2) x^{ij} + 1 - 2x^{ik} &= (n - 2) x^{ik} + 1 - 2x^{ij} \Leftrightarrow \\ x^{ij} &= x^{ik} \end{aligned}$$

Thus, there exists $x \in \mathbb{R}$ such that for each $\{i,j\} \subset N$,

$$\begin{aligned} R_i\left(N, 1^{ij}\right) &= R_j\left(N, 1^{ij}\right) = x, \text{ and} \\ R_l\left(N, 1^{ij}\right) &= \frac{1-2x}{n-2} \text{ for each } l \in N \setminus \{i,j\}. \end{aligned}$$

Let $k \in N$. By (2),

$$\begin{aligned} R_k(N,A) &= \alpha_k x + (||A|| - \alpha_k)\frac{1-2x}{n-2} \\ &= \alpha_k x + (2x-1)\left[\frac{(n-1)\alpha_k - ||A||}{n-2} - \alpha_k\right] \\ &= \alpha_k x + (2x-1)\, CD_k(N,A) - (2x-1)\,\alpha_k \\ &= \frac{\alpha_k}{2} 2(x - 2x + 1) + (2x-1)\, CD_k(N,A) \\ &= (2-2x)\, ES(N,A) + (2x-1)\, CD_k(N,A). \end{aligned}$$

Let $\{i,j,l\} \subset N$ be a set of three different teams. By *maximum aspirations*,

$$\begin{aligned} x &= R_i\left(N,1^{ij}\right) \le \alpha_i\left(1^{ij}\right) = 1 \text{ and} \\ \frac{1-2x}{n-2} &= R_l\left(N,1^{ij}\right) \le \alpha_l\left(1^{ij}\right) = 0. \end{aligned}$$

Thus, $\frac{1}{2} \le x \le 1$. Let $\lambda = 2 - 2x$. Then, $1 - \lambda = 2x - 1$. As $x$ ranges from 1/2 to 1, it then follows that $\lambda$ ranges from 0 to 1. Consequently,

$$R_k(N,A) = \lambda ES_k(N,A) + (1-\lambda)\, CD_k(N,A) = EC_k^{\lambda}(N,A),$$

as desired. ■

**Remark 1** *The axioms of Theorem 1 are independent.*

*Let $R^1$ be the rule that arises as a convex combination between the equal split rule and concede-and-divide, but with the (endogenous) weight obtained by the ratio between the maximum audience and the overall audience. Formally, for each problem $(N,A) \in \mathcal{P}$, let $\bar{A} = max_{i,j \in N} a_{ij}$. Then, for each $i \in N$,*

$$R_i^1(N,A) = \frac{\bar{A}}{||A||} ES_i(N,A) + \left(1 - \frac{\bar{A}}{||A||}\right) CD_i(N,A).$$

*$R^1$ satisfies equal treatment of equals and maximum aspirations, but not additivity.*

*Let $R^2$ be the rule in which, for each game $(i,j) \in N \times N$, the revenue $a_{ij}$ goes to the team with the lowest number of the two. Namely, for each problem $(N,A) \in \mathcal{P}$, and each $i \in N$,*

$$R_i^2(N,A) = \sum_{j \in N : j > i} (a_{ij} + a_{ji}).$$

$R^2$ *satisfies maximum aspirations and additivity, but not equal treatment of equals.*

*The uniform rule, which divides the total audience equally among the teams, satisfies additivity and equal treatment of equal, but not maximum aspirations.*

Theorem 1 shows that the family of $EC$ rules is characterized only by three basic and intuitive axioms, which, when combined, have strong implications to single out a one-parameter family ranging from the *equal-split* rule to *concede-and-divide.*

In Bergantiños and Moreno-Ternero (2019), we characterize the *equal-split* rule as the unique rule satisfying *equal treatment of equals*, *additivity*, and the so-called *null team axiom*, which states that teams generating null audiences in all the games receive nothing. We also characterize *concede-and-divide* as the unique rule satisfying equal treatment of equals, *additivity*, and the so-called *essential team axiom*, which states that teams without whom game audiences are null receive their whole audience. Obviously, it follows from Theorem 1 that no other rule within the family of $EC$ rules, different from the *equal-split* rule, satisfies null team. Likewise, no other rule within the family, different from *concede-and-divide*, satisfies essential team.

As a consequence of Theorem 1, we can give a characterization of the *equal-split rule* alternative to the one provided in Bergantiños and Moreno-Ternero (2019) by replacing the *null team axiom* by the combination of *maximum aspirations* and *non negativity.* Formally,

**Non negativity**. For each $(N, A) \in \mathcal{P}$ and each $i \in N$,

$$R_i(N, A) \geq 0.$$

**Corollary 1** *A rule satisfies additivity, equal treatment of equals, maximum aspirations and non negativity if and only if it is the equal-split rule.*

**Proof.** By Theorem 1, we know that the *equal-split* rule satisfies *equal treatment of equals*, *additivity* and *maximum aspirations.* It is obvious that it also satisfies *non negativity.*

Conversely, let $R$ be a rule satisfying the four properties. By Theorem 1, $R$ belongs to the $EC$-family. Thus, there exists $\lambda \in [0,1]$ such that, for each $(N, A) \in \mathcal{P}$,

$$R(N, A) = \lambda ES(N, A) + (1 - \lambda) CD(N, A).$$

Suppose, by contradiction, that $\lambda < 1$. Then,

$$R_3\left(\{1, 2, 3\}, 1^{12}\right) = (1 - \lambda)(-1) < 0,$$

which contradicts *non negativity*. Thus, $\lambda = 1$ and, hence, $R \equiv ES$. ■

Now, given a problem $(N, A) \in \mathcal{P}$, and $i \in N$, one might be interested in identifying the set of rules within the family that yield a positive amount to team $i$. Here is a clear-cut answer to that question:

**Proposition 1** *For each $(N, A) \in \mathcal{P}$ and each $i \in N$, we have the following:*

*(a) If $\alpha_i \geq \overline{\alpha}$, then $EC_i^{\lambda}(N, A) \geq 0$, for each $\lambda \in [0, 1]$.*

*(b) If $\alpha_i < \overline{\alpha}$, then $EC_i^{\lambda}(N, A) \geq 0$ if and only if*

$$\lambda \geq 1 - \frac{(n-2)\alpha_i}{n\left(\overline{\alpha} - \alpha_i\right)}.$$

Proposition 1 says that for each rule within the family, teams with an audience above average will get a non-negative amount under. Teams with an audience below average will get a non-negative amount depending on the relationship between $\alpha_i$ and $\overline{\alpha}$. When $\alpha_i$ is relatively small with respect to $\overline{\alpha}$, we need a larger $\lambda$ for *non-negativity.* The only case always guaranteeing a non-negative allocation to each agent is $\lambda = 1$, as stated in Corollary 1.

**Proof.** Let $(N, A) \in \mathcal{P}$, $i \in N$, and $\lambda \in [0, 1]$. By equation $(1)$, $EC_i^{\lambda}(N, A) \geq 0$ if and only if

$$\lambda \frac{\alpha_i}{2} + (1-\lambda) \frac{(n-1)\,\alpha_i - ||A||}{n-2} \geq 0.$$

Or, equivalently,

$$(n-2)\,\lambda\alpha_i + 2\,(1-\lambda)\,[(n-1)\,\alpha_i - ||A||] \geq 0.$$

As

$$||A|| = \frac{\sum\limits_{i \in N} \alpha_i}{2}, \text{ and } \overline{\alpha} = \frac{\sum\limits_{i \in N} \alpha_i}{n},$$

we deduce that

$$||A|| = \frac{n\overline{\alpha}}{2}.$$

Then, $EC_i^{\lambda}(N, A) \geq 0$ if and only if

$$(n-2)\,\lambda\alpha_i + 2\,(1-\lambda)\left[(n-1)\,\alpha_i - \frac{n\overline{\alpha}}{2}\right] \geq 0.$$

Equivalently,

$$\lambda n\alpha_i - 2\lambda\alpha_i + 2n\alpha_i - 2\alpha_i - 2\lambda n\alpha_i + 2\lambda\alpha_i - n\overline{\alpha} + \lambda n\overline{\alpha} \geq 0,$$

or

$$\lambda n\left(\overline{\alpha}-\alpha_i\right)\geq n\overline{\alpha}-2n\alpha_i+2\alpha_i. \tag{3}$$

We now consider three cases:

**Case** $\alpha_i > \overline{\alpha}$.

In this case, (3) is equivalent to

$$\lambda \leq \frac{n\overline{\alpha}-2n\alpha_i+2\alpha_i}{n\left(\overline{\alpha}-\alpha_i\right)} = 1-\frac{(n-2)\,\alpha_i}{n\left(\overline{\alpha}-\alpha_i\right)}.$$

As $\overline{\alpha}-\alpha_i<0$ we deduce that

$$1-\frac{(n-2)\,\alpha_i}{n\left(\overline{\alpha}-\alpha_i\right)}\geq 1,$$

and hence (3) holds for any $\lambda \in [0,1]$.

**Case** $\alpha_i = \overline{\alpha}$.

In this case, (3) is equivalent to $0 \geq (2-n)\,\alpha_i$, which always holds.

**Case** $\alpha_i < \overline{\alpha}$.

In this case, (3) is equivalent to

$$\lambda \geq \frac{n\overline{\alpha}-2n\alpha_i+2\alpha_i}{n\left(\overline{\alpha}-\alpha_i\right)} = 1-\frac{(n-2)\,\alpha_i}{n\left(\overline{\alpha}-\alpha_i\right)},$$

as stated in the proposition. ■

# 4 Further insights

We concentrate in this section on the family just characterized and explore further properties of it. We first study teams' preferences with respect to the rules within the family. Then, we turn to the distributional effects of those rules.

In what follows, we assume, without loss of generality, that, for each $(N,A)\in\mathcal{P}$, $N=\{1,\ldots,n\}$ and $\alpha_1\leq\alpha_2\leq\cdots\leq\alpha_n$, with at least one strict inequality.[9]

[9] Otherwise, the problem would be trivially solved, as all rules within our family would yield the same allocation.

## 4.1 Majority preferences

We begin by showing that the rules within the $EC$-family satisfy the so-called *single-crossing* property.[10] Formally,

**Proposition 2** *Let $0 \leq \lambda_1 \leq \lambda_2 \leq 1$, and $(N, A) \in \mathcal{P}$. Then, there exists $i^* \in N$ such that:*

*(i) $EC_i^{\lambda_1}(N, A) \leq EC_i^{\lambda_2}(N, A)$ for each $i = 1, ..., i^*$ and*

*(ii) $EC_i^{\lambda_1}(N, A) \geq EC_i^{\lambda_2}(N, A)$ for each $i = i^* + 1, ..., n$.*

**Proof.** Let $0 \leq \lambda_1 \leq \lambda_2 \leq 1$, and $(N, A) \in \mathcal{P}$.

We consider two cases:

1. **Case** $\alpha_i \leq \overline{\alpha}$. In this case,

$$\begin{aligned} EC_i^{\lambda_1}(N, A) &= \frac{\alpha_i}{2} + \frac{n(1-\lambda_1)}{2(n-2)}(\alpha_i - \overline{\alpha}) \\ &= \frac{\alpha_i}{2} + \frac{n(\lambda_1 - 1)}{2(n-2)}(\overline{\alpha} - \alpha_i) \\ &\leq \frac{\alpha_i}{2} + \frac{n(\lambda_2 - 1)}{2(n-2)}(\overline{\alpha} - \alpha_i) \\ &= EC_i^{\lambda_2}(N, A). \end{aligned}$$

2. **Case** $\alpha_i > \overline{\alpha}$. In this case,

$$\begin{aligned} EC_i^{\lambda_1}(N, A) &= \frac{\alpha_i}{2} + \frac{n(1-\lambda_1)}{2(n-2)}(\alpha_i - \overline{\alpha}) \\ &\geq \frac{\alpha_i}{2} + \frac{n(1-\lambda_2)}{2(n-2)}(\alpha_i - \overline{\alpha}) \\ &= EC_i^{\lambda_2}(N, A). \end{aligned}$$

It turns out that $i^*$ is precisely the team whose overall audience is closest (from below) to the average overall audience. ■

Given a problem $(N, A) \in \mathcal{P}$ we say that $EC^{\lambda}(N, A)$ is a *majority winner* (within the $EC$-family) for $(N, A)$ if there is no other rule $EC^{\lambda'}$ (within the family) such that $EC_i^{\lambda'}(N, A) > EC_i^{\lambda}(N, A)$ for a majority of teams. That is, there is no other rule $EC^{\lambda'}$ (within the family) such that

$$\left|\left\{i \in N : EC_i^{\lambda'}(N, A) > EC_i^{\lambda}(N, A)\right\}\right| > \left|\left\{i \in N : EC_i^{\lambda'}(N, A) \leq EC_i^{\lambda}(N, A)\right\}\right|.$$

[10] This feature is also shared by the one-parameter rule of taxation methods introduced by Moreno-Ternero (2011).

We say that the family of $EC$ rules has a *majority voting equilibrium* if there is at least one majority winner (within the $EC$-family) for each problem $(N, A) \in \mathcal{P}$.

It is well known that the single-crossing property of preferences is a sufficient condition for the existence of a majority voting equilibrium (e.g., Gans and Smart, 1996). Thus, we have the following corollary from Proposition 2.

**Corollary 2** *There is a majority voting equilibrium for the family of EC rules.*

We now study which specific $EC$ rule could be a majority winner for each problem. We obtain three different scenarios, depending on the characteristics of the problem at stake. For some problems, only the *equal-split* rule is a majority winner. For some problems, only *concede-and-divide* is a majority winner. For the remainder of the problems, each $EC$ rule is a majority winner.

For each $(N, A) \in \mathcal{P}$, we consider the following partition of $N$, with respect to the average claim $(\bar{\alpha})$:

$$\begin{aligned} N_l(A) &= \{i \in N : \alpha_i < \bar{\alpha}\}, \\ N_u(A) &= \{i \in N : \alpha_i > \bar{\alpha}\}, \text{ and} \\ N_e(A) &= \{i \in N : \alpha_i = \bar{\alpha}\}. \end{aligned}$$

When no confusion arises, we simply write $N_l$, $N_u$, and $N_e$.

**Proposition 3** *Let $(N, A) \in \mathcal{P}$. The following statements hold:*
*(i) If $|N_l| > |N_u| + |N_e|$, then $ES(N, A)$ is the unique majority winner.*
*(ii) If $|N_u| > |N_l| + |N_e|$, then $CD(N, A)$ is the unique majority winner.*
*(iii) Otherwise, each $EC^{\lambda}(N, A)$ is a majority winner.*

**Proof.** Let $0 \leq \lambda \leq 1$, and $(N, A) \in \mathcal{P}$. By (1), for each $i \in N$,

$$EC_i^{\lambda}(N, A) = \frac{\alpha_i}{2} + \frac{n(1-\lambda)}{2(n-2)}(\alpha_i - \overline{\alpha}).$$

If $\alpha_i < \bar{\alpha}$, then $EC_i^{\lambda}(N, A)$ is an increasing function of $\lambda$, thus maximized at $\lambda = 1$. This implies that, for each $i \in N_l$, $ES_i(N, A)$ is the most preferred outcome (among those provided by the family).

If $\alpha_i > \bar{\alpha}$, then $EC_i^{\lambda}(N, A)$ is a decreasing function of $\lambda$, thus maximized at $\lambda = 0$. This implies that, for each $i \in N_u$, $CD(N, A)$ is the most preferred outcome (among those provided by the family).

If $\alpha_i = \bar{\alpha}$, then $EC_i^{\lambda}(N, A) = \frac{\alpha_i}{2}$ for each $\lambda \in [0, 1]$. this implies that, for each $i \in N_e$, all rules in the family yield the same outcome.

From the above, statements $(i)$ and $(ii)$ follow trivially. Assume, by contradiction, that statement $(iii)$ does not hold. Then, there exists $(N, A) \in \mathcal{P}$ and $\lambda \in [0, 1]$ such that $EC^{\lambda}$ is not a majority winner for $(N, A)$. Thus, we can find $\lambda' \in [0, 1]$ such that $EC_i^{\lambda'}(N, A) > EC_i^{\lambda}(N, A)$ holds for the majority of the teams. We then consider two cases:

**Case** $\lambda' > \lambda$.

In this case, $EC_i^{\lambda'}(N, A) > EC_i^{\lambda}(N, A)$ if and only if $i \in N_l$. Now,

$$\begin{aligned}
|N_l| &= \left|\left\{i \in N : EC_i^{\lambda'}(N, A) > EC_i^{\lambda}(N, A)\right\}\right| \\
&> \left|\left\{i \in N : EC_i^{\lambda'}(N, A) \leq EC_i^{\lambda}(N, A)\right\}\right| \\
&= |N_u| + |N_e|
\end{aligned}$$

which is a contradiction.

**Case** $\lambda' < \lambda$.

In this case, $EC_i^{\lambda'}(N, A) > EC_i^{\lambda}(N, A)$ if and only if $i \in N_u$. Now,

$$\begin{aligned}
|N_u| &= \left|\left\{i \in N : EC_i^{\lambda'}(N, A) > EC_i^{\lambda}(N, A)\right\}\right| \\
&> \left|\left\{i \in N : EC_i^{\lambda'}(N, A) \leq EC_i^{\lambda}(N, A)\right\}\right| \\
&= |N_l| + |N_e|
\end{aligned}$$

which is a contradiction. ■

The previous results imply that if the distribution of claims is skewed to the left, then the *equal-split* allocation is the majority winner, whereas if it is skewed to the right, then the *concede-and-divide* allocation is the majority winner. If it is not skewed, then any allocation within the family can be a majority winner.

The single-crossing property of preferences also guarantees that the social preference relationship obtained under majority voting is transitive, and corresponds to the median voter's. In our setting, the median voter corresponds to the team with the median overall audience (claim).

Depending on whether the number of teams is odd or even, the median can be uniquely determined or not. To avoid ambiguity, we consider in each case the median to be the mean of the two middle values. Formally, the *median overall audience* is defined by

$$\alpha_m = \begin{cases} \alpha_{\frac{n+1}{2}} & \text{if } n \text{ is odd} \\ \frac{1}{2}\left(\alpha_{\frac{n}{2}} + \alpha_{\frac{n+2}{2}}\right) & \text{otherwise.} \end{cases}$$

Depending on whether this median overall audience is below or above the average audience, the median voter's preferred rule (and, thus, the majority winner) will either be the *equal-split* rule or *concede-and-divide*. More precisely,

**Corollary 3** *Let $(N, A) \in \mathcal{P}$ be such that $n$ is odd. The following statements hold:*

*$(i)$ If $\alpha_m < \bar{\alpha}$, then $ES(N, A)$ is the unique majority winner.*

*$(ii)$ If $\alpha_m > \bar{\alpha}$, then $CD(N, A)$ is the unique majority winner.*

*$(iii)$ If $\alpha_m = \bar{\alpha}$, then any $EC^{\lambda}(N, A)$ is a majority winner.*

**Proof.** If $\alpha_m < \bar{\alpha}$, then $|N_l| \geq m$. Hence $|N_l| > |N_u| + |N_e|$. By Proposition 3, statement $(i)$ holds.

If $\alpha_m > \bar{\alpha}$, then $|N_u| \geq m$. Hence $|N_u| > |N_l| + |N_e|$. By Proposition 3, statement $(ii)$ holds.

If $\alpha_m = \bar{\alpha}$, then $|N_l| < m$, $|N_u| < m$, and $|N_e| > 0$. Hence, we are in case $(iii)$ of the statement of Proposition 3, which concludes the proof. ■

**Corollary 4** *Let $(N, A) \in \mathcal{P}$ be such that $n$ is even. The following statements hold:*

*$(i)$ If $\alpha_{\frac{n+2}{2}} < \bar{\alpha}$, then $ES(N, A)$ is the unique majority winner.*

*$(ii)$ If $\alpha_{\frac{n}{2}} > \bar{\alpha}$, then $CD(N, A)$ is the unique majority winner.*

*$(iii)$ If $\alpha_{\frac{n}{2}} \leq \bar{\alpha} \leq \alpha_{\frac{n+2}{2}}$, then any $EC^{\lambda}(N, A)$ is a majority winner.*

**Proof.** If $\alpha_{\frac{n+2}{2}} < \bar{\alpha}$, then $|N_l| \geq m$. Hence $|N_l| > |N_u| + |N_e|$. By Proposition 3, statement $(i)$ holds.

If $\alpha_{\frac{n}{2}} > \bar{\alpha}$, then $|N_u| \geq m$. Hence $|N_u| > |N_l| + |N_e|$. By Proposition 3, statement $(ii)$ holds.

Suppose now that $\alpha_{\frac{n}{2}} \leq \bar{\alpha} \leq \alpha_{\frac{n+2}{2}}$. Then, it is enough to prove that we are in case $(iii)$ of the statement of Proposition 3. That is, we have to prove that neither $|N_l| > |N_u| + |N_e|$ nor $|N_u| > |N_l| + |N_e|$ hold. We consider several subcases:

1. If $\bar{\alpha} = \alpha_{\frac{n}{2}}$, then $|N_l| < \frac{n}{2}$, $|N_u| \leq \frac{n}{2}$ and $|N_e| > 0$.

2. If $\alpha_{\frac{n}{2}} < \bar{\alpha} < \alpha_{\frac{n+2}{2}}$, then $|N_l| = \frac{n}{2}$, $|N_u| = \frac{n}{2}$ and $|N_e| = 0$.

3. If $\bar{\alpha} = \alpha_{\frac{n+2}{2}}$, then $|N_l| \leq \frac{n}{2}$, $|N_u| < \frac{n}{2}$ and $|N_e| > 0$.

In either case, the desired conclusion holds. ■

## 4.2 On the distributive power of the rules

We now turn to the distributional effects of the rules within the family. More precisely, we show that the rules within the family are completely ranked according to the so-called Lorenz dominance criterion, the most fundamental criterion of income inequality.

Formally, given $x, y \in \mathbb{R}^n$ satisfying $x_1 \leq x_2 \leq ... \leq x_n$, $y_1 \leq y_2 \leq ... \leq y_n$, and $\sum_{i=1}^n x_i = \sum_{i=1}^n y_i$, we say that $x$ *is greater than* $y$ *in the Lorenz ordering* if $\sum_{i=1}^k x_i \geq \sum_{i=1}^k y_i$, for each $k = 1, ..., n-1$, with at least one strict inequality. This criterion induces a partial ordering on allocations which reflects their relative spread. When $x$ is greater than $y$ in the Lorenz ordering, the distribution $x$ is unambiguously "more egalitarian" than the distribution $y$ (e.g., Dasgupta, Sen and Starret, 1973).

In our setting, we say that a rule $R$ is **more egalitarian** than another $R'$ if for each $(N, A) \in \mathcal{P}$, $R(N, A)$ is greater than $R'(N, A)$ in the Lorenz ordering.

As mentioned above, the Lorenz ordering is only a partial ordering. Thus, one should not expect many rules to be ranked according to the egalitarian criterion just described. Nevertheless, as the next result shows, the rules within the $EC$-family are fully ranked according to the parameter that defines the family.[11] This parameter can therefore be interpreted as an index of the distributive power of the rule.

**Proposition 4** *If* $0 \leq \lambda_1 \leq \lambda_2 \leq 1$ *then,* $EC^{\lambda_2}$ *is more egalitarian than* $EC^{\lambda_1}$.

**Proof.** Let $(N, A) \in \mathcal{P}$.

We first prove that $ES(N, A)$ is greater than $CD(N, A)$ in the Lorenz ordering.

Let $i \in N$. By equation (1),

$$CD_i(N, A) = \frac{\alpha_i}{2} + \frac{n}{2(n-2)}(\alpha_i - \overline{\alpha}).$$

[11] Although we provide a direct proof for this result, it can also be derived as a consequence of Proposition 2.

Thus,

$$\begin{aligned} ES_1(N,A) &\leq ES_2(N,A) \leq \ldots \leq ES_n(N,A) \text{ and} \\ CD_1(N,A) &\leq CD_2(N,A) \leq \ldots \leq CD_n(N,A). \end{aligned} \tag{4}$$

It then suffices to show that, for each $k = 1, \ldots, n-1$,

$$\sum_{i=1}^{k} \frac{\alpha_i}{2} \geq \sum_{i=1}^{k} \left( \frac{\alpha_i}{2} + \frac{n}{2(n-2)} (\alpha_i - \overline{\alpha}) \right).$$

But this is simply a consequence of the fact that

$$\sum_{i=1}^{k} \alpha_i \leq k\overline{\alpha},$$

for each $k = 1, \ldots, n-1$.

We now prove that $EC^{\lambda_2}(N,A)$ is greater than $EC^{\lambda_1}(N,A)$ for each $0 \leq \lambda_1 \leq \lambda_2 \leq 1$. By (4), we have that

$$\begin{aligned} EC_1^{\lambda_1}(N,A) &\leq EC_2^{\lambda_1}(N,A) \leq \ldots \leq EC_n^{\lambda_1}(N,A) \text{ and} \\ EC_1^{\lambda_2}(N,A) &\leq EC_2^{\lambda_2}(N,A) \leq \ldots \leq EC_n^{\lambda_2}(N,A). \end{aligned}$$

Then, it suffices to show that, for each $k = 1, \ldots, n-1$,

$$\sum_{i=1}^{k} EC_i^{\lambda_2}(N,A) \geq \sum_{i=1}^{k} EC_i^{\lambda_1}(N,A).$$

Now,

$$\begin{aligned} \sum_{i=1}^{k} \left[ \frac{\alpha_i}{2} + \frac{n(1-\lambda_2)}{2(n-2)} (\alpha_i - \overline{\alpha}) \right] &\geq \sum_{i=1}^{k} \left[ \frac{\alpha_i}{2} + \frac{n(1-\lambda_1)}{2(n-2)} (\alpha_i - \overline{\alpha}) \right] \Leftrightarrow \\ \sum_{i=1}^{k} \frac{n(1-\lambda_2)}{2(n-2)} (\alpha_i - \overline{\alpha}) &\geq \sum_{i=1}^{k} \frac{n(1-\lambda_1)}{2(n-2)} (\alpha_i - \overline{\alpha}) \Leftrightarrow \\ (1-\lambda_2) \sum_{i=1}^{k} (\alpha_i - \overline{\alpha}) &\geq (1-\lambda_1) \sum_{i=1}^{k} (\alpha_i - \overline{\alpha}). \end{aligned}$$

As $\sum_{i=1}^{k} (\alpha_i - \overline{\alpha}) \leq 0$ and $\lambda_1 \leq \lambda_2$, the above follows. ∎

# 5 An empirical application

In this section, we present an empirical application of our model resorting to La Liga, the Spanish Football League.

La Liga is a standard round robin tournament involving 20 teams. Thus, each team plays 38 games, facing each time one of the other 19 teams (once home, another away). The 20 teams, and the overall audience (in millions) of each team during the season 2017-2018, are listed in the first two columns of Table 1.[12]

Insert Table 1 about here

Note that the total audience of the entire season is $197,05$ millions, and the total revenue was $1325,6$ millions of euros. Thus, in order to accommodate the premises of our model and identify total audience with total revenue, we have to assume that each viewer paid a pay-per-view fee of 6.73 euros (instead of only one) per game. This normalizing assumption appears in Column 3. The resulting scaling will be implicit in the next tables describing the allocations.

Columns 4 and 5 give the allocation put in practice for the season 2017-18 (in millions of euros and in percentage terms).[13] As we can see, two teams (Barcelona and Real Madrid) dominated the sharing collecting (when combined) almost 23% of the pie.

Table 2 lists again the allocation put in practice for the season 2017-18, but now together with the ones proposed by the two extreme rules of the $EC$-family (the *equal-split* rule and *concede-and-divide*). In the last column of this table we explore whether the amount obtained by each team in the allocation used in practice corresponds to some rule in the $EC$ family. For instance, Barcelona receives the amount that the rule $EC^{0.98}$ would yield for this setting. In contrast, Real Madrid receives less than the amount proposed by any rule within the family because $148 < \min\{158.43, 260.81\}$. On the other hand, Atlético de Madrid receives more than the amount proposed by any rule within the family because $110.60 > \max\{85.77, 107.43\}$.

Insert Table 2 about here

[12] The source for most of the data provided here is Palco 23, the leading newspaper in economic information of the sport business in Spain. Palco 23 refers itself to Havas Sports and Entertainment as its source. See, for instance, https://www.palco23.com/competiciones/del-barca-al-numancia-que-clubes-cobraron-mas-de-laliga-por-tv.html

[13] The source is La Liga's website. See, for instance, http://www.laliga.es/lfp/reparto-ingresos-audiovisuales

Several conclusions can be derived from Table 2. Maybe the most obvious one is that, contrary to what some might argue, the actual revenue sharing seems to be biased against the two powerhouses. Barcelona receives approximately the minimum it could receive, whereas Real Madrid receives even less than the minimum. With *concede-and-divide* (one of the extreme rules within the family), Barcelona and Real Madrid together would receive 38.28% of the pie (instead of the 22.78% they actually receive).

Another conclusion is that nine teams are favored by the actual allocation, in the sense that the amount each gets is above the amounts suggested by all the rules within the family.

Apart from Real Madrid, only one team (Betis) obtains amounts below those suggested by the two rules. It is actually a remarkable case, as the allocation yields 3.99%, whereas the two rules would recommend 7.1% and 9.44%, respectively.

The remaining nine teams obtain amounts that can be rationalized by some rule within the $EC$-family. However, the rule would be different for each team. For instance, for Barcelona, it would be the rule corresponding to $\lambda = 0.98$ (which means that it receives something quite similar to the *equal-split* outcome) but for Celta, it would be the rule corresponding to $\lambda = 0.02$ (which means that it receives something quite similar to the *concede-and-divide* outcome).

In Bergantiños and Moreno-Ternero (2019) we essentially divide the viewers of each game in two categories: *fans* and *no fans*. As the name suggests, the former are those watching the game because they are fans of one of the teams playing. The latter are those watching the game because they thought that the specific combination of teams rendered the game interesting. We argue that the revenue generated by a fan should be allocated to the corresponding team, whereas the revenue generated by the no fans should be divided equally between both teams.

The *equal-split* rule and *concede-and-divide* are two extreme rules from the point of view of treating fans. The former assumes that there are no fans. The latter assumes that there are as many fans as possible (compatible with the real data). Thus, the allocation obtained by a team should be somewhat in between the allocations proposed by both rules to such a team.

In practice, we know the total number of viewers of each game, but not the partition in the two categories mentioned above. Now, it is feasible to estimate the average number of fans and no fans watching the games. For instance, we can take a sample of viewers and ask them to report the games they have watched, and if they are fans of some team. Let $f$ denote the number of people who have watched a game being a fan of some of the teams. Let $f^n$ denote the number of people who have watched a game without being a fan of any of the teams. Let us define $\bar{\lambda} = \frac{f^n}{f+f^n}$ as the percentage of no fans watching a game. Similarly, $1 - \bar{\lambda} = \frac{f}{f+f^n}$ is the percentage of fans watching a game. We argue that $EC^{\bar{\lambda}}$ should be a salient rule among those within the family.

Based on the above, the parameter $1 - \lambda$ could be interpreted as the percentage of viewers who watch a game because they are fans of one of the teams playing the game. Similarly, $\lambda$ could be interpreted as the percentage of viewers who watch a game without being a fan of one of the teams playing the game. In the case of Barcelona mentioned above, $\lambda = 0.98$ indicates that, among those watching a Barcelona game, there is approximately the same number of Barcelona fans as of fans for the opposite team. This is quite counterintuitive because the audiences of Barcelona games are much larger than the audiences of all other games (excluding those involving Real Madrid).

Table 3 compares the allocation implemented by La Liga with two allocations selected by rules in our family: $ES$ and $EC^{0.25}$.

Insert Table 3 about here

The rule $EC^{0.25}$ was simply chosen based on our intuition. We believe that most of the viewers of a game are fans of one of the teams. Thus, we chose a relatively small $\lambda$. Nevertheless, and somewhat surprisingly, we obtain that the rule within the $EC$-family yielding a closer allocation to the allocation of La Liga (according to the Euclidean distance) is the rule corresponding to $\lambda = 1$, i.e., the *equal-split* rule.

If we compare $ES$ with the allocation implemented in La Liga we realize that one team (Betis) obtains much less (41 millions of euros). Other nine teams (including Real Madrid) also obtain less (between 1 and 10 millions). The remaining ten teams (including Barcelona) obtain more (between 0 and 25 millions).

If we compare $EC^{0.25}$ with the allocation implemented in La Liga the situation is even more extreme. Three teams (Barcelona, Betis and Real Madrid) obtain much less (between 64 and 87 millions of euros). One team (Celta) obtains 1.63 millions less. The remaining sixteen teams obtain more (between 0 and 35 millions). Thus, according to $EC^{0.25}$, the allocation implemented by La Liga favors teams with lower audiences.

It has been argued that an extremely unequal sharing of the broadcasting revenues would be detrimental to the overall quality of the tournament. Thus, we consider alternative schemes with our database. More precisely, we present hybrid schemes in which a portion of the overall revenue is divided equally, another is divided according to performance, and the residual is divided according to one of our two rules (thus, only taking into account audiences). Note that this is indeed what happens in most important European football leagues. La Liga itself implemented a new scheme along those lines, in which half of the overall revenue was shared equally, whereas one quarter was shared according to league performance and the remaining quarter according to what they dubbed social relevance. The details of this new scheme, which was actually sanctioned by the Spanish government, appeared in the Official Bulletin of the Spanish State on May 1st, 2015.

Table 4 summarizes the outcomes that the *equal-split* and *concede-and-divide* rules would yield when modified to endorse the hybrid scheme implemented by La Liga. More precisely, we assume that half of the overall revenue is shared equally (that would represent 33.14 million euros for each team), whereas one quarter is shared according to league performance and the remaining quarter according to social performance (where we apply our two rules). By league performance, La Liga refers to the standings at the end of the previous five seasons (where a zero score is given to those teams that played in the second division, or below, in one of those years). One quarter of the budget is then allocated proportionally to those 5-year standings. By social performance, La Liga assigns one third (of the corresponding one quarter) proportionally to the revenues generated from ticket sales in the last five seasons.[14] The other two thirds (of that one quarter) are supposed to be assigned according to audiences. We then consider our *equal-split* and *concede-and-divide* rules for that portion of the budget. More precisely, the fifth and sixth columns of Table 4 provide the amounts suggested by each rule for the division of one sixth of the budget, whereas the last two columns are the result of aggregating (for each team) those amounts with the fixed amount (33.14 million) and the proportional amounts to league performance and ticket sales.

Insert Table 4 about here

[14] For this, we consider data on season tickets for the last two seasons, which are the only ones available (again, obtained from Palco 23). See, for instance, https://www.palco23.com/clubes/los-clubes-arrancan-la-liga-santander-con-cerca-de-600-000-abonados.html and https://www.palco23.com/clubes/los-clubes-de-primera-y-segunda-rozan-los-800000-abonados-en-2017-2018.html

An obvious observation to make from Table 4 is that the hybrid schemes become more egalitarian. More precisely, under the *equal-split rule* itself, the two powerhouses obtain (combined) 23.39% of the pie. The hybrid scheme lowers this to 20.46%. Under *concede-and-divide* itself, the two powerhouses obtain (combined) 38.27%, which now drastically moves down (under the hybrid scheme) to 22.95%.

As mentioned above, one sixth of the total budget (around 221 millions of euros) is assigned according to audiences. In such a case, we can compute directly the *equal-split* and *concede-and-divide* allocations. Besides, if we subtract from the total allocation obtained by each team the corresponding amounts listed in Columns 2, 3 and 4 from Table 4, we obtain the way in which La Liga allocates the amount corresponding to audiences among the teams.[15] This appears in Column 4 of Table 5.

Insert Table 5 about here

In the last column of Table 5 we perform the same exercise as in Table 2 (mentioned above). In this case, we observe that six teams are favored by the actual allocation, in the sense that the amount each gets is above the amounts suggested by any member of the $EC$-family. Five teams obtain amounts below those suggested by the members of the $EC$-family. The remaining nine teams obtain amounts suggested by the member of the $EC$-family given by the corresponding cell in the last column. In this case, the allocation implemented by La Liga does not favor teams with lower audiences. It seems to be quite uniform in that aspect.

We now obtain that the rule within the $EC$-family that yields a closer allocation to the allocation given by Column 4 in Table 5 (according to the Euclidean distance) is the rule corresponding to $\lambda = 0.29$. Thus, we compare in Table 6 the allocation being implemented by La Liga with that provided by $EC^{0.29}$. One team (Betis) obtains in the allocation implemented by La Liga around 14 millions of euros less than with $EC^{0.29}$. Other seven teams also obtain less (between 1 and 7 millions). The remaining twelve teams (including Barcelona and Real Madrid) obtain more (between 0 and 5 millions).

Insert Table 6 about here

[15] Note that the data from Columns 2 and 3 in Table 4 are the ones used by La Liga, but the data from Column 4 are estimations.

# 6 Discussion

We have studied the problem of sharing the revenues from broadcasting sport events, as recently introduced by Bergantiños and Moreno-Ternero (2019). We have considered three basic and intuitive axioms for such a problem. Together, the three axioms characterize a family of rules that offer a compromise between two focal and somewhat polar rules: the *equal-split* rule and *concede-and-divide*. As such, the family is flexible enough to accommodate a wide variety of views regarding the existence of fans associated to each participating team. It ranges from the extreme view that, de facto, dismisses the existence of those fan bases (as exemplified by the *equal-split* rule) to the polar (and, thus, extreme too) view that minimizes the number of *casual viewers*, who simply watch a game because they are interested into the specific pair of teams involved in it (as exemplified by *concede-and-divide*).

We have also shown that the family has other merits. For instance, it constitutes a domain of rules for which majority voting equilibrium exists. Also, the rules within the family are fully ranked according to the Lorenz dominance criterion.

Our family of rules is reminiscent of some other families that have been considered in the literature on related topics (such as income inequality measurement, surplus sharing, cost allocation, or claims problems). Some of these families also offer compromises between focal and somewhat polar rules. Others share with ours the structure regarding the order of their members (according to the spread of the outcomes they yield), or the majority preferences (with respect to the members of the family).

We have also applied the rules within our family to a real-life situation. More precisely, we have explored the allocation of the (joint) revenues collected from selling broadcasting rights in the case of La Liga, the Spanish Football League. Our analysis indicates that the family can essentially accommodate the real-life outcomes we observe, especially when the rules within our family are combined with performance measures and lower bounds guaranteed for each participating team.

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