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Yongmin Chen and Jianpei Li and Marius Schwartz

University of Colorado Boulder, University of international business and economics, Georgetown university

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Yongmin Chen* Jianpei Li† Marius Schwartz‡

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Abstract. This paper analyzes welfare under differential versus uniform pricing across oligopoly markets that differ in costs of service. We establish necessary and sufficient conditions on demand properties—cross/own elasticities and curvature—for differential pricing by symmetric firms to raise aggregate consumer surplus, profit, and total welfare. The analysis reveals intuitively why differential pricing is generally beneficial though not always—including why profit can fall, unlike for monopoly—and why it is more beneficial than oligopoly third-degree price discrimination. When firms have asymmetric costs, however, differential pricing can reduce profit or consumer surplus even with 'simple' demands such as linear.

Keywords: differential pricing, price discrimination, demand curvature, cross-price elasticity, pass-through, oligopoly

*University of Colorado Boulder; yongmin.chen@colorado.edu

†University of International Business and Economics (UIBE); lijianpei2007@gmail.com

‡Georgetown University; mariusschwartz@mac.com

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1. Introduction

Distinct consumer groups—or 'markets'—for a product frequently differ in their costs of service or demands. A large literature studies the welfare effects, relative to uniform pricing, of differential pricing across markets based solely on demand elasticities—classic third-degree price discrimination—under monopoly or oligopoly. See, for example, Aguirre, Cowan and Vickers (2010) and references therein for monopoly third-degree price discrimination; and Holmes (1989) and Stole (2007) for the oligopoly case. Very little work has compared uniform pricing (UP) and differential pricing (DP) when, instead, markets vary in costs of service. Yet DP motivated (at least partly) by cost differences is controversial and frequently subject to various constraints in monopoly or oligopoly markets, such as gender-neutral requirements in insurance or pensions, universal-service mandates on utilities, antidumping rules in international trade, and consumer resistance to add-on pricing such as airline bag fees.[1]

This paper analyzes the welfare effects of (cost-based) differential pricing by oligopoly firms. If markets were perfectly competitive, DP obviously would be desirable, as prices would equal marginal costs in each market hence maximize welfare, whereas UP would distort the output allocation. However, the ranking is no longer clear when prices exceed marginal costs, as Chen and Schwartz (2015) show for the case of monopoly: moving from UP to DP there can reduce consumer surplus (aggregated across markets) and—while raising profit—can also reduce total welfare, albeit under rather stringent demand curvature conditions. In oligopoly, the welfare analysis of DP is even richer due to at least two new forces: (1) when firms supply differentiated products, the pricing equilibria will depend also on cross-price elasticities[2]; and (2) firms may differ in their costs within each market, and the pricing equilibria then will depend on the pattern of cost asymmetry even with homogeneous products.

We consider a setting with two competing firms, each selling a single product in two distinct markets. To isolate the role of cost differences, we assume the markets have equal demand

[1] The constraints on cost-based DP can stem from various sources: government policy, contractual restrictions, consumer perceptions of the likely effects, or transaction costs. For further discussion and examples, see Chen and Schwartz (2015), Edelman and Wright (2015), and Nassauer (2017). On add-on pricing generally see Ellison (2005) and Brueckner et al. (2015).

[2] Mrázová and Neary (2017) show that any well-behaved demand function for a single product can be represented by its elasticity and curvature. With differentiated products, cross-price elasticity is additionally needed.

elasticities but different marginal costs of service. Firms sell symmetrically differentiated or homogeneous products and compete in prices under the alternative UP and DP regimes.[3] In our main model, the firms also have symmetric costs—the same cost within a market—and sell differentiated products. This environment serves two purposes. It reveals how the welfare properties of DP under monopoly (Chen and Schwartz, 2015) may differ in oligopoly solely due to the cross-elasticity/substitution effect; and it permits a natural comparison to *price discrimination* in symmetric oligopoly analyzed by Holmes (1989). An extension of the model retains demand symmetry between firms but allows firms to have asymmetric costs.

In our main model, a major factor is the pass-through rate from firms' common marginal cost to their symmetric equilibrium price.[4] Under UP each firm sets price based on the average of its marginal costs across markets, whereas under DP it sets prices based on each market's specific marginal cost. Moving to DP effectively lowers firms' marginal cost in one market and raises cost in the other market, with the equilibrium price adjustments determined by the pass-through rate. Our unifying approach to analyzing the welfare effects of DP examines when each welfare measure is convex or concave in marginal cost. We obtain necessary and sufficient conditions for DP to raise consumer surplus, profit, or total welfare (Propositions 1-3), based on general properties of the demand system (which also determine the pass-through rate): curvature and own- and cross-price elasticities, and how they vary as firms change price equally. These conditions reduce to their counterparts for monopoly DP—and hence neatly nest the latter—when cross-price effects vanish. Throughout, we trace the welfare changes to endogenous forces, such as the change in average price and extent of output reallocation between markets.

Consumer surplus is subject to the same forces as under monopoly when moving to DP. Consumers in the aggregate benefit from the price dispersion. But average price can rise with DP, as occurs if the pass-through rate is greater at higher than at lower prices along the common demand function of the two markets. Potentially, consumers can lose on balance (though we have not found such an example for differentiable demands).

[3] Hereafter, unless stated otherwise, DP refers to 'cost-based'. These alternative pricing regimes can be attained through interventions that do not require knowledge of costs: laissez faire yields DP, whereas prohibiting any price differences yields UP. Cowan (2018) analyzes regulatory schemes that constrain a monopolist's price-cost margins, schemes that improve welfare but require the regulator to know costs.

[4] For a general analysis of pass-through in various applications see Weyl and Fabinger (2013).

Unlike for monopoly, DP can reduce profits in oligopoly.[5] This can occur in two ways. First, DP induces excessive output reallocation between markets when pass-through exceeds one, as the price difference 'overshoots' the cost difference so the profit margin becomes smaller in the (low-cost) market that gains output. Under monopoly, DP nevertheless raises profit for any pass-through rate, because a rate above one requires demand to be highly convex, in which case the price dispersion chosen by a monopolist yields a large output expansion (Chen and Schwartz, 2015). In oligopoly, however, pass-through can be high not only due to demand curvature but also due to cross-price effects between firms. Consequently, DP can reallocate output excessively *and* without expanding total output enough to outweigh this misallocation effect (though in our CES demand example, profit does not increase only in limiting cases). Second, DP can reduce average price, while also reducing output. If the products are closer substitutes at lower prices than at higher prices, moving to DP can reduce price by more in the low-cost market than it raises price in the other market—and not due to greater demand curvature at lower prices, which explains why output can decrease. In some such cases, total welfare also declines, as we show with an example. Interestingly, Holmes (1989) finds that classic third-degree *price discrimination* in oligopoly also can reduce average price without expanding output relative to UP, but due to different demand properties.

Overall, our analysis suggests that—while there are exceptions—cost-based DP in symmetric oligopoly is broadly beneficial. Furthermore, DP is more beneficial for consumers and total welfare than classic third-degree price discrimination. In both cases, consumers gain from the price dispersion by adjusting quantities. But price discrimination has a bias to raise average price, which harms consumers, whereas cost-based DP does not, for a broad class of demand functions. Regarding total welfare, classic price discrimination misallocates output while UP does not, whereas when markets differ in costs of service, UP misallocates output and DP can improve the allocation. We provide an example where markets differ both in costs of service and in demand elasticities and, as expected, differential pricing is beneficial for consumers as well as firms if the cost differences are large relative to the demand differences.

Do the generally favorable effects of DP persist when firms are asymmetric? An extension

[5] In such cases, firms would jointly gain from committing to uniform pricing, but such a commitment would not be unilaterally optimal.

of our main model introduces cost asymmetries between firms in a given market. We start with the case where firms sell homogeneous products.[6] In one scenario, the same firm has a cost advantage in both markets. It captures both markets while pricing at the rival's relevant marginal cost—the market-specific cost under DP or the average of marginal costs under UP. Average price under DP then equals the uniform price, hence consumer surplus must rise due to price dispersion, but profit of the lower-cost firm can readily fall if the difference in its costs across markets is lower than for the rival (Proposition 4). Intuitively, the firm suffers from having to set a price differential that exceeds its cost differential.

In an alternative scenario, each firm has a cost advantage in one market (but under UP each firm must serve both markets). Under DP the equilibrium price in each market now equals the marginal cost of the higher-cost firm for that market; whereas under UP the single equilibrium price equals the marginal cost of the firm with the higher marginal cost *averaged* across the markets. Average price across markets under DP then exceeds the uniform price, because cost dispersion—which determines equilibrium markups—is higher under DP than under UP. If cost heterogeneity between firms is large relative to that across markets, DP reduces consumer surplus as the price-increasing effect dominates the beneficial price dispersion effect (Proposition 5).

When firms have asymmetric costs and supply homogeneous products, therefore, DP can reduce either profit or consumer surplus even for 'simple' demand functions such as linear. As a robustness check, we extend these findings to symmetrically differentiated products with a linear demand system from Shubik and Levitan (1980), showing that DP can reduce either profit or consumer surplus if products are sufficiently close substitutes (Proposition 6).

To our knowledge, the only other analysis addressing cost-based DP in oligopoly is by Adachi and Fabinger (2019). Our contributions are complementary. Adachi and Fabinger add cost differences between markets to Holmes' (1989) symmetric oligopoly setting. They provide sufficient conditions for DP to lower or raise total welfare, using similar techniques as Aguirre, Cowan and Vickers (2010; 'ACV'), who study monopoly price discrimination with no cost differences. Their conditions resemble ACV's, in comparing weighted markups between markets at the equilibrium price(s), but are more complex due to the role of cross-price effects in the weights. Our analysis

[6] Since DP by symmetric firms is always beneficial with homogeneous products (Bertrand competition then yields the same first-best outcome as perfect competition), this case sharply highlights the role of cost asymmetries.

of symmetric oligopoly assumes equally-elastic demands across markets in order to focus sharply on the role of cost differences. By analyzing when each welfare measure is a convex function of marginal cost, we provide transparent necessary and sufficient conditions on the demand system for DP to raise or lower consumer surplus, profit, or total welfare, and decompose the underlying forces. In addition, we highlight the effects of cost asymmetries between firms.

The next section presents the main model and some preliminary results. Section 3 analyses the effects of DP with symmetric firms. Section 4 considers the extension where firms have asymmetric costs. We conclude in Section 5, and gather all proofs in the Appendix.

2. A Model With Symmetric Firms

Two firms each produce one product. The demand function for firm i, $i = 1, 2$, is $D_i(p_1, p_2)$. There are two distinct groups of consumers or markets, L and H. Each firm's constant marginal cost is c_L to serve group L and c_H to serve group H, with $0 \leq c_L < c_H$. Group L's demand for firm i's product is $\lambda D_i(p_1, p_2)$ and group H's demand is $(1 - \lambda) D_i(p_1, p_2)$ for $\lambda \in (0, 1)$. Since these demand functions differ only in scale, they have equal price elasticities at any common prices, but the (constant) marginal costs of serving the groups differ.

Firms compete by simultaneously choosing prices, possibly in one of two pricing regimes. Under uniform pricing (UP) each firm can only set a single price for all consumers, whereas under differential pricing (DP) each firm can charge two different prices for the two distinct consumer groups.

Following Holmes (1989), the firms produce symmetrically differentiated substitute products with $D_i(p_1, p_2)$ being a continuous and differentiable function[7] satisfying

$$D_i(x, y) = D_j(y, x) \quad \text{for firm } i \neq j = 1, 2. \tag{1}$$

At equal prices $p_i = p_j \equiv p$, we further define the industry demand as

$$D(p) \equiv 2D_i(p, p) \quad \text{for } i = 1, 2. \tag{2}$$

[7] We shall also briefly address the case where the two symmetric firms produce a homogeneous product so that $D_i(p_i, p_j)$ is not continuous at $p_i = p_j$. The analysis is then straightforward.

This setting lets us compare for symmetric duopoly the welfare properties of differential pricing motivated solely by cost differences between markets instead of solely by demand differences as in Holmes (1989).[8]

Given firms' symmetry, we will analyze the symmetric equilibria in which both firms charge equal prices under UP or under DP. Under DP, each firm charges market-specific prices p_L and p_H that maximize its profit for marginal costs c_L and c_H, respectively. Under UP, each firm draws customers from groups L and H in proportion to their relative masses λ and $1-\lambda$, hence its virtual marginal cost will be a weighted average of c_L and c_H,

$$\bar{c} \equiv \lambda c_L + (1-\lambda)\, c_H, \tag{3}$$

and the symmetric uniform price p^u maximizes a firm's profit for marginal cost $\bar{c}$.[9]

We assume standard demand conditions such that DP raises price in the high-cost market and reduces price in the low-cost market: $p_L < p^u < p_H$. Let $q^u = D(p^u)$, $q_L = D(p_L)$, and $q_H = D(p_H)$. Then $q_H < q^u < q_L$, and $\Delta q_L \equiv q_L - q^u > 0$ while $\Delta q_H \equiv q_H - q^u < 0$. Define

$$p^d \equiv \lambda p_L + (1-\lambda)\, p_H \tag{4}$$

as the *average* price under DP weighted by the relative sizes of the two groups, which equal their relative consumption quantities under UP. If $p^d = p^u$, and consumers in the two groups were to maintain the same consumption quantities as under UP, their total expenditure and welfare would be unchanged. Under monopoly, or a homogeneous-product oligopoly with downward-sloping market demand $D(p)$, $p^d \leq p^u$ is a sufficient condition for DP to raise aggregate consumer surplus because consumers can advantageously adjust quantities to exploit price dispersion—purchasing more where price is lower and less where price is higher (Waugh, 1944). This sufficient condition extends to symmetric oligopoly under product differentiation as follows.

[8] Holmes assumes that marginal cost is the same across markets, but markets differ in demand elasticities such that in symmetric equilibrium both firms charge a higher price in the same market (the 'strong' market).

[9] At a common uniform price p^u, firm i's profit is

$$(p^u - c_L)\lambda D_i(\cdot) + (p^u - c_H)(1-\lambda) D_i(\cdot) = (p^u - \bar{c}) D_i(\cdot).$$

Suppose consumers have quasi-linear utility, denoted $V(q_1, q_2) + q_0$, where q_0 is consumption of the numeraire good.[10] Then, consumer surplus is

$$v(p_1, p_2) = \max_{q_1, q_2} \{V(q_1, q_2) - (p_1 q_1 + p_2 q_2)\},$$

with

$$dv(p_1, p_2) = -D_1(p_1, p_2)\, dp_1 - D_2(p_1, p_2)\, dp_2.$$

When both firms charge the same price p, with $D(p, p) = 2D_i(p, p)$ from (2), we can rewrite $v(p, p) \equiv S(p)$ so that aggregate consumer surplus is

$$S(p) \equiv \int_p^\infty D_1(x, x)dx + \int_p^\infty D_2(x, x)dx = \int_p^\infty D(x)dx. \tag{5}$$

Then, the consumer surplus under uniform pricing and differential pricing are respectively

$$S^u = S(p^u), \qquad S^d = \lambda S(p_L) + (1 - \lambda)S(p_H). \tag{6}$$

Since $S(p)$ is convex in p, we have

$$\begin{aligned} S^d - S^u &= \lambda S(p_L) + (1 - \lambda)S(p_H) - S(p^u) \\ &> S(\lambda p_L + (1 - \lambda)p_H) - S(p^u) = S(p^d) - S(p^u) \\ &\geq 0 \quad \text{if} \quad p^d \leq p^u. \end{aligned}$$

We thus have the following result, where firms' products can be either homogeneous or symmetrically differentiated:[11]

Remark 1 *DP increases consumer surplus if average price does not rise ($p^d \leq p^u$).*

[10] Quasi-linear utility ensures that demand functions have equal cross-partial derivatives, $\partial D_1 / \partial p_2 = \partial D_2 / \partial p_1$, hence consumer surplus is an unambiguous measure of consumer welfare with multiple products (its magnitude is independent of the integration path along which prices are changed).

[11] If there were $n > 2$ firms producing n differentiated goods, consumer surplus could be similarly defined as $v(\mathbf{p})$ for price vector $\mathbf{p} = (p_1, p_2, ..., p_n)$. Then, under symmetry, we would have $D(\mathbf{p}) = nD_i(\mathbf{p})$, and again $S(p) = \int_p^\infty D(x)dx$. Thus, both Remark 1 and the profit decomposition in (7) below generalize to symmetric oligopoly with $n > 2$ firms.

Next, let the industry output under DP be $q^d \equiv \lambda q_L + (1-\lambda) q_H$, and also let $\Delta q = q^d - q^u$. Furthermore, let $m_L = p_L - c_L$ and $m_H = p_H - c_H$ be the price-cost margins for the two consumer groups under DP. The difference of industry profit under DP and UP is

$$\Pi^d - \Pi^u = [\lambda (p_L - c_L) q_L + (1-\lambda)(p_H - c_H) q_H] - [p^u - \lambda c_L - (1-\lambda) c_H] q^u.$$

The effect of DP on industry profit can then be decomposed as follows:

$$\Delta\Pi \equiv \Pi^d - \Pi^u = \underbrace{\left(p^d - p^u\right) q^u}_{\text{Average-P Effect}} + \underbrace{\lambda (m_L - m_H) \Delta q_L}_{\text{Reallocation Effect}} + \underbrace{m_H \Delta q}_{\text{Output Effect}} . \tag{7}$$

Since $\Delta q_L > 0$, the reallocation effect will be positive if the margin is higher in market L than in H at the differential prices (as was true under UP) and negative if the reverse holds. Under classic third-degree price discrimination, i.e., markets face different prices but have the same cost of service, the reallocation effect is necessarily *negative*: output shifts to the market where price fell and, hence, where the margin is lower. Thus, profitable price discrimination requires an increase in output or in the average price. In contrast, (cost-based) differential pricing can be profitable even when output and average price fall, because the reallocation effect will be positive as long as the price difference between markets remains less than the cost difference. This distinction will prove useful in Subsection 3.5.

3. Welfare Analysis

If the firms supply a homogeneous product, then under UP each charges $p^u = \bar{c} = \lambda c_L + (1-\lambda) c_H$, from (3); whereas under DP each firm charges $p_L = c_L$ and $p_H = c_H$, with average price $p^d = \lambda p_L + (1-\lambda) p_H = p^u$. Thus, DP obviously is beneficial: consumer surplus is higher under DP due to price dispersion, while profit is zero under both regimes.

However, the results are no longer obvious when products are (symmetrically) differentiated. The remainder of this section addresses that case. Subsections 3.1-3.3 analyze the effects of DP compared to UP on consumer surplus, profits, and total welfare, respectively, using general properties of the demand system. Subsection 3.4 provides illustrative examples using specific demand functions. Subsection 3.5 compares the welfare effects of our cost-based differential pricing to classic price discrimination (Holmes, 1989).

3.1 Equilibrium Prices and Consumer Surplus

Since market demands are proportional, markets essentially differ only in marginal cost c, assumed symmetric between firms. Therefore, we can analyze the properties of all relevant variables as functions of c. Firm i chooses p_i to maximize

$$\pi_i\left(p_1, p_2\right) = \left(p_i - c\right) D_i\left(p_1, p_2\right),$$

taking as given p_j, with $c = \bar{c}$ under UP, and $c = c_L$ or c_H under DP. A sufficient condition for the existence of a unique equilibrium, which we shall maintain, is

$$-\frac{\partial^2 \pi_i}{\partial p_i^2} > \frac{\partial^2 \pi_i\left(p_1, p_2\right)}{\partial p_i \partial p_j} > 0. \tag{8}$$

The second inequality implies that firms' prices are strategic complements.

The symmetric equilibrium price of both firms, $p^* \equiv p^*\left(c\right)$, satisfies the first-order condition

$$\frac{\partial \pi_i\left(p^*, p^*; c\right)}{\partial p_i} = D_i\left(p^*, p^*\right) + \left(p^* - c\right) \frac{\partial D_i\left(p^*, p^*\right)}{\partial p_i} = 0. \tag{9}$$

Substituting the relevant value of c in (9) yields the equilibrium prices as

$$\text{UP:} \quad p^u = p^*\left(\bar{c}\right), \qquad\qquad \text{DP:} \quad p_L = p^*\left(c_L\right), \quad p_H = p^*\left(c_H\right).$$

Moving from UP to DP therefore can be analyzed as if marginal cost fell in market L from the virtual level $\bar{c}$ to c_L and rose in market H from $\bar{c}$ to c_H. Profits per firm under UP and DP are

$$\pi^u \equiv \pi_i(p^u, p^u), \qquad\qquad \pi^d \equiv \lambda \pi_i(p_L, p_L) + (1 - \lambda)\pi_i(p_H, p_H). \tag{10}$$

By the symmetry of demand, without loss of generality we can conduct our analysis for all variables associated with one firm. For firm 1, say, define

$$\text{own-price elasticity:}\quad \eta_{11} = -\frac{\partial D_1(p,p)}{\partial p_1}\frac{p}{D_1(p,p)} > 0 \tag{11}$$

$$\text{cross-price elasticity:}\quad \eta_{12} \equiv \frac{\partial D_1(p,p)}{\partial p_2}\frac{p}{D_1(p,p)} > 0 \tag{12}$$

$$\text{elasticity ratio:}\quad \eta_r \equiv \frac{\eta_{12}}{\eta_{11}} > 0. \tag{13}$$

As is customary, we assume $-\frac{\partial D_i(p,p)}{\partial p_i} > \frac{\partial D_i(p,p)}{\partial p_j}$, hence $\eta_r \in (0,1)$. A larger η_r reflects greater substitutability between the products.[12]

For a firm's demand, define also the

$$\text{(margin-adjusted) curvature:}\quad \alpha(p) \equiv \frac{(p-c)}{p}\left[\frac{p}{-\frac{\partial D_1(p,p)}{\partial p_1}}\frac{d}{dp}\frac{\partial D_1(p,p)}{\partial p_1}\right]. \tag{14}$$

The square-bracketed term is the elasticity of the slope of firm 1's demand with respect to an equal change in both prices. Thus, $\alpha(p) = 0$ if D_1 is linear, and $\alpha(p) > (<)\, 0$ if D_1 is convex (concave) in symmetric price p. For a monopolist with demand $q = D(p)$, $\sigma \equiv \frac{p^m - c}{p^m}\frac{p^m}{-D'(p^m)}D''(p^m)$ is the curvature of the inverse demand function $P(q) = D^{-1}(q)$ at monopoly price p^m (e.g., Chen and Schwartz, 2015). Thus the equilibrium oligopoly version of demand curvature $\alpha(p)$ would reduce to σ under monopoly. Making use of (9), $\alpha(p)$ can be rewritten solely in terms of demand parameters as

$$\alpha(p) = \frac{D_1(p,p)}{\left[\frac{\partial D_1(p,p)}{\partial p_1}\right]^2}\frac{d}{dp}\frac{\partial D_1(p,p)}{\partial p_1}. \tag{15}$$

The equilibrium price satisfies the familiar inverse elasticity rule: $\frac{p^* - c}{p^*} = \frac{1}{\eta_{ii}(p^*)}$. Using (9), the pass-through rate from marginal cost to equilibrium price is

$$\begin{aligned} p^{*'}(c) &= -\frac{-\frac{\partial D_1(p^*,p^*)}{\partial p_1}}{2\frac{\partial D_1(p^*,p^*)}{\partial p_1} + \frac{\partial D_1(p^*,p^*)}{\partial p_2} + (p^*(c) - c)\frac{d}{dp^*}\frac{\partial D_1(p^*,p^*)}{\partial p_1}} \\ &= \frac{1}{2 - (\alpha + \eta_r)}, \end{aligned} \tag{16}$$

[12]In symmetric equilibrium, η_r equals the *diversion ratio,* a measure of substitutability commonly used in antitrust (e.g., Chen and Schwartz, 2016). The diversion ratio from product 1 to product 2 is $-\frac{(\partial q_2/\partial p_1)dp_1}{(\partial q_1/\partial p_1)dp_1} = \frac{\eta_{12}q_2}{\eta_{11}q_1}$, and $q_2 = q_1$ under symmetry.

where α and η_r (and later also α' and η_r') are evaluated at symmetric equilibrium prices $p = p^*(c)$.[13] Notice that $p^{*'}(c) > 0$ under assumption (8).

Equation (16) shows that a marginal increase in c affects equilibrium price under competition through two channels. One is the curvature of a firm's demand: $p^{*'}(c)$ is larger when the firm's demand is convex ($\alpha > 0$) rather than concave ($\alpha < 0$), as under monopoly (Bulow and Pfleiderer, 1983). The second channel, specific to oligopoly, is the degree of product substitutability: *ceteris paribus,* a higher η_r raises $p^{*'}(c)$. A common increase in marginal cost c will raise also the rival's price, which magnifies a firm's own price increase assuming that prices are strategic complements, and this cross-effect increases with η_r.

We now can compare the uniform price, p^u, with the (group-weighted) average price under DP, defined in (4): $p^d \equiv \lambda p^*(c_L) + (1-\lambda)p^*(c_H)$. The difference between p^u and p^d will affect the change in consumer surplus and in profits from a move to DP. Since

$$p^u = p^*(\bar{c}) = p^*(\lambda c_L + (1-\lambda)c_H),$$

we have $p^u \gtreqless p^d$ when

$$p^u = p^*(\lambda c_L + (1-\lambda)c_H) \gtreqless \lambda p^*(c_L) + (1-\lambda)p^*(c_H) = p^d.$$

That is, DP lowers average price if $p^*(c)$ is concave, and raises average price if $p^*(c)$ is convex. From (16),

$$p^{*''}(c) = \left[\alpha'(p^*) + \eta_r'(p^*)\right]\left[p^{*'}(c)\right]^3. \tag{17}$$

Thus, $p^{*''}(c)$ has the same sign as $[\alpha' + \eta_r']$, implying:

Remark 2 *For all $p^*(c)$ over $c \in [c_L, c_H]$: (i) If $\alpha' + \eta_r' > 0$, then DP raises average price, $p^d > p^u$; (ii) if $\alpha' + \eta_r' \leq 0$, then $p^d \leq p^u$.*

Note that if $[\alpha'(p^*(c)) + \eta_r'(p^*(c))]$ has a consistent sign over the relevant range $c \in [c_L, c_H]$, then the condition in (i), i.e., $\alpha' + \eta_r' > 0$, is sufficient and *also necessary* for $p^d > p^u$, and

[13] For a monopolist, with $\eta_r = 0$ and $\alpha = \sigma$, the pass-through would reduce to $p^{m'}(c) = \frac{1}{2-\sigma}$. It is straightforward to extend (9) to an n-firm symmetric oligopoly and obtain the pass-through rate in this more general case. Remark 2 and Propositions 1-3 below can also be extended to n-firm symmetric oligopoly, with proper modifications.

similarly the condition in (ii) is also necessary for $p^d \leq p^u$. This observation also applies to the subsequent Propositions 1 through 3.

If $\alpha' + \eta_r' \leq 0$, the pass-through weakly decreases with price, because at higher prices the demand curvature term is lower ($\alpha' \leq 0$) and/or product substitutability is weaker ($\eta_r' \leq 0$). Pass-through then will be smaller in market H, where a move to DP raises marginal cost (from $\bar{c}$ to c_H) and, hence, price, than in market L where marginal cost falls, explaining why the average price weakly falls. Both α and η_r are constant for some familiar classes of demand functions—including linear and CES demands (see Subsection 3.4 below). For these classes of demands, cost-based DP leaves average price unchanged.

The condition $\alpha' + \eta_r' \leq 0$, implying $p^d \leq p^u$, is sufficient for DP to raise consumer surplus (Remark 1) but is not necessary since consumers gain from price dispersion. A tighter sufficient condition is derived next. Rewrite consumer surplus (5) as a function of c

$$s(c) \equiv S(p^*) = \int_{p^*(c)}^{\infty} D(x)dx. \tag{18}$$

DP raises or lowers consumer surplus as $s(c)$ is convex or concave:

$$S^d = \lambda s(c_L) + (1-\lambda)s(c_H) \gtreqless s(\bar{c}) = S^u \qquad \Longleftrightarrow \qquad s''(c) \gtreqless 0.$$

When the two firms charge a symmetric price p, define the price elasticity of market demand by

$$\eta(p) \equiv -\frac{D'(p)}{D(p)}p. \tag{19}$$

Then $\eta(p) = \eta_{11}(p) - \eta_{12}(p)$, the difference between firm 1's own and cross price elasticities when two firms have equal price p.[14]

Analyzing the sign of $s''(c)$ yields a tighter condition than $\alpha' + \eta_r' \leq 0$, on the demand parameters $\eta_r = \eta_r(p)$, $\alpha = \alpha(p)$, and $\eta = \eta(p)$, for DP to raise consumer surplus:

[14] Notice that our η_{11}, η_{12}, and η correspond to ε_i^F, ε_i^C, and ε^I, respectively, in Holmes (1989), with all elasticities again defined as positive. In his notation, our elasticity ratio η_r, instead, is e_i^C / e_i^F.

Proposition 1 *Consumer surplus is higher under differential pricing than under uniform pricing if (20) holds, and is lower if the inequality in (20) is reversed.*

$$\frac{-(\alpha' + \eta_r')}{2 - (\alpha + \eta_r)} + \frac{\eta}{p} > 0. \tag{20}$$

The first term in (20) is the average price effect when moving from UP to DP. Since $\frac{1}{2-(\alpha+\eta_r)} = p^{*'}(c) > 0$ from (16), the first term takes the sign of $-(\alpha' + \eta_r')$. When $\alpha' + \eta_r' \leq 0$, DP weakly lowers the average price, which benefits consumers. The second term, $\frac{\eta}{p} > 0$, reflects the price-dispersion effect: when the price elasticity of market demand η is higher, consumers are more capable of making quantity adjustments and thus benefit more from the price dispersion. On balance, DP raises consumer surplus if it does not raise average price too much, i.e., if $[\alpha(p) + \eta_r(p)]$ does not increase too fast.

Under monopoly, the corresponding condition for DP to raise consumer surplus (Chen and Schwartz, 2015) is

$$\frac{\sigma'(q)}{2 - \sigma(q)} + \frac{1}{q} > 0 \Longleftrightarrow \frac{-\sigma'(q)\, q'(p)}{2 - \sigma(q)} + \frac{\eta}{p} > 0.$$

We can view $\sigma(q) \equiv -qP''(q)/P'(q)$, the curvature of inverse demand, as corresponding to $\alpha(p)$, and $-\sigma'(p)\, q'(p)$ as corresponding to $-\alpha'(p)$. Thus, condition (20) in oligopoly reduces to its monopoly counterpart for $\eta_r' = \eta_r = 0$ and $\alpha = \sigma$. Condition (20) holds for a broad class of demand functions under oligopoly or monopoly (see examples in Subsection 3.4 and in Chen and Schwartz, 2015) and, hence, DP tends to benefit consumers.

3.2 Profit

Equilibrium profit for firm 1 under marginal cost c is

$$\pi^*(c) \equiv \pi_1(p^*(c), p^*(c)) = [p^*(c) - c]\, D_1(p^*(c), p^*(c)).$$

Thus, using the envelope theorem:

$$\pi^{*'}(c) = -\underbrace{D_1(p^*(c), p^*(c))}_{\text{direct effect}} + \underbrace{[p^*(c) - c]\,\frac{\partial D_1(p^*, p^*)}{\partial p_2} p^{*'}(c)}_{\text{rival's effect}},$$

where the second term, the rival's effect, can be rewritten as

$$D_1(p^*,p^*)\left[\frac{\partial D_1(p^*,p^*)}{\partial p_2}\frac{p^*(c)}{D_1(p^*,p^*)}\right]\left[\frac{p^*(c)-c}{p^*(c)}\right]p^{*'}(c)=D_1(p^*,p^*)(\eta_{12}/\eta_{11})p^{*'}(c),$$

using the inverse-elasticity rule $(p^*(c)-c)/p^*(c)=1/\eta_{11}$. Since $\eta_{12}/\eta_{11}\equiv\eta_r$, we can express $\pi^{*'}(c)$ as

$$\pi^{*'}(c)=-D_1\left(p^*(c),p^*(c)\right)\left[1-p^{*'}(c)\eta_r\right]. \tag{21}$$

For a monopolist, an increase in c lowers profit because its profit margin for each unit of sales is reduced. Under competition, this margin reduction is alleviated by the rise in the rival's price due to the common increase in c, as reflected in the additional term $-p^{*'}(c)\eta_r$ in (21). Thus, $\pi^{*'}(c)<0$ if and only if $p^{*'}(c)\eta_r<1$. Condition (8) for a unique equilibrium does not ensure $p^{*'}(c)\eta_r<1$, and we will consider also $p^{*'}(c)\eta_r\geq 1$.

The result below is derived by analyzing when $\pi^*(c)$ is convex or concave.

Proposition 2 *Profit is higher under differential pricing than under uniform pricing if (22) holds, and is lower if the inequality in (22) is reversed.*

$$\frac{\eta_r(\alpha'+\eta_r')}{(2-\alpha-\eta_r)}+\eta_r'+\frac{\eta}{p}\left[(2-\alpha-\eta_r)-\eta_r\right]>0. \tag{22}$$

For a monopolist, in (22) $\eta_r=\eta_r'=0$, yielding simply $\eta\frac{2-\alpha}{p}>0$, which always holds. It represents a monopolist's gain from adjusting outputs across markets in response to mean-preserving cost dispersion[15] and is proportional to demand elasticity, akin to the flexibility gain for consumer surplus. As with consumer surplus, the condition for DP to raise profit in oligopoly embeds and generalizes the condition under monopoly. Next, consider the three terms in (22) for our oligopoly case.

The first term in (22), $\frac{\eta_r(\alpha'+\eta_r')}{2-\alpha-\eta_r}$, takes the sign of $(\alpha'+\eta_r')$ which determines the direction of change in average price moving to DP. It affects the firm's profit via the *rival's* price response to the common cost shocks ($p^{*'}(c)=1/(2-\alpha-\eta_r)$), and in proportion to the substitutability term, η_r. An increase in average price due to DP boosts industry profit because competition

[15] Recall that moving from UP to DP can be analyzed as a virtual decrease in marginal cost from $\bar{c}=\lambda c_L+(1-\lambda)c_H$ to c_L in market L and an increase from $\bar{c}$ to c_H in market H, with respective weights λ and $1-\lambda$.

under uniform pricing forces price too low from the standpoint of the industry.

The middle term, η_r', reflects an output externality. Each firm sets its price based on the firm elasticity of demand, η_{11}, but when both firms adjust prices equally, output is determined by market elasticity, $\eta = \eta_{11} - \eta_{12}$. Since $\eta_r \equiv \eta_{12}/\eta_{11} = 1 - \eta/\eta_{11}$, if $\eta_r'(p) > 0$, market elasticity relative to firm elasticity is smaller at higher prices than at lower prices. Moving to DP then induces a positive output externality on the rival firm: each firm ignores that (a) its price increase in market H expands the rival's output and (b) its price decrease in market L reduces the rival's output—but effect (a) exceeds (b) when $\eta_r' > 0$. Oligopoly DP then yields a larger output than predicted based on each firm's own-elasticity, boosting profit.[16]

The last term, $\frac{\eta}{p}\left[(2-\alpha-\eta_r)-\eta_r\right]$, can be written as $\frac{\eta}{p}\frac{1}{p^{*'}(c)}\left[1-p^{*'}(c)\eta_r\right]$: a monopolist's gain from adjusting prices and outputs across markets in response to cost dispersion, modified in oligopoly by the impact of the rival's symmetric price responses ($p^{*'}(c)\eta_r > 0$). Suppose $p^{*'}(c)\eta_r \in (0,1)$, or $\left[1-p^{*'}(c)\eta_r\right] > 0$. In market H, where moving to DP effectively raises marginal cost, the firm loses from the cost increase, but less than if it were a monopolist. The reverse occurs in market L, where moving to DP effectively lowers marginal cost. Since the rival's response in each market does not outweigh the own-cost effect, cost dispersion still benefits the firm, in proportion to the elasticity of market demand.

However, it is possible to have $p^{*'}(c)\eta_r \geq 1$ so that DP (weakly) decreases profit, unlike for monopoly. The condition $p^{*'}(c)\eta_r \geq 1$ requires $p^{*'}(c) > 1$ (since $\eta_r < 1$), hence the virtual cost decrease in market L reduces the profit margin there; the reverse pattern occurs in market H, where DP now increases the profit margin. Why, then, does overall profit fall? Observe that

$$m_L - m_H = c_H - c_L - \int_{c_L}^{c_H} p^{*'}(c)\,dc \;>(=)<0 \;\text{ if }\; p^{*'}(c) <(=)> 1. \tag{23}$$

Thus, when $p^{*'}(c) > 1$ the profit margin in market L under DP is lower than in H, hence the output reallocation to L harms profit, by (7). For a monopolist, DP nevertheless raises profit because $p^{*'}(c) > 1$ requires demand to be sufficiently convex that the monopolist's price dispersion expands output enough to outweigh the harmful misallocation (Chen and Schwartz,

[16] In addition, when $\eta_r' > 0$ DP tends to raise average price (see first term in (22)), which also boosts profit. Both effects are reversed if $\eta_r' < 0$, and DP then can reduce profit, as we will show.

2015).[17] In oligopoly, however, $p^{*'}(c) = \frac{1}{2-\alpha-\eta_r} > 1$ can arise not only from demand convexity ($\alpha > 0$), but also from the cross-elasticity term, η_r. Indeed, the condition $p^{*'}(c)\eta_r \geq 1$ requires both $p^{*'}(c) > 1$ and η_r large enough (though still < 1), so that the price fall in market L due to firms' virtual cost reduction moving to DP is driven sufficiently by product substitutability rather than demand curvature, hence industry output does not rise too much.[18] Example 2 in Subsection 3.4 illustrates a case with $p^{*'}(c)\eta_r \to 1$ in the limit, hence DP can fail to raise profits.

A second way that DP can reduce profit arises when $\eta_r' < 0$. DP then can lower both average price and total output, and reduce profit due to the first two terms in (22) (tracking the first and third terms in decomposition (7)). See Example 3 in Subsection 3.4, and Subsection 3.5.

Although DP by symmetric firms may reduce profit, the required demand conditions seem rather special. In the 'normal' case ($p^{*'}(c)\eta_r < 1$), there is a systematic force pushing towards greater profit: the beneficial output reallocation effect captured by the last term in (22). Profit and consumer surplus both benefit from greater scope for output reallocation under DP, a larger elasticity of market demand η. When firms have asymmetric costs, however, new forces will emerge that can make DP harmful to consumers or profits even with simple demand conditions (see Section 4).

3.3 Total Welfare

Given marginal cost c and the associated equilibrium price $p^*(c)$, the equilibrium total welfare in a market can be written as

$$\begin{aligned} W(p^*(c)) &\equiv w(c) = s(c) + 2\,[p^*(c) - c]\, D_1(p^*(c), p^*(c)) \\ &= s(c) + [p^*(c) - c]\, D(p^*(c)). \end{aligned} \tag{24}$$

Analyzing when $w(c)$ is convex, we obtain the following condition for DP to raise or lower total welfare.

[17] From revealed preference, monopoly profits must be no lower under DP than under the constraint of UP. The text explained why this holds even when $p^{*'}(c) > 1$.

[18] For a given own-price elasticity η_{11}, a larger cross-elasticity η_{12} implies a lower market-demand elasticity η. Thus, for $\eta_r \equiv \eta_{12}/\eta_{11}$ large (but still < 1), η_{11} can be high enough to render a unilateral price decrease profitable, yet η can be low enough that a price decrease by all firms is unprofitable.

Proposition 3 *DP increases total welfare if (25) holds, and DP reduces total welfare if (25) is reversed.*

$$\frac{-\left(\alpha' + \eta_r'\right)\left(1-\eta_r\right)}{\left(2-\alpha-\eta_r\right)} + \eta_r' + \frac{\eta}{p}\left[\left(2-\alpha-\eta_r\right) + \left(1-\eta_r\right)\right] > 0. \tag{25}$$

The first term corresponds to the first terms in (20) and (22), reflecting the net effect of change in average price on consumer surplus plus profit: when $(\alpha' + \eta_r') \leq 0$, average price weakly falls, hence the net effect is weakly positive due to industry output expansion given $1 - \eta_r > 0$.[19] The second term, η_r', is the same as the middle term in (22) and captures the net output externality under competition across the two markets, H and L. Together, the first and second terms in (25) reflect how DP affects total welfare through the change in total output. The last term, $\frac{\eta}{p}\left[\left(2-\alpha-\eta_r\right) + \left(1-\eta_r\right)\right]$, corresponds to the output adjustment effect: moving from UP to DP creates price dispersion for consumers and cost dispersion for firms, leading to beneficial output adjustments in total, and a higher η magnifies this effect. DP can raise total welfare due to both the (beneficial) output reallocation and output expansion, but neither of them alone is necessary for DP to raise welfare.[20] And like its counterparts for consumer surplus and profit, (20) and (22), condition (25) is met for a broad class of demands, such as those with constant α and η_r, but not always; see Example 3 below.

The condition for DP to raise total welfare under monopoly (Chen and Schwartz, 2015, condition (A1B)) can be written as

$$\frac{-\sigma' D'(p)}{(2-\sigma)} + \frac{\eta}{p}\left[(2-\sigma) + 1\right] > 0,$$

and monopoly DP lowers total welfare if the above inequality is reversed. As with consumer surplus and profit, the condition for DP to raise total welfare under oligopoly, (25), embeds and generalizes that under monopoly: $\eta_r = \eta_r' = 0$ for the single-product monopolist, while α in oligopoly corresponds to σ under monopoly and α' corresponds to $\sigma' D'(p)$.

Finally, observe that under monopoly, DP always increases profit, hence total welfare rises under broader conditions than does consumer surplus (compare Propositions 1 and 2 of Chen

[19] Recall that $1 - \eta_r = (\eta_{11} - \eta_{12})/\eta_{11} = \eta/\eta_{11}$, the market elasticity of demand divided by the firm's elasticity, and $\eta > 0$ since market demand is sensitive to price.

[20] Importantly, output reallocation under $p_H - c_H < p_L - c_L$ can improve total welfare even when total output decreases, a key difference between cost-based DP and price discrimination.

and Schwartz, 2015). In oligopoly, DP can reduce profit, hence consumer surplus may rise yet total welfare fall, as in Example 3. Thus, the conditions for DP to raise consumer surplus or total welfare, (20) and (25), are no longer nested.

3.4 Examples

The ensuing examples show that the conditions in Propositions 1-3 for differential pricing to benefit consumers, profits, and overall welfare, are met by familiar demand functions—though not always—and illustrate the underlying economic forces.[21] To that end, recall from (7) that the output reallocation induced by DP affects profit positively when $m_L > m_H$ and negatively when $m_L < m_H$; and that $m_L > (<)\ m_H$ if $p^{*'}(c) < (>)\ 1$, from (23).[22] For abbreviation, we use q_i to denote demand for firm $i's$ product.

Example 1 *Linear demand (DP increases consumer surplus and profit):*

$$q_i = a - p_i + \gamma(p_j - p_i), \quad a > 0,\ \gamma > 0.$$

Then, $\eta_r = \frac{\gamma}{1+\gamma}$, $\eta = \frac{p}{a-p}$, *and* $\alpha = 0$.

It follows that both (20) and (22) hold, hence DP increases consumer surplus and profit. Average price and total output are the same under UP and DP, so the gains come solely from reallocating output between markets. The gains can be significant. For instance, if $\{a, \gamma, c_L, c_H, \lambda\} = \{8, 1.5, 3, 5, 0.5\}$, profit and consumer surplus both rise by 6.25% when moving from UP to DP; and if $\{a, \gamma, c_L, c_H, \lambda\} = \{8, 1, 2, 5, 0.4\}$, they rise by 12.24%.[23]

Example 2 *CES demand (DP increases consumer surplus and profit):*

$$q_i = m^{\frac{1}{1-m}} p_i^{-\rho} \left(p_1^{1-\rho} + p_2^{1-\rho}\right)^{\frac{\frac{1}{1-m}-\rho}{\rho-1}}, \quad \rho > 1, \quad 0 < m < \frac{\rho - 1}{\rho}.$$

Then, $p^*(c) = \frac{(1+\rho-m\rho)c}{2m+\rho-m\rho-1}$, $p^{*\prime}(c) = \frac{1+\rho-m\rho}{2m+\rho-m\rho-1} > 1$, $\eta_r = \frac{\rho-m\rho-1}{1+\rho-m\rho}$, $\eta = \frac{1}{1-m}$, $\alpha = \frac{4-2m}{1+\rho-\rho m}$.

[21] Armstrong and Vickers (2018) characterize an important class of demand systems in which consumer surplus is a homothetic function of quantities. The demand functions in our examples below are all part of this class.

[22] More details on the examples in this section are available upon request.

[23] This linear demand system is adapted from Shubik and Levitan (1980) and will be used again in Example 4 and in Subsection 4.2.

Once again, both (20) and (22) hold, so that DP increases consumer surplus and profits. Since α and η_r are constant, average price does not change, and DP increases consumer surplus due to output expansion and reallocation. Although $p^{*\prime}(c) > 1$, hence the output reallocation is excessive for profit, $p^{*\prime}(c)\eta_r < 1$ holds for all feasible parameters so profit still increases due to the output expansion. However,

$$p^{*\prime}(c)\eta_r = \frac{1+\rho-m\rho}{2m+\rho-m\rho-1}\frac{\rho-m\rho-1}{1+\rho-m\rho} = \frac{1-\rho+m\rho}{1-\rho+m(\rho-2)} \to 1 \text{ as } m \to 0 \text{ or as } \rho \to \infty.$$

Therefore, in this example DP can fail to raise profit in the limit as $m \to 0$ or as $\rho \to \infty$.

Example 3 *Binomial Logit demand with outside option (DP increases consumer surplus but can reduce total output, profit, and total welfare):*

$$q_i = \frac{e^{\frac{a-p_i}{\mu}}}{e^{\frac{a-p_i}{\mu}} + e^{\frac{a-p_j}{\mu}} + A}, \qquad A > 0, \quad \mu > 0.$$

Then,

$$p^* = c + \mu + \frac{\mu}{Ae^{\frac{-a+p^*}{\mu}} + 1}, \quad p^{*\prime}(c) = \frac{\left(e^{\frac{a}{\mu}} + Ae^{\frac{p^*}{\mu}}\right)^2}{e^{\frac{2a}{\mu}} + A^2 e^{\frac{2p^*}{\mu}} + 3Ae^{\frac{a+p^*}{\mu}}};$$

$$\eta_r = \frac{1}{1+Ae^{\frac{-a+p^*}{\mu}}}, \quad \eta_r' < 0; \quad \alpha = \frac{A^2 e^{\frac{2p^*}{\mu}}}{\left(e^{\frac{a}{\mu}} + Ae^{\frac{p^*}{\mu}}\right)^2} > 0, \quad \alpha' > 0; \quad \eta = \frac{Ae^{\frac{p^*}{\mu}} p^*}{2e^{\frac{a}{\mu}}\mu + Ae^{\frac{p^*}{\mu}}\mu}.$$

In Example 3, condition (20) always holds, hence DP raises consumer surplus. However, conditions (22) and (25) can be reversed, so DP can lower profit and total welfare. For instance, let $A = 0.01$, $c_L = 0$, $c_H = 2$, $\mu = 1$, $a = 0$, and $\lambda = 0.5$. Then: $m_L^d = 1.94 > m_H^d = 1.71$, so the output reallocation benefits profit and total welfare. But DP lowers average price, from $p^u = 2.85$ to $p^d = 2.82$, and lowers total output, from $q^u = 0.92$ to $q^d = 0.90$, causing profit to fall: $\pi^d = 1.65 < \pi^u = 1.71$. The output reduction also reduces welfare, albeit slightly (by 0.067%). When $\mu = 0.3$, $a = 0.5$ instead, DP lowers profit, but the incremental gain in consumer surplus dominates the reduction in profit, and total welfare is higher under DP.

The price and output changes can be understood as follows. Demand is more convex at higher prices ($\alpha' > 0$), which pushes the pass-through $p^{*'}(c)$ to be increasing; however, the products' substitutability is smaller at higher prices ($\eta_r' < 0$), which pushes $p^{*'}(c)$ to be decreasing. On balance, $\alpha' + \eta_r'$ can be either positive or negative, depending on parameter values. As a result, the average price may either increase or decrease under DP; and, associated with this, output may either increase or decrease. But consumer surplus always increases under DP, primarily due to the positive output reallocation effect.

Summarizing our findings, DP raises both consumer surplus and profit when the demand curvature (α) and the elasticity ratio (η_r) do not change too fast relative to the elasticity of market demand (η). In particular, DP raises both consumer surplus and profit when α and η_r are constant, such as for linear and CES demand. There are demands for which, under some parameter values, profit or total welfare can be lower under DP than under UPs, but such cases appear to be unusual.[24]

3.5 Comparison to Oligopoly Price Discrimination

For symmetric oligopoly, Holmes (1989) analyzed price discrimination rather than our cost-based differential pricing. Our results exhibit similarities to his findings as well as differences.

In both settings, differential pricing—cost-based or demand-based—may reduce profit relative to uniform pricing, unlike for monopoly. Holmes shows this can occur if the market with the smaller elasticity of market demand has the larger cross-price elasticity between firms. Price discrimination then lowers price in the 'wrong market' and can reduce total output. In our setting, markets differ only in costs, but DP still can reduce output and profit if cross-price elasticity relative to a firm's own-price elasticity, for the common demand function across markets, is greater at lower prices than at higher prices (Example 3). Then price can fall by more in the low-cost market than it rises in the other market, while still reducing total output.[25]

Turning to consumer surplus, cost-based DP is more likely to benefit consumers than is price

[24] Under our symmetric setting, we have not found an example in which DP lowers consumer surplus, even though it is conceivably possible. However, as we show in Section 4, DP can lower consumer surplus if costs differ between firms. Thus, our broad message is that DP is often, but not always, beneficial to consumers.

[25] Additionally, cost-based DP can potentially fail to increase profit due to a second force: excessive output reallocation between markets when $p^{*'}(c) > 1$, as in Example 2 when $m \to 0$ or when $\rho \to \infty$.

discrimination. From our profit decomposition (7), profitable price discrimination requires an increase in total output or in average price; indeed, price discrimination has a tendency to raise average price, which of itself harms consumers.[26] By contrast, DP can raise profit even if average price does not rise (Examples 1-2), which ensures that consumers also benefit. The cost savings achieved by reallocating output to the lower-cost market provide firms an incentive to adopt DP also under demand conditions that do not yield an increase in average price, and consumers benefit from the price dispersion by adjusting their consumption patterns.

Total welfare also is more likely to rise with cost-based DP than with price discrimination. Discrimination misallocates output between markets, hence an increase in total output is necessary for total welfare to rise.[27] In contrast, cost-based DP can increase welfare when output remains constant, as with linear demand (Example 1),[28] or potentially even when output falls, due to the favorable reallocation to the lower-cost market (as in Example 3 for some parameters).

As a robustness exercise, we consider an example where markets differ both in their costs of service, as until now, and in their demand elasticities.

Example 4 *Markets differ in costs and demands (DP is beneficial if the difference in costs is large relative to that in demands):*
Firm i faces the following demand system (an extension of Example 1) in market $k \in \{L, H\}$,

$$q_{ik} = a + b_k - p_{ik} + \gamma(p_{jk} - p_{ik}). \tag{26}$$

Let $b_L = 0$ and $b_H = b > 0$ so that H is the 'strong' market.[29] As before, consumers are distributed between markets L and H in proportions λ and $1-\lambda$, and the firms are symmetric in each market with marginal costs c_L and c_H. In the Appendix we show the following when moving from UP to DP: average price weighted by the consumption quantities under UP increases; total

[26] "There is a sense in which discrimination increases 'average' price; the increase in price in the strong market above the uniform price is 'large' relative to the decrease in the weak-market price." (Holmes 1989, p. 248.) Under monopoly, this price bias of price discrimination is discussed by Chen and Schwartz (2015, pp. 449-451).

[27] Holmes (1989, fn.2) notes that this well-known result under monopoly also "holds for this oligopoly analysis."

[28] In Holmes' setting, price discrimination can increase or decrease output even with linear demand, depending on his elasticity-ratio condition, which compares relative market-demand elasticities to relative cross-price elasticities. When output falls, total welfare also must fall.

[29] Cowan (2007) also uses a shifting parameter in the demand function to capture different demand elasticity across markets.

output does not change; and profit increases. Furthermore, there exist critical values b_1 *and* b_2 *such that: i) consumer surplus increases if* $b \leq b_1$*; ii) total welfare increases if* $b \leq b_2$.

The weighted average price is analogous to p^d defined in(4), but the weights now reflect both the relative sizes of markets L and H (λ and $1-\lambda$) *and* that per capita quantity under UP is higher in market H. This weighted average price exceeds p^u, unlike in the base model where only costs differ, because price discrimination is now present. Profit rises for two reasons: the rise in average price, and cost savings from reallocating output. Consumer surplus rises (due to price dispersion) as does total welfare (due to cost savings) if the demand difference between markets is small relative to the cost difference, so that differential pricing is driven predominantly by cost differences. This generalizes the results from our main model where DP is driven solely by cost differences: there, DP always raises total welfare and its components under linear demand. Notice that a similar result also holds under monopoly in Chen and Schwartz (2015).

4. Firms With Asymmetric Costs

Do asymmetries between firms introduce new forces, beyond demand-side factors such as pass-through, that alter the welfare properties of DP relative to UP? We extend our model to allow cost asymmetries between firms for a given market, in addition to cost differences across markets as assumed until now. Thus, firm i has costs (c_{iL}, c_{iH}), where $c_{iH} > c_{iL}$, for $i = 1, 2$. The firms are still assumed symmetric in demand: they produce either homogeneous products (Subsection 4.1 below) or symmetrically differentiated products (Subsection 4.2 below).

4.1 Homogeneous Products

Consider two scenarios of cost asymmetries:

(1) *Global Cost Advantage*: the same firm, say firm 1, has a cost advantage in serving both consumer groups: $c_{1L} < c_{2L}$ and $c_{1H} < c_{2H}$;

(2) *Local Cost Advantage*: each firm has a cost advantage in serving a different group. Without loss of generality, let $c_{1L} < c_{2L}$ and $c_{1H} > c_{2H}$, with $\bar{c}_1 \equiv \lambda c_{1L} + (1-\lambda)c_{1H} \leq \bar{c}_2 \equiv \lambda c_{2L} + (1-\lambda)c_{2H}$.

Global Cost Advantage

We adopt the standard assumption for Bertrand competition with asymmetric costs: the lower-cost firm can capture the market by pricing at the rival's marginal cost. Assume also that firms' costs are not too far apart, so the lower-cost firm sets price below its monopoly level, at the rival's cost. Under DP, competition occurs market-by-market and the equilibrium prices in the two markets are therefore:

$$p_L = \max\{c_{1L}, c_{2L}\} = c_{2L}; \qquad p_H = \max\{c_{1H}, c_{2H}\} = c_{2H}. \tag{27}$$

Under UP, we assume that the firm with the lower average cost can capture both markets by pricing at the other firm's *average cost.* Therefore, the equilibrium uniform price is given by

$$p^u = \max\{\bar{c}_1, \bar{c}_2\} = \bar{c}_2. \tag{28}$$

The next result shows that, while DP benefits consumers, profits can readily fall. The profit comparison depends on the difference in marginal costs of serving the two markets for firm 1 ($\Delta c_1 \equiv c_{1H} - c_{1L} > 0$) relative to firm 2 ($\Delta c_2 \equiv c_{2H} - c_{2L} > 0$).

Proposition 4 *For any given pair of costs* $\{(c_{1L}, c_{1H}), (c_{2L}, c_{2H})\}$ *with* $c_{1L} < c_{2L}$, $c_{1H} < c_{2H}$, *and* $\Delta c_i \equiv c_{iH} - c_{iL} > 0$, $i = 1, 2$:
(i) $p^d = p^u$, *and hence* $S^d > S^u$;
(ii) with linear demand, $\Pi^d > \Pi^u$ *if* $\Delta c_1 > \Delta c_2$ *and* $\Pi^d < \Pi^u$ *if* $\Delta c_1 < \Delta c_2$;
(iii) relative to linear demand, $\Pi^d - \Pi^u$ *and* $W^d - W^u$ *are higher if demand is strictly convex and lower if demand is strictly concave.*

Part (*i*) is straightforward. Consider part (*ii*). With linear demand, total output as well as average price are the same under DP and UP, hence the change in firm 1's profit is determined entirely by the reallocation effect in (7). Firm 1's prices are set equal to firm 2's costs: $p_L = c_{2L}$ and $p_H = c_{2H}$. Thus, the industry profit is the same as firm 1's profit, and the difference in firm 1's profit margins under DP between markets L and H is $(c_{2L} - c_{1L}) - (c_{2H} - c_{1H}) = \Delta c_1 - \Delta c_2$. The output reallocation under DP raises profit if the margin is higher in market L, which occurs if $\Delta c_1 > \Delta c_2$, and lowers profit if $\Delta c_1 < \Delta c_2$.[30] Intuitively, firm 1 is harmed by being constrained to adopt a price differential larger than the difference in its costs.

Turning to part (*iii*), in market H where price rises under DP, output decreases less if demand is strictly convex instead of linear, while in market L where price falls under DP, output increases by more if demand is strictly convex instead of linear. Relative to linear demand, therefore, $\Pi^d - \Pi^u$ and $W^d - W^u$ are both higher if demand is strictly convex, and the conclusion is reversed if demand is strictly concave.

Local Cost Advantage

Now suppose firm 1 has the cost advantage for market L and firm 2 has the advantage for H: $c_{1L} < c_{2L} < c_{2H} < c_{1H}$, with $\bar{c}_1 \leq \bar{c}_2$. Under DP, firm 1 serves market L at price $p_L = c_{2L} < \bar{c}_2$ and firm 2 serves market H at price $p_H = c_{1H} > c_{2H} > \bar{c}_2$. Under UP, we assume that each firm cannot refuse to serve its higher-cost market; it must be willing to sell in both markets or none. Suppose also that at equal prices ($p_1 = p_2$), if $\bar{c}_1 < \bar{c}_2$, firm 1 can capture both markets at price $\bar{c}_2$, while if $\bar{c}_1 = \bar{c}_2$, the firms split both markets equally. In both cases, the equilibrium uniform price is $p^u = \bar{c}_2$, and moving to DP lowers price in market L and raises price in market H, but raises average price.[31] The next result shows that while profit necessarily rises, consumer surplus can fall without requiring unusual demand conditions.

Proposition 5 *For any given pair of costs* $\{(c_{1L}, c_{1H}), (c_{2L}, c_{2H})\}$ *with* $c_{1L} < c_{2L} < c_{2H} < c_{1H}$, *and* $\bar{c}_1 \leq \bar{c}_2$*:*

(i) average price is higher under DP than under UP: $p^d > p^u$*;*

(ii) consumer surplus is higher under DP ($S^d > S^u$) *if cost differences within markets,* $\Delta_L \equiv c_{2L} - c_{1L}$ *and* $\Delta_H \equiv c_{1H} - c_{2H}$*, are small, but* $S^d < S^u$ *if* $c_{2H} - c_{2L}$ *is small;*

(iii) profits are always higher under DP: $\Pi^d > \Pi^u$*.*

Result (i) above can be understood as follows. The uniform price is determined by the firm with the higher *average* of the marginal costs across the two markets. Under DP, each

[30] With linear demand, total welfare rises if $\Delta c_1 \geq \Delta c_2$, but can fall if $\Delta c_1 < \Delta c_2$. As $\Delta c_1 \to 0$, the output allocation under uniform pricing converges to the first-best, but is inefficient under differential pricing, since $p_{1H} - p_{1L} = \Delta c_2 > 0$, hence $W^d < W^u$.

[31] We have considered a variant of this scenario, where firm 1 has lower cost to serve market A than market B and the reverse holds for firm 2. Differential pricing then raises price in both markets.

market's price is set by the higher of the two firms' marginal costs for that market. Since cost heterogeneity is greater market-by-market than on average, the average price is higher under DP. The effect is similar to one noted by Dana (2012).

Consumer surplus is subject to opposing effects: it increases due to the price dispersion, but decreases due to the rise in average price. When the cost difference between firms within each market ($c_{2k} - c_{1k}$, $k = L, H$) is sufficiently small, the average price under DP converges to the uniform price, hence the price dispersion effect dominates and DP raises consumer surplus. The opposite arises if the cost difference between markets for firm 2 ($c_{2H} - c_{2L}$) is small: then $p^u = \bar{c}_2$ is close to c_{2L}, so that moving to DP lowers price in market L only slightly but raises price in market H substantially. However, DP can lower consumer surplus even when $c_{2H} - c_{2L}$ is not 'small'. For example, suppose $\lambda = 1/2$, $D(p) = 10 - p$, $c_{1L} = 3, c_{2L} = 4$, $c_{2H} = 6, c_{1H} = 7$. Then $\bar{c}_1 = \bar{c}_2 = 5 = p^u$, $p_L = 4$, $p_H = 7$, $p^d = 5.5 > p^u$, and $S^d = 11.25 < S^u = 12.5$.

Industry profits always rise with DP, for three reasons: as with monopoly, output is reallocated to the lower-cost market; in addition, DP now leads to each market being served by the efficient firm in that market (firm 2 replaces 1 in market H) and, furthermore, DP raises average price by relaxing the competitive constraint.

Summarizing, when a different firm has the cost advantage in each market, DP has opposing effects on consumer surplus. The price dispersion is beneficial, but the increase in average price is harmful. The first effect dominates if the cost heterogeneity across markets is large relative to that between firms in a given market, while the second effect dominates in the reverse case.

4.2 Differentiated Products With Linear Demands

Propositions 4 and 5 showed that DP can reduce profit or consumer surplus even with 'simple' demand functions—such as linear demand—when firms have asymmetric costs and produce homogeneous products, i.e., perfect substitutes. To check whether these findings may extend to imperfect substitutes, we consider differentiated products with the following linear demand system:

$$q_i = a - p_i + \gamma(p_j - p_i) \quad \text{for } i,\, j \in \{1, 2\} \ (j \neq i)\,, \tag{29}$$

where $a > 0$, and $\gamma \in (0, +\infty)$ measures the degree of product substitutability. The products become unrelated (highly differentiated) as $\gamma \to 0$ and highly substitutable as $\gamma \to \infty$; with equal marginal costs c, firms' equilibrium price converges to c as $\gamma \to \infty$. This system reflects the demands of a representative consumer with utility from the two differentiated goods:

$$V(q_1, q_2) = a(q_1 + q_2) - \frac{1}{2}\left[\frac{\gamma+1}{2\gamma+1}q_1^2 + \frac{2\gamma}{2\gamma+1}q_1 q_2 + \frac{\gamma+1}{2\gamma+1}q_2^2\right]. \tag{30}$$

As in Subsection 4.1, the two firms may have different costs of serving the same market; c_{1k} may differ from c_{2k}, for $k = L, H$, in arbitrary ways—including the cases of global or local cost advantage.

Proposition 6 *Suppose the demand system is given in (29). There exist critical values γ_1 and γ_2 such that:*
(i) when $\gamma < \gamma_1$, consumer surplus and industry profit are both higher under DP than under UP regardless of the cost asymmetry between firms;
(ii) when $\gamma > \gamma_2$, the following results in Propositions 4 and 5 hold: with global cost advantage ($c_{1L} < c_{2L}$ and $c_{1H} < c_{2H}$), DP reduces profit if $c_{1H} - c_{1L} < c_{2H} - c_{2L}$, whereas with local cost advantage ($c_{1L} < c_{2L}$, $c_{1H} > c_{2H}$ and $\bar{c}_1 \leq \bar{c}_2$), DP reduces consumer surplus if $c_{2H} - c_{2L}$ is sufficiently small.

When the products are sufficiently differentiated ($\gamma < \gamma_1$), under both UP and DP the equilibrium is *interior,* with both firms producing positive outputs and the prices determined by the standard first-order conditions. The average prices under DP and under UP are then equal, as in the case of symmetric costs and linear demands (Example 1). Hence, consumer surplus is higher under DP, and industry profit also is higher because it is a convex function of (c_1, c_2).

When products are sufficiently close substitutes ($\gamma > \gamma_2$), under both regimes we have a *corner equilibrium.* Under UP, firm 2 (the higher-cost firm here) sets price at marginal cost $\bar{c}_2$ while firm 1 captures the market by setting a limit price below $\bar{c}_2$ that induces zero demand for firm 2, and this limit price $\to \bar{c}_2$ as $\gamma \to \infty$; and similarly under DP the lower-cost firm in each market, L or H, sets a limit price to capture that market. Thus, as the products converge to perfect substitutes, the outcome converges to the homogeneous-products case, described in

Proposition 4 for global cost advantage and in Proposition 5 for local cost advantage.[32]

5. Conclusion

The welfare properties of uniform versus differential pricing in oligopoly when markets differ in costs of service have gone largely unexplored, despite the prevalence of industries where firms are constrained from adopting cost-based differential pricing. In a standard setting where firms face symmetric demands, we showed that the effects of purely cost-based differential pricing on consumer welfare and profits depend on whether products are homogeneous or differentiated and whether firms are symmetric in costs or not.

With symmetric firms, if products are homogeneous then differential pricing obviously maximizes consumer welfare whereas uniform pricing does not, while profits are zero in both regimes. If products are differentiated, then differential pricing increases consumer surplus and profits under conditions met by many standard demand functions. The systematic force driving higher profit is cost savings from reallocating output between markets by adjusting prices; consumers benefit from this price dispersion provided average price does not rise too much. Although profit can fall with differential pricing—unlike for monopoly—and potentially consumer surplus too, such outcomes require demand conditions that seem rather stringent.

When firms have asymmetric costs, however, differential pricing can reduce profit or, under an alternative cost configuration, reduce consumer surplus even for standard demand functions such as linear demands.

Thus, cost-based differential pricing in oligopoly can have subtle welfare effects. By elucidating these effects and the underlying economic forces, this paper advances our understanding of a significant issue in economics—in parallel to the extensive studies on third degree price discrimination—and helps evaluate prevalent constraints on a common business practice.

[32] Chen, Li and Schwartz (2017, Proposition 5) considered an alternative linear-demands system: $q_i = a - p_i + \gamma p_j$ for $i \neq j = 1, 2$, $\gamma \in (0, 1)$. The corner solution is absent in that model because the products do not converge to perfect substitutes as $\gamma \to 1$. The equilibrium then is always interior, and DP increases consumer surplus and profits regardless of the cost configuration, consistent with our Proposition 6 for $\gamma < \gamma_1$. The qualitative difference between the two types of equilibria seems to be that in an interior equilibrium prices depend on both firms' costs, whereas in a corner equilibrium the price of the firm that makes positive sales is determined solely by the rival's marginal cost.

Appendix

Proof of Proposition 1. Using (18), we have

$$s'(c) = S'(p^*)\, p^{*'}(c) = -D(p^*)\, p^{*'}(c).$$

It follows that

$$\begin{aligned}
s''(c) &= -D'(p^*)\left[p^{*'}(c)\right]^2 - D(p^*)\, p^{*''}(c) \\
&= -D'(p^*)\left[p^{*'}(c)\right]^2 - D(p^*)\left(\eta_r' + \alpha'\right)\left[p^{*'}(c)\right]^3 \\
&= \left[\frac{-D'(p^*)}{D(p^*)} - \left(\eta_r' + \alpha'\right) p^{*'}(c)\right] D(p^*)\left[p^{*\prime}(c)\right]^2 \\
&= \left[\frac{\eta}{p^*} - \frac{\eta_r' + \alpha'}{2 - \alpha - \eta_r}\right] D(p^*)\left[p^{*\prime}(c)\right]^2
\end{aligned}$$

Thus,

$$\text{Sign } s''(c) = \text{Sign } \left[\frac{\eta}{p^*} - \frac{\eta_r' + \alpha'}{2 - \alpha - \eta_r}\right]$$

Therefore, if (20) holds, $s(c)$ is convex and

$$\begin{aligned}
S^u &= S(p^u(\bar{c})) = s(\bar{c}) = s(\lambda c_L + (1-\lambda) c_H) \\
&< \lambda s(c_L) + (1-\lambda) s(c_H) = \lambda S(p^*(c_L)) + (1-\lambda) S(p^*(c_H)) = S^d.
\end{aligned}$$

Similarly, if (20) is reversed, then $s(c)$ is concave and DP lowers consumer surplus. ■

Proof of Proposition 2. We derive the condition for equilibrium profit $\pi^*(c)$ to be convex as follows:

$$\begin{aligned}\pi^{*''}(c) &= \frac{dD_1(p^*,p^*)}{dp^*}p^{*'}(c)\left[p^{*'}(c)\eta_r - 1\right] + D_1(p^*,p^*))\left[p^{*''}(c)\eta_r + \left[p^{*'}(c)\right]^2\eta_r'\right] \\
&= \frac{D_1(p^*,p^*)p^{*\prime}(c)}{p^*}\left\{\underbrace{\frac{-D'(p^*(c))p^*}{D(p^*(c))}}_{\eta(p^*)}\left[1 - p^{*'}(c)\eta_r\right] + p^*\left[\left(\eta_r' + \alpha'\right)\left[p^{*'}(c)\right]^2\eta_r + p^{*'}(c)\eta_r'\right]\right\} \\
&= \frac{D_1(p^*,p^*)p^{*\prime}(c)}{p^*}\left\{\eta\left[1 - \frac{\eta_r}{2-\alpha-\eta_r}\right] + p^*\left[\frac{(\eta_r'+\alpha')\,\eta_r}{(2-\alpha-\eta_r)^2} + \frac{\eta_r'}{2-\alpha-\eta_r}\right]\right\} \\
&= \frac{D_1(p^*,p^*)\left[p^{*\prime}(c)\right]^2}{p^*}\left\{\eta\left[(2-\alpha-\eta_r)-\eta_r\right] + p^*\left[\frac{(\eta_r'+\alpha')\,\eta_r}{(2-\alpha-\eta_r)} + \eta_r'\right]\right\}.\end{aligned}$$

Therefore, we have

$$\text{Sign } \pi^{*''}(c) = \text{Sign }\left\{\frac{\eta_r(\alpha'+\eta_r')}{(2-\alpha-\eta_r)} + \eta_r' + \frac{\eta}{p^*}\left[(2-\alpha-\eta_r)-\eta_r\right]\right\}.$$

Denote the equilibrium profits of each firm under uniform pricing by $\pi^u = \pi^*(\bar{c})$, and under differential pricing by $\pi^d = \lambda\pi^*(c_L) + (1-\lambda)\,\pi^*(c_H)$. Then, when $\pi^{*''}(c) \gtreqless 0$,

$$\pi^u = \pi^*(\bar{c}) \lesseqgtr \lambda\pi^*(c_L) + (1-\lambda)\,\pi^*(c_H) = \pi^d.$$

Therefore, for $\eta_r = \eta_r(p)$, $\alpha = \alpha(p)$, and $\eta = \eta(p)$: $\pi^{*''}(c) > 0$ or $\pi^d > \pi^u$ if (22) holds, and $\pi^d < \pi^u$ if (22) is reversed. ■

Proof of Proposition 3. From (24),

$$\begin{aligned}w'(c) &= -D(p^*)\,p^{*'}(c) + \left[p^{*'}(c) - 1\right]D(p^*(c)) + \left[p^*(c) - c\right]D'(p^*(c))p^{*'}(c) \\
&= p^{*\prime}(c)D'(p^*(c))(p^*(c)-c) - D(p^*(c)).\end{aligned}$$

But because $p^*\frac{D'(p^*(c))}{D(p^*(c))} = -\eta_{11} + \eta_{12}$ and $\frac{p^*(c)-c}{p^*} = \frac{1}{\eta_{11}}$, we have

$$w'(c) = -D(p^*(c))\left[p^{*\prime}(c)\,(1-\eta_r) + 1\right].$$

It follows that

$$w''(c) = -D'(p^*(c))p^{*'}(c)\left[1+p^{*\prime}(c)(1-\eta_r)\right] - D(p^*(c))\left[p^{*''}(c)(1-\eta_r) - \left[p^{*\prime}(c)\right]^2\eta_r'\right]$$
$$= -D'(p^*(c))p^{*'}(c)\left[1+p^{*\prime}(c)(1-\eta_r)\right] - D(p^*(c))\left[\left(\eta_r'+\alpha'\right)\left[p^{*\prime}(c)\right]^3(1-\eta_r) - \left[p^{*\prime}(c)\right]^2\eta_r'\right]$$
$$\gtreqless 0$$

$$\Longleftrightarrow \frac{-D'(p^*(c))}{D(p^*(c))}\left[1+p^{*\prime}(c)(1-\eta_r)\right] - \left[\left(\eta_r'+\alpha'\right)\left[p^{*'}(c)\right]^2(1-\eta_r) - p^{*'}(c)\eta_r'\right] \gtreqless 0 \Longleftrightarrow$$
$$\eta\left[1+\frac{1-\eta_r}{2-\eta_r-\alpha}\right] + p^*\left[\frac{-\left[\alpha'+\eta_r'\right](1-\eta_r)}{\left[2-\eta_r-\alpha\right]^2} + \frac{\eta_r'}{2-\eta_r-\alpha}\right] \gtreqless 0.$$
$$\frac{-\left(\alpha'+\eta_r'\right)(1-\eta_r)}{(2-\eta_r-\alpha)} + \eta_r' + \frac{\eta}{p^*}\left[(2-\alpha-\eta_r)+(1-\eta_r)\right] \gtreqless 0.$$

When $w''(c) \gtreqless 0$, we have

$$\begin{aligned} W\left(p^*\left(\bar{c}\right)\right) &= w(\bar{c}) = w(\lambda c_L + (1-\lambda)c_H) \\ &\lesseqgtr \lambda w(c_L) + (1-\lambda)w(c_H) = \lambda W\left(p^*\left(c_L\right)\right) + (1-\lambda)W\left(p^*\left(c_H\right)\right). \end{aligned}$$

■

Proof of Example 4. Under DP, the symmetric equilibrium price and each firm's output and profit in market $k \in \{L, H\}$ can be written as:

$$p_k = \frac{a+b_k+c_k(1+\gamma)}{2+\gamma}, \quad q_k = \frac{\lambda_k(a+b_k-c_k)(1+\gamma)}{2+\gamma}, \quad \pi_k = \frac{\lambda_k(a+b_k-c_k)^2(1+\gamma)}{(2+\gamma)^2}$$

in which $\lambda_k = \lambda$ for $k = L$ and $\lambda_k = 1-\lambda$ for $k = H$.

Total consumer surplus in market k is[33]

$$s_k = \frac{\lambda_k(a+b_k-c_k)^2(1+\gamma)^2}{(2+\gamma)^2}.$$

[33] The consumer surplus in market k can be calculated using the representative consumer's utility in market k

$$V_k\left(q_1, q_2\right) = (a+b_k)(q_1+q_2) - \frac{1}{2}\left[\frac{\gamma+1}{2\gamma+1}q_1^2 + \frac{2\gamma}{2\gamma+1}q_1q_2 + \frac{\gamma+1}{2\gamma+1}q_2^2\right]. \quad (31)$$

Under UP, at symmetric price (p, p), each firm's demand in market k is proportional to $a + b_k - p$. Therefore, a firm's demand across both markets is $\lambda(a - p) + (1 - \lambda)(a + b - p)$ if $p \leq a$. If $a + b > p > a$, market L is not served and each firm's demand is $(1 - \lambda)(a + b - p)$. Thus, both markets will be served if and only if the equilibrium symmetric price satisfies $p \leq a$. We shall analyze this case, which holds when $b \leq \frac{(a-\bar{c})(1+\gamma)}{1-\lambda}$, where $\bar{c}$ was defined in (3).

With both markets served, the symmetric equilibrium price and each firm's output are:

$$p^u = \frac{a + (1-\lambda)b + \bar{c}(1+\gamma)}{2+\gamma}, \qquad q^u = \frac{(a + b(1-\lambda) - \bar{c})(1+\gamma)}{2+\gamma};$$

and each firm's equilibrium profit is

$$\begin{aligned} \pi^u &= \lambda(a - p^u)(p^u - c_L) + (1-\lambda)(a + b - p^u)(p^u - c_H) \\ &= (a - p^u)(p^u - \bar{c}) + (1-\lambda)b(p^u - c_H) \\ &= \frac{\left((a-\bar{c})(1+\gamma) - b(1-\lambda)\right)\left(a + b(1-\lambda) - \bar{c}\right)}{(1+\gamma)^2} + b(1-\lambda)\left(-c_H + \frac{a + b(1-\lambda) + \bar{c}(1+\gamma)}{2+\gamma}\right). \end{aligned}$$

Total consumer surplus under UP is

$$\begin{aligned} S^u &= -a^2 + b^2(1-\lambda) + \frac{2\left((a-\bar{c})(1+\gamma) - b(1-\lambda)\right)\left(a + b(1-\lambda)\right)}{(2+\gamma)} \\ &\quad + \frac{\left(a + b(1-\lambda) + \bar{c}(1+\gamma)\right)^2}{(2+\gamma)^2}. \end{aligned}$$

Note that the per-capita equilibrium quantity in market k under UP is given by

$$\widehat{q}_L \equiv q_{iL}\mid_{(p^u,p^u)} = a - p^u, \qquad \widehat{q}_H \equiv q_{iH}\mid_{(p^u,p^u)} = a + b - p^u. \tag{32}$$

Then $\frac{\lambda \widehat{q}_k}{q^u} \equiv \widehat{\lambda}_k$, $k \in \{L, H\}$, is the share of total quantity demanded in market k under UP in equilibrium. Using these shares as weights, the weighted average price under DP exceeds the uniform price:

$$\widehat{\lambda}_L p_L + \widehat{\lambda}_H p_H - p^u = \frac{b\left(b + (c_H - c_L)(1+\gamma)\right)(1-\lambda)\lambda}{(1+\gamma)(a + b(1-\lambda) - \bar{c})} > 0.$$

Observing that $q^u = q_L + q_H$, total quantity demanded from each firm remains unchanged.

Industry profit is higher under DP:

$$\Pi^d - \Pi^u = 2\left(\pi_L + \pi_H - \pi^u\right) = \frac{2\left(b(1+\gamma) + c_H - c_L\right)\left(b + (c_H - c_L)(1+\gamma)\right)(1-\lambda)\lambda}{(2+\gamma)^2} > 0.$$

The change in consumer surplus is

$$\begin{aligned} S^d - S^u &= s_L + s_H - S^u \\ &= \frac{1}{(2+\gamma)^2}\left(b + (c_H - c_L)(1+\gamma)\right)\left((c_H - c_L)(1+\gamma) - b(3+2\gamma)\right)(1-\lambda)\lambda. \end{aligned}$$

Let $b_1 = \min\{\frac{(a-\bar{c})(1+\gamma)}{1-\lambda}, \frac{(c_H - c_L)(1+\gamma)}{(3+2\gamma)}\}$. Therefore, $S^d \geq S^u$ if $b \leq b_1$.

Comparing total welfare gives

$$\begin{aligned} W^d - W^u &= \Pi^d - \Pi^u + S^d - S^u \\ &= \frac{\lambda(1-\lambda)\left(b + (c_H - c_L)(1+\gamma)\right)\left((3+\gamma)(c_H - c_L) - b\right)}{(2+\gamma)^2}. \end{aligned}$$

Let $b_2 = \min\{\frac{(a-\bar{c})(1+\gamma)}{1-\lambda}, (c_H - c_L)(3+\gamma)\} > b_1$. Therefore, $W^d \geq W^u$ if $b \leq b_2$. ■

Proof of Proposition 4. (i) From (27) and (28):

$$p^d = \lambda p_L + (1-\lambda)p_H = \lambda c_{2L} + (1-\lambda)c_{2H} = p^u.$$

Then $S^d > S^u$ holds, from Remark 1.

(ii) For profit we only need to consider firm 1, since the higher-cost rival earns no profit under either pricing regime. Given $p^d = p^u$, linear demand implies that total output also remains unchanged:

$$q^d = \lambda D(p_L) + (1-\lambda)D(p_H) = D(\lambda p_L + (1-\lambda)p_H) = D(p^u) = q^u.$$

With $p^d = p^u$ and $q^d = q^u$, (7) implies that $sign(\Pi^d - \Pi^u) = sign(m_L - m_H)$. Since $p_L = c_{2L}$

and $p_H = c_{2H}$, we have

$$m_L - m_H = (c_{2L} - c_{1L}) - (c_{2H} - c_{1H}) = c_{1H} - c_{1L} - (c_{2H} - c_{2L}) \equiv \Delta c_1 - \Delta c_2.$$

(iii) From (27) and (28), the prices p_L, p_H and p^u are determined by firm 2's marginal costs independent of the curvature of $D(p)$. Suppose $D(p)$ is strictly convex. Consider the linear demand $L(p)$ that is tangent to $D(p)$ at p^u.[34] Uniform pricing yields the same price and output with $L(p)$ or $D(p)$, hence the same profit and welfare. But under differential pricing, since $p^d = p^u$, outputs in both markets will be greater with $D(p)$ than with $L(p)$. Since firm 1's margins in both markets are positive ($p_L = c_{2L} > c_{1L}$, $p_H = c_{2H} > c_{1H}$), profit and total welfare will be higher with $D(p)$ than with $L(p)$. The reverse holds if $D(p)$ is strictly concave. ■

Proof of Proposition 5. (i) Price: $p^d = \lambda c_{2L} + (1-\lambda) c_{1H} > \lambda c_{2L} + (1-\lambda) c_{2H} = \bar{c}_2 = p^u$.
(ii) Consumer Surplus:

$$S^u = s(\bar{c}_2) = \int_{\bar{c}_2}^{\infty} D(p)\, dp,$$

$$S^d = \lambda s(c_{2L}) + (1-\lambda) s(c_{1H}) = \lambda \int_{c_{2L}}^{\infty} D(p)\, dp + (1-\lambda) \int_{c_{1H}}^{\infty} D(p)\, dp.$$

When $\Delta_L \equiv c_{2L} - c_{1L} \to 0$ and $\Delta_H \equiv c_{1H} - c_{2H} \to 0$, $\lambda c_{2L} + (1-\lambda) c_{1H} \to \bar{c}_2$, and hence, because $s(p)$ is strictly convex,

$$S^u = s(\bar{c}_2) \to s(\lambda c_{2L} + (1-\lambda) c_{1H}) < \lambda s(c_{2L}) + (1-\lambda) s(c_{1H}) = S^d.$$

On the other hand, when $c_{2L} \to c_{2H}$ so that $c_{2L} \to \bar{c}_2$,

$$S^d - S^u = \lambda \int_{c_{2L}}^{\bar{c}_2} D(p)\, dp - (1-\lambda) \int_{\bar{c}_2}^{c_{1H}} D(p)\, dp < 0.$$

(iii) Profits**:** Under UP, firm 2's profit is zero, but under DP, each firm earns positive profit.

[34] The ensuing argument is inspired by Malueg (1993).

Total profits under the two regimes are

$$\Pi^u = (\bar{c}_2 - \bar{c}_1)D(\bar{c}_2),$$
$$\Pi^d = \pi_1^d + \pi_2^d = \lambda(c_{2L} - c_{1L})D(c_{2L}) + (1-\lambda)(c_{1H} - c_{2H})D(c_{1H}).$$

Thus,

$$\begin{aligned}\Pi^d - \Pi^u &= \lambda(c_{2L} - c_{1L})D(c_{2L}) - (\bar{c}_2 - \bar{c}_1)D(\bar{c}_2) + (1-\lambda)(c_{1H} - c_{2H})D(c_{1H}) \\ &> \lambda(c_{2L} - c_{1L})D(c_{2L}) - \lambda(c_{2L} - c_{1L})D(\bar{c}_2) - (1-\lambda)(c_{2H} - c_{1H})D(\bar{c}_2) \\ &= \lambda(c_{2L} - c_{1L})\left[D(c_{2L}) - D(\bar{c}_2)\right] + (1-\lambda)(c_{1H} - c_{2H})D(\bar{c}_2) > 0.\end{aligned}$$

■

Proof of Proposition 6. Following Section 4.1, suppose $\bar{c}_1 < \bar{c}_2$. Under UP, firm $i's$ profit function is:

$$\pi_i = (p_i - \bar{c}_i)\left(a - p_i + \gamma(p_j - p_i)\right).$$

Suppose $\gamma \leq \gamma^u \equiv \frac{3a+\bar{c}_1-4\bar{c}_2+\sqrt{9a^2-2a\bar{c}_1+\bar{c}_1^2-16a\bar{c}_2+8\bar{c}_2^2}}{2(\bar{c}_2-\bar{c}_1)}$. Using the first order conditions, the equilibrium prices and outputs of firms $i \neq j = 1, 2$ are:

$$p_i^u = \frac{a(2+3\gamma) + (1+\gamma)\left(\bar{c}_j\gamma + 2\bar{c}_i(1+\gamma)\right)}{4+8\gamma+3\gamma^2},$$
$$q_i^u = \frac{(1+\gamma)\left(\bar{c}_j\gamma(1+\gamma) + a(2+3\gamma) - \bar{c}_i(2+4\gamma+\gamma^2)\right)}{4+8\gamma+3\gamma^2} \geq 0.$$

Note that if $\gamma > \gamma^u$, then $q_2^u < 0$ and the above p_i^u and q_i^u no longer form an equilibrium. Instead, the equilibrium will be a corner solution, described shortly.

Similarly, under DP, for $k = L, H$, there exists γ_k such that if $\gamma \leq \gamma_k$, the equilibrium prices and quantities of firms $i \neq j = 1, 2$ are:

$$p_{ik} = \frac{a(2+3\gamma) + (1+\gamma)\left(c_{jk}\gamma + 2c_{ik}(1+\gamma)\right)}{4+8\gamma+3\gamma^2},$$
$$q_{ik} = \frac{(1+\gamma)\left(c_{jk}\gamma(1+\gamma) + a(2+3\gamma) - c_{ik}(2+4\gamma+\gamma^2)\right)}{4+8\gamma+3\gamma^2} \geq 0,$$

while if $\gamma > \gamma_k$, and $c_{ik} > c_{jk}$, then $q_{ik} < 0$ and the equilibrium instead will be a corner solution.

Hence, when $\gamma \leq \gamma_1 \equiv \min\{\gamma^u, \gamma_L, \gamma_H\}$, the average prices under DP and UP are equal:

$$p_1^d = \lambda p_{1L} + (1-\lambda)\, p_{1H} = p_1^u; \qquad\qquad p_2^d = \lambda p_{2L} + (1-\lambda)\, p_{2H} = p_2^u.$$

Furthermore, it is straightforward to verify that when $\gamma \leq \gamma_1$, in equilibrium consumer surplus as a function of (q_1, q_2) and industry profit as a function of (c_1, c_2) are convex, implying that consumer surplus, industry profit and total welfare are all higher under DP than under UP.

Now turn to the case where $\gamma > \gamma_2 \equiv \max\{\gamma^u, \gamma_L, \gamma_H\}$. Suppose $\bar{c}_i < \bar{c}_j$. Under UP, the equilibrium is a corner solution in which $p_j^u = \bar{c}_j$ and firm i captures all consumers in both markets by setting a limit price p_i^u that induces zero demand from firm j:

$$q_j^u = a - p_j^u + \gamma(p_i^u - p_j^u) = 0,$$

where $p_i^u = \frac{(\gamma+1)\bar{c}_j - a}{\gamma}$ and $q_i^u = \frac{(a-\bar{c}_j)(1+2\gamma)}{\gamma}$.

Similarly, under DP, the equilibrium is a corner solution in which the higher cost firm sets price $p_{jk} = c_{jk}$ and the lower cost firm chooses price $p_{ik} = \frac{(\gamma+1)c_{jk} - a}{\gamma}$ that induces zero demand from firm j in market k.

Suppose firm 1 has global cost advantage with $c_{1k} < c_{2k}$ as in Proposition 4. Then, $p_2^u = \bar{c}_2$ and $p_1^u = \frac{(\gamma+1)\bar{c}_2 - a}{\gamma}$ under UP, and $p_{2k} = c_{2k}$ and $p_{1k} = \frac{(\gamma+1)c_{2k} - a}{\gamma}$ in market k under DP. Firm 2 receives zero profit under both DP and UP. For firm 1, we have $p_1^d - p_1^u = 0$ and $\Delta q_1 = 0$. Thus, using (7), $\Delta\Pi = \Pi^d - \Pi^u = \pi_1^d - \pi_1^u$ has the same sign as $m_{1L} - m_{1H}$. Note that

$$\begin{aligned} m_{1L} - m_{1H} = (p_{1L} - c_{1L}) - (p_{1H} - c_{1H}) &= \left[\frac{(\gamma+1)c_{2L} - a}{\gamma} - c_{1L}\right] - \left[\frac{(\gamma+1)c_{2H} - a}{\gamma} - c_{1H}\right] \\ &= (c_{1H} - c_{1L}) - \frac{\gamma+1}{\gamma}(c_{2H} - c_{2L}) < 0 \end{aligned}$$

holds if $\Delta c_1 < \frac{\gamma+1}{\gamma}\Delta c_2$. Thus, $\Pi^d > \Pi^u$ if $\Delta c_1 > \frac{\gamma+1}{\gamma}\Delta c_2$ and $\Pi^d < \Pi^u$ if $\Delta c_1 < \frac{\gamma+1}{\gamma}\Delta c_2$.

Next consider local cost advantage with $c_{1L} < c_{2L}$, $c_{1H} > c_{2H}$, and $\bar{c}_1 \leq \bar{c}_2$. Under UP,

$q_2^u = 0$ and $q_1^u = \frac{(a-\bar{c}_2)(1+2\gamma)}{\gamma}$. Consumer surplus can be computed as

$$S^u = U(q_1^u, q_2^u) - p_1^u q_1^u - p_2^u q_2^u = \frac{(a-\bar{c}_2)(1+\gamma)(1+2\gamma)}{2\gamma^2}.$$

Similarly, consumer surplus under DP can be computed as

$$s_L^d = \frac{(a-c_{2H})(1+\gamma)(1+2\gamma)}{2\gamma^2}, \qquad s_H^d = \frac{(a-c_{1H})(1+\gamma)(1+2\gamma)}{2\gamma^2}.$$

Therefore,

$$\begin{aligned}
\Delta S = S^d - S^u &= \lambda s_L^d + (1-\lambda)s_H^d - S^u \\
&= \lambda\frac{(a-c_{2H})(1+\gamma)(1+2\gamma)}{2\gamma^2} + (1-\lambda)\frac{(a-c_{1H})(1+\gamma)(1+2\gamma)}{2\gamma^2} - \frac{(a-\bar{c}_2)(1+\gamma)(1+2\gamma)}{2\gamma^2} \\
&= \frac{(1+\gamma)(1+2\gamma)(1-\lambda)\left[(c_{2H}-c_{2L})^2\lambda - (2a-c_{1H}-c_{2H})(c_{1H}-c_{2H})\right]}{2\gamma^2},
\end{aligned}$$

with $\Delta S < 0$ if $c_{2H} - c_{2L}$ is sufficiently small, and $\Delta S > 0$ if $c_{1H} - c_{2H}$ is sufficiently small. ■

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