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Effect of Aging on Housing Prices: A Perspective from an Overlapping Generation Model

Tianyu Sun*, Satish Chand and Keiran Sharpe

ABSTRACT

This paper investigates the effect of aging on housing prices. It provides a theoretical explanation to address the on-going debate about the impact of aging on house prices. The analysis demonstrates that aging has divergent effects on housing prices, depending on the net effects of a fall in fertility vis-à-vis a rise in longevity on demand for housing. In addition, the results suggest that aging can produce a turning point in the price dynamics. To the left of the peak, aging boosts prices while to the right, it has the opposite effect. Furthermore, inequality of household utility is enlarged during the aging processes.

JEL classification: E21, E31, J11, R21, R31

Keywords: Aging Population; OLG Model; House Prices; Land Prices; Turning Point

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1. Introduction

Will an aging population affect housing prices? The extant literature provides divergent answers to this question. The first argues that an aging population will have a significant effect on housing prices (Mankiw and Weil 1989, 1991, Bergantino 1998, Takáts 2012), and the second opined that aging would have little or mixed impact (Engelhardt and Poterba 1991, Hendershott 1991, Poterba 2001, Eichholtz and Lindenthal 2014, Hiller and Lerbs 2016). The debate on the impact of aging on housing prices is largely drawn from empirical studies, and remains alive (Poterba 2014). This paper contributes to the extant literature by investigating the effects of aging within the context of an overlapping generations model (OLG) to explain the mixed results from empirical studies estimating the effects of aging on housing prices.

The results show that an aging population can result from a fall in fertility and/or an increase in longevity: housing prices increase only when the net effect of the above is positive. More specifically, a fertility rate lower than the replacement level will depress housing prices while an increase in longevity has the opposite effect. The above explains some of the divergent effects of an aging population on housing prices in different contexts. From this perspective, the seemingly contradictory views from previous empirical research can be reconciled.

Moreover, an increase in longevity with a synchronized fall in fertility can lead to an increase in prices initially with the effect being overturned later, leading to a turning point in price dynamics over time. Specifically, an aging population resulting from an increase in longevity with a decline in fertility can initially boost demand for housing and lead to the increase in prices, but this effect will peter out as the working-age population begins to shrink. This finding casts fresh light on the relationship between demographic changes and economic fluctuations.

The results presented above have been obtained using an OLG model that has been drawn from two separate strands in the literature. The first strand is drawn from Iacoviello and Neri (2010), in whose work the housing market is analysed using a Dynamic Stochastic General Equilibrium (DSGE) model. The settings of our model are mainly based on their approach, but are transformed to an OLG framework to incorporate the age structure of the population. The second strand of our model comes from the research into land prices (Davis and Heathcote 2007, Liu, Wang, and Zha 2013). Their insights are to separate out the effects of land and construction on the price of a house.

Our paper fits into the emerging literature on housing prices using theoretical models.[1] While this literature has focused on various factors underscoring the demand for housing, it has not addressed the impact of aging that we focus on here. In addition, our paper also contributes to the literature on the consequences of aging on the accumulation of assets and savings.[2] The bulk of theoretical work in this area has ignored housing as an asset, a major deficiency given that housing is the largest component of households' wealth. We address this void in the literature.

We first model the long-term effect of an aging population on housing prices, and then simulate the dynamics of housing prices due to demographic changes. Although our work aims to reach general conclusions about the effects of aging, the results are based on simulations that in turn are sensitive to the parameters of the model. Our parameters are calibrated for China for two compelling reasons: (i) housing assets constitute more than 70 percent of Chinese households' wealth (Xie and Jin 2015); and, (ii) China is undergoing rapid aging of its population (Lutz, Sanderson, and Scherbov 2008). Thus, choosing the parameters for China provides us with a typical example without losing the generality of the conclusions.

Our results show that demographic change has a bigger impact on land prices than that for construction, a fact revealed only when the two are looked at separately. This result is consistent with the empirical findings of Davis and Heathcote (2007). Specifically, our simulations show that a decline in the fertility rate of 10 percent from the replacement level for one generation (i.e. 30 years in our model) leads to a fall in the price of construction by 1 percent whereas the price of land falls by 10 percent. In contrast, when longevity increases by 6 percent then the corresponding house and land prices increase by 1 percent and 7 percent, respectively.

We next investigate the effects of a simultaneous change in fertility and longevity on households' utility. Anticipating the results, workers would have greater utility because of higher house and land ownership per capita; however, the utility of the retirees is worse. Assuming perfect foresight in the model, the consequential housing prices would increase immediately before the demographic changes and decline as the effects of the fall in fertility take over.

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[1] See, for example, Iacoviello (2005); Iacoviello and Neri (2010); Liu, Wang, and Zha (2013); Justiniano, Primiceri, and Tambalotti (2015); Ng (2015); Chen and Wen (2017).

[2] See, for example, Ando and Modigliani (1963); Brooks (2000); Brooks (2002); Abel (2001); Abel (2003); Modigliani and Cao (2004); and Curtis, Lugauer, and Mark (2015).

The remainder of this paper is organized as follows. Section 2 describes the OLG model, Section 3 presents the assumptions and the calibration of parameters, while section 4 shows the solution method. Section 5 presents the long-term effects of aging on housing prices, and section 6 investigates household utility and the housing price dynamics. Section 7 concludes.

2. The Model

2.1. Demography

The model's time structure is the same as the classical model of Diamond (1965). The economy consists of two overlapping generations, namely, workers and retirees, who are alive in both periods. For each generation, we assume that the households are identical.[3]

The population of workers is affected only by the rate of fertility. The fertility rate at time t, denoted by n_t, is given as equation (1), where $N_{t,1}$ and $N_{t-1,1}$ represent the population of workers at time t and its previous period, respectively. Note that in this stylized model, the fertility rate is equal to the rate of growth of workers.

$$n_t = \frac{N_{t,1}}{N_{t-1,1}} \tag{1}$$

The population of retirees, in contrast, is influenced by both the fertility rate and longevity. Following Blackburn and Cipriani (2002) and Cipriani (2013), longevity is defined by the share of households that survive to retirement stage. Specifically, households are alive as workers with certainty, but a fraction die at the beginning of their retirement, while the rest live for the remaining period of retirement. This survival function is given as equation (2), where $N_{t-1,1}$ and $N_{t,2}$ represent the number of workers in period $t-1$ and the retirees in period t respectively.

$$\pi_t = \frac{N_{t,2}}{N_{t-1,1}} \tag{2}$$

Based on (1) and (2), the population dynamics of retirees is given as:

$$\frac{N_{t,2}}{N_{t-1,2}} = \frac{n_{t-1}\pi_t}{\pi_{t-1}} \tag{3}$$

[3] Therefore, if we know the population of each generation, then the aggregate and per-capita values can be transformed from one to the each other. For the purposes of illustration, we will present the model mainly using aggregate variables (except the section about households). The equations for individuals are listed in Appendix A.

2.2. Households

Utility

The households' utility is characterized by (4) as follows:

$$U_t = U\left(U_{t,1}, U_{t+1,2}\right) = U_{t,1} + \pi_{t+1}\, \beta\, E\left(U_{t+1,2}\right) \tag{4}$$

Following Diamond (1965), the expected life span utility of a household (U_t) is time separable and is determined by the sum of utility when working, $U_{t,1}$ and that when retired, $U_{t+1,2}$. Two discount factors are 1) the time preference β ($0 < \beta < 1$) and 2) the survival rate π. The form of equation (4) is consistent with the practice that survival rates are involved.[4]

The utility during work ($U_{t,1}$) is given as:

$$U_{t,1} = \ln\left(c_{t,1}\right) + j_h \ln\left(h_{t,1}\right) + j_l \ln\left(l_{t,1}\right) - \frac{\tau}{1+\eta}\left(n_{c,t}^{1+\xi} + n_{h,t}^{1+\xi}\right)^{\frac{1+\eta}{1+\xi}} \tag{5}$$

Following Iacoviello and Neri (2010) and Liu, Wang, and Zha (2013), the terms providing positive utility in (5) are per capita consumption of goods, $c_{t,1}$, structure of houses, $h_{t,1}$, and land, $l_{t,1}$. Variables $n_{c,t}$ and $n_{h,t}$ represent hours worked in non-housing and housing sectors respectively. The negative term on the far right of (5) represents the disutility of work.

First, we explain why both the building and land are in the utility function. Housing provides utility and consists of the construction and land. However, the literature includes either the structure or the land in the utility function. We include both, and each contributes to utility separately. We assume that land provides utility above and beyond the building that sits on it. For example, the houses that occupy bigger land (e.g. villas) provide additional utility compared to apartments of identical residential space. This may be rationalised as green space providing utility of its own. This preference is supported by scholarship within psychology.[5]

Back to the equation (5), the parameters j_h and j_l represent the preference for house and land respectively, and the parameter τ denotes the disutility of work. The parameter $\eta > 0$ ensures that the utility function is concave with respect to labour supply. In addition, the parameter $\xi > 0$ indicates the labour mobility between the two production sectors is imperfect (Horvath 2000, Iacoviello and Neri 2010).

The utility function for the retirees, who do not participate in the labour market, is analogous to (5); that is,

$$U_{t+1,2} = \ln\left(c_{t+1,2}\right) + j_h \ln\left(h_{t+1,2}\right) + j_l \ln\left(l_{t+1,2}\right) \tag{6}$$

[4] See, for example, Blackburn and Cipriani (2002), Cipriani (2013) and Muto, Oda, and Sudo (2016)

[5] See, for example, Griffit and Veitch (1971) and Sundstrom (1975).

, where the consumption, house and land per capita of this generation are denoted by $c_{t+1,2}$, $h_{t+1,2}$ and $l_{t+1,2}$ respectively.

Budget Constraint

The budget constraint for individual worker is as follows:

$$c_{t,1} + q_t h_{t,1} + p_{l,t} l_{t,1} = (1 - T)\left(w_{c,t} n_{c,t} + w_{h,t} n_{h,t} + d_t\right) \tag{7}$$

, where the right side is the income of the workers and the left side is the expenditures. The income can be divided into wage, $w_{c,t} n_{c,t} + w_{h,t} n_{h,t}$, and dividend, d_t, income. Workers pay tax of T, and thus the fraction $(1 - T)$ is the share of the income kept by the workers while the remainder accrues to the retirees. One could rationalize the specification of 'T' in (7) as a pay-as-you-go pension system where the current generation of workers are taxed to fund consumption of the retirees.

The expenditure is $c_{t,1} + q_t h_{t,1} + p_{l,t} l_{t,1}$, where q_t and $p_{l,t}$ denote the prices of structure and land respectively. The households do not have deposits because it equals the loans to the firms. However, the firms are assumed to be owned by all the households collectively. Thus, for any individual, one cannot retrieve the capital of firms for consumption. Instead, households have the right to claim for the profits of the firms, i.e. the dividend d_t in equation (7). Further illustration can be found in the section of firms.

The budget constraint for individual retiree is as follows:

$$\begin{gathered} c_{1+t,2} + q_{1+t} h_{1+t,2} + p_{l,1+t} l_{1+t,2} = \\ T\left(d_{1+t} + n_{c,1+t} w_{c,1+t} + n_{h,1+t} w_{h,1+t}\right) \frac{n_{1+t}}{\pi_{1+t}} + \\ q_{1+t}(1 - \delta_h)\left(\frac{h_{t,1}}{\pi_{1+t}} + \frac{\pi_t h_{t,2}}{\pi_{1+t} n_t}\right) + p_{l,1+t}\left(\frac{l_{h,t,1}}{\pi_{1+t}} + \frac{\pi_t l_{t,2}}{\pi_{1+t} n_t}\right) \end{gathered} \tag{8}$$

The right part of (8) is the income of retirees, while the left part is expenditures. The revenues accruing to the retirees have some differences with that of workers because the revenues consist of three parts: 1) the pension payments; 2) the wealth accumulated previously; and 3) inheritances. The aggregate values of these revenues are listed in Table 1, and the per capita values can be derived according to the equations (1), (2) and (3) by dividing the totals by the population of retirees, $N_{1+t,2}$.

For the pension system, the retirees receive a share of the wage and profit revenues of all the households as explained above. They also have income from accumulated wealth in the form of houses and lands that they purchased while working. Moreover, inheritance being equal to the left-over wealth of the previous generation is an additional component of income of retirees.

Table 1—Aggregate market values of retirees' income

Pensions	$\boldsymbol{T(d_{1+t}N_{1+t,1} + n_{c,1+t}w_{c,1+t}N_{1+t,1} + n_{h,1+t}w_{h,1+t}N_{1+t,1})}$
Wealth	$\pi_{t+1}(q_{t+1}(1-\delta_h)h_{t,1}N_{t,1} + p_{l,t+1}l_{t,1}N_{t,1})$
Inheritances	$q_{t+1}(1-\delta_h)h_{t,2}N_{t,2} + p_{l,t+1}l_{t,2}N_{t,2} +$ $(1-\pi_{t+1})q_{t+1}(1-\delta_h)h_{t,1}N_{t,1} + (1-\pi_{t+1})p_{l,t+1}l_{t,1}N_{t,1}$

Note: δ_h represents depreciation rate of houses.

In particular, the inheritance is in the form of houses (and land) and assumed to be so for three reasons. First, housing is needed by every individual, including the retired. Second, houses cannot be fully consumed by their owners, even at the time of their death. Third, houses are bequeathed to the next generation. For example, during the study of the bequest decision of Australians, Ding (2012) stated that housing assets constitute the bulk of bequests.

In our model, inheritance comes from two sources: 1) the previous generation of retirees, whose population is $N_{t,2}$; and 2) the workers who do not survive to retirement, whose population is $(1-\pi_{t+1})N_{t,1}$. The market values of these two parts are $q_{t+1}(1-\delta_h)h_{t,2}N_{t,2} + p_{l,t+1}l_{t,2}N_{t,2}$ and $(1-\pi_{t+1})q_{t+1}(1-\delta_h)h_{t,1}N_{t,1} + (1-\pi_{t+1})p_{l,t+1}l_{t,1}N_{t,1}$ respectively, where δ_h represents that rate of depreciation of houses. Meanwhile, the market value of house and land purchased during their working period is $\pi_{t+1}(q_{t+1}(1-\delta_h)h_{t,1}N_{t,1} + p_{l,t+1}l_{t,1}N_{t,1})$. Thus, the total market value of the housing wealth is obtained by adding these parts together and is represented as $q_{t+1}(1-\delta_h)h_{t,1}N_{t,1} + p_{l,t+1}l_{t,1}N_{t,1}$.

2.3. Firms

The other agent in the economy is the firms. We adapt the model in Liu, Wang, and Zha (2013) where firms are assumed to exist forever, and they have three important functions. The first is that they produce goods for consumption and housing, and this function will be illustrated in the section on technology. Furthermore, the firms invest in and maintain plant and equipment. Finally, firms maximize profits with the proceeds paid to households in the form of dividends.

Profit maximization

Following Liu, Wang, and Zha (2013), firms maximize long-run profits, denoted as follows:

$$Max \sum_t \beta_e^t \ln(D_t) \tag{9}$$

, where $D_t = d_t N_{t,1}$ represents the total profits of the firms in period t. In addition, the parameter β_e represents the time preference of firms. Additionally, the time preference of firms is less than that of the households, i.e. $\beta_e < \beta$, lending the incentive to invest (Iacoviello 2005, Liu, Wang, and Zha 2013, Iacoviello 2015). By using the log-function in (9), we assume that the firm prefers stable profit flow. This assumption follows from risk-aversion by households who own these firms (Sandmo 1971, Leland 1972, Oh, Rhodes, and Strong 2016).

A major modification from Liu, Wang, and Zha (2013) is that, in our model, the firms are assumed to be owned by all the households instead of a certain fraction of them. This modification allows us to focus on the households' heterogeneity of age structures without the complication of income distribution. Without this modification, as shown in Liu, Wang, and Zha (2013), the entrepreneurs would be introduced as a distinct class from the employees.

Here, we further provide microeconomic explanations about the assumed ownership of the firms above. To begin with, the economy consists of many firms. When viewing the firms together, it works as the firms in our model. In addition, every firm is owned by some households, while every household is one of the owners of some firms because we have no distinction between entrepreneurs and employees. Lastly, the households have full insurance on their wage and profits, thus their income is subject to the performance of the aggregate economy.

One of the consequences of the setting above is that households do not have deposits. Because the deposit is held as capital of firms, the households as owners of firms do not need separate deposit accounts. According to (16), the capital can be adjusted and distributed to households in the form of profit. From this point of view, the personal deposits are not disappeared but pooled in the total capitals. A possible drawback is that this saving is the same for all the households; nevertheless, modelling wealth distribution aside from the housing asset is beyond the scope of this paper.

Furthermore, we will explain why we have $\beta_e < \beta$ based on the setting above. Although each firm is owned by a panel of households, the time that the panel can own this firm will be no more than the longevity of households (because only living people can be on the panel). Thus, the firm owners, although they are households, will be more impatient about the future of firms than their own lives. This explanation is consistent with the reality that the survival rates of firms are less than that of the households.

Technologies

An often cited omission from previous research on the effect of aging population on housing prices is the supply side of the market (Swan 1995). We follow Iacoviello and Neri (2010) and treat the supply of housing as a separable production sector from the non-housing productions.

Non-housing production sector: The non-housing sector uses Cobb-Douglas technology of the form:

$$Y_t = A_{c,t}K_{c,t-1}^{\mu_c}N_{c,t}^{1-\mu_c} \tag{10}$$

The non-housing production is denoted by $Y_t = y_t N_{t,1}$, and there are three types of variables involved in the production process: the aggregate capital of non-housing production sector, $K_{c,t-1} = k_{c,t-1}N_{t-1,1}$; the labour employed, $N_{c,t} = n_{c,t}N_{t,1}$; and the Total Factor Productivity (TFP) in the non-housing production sector, $A_{c,t}$.

Housing production sector: Similar to that of the non-housing production sector, housing production takes the form:

$$\mathrm{IH}_t = A_{h,t}K_{h,t-1}^{\mu_h}L_{e,t-1}^{\mu_l}N_{h,t}^{1-\mu_h-\mu_l} \tag{11}$$

The houses built by the housing production sector is denoted by $\mathrm{IH}_t = ih_t N_{t,1}$. The construction of houses needs capital $K_{h,t-1} = k_{h,t-1}N_{t-1,1}$, land (which is owned by the firms) $L_{e,t-1} = l_{e,t-1}N_{t-1,1}$, and labour $N_{h,t} = n_{h,t}N_{t,1}$ while TFP growth is denoted by $A_{h,t}$. Following the work of Iacoviello and Neri (2010), we assume the land is indispensable for the production of housing.

Capital Accumulation

We denote the investments in non-housing and housing sectors in period *t* as $\mathrm{IK}_{c,t} = ik_{c,t}N_{t,1}$ and $\mathrm{IK}_{h,t} = ik_{h,t}N_{t,1}$ respectively. Following Liu, Wang, and Zha (2013), the capital accumulation processes for the non-housing and housing sectors are assumed to have the following specifications:

$$K_{c,t} = (1-\delta_{\mathrm{kc}})K_{c,t-1} + \mathrm{IK}_{c,t} - \Phi_{c,t} \tag{12}$$

$$K_{h,t} = (1-\delta_{\mathrm{kh}})K_{h,t-1} + \mathrm{IK}_{h,t} - \Phi_{h,t} \tag{13}$$

, where parameters δ_{kc} and δ_{kh} represent the depreciation rates of capital in the non-housing and housing sectors respectively.

In addition, when facing external shocks, the capital accumulations may accrue losses because of adjustment cost. The adjustment cost of capital includes opportunity costs of underutilized capital, the costs of obsolescence, and transition costs among activities (de Córdoba et al. 2006). The adjustment cost can exist in a long period as 30 years in our model because it is the sum of the cost that happens within this period. We use $\Phi_{c,t}$ and $\Phi_{h,t}$ to denote the accumulated adjustment cost of non-housing and housing sectors in period *t*. Following Iacoviello and Neri (2010) and Liu, Wang, and Zha (2013), they have the following specifications:

$$\Phi_{c,t} = \Phi_c(K_{c,t}, K_{c,t-1}) = \frac{\phi_{kc}}{2}\left(\frac{K_{c,t}}{K_{c,t-1}} - \exp(g_{KC,t})\right)^2 K_{c,t-1} \tag{14}$$

$$\Phi_{h,t} = \Phi_h(K_{h,t}, K_{h,t-1}) = \frac{\phi_{kh}}{2}\left(\frac{K_{h,t}}{K_{h,t-1}} - \exp(g_{KH,t})\right)^2 K_{h,t-1} \tag{15}$$

, where ϕ_{kc} and ϕ_{kh} are the parameters that represent the specific frictions in adjusting the capital stocks in non-housing and housing sectors respectively. In addition, variables $g_{KC,t}$ and $g_{KH,t}$ denote the trend capital growth rates on which the corresponding adjustment cost would be zero. The trend growth rate will be illustrated in the section of solution method.

Budget Constraint

The aggregate budget constraint of the firms is as follows:

$$\begin{aligned} D_t + \frac{K_{c,t}}{A_{k,t}} + K_{h,t} + w_{c,t}N_{c,t} + w_{h,t}N_{h,t} + p_{l,t}L_{e,t} + \frac{\Phi_{c,t}}{A_{k,t}} + \Phi_{h,t} \\ = Y_t + q_t\,\mathrm{IH}_t + \frac{1-\delta_{\mathrm{kc}}}{A_{k,t}}K_{c,t-1} + (1-\delta_{\mathrm{kh}})K_{h,t-1} + p_{l,t}L_{e,t-1} \end{aligned} \tag{16}$$

The right side of the (16) is the resources available to the firms while the left side is the payments for the above. Firms have revenues from: 1) selling non-housing and housing productions; 2) the balance of capital after depreciation, and 3) the market value of land owned by the firm. For the capital in the non-housing sector, as in Iacoviello and Neri (2010), the investment specific technology shock is introduced to the model and denoted by $A_{k,t}$.

The distribution of this wealth can be divided into four parts. Firstly, firms pay profits to households, as denoted by D_t in (16). Secondly, the capital stocks for next period are decided by the firms, which are denoted as $K_{c,t}$ and $K_{h,t}$. Along with this process, the capital adjustment costs are involved ($\Phi_{c,t}$ and $\Phi_{h,t}$). Thirdly, wages are paid to the workers. Lastly, the firms will decide on the amount of land owned in this period.

2.4. Equilibrium

There are four markets in our model, which are 1) the non-housing production market, 2) the housing market, 3) the land market, and 4) the labour market. These markets are assumed to be perfectly competitive, meaning that the prices have no mark-up and the payments of production factors are equal to their marginal contributions. The equilibrium conditions of the first three markets are:

$$Y_t = C_t + \frac{\mathrm{IK}_{c,t}}{A_{k,t}} + \mathrm{IK}_{h,t} + \Phi_t \tag{17}$$

$$H_t = \mathrm{IH}_t + (1 - \delta_h)H_{t-1} \tag{18}$$

$$L_t = L_{h,t} + L_{e,t} \tag{19}$$

, where $C_t = c_{t,1}N_{t,1} + c_{t,2}N_{t,2}$, $H_t = h_{t,1}N_{t,1} + h_{t,2}N_{t,2}$, $L_{h,t} = l_{h,t,1}N_{t,1} + l_{h,t,2}N_{t,2}$, $Y_t = y_tN_{t,1}$, $H_t = h_tN_{t,1}$, $L_t = l_tN_{t,1}$.

For the labour market, its equilibrium means the labour supply of households would be equal to the labour demand of firms. This equilibrium amount of working hours has been denoted by $N_{c,t}$ and $N_{h,t}$ in the previous discussion (or per worker, $n_{c,t}$ and $n_{h,t}$).

3. Assumptions and Calibration

3.1. Demographic Assumptions

In this research, we study both the effects of a decline in the fertility rate and an increase in longevity on housing prices. The levels of the decline and increase are arbitrarily chosen; however, the analysis on them will be sufficient to show the general housing price dynamics before, during and after the aging population process.

Specifically, at the period 1 and 2, the fertility rate is set at the replacement level. Using variable $\gamma_{N,t}$ to represent the rate of growth of the worker population, then $\gamma_{N,1} = \gamma_{N,2} = 0$. The first shock is a fall in the fertility rate of 10 percent in period 3 (Figure 1a), and this fall is foreseen by the agents of the economy beforehand. Correspondingly, we have $\gamma_{N,3} = -0.1$. In period 4, the fertility rate is returned to the replacement level as its original state and kept stable from then on (Figure 1a). For worker population, it means that $\gamma_{N,t} = 0, t \geq 4$.

The second shock is an increase in the survival rates π_t. Correspondingly, the longevity increases. In period 1 and 2, we assume that $\pi_1 = \pi_2 = 0.8$. In period 3, the survival rate increases to 0.9 and remains constant thereafter, i.e. $\pi_t = 0.9, t \geq 3$. Again, the longevity changes are known by the agents of the economy beforehand. With the assumed increase in survival rate, total longevity increases by 5.56 percent[6] (see Figure 1b).

By assuming both the changes above, the overall demographic changes regarding with the aging population are shown in Figure 1. The population of workers has declined (Figure 1c),

[6] The calculation is that $(30 \times 0.9 + 30)/(30 \times 0.8 + 30) = 1.0556$.

while the population of retirees spikes at period 3, before stabilising after period 4 (Figure 1d). During this process, the proportion of retirees rises from period 2 to 3. After that, it declines and then stays the same (Figure 1f).

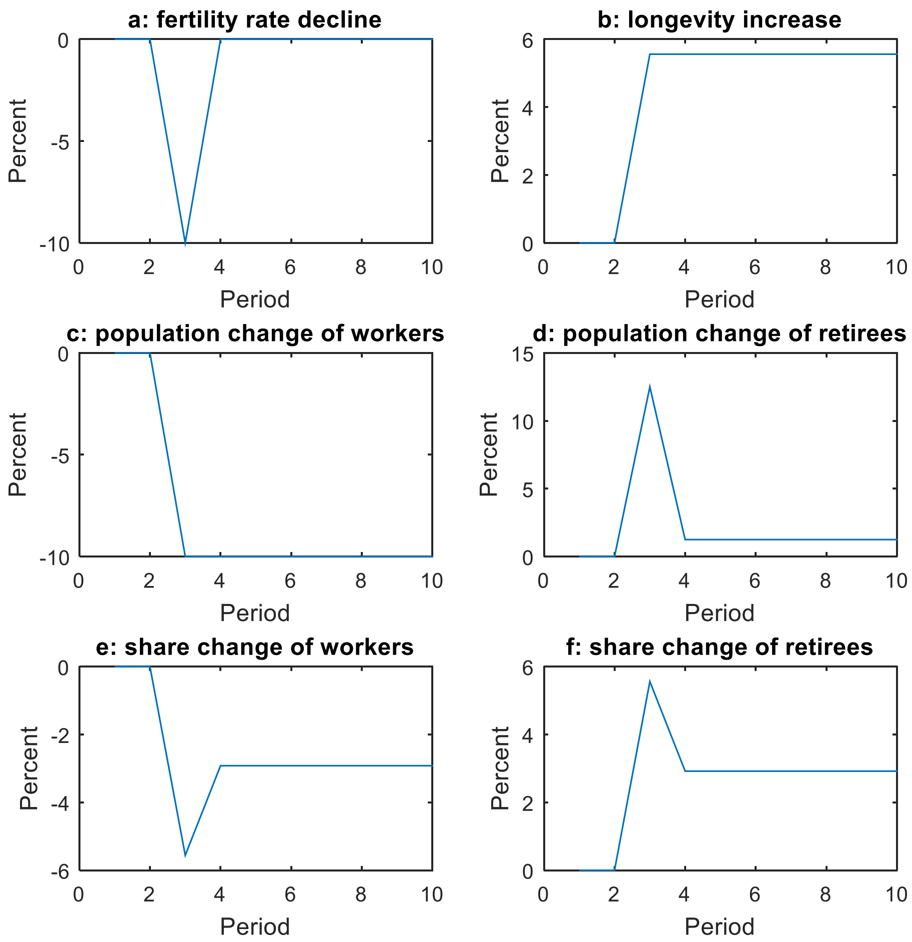

Figure 1. Overall Effect: Demography Changes

3.2. Parameter Calibration

To present the result numerically, we need assign values for the parameters. In this paper, we seek for feasible parameter values that fit the case of China because it will be a typical example when examining the effects of ageing on housing prices. For the parameters that has not been found under the context of China, we use the ones of America as a substitution.

There are 16 parameters in our model. In Table 2, their calibrated values are listed, as well as the sources of the calibration.

The time preference of households β is calibrated to match China's average real interest rates during 1980–2015[7]. According to World Development Indicators database[8], this rate, rounded to two decimal places, is 2 percent, which indicates the annual discount factor of 0.98, and for a period of thirty years, we set $\beta = 0.55$. Besides, following Iacoviello and Neri (2010) and Ng (2015), the annual discount rate of firms is set as 0.958 (=0.98/1.0232), lower than that of the households. In thirty years, the corresponding discount rate of firms β_e is 0.275.

Table 2—Parameter Calibration of the Model

Description	Symbols	Values	Sources
Time preference of households	β	0.55	See text
Time preference of firms	β_e	0.275	See text
Capital share (non-housing sector)	μ_c	0.46	Ng (2015)
Capital share (housing sector)	μ_h	0.24	See text
Land share (housing sector)	μ_l	0.10	Ng (2015)
Adjustment Friction (non-housing sector)	ϕ_{kc}	11	Ng (2015)
Adjustment Friction (housing sector)	ϕ_{kh}	10	Ng (2015)
Capital depreciation (non-housing sector)	δ_{kc}	1	Ng (2015)
Capital depreciation (housing sector)	δ_{kh}	1	Ng (2015)
House depreciation	δ_h	0.7	Iacoviello and Neri (2010)
Weight on housing in utility	j_h	0.12	Ng (2015)
Weight on land in utility	j_l	0.045	Liu, Wang, and Zha (2013)
Dis-preference on Labour supply	τ	1	Ng (2015)
See text	η	0.5	Ng (2015)
See text	ξ	1	Ng (2015)
Pension share	T	0.4	Iacoviello and Pavan (2013)

There are three depreciation rates in our model. For the two types of capital in the production sectors, their annual depreciation rates are about 10 percent (Iacoviello and Neri 2010, Ng 2015). In thirty years, this depreciation rate implies that the initial capital stock is fully depreciated. Thus, the corresponding parameters δ_{kc} and δ_{kh} are calibrated as 1. The depreciation rate of houses is calibrated according to Iacoviello and Neri (2010) as 4 percent annually[9], indicating 70 percent depreciation of the stock in 30 years.

[7] Follow Blackburn and Cipriani (2002) and Muto, Oda, and Sudo (2016), the survival rates in the utility function do not influence the calibration methods of the parameter of time preference.

[8] World Development Indicator of the World Bank: Real interest rate.
Website: http://data.worldbank.org/indicator/FR.INR.RINR. Date of Access: 21/Apr/2017

[9] We did not use this parameter value in Ng (2015) because that value indicates the houses will close to be fully depreciated in 30 years, and this is not true in the reality. Thus, we use the parameter value in Iacoviello and Neri (2010) instead.

The parameters denoting income shares and weight of utility are calibrated according to Ng (2015) and Liu, Wang, and Zha (2013). Their values are listed in Table 2: the capital share in housing sector (μ_h) is set at 0.24[10]. The capital adjustment cost in the non-housing and housing sector is calibrated according to Ng (2015), and the values are reported in Table 2. Among all the parameters, the value of pension share T of China has not been found in similar studies. Here, we use the case of America as an alternative, and the value is calibrated according to the work of Iacoviello and Pavan (2013).

4. Solution method

Regarding the demographic changes, there are two exogenous variables in our model, i.e. the worker population growth rate, $\gamma_{N,t}$, and the survival rate, π_t. When the values of these exogenous variables change, their effects can be captured by the simulation[11].

However, when the worker population growth rate is negative as in our assumptions, the worker population declines permanently (see figure 1c). Because the worker population is not a variable in the model of per capita variables, we cannot use the simulations to detect the effect of this permanent change.

To see this, note that the worker population growth rates are zero in both the original and end state. Thus, we will get the same value of the per worker land area l for both the states from the simulation (if consider the changes in $\gamma_{N,t}$ only). Nevertheless, when the total land area is a constant, we know that the per worker land area l will increase by 10 percent if the worker population declines by 10 percent. The values of other variables may also be affected, but the influence cannot be shown by the simulation. In this study, the influences are denoted by the trends of variables because they are determining the levels of variables.

Besides the worker population, the changes in TFPs ($A_{c,t}, A_{h,t}, A_{k,t}$) and land area L_t will also affect the levels of variables, and these effects are captured by the trend. Incorporating these factors are for general concerns to reveal their effects, and they will be assumed to be constants afterward. Correspondingly, we use the steady states to denote the values of the de-trended variables that can be calculated by the simulation. Formally, for a variable X_t, its trend and steady state have the following relationship:

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[10] Comparing with Iacoviello and Neri (2010) and Ng (2015), the intermediate input has been omitted here, and the share of this input is added to the capital.

[11] The simulation is conducted with Dynare 4.5.6.

$$\tilde{X}_t = \frac{X_t}{G_t} \tag{20}$$

, where $\tilde{X}_t$ and G_t denote the steady state and trend of the variable X at period t. The model equations of de-trended variables are listed in the Appendix. In addition, we define the trend growth rate of G_t as $g_t = \ln(G_t) - \ln(G_{t-1})$.

The algorithm calculating the trend of each variable is the same as that of calculating the balanced growth in the literature[12]. We demonstrate that the algorithm that has traditionally relied on the assumptions of constant growth rate in the exogenous variable applies to the more realistic case where the growth rates vary with time. This demonstration is under the following two assumptions:

1) The steady states do not become zero or infinity.
2) Trend growth rates are determined by changes in the contemporary exogenous variable.

For the first assumption, the steady states do not become zero means that, in the model, every variable is a necessary component, and none of them can be neglected. Meanwhile, forbidding the steady states turning to infinity is to keep their economic meanings. For example, no consumption (in a finite period) could go infinity.

In addition, the second assumption implies that the trend growth rates are time-varying with respect to the exogenous changes, and they are constants when the exogenous changes are fixed. Here, we say that two trend growth rates are different if there is a change in exogenous variables such that the two are unequal. In contrast, we say two trend growth rates are equal if they are equal for any change in exogenous variables.

Based on the above assumptions, we present the algorithm in propositions, and the proofs are provided in the Appendix. In particular, we discuss the relationship of trend growth rates in two kinds of equations: linear and Cobb Douglas forms. The two kinds of equations are basic building-bricks in the Diamond family of OLG models, and they cover basic algebra including addition, subtraction, multiplication, and power.

The linear equation has the form as follows:

$$X_t = aX_{1,t} + bX_{2,t} \tag{21}$$

, where a and b are non-zero parameters. Assuming variables X_t, $X_{1,t}$ and $X_{2,t}$ have trends $G_t, G_{1,t}$ and $G_{2,t}$, and the trend growth rates are $g_t, g_{1,t}$ and $g_{2,t}$.

The Cobb-Douglas form equation that we are focusing on is shown as follows:

[12] See, for example, Jones (1995), İmrohoroğlu, İmrohoroğlu, and Joines (1999) and Iacoviello and Neri (2010). A clear illustration of the algorithm could be found in the appendix of Iacoviello and Neri (2010).

$$X_t = cX_{1,t}^a X_{2,t-1}^b \tag{22}$$

, where a, b and c are non-zero parameters.

Proposition 1

For the linear equation above, we have $G_{1,t} = G_{2,t} = G_t$ and $g_{1,t} = g_{2,t} = g_t$.

Proposition 2

For the Cobb-Douglas form above, we have $G_t = G_{1,t}^a G_{2,t}^b$ and $g_t = a\, g_{1,t} + b\, g_{2,t}$.

Recall that the steady states are calculated by simulations, and the trends can be derived according to the algorithm above. After having both, the growth rate of a variable is the sum of the growth rates of the steady state and the trend. To see this, we can rewrite the equation (20) into $X_t = \tilde{X}_t G_t$, and take log-difference on both sides. Then, we have:

$$\ln(X_t) - \ln(X_{t-1}) = \ln(\tilde{X}_t) - \ln(\tilde{X}_{t-1}) + g_t \tag{23}$$

, where $\ln(X_t) - \ln(X_{t-1})$ is the growth rate of the variable x at period t. The terms $\ln(\tilde{X}_t) - \ln(\tilde{X}_{t-1})$ and g_t are growth rates of the steady state and trend respectively.

5. Long-Term Projection

A decline in fertility rate and an increase in longevity are both important underlying forces for aging of the population. We investigate the long-term effects of these two separately at first, and then combine them together to assess the aggregate changes. More concretely, since the long run effect could be reflected by trend or steady state changes, we are here looking for the specific values of these changes.

5.1. Fertility Rate Decline

As illustrated above, the fertility rate is the same in both the original and end states, so the steady states of variables are the same. Therefore, in the long run, the effect of the fertility rate decline would be revealed by the trends only.

In particular, the trend growth rates of house and land prices have the following closed form solutions (the derivations are provided in Appendix B):

$$g_{q,t} = \frac{1-\mu_h}{1-\mu_c}\gamma_{\text{ac,t}} - \gamma_{\text{ah,t}} + \frac{\mu_c(1-\mu_h)}{1-\mu_c}\gamma_{\text{ak,t}} + \mu_l(\gamma_{N,t} - \gamma_{L,t}) \tag{24}$$

$$g_{pl,t} = \frac{1}{1-\mu_c}\gamma_{ac,t} + \frac{\mu_c}{1-\mu_c}\gamma_{ak,t} + (\gamma_{N,t} - \gamma_{L,t}) \tag{25}$$

The meanings of the variables are listed in Table 3:

Table 3—Exogenous Variables in Trends

γ_N	Growth rate of worker population
γ_L	Growth rate of residential land area
γ_{ac}	Growth rate of TFP in non-housing sector
γ_{ah}	Growth rate of TFP in housing sector
γ_{ak}	Growth rate of investment specific technology

Specifically, the trend caused by fertility rate changes is reflected by the following equations, which neglect other exogenous variables from equations (24) and (25).

$$g_{q,t} = \mu_l \gamma_{N,t} \tag{26}$$

$$g_{pl,t} = \gamma_{N,t} \tag{27}$$

The only parameter in equation (26) is μ_l, which represents the share of land in house construction. This parameter, as assumed in the previous section, has the range of $0 < \mu_l < 1$. Therefore, the effect of adjusting γ_N would be bigger on the land price than that of the house.

The fertility rate n_t is not shown in these formulas directly; however, the variables $\gamma_{N,t}$ and n_t have the relationship $n_t = \exp(\gamma_{N,t})$. Thus, when $\gamma_{N,t} = 0$, the fertility rate is at replacement level, meaning that the worker population is stable. A negative γ_N implies a decline in fertility rate, and so the worker population contracts. Accordingly, the proportion of retirees would increase commensurately.

In this circumstance, the replacement level of fertility rate is essential for the price trends. If the fertility rate remains higher than the replacement level, then the prices would rise as a result. In contrast, if the fertility rate stays lower than the replacement level, then the price falls.

The following result derived from (26) and (27) would sum up the discussion above:

Result 1: *A decline in the fertility rate lowers the house and land prices in the long run. Moreover, when* $0 < \mu_l < 1$*, the effect on the land price would be bigger than that on the house.*

The mechanisms from fertility rate to prices is presented in Figure 2.

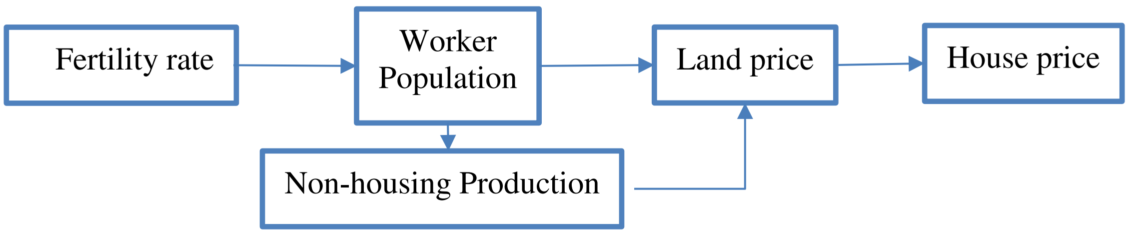

Figure 2. Causal Chain from Fertility Rate to Prices

In the first step, the fertility rate decline would lead to a contraction in the population of workers which then affects land and house prices. We will take the land price as an example to illustrate this effect. First, when worker population declines, the land price would fall because of reduced demand. Second, the decline in the population of workers will lead to a decline in non-housing production. Note that the land price is a relative price and is denoted by the units of non-housing production. Thus, when the output declines, the land price would decline accordingly.

What about the price of houses? First note that house construction is endogenous in our model. Thus, the changes in population and productions may not influence the house price because of the flexibility of supply. However, land is an input in house construction and thus a fall in the price of land will be transmitted to the house price. As shown in (26), the parameter μ_l denotes the share of land, which is also the share of price transmission. Since $0 < \mu_l < 1$, the decline in house price is less than that of land.

Finally, the mechanisms illustrated above would indicate the price trend decline would happen along with the shrinkage of worker population, and the value of the decline can be calculated from equations (26) and (27). Substituting in the parameters from the previous section, the trends of house/land price are shown in Figure 3, where their values decline by 1 percent and 10 percent, respectively.

Figure 3. Price Changes: Long-Term Effect of Fertility Rate Decline

Notes: The corresponding demographic changes are shown in Figure 1a.

5.2. Longevity Increase

As explained in the section on the solution method, the effect of an increase in longevity impacts on steady states instead of trends.

There are three ways of calculating the effect on steady states: derive the analytical solution, calculate the partial derivative from the above, or use numerical simulations when the above is not practical. The first two have proved difficult thus numerical simulations have been employed. By using parameter values and the changes in longevity, this method permits the calculation of steady-state values for the endogenous variable.

Here, by using the calibrated parameters and assumptions in the previous section, the numerical solution is presented to show the long-term effect of an increase in longevity. The robustness of the results is checked as explained in Appendix D. The results of the numerical solution are shown in Figure 4. The effects of an increase in the survival rate from 0.8 to 0.9 are calculated and plotted in this figure: it shows a positive impact on price of an increase in longevity.

This result indicates that, for plausible calibrations of the structural parameters of the economy, the long-term effects of an increase in longevity on the prices is positive. In addition, as shown in Figure 4, the rise in the price of land is higher than that of the house. Thus, for the given parameters, the long-term effect of an increase in longevity on land price is larger than that on the house.

Figure 4. Price Changes: Long-Term Effects of Longevity Increase

Notes: the survival rate is incremented by 0.1 to derive the corresponding steady state prices for house and land separately.

Result 2: *the long-run effect of an increase in longevity is positive on both house and land prices. Moreover, its effect on the land price is bigger than that on the house price.*

5.3. Overall

We next investigate combined effects of a fall in the fertility rate and an increase in longevity. More concretely, when using the assumed demographic changes in the previous section, their overall effects on the prices are shown in Table 4.

In the long run, the house price rises by 0.07 percent. This rise indicates that, conditional on the values of the assumed parameters, the positive effect on house price from an increase in longevity is greater than the negative effect from the fall in fertility. In contrast, the land price declines by 2.39 percent. Thus, the negative effect of the simulated fall in fertility outweighs the positive effect from the simulated increase in longevity.

Table 4—Prices Changes: Long-Term Effect of Aging Population (in percent)

	effect of decline in fertility rate	**effect of increase in longevity**	**overall effect**
house price	-1.00	1.07	0.07
land price	-10.00	7.61	-2.39

Note: the numbers come from Figure 3 and 5, and the overall effect is the sum of the two. The justification of this method is shown in Appendix D.

These offsetting effects can be deduced from Result 1 and Result 2. Recall from Result 1, the long-term effect of fertility rate decline on prices is negative; however, in Result 2, the long-term effect on prices of an increase in longevity is positive. Because these two effects move in opposite directions, the net effect depends on which of the above is overwhelming. Furthermore, if we manipulate the extent of fertility rate decline, these opposing effects could be studied more closely for specific parameter values.

As shown in Figure 5, the Cartesian plane consists of the fertility rate decline (y-axis) and longevity increase (x-axis). The line pictured in the plane is a set of points, denoting combinations of fertility rate decline and longevity increase. For house price, on this line, the upward effect from longevity increase equals the downward effect from fertility rate decline. That is, the net effect of an aging population on house prices is zero on this line.

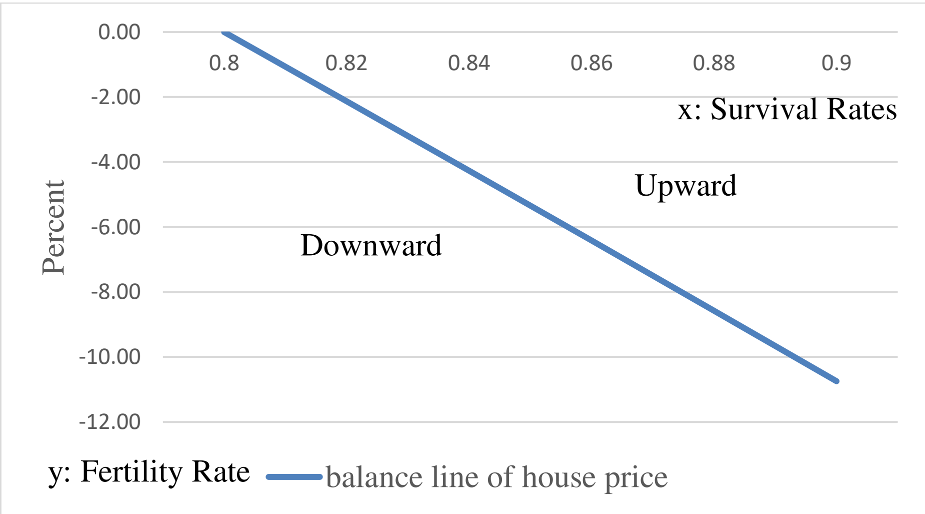

Figure 5. Plane of Aging Population and Balance Line of House Price

Notes: The effect of longevity increase on house price comes from Figure 4, and the effect of fertility rate decline is calculated according to (26).

Moreover, the line pictured in Figure 5 is also a dividing line. It divides the quadrant of the plane into two parts. On the right side of the line, the house price will rise in the long run, because of the stronger effect from an increase in longevity via-a-vis a fall in fertility. However, on the left side of the line, the house price will decline due to the weight of fertility rate decline.

For land price, a similar plane is shown in Figure 6, together with a line, on which the long-term effect of aging population is zero. Similar to that of the house price, the quadrant is also divided into two parts by this balance line, and its right / left side would be the upward / downward area for the land price.

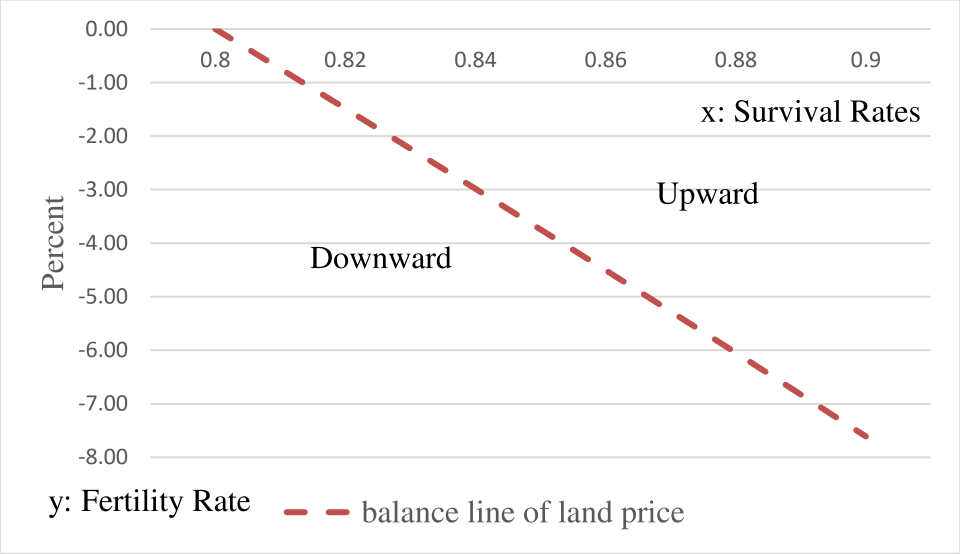

Figure 6. Plane of Aging Population and Balance Line of Land Price

Notes: The effect of longevity increase on land price comes from Figure 4, and the effect of fertility rate decline is calculated according to (27).

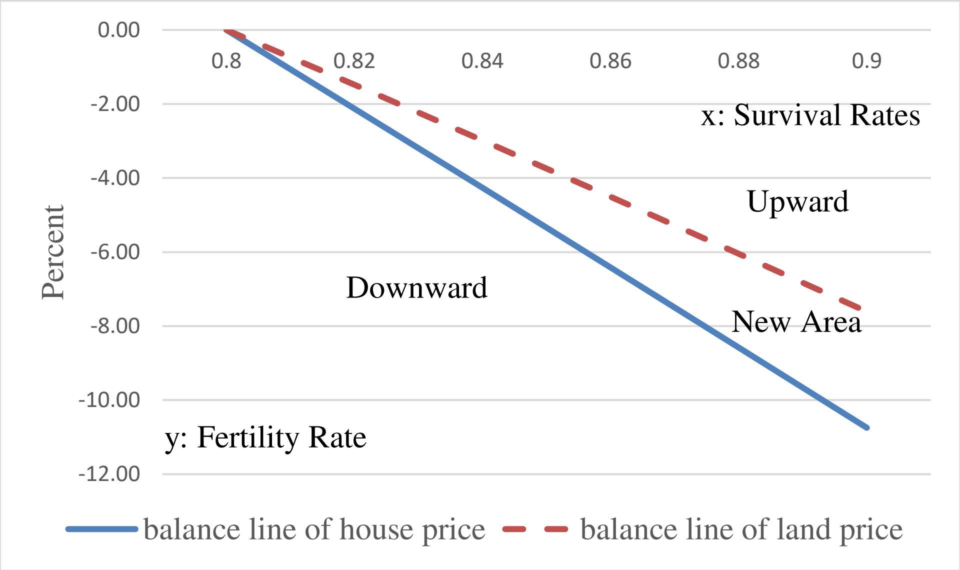

Figure 7. Plane of Aging Population and Balance Line of Land Price

Notes: source from Figure 5 and 7.

Lastly, notice that the balance lines of house and land prices do not coincide with each other, which is shown in Figure 7. Thus, the zero long-run effect cannot be achieved for both the prices simultaneously. Moreover, the quadrant is divided into three areas by two different lines. Because the areas preserved the properties as in Figure 5 and 7, the right most area is where both land and house prices rise while left-most region is where both prices fall. The middle area lends room for the two prices to diverge: specifically where the price of land drops while that for houses increase. The parameters assumed in the calculations are those for this specific region as shown in Table 4.

To sum up, the discussion above could be highlighted by the following result:

Result 3: *the aging population could have zero effect on either house or land price in the long run.*

6. Simulation

In this section, we report on simulations of the economic effects of the demographic changes presented in Figure 1. Specifically, the simulations provide the dynamics in terms of household utility and housing prices during the transition periods. From these dynamics, we explain how the demographic changes and economic fluctuations are interconnected. The results have had trends involved according to (23). Thus, they are the overall dynamics of variables instead of steady states.

6.1. Households' Utility

Aside from the negative utility from working, the household utility is determined by the non-housing consumption and house and land owned per capita. If we divide the households into workers and retirees, their per capita utility is shown in Figure 8. The per capita consumption of workers is greater from that of period 1 in periods 2 to 4 (see Figure 8a) when workers own more houses and land both in the long run and short run (see Figure 8c, e). In contrast, the consumption of retirees decreases from period 2 and reaches its lowest point in period 3 (see Figure 8b). Although consumption rises from period 3 to 4, the long run per capita consumption is still below that of period 1 (the original state). Similar situations also happen in both the house and land owned by retirees on a per capita basis (see Figure 8d, f).

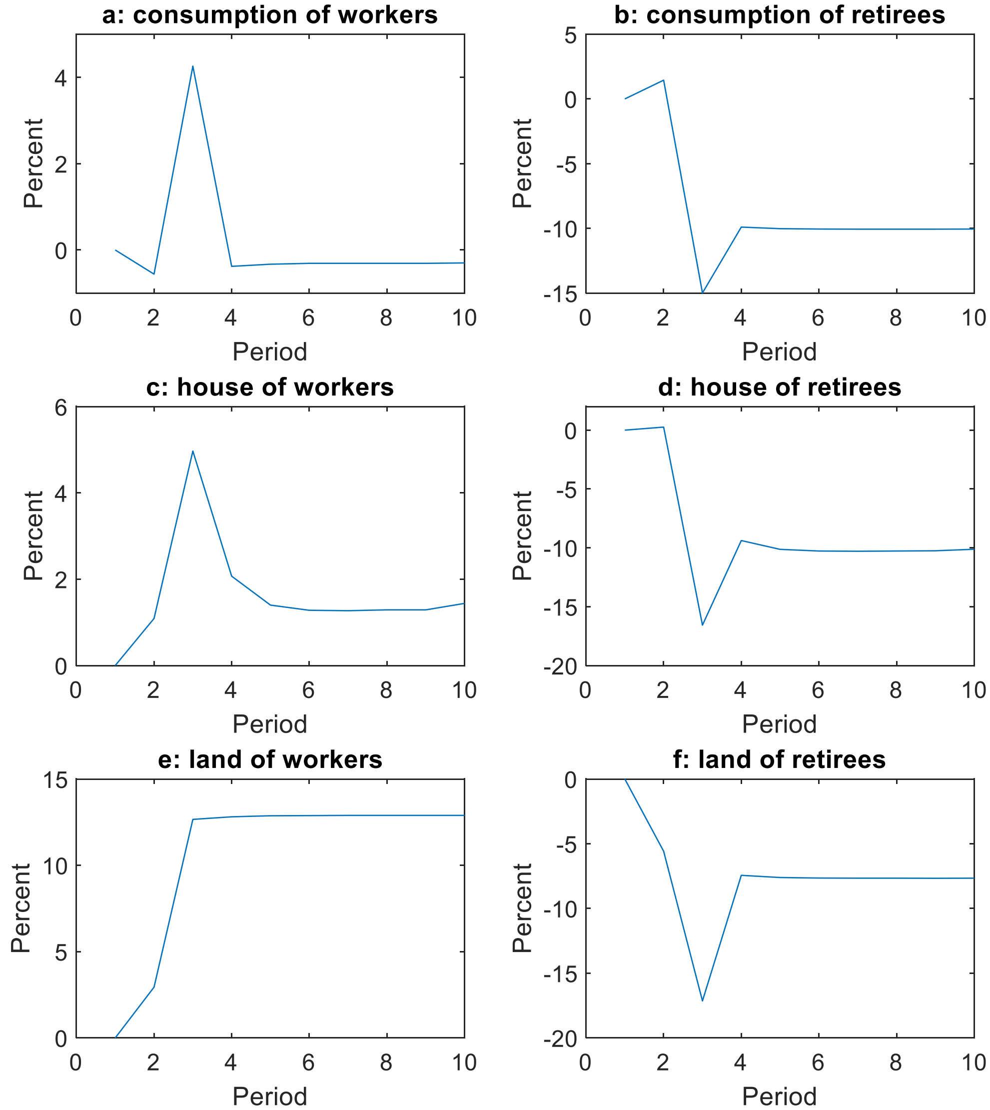

Figure 8. The Utility Dynamics (per capita)

Note: the corresponding demographic changes are presented in Figure 1, both the fertility rate decline and longevity increase happen from period 2 to 3.

The results in Figure 8 show that, relative to period 1 (the original state), the utility of workers is higher; however, the retirees' utility is worse than their original state. We next discuss the retirees' utility since workers' behaviour is influenced by the expectations of their life in retirement.

For retirees, in the long run, the decrease in utility is due to the increase in longevity which in turn raises the population of retirees. Compared with the case of no increase in longevity, the population of retirees rises by 12.5 percent. When the population rises, to keep the same utility level as before, it requires a rise of total income. However, the simulation indicates that in the long-run total income of retirees will rise by less than 1 percent thus per capita income and the utility of retirees fall.

During the transition period, the significant utility loss of retirees is due to the decline in the fertility rate. This decline leads to fewer workers and a concomitant rise in the proportion of retirees. Especially in period 3, the proportion of retirees would reach its peak (see Figure 1f), meaning that the pension income from workers would be shared by more retirees, leading to the most drastic losses in utility. After that, this utility will rise along with a decline in the proportion of retirees (see Figure 1f), and move towards its long run level.

For the workers, their behaviour would be influenced by their expectations about the living standard of their retirement. In the long-run, the higher utility is due to the increased savings of workers. Recall that the utility loss would happen in households' retirement period, the workers with rational expectation would increase their savings to fund their retirement so as to maximize their life-span utility. Specifically, the households, having perfect foresight of their future utility loss, would purchase more house and land when they are workers, and sell them when they retire. This purchase behaviour against the future utility loss raises the house and land owned by workers and with it their utility.

During the transition period, the increased utility of workers comes from the fall in the fertility rate. Because of the fertility rate decline, the worker population decreases accordingly. However, the total income of workers will not decrease to the same extent. The key driver here is the wealth stock of the firms, including $K_{c,t-1}$, $K_{h,t-1}$ and $L_{e,t-1}$ (see equation (10) and (11)). When worker population declines, the adjustments of the wealth are not immediate, indicating higher per worker output in goods and houses. According to equation (16), higher per worker output and wealth stock translates into the income of workers through wages and profits. Thus, higher per capita income would accrue to workers, and their utility increases. Along with the adjustment of the wealth stock, workers' utility will tend to converge towards its long run level.

Based on the discussion above, we note that an aging population would increase inequality across generations. The utility of workers increases whereas that of the retirees falls. Especially in the transition periods, this growth in inequality would be significant. In these periods, as has been illustrated above, the significant aging population and the wealth stock adjustment would drive a wedge between the two generations.

The results of this section can be concluded as follows:

Result 4: *the aging population could increase inequality across generations. Specifically, the utility of workers would rise while that of retirees would fall.*

6.2. Prices

The price dynamics following the demographic changes presented in Figure 1 is explained next. The house price rises from period 1 to 3, and then declines from period 3 to 4 (see Figure 9). Meanwhile, the land price also rises from period 1 to 2, and declines from period 2 to 4 (see Figure 10).

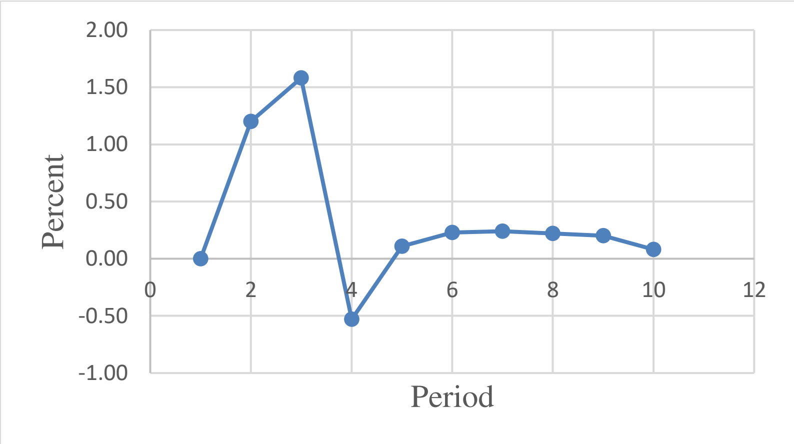

Figure 9. the Overall Dynamics of House Price

Note: the corresponding demographic changes are presented in Figure 1, both the fertility rate decline and longevity increase happen from period 2 to 3. After that, the fertility rate moves back to the replacement level, while the longevity stays stable.

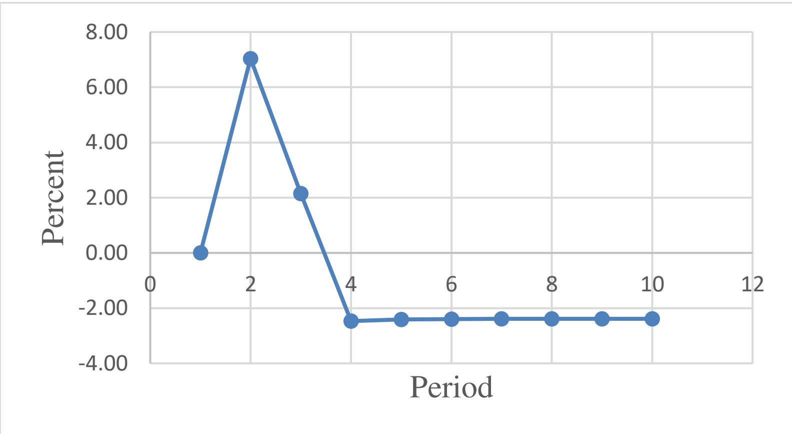

Figure 10. the Overall Dynamics of Land Price

Note: the corresponding demographic changes are presented in Figure 1, both the fertility rate decline and longevity increase happen from period 2 to 3. After that, the fertility rate moves back to the replacement level, while the longevity stays stable.

As shown in Figure 9 and 10, the land price dynamics produce a turning point at period 2, while the turning point of the house price happens at period 3. Before these points, the prices are rising; however, after these points, the prices fall. What is the mechanism that connects the demographic change to the price movements?

To explain these dynamics, the Euler Equations of house and land are listed:

$$\tilde{q}_t = \beta E_t(\exp(g_{q,1+t})\,\tilde{q}_{1+t}(1-\delta_h)\frac{c_{1,t}}{c_{2,1+t}}) + j_h\frac{c_{1,t}}{\tilde{h}_{1,t}} \qquad (28)$$

$$\tilde{p}_{l,t} = \beta E_t \left(\exp(g_{\text{pl},1+\text{t}})\, \tilde{p}_{l,1+t}\, \frac{c_{1,t}}{c_{2,1+t}} \right) + j_l \frac{c_{1,t}}{\tilde{l}_{1,t}} \tag{29}$$

, where the variables with a tilde denote their de-trended values.

According to (28) and (29), the price dynamics are explained by the utility changes of households, which are presented in Figure 8. In the first step, we will emphasize that the utility changes and perfect foresight of households are driving prices to rise from period 1 to 2.

As shown in Figure 8b, the per capita consumption of retirees would decline significantly from period 2 to 3. Given the assumption of perfect foresight of households, their behaviours would change accordingly and beforehand. The expected decline in consumption is denoted by $E_t(c_{2,1+t})$. According to (28) and (29), the decline of $E_t(c_{2,1+t})$ will raise the house and land prices ($\tilde{q}_t$ and $\tilde{p}_{l,t}$) beforehand. Therefore, if the expected consumption decline would happen from period 2 to 3, then the price rises would precede period 2. This mechanism explains the price rise before the demographic changes.

Intuitively, as explained in the household utility section, expectations drive the price rise. When a decline in consumption is anticipated, the households raise their savings by purchasing more housing and land when they are workers (see figure 8c, e). It is these purchases which drive the price rise.

After period 2, the drop in the fertility rate tends to drag the prices down as discussed in the previous section. Specifically, the house and land prices should be lowered by 1 and 10 percent respectively from period 2 to 3 in terms of the fertility rate decline (see figure 3). However, the price dynamics show that the land price falls by 5 percent instead, and the house price even rises (see figure 9 and 10). What is the force supporting the prices?

Here, this force comes from the character of stock of wealth. As discussed previously, the stock of wealth would not adjust immediately against a decline in the population of workers. Therefore, with these wealth stocks, the per capita income of workers in period 3 rises. More house and land would be purchased by using these incomes, and these purchases are supporting the price of these assets. This supporting force shapes the price dynamics by preventing the land price from sharp decline and postponing the turning point of the house price.

Nevertheless, this supporting force fades away as the wealth stock is adjusted, and thus both the prices fall at period 4. The fading of this supporting force can be reflected by the workers' consumption at period 4, which drops to its long run level (see figure 8a). According to (28) and (29), the per capita worker consumption $c_{1,t}$ is positively related to the housing prices. Thus, when the consumption declines, the housing prices would fall along with it.

In particular, the land price reaches its long run level at period 4, but the house price overshoots (see figure 9 and 10). Why? This further decline of house price is explained from changes in the house stock. To illustrate, we should notice that the house stock $\tilde{h}_{1,t}$ is not adjusting simultaneously with the consumption (see Figure 8c). Because of the depreciation character of house stock, the adjustment would not be immediate, but take place gradually. Based on (28), a higher $\tilde{h}_{1,t}$ indicates a lower house price. Thus, in period 4, because $\tilde{h}_{1,t}$ is higher than its long run level, the price would have a further fall. This mechanism enriches the discussion about overshooting phenomena that started with Dornbusch (1976).

In sum, connections between demographic changes and price dynamics are presented above, and these connections could be summarized by the following result:

Result 5: *aging population could cause turning points in housing prices.*

Although this result is based on the specific demographic changes presented in the previous section, it is theoretically grounded as shown in Figure 7– an issue under current investigation.

7. Conclusion

The effect of an aging population on housing prices remains an unresolved issue with mixed empirical findings. This debate began with Mankiw and Weil (1989) and has been ongoing since. Here we build an overlapping generations model where aging results from a combination of a decline in fertility and an increase in longevity to simulate effects of the above on housing prices. Our simulations for plausible parameter values give mixed results, depending on which of the above-mentioned overwhelms in terms of the impact of aging on house (and land) prices.

This paper has tackled the impact of aging on housing prices from a theoretical perspective. Our result provides reasons for the mixed results shown through empirical studies. It shows that a decline in the fertility rate depresses housing prices while an increase in longevity does the opposite – the net effect of a simultaneous change in the above two factors depends on which effect is overwhelming. Note that aging is caused by a combination of a fall in fertility with an increase in longevity, but the exact magnitude of the afore-mentioned differs across contexts. More concretely, in the long run, housing prices will decline if the drag of a fall in fertility outweighs the push from an increase in longevity, and vice versa. This result may explain the mixed findings from existing empirical research.

If indeed true, then what does the above imply for the price trends for housing in the future? Our simulations predict sharp turning points in housing prices when the upward effect is overwhelmed by the current declines in the rate of fertility. The turning points are revealed by the simulations and explained theoretically. Before the turning points, the prices would rise in anticipation of a longer life span by existing population of workers and thus the need for more wealth to fund retirement. Nevertheless, the decline afterwards is due to the decrease of worker population caused by the lower fertility rate. Although the wealth stock would temporary support the housing prices, the price declines would continue when this support effect fades away.

Furthermore, household behaviour has also been discussed in this paper, and the analysis showed that the demographic changes would lead to welfare inequality across generations. More concretely, the utility of workers will be higher; however, that of retirees will be lower (see Figure 8). This inequality would be most significant during the transition periods.

In sum, this paper contributes to the literature on the effects of aging – a phenomenon that is spreading across the world – which will affect housing prices but the exact magnitudes will differ by context and change with time. The next challenge, which is part of ongoing research, is to test these predictions using data on aging societies. These findings will deepen our understanding of the relationship between aging and the economy.

Appendix A. per capita Version Model Equations

The per capita version model equations are transformed from the corresponding aggregate equations, and the derivation is based on Eq. (1), (2), (3) in the demography section.

Households: For each generation, the budget constraint is shown by variables in lower case denoting per capita amount of that generation.

$$c_{t,1} + q_t h_{t,1} + p_{l,t} l_{t,1} = (1 - T)\left(w_{c,t} n_{c,t} + w_{h,t} n_{h,t} + d_t\right) \tag{A1}$$

$$\begin{gathered} c_{1+t,2} + q_{1+t} h_{1+t,2} + p_{l,1+t} l_{1+t,2} = \\ q_{1+t}(1 - \delta_h)\left(\frac{h_{t,1}}{\pi_{1+t}} + \frac{\pi_t h_{t,2}}{\pi_{1+t} n_t}\right) + T\left(d_{1+t} + n_{c,1+t} w_{c,1+t} + n_{h,1+t} w_{h,1+t}\right)\frac{n_{1+t}}{\pi_{1+t}} \\ + p_{l,1+t}\left(\frac{l_{h,t,1}}{\pi_{1+t}} + \frac{\pi_t l_{t,2}}{\pi_{1+t} n_t}\right) \end{gathered} \tag{A2}$$

Firms: For firms, the variables in lower case represent the amount per worker.

$$y_t = A_{c,t}\left(\frac{k_{c,t-1}}{n_t}\right)^{\mu_c} n_{c,t}^{1-\mu_c} \tag{A3}$$

$$\mathrm{ih}_t = A_{h,t}\left(\frac{k_{h,t-1}}{n_t}\right)^{\mu_h}\left(\frac{l_{e,t-1}}{n_t}\right)^{\mu_l} n_{h,t}^{1-\mu_h-\mu_l} \tag{A4}$$

$$k_{c,t} = (1 - \delta_{\mathrm{kc}})\frac{k_{c,t-1}}{n_t} + \mathrm{ik}_{c,t} - \phi_{c,t} \tag{A5}$$

$$k_{h,t} = (1 - \delta_{\mathrm{kh}})\frac{k_{h,t-1}}{n_t} + \mathrm{ik}_{h,t} - \phi_{h,t} \tag{A6}$$

$$\phi_{c,t} = \phi_c\left(k_{c,t}, k_{c,t-1}\right) = \frac{\phi_{kc}}{2}\left(\frac{k_{c,t} n_t}{k_{c,t-1}} - \exp(g_{KC,t})\right)^2 \frac{k_{c,t-1}}{n_t} \tag{A7}$$

$$\phi_{h,t} = \phi_h\left(k_{h,t}, k_{h,t-1}\right) = \frac{\phi_{kh}}{2}\left(\frac{k_{h,t} n_t}{k_{h,t-1}} - \exp(g_{KH,t})\right)^2 \frac{k_{h,t-1}}{n_t} \tag{A8}$$

$$\begin{gathered} d_t + \frac{k_{c,t}}{A_{k,t}} + k_{h,t} + w_{c,t} n_{c,t} + w_{h,t} n_{h,t} + p_{l,t} l_{e,t} + \frac{\phi_{c,t}}{A_{k,t}} + \phi_{h,t} = \\ y_t + q_t\,\mathrm{ih}_t + \frac{1 - \delta_{\mathrm{kc}}}{A_{k,t}}\frac{k_{c,t-1}}{n_t} + (1 - \delta_{\mathrm{kh}})\frac{k_{h,t-1}}{n_t} + p_{l,t}\frac{l_{e,t-1}}{n_t} \end{gathered} \tag{A9}$$

Equilibrium:

$$y_t = c_t + \frac{\mathrm{ik}_{c,t}}{A_{k,t}} + \mathrm{ik}_{h,t} - \phi_t \quad (A10)$$

$$h_t = \mathrm{ih}_t + (1-\delta_h)\frac{h_{t-1}}{n_t} \quad (A11)$$

$$l_t = l_{h,t} + l_{e,t} \quad (A12)$$

, where $c_t = c_{t,1} + \frac{\pi_t}{n_t}c_{t,2}$, $h_t = h_{t,1} + \frac{\pi_t}{n_t}h_{t,2}$, $l_{h,t} = l_{h,t,1} + \frac{\pi_t}{n_t}l_{h,t,2}$.

Appendix B. Trends

The proof of the algorithm

Proposition 1

For the linear equation (21), we have $G_{1,t} = G_{2,t} = G_t$ and $g_{1,t} = g_{2,t} = g_t$.

Proof

We will prove that the unequal trend growth rates cannot exist according to our assumptions. If not, then there are exogenous variable changes such that the growth rates are not equal. In this circumstance, we fix the exogenous change so that the unequal trend growth rates are constants, i.e. $G_t = (\exp(g))^t > 0, G_{1,t} = (\exp(g_1))^t > 0, G_{2,t} = (\exp(g_2))^t > 0$. The inequality implies that there are some growth rates smaller than the others. Without loss of generality, we assume that $g \leq g_i$ and at least one of the inequality is strict[13].

We rewrite the equation (21) as follows:

$$\frac{X_t}{G_t} = a\frac{X_{1,t}}{G_{1,t}}\frac{G_{1,t}}{G_t} + b\frac{X_{2,t}}{G_{2,t}}\frac{G_{2,t}}{G_t}$$

$$\tilde{X}_t = a\tilde{X}_{1,t}\frac{G_{1,t}}{G_t} + b\tilde{X}_{2,t}\frac{G_{2,t}}{G_t}$$

$$\tilde{X}_t = a\tilde{X}_{1,t}\left(\frac{\exp(g_1)}{\exp(g)}\right)^t + b\tilde{X}_{2,t}\left(\frac{\exp(g_2)}{\exp(g)}\right)^t \quad (B1)$$

In equation (B1), at least one of the terms $(\frac{\exp(g_i)}{\exp(g)})^t$ would go infinity when $t \to +\infty$. Consequently, the steady state $\tilde{X}_t$ would become infinity if $\tilde{X}_{i,t}$ are not zero. Either the way, it

13 In case that it is g_1 smaller than the others, we can rewrite equation (4) into $X_{1,t} = a'X_t + b'X_{2,t}$, where $a' = \frac{1}{a}$ and $b' = -\frac{b}{a}$. The other situations are similar.

violates the assumption one. Thus, for any exogenous change, we have $g_{1,t} = g_{2,t} = g_t$ and $G_{1,t} = G_{2,t} = G_t$. #

Proposition 2

For the Cobb-Douglas form equation (22), we have $G_t = G_{1,t}^a G_{2,t}^b$ and $g_t = a\, g_{1,t} + b\, g_{2,t}$.

Proof

The logic is the same as the proof of the proposition 1. Divide both sides of the equation (22) by G_t and we derive the equation as follows:

$$\tilde{X}_t = \frac{X_t}{G_t} = c\frac{X_{1,t}^a X_{2,t-i}^b}{G_t} = c\left(\frac{X_{1,t}}{G_{1,t}}\right)^a \left(\frac{X_{2,t-1}}{G_{2,t}}\right)^b \frac{G_{1,t}^a G_{2,t}^b}{G_t} = c\tilde{X}_{1,t}^a \left(\frac{\tilde{X}_{2,t-1}}{\exp(g_{2,t})}\right)^b \frac{G_{1,t}^a G_{2,t}^b}{G_t}$$

If $G_t \neq G_{1,t}^a G_{2,t}^b$, then there are exogenous variable changes satisfy this inequality. Similar to the proof in proposition 1, we fix the exogenous change and the unequal trend growth rates would be constants. Then, the term $\frac{G_{1,t}^a G_{2,t}^b}{G_t}$ will turn to zero or infinity when $t \to +\infty$. To hold the equation, the steady states would become either zero or infinity, and thus violate the assumption one. Therefore, for any change in exogenous variables, we have $G_t = G_{1,t}^a G_{2,t}^b$.

Taking log-difference operation on both sides of $G_t = G_{1,t}^a G_{2,t}^b$, we have:

$$lnG_t - lnG_{t-1} = a\big(lnG_{1,t} - lnG_{1,t-1}\big) + b(lnG_{2,t} - lnG_{2,t-1})$$

According to equation (3), the above equation can be written as $g_t = a\, g_{1,t} + b\, g_{2,t}$. #

Remark 1

The conclusion in proposition 1 can be extended when there are more linear terms in equation (4), i.e. $G_t = G_{1,t} = G_{2,t} = \cdots$.

Remark 2

The conclusion in proposition 2 can be extended when there are more terms multiplied in the right-hand side of the equation (7). For example, when $X_t = dX_{1,t}^a X_{2,t-1}^b X_{3,t-1}^c$, we have $g_t = a\, g_{1,t} + b\, g_{2,t} + c\, g_{3,t}$.

Remark 3

The relationship of the trend growth rates is contemporaneous, and thus the time subscript can be omitted.

So far, the algorithm that calculates balanced growth rates in literature is shown to be held in more general circumstance under the two assumptions we made.

The Trends in the Model

The exogenous variables that causing trend changes have been listed in Table 3. In addition, we assume that the per worker labour supplies, $n_{c,t}$ and $n_{h,t}$, do not have trends. The trends of other variables are calculated as follows.

The per worker land area is calculated as:

$$l_t = \frac{L_t}{N_{1,t}}$$

Denote the growth rates of worker population and land area by $\gamma_{N,t}$ and $\gamma_{L,t}$ respectively, the per worker land area growth rate can be calculated according to the proposition 2 as:

$$g_{l,t} = \gamma_{L,t} - \gamma_{N,t} \tag{B2}$$

According to Eq. (A12) and the proposition 1, the variables $l_{e,t}$ have the same trend growth rate as l_t, i.e. $g_{l,t}$. Based on Eq. (A9) and the proposition 1, the terms $p_{l,t}l_{e,t}$ and $\frac{k_{c,t}}{A_{k,t}}$ have the same growth rate as the variable y_t, and this relationship can be described by:

$$g_{y,t} = g_{pl,t} + g_{l,t} = g_{kc,t} - \gamma_{ak,t} \tag{B3}$$

We have known $g_{l,t}$ (see Eq. (B2)), if we can describe $g_{y,t}$ using exogenous variables, the trend growth rate of land price $g_{pl,t}$ would be calculated according to the above equation.

Use Eq. (A3) and the proposition 2, $g_{y,t}$ can be calculated as:

$$g_{y,t} = \gamma_{ac,t} + \mu_c g_{kc,t} \tag{B4}$$

Use Eq. (B3) and (B4), the $g_{y,t}$ can be described as:

$$g_{y,t} = \frac{1}{1-\mu_c}\gamma_{ac,t} + \frac{\mu_c}{1-\mu_c}\gamma_{ak,t} \tag{B5}$$

Thus, according to Eq. (B3), the trend growth rate of land price is:

$$g_{pl,t} = \frac{1}{1-\mu_c}\gamma_{ac,t} + \frac{\mu_c}{1-\mu_c}\gamma_{ak,t} + \left(\gamma_{N,t} - \gamma_{L,t}\right) \tag{B6}$$

Next, we will derive the trend growth rate of house price. According to Eq. (A4) and the proposition 2, the trend growth rate of ih_t is:

$$g_{ih,t} = \gamma_{ah,t} + \mu_h g_{kh,t} + \mu_l g_{l,t} \tag{B7}$$

From Eq. (A9), propositions 1 and 2, we can get:

$$g_{y,t} = g_{kh,t} = g_{ih,t} + g_{q,t} \tag{B8}$$

Based on Eq. (B3), (B5), (B7) and (B8), the trend growth rates of house and house price are:

$$g_{ih,t} = \gamma_{\mathrm{ah,t}} + \frac{\mu_h}{1-\mu_c}\gamma_{\mathrm{ac,t}} + \frac{\mu_c\,\mu_h}{1-\mu_c}\gamma_{\mathrm{ak,t}} - \mu_l\left(\gamma_{N,t} + \gamma_{L,t}\right) \tag{B9}$$

$$g_{q,t} = \frac{1-\mu_h}{1-\mu_c}\gamma_{\mathrm{ac,t}} - \gamma_{\mathrm{ah,t}} + \frac{\mu_c(1-\mu_h)}{1-\mu_c}\gamma_{\mathrm{ak,t}} + \mu_l\left(\gamma_{N,t} - \gamma_{L,t}\right) \tag{B10}$$

Assuming that all the exogenous variables are zero except the worker population growth rate $\gamma_{N,t}$, the trend growth rates above can be written as follows:

$$g_{l,t} = -\gamma_{N,t} \tag{B11}$$

$$g_{pl,t} = \gamma_{N,t} \tag{B12}$$

$$g_{ih,t} = -\mu_l \gamma_{N,t} \tag{B13}$$

$$g_{q,t} = \mu_l \gamma_{N,t} \tag{B14}$$

Because $g_{y,t} = 0$ in this circumstance, according to budget constraint equations in Appendix A, the per capita variables such as wages, profits, consumptions and capitals also have no trend. Meanwhile, based on the equilibrium equations in the appendix A, the variables $l_{h,t,1}$, $l_{h,t,2}$ and $l_{h,t}$ have the same trend growth rate as l_t. Similarly, the variables $h_{t,1}$, $h_{t,2}$ and h_t have the same trend growth rate as ih_t.

Appendix C. De-trended Equations and First Order Conditions

Using the equations in the appendix B, the equations in the appendix A can be written in de-trended form. We list these equations as follows, as well as the de-trended first order conditions of households and firms.

Households:

$$\tilde{c}_{t,1} + \tilde{q}_t \tilde{h}_{t,1} + \tilde{p}_{l,t} \tilde{l}_{t,1} = (1 - T)\left(\tilde{w}_{c,t} \tilde{n}_{c,t} + \tilde{w}_{h,t} \tilde{n}_{h,t} + \tilde{d}_t\right) \tag{C1}$$

$$\begin{aligned} &\tilde{c}_{1+t,2} + \tilde{q}_{1+t} \tilde{h}_{1+t,2} + \tilde{p}_{l,1+t} \tilde{l}_{1+t,2} = \\ &\tilde{q}_{1+t}(1 - \delta_h)\left(\frac{\tilde{h}_{t,1}}{\pi_{1+t} \exp\left(g_{h,t+1}\right)} + \frac{\pi_t \tilde{h}_{t,2}}{\pi_{1+t} n_t \exp\left(g_{h,t+1}\right)}\right) + \\ &T\left(\tilde{d}_{1+t} + \tilde{n}_{c,1+t} \tilde{w}_{c,1+t} + \tilde{n}_{h,1+t} \tilde{w}_{h,1+t}\right) \frac{n_{1+t}}{\pi_{1+t}} + \\ &\tilde{p}_{l,1+t}\left(\frac{\tilde{l}_{h,t,1}}{\pi_{1+t} \exp\left(g_{l,t+1}\right)} + \frac{\pi_t \tilde{l}_{t,2}}{\pi_{1+t} n_t \exp\left(g_{l,t+1}\right)}\right) \end{aligned} \tag{C2}$$

First Order Conditions of Households:

The households choose the following variables: $c_{t,1}, h_{t,1}, l_{t,1}, n_{c,t}, n_{h,t}, c_{1+t,2}, h_{1+t,2}, l_{1+t,2}$. The utility maximization of households have the first-order conditions as follows:

$$\frac{\tilde{q}_t}{\tilde{c}_{t,1}} = \frac{j_h}{\tilde{h}_{t,1}} + \frac{\beta(1 - \delta_h)\, exp\left(g_{q,t+1}\right) \tilde{q}_{1+t}}{\tilde{c}_{1+t,2}} \tag{C3}$$

$$\frac{\tilde{q}_{1+t}}{\tilde{c}_{1+t,2}} = \frac{j_h}{\tilde{h}_{1+t,2}} \tag{C4}$$

$$\frac{\tilde{p}_{l,t}}{\tilde{c}_{t,1}} = \frac{j_l}{\tilde{l}_{h,t,1}} + \frac{\beta\, exp\left(g_{pl,t+1}\right) \tilde{p}_{l,1+t}}{\tilde{c}_{1+t,2}} \tag{C5}$$

$$\frac{\tilde{p}_{l,1+t}}{\tilde{c}_{1+t,2}} = \frac{j_l}{\tilde{l}_{h,1+t,2}} \tag{C6}$$

$$\frac{(1-T)\widetilde{w}_{c,t}}{\tilde{c}_{t,1}} = \tau \tilde{n}_{c,t}^{\xi} \left(\tilde{n}_{c,t}^{1+\xi} + \tilde{n}_{h,t}^{1+\xi}\right)^{\frac{\eta-\xi}{1+\xi}} \tag{C7}$$

$$\frac{(1-T)\widetilde{w}_{h,t}}{\tilde{c}_{t,1}} = \tau \tilde{n}_{h,t}^{\xi} \left(\tilde{n}_{c,t}^{1+\xi} + \tilde{n}_{h,t}^{1+\xi}\right)^{\frac{\eta-\xi}{1+\xi}} \tag{C8}$$

Firms:

$$\tilde{y}_t = A_{c,t} \left(\frac{\tilde{k}_{c,t-1}}{n_t}\right)^{\mu_c} \tilde{n}_{c,t}^{1-\mu_c} \tag{C9}$$

$$\tilde{\mathrm{ih}}_t = A_{h,t} \left(\frac{\tilde{k}_{h,t-1}}{n_t}\right)^{\mu_h} \left(\frac{\tilde{l}_{e,t-1}}{n_t \exp\left(g_{l,t+1}\right)}\right)^{\mu_l} \tilde{n}_{h,t}^{1-\mu_h-\mu_l} \tag{C10}$$

$$\tilde{k}_{c,t} = (1-\delta_{\mathrm{kc}}) \frac{\tilde{k}_{c,t-1}}{n_t} + \tilde{\mathrm{ik}}_{c,t} \tag{C11}$$

$$\tilde{k}_{h,t} = (1-\delta_{\mathrm{kh}}) \frac{\tilde{k}_{h,t-1}}{n_t} + \tilde{\mathrm{ik}}_{h,t} \tag{C12}$$

$$\tilde{\phi}_{c,t} = \phi_c\left(\tilde{k}_{c,t}, \tilde{k}_{c,t-1}\right) = \frac{\phi_{kc}}{2} n_t \left(\frac{\tilde{k}_{c,t}}{\tilde{k}_{c,t-1}} - 1\right)^2 \tilde{k}_{c,t-1} \tag{C13}$$

$$\tilde{\phi}_{h,t} = \phi_h\left(\tilde{k}_{h,t}, \tilde{k}_{h,t-1}\right) = \frac{\phi_{kh}}{2} n_t \left(\frac{\tilde{k}_{h,t}}{\tilde{k}_{h,t-1}} - 1\right)^2 \tilde{k}_{h,t-1} \tag{C14}$$

Here, $\exp(g_{KC,t})$ and $\exp(g_{KH,t})$ are equal to the fertility rate n_t because $g_{KC,t} = g_{kc,t} + \gamma_{n,t}$ and $g_{kc,t} = 0$ according to the above discussion.

$$\begin{aligned} &\tilde{d}_t + \tilde{k}_{c,t} + \tilde{k}_{h,t} + \widetilde{w}_{c,t}\tilde{n}_{c,t} + \widetilde{w}_{h,t}\tilde{n}_{h,t} + \tilde{p}_{l,t}\tilde{l}_{e,t} + \tilde{\phi}_{c,t} + \tilde{\phi}_{h,t} = \\ &\tilde{y}_t + \tilde{q}_t\, \tilde{\mathrm{ih}}_t + (1-\delta_{\mathrm{kc}}) \frac{\tilde{k}_{c,t-1}}{n_t} + (1-\delta_{\mathrm{kh}}) \frac{\tilde{k}_{h,t-1}}{n_t} + p_{l,t} \frac{\tilde{l}_{e,t-1}}{n_t \exp\left(g_{l,t}\right)} \end{aligned} \tag{C15}$$

First Order Conditions of Firms:

The firms will decide on the following variables: $k_{c,t}, k_{h,t}, d_t, n_{c,t}, n_{h,t}, l_{e,t}$. The first order condition with respect to $k_{c,t}$ is:

$$\frac{\beta_e \left(\frac{1-\delta_{\mathrm{kc}}}{n_{t+1}} - \tilde{\phi}^{(0,1)}\left(\tilde{k}_{c,t+1}, \tilde{k}_{c,t}\right) + \tilde{r}_{c,t+1}\right)}{\tilde{d}_{t+1}} = \frac{\frac{1}{A_{k,t}} + \tilde{\phi}^{(1,0)}\left(\tilde{k}_{c,t}, \tilde{k}_{c,t-1}\right)}{\tilde{d}_t} \tag{C16}$$

Where:

$$\tilde{r}_{c,t+1} = \tilde{y}^{(1,0)}\left(\tilde{k}_{c,t}, \tilde{n}_{c,t+1}\right) = \mu_c \frac{\tilde{y}_{t+1}}{\tilde{k}_{c,t}} \tag{C17}$$

$$\tilde{\phi}^{(0,1)}\left(\tilde{k}_{c,t+1},\tilde{k}_{c,t}\right) = -\frac{\phi_{kc}n_{t+1}}{2}\left(\left(\frac{\tilde{k}_{c,t+1}}{\tilde{k}_{c,t}}\right)^2 - 1\right) \qquad (C18)$$

$$\tilde{\phi}^{(1,0)}\left(\tilde{k}_{c,t},\tilde{k}_{c,t-1}\right) = \phi_{kc}n_t\left(\frac{\tilde{k}_{c,t}}{\tilde{k}_{c,t-1}} - 1\right) \qquad (C19)$$

Similarly, the first order condition with respect to $k_{h,t}$ is:

$$\frac{\beta_e\left(\frac{1-\delta_{\mathrm{kh}}}{n_{t+1}} - \tilde{\phi}^{(0,1)}\left(\tilde{k}_{h,t+1},\tilde{k}_{h,t}\right) + \tilde{r}_{h,t+1}\right)}{\tilde{d}_{t+1}} = \frac{1+\tilde{\phi}^{(1,0)}\left(\tilde{k}_{h,t},\tilde{k}_{h,t-1}\right)}{\tilde{d}_t} \qquad (C20)$$

Where:

$$\tilde{r}_{h,t+1} = q_{1+t}\iota\tilde{h}^{(1,0,0)}\left(\tilde{k}_{h,t},\tilde{n}_{h,t+1},\tilde{l}_{e,t}\right) = \mu_h\frac{\tilde{q}_{1+t}\iota\tilde{h}_{t+1}}{\tilde{k}_{h,t}} \qquad (C21)$$

$$\phi^{(0,1)}\left(\tilde{k}_{h,t+1},\tilde{k}_{h,t}\right) = -\frac{\phi_{kh}n_{t+1}}{2}\left(\left(\frac{\tilde{k}_{h,t+1}}{\tilde{k}_{h,t}}\right)^2 - 1\right) \qquad (C22)$$

$$\phi^{(1,0)}\left(\tilde{k}_{h,t},\tilde{k}_{h,t-1}\right) = \phi_{kh}n_t\left(\frac{\tilde{k}_{h,t}}{\tilde{k}_{h,t-1}} - 1\right) \qquad (C23)$$

The first order condition with respect to $\tilde{l}_{e,t}$ is:

$$\frac{\tilde{p}_{l,t}}{\tilde{d}_t} = \frac{\beta_e\, exp\left(g_{pl,t+1}\right)\left(\frac{\tilde{p}_{l,1+t}}{n_{1+t}} + \tilde{r}_{l,t+1}\right)}{\tilde{d}_{1+t}} \qquad (C24)$$

Here,

$$\tilde{r}_{l,t+1} = \tilde{q}_{1+t}\mathrm{\iota}\tilde{\mathrm{h}}^{(0,0,1)}\left(\tilde{k}_{h,t},\tilde{n}_{h,1+t},\tilde{l}_{e,t}\right) = \mu_h\, exp\left(g_{pl,t+1}\right)\,\frac{\tilde{q}_{1+t}\iota\tilde{h}_{t+1}}{\tilde{l}_{e,t}} \qquad (C25)$$

e first order conditions with respect to $n_{c,t}$ and $n_{h,t}$ are:

$$\tilde{w}_{c,t} = (1-\mu_c)\frac{\tilde{y}_t}{\tilde{n}_{c,t}} \qquad (C26)$$

$$\tilde{w}_{h,t} = (1-\mu_h-\mu_l)\frac{\tilde{q}_t\iota\tilde{h}_t}{\tilde{n}_{h,t}} \qquad (C27)$$

Equilibrium:

$$\tilde{y}_t = \tilde{c}_t + \mathrm{\iota}\tilde{\mathrm{k}}_{c,t} + \mathrm{\iota}\tilde{\mathrm{k}}_{h,t} + \tilde{\phi}_t \qquad (C28)$$

$$\tilde{h}_t = \mathrm{\iota}\tilde{\mathrm{h}}_t + (1-\delta_h)\frac{\tilde{h}_{t-1}}{n_t\exp\left(g_{h,t}\right)} \qquad (C29)$$

$$\tilde{l}_t = \tilde{l}_{h,t} + \tilde{l}_{e,t} \qquad (C30)$$

, where $\tilde{c}_t = \tilde{c}_{t,1} + \frac{\pi_t}{n_t}\tilde{c}_{t,2}$, $\tilde{h}_t = \tilde{h}_{t,1} + \frac{\pi_t}{n_t}\tilde{h}_{t,2}$, $\tilde{l}_{h,t} = \tilde{l}_{h,t,1} + \frac{\pi_t}{n_t}\tilde{l}_{h,t,2}$.

Appendix D. Robustness Check

In this section, we will examine the robustness of long-term effect of an increase in longevity on housing prices. Specifically, we change the calibrated parameter values to assess their effects on the numerical outcomes. More concretely, each calibrated parameter incremented by 10 percent and the effects on house (and land) prices assessed. To make the results comparable to the original one, the same demographic change would be applied, i.e. survival rate rise from 0.8 to 0.9. If the above results about longevity increase is robust, then the parameter changes should only make modest changes on the result.

For the land price, the check for robustness is shown in Figure D1. Comparing with the original result, which is shown at the left most, the price changes are modest. The biggest difference lay on the time preference β and tax rate T, however, none of their effects is bigger than 2 percent. Thus, the numerical solution of land price changes is considered to be robust.

Similarly, the check for robustness of house price to changes in longevity is shown in Figure D2. Like that of the land price, the longevity increase effect on house price would only be affected modestly by parameter value changes. Therefore, the result on the house price is also robust in this circumstance.

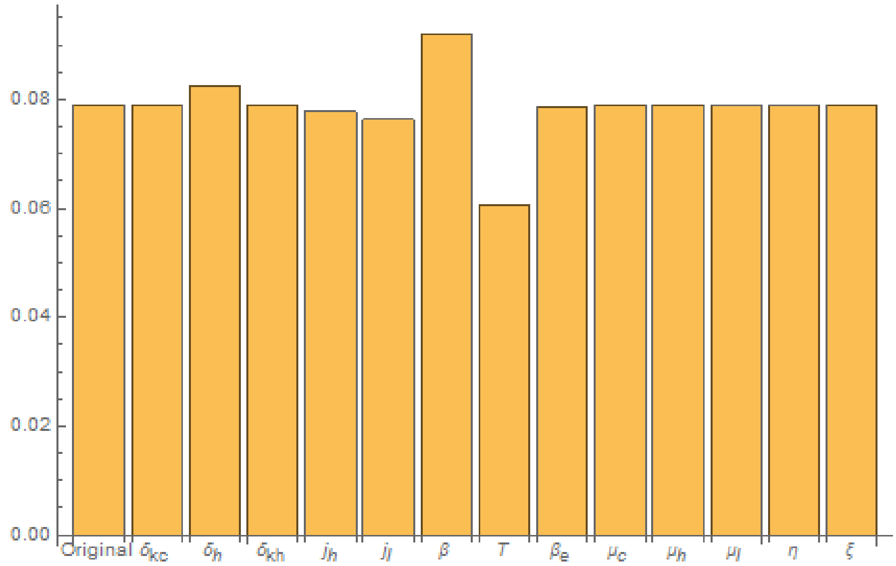

Figure D1. Robustness check: Effect of Longevity Increase on Land Price

Notes: From left to right, the first bar in chart is the land price change when parameters are calibrated to their original values. Then, we change the parameter values one by one. Each parameter is increased by 10 percent comparing with their original values. For example, we could increase the depreciation rate of houses by 10 percent, and the other parameter values are unchanged. In this situation, the effect of longevity increase on land price is shown in the third bar.

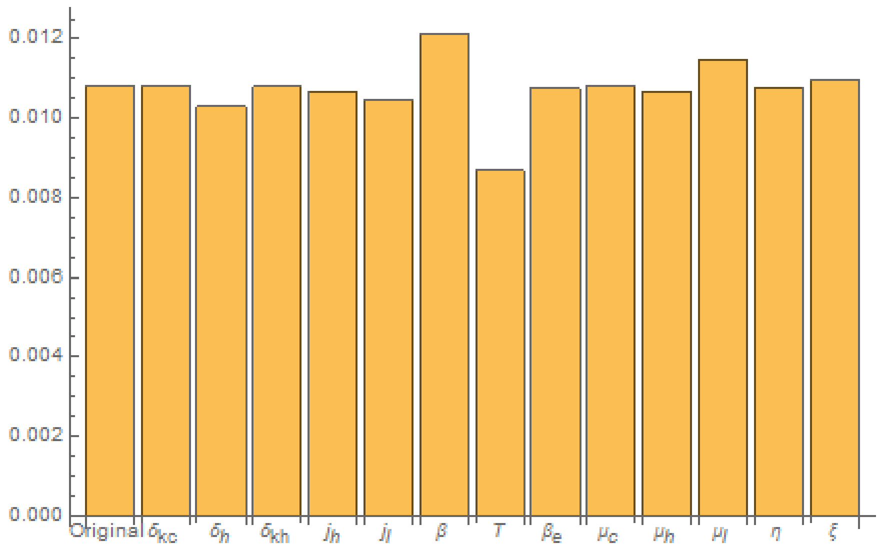

Figure D2. Robustness check: Effect of Longevity Increase on House Price

Notes: From left to right, the first bar in chart is the house price change when parameters are calibrated to their original values. Then, we change the parameter values one by one. Each parameter is increased by 10 percent comparing with their original values. For example, we could increase the depreciation rate of houses by 10 percent, and the other parameter values are unchanged. In this situation, the effect of longevity increase on house price is shown in the third bar.

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