Fiat Money as a Public Signal, Medium of Exchange, and Punishment

Gomis-Porqueras, Pedro and Sun, Ching-jen

Department of Economics, Deakin University, Department of Economics, Deakin University

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Fiat Money as a Public Signal, Medium of Exchange, and Punishment*

Pedro Gomis-Porqueras[†] Ching-Jen Sun[‡]

Deakin University Deakin University

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Abstract

This paper studies different welfare-enhancing roles that fiat money can have. To do so, we consider an indivisible monetary framework where agents are randomly and bilaterally matched and the government has weak enforcement powers. Within this environment, we analyze state contingent monetary policies and characterize the resulting equilibria under different government record-keeping technologies. We show that a threat of injecting fiat money, conditional on private actions, can improve allocations and achieve efficiency. This type of state contingent policy is effective even when the government cannot observe any private trades and agents can only communicate with the government through *cheap talk*. In all these equilibria fiat money and self-enforcing credit are complements in the off equilibrium. Finally, this type of equilibria can also emerge even when the injection of fiat money is not a public signal.

JEL Codes: E00, E40, C73, D82.

Keywords: cheap talk; record-keeping; fiat money.

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[†]Address: Deakin University, Department of Economics, 70 Elgar Road, Burwood, VIC 3125, Australia. E-mail: peregomis@gmail.com.

[‡]Address: Deakin University, Department of Economics, 70 Elgar Road, Burwood, VIC 3125, Australia. E-mail: cjsun@deakin.edu.au.

"The master does nothing, yet he leaves nothing undone. The Tao never does anything, yet through it all things are done."

Tao Te Ching by Laozi

1 Introduction

In Friedman's (1969) influential paper, the notion of the optimum quantity of money has become one of the most widely debated propositions in monetary economics. Since then the optimality and efficiency of the Friedman rule has been shown to be robust across different monetary environments.[1] In particular, in environments with divisible fiat money and some decentralized trades, efficiency can be achieved if the monetary authority can generate deflation and the trading protocol in anonymous markets is monotone in the buyer's surplus.[2] First best allocations can also be obtained without taxation, when agents are sufficiently patient and the equilibrium has the coalition-proof implementability property.[3] Thus, unless the previous conditions are satisfied, pure monetary equilibria is then inefficient.

By exploiting the fact that pure monetary equilibria can be inefficient when agents are not sufficiently patient and taxation is not feasible, we highlight different welfare-enhancing roles that fiat money can have on and off the equilibrium path. To do so, we consider the indivisible monetary framework of Kiyotaki and Wright (1993). Agents are randomly and bilaterally matched and discount the future. The discount rate is not high enough so that Araujo's (2004) gift-giving (no self-enforcing credit) equilibrium cannot be supported. There is also a government that issues indivisible fiat money, however, it

[1]We refer the reader to Woodford (1990), Chari, Christiano and Kehoe (1996) and Williamson and Wright (2010a,b) for more on this topic.

[2]Aruoba, Rocheteau and Waller (2007) show how efficiency, in the Lagos and Wright (2005) framework, can be attained under the Friedman rule if the trading protocol is monotone in the buyer's surplus. When the surplus is not monotone, Gomis-Porqueras et al (2010) show the need of fiscal policy to obtain the efficient allocation. We refer to Nosal and Rocheteau (2010) for an excellent exposition of these results.

[3]Hu, Kennan and Wallace (2009) show that one can obtain the first best without Friedman rule in the Lagos and Wright (2005) when the equilibrium is coalition-proof implementable and when agents are sufficiently patient. When policies are incentive compatible, Andolfatto (2010) shows that a government that pays interest on money holdings and can record the identity of agents that receive such payment delivers efficient allocations.

can not collect taxes, making all trades voluntary. Finally, following Kandori (1992), we assume that private agents cannot observe the trade history of other agents. Within this environment, we characterize the equilibrium consistent with state contingent monetary policies under different government record-keeping technologies.[4]

The search-theoretic approach to monetary economics argues that the important role of fiat money is its role as a medium of exchange as it increases welfare (Kiyotaki and Wright (1991,1993)). The rationale is that fiat money, when functions as a medium of exchange, can (partially) resolve the double coincidence of wants problem and hence enhance welfare. Under various information structures, this paper demonstrates that it is *not essential* for fiat money to function as a medium of exchange on the equilibrium path for it to play a welfare-enhancing role in society. Specifically, we show that contingent monetary policies that threaten to inject fiat money based on different information sets available to the government can help sustain efficient allocations. Under perfect and partial record-keeping, gift-exchange can be sustained as a sequential equilibrium, when fiat money is injected whenever the government detects a deviation from gift-giving. When the government can not observe any private trade but agents can send costless and non-verifiable signals to the government at the end of each bilateral meeting (cheap talk), a slightly more elaborate contingent money injection policy, based on the received cheap talk from agents, is required to achieve efficient allocations. Since reporting a deviation triggers a monetary injection, proper incentives have to be given to agents, ensuring that truth-telling is optimal. This requires that when there is no deviation, an agent should not report it to the government. This is the case as moving to the monetary equilibrium makes the reporting agent worse off. Moreover, the policy has to provide incentives such that when an agent sees a deviation, this agent should immediately report to the government. For that to be the case, reporting has to be rewarded. Such contingent monetary policy delivers an equilibrium where self-enforcing credit can be sustained as an equilibrium outcome. We also show that the limiting discount rate consistent with the

[4]Note that the equilibrium where the government injects fiat money after it observes a deviation from gift-giving can be thought as a harsh bankruptcy law, whereby agents that default on their promises are excluded from the credit market and can only have access to their own savings.

gift-giving equilibrium is independent of the population size. In all the previous monetary equilibria, fiat money enhances welfare not because of its medium of exchange functions, but because it plays a role as a signal for agents to coordinate their equilibrium behavior. In our environment fiat money is a medium of exchange in the off equilibrium and a punishment, but it never circulates in equilibrium.[5] Thus, fiat money and self-enforcing credit are complements in the off equilibrium.[6] We also argue that for our contingent monetary policy to work, it is not important whether money injection is a public signal or not.

2 Environment

Our environment is based on Kiyotaki and Wright (1993). Time is discrete and indexed by $t \geq 1$. The economy has a large number of indivisible consumption goods of unit size. $N > 3$ infinitely lived agents discount the future at a rate $\beta \in (0, 1)$, where $\beta < \beta_{SN}$ so that supporting a gift-giving social norm is not feasible.[7] Agents derive utility $u > 0$ from a fraction $x \in (0, 1)$ of these goods.[8] However, which goods they receive a payoff from is agent specific. In each period, agents are bilaterally matched under a uniform random matching technology. Let $m_t(i)$ be agent $i's$ trading partner in period t. Double-coincidence of wants meetings occur with probability $\delta = x^2$, while single-coincidence meetings happen with probability $\sigma = x(1 - x)$. Agents have access to a technology whereby an agent can not produce the good that she wants to consume. When producing one unit of a commodity, drawn randomly from the set of all commodities, agents incur a cost $c > 0$, where $u > c$.

Agents cannot observe the past actions of their trading partners, as in Kandori (1992). Once bilateral matches $\{(i, m_t(i))\}$ are formed, i and $m_t(i)$ play a stage game. In the

[5] As Gu et al (2016) point out, when the distribution of fiat money is degenerate, it is challenging to have self-enforcing credit and fiat money coexist.

[6] We refer the reader to Gomis-Porqueras and Sanches (2013), Araujo and Hu (2018), Lotz and Zhang (2016), among others, for instances where fiat money and credit are complements on the equilibrium path.

[7] β_{SN} denotes the lowest discount rate for a social norm to sustain gift-giving. We refer the reader to Araujo (2004) for more on the derivation of this threshold discount rate and the gift-giving equilibrium.

[8] N is assumed to be even for simplicity.

stage game, agent i chooses whether to produce for agent $m_t(i)$, and vice versa. From now on, let $U_{it}(a_{it}, a_{m_t(i)t})$ be $i's$ stage payoff, where $a_{it} \in A_i$ is $i's$ action in period t. Then $i's$ total payoff in the repeated matching game, where the payoff is the expected sum of his stage payoffs discounted by β. Let a_i^{t-1} denote $i's$ record of play up to period $t-1$.

Finally, there is a government that can issue indivisible fiat money, and agents can only hold at most one unit per period.[9] Moreover, the government has no enforcement power so that taxation is not feasible, making all trades voluntary. Lastly, we allow the government to have different information sets by having access to different record-keeping technologies.[10] In particular, we consider the following scenarios: *(i)* perfect observability and record-keeping of all private trades, as in Kocherlakota (1998), *(ii)* partial observability and record-keeping of private trades, as in Takahashi (2010) and Araujo and Camargo (2015), and *(iii)* inability to observe *any* private trades, but agents can communicate with the government through cheap talk.[11]

2.1 Government Policies

In the original Kiyotaki and Wright (1993), the government simply injects fiat money in the initial period and does not subsequently change the economy's money supply. We denote such operating procedure for monetary policy as Policy I. An alternative is a state contingent monetary policy where there is an injection of fiat money conditional on an agent deviating from gift-giving. We denote such procedure as Policy II. When money injection is a public signal, it is important to highlight that regardless of the policy stance (Policy I or Policy II), there always exist a monetary and a non-monetary equilibrium. In this later one, agents revert to autarky after they observe money injection.[12]

From now on, we study monetary equilibrium after fiat money is injected. We do so for

[9]We are implicitly assuming that the government can commit to future actions.

[10]This is in contrast to Kiyotaki and Wright (1989, 1991, 1993), Araujo (2004), Araujo and Camargo (2015) and Wiseman (2015), among others, where the government does not have access to any record-keeping service.

[11]The case where the government has an inability to observe private trades is related to that of Awaya and Fukai (2015).

[12]For more on this type of equilibria, we refer the reader to Kocherlakota (1998).

the following reasons. First, whenever fiat money is injected, the monetary equilibrium delivers a higher expected payoff, when compared to the non-monetary equilibrium. Thus, using payoff dominance as an equilibrium refinement, the monetary equilibrium would be the predicted outcome of the subgame following money injection.[13] Second, by focusing on monetary equilibria, we can have a better understanding of the various roles fiat money can have under different government information sets and how it relates to self-enforcing credit.

Through out the rest of the paper, we explore how a government can induce cooperation among private agents by threatening to inject fiat money after a deviation from gift-giving has been observed.[14] This type of policy can be seen as a punishment, as injecting fiat money results in lower social welfare relative to gift-giving equilibria. Whenever such policy works, it is better than Policy I, as gift-giving Pareto dominates the monetary equilibrium. The key question that this paper addresses is how much information the government needs in order to implement such state contingent policy. We show that the information requirement is much weaker than one a *priori* would expect.

3 Monetary Equilibrium

In this section we analyze the resulting monetary equilibrium of a government that follows a policy of injecting fiat money after private agents have deviated from gift-giving. More precisely, monetary policy is a one-off injection of fiat money contingent on the history of action profiles in stage games $\{(a_{is}, a_{m_s(i)s})\}_{s=1,..,t-1}$ for $i \in N$. Therefore, such policy can be viewed as a *machine* or *automaton* that determines the timing and the amount of the (one-off) injection of fiat money contingent on history. Such injection is a public signal. We examine this type of policy under various government informations sets and record-keeping technologies.

[13] For more on this refinement, we refer the reader to Harsanyi and Selten (1988).
[14] We are implicitly assuming that the government can commit to future actions.

3.1 Perfect Observability

We first consider a situation where the government can perfectly observe and record all trades by private agents. Suppose the government's monetary policy is such that $M<N$ units of money are randomly distributed among agents at the beginning of period t, if there exists a bilateral match $\{(i, m_{t-1}(i))\}$ in period $t-1$ where i likes $m_{t-1}(i)'s$ good, but $m_{t-1}(i)$ does not produce the good to i. Given this monetary policy, consider the following profile of strategies for agents:

> S_{WoM} (Without Money): Whenever an agent meets someone who likes his good, he produces the good for her (unconditional gift-giving).
>
> S_{WM} (With Money): Once M units of money are injected, agents follow the strategies prescribed in the monetary equilibrium of Kiyotaki and Wright (1993), where agents only produce if their counter-party exchanges fiat money for goods.

Proposition 1. *Under a state contingent policy of threatening to inject fiat money whenever a deviator is observed, S_{WoM} and S_{WM} constitute a sequential equilibrium if $\beta \geq \beta_{PO}$, where $\beta_{PO} \equiv \frac{c}{c+\sigma \min\{1-\frac{M(N-M)}{N(N-1)}, \frac{N-M}{N-1}\}(u-c)} < 1$.*

All proofs can be found in the Appendix.

As we can see, Proposition 1 highlights that if the government is able to actively and perfectly monitor market activities and take contingent actions, fiat money can be used as an effective threat to induce cooperation. We also note that the policy considered is not individual-specific, as money is injected randomly among agents after a deviation is observed. Hence punishment is not targeted to any specific individual. To implement this policy, knowing the identity of the deviator is not necessary. Finally, note that fiat money and self-enforcing credit are complements.

3.2 Partial Observability

We now relax the information and records that the government can have at its disposal. Following Araujo and Camargo (2015), we assume that the record-keeping technology is only able to store information of all last-period bilateral trades. In this new environment, the government randomly injects M units of money in period t if there exists a bilateral match $(i, m_{t-1}(i))$ in period $t-1$ where i likes $m_{t-1}(i)'s$ good, but $m_{t-1}(i)$ does not produce the good to i. We note that in order to implement this government policy, one does not need to recover the whole history of past trades. It only requires that the action taken by agents in the last period is observed. This requirement is consistent with the conditions outlined in Proposition 1. Thus, as long as $\beta \geq \beta_{PO}$, a policy of threatening to injecting fiat money can support gift-giving when the government has partial observability. As in the previous environment, fiat money acts as a public signal, as a medium of exchange in the off equilibrium and as a punishment. Moreover, fiat money and self-enforcing credit are complements in the off equilibrium.

3.3 Cheap Talk

The literature has shown that fiat money can enhance welfare when used as a medium of exchange, as it functions as a form of record-keeping or memory to (practically) resolve the double coincidence of wants problem.[15] This type of equilibria, however, does not exploit all of the information that each agent has. As Hayek (1945) highlighted: "... *and the problem of what is the best way of utilizing knowledge initially dispersed among all the people is at least one of the main problems of economic policy or of designing an efficient economic system*". Here we exploit the fact that private trading generates some information. We then explore how it can be used by the monetary authority to induce better allocations, even when the government cannot observe any private trades.

In this limited information setting, the only way to obtain information is for agents to inform the government about her trading partner's behavior. We assume that the communication between the government and agents is cheap talk as we have that: *(i)* sending a message is costless, *(ii)* the message is non-binding and *(iii)* the correctness

[15]The literature on the essentially of fiat money mainly focuses on the substitutability between memory and money. For instance, in Kocherlakota (1998), memory is defined as knowledge on the part of an agent of the full histories of all agents with whom he has had direct or indirect contact in the past. Kocherlakota shows that any allocation that is feasible in an environment with money is also feasible in the same environment with memory. Hence he concludes that money is equivalent to a primitive form of memory.

of the message cannot be verified; i.e., there is no hard evidence to verify whether an agent tells the truth or not. After receiving messages from agents, the government can implement monetary policy. In particular, the monetary authority decides whether or not to inject fiat money conditional on whether an agent(s) reports to the government or not.

In this new environment, the timing of the game with cheap talk is as follows. There are three stages in each period. In the first stage, agents are matched and they decide whether to produce or not. In the second stage, each agent has the option to report to the government if her trading partner does not produce for her. In the third stage, after receiving the messages from agents (if any), the government decides whether to inject fiat money or not.

Within this environment, we ask whether it is possible to design a policy to support gift-giving in this cheap talk game. Since the government is not able to observe any economic activity, it relies on agents to truthfully report the actions of her trading partners. Hence for a money injection threat policy to work, proper incentives must be provided such that: *(i)* when seeing a deviation, an agent has an incentive to report to the government and *(ii)* when there is no deviation, an agent has no incentive to lie that someone has deviated. Note that in this environment it is sufficient to consider the following simple binary message space: report or not report.

Suppose the government's monetary policy is such that if no one reports, no fiat money is injected. If there is only one reporter, the central bank hands out one unit of money to the reporter. If there are two or more reporters, 2 units of money will be randomly distributed among reporters. Given this monetary policy, consider agents' production and reporting strategies as follows:

> S_A: Suppose there is no fiat money in the economy. Irrespective of their previous private history, agents exchange goods simultaneously in double-coincidence of wants meetings. When an agent meets another in a single-coincidence meeting where the later likes his good, he gives the good to that agent. If in a meeting an agent fails to give a good that the other agent likes,

both agents report to the government in that round. Otherwise, there is trade and no one reports to the government. However, when the government injects fiat money, agents simply follow the strategies prescribed in the monetary equilibrium of Kiyotaki and Wright (1993), where agents only produce if their counter-party exchanges fiat money for goods.

Proposition 2. *Under the government's contingent money injection policy, S_A constitutes a sequential equilibrium if $\beta \geq \beta_{CT}$, where $\beta_{CT} < 1$ and $\lim_{N\to\infty} \beta_{CT} < 1$.*

We refer the reader to the appendix for β_{CT}'s explicit expression.

As we can see, money and (minimal) information resulting from private trades can complement each other and achieve the first best. Without money, the economy reverts to autarky. On the other hand, injecting money without utilizing the knowledge possessed by each agent, the outcome is not efficient. Three additional remarks are noteworthy. First, Proposition 2 establishes that even when the government can not observe private trades, as long as agents are able to communicate the underlying economic activities through cheap talk, gift-giving can be sustained under a money injection threat policy. As in the previous scenarios, in equilibrium money never circulates, yet the first best outcome is achieved. Second, the government will inject no more than two units of money. When an agent deviates from gift-giving, only two agents are aware of the deviation. Injecting one unit of money to induce truth-reporting from the agent who sees the deviation is not incentive compatible. This is the case as she finds it optimal to not report to the government as the expected payoff following the monetary equilibrium is lower than what she can get by not reporting. This is the case as others still follow unconditional gift-giving towards her. On the other hand, injecting two units of fiat money ensures that both agents, in the match where deviation is observed, will report to the government. This is the case as each agent knows that the other agent is going to report to the government, so it is optimal for her to report to the government as well. Then both agents will get a higher expected payoff in the subsequent monetary equilibrium.[16] Finally, Proposition 2

[16]Like any coordination game, in the game with cheap talk there are other possible equilibria. In

highlights that without government intervention, given a discount factor β, a social norm to support gift-giving constructed in Araujo (2004) (without government intervention) eventually breaks down when the population size is large as $\lim_{N\to\infty}\beta_{SN} = 1$. Instead with our state contingent monetary policy, gift-giving can be sustained in economies with any arbitrarily large population size provided that $\beta \geq \lim_{N\to\infty}\beta_{CT}$.

Private Signal

We now consider a situation where money injection is not a public signal. As a result, private agents do not immediately know when a deviation from gift-giving has taken place. Nevertheless, fiat money still conveys information when agents trade with each other. This is the case as agents can not hide fiat money from each other. As a result, agents learn about a deviation through contagion as in Kandori (1992). For finite populations, all agents in the economy eventually learn that a deviation has taken place. Thus, in a stationary equilibrium and in the off equilibrium path all agents play a Kiyotaki and Wright (1993) monetary equilibrium with M=2. Moreover, we could design a contingent monetary policy that distributes $M > 2$ units of money to the society when two agents report to the government. In this way, we can speed up the contagious process towards the Kiyotaki and Wright monetary equilibrium, which further weakens agents' incentives to deviate from equilibrium. Depending on the choice of M, fiat money under our construction serves as a signal for agents to coordinate their equilibrium behavior – it is a purely private signal when $M = 2$ to a public signal when $M = N$.

4 Conclusions

Typically, pure monetary equilibrium, without active government intervention, does not achieve first-best allocations. In this paper, we exploit the monetary inefficiency property when thinking about monetary policy in the Kiyotaki and Wright (1993) environment with a government that has weak enforcement powers and agents are not too patient. In

particular, there is always an equilibrium with autarky regardless of what has happened before, agents never produce. Since this equilibrium is pay off dominated by the monetary one, we focus on this latter one.

particular, we consider an active monetary policy whereby the government threatens to inject fiat money if a deviation from gift-giving is observed. We show that in equilibrium fiat money is never injected and first best allocations are obtained even when the government does not have access to any private records and agents can only communicate with the government through cheap talk. Moreover, we show that gift-giving can be sustained in a large society with any population size provided that agents are sufficiently patient. Finally, all the equilibria analyzed in this paper are such that fiat money and self-enforcing credit (gift-giving) are complements on the off equilibrium path. Moreover, even when money injection is not a public signal, agents can learn about a deviation through contagion as in Kandori (1992), making our state contingent monetary policy useful.

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Appendix

Proof of Proposition 1.

First we observe that on the equilibrium path (where agents follow unconditional gift-giving), the expected payoff for each agent is

$$V_G = \frac{\sigma + \delta}{1 - \beta}(u - c).$$

Once the government sees a deviation, M units of money are randomly injected into the economy. Let V_M^0 denote the expected payoff of an agent without money and V_M^1 the expected payoff of an agent with money. Following Kiyotaki and Wright (1993), we have

$$\begin{aligned} V_M^0 &= \beta V_M^0 + \sigma \frac{M}{N-1}[-c + \beta(V_M^1 - V_M^0)] + \delta(u - c) \\ &= \frac{\sigma \frac{M}{N-1}[-c + \beta(V_M^1 - V_M^0)] + \delta(u - c)}{1 - \beta}, \text{ and} \end{aligned}$$

$$\begin{aligned} V_M^1 &= \beta V_M^1 + \sigma \frac{N-M}{N-1}[u + \beta(V_M^0 - V_M^1)] + \delta(u - c) \\ &= \frac{\sigma \frac{N-M}{N-1}[u + \beta(V_M^0 - V_M^1)] + \delta(u - c)}{1 - \beta}. \end{aligned}$$

The expected payoff for an agent who deviates is given by

$$\begin{aligned} V_d &= \frac{M}{N} V_M^1 + \frac{N-M}{N} V_M^0 \\ &= \frac{M}{N} \frac{\sigma \frac{N-M}{N-1}[u + \beta(V_M^0 - V_M^1)] + \delta(u - c)}{1 - \beta} + \frac{N-M}{N} \frac{\sigma \frac{M}{N-1}[-c + \beta(V_M^1 - V_M^0)] + \delta(u - c)}{1 - \beta} \\ &= \frac{\sigma \frac{M(N-M)}{N(N-1)} + \delta}{1 - \beta}(u - c). \end{aligned}$$

Hence no agent will deviate from gift-giving if $-c + \beta V_G \geq \beta V_d$, which implies that the discount rate has to satisfy

$$\beta \geq \frac{c}{c + \sigma(1 - \frac{M(N-M)}{N(N-1)})(u-c)}.$$

On the other hand, after a deviation, in order for agents to follow a monetary equilibrium, we require $\beta \geq \beta_M \equiv \frac{c}{c+\sigma\frac{N-M}{N-1}(u-c)}$ as derived in Kiyotaki and Wright (1993). The two inequalities hold simultaneously if

$$\beta \geq \frac{c}{c + \sigma \min\{1 - \frac{M(N-M)}{N(N-1)}, \frac{N-M}{N-1}\}(u-c)} = \beta_{PO}.$$

Proof of Proposition 2.

In order to show that the strategy profile supports gift exchange as an equilibrium, we need to establish the following two conditions:

I. No agent has an incentive to deviate on the equilibrium path

There are two conditions to be established:

(i) As agents follow unconditional gift-giving on the equilibrium path, the expected payoff for each agent is

$$V_G = \frac{\sigma + \delta}{1 - \beta}(u - c).$$

Pick any matched pair $(i, m_t(i))$ in period t, and suppose (without loss of generality) that $m_t(i)$ likes $i's$ good. If agent i follows the equilibrium path and produces for $m_t(i)$, his expected payoff is

$$-c + \beta V_G.$$

If i deviates from the equilibrium path and does not produce for $m_t(i)$, both i and $m_t(i)$ will report to the central bank. The central bank then hands out one unit of money to both i and $m_t(i)$. Following the monetary equilibrium after deviation, $i's$ expected payoff is $V^1_{M=2}$. Accordingly, i has no incentive to deviate whenever we satisfy

$$-c + \beta V_G \geq \beta V^1_{M=2}.$$

First we derive $V^1_{M=2}$ explicitly. Again, following Kiyotaki and Wright (1993), $V^0_{M=2}$ and $V^1_{M=2}$ satisfy the following system of equations:

$$\begin{bmatrix} 1-\beta+\frac{2\sigma\beta}{N-1} & -\sigma\frac{2}{N-1}\beta \\ -\sigma\beta\frac{N-2}{N-1} & 1-\beta+\sigma\beta\frac{N-2}{N-1} \end{bmatrix} \begin{bmatrix} V^0_{M=2} \\ V^1_{M=2} \end{bmatrix} = \begin{bmatrix} \delta(u-c)-\sigma\frac{2}{N-1}c \\ \delta(u-c)+\sigma\frac{N-2}{N-1}u \end{bmatrix}$$

Accordingly, $V^1_{M=2}$ can be solved as

$$\begin{aligned} V_{M,2} &= \frac{(1-\beta+\frac{2\sigma\beta}{N-1})(\delta(u-c)+\sigma\frac{N-2}{N-1}u)+\sigma\beta\frac{N-2}{N-1}(\delta(u-c)-\sigma\frac{2}{N-1}c)}{(1-\beta+\frac{2\sigma\beta}{N-1})(1-\beta+\sigma\beta\frac{N-2}{N-1})-\sigma^2\beta^2\frac{2(N-2)}{(N-1)^2}} \\ &= \frac{(u-c)[\delta(1-\beta+\sigma\beta\frac{N}{N-1})+\frac{2\sigma^2\beta(N-2)}{(N-1)^2}]+(1-\beta)\sigma u\frac{N-2}{N-1}}{(1-\beta)(1-\beta+\sigma\beta\frac{N}{N-1})}. \end{aligned}$$

Hence the no-deviation condition becomes

$$-c+\beta\frac{\sigma}{1-\beta}(u-c) \;\; \geq \;\; \frac{\beta}{1-\beta}\frac{(u-c)\frac{2\sigma^2\beta(N-2)}{(N-1)^2}+(1-\beta)\sigma u\frac{N-2}{N-1}}{1-\beta+\sigma\beta\frac{N}{N-1}},$$

which can be rearranged as:

$$\frac{(u-c)\beta\sigma-(1-\beta)c}{\beta\sigma}-\frac{N-2}{N-1}\frac{(u-c)\frac{2\sigma\beta}{N-1}+(1-\beta)u}{1-\beta+\sigma\beta\frac{N}{N-1}} \geq 0.$$

This inequality holds if

$$\frac{(u-c)\beta\sigma-(1-\beta)c}{\beta\sigma}-\frac{(u-c)\frac{2\sigma\beta}{N-1}+(1-\beta)u}{1-\beta+\sigma\beta} \geq 0.$$

After some simplifications, we get the following quadratic inequality in β :

$$\left[c(2\sigma-1)+\sigma^2(u-c)(1-\frac{2}{N-1})\right]\beta^2+2c(1-\sigma)\beta-c \geq 0.$$

Let σ^* denote the unique $\sigma \in (0,1)$ such that $c(2\sigma-1)+\sigma^2(u-c)(1-\frac{2}{N-1})=0$. The coefficient of β^2

$$c(2\sigma-1)+\sigma^2(u-c)(1-\frac{2}{N-1})\begin{cases} >0 & \sigma>\sigma^* \\ =0 & \sigma=\sigma^* \\ <0 & \sigma<\sigma^* \end{cases}.$$

Consider $N>3$ and let β_1^* denote the smallest value of $\beta \in (0,1)$ that satisfies the above quadratic inequality. It can be readily verified that

$$\beta_1^* = \begin{cases} \frac{-2c(1-\sigma)+\sqrt{(2c(1-\sigma))^2+4c\left[c(2\sigma-1)+\sigma^2(u-c)(1-\frac{2}{N-1})\right]}}{2\left[c(2\sigma-1)+\sigma^2(u-c)(1-\frac{2}{N-1})\right]} & \sigma \neq \sigma^* \\ \frac{1}{2(1-\sigma)} & \sigma=\sigma^* \end{cases}.$$

(ii) When seeing no deviations, an agent has no incentive to unilaterally deviate from the equilibrium path and report to the central bank. This requires

$$V^1_{M=1} \leq V_G.$$

Recall that

$$-c+\beta V^1_{M=1} \geq \beta V^0_{M=1}, \text{ and } V^1_{M=1} = \frac{\sigma\frac{N-1}{N-1}[u+\beta(V^0_{M=1}-V^1_{M=1})]+\delta(u-c)}{1-\beta}.$$

Therefore the above inequality holds as

$$V^1_{M=1} = \frac{\sigma\frac{N-1}{N-1}[u+\beta(V^0_{M=1}-V^1_{M=1})]+\delta(u-c)}{1-\beta} \leq \frac{\sigma+\delta}{1-\beta}(u-c)=V_G.$$

II. No agent has an incentive to deviate off the equilibrium path

There are two conditions to be established:

(i) Since the government can not observe the actions of private agents, it is important to determine whether agents have been provided proper incentives to report a deviation. To determine when that will be the case, we need to establish, for any given matched pair, it is optimal for both agents to report to the central bank after seeing a deviation. We note that after seeing a deviation from his trading partner, an agent must believe that his trading partner is the only one who deviates. The following condition makes sure that each agent in a matched pair will report to the central bank after seeing a deviation:

$$V^1_{M=2} \geq V^0_{M=1}.$$

When there is only one unit of money in the economy, we have the following system of equations on $V^0_{M=1}$ and $V^1_{M=1}$:

$$\begin{bmatrix} 1-\beta+\frac{\sigma\beta}{N-1} & -\sigma\frac{1}{N-1}\beta \\ -\sigma\beta & 1-\beta+\sigma\beta \end{bmatrix} \begin{bmatrix} V^0_{M=1} \\ V^1_{M=1} \end{bmatrix} = \begin{bmatrix} \delta(u-c)-\sigma\frac{1}{N-1}c \\ \delta(u-c)+\sigma u \end{bmatrix}$$

Hence $V^0_{M=1}$ can be solved as

$$V^0_{M=1} = \frac{(\delta(u-c)-\sigma\frac{1}{N-1}c)(1-\beta+\sigma\beta)+(\delta(u-c)+\sigma u)\sigma\frac{1}{N-1}\beta}{(1-\beta+\frac{\sigma\beta}{N-1})(1-\beta+\sigma\beta)-\frac{\sigma^2\beta^2}{N-1}}.$$

The no-deviation inequality becomes

$$\begin{aligned} & \frac{(u-c)[\delta(1-\beta+\sigma\beta\frac{N}{N-1})+\frac{2\sigma^2\beta(N-2)}{(N-1)^2}]+(1-\beta)\sigma u\frac{N-2}{N-1}}{(1-\beta)(1-\beta+\sigma\beta\frac{N}{N-1})} \\ \geq \; & \frac{(\delta(u-c)-\sigma\frac{1}{N-1}c)(1-\beta+\sigma\beta)+(\delta(u-c)+\sigma u)\sigma\frac{1}{N-1}\beta}{(1-\beta+\frac{\sigma\beta}{N-1})(1-\beta+\sigma\beta)-\frac{\sigma^2\beta^2}{N-1}} \end{aligned}$$

As $(1-\beta)(1-\beta+\sigma\beta\frac{N}{N-1}) = (1-\beta+\frac{\sigma\beta}{N-1})(1-\beta+\sigma\beta)-\frac{\sigma^2\beta^2}{N-1}$, the inequality holds if and only if

$$\begin{aligned} & (u-c)[\delta(1-\beta+\sigma\beta\frac{N}{N-1})+\frac{2\sigma^2\beta(N-2)}{(N-1)^2}]+(1-\beta)\sigma u\frac{N-2}{N-1} \\ \geq \; & (\delta(u-c)-\sigma\frac{1}{N-1}c)(1-\beta+\sigma\beta)+(\delta(u-c)+\sigma u)\sigma\frac{1}{N-1}\beta. \end{aligned}$$

After some simplifications, we get

$$\beta \leq \frac{u(N-2)+c}{u(N-2)+c+\sigma(u-c)(\frac{2}{N-1}-1)} \equiv \beta_2^*.$$

Observe that $\beta_2^* > 1$ when $N > 3$. Hence this condition holds for all $\beta \in (0,1)$ whenever $N > 3$.

(ii) If someone deviates in a bilateral meeting, both agents will report to the central bank, and two units of money will be injected into the economy. We need to make sure that agents follow a monetary equilibrium after money injection, which requires

$$\beta \geq \beta_{M=2} = \frac{c}{c+\sigma\frac{N-2}{N-1}(u-c)}.$$

Now, combining all conditions in I and II, we conclude that there exists an equilibrium where a threat of money injection can support gift-giving when $N > 3$ and

$$\beta \geq \beta_{CT} \equiv \max\{\beta_1^*, \beta_{M=2}\}.$$

Note that $\lim_{N\to\infty}\beta_1^* < 1$ and $\lim_{N\to\infty}\beta_{M=2} < 1$ thus we have that $\lim_{N\to\infty}\beta_{CT} < 1$.