Synthetic Estimation of Dynamic Panel Models When Either N or T or Both Are Not Large: Bias Decomposition in Systematic and Random Components

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Abstract

By increasing the dimensions N or T, or both, in data panel analysis, bias can be reduced asymptotically to zero. This research deals with an econometric methodology to separate and measure bias through synthetic estimators without altering the data panel dimensions. This is done by recursively decomposing bias in systematic and random components. The methodology provides consistent synthetic estimators.

JEL Classification: C23, B23.
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1. Introduction

Panel data involves two dimensions: the first dimension is N representing the number of individuals; and the second dimension is T standing for the number of time periods. In the literature there abundant papers related to panel data dynamic estimators and their asymptotic properties; see, for instance: Hsiao and Zhang (2015), Hsiao, *et al*. (2002), Hsiao and Tahmiscioglu (2008), Abadie and Imbens (2011), and MacKinnon and Smith (1998). Most of these studies measure the sensitivity of panel data estimators when either N or T, or both, are large by using Monte Carlo simulations or bootstrap experiments. These experiments increase the size N or T, or both, to obtain the asymptotic bias distribution and state a procedure to reduce bias. In this asymptotic bias approach literature, it can be found different panel data dynamic asymptotic estimators properties. The differences rely on different initial assumptions; functional forms; sample size; endogeneity treatments, and econometric techniques (MLE, maximum likelihood, or GMM, general method of moments). In this regard, Hsiao (2003) considers that if y_{i0} is fixed and α_i measures the individual specific effects, then the maximum likelihood estimator can be viewed as a covariance estimator (β_{cv}). This author finds β_{cv} asymptotically normally distributed with mean 0 if N is fixed and T is large. On the other hand, Hahn and Kuersteiner (2002) show that β_{cv} is asymptotically biased of order $\sqrt{\mathrm{N/T}}$, if N and T tend to infinite, and T/N goes to a finite constant different to zero.

Arellano and Bond (1991) find a GMM panel consistent estimator, and asymptotically unbiased if T is fixed and N goes to infinite. Alvarez and Arellano (2003) report a GMM panel estimator[1] that is asymptotically biased of order $\sqrt{c^*}$, $0 < c^* < \infty$, where c^* is an optimal constant such that $\lim_{\mathrm{N}\to\infty} \mathrm{T/N} = c^*$.

The asymptotic bias approach properties are studied in the literature because consistent estimators have an important role. Consider first that consistent estimators are obtained when asymptotic bias is reduced. The central limit theorem postulates that as the sample size increases the sample estimators converges toward the population parameters. The asymptotic bias approach assumed that the central limit theorem holds, and then bias goes to zero as N or T, or both, go to infinite and that estimators become consistent. Thus, consistent estimators are important since they can be used to carry out statistic inference or hypothesis testing, which depends on *t*-tests power, and confidence interval size.

This paper proposes a methodology to obtain consistent synthetic estimators, without the need to increase panel data dimensions. This method treats bias as a type of serial correlation problem.. This solution separates consistent estimators and their biases in a lineal fashion.

Finally, this paper is organized as follows: section 2 presents the methodology and the proposed novel method; section 3 discusses the findings; and section 4 concludes.

2. Proposed methodology

Suppose that a dynamic panel data model has the following form:

$$(1) \qquad y_{it} = \alpha_i + \beta y_{i,t-1} + u_{it,} \;\; i = 1, \dots, N, t = 1, \dots, T,$$

which is supposed to be stationary. Also suppose that:

Assumption 1: u_{it} is a random variable with distribution $N(0, \sigma_u^2)$.

Assumption 2: $\mathrm{E}[\beta|\alpha_i]$ and its bias and moving average terms can be decomposed on systematic and random parts. The systematic part is represented by its mean. The random part is represented as a serial correlation problem.

A proposed analytical solution is next implemented to separate efficient synthetic estimators from their bias. In equation (1), the computation of the parameters introduces a serial correlation problem. This is because the specific individual-effects are presented in both estimators α_i and β. The estimator β considers time and individual effects since

[1] Several applications of GMM estimation in Dynamic Panel Data can be found in Aali-Bujari *et al.* (2017a), Aali-Bujari *et al.* (2017b), Aali-Bujari *et al.* (2016), Salazar-Núñez y Venegas-Martínez (2018a) and (2018b).

$y_{i,t-1}$ has the two panel data dimensions (i and $t-1$). This leads to a measurement error given the double accounting (on α_i and β) a specific individual-effects. If this measurement error were closed to zero, then bias would not exist. Thus, bias is a result of an estimation error (double accounting for individual effects). The omitted variable formula is then used to represent the expected value for the estimator β and its bias, as follows:[2]

(2) $$E\left[\hat{\beta}\middle|\alpha_i\right] = \beta + \frac{\mathrm{Cov}[\alpha_i, y_{i,t-1}]}{\mathrm{Var}[\alpha_1]} u_{it}$$

where $E\left[\hat{\beta}\middle|\alpha_i\right]$ expresses the expected value of β conditional to α_i. In this case, β is a consistent estimator, and $\frac{\mathrm{Cov}[\alpha_i, y_{i,t-1}]}{\mathrm{Var}[\alpha_1]} u_{it}$ represents bias.[3] Assumption 2, in equation (2), states a systematic part, β, and a random part, $\frac{\mathrm{Cov}[\alpha_i, y_{i,t-1}]}{\mathrm{Var}[\alpha_1]} u_{it}$.

Equation (2) is nonlinear in its bias component. For tractability purposes bias will be treated through Beverigde-Nelson decomposition. This decomposition linearizes bias into two parts, mean and error parts. Assume now that bias is equal to ξ_{it}, and that it has the following linear representation:

(3) $$\xi_{it} = \delta_i + u_{it}$$

where $\xi_{it} = \frac{\mathrm{Cov}[\alpha_i, y_{i,t-1}]}{\mathrm{Var}[\alpha_1]} u_{it}$, δ_i is the mean, and u_{it} stands for the error term. Assumption 2 applied to equation (3) states that the systematic part is δ_i, and the random part is u_{it}. Next, consider the following functional moving average form for u_{it}.

(4) $$u_{it} = \psi(L)\xi_{it}.$$

Expanding the lag polynomial $\psi(L)$ leads to

(5) $$u_{it} = \psi(1)\xi_{it} + \psi(2)\xi_{i,t-2} + \cdots + \psi(T)\xi_{i,0}$$

where $\psi(i)$ represents a moving average of order $\boldsymbol{i}$, and $\psi(T)$ represents a moving average of order T. Plugging equation (5) into equation (4) gives

(6) $$\xi_{it} = \delta_i + \psi(1)\xi_{i,t-1} + \psi(2)\xi_{i,t-2} + \cdots + \psi(T)\xi_{i,0}$$

Thus, equation (2) can be rewritten as

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[2] For an endogeneity treatments using synthetic data based on the omitted variable formula see, for instance, Carbajal (2014).

[3] For Makowski *et al*. (2006) the omission bias is illustrated with the following model $y = \alpha_1 x_1 + \alpha_2 x_2 + \varepsilon$; $\hat{\alpha}_1 = \frac{\sum_{i=1}^{N} x_{1i} y_i}{\sum_{i=1}^{N} x_{1i}^2}$, and $E(\hat{\alpha}_1) = \alpha_1 + \frac{\sum_{i=1}^{N} x_{1i} x_{2i}}{\sum_{i=1}^{N} x_{1i}^2} \alpha_2$.

(7) $$\mathrm{E}\big[\hat{\beta}\big|\alpha_i\big] = \beta + \delta_i + \psi(1)\xi_{i,t-1} + \psi(2)\xi_{i,t-2} + \cdots + \psi(T)\xi_{i,0}$$

where $\mathrm{E}\big[\hat{\beta}\big|\alpha_i\big] = \mathrm{E}[\beta|\alpha_i] = \beta$ is a consistent estimator. Here, bias is $\delta_i + \psi(1)\xi_{it} + \psi(2)\xi_{i,t-1} + \cdots + \psi(T)\xi_{i,0}$. As the sample size increases, and if the assumptions of the Central Limit Theorem hold, then it is expected that bias converges to zero:

(8) $$\lim_{T\to\infty} \delta_i + \psi(1)\xi_{it} + \psi(2)\xi_{i,t-1} + \cdots + \psi(T)\xi_{i,0} = 0$$

If equation (8) holds, then equation (7) reduces to $\mathrm{E}\big[\hat{\beta}\big|\alpha_i\big] = \beta$, where β is a consistent estimator. This is the result that the asymptotic approach looks for, when N or T, or both, are large.

Theorem 1. An efficient synthetic estimator in presence of specific individual-effects correlation is obtained by estimating its bias component.

Proof:

Plugging (7) into equation (1) provides:

(9) $$y_{it} = \alpha_i + \big[\beta + \delta_i + \psi(1)\xi_{i,t-1} + \psi(2)\xi_{i,t-2} + \cdots + \psi(T)\xi_{i,0}\big]y_{i,t-1} + u_{it}.$$

Distributing the $y_{i,t-1}$ term gives:

(10) $$y_{it} = \alpha_i + \beta y_{i,t-1} + \delta_i y_{i,t-1} + \psi(1)\xi_{i,t-1}y_{i,t-1} + \psi(2)\xi_{i,t-2}y_{i,t-2} + \cdots + \psi(T)\xi_{i,0}y_{i,0} + u_{it.}$$

Collecting the individual effects estimators in one term, $\eta_i = \alpha_i + \delta_i y_{i,t-1}$, yields[4]

(11) $$y_{it} = \eta_i + \beta y_{i,t-1} + \psi(1)\xi_{i,t-1}y_{i,t-1} + \psi(2)\xi_{i,t-2}y_{i,t-2} + \cdots + \psi(T)\xi_{i,0}y_{i,0} + u_{it}$$

Equation (11) represents the first iteration of the proposed methodology to separate and quantify bias components. For the time being, consider the term $\psi(1)\xi_{i,t-1}y_{i,t-1}$. Its synthetic estimator could be decomposed in systematic and random components.

(12) $$\mathrm{E}\big[\hat{\psi}(1)\big|\eta_i\big] = \psi(1) + \frac{\mathrm{Cov}[\eta_i, \xi_{i,t-1}y_{i,t-1}]}{\mathrm{Var}[\eta_i]}u_{it}$$

Assumption 2 applied to equation (12) stating that the systematic part is $\psi(1)$, and the random part is $\frac{\mathrm{Cov}[\eta_i, \xi_{i,t-1}y_{i,t-1}]}{\mathrm{Var}[\eta_i]}u_{it}$. Next, the analogs of equations (3)-(10) are presented for $\psi(1)$ estimator. For the sake of simplicity, and for comparison purposes, let $\psi(1)$ be represented as follows $\underline{\xi}_{it} = \frac{\mathrm{Cov}[\eta_i, \xi_{i,t-1}y_{i,t-1}]}{\mathrm{Var}[\eta_i]}u_{it}$. Hence,

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[4] Here, δ_i is a mean individual effects estimator and although $y_{i,t-1}$ contains both data panel dimensions, δ_i takes into account only specific individual-effects.

(13) $\underline{\xi}_{it} = \underline{\delta}_i + u_{it}$

(14) $u_{it} = \underline{\psi}(L)\underline{\xi}_{it}$

(15) $u_{it} = \underline{\psi}(1)\underline{\xi}_{i,t-1} + \underline{\psi}(2)\underline{\xi}_{i,t-2} + \cdots + \underline{\psi}(T)\underline{\xi}_{i,0}$

(16) $\underline{\xi}_{it} = \underline{\delta}_i + \underline{\psi}(1)\underline{\xi}_{i,t-1} + \underline{\psi}(2)\underline{\xi}_{i,t-2} + \cdots + \underline{\psi}(T)\underline{\xi}_{i,0}$

(17) $\mathrm{E}\left[\hat{\psi}(1)\middle|\eta_i\right] = \psi(1) + \underline{\delta}_i + \underline{\psi}(1)\underline{\xi}_{i,t-1} + \underline{\psi}(2)\underline{\xi}_{i,t-2} + \cdots + \underline{\psi}(T)\underline{\xi}_{i,0}$

where $\lim_{T\to\infty} \mathrm{E}\left[\hat{\psi}(1)\middle|\eta_i\right] = \psi(1)$ and $\psi(1)$ is a consistent estimator. By symmetry, equation (17) can be generalized for the estimators $\psi(2), \ldots, \psi(T)$:

(18) $\mathrm{E}\left[\hat{\psi}(2)\middle|\eta_i\right] = \psi(2) + \underline{\underline{\delta}}_i + \underline{\psi}(2)\underline{\xi}_{i,t-2} + \underline{\psi}(3)\underline{\xi}_{i,t-3} + \cdots + \underline{\psi}(T)\underline{\xi}_{i,0}$

$\vdots$

(19) $\mathrm{E}\left[\hat{\psi}(T)\middle|\eta_i\right] = \psi(T) + \underline{\underline{\underline{\delta}}}_i + \underline{\psi}(T)\underline{\xi}_{i,0}.$

Plugging equations (17), (18) and (19) into equation (11) yields:

(20) $$\begin{aligned} y_{it} = {} & \eta_i + \beta y_{i,t-1} + \\ & \psi(1) + \underline{\delta}_i y_{i,t-1} + \underline{\psi}(1)\underline{\xi}_{i,t-1}y_{i,t-1} + \underline{\psi}(2)\underline{\xi}_{i,t-2}y_{i,t-2} + \cdots + \underline{\psi}(T)\underline{\xi}_{i,0}y_{i,0} + \\ & \psi(2) + \underline{\underline{\delta}}_i y_{i,t-2} + \underline{\psi}(2)\underline{\xi}_{i,t-2}y_{i,t-2} + \underline{\psi}(3)\underline{\xi}_{i,t-3}y_{i,t-3} + \cdots + \underline{\psi}(T)\underline{\xi}_{i,0}y_{i,0} + \\ & \vdots \\ & \psi(T) + \underline{\underline{\underline{\delta}}}_i y_{i,0} + \underline{\psi}(T)\underline{\xi}_{i,0}y_{i,0} + u_{it}. \end{aligned}$$

Collecting again the individual effects terms in only one term, *i.e.*, $\underline{\eta}_i = \eta_i + \underline{\delta}_i y_{i,t-1} + \underline{\underline{\delta}}_i y_{i,t-2} + \cdots + \underline{\underline{\underline{\delta}}}_i y_{i,0}$ leads to

(20) $$\begin{aligned} y_{it} = {} & \underline{\eta}_i + \beta y_{i,t-1} + \\ & \psi(1) + \underline{\psi}(1)\underline{\xi}_{i,t-1}y_{i,t-1} + \underline{\psi}(2)\underline{\xi}_{i,t-2}y_{i,t-2} + \cdots + \underline{\psi}(T)\underline{\xi}_{i,0}y_{i,0} + \\ & \psi(2) + \underline{\psi}(2)\underline{\xi}_{i,t-2}y_{i,t-2} + \underline{\psi}(3)\underline{\xi}_{i,t-3}y_{i,t-3} + \cdots + \underline{\psi}(T)\underline{\xi}_{i,0}y_{i,0} \\ & + \cdots + \\ & \psi(T) + \underline{\psi}(T)\underline{\xi}_{i,0}y_{i,0} + u_{it} \end{aligned}$$

Now collecting similar terms yields:

(21) $$\begin{aligned} y_{it} = {} & \underline{\eta}_i + \beta y_{i,t-1} + \psi(1) + \psi(2) + \cdots + \psi(T) + \\ & \underline{\psi}(1)\underline{\xi}_{i,t-1}y_{i,t-1} + 2\underline{\psi}(2)\underline{\xi}_{i,t-2}y_{i,t-2} + \cdots + T\underline{\psi}(T)\underline{\xi}_{i,0}y_{i,0} + u_{it} \end{aligned}$$

Consider that the estimated variables for $\psi(1), \psi(2), \dots, \psi(T)$ are vectors of ones in each case. In consequence, they are the means at each lag value. If panel data is stacked by individuals, then moving average means also estimate specific individual effects. For the sake of simplicity, the moving average terms could be collected together with the specific individual-effects means. Thus, these terms can be compiled together with $\underline{\eta}_i$ in only one term representing all individual effects in equation (21), *i.e.*, $\underline{\underline{\eta}}_i = \underline{\eta}_i + \psi(1) + \psi(2) + \cdots + \psi(T)$. Thus, equation (21) can be rewritten as:

$$y_{it} = \underline{\underline{\eta}}_i + \beta y_{i,t-1} + \underline{\psi}(1)\underline{\xi}_{i,t-1} y_{i,t-1} + 2\underline{\psi}(2)\underline{\xi}_{i,t-2} y_{i,t-2} + \cdots + T\underline{\psi}(T)\underline{\xi}_{i,0} y_{i,0} + u_{it} \tag{22}$$

The above equation represents the second iteration of the proposed methodology to separate and quantify bias components.

Theorem 2. Efficient estimators can be computed at any panel data dimensions size.

Proof:

Theorem 1 provides a recursive method to decompose bias components to be estimated, which recursively converges to their consistent estimators. Then, the following equality follows:

$$\begin{aligned}\frac{\mathrm{Cov}\left[\alpha_i, y_{i,t-1}\right]}{\mathrm{Var}\left[\alpha_1\right]} u_{it} &= \left[\left(\underline{\underline{\eta}}_i - \alpha_i\right)\right. \\ &\quad \left. + \left(\underline{\hat{\psi}}(1)\underline{\xi}_{i,t-1} y_{i,t-1} + 2\underline{\hat{\psi}}(2)\underline{\xi}_{i,t-2} y_{i,t-2} + \cdots + T\underline{\hat{\psi}}(T)\underline{\xi}_{i,0} y_{i,0}\right)\right]\end{aligned}$$

where the left hand side is equation (2) bias, and the right hand side is the systematic component, $\underline{\underline{\eta}}_i - \alpha_i$, and $\underline{\hat{\psi}}(1)\underline{\xi}_{i,t-1} y_{i,t-1} + 2\underline{\hat{\psi}}(2)\underline{\xi}_{i,t-2} y_{i,t-2} + \cdots + T\underline{\hat{\psi}}(T)\underline{\xi}_{i,0} y_{i,0}$ is the random component. Thus, after estimating equation (22), the following subtraction can be applied to equation (2)[5]

$$\begin{aligned}E\left[\hat{\beta}\middle|\alpha_i\right] = \beta + \frac{\mathrm{Cov}\left[\alpha_i, y_{i,t-1}\right]}{\mathrm{Var}\left[\alpha_1\right]} u_{it} & \\ - \left[\left(\underline{\underline{\eta}}_i - \alpha_i\right)\right. & \\ \left. + \left(\underline{\hat{\psi}}(1)\underline{\xi}_{i,t-1} y_{i,t-1} + 2\underline{\hat{\psi}}(2)\underline{\xi}_{i,t-2} y_{i,t-2} + \cdots + T\underline{\hat{\psi}}(T)\underline{\xi}_{i,0} y_{i,0}\right)\right] &\end{aligned}$$

Therefore,

[5] The addition of the mean specific individual-effects bias systematic components decomposition also applies.

$$E[\hat{\beta}|\alpha_i] = \beta$$

where β is a synthetic consistent estimator. As it can be seen, panel dimensions remain without change. This means that either N or T or both are not large, as the asymptotic bias properties are not needed in the consistent estimator computation.

3. Discussion of findings

Lliterature addressing a recursive bias approach is scarce. This is because most papers focus on the asymptotic bias approach to quantify asymptotic estimators sensitivities when panel data dimensions increase (N, T or both) in order to reduce asymptotically bias to zero, and increase statistic inference performance. On the one hand, the recursive bias approach is not made explicit theoretically in the literature, *i.e.*, Hsiao and Zhang (2015) consider bias as the result of using instruments to purge correlations between estimators and equation errors; this implies that bias has no other meaning. On the other hand, recursive bias approach literature is also neglected. In most cases, the recursive bias properties are not implemented in the first place; *v.g.*: Arellando and Bond (1991), Alvarez and Arellano (2003), Anderson and Hsiao (1981), and Hsiao *et al*. (2002).

4. Conclusions

This paper has proposed a methodology to separate efficient estimators from their bias under certain assumptions. The propose modeling addresses bias as an independent random process. That is to say, it is seen as a serial correlation problem, which is also a non-linear problem that could be disentangled in a succession of linear steps. The aim of this is to separate, and measure bias components. Therefore, consistent estimators would be assured. The systematic part represented by the mean also represents a consistent estimator. Moreover, equations (11) and (22) represent the first and second iteration of the proposed methodology to separate and quantify bias components, and after T iterations converges to consistent estimators of the bias components.

Finally, it is important to point out that the recursive bias approach does not need to increase N or T or both, in order to obtain an efficient synthetic estimator of β. Monte Carlo and bootstrap experiments find asymptotic convergence with a consistent estimator whose value is provided before the simulation experiments begin. In the recursive bias approach a consistent estimator does not have to be predetermined.

References

Aali-Bujari, A. F. Venegas-Martínez and A. O. Palafox-Roca (2017). Impact of Energy Consumption on Economic Growth in Major OECD Economies (1977-2014): A Panel Data Approach. *International Journal of Energy Economics and Policy*. 7(2), 18-25.

Aali-Bujari, A., F. Venegas-Martínez, and G. Pérez-Lechuga (2017). Impact of the Stock Market Capitalization and the Banking Spread in Growth and Development in Latin American: A Panel Data Estimation with System GMM. *Revista Contaduría y Administración*, 62(5), 1427–1441.

Aali-Bujari, A., F. Venegas-Martínez and G. Pérez-Lechuga (2016). Impact of Derivatives Markets on Economic Growth in Some of the Major World Economies: A Difference-GMM Panel Data Estimation (2002-2014). *AESTIMATIO: The International Journal of Finance*, 12, 110-127.

Abadie, A. and G.W. Imbens (2011). "Bias-Corrected Matching Estimators for Average Treatment Effects." *Journal of Business and Economic Statistics,* 29(1), 1-11.

Alvarez, J. and M. Arellano (2003). "The Time Series and Cross-Section Asymptotic of Dynamic Panel Data Estimators." *Econometrica,* 71(4), 1121-1159.

Anderson, T.W. and C. Hsiao (1981). "Estimation of Dynamic Models with Error Components." *Journal of the American Statistical Association,* 76(375), 598-606.

Arellano, M. and S. Bond (1991). "Some Test of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations." *The Review of Economic Studies,* 58(2), 277-297.

Atenafu, E., J.S. Hamid, T. To, A.R. Willan, B.M. Feldman, and J. Beyene (2012). "Bias-Corrected Estimator for Intraclass Correlation Coefficient in the Balanced one-way Random Effects Model." *BMC Medical Research Methodology,* 1471-2288(12), 1-8.

Carbajal-De-Nova, C. (2014). Synthetic Data: An Endogeneity Simulation *Working Paper* Universidad Autónoma Metropolitana, Iztapalapa.

Douc, R, E. Moulines, and David Stoffer (2014). *Nonlinear Time Series: Theory, Methods and Applications with R Examples* Boca Raton: CRC Press.

Hahn, J. and G. Kuersteiner (2002). "Asymptotically Unbiased Inference for a Dynamic Panel Model with Fixed Effects when both N and T are large." *Econometrica,* 70(4), 1639-1657.

Hsiao, C., M.H. Pesaran and A.K. Tahmiscioglu (2002). "Maximum Likelihood Estimation of Fixed Effects Dynamic Panel Data Model Covering Short Time Periods." *Journal of Econometrics,* 109(1), 107-150.

Hsiao, C. 2003. *Analysis of Panel Data*. UK, Cambridge University Press.

Hsiao, C. and A.K. Tahmiscioglu (2008). "Estimation of Dynamic Panel Data Models with both Individual and Time-specific effects." *Journal of Statistics Planning Inference,* 138(9), 2698-2721.

Hsiao, C., and J. Zhang (2015). "IV, GMM or Likelihood Approach to Estimate Dynamic Panel Models when either N or T or both are Large." *Journal of Econometrics,* 187(1), 312-322.

MacKinnon, J.G., and Jr. A.A. Smith (1998). "Approximate Bias Correction in Econometrics." *Journal of Econometrics,* 85(2), 205-230.

Makowski, D., Daniel Wallach, Bruno Andrieu, and Marie-Helene Jeuffroy (1999). "Parameter Estimation for Crop Models." *Working with Dynamic Crop Models*. (Daniel Wallach, David Makowski, James W. Jones, and François Brun editors) San Francisco: Elsevier.

Salazar-Núñez, H. F. y F. Venegas-Martínez (2018). Impacto del uso de energía y la formación bruta de capital en el crecimiento económico: análisis de datos panel en 73 países agrupados por nivel de ingreso y producción de petróleo. *El Trimestre Económico*, 85(2-338), 341-364.

Salazar-Núñez, H. F. y F. Venegas-Martínez (2018). Impacto del consumo de energía y del valor agregado de las manufacturas en el crecimiento económico de los países que integran el TLCAN: un modelo de datos panel cointegrado con cambio estructural. *Economía Teoría y Práctica*, 4(Número Especial), 77-102.

Spanos, A. 1999. *Probability Theory and Statistical Inference. Econometric Modeling with Observational Data*. Cambridge: Cambridge University Press.