Capital, technical progress and international trade

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An oddity of economic theory nowadays is that the disputes in the 1950s and 1960s over a fundamental element of economic theory, the concept of capital, should not only have been virtually forgotten, but that its unambiguous outcome should have been ignored by what is now the mainstream. Few who teach or write about economic theory now are aware that there had been a dispute and, apart from some small circles, those who are aware mostly think of it as an obscure quarrel from a bygone era.

At issue was whether or not the conventional representation of production by a function of quantities of capital, labour and, perhaps, other factors could be justified, and the outcome was that it could not, except with restrictions that rendered it useless. As a matter of logic, the outcome was unambiguous. Perhaps issue and outcome would have been more widely known if more had been done to explain the implications. After all, Sraffa, who first discerned the issue, gave his book[1] the subtitle, "Prelude to a Critique of Economic Theory", which, in spite of its apparent modesty, shows his awareness that it concerned the foundations of economic theory. But the Prelude has not been followed by a systematic critique. Those who had taken part in the discussions, understood them best and had made the crucial points seem not to have taken the matter further.

This paper's purpose is to demonstrate further that the issue is fundamental to economic theory by showing that there are two more reasons for the outcome. One is technical progress. It is an elaboration of a point made by Pasinetti[2] in 1959, that technical progress has to include the production of capital as well as final output, a point taken up later by Read[3] and Rymes[4]. The other is international trade; if capital goods are tradable and can be made in different countries with different wage and profit rates, there can be no such thing as a quantity of capital without first specifying the pattern of trade and the costs of all countries. Section 1 is an attempt to provide a concise, clear exposition of the original disputes in a way that makes a logically complete case. Much of it has to be examination of several fallacious arguments; if there had not been any, there would have been no dispute. Section 2 is devoted to technical progress. It is intended to show that representing production by functions of capital and other factors cannot provide realistic or plausible descriptions or explanations of technical progress, but that a proper representation can. Section 3, devoted to international trade, is shorter; it sets out the objection just mentioned to quantifying capital and, therefore, to the Heckscher-Ohlin theory, and shows how a proper representation of production naturally connects trade to technical progress and, thence, how competitiveness is different to economic viability.

I. Capital Goods and Determinacy

[1] Sraffa, "Production of Commodities by Means of Commodities: Prelude to a Critique of Economic Theory."
[2] Pasinetti, "On Concepts and Measures of Changes in Productivity."
[3] Read, "The Measure of Total Factor Productivity Appropriate to Wage-Price Guidelines."
[4] Rymes, *On Concepts of Capital and Technical Change*.

Economic theory and depressions

Piero Sraffa seems to have been alone among economists before the Great Depression, if some of Marx's followers are left aside, in believing that there was some flaw common to the various economic theories of his time. According to the various schools the economic system was inherently stable; slumps and booms could occur, but, left to itself, every economy reverted to the full employment of its labour and productive capacity. Thus, Marshall could describe a crash and the argue that recovery would follow quickly, Wicksell could attribute unemployment to the wrong choice of interest rates, Walras could practically ignore the subject and Schumpeter maintain that the disruptions caused by inventions were temporary. Crashes and slumps were familiar to all, but the very term used for their discussion, the "trade cycle", implied that they were transient. Jevons's speculation, that sunspots, which are cyclical, could have something to do with the trade cycle, was in this spirit.

Cyclical is a misleading term for the prolonged periods of depression and unemployment before the First World War and more so for the years immediately after. There was good reason to believe that something was wrong with the prevailing theories. It had to be with the theories, for the politicians in power could not be blamed; they had never questioned the orthodoxy before the War and in the new state of affairs after the War they sought and followed the guidance of the experts (but not Keynes).

Sraffa seems to have thought for a while that he had found that "something" in the neglect of the implications for perfect competition of increasing returns. In his paper of 1926[5] he argued that supply and demand could not be considered symmetrical in determining prices, as with Marshall's analogy of the two blades of scissors. Sraffa did not mention the analogy, but argued, first, that the primary determinant of price was the cost of production and, second, that the perfect competition of atomistic producers, none of whom could affect prices, had to be replaced by competition among quasi-monopolies. Increasing returns at the level of the factory were normal and firms producing competing goods could reduce their costs of production by expanding. But more sales by one firm entailed less sales by the others, so, if firms were restrained from expansion, it was by the need to spend on marketing and by the reactions to be expected of the competition. Even with constant returns there was nothing to stop individual atomistic producers from expanding until they could affect prices. Competition consisted, then of producers differentiating and branding their goods to become quasi-monopolists of their brands.

This argument had some success in that it was followed by several works on imperfect or monopolistic competition, notably the books by Robinson[6] and Chamberlin[7]. But it did not result in the kind of critique of economic theory Sraffa seems to have had in mind. One reason can be seen from the mathematical representation of production by production functions with labour and capital as arguments. Wicksteed had pointed out that, if factors are paid their marginal products, as competition demands, the returns to scale must be constant, at least at the factory level. Increasing returns result in the pay exceeding the output and decreasing returns leave a surplus. Returns to scale are not necessarily constant and when they are not there is a mismatch between factor payments and the value of production. So, is there to be one theory of prices when returns to scale are constant and another when they are not? Wicksteed's point shows that something else is wrong with the theories and that the

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[5] "The Laws of Returns under Competitive Conditions."

[6] Robinson, *The Economics of Imperfect Competition*.

[7] Chamberlin, *The Theory of Monopolistic Competition*.

concern with increasing returns is more concern with symptoms than with causes. Sraffa knew Wicksteed's point, though he did not need to refer to it explicitly in his paper.

The concept of capital and indeterminacy

Eventually Sraffa found that "something else" and explained it in his book, "The Production of Commodities by Means of Commodities". It was the notion of capital as a factor of production. In this paper the discussion is limited to capital as produced goods used for production. Natural resources and investments in exploiting and improving them count as capital and are accommodated in Sraffa's schema, but were, for simplicity, left out of the disputes and are left out here. Education, too, counts as capital and is left out, because it is imparted to people, and people cannot be treated like machines because of an investment in them. Capital, considered as objects used for production in the production of which the state and firms invest, consists, therefore, of a variety of goods; some, like infrastructure, such as roads and ports, often built without a specific duration in mind; some, like much of the machinery and rolling stock of factories, expected to last a few years, and others, like inventories of raw materials and finished products, turned over in days or weeks.

If the quantity of capital is the value of the produced goods used for production, the common argument that profits and wages are determined by the supply of and demand for capital and labour cannot be used. At any moment the physical capital and its cost are the result of the wage and profit rates of the past and the expectations, when the investment decisions were made, of what those rates and prices would be later, and its economic value depends on what these quantities are expected to be in the future. This is obvious if the physical capital includes durable products that cannot be transferred between uses except at costs that cannot be ignored, what will be termed here as "heterogeneous". It holds as well if the production process is represented as using only goods that are used up in the process and have no durable capital, provided the goods used in the production process are heterogeneous and the process takes time, which is how Sraffa began his exposition.

This contradicts the common argument that an additional unit of capital with the same amount of labour increases output by the marginal product of capital and that this determines the rate of profit. By the same common argument, competition causes the labour used with a given quantity of capital to be paid the value of the output that would be produced by an additional unit of labour and the same amount of capital, labour's marginal value product. It is as though the stock of capital were a malleable quantity and that production were a function of the quantities of capital and labour. The partial derivative of the function with respect to capital is the marginal product of capital and that with respect to labour is the marginal product of labour. Production alone determines wage and profit rates in a competitive economy, for they adjust to the marginal products of the given quantities of labour and capital so that both are fully employed. Both wage and profit rates, which really determine the value of the capital stock, are supposedly determined by the supplies of labour and capital.

Sraffa demonstrated that, when production is considered alone, there is an indeterminacy: either the wage or the profit rate must be given for the other, hence prices, to be determined. He depicted production as a self-contained system, unchanging through time, each round of production being labour working on produced goods, which are used up in the production process and are, therefore, circulating capital. In this system the price of any good is the sum of the wage cost, the cost of the circulating capital and the profit on it. Each round's cost of the circulating capital is calculated from the prices of the goods from the previous round, which, in a static system, gives the prices of goods as functions of the nominal wage and the profit rate by inverting a matrix. Whatever is taken as the numeraire or standard of value, there is one degree of freedom, meaning that something must be given from outside the system. It can be the real wage, if an unambiguous measure of it exists, and it can be the profit rate, and the two are inversely related lying anywhere between a zero wage and zero profit. As long as both wage and profit enter into the cost of production of capital goods there will be indeterminacy.

Durable, heterogenous capital goods were left out to simplify the reasoning, but can be included by allowing joint production, a device, as Sraffa mentioned, first used by Torrens; each time a capital good is used to produce a given good it becomes the same capital good one period older as a joint output with the good it was used to produce, until it reaches the end of its use. Wage and profit rates are still inversely related, but the mathematics is more complicated, because capital goods of different ages are different goods and each activity has, therefore, correspondingly many outputs. Whether capital goods are durable or not, their prices include both profit and wages. In none of this is there any need to assume constant returns and the process through infinite time can be replaced by a beginning in which the industries are put in place by some other technique of production, which adds to the complication without changing the conclusion.

Disputes about what was meant by capital, how to value it and how it was related to production, prices, wages and profit were nothing new. There is no need to describe here the various schools of thought beyond mentioning that Wicksell had already argued that the profit rate was equal to the marginal product of waiting, which was not equal to the marginal product of capital, and had shown that a change of wage rate could alter both the prices of any capital goods and the composition of the capital stock. What was new was the indeterminacy, and Sraffa seems to have been the first to spot it, or, at least, to state it explicitly. A letter Sraffa wrote to Joan Robinson in 1936[8] seems to show that he had begun to think along these lines by then. Perhaps, in addition to his earlier questioning of constant returns, it was the discussions of Keynes and his circle, as they tried to see how theory could explain prolonged unemployment, that led Sraffa to the question of how manufactures were priced when the capital equipment for producing them was not fully used and from that to his final argument. But it took twenty-three more years for his book to appear.

Robinson's objections to aggregate capital and production functions

By then the controversy had begun. Joan Robinson's article "The Production Function and the Theory of Capital" had appeared in 1953 and was a criticism of the notion of the production function with capital as one of the arguments. She assumed a closed economy that could use any of a given set of discrete production techniques, along with the usual assumptions of competition, constant returns to scale and an unambiguous measure of the real wage. Comparing stationary states, there is for a given wage a technique that yields the highest rate of profit and each technique, taken alone, gives an inverse relation between wage and profit rates ranging from a zero wage to a zero profit rate. Each technique can be assumed to give the highest profit rate for a range of values of the real wage and at either end of this range, if both wage and the profit rate are positive, some other technique yields the same wage and profit rates as the given technique and then yields the highest profit rate for an adjacent range of values of the real wage.

Robinson posed the question as to how capital was to be valued for each technique. Since capital goods are made with the use of capital goods and take time to make, the value of the

[8] Cohen and Harcourt. (p.203)

capital stock consists of a profit and a wage component. Robinson took the wage as the unit of value, so that the value of a given capital stock is lower when the real wage is higher and the profit rate lower. Other ways of providing a unit of value (normalising prices) are to take a good or a basket of goods as the unit or numeraire.

Regardless of what is chosen as numeraire, when two different techniques yield the same wage and profit rates, the one with a greater value of capital per worker has the greater output per worker and the ratio of the difference of output per head to the difference of capital per head is the rate of profit. These two techniques can be used simultaneously in different proportions and it follows that, comparing different proportions, the profit rate continues to be the ratio of the differences of output per head to the differences of total capital per head. Usually the technique giving the greater output per head is expected to yield a lower rate of profit, but it does not have to; a technique can yield a higher wage and lower rate of profit than another and have a lower value of capital per head (capital intensity reversal). There is also the possibility that a technique yields the highest rates of profit for one range of real wages and does so again for another range, with other techniques being used in between, what is referred to as "reswitching". If the wage is the unit of value, the function relating output per head to capital per head is not as usually depicted, a curve with a positive but decreasing slope, even if techniques giving more output per head are assumed to have more capital per head, for, in the range that a given technique is used, as the real wage rises and the profit rate falls the value of the capital stock falls.

From this Robinson argued that 'the comparison between equilibrium positions with different factor ratios cannot be used to analyse changes in the factor ratio taking place through time, and it is impossible to discuss changes (as opposed to differences) in neo-classical terms'[9] and that it does not follow, as in the neo-classical doctrine, 'that the level of wages determines the amount of employment, and that, when unemployment occurs, workers (unless frustrated by the misguided policy of trade unions) offer themselves at a lower real wage rate than that ruling, and go on doing so till all are employed', even though, given the techniques of production available and 'the quantity of capital (in terms of product), there is one value of the wage rate which is compatible with full employment of any given labour force'[10].

Controversy over capital

After that the controversy concerned two questions, though they were often discussed together. One, the narrower one, was if there were ways around the problems of quantifying or aggregating capital. The other, broader controversy was whether economic theory would have to dispense with production functions and aggregate capital and what the implications could be, but did not extend to technical progress or international trade.

Several efforts to qualify Robinson's argument followed, the only logically sound one being that of Champernowne, which appeared as a "Comment" along with Robinson's article.[11] Champernowne showed that a measure of the quantity of capital and a production function with the desired properties of partial derivatives equal to wage and profit rates can be constructed by comparing stationary states and using a chain index. He assumed a single consumption good and constructed his index from the value, as given by Robinson's method, of the capital per worker of each technique at the lowest wage of the range in which it can be

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[9] Robinson, "The Production Function and the Theory of Capital," 100.)
[10] Robinson, 100.
[11] Champernowne, "The Production Function and the Theory of Capital: A Comment."

used and obtained his index from the ratio of this value of capital for each technique to the value of the preceding technique when all techniques used are ranked in order of rising wages and one technique is chosen as the base. When two techniques can be used at the same wage and profit rates they can be used simultaneously in any proportion and the amount of capital is the sum of the amounts of capital per worker multiplied by the number of workers used with each technique. Champernowne gave examples to show that capital intensity reversal and reswitching are possible and that the latter must be excluded by assumption for a consistent measure of the quantity of capital.

Champernowne's chain index does not purport to be usable for actual changes between different steady states; it only compares one steady state with another. Hence, it cannot be used as a production function as commonly done to describe actual changes or to explain cause and effect. It gives the appearance of a production function, as long as there is no reswitching, and has, what Champernowne, himself, and some others considered an advantage, the property that a given stock of capital has the same value whatever the wage and profit rates, again provided there is no reswitching. This last property is not necessarily an advantage; it seems to relate capital as a quantity to the physical stock, whereas profit maximisation is in theory related to the cost.

Later, in 1955, Solow posed the question as an index number problem; how to aggregate the services of different types of capital into one index.[12] This was to misstate the problem, as Robinson pointed out.[13] Her criticism of the notion of aggregate capital began with the observation that no unit for measuring the quantity of capital had been specified, except value, which depends on prices and, therefore, on the profit and wage rates it is supposed to determine. The question is about how much capital there is in a capital good and presenting it as an index number problem ignores that.

Robinson's point is elaborated on here because it is sometimes misunderstood. Solow presented his index number at first as the question, what are the conditions under which two or more variables in a function can be combined as one variable without altering the function? Solow referred to a theorem by Leontief that gives the necessary and sufficient conditions, which are that the partial derivatives of the function with respect to these variables should be independent of any other variables, a condition too restrictive to be considered to occur normally.

Solow's procedure was not what would have been expected; it was to assume a production function with, not capital goods, but two or more types of capital services. Flows of service of capital goods are presumably quantities independent of prices and Solow's production function represents production as a flow coming from flows. Solow may seem to avoid capital goods, but, then, what restricts the size of the flows? Presumably it is the stocks of capital goods, each of which has a capacity that limits the service it can provide in a period. For example, the services can be provided in fixed proportions to the capacities of the capital goods, say to full capacity, in which case the capacities are known from the flows of services. A production function with the services of capital goods, rather than the goods, themselves, as variables has, therefore, to be accompanied by information on the capacities of the capital goods and the proportions of the capacities used. Under competitive conditions the marginal value product of a service equals the price of that service. In equilibrium the price of a service yields the profit on the associated capital good, of which the capacity is known, so that the cost of the capital good can be calculated from the prices of services,

[12] "The Production Function and the Theory of Capital."

[13] Robinson, "The Production Function and the Theory of Capital--A Reply."

themselves calculated from the prices of the goods produced. Or, the prices of the services of capital goods and the goods produced can be calculated from the rate of profit and the values of the stocks of capital goods. Solow does not explain how the rate of profit is determined by a production function that only has services as inputs.

Solow does not eschew capital in this paper; he says earlier, 'For many purposes it is remarkably useful to assume that there exists only one physical commodity which can either be consumed or used as capital in the production of more of itself. Then Q and C are measured in the *same* units except that Q is a flow and C is a stock.'[14] Q is the output of the production function and C capital, which he says is measured in "unambiguous physical units", though what the unit is remains unstated. If capital goods are put in place of services in Solow's argument above, each type of capital good is a variable in his production function. Assuming indivisibilities and divergence from constant returns to scale away, there can be blast furnaces of all sizes and there is a measure that indicates the amount of blast furnace, the unit of measurement being a standardised blast furnace. The same applies to a bridge over a river in a particular place, winches, looms and lathes of various kinds in factories and all other forms of capital stock. Complementarities among items would have to be allowed for; more blast furnace requires more hopper, more storage space for inputs and outputs, more equipment for conveyance, etc. which, together, give more steel mill.

Solow's argument seems to lead to the conclusion that, with a stretch of the imagination, the index number problem can be avoided by making all capital goods arguments of macroeconomic production functions. But the same index number problem makes aggregation of separate production units impossible except under special conditions. As explained by Felipe and Fisher[15], the mathematical restrictions on the production functions of the individual production units needed to allow the units to be aggregated into a production function in which the endowments of factors are the sums of the endowments of the units are too restrictive to be thought of as likely to be met or to be construed as representing a genuine economy. Assuming factors to be malleable, therefore, prevents the use of production functions for the whole economy if the same factors are used by the different production units, which they presumably are in different proportions. In contrast, production units can be aggregated if, instead of malleable factors, the capital goods are conceived of as heterogeneous manufactures, as long as indivisibilities and divergences from constant returns can be assumed away.

Movements along Solow's production function are assumed to incur no costs, as if capital stock were malleable. This is standard practice going back to J. B Clark, at least, and Solow's assessment of it is: 'The kernel of useful truth in' Clark's 'picture of capital as a kind of jelly that transforms itself over time is that indeed, over time something like this does happen as capital goods wear out and are replaced by different capital goods'[16] Since the capital goods that replace those worn out have to be made and have a cost, the simile of a jelly, as if the same goods were somehow transformed at no cost, is a bad one. Installing new capital goods is an investment separate from the investment in the goods wearing out and does not necessarily occur.

Various ways of ascribing a quantity other than value to capital were devised, none cogent. Solow, himself, presented one model in which machines are made by labour alone and another in which capital goods are made from a mass of putty, which, once formed into

[14] "The Production Function and the Theory of Capital," 101. Italics in original.
[15] "Aggregation in Production Functions: What Applied Economists Should Know."
[16] Solow, *Capital Theory and the Rate of Return*, 27.

machines, is unalterable, and Swan proposed that machines be made from a set of elementary components, each of which embodies a quantity of capital. In these no capital is used for forming capital goods, so that they are ways of having a quantity of capital without profit in its cost of production and, since different capital goods can be made from the same labour, putty or components without other costs, they are various forms of malleable capital.

Somewhat later Samuelson proposed the notion of "surrogate capital", a quantity that could be deduced from a model he devised and yielded a production function with the properties desired.[17] His model had a single consumption good and several types of durable machines, each of which could, in combination with a fixed amount of labour, produce the consumption good or machines of the same type. The highest rate of profit, given the real wage, (or *vice versa*) that could be obtained in each stationary state defined the "factor price frontier", which could be approximated by a production function with surrogate capital. There would be no need for approximation if the machine types were a continuum. But he also pointed out in the same paper that Garegnani had shown that the condition for surrogate capital is that each type of machine use the same amount of labour for producing the consumption good as it uses for making the machine. Garegnani showed later that the condition is necessary and sufficient and, in effect, results in a single good economy, for, in stationary states, the machines are just intermediate steps in the production of the consumption good by itself and labour.[18]

Finally, the first question of the controversy was considered settled after Levhari had claimed to have proved the assertion that, if production can be described by an indecomposable matrix, meaning that every good enters directly or indirectly into the production of every good, reswitching cannot occur for the whole matrix.[19] Pasinetti showed that there was a mistake in the proof and soon several counterexamples were given to show the assertion to be wrong.[20] Champernowne's production function and the associated measure of capital can be used to compare steady states if it is known there is no reswitching, but the common practice of using production functions with capital as an aggregate quantity to describe change over time has no justification.

Attempts to remove the indeterminacy: fallacies with capital and production functions

With the first question settled, the second question of the implications for economic theory became, what was the critique to follow the prelude? Several economists have claimed that no critique need follow, for, beyond showing the need for care with some simplifications, the point at issue does not affect the validity of neoclassical economic theory or of the applicability of marginal costs. They have made various attempts to justify the claim, all either fallacious or mathematical abstractions remote from reality.

First are arguments intended to remove indeterminacy, but which proceed by assuming what is in dispute. Of these the earliest seems to have been Solow's attempt, in his De Vries lectures of 1963, to show that conclusions drawn from assuming malleable capital can be reached without that assumption and that a rate of return on capital can be calculated from a

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[17] Samuelson, "Parable and Realism in Capital Theory: The Surrogate Production Function."

[18] Garegnani, "Heterogeneous Capital, the Production Function and the Theory of Distribution."

[19] "A Nonsubstitution Theorem and Switching of Techniques."

[20] "Switches of Technique and the 'Rate of Return' in Capital Theory."

reduction of consumption in one period to allow an increase in capital goods so as to yield an increase of consumption the next period. Beginning from an efficient allocation, so that the production of no good could be increased without reducing the production of some other good, he considered a reduction of consumption and asserted 'Because all the allocations considered are efficient, those which produce less consumption must also produce more of at least some kinds of capital goods'.[21] Here the logic is faulty. He did not and could not assume all allocations to be efficient. Simply reducing consumption results in an inefficient allocation if the capital goods producing the consumption cannot be used for producing capital goods. Solow is either tacitly assuming that capital is malleable, though he states in the same work the assumption is 'obviously absurd' and is not needed for 'neo-classical capital theory'[22], or he is assuming that the labour released by reducing consumption can make capital goods without using capital goods or that the existing stock of capital goods was not being used to capacity and the initial state was inefficient.

Solow purports to give an example that does not have malleable capital but shows how 'in competitive equilibrium the rate of interest must equal the rate of return on investment'[23], the example being a model of Worswick, in which capital goods are assumed to be made with labour alone. He claims the assumption is a simplification 'without being in the least necessary'(Hahn 1982, 31), though, as already mentioned, it removes profit from the cost of capital and allows capital to be quantified as an amount of labour. Böhm-Bawerk already knew better.

Later, in 1982, Hahn tried to argue, firstly, that Sraffa's static model is merely a special case of more general neoclassical theory and adds nothing that cannot be obtained from the latter and, secondly, that Sraffa was wrong to assume a uniform rate of profit in a static system and thus to restrict unduly what should be an inter-temporal equilibrium. Hahn points out that Sraffa's main argument is made with a model that has one technique of production and is adapted to several techniques only towards the end of the book, whereas neoclassical theory allows for an unlimited number of techniques, and he asserts that, correctly formulated, the standard marginal relations still hold.(Hahn 1982)

For his first argument Hahn used a model with two goods and a choice of production activities, each activity using both goods and labour to produce one of the goods. Different techniques of production use the two goods and labour in different proportions. Taking at first a single period and assuming that the production techniques can be represented by differentiable functions of the inputs of the goods and labour, there is a consistent set of equations relating the marginal products to the prices and wage, given the rate of profit.

From this Hahn concludes that, 'under assumptions no more stringent that Sraffa's'(Hahn 1982, 360), where 'you cannot get more neoclassical than differentiable' functions representing production techniques, the rate of profits is still not determined though '*every* possible marginal product had been used'(Hahn 1982, 361. Italics in original.). One more equation is needed for complete determinacy, that is to determine the profit rate, just as with Sraffa, and that implies 'the meaninglessness of a sentence like: 'the marginal product of labour determines the real wage''(Hahn 1982, 361) since all the equations must be solved simultaneously.

[21] *Capital Theory and the Rate of Return*, (p.18).
[22] Solow, (p.26).
[23] Solow, (p.33).

One of Hahn's assumptions, however, is more stringent than Sraffa's and that is that the same goods are used in different combinations to produce the same goods. If one of them were to be called "capital" and assumed not to be consumed, the assumption would be seen to be that of the production function with malleable capital that Sraffa had criticised plus a second input. Hahn avoids the term "production functions" when referring to the functions representing the production techniques, though that is what they are. This assumption also provides the equation needed to determine the rate of profit by the standard neoclassical argument that competition results in the full employment of labour; the wage falls if there is unemployment and rises if there is a shortage of labour until it and the marginal product of labour are equal. In Hahn's model, therefore, the marginal product of labour does determine the real wage, even if Hahn, himself, omits this piece of neoclassical theory.

So, when Hahn refers to the problem of finding a measure of the capital of his model, assuming both goods are inputs into their own production, he merely restates Solow's index number problem. To obtain his measure of capital he modifies his model for a two period variant with an initial endowment of the two goods, which, since it is used for production, can be thought of as capital and a function of it 'may be thought of as a measure of capital stock'(Hahn 1982, 361. Footnote). But, knowing the value of the function 'would not allow us to 'determine' equilibrium',(Hahn 1982, 369) since different combinations of the two goods can give the same value of the function, but different equilibria. Then, other values of the measure of capital stock given by other combinations of the quantities of the goods are possible, which is to say that it is not possible to combine the two goods without altering the production functions of the model, except in special cases.

Nevertheless, Hahn's procedure of assuming that production can be represented as using the same goods as inputs in different proportions is common. Ethier used a similar model for the same purpose.[24] Both Samuelson and Solow described a procedure of supposing a finite number of techniques with no reswitching and then supposing that the number increases in such a way that the set of profit rates at which two techniques are used becomes everywhere dense. They also needed the assumptions that a technique used at a lower profit rate than another has a greater value of capital per head and that the difference of capital per head between two techniques tends to zero if the difference in profit rates tends to zero. Samuelson used this procedure to make his continuously differentiable "surrogate" production function from a beginning with a finite number of techniques. Another of several examples is that of Burmeister.[25] Unlike these, Champernowne's production function has the desired properties of partial derivatives with respect to labour and capital being equal to the wage and profit rates respectively, yet does not need that procedure because it is confined to a finite number of techniques.

Less stringent and more realistic is to accept that different techniques for producing the same good use different varieties of capital equipment. If a firm does change its technique for producing a specific good, some capital equipment is changed, with the concomitant costs, though some, like buildings and roads, may not be. Equipment cannot be supposed to consist only of goods that can be combined in different proportions. This is obvious when the production possibilities are depicted as a finite number of activities, which must, then, be discrete. It is obscured by the assumption of continuity, when it becomes harder to imagine that the equipment may be different if the change of wage or profit is infinitesimal.

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[24] Ethier, "The Theorems of International Trade in Time-Phased Economies."
[25] Burmeister, "Wicksell Effects."

This point, that different production techniques may have different capital goods, has a counterpart with consumption goods. If it is no longer assumed that there is only one such good, the possibility of different compositions of consumption must be allowed for. Along with differences of production techniques there are differences of consumption patterns and the comparisons are as of two countries that can use the same production techniques, but of which the peoples have different preferences. Production techniques can still be ranked by their profit rates, but when two countries with different profit rates have different patterns of consumption and, consequently, different stocks of capital goods, the procedure of combining two techniques in different proportions, which posed no problem with one consumption good, now must cope with differences of people. The counterpart to the combination of different production techniques involves peculiar assumptions about how to combine different peoples.

Attempts to remove the indeterminacy: demand and equilibrium

Second, there are the arguments that the indeterminacy between wage and profit can be removed by bringing in demand. If demand determines prices, it determines wages and profits. This does not contradict Sraffa's argument, which is, that to consider production alone is not enough to determine the economy and which refutes the common, textbook argument using production functions that seems to give determinacy without reference to demand, namely that competition causes wage and profit rates to become equal to their marginal products. But it adds the condition of consistency, for wages and profits also determine demand.

In the simplest case, that of the steady state, the indeterminacy is evident when demand is left out. For a given set of available techniques of production and a given constant population growth rate, there is a range of steady states, each with its own wage and profit rates and technique of production, and there is no reason to select one state rather than another. Demand can be brought in to determine the state by making assumptions about saving, for instance by assuming constant rates of saving out of wages and profits. It is possible for saving not to suffice for equipping all workers as the population grows and old equipment is discarded, even with a constant population, yet for the economy to be in a steady state. Since the gross investment in each period that keeps a given proportion of a constant or steadily growing population employed replaces worn out equipment and equips any additions to the workforce, the technique of production being used, which determines the distribution of income between wages and profits, must be one that gives the right amount of saving. Since returns to scale are constant, any proportion up to that giving full employment meets the condition of consistency, which adds another form of indeterminacy. This second form of indeterminacy can be accepted as a concern of the country's economic authorities, or assumptions can be made about the behaviour of wages and investment in addition to those about saving rates in an attempt to remove it. In either case the economy moves out of the steady state.

In theory an economy can be in equilibrium, though not in a steady state, the criterion for equilibrium being that no firm or individual gains from change if none of the others change, which implies that all expectations are fulfilled. With capital goods that are durable and heterogeneous changes of prices, capital stock and production become too complicated to be explicitly described. Nonetheless, firms and households know what to expect, for fulfilled expectations means, among other things, that the profits on an investment in equipment discounted over the time that the equipment is used yields the anticipated rate of profit, which implies both a constraint on the changes of prices of output and capital goods and that the changes are foreseen. Competition implies uniformity of prices at any time and, therefore, that the same discount rate has been used for all investments at the time, though that rate can change over time.

This equilibrium is general and inter-temporal. Prices can change from period to period and the profit rates they yield for any one period can, therefore, differ from good to good. Hahn argues that, because of this, a uniform rate of profit each period, as assumed by Sraffa, would, at best, be a special case of equilibrium.(Hahn 1982, 363–64) His point is valid and he illustrates it by his model with two malleable goods, now called wheat and barley, and two periods. But that is only to repeat that Sraffa's indeterminacy does not occur in models of inter-temporal general equilibrium, which are necessarily determinate.

Models like that of Arrow and Hahn(Arrow and Hahn 1983), which had its origin with Debreu, have assumptions about production and consumption at least as general as those of the steady state and do have such inter-temporal general equilibria. Arrow and Hahn assume consumers to have consistent preferences for the various goods over the various periods, whilst the assumptions about production are general enough to accommodate durable, heterogeneous capital goods. Consumers and producers behave rationally, which, for the former, is defined as maximising their welfare over time within the limits of their incomes and, for the latter, maximising profits. That there is an equilibrium is proved using a fixed point or a separating hyperplane theorem, both of which are statements that something exists without indication of how it is to be found. Equilibrium is shown to be efficient in the sense that no individual or firm can be better off without some other individual or firm being worse off.

These are, however, purely mathematical results that leave open the questions as to how they can be applied to the real world and how they can be extended to international trade. In his criticism of Sraffa, Hahn, refers to a similar '… crucial and beautiful theorem in neoclassical economics …' and states, after listing the conclusions, 'These results are theorems and they are not at risk.'(Hahn 1982, 371) Arrow and Hahn do not assert that their model is realistic; their justification is that it shows that '… an economy motivated by individual greed and controlled by a very large number of different agents …' does not end in chaos but is '… compatible with a coherent disposition of economic resources that could be regarded, in a well-defined sense, as superior to a large class of possible alternative dispositions'(Arrow and Hahn 1983, vii). Taken literally, they seem to imply that the problems of economic development can be solved by starting with anarchy. No need for institutions, government or economic theory, and no need to study economic instability.

More reasonable would be to argue that the impossibility of the requirements is proof to the contrary; that equilibrium in the sense used here cannot possibly occur. Few take such models literally and it may seem superfluous to give reasons as to why they are too far-fetched to be used to draw conclusions about how economies work in practice, especially as neither Hahn nor any other proponents of models of this sort claim these models can describe actual economies. Despite their apparent generality, the assumptions require too much that is impossible, including perfect foresight, perfect forward markets for everything and individual consumers whose preferences ignore birth, death and family. And no model of comparable generality seems to have been formulated for several open economies; they are all confined to the closed economy. These models are also vague; their proponents do not say in what way they are relevant to reality, but seem to imply that they are relevant in some way, which is why it is not superfluous to make some obvious points in what follows.

Their very generality is a source of vagueness. The assumptions about production may be general enough to allow for some deviation from constant returns, externalities and indivisibilities, as well as for some technical progress, but there seems to be no way of telling how much and in what form. Similarly, the assumptions about consumption perhaps allow for the welfare of an individual, say a child, to influence the welfare of another, say a parent, but it is not clear to what extent education and upbringing can be allowed for. Perhaps mathematical research would yield information as to the extent that these and other complications can be accommodated, but it would only be of academic interest and probably uninteresting mathematics.

Its complexity alone makes inter-temporal general equilibrium impossible. Equilibrium results from the rational behaviour of each household and firm and, at the same time, determines that behaviour by providing the information households and firms need to decide how to act, notably prices, demand and the production possibilities of firms through the future. Each household and firm knows how all the others intend to act or is somehow provided all the data it needs to act consistently with the others. Among the reasons that this is impossible is that any but the simplest optimisation requires an expertise in programming that few households can have and optimisation of this complexity is impossible, even if the data needed are available.

Behaviour when coping with unmanageable complexity and uncertainty is qualitatively different to the rational behaviour of models of general equilibrium. Households must find methods other than intertemporal maximisation of welfare to cope with the complexity and uncertainty of their economic lives. Usually they make budgets. It is what practically all institutions, including those constituting the government of the country, and big firms do. When households seek expert advice on how to manage their incomes and daily finances, they are told to prepare budgets. Even households that do not explicitly plan their expenditures by budgeting can have budgets from habit or from rules they have in mind, and some kinds of behaviour that seem irrational because they do not conform to inter-temporal maximisation of welfare may be explicable as attempts to stick to budgets. Not all households can be assumed to use budgets regularly; some are less careful with their spending and others too rich to bother. But they cannot be imagined to optimise their expenditure. Household budgets are easily revised with changes of pay, employment, inflation and so on, in contrast to the budgets of government and many firms and institutions, which must also have budgets because they are answerable to others for the monies with which they are entrusted.

Some economists argue that complexity does not pose a difficulty. They compare the household or firm to the billiard player, who does not need to know Newton's laws of mechanics. As an analogy it is misleading by being vague, since, if it applies to anything, it applies to the knowledge involved, not the execution. It implies that mathematicians are good at billiards because they know the theory. It is also irrelevant, for, whether or not households and firms know economic theory, it is the complexity of the calculations they cannot cope with; though most people can learn to play billiards and improve with practice, general equilibrium implies they make perfect shots straight away. Not even the best player performs perfect shots all the time, and players vary in their abilities and in the time they devote to practice. If the analogy were valid computers would not be needed for the design of aeroplane wings or bridges, or, to put it differently, this recourse to analogy shows there is no logical argument.

Economics without equilibrium

Presumably the proponents of inter-temporal general equilibrium believe actual economies approximate their models, or would do so if left to themselves, for the models would otherwise be no more than riders to theorems in topology and Hahn's references to such a model in arguing against Sraffa would be pointless. But the belief that such models resemble reality is self-contradictory. Expectations are often mistaken and plans of households, firms and governments are almost always modified, if not changed altogether, to match actual outcomes. To say that this is in some way an approximation to an inter-temporal general equilibrium is to say that the equilibrium is not determinate. Some objections to the argument that the equilibrium would occur if the economy were left to itself have already been given.

Models devised for forecasting an economy's behaviour are, therefore, necessarily conjectural; they must rely on assumptions as to how households and firms behave and on simplified descriptions of production. They are not simplifications of or approximations to general equilibrium models with durable, heterogeneous capital goods applied to available data, nor can they reproduce the optimisation by households and firms that general equilibrium models presuppose. As an example, no satisfactory way of predicting saving has yet been found. In theory a forecasting model should depict production as flows of current inputs (raw materials and intermediate goods) into production processes constrained by capital stocks to produce consumption goods, raw materials, intermediate goods and capital goods. It should, therefore, have two input-output matrices, one for current inputs and the other for the capital stock, which Schwartz seems to have been the first and practically the only one to propose.[26]

No model of this kind with two matrices seems to be in use for economic forecasting. One reason may be that such models are more onerous to put together than conventional ones. Getting the data for the input-output matrices should not be as difficult nowadays as it was some time ago, but investment and the corresponding production need to be specified to the same level of disaggregation and, so, forecasts must rely on information about their intentions that firms, domestic and foreign, and government agencies provide. Production functions have the apparent advantage of not needing so much information; once the type of the function has been chosen, it takes relatively little to fit its few parameters. They also simplify investment, which becomes a quantity of capital. It is the hope that convenience can substitute for reality.

Pragmatism and production functions

Considering the quantities of data that are routinely collected by various agencies, including central banks, this does not seem enough to explain the prevalence of models with production functions. Thus, all central banks that have models for their countries' economies use production functions, mostly CES functions, though the German central bank and some others use Cobb-Douglas functions. A more likely reason for the prevalence is that the people who make the models are not aware of the objections to production functions. Forecasts are routinely tested by the outcomes and experience shows the models used for them to be unreliable, so it would be expected that the models would be rejected, as they would be in the natural sciences. But, in practice, the standard of comparison for economic models is the accuracy of the forecasts of other models and, since they are all more or less alike, they all give equally unreliable results. As a matter of experience, a model that gave a

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[26] Schwartz, *Lectures on the Mathematical Method in Analytical Economics*.

more accurate result than others in one period cannot be relied on to give an equally good result the next. Economies are hard to predict, even in the short run, and it is accepted that those who make and use the models are not to be held accountable for the accuracy of their forecasts. Instead of the models being rejected, forecasts are adjusted as the actual outcomes become known.

Economic models for forecasting only became common as computers made their calculations feasible, but fitting Cobb-Douglas production functions to national output has been going on for longer.[27] In these early exercises the functions fitted the data well and, since it seemed that some kinds of theoretical and empirical work would be unmanageably complicated if the heterogeneity and durability of capital goods were allowed for, some argued in favour of using production functions and treating capital as though it were a malleable substance on the grounds of being pragmatic and practical. In his De Vries lectures Solow likened his empirical work on the 'social return on investment' to '… what the inveterate gambler said about the dishonest roulette wheel, 'I know the wheel is crooked, but it's the only game in town'.'[28]

The gambler's fate is assured, as Brown had already shown in 1957.[29] Brown showed that the fit of the production function to the data was good because of the regularities of the data, notably similar growth rates of factors and similar rates of pay and returns on investment across the economy. He also agreed with others who had pointed out that it was improbable that 'one unchanging production function should fit a growing, changing economy over a run of years' and remarked that attempts to add time as a variable to the production functions had given results that were not acceptable, rather than better.[30]

2. Capital Goods and Technical Progress

Defining technical progress.

For present purposes and without attempting to give a precise definition, technical progress is taken in the broadest sense allowed by the assumptions that there is at any time a set of known goods that can be made in one or more versions and a set of known production techniques for making them, and that there are several countries with different nominal wages; it is the addition of new goods or of new versions of existing ones and the addition of new production techniques that can yield positive profit for some combination of nominal wages among the different countries whenever potential producers have the requisite training and knowledge. If a new production technique cannot be profitable with any consistent set of wage and profit rates in the various countries, it is not to count as progress. It is taken for granted that countries do not differ in how the requisite training and knowledge can be imparted, though the imparting may not occur, nor in how well any technique can operate. Goods can be for consumption, can be capital goods and can be intermediate goods, including raw materials, and they can be durable or used once only. New production techniques can be expected to involve new types of capital or intermediate goods, but allowance is made for the possibility that they do not. In this, the common sense view, technical progress is equivalent

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[27] Brown, E. H. Phelps, "The Meaning of the Fitted Cobb-Douglas Function," 546.
[28] Solow, *Capital Theory and the Rate of Return*, 68.
[29] Brown, E. H. Phelps, "The Meaning of the Fitted Cobb-Douglas Function."
[30] Brown, E. H. Phelps, 548–50.

to acquisition of knowledge by which new and better production techniques and products are made possible and allows for such knowledge to be generated deliberately and at a cost through R&D, though not necessarily a cost that is recovered.

Skill acquired from practice, as with Adam Smith's pin maker, does not count as technical progress; it is not a new technique, but is confined to the individual and is an improvement of efficiency that must be supposed to have a limit. Arrow's argument for "learning by doing" is that it is improvement with experience, which seems to be the same as with Smith. But he implicitly assumes that experience is not confined to individuals and results in improvements of capital, newer capital requiring fewer workers per unit than older capital, the reverse of what would be expected from Smith's pin maker; older capital would be more efficiently operated, unless the workers operating it are transferred to newer capital, for which there is no reason if there is no other difference between the capitals. There is also no limit to the improvement possible. Hence, learning by doing as defined by Arrow does count as technical progress.

Technical progress and product differentiation.

Most goods have several characteristics that users like or dislike and there can be several versions with different combinations of these characteristics at any time. Lancaster had already proposed associating a combination of characteristics with each good in 1966.[31] For present purposes such goods are termed differentiated, whilst those that are not differentiated are referred to as homogeneous. In some cases competing firms may make the same version of a differentiated good and compete solely on price. But often versions of the good differ because of design and the designs are protected by intellectual property rights, like patents, copyright and trademarks, which allow each owner of a design to prevent others from copying it or imitating it too closely. Furniture, clothing and houses, for example, are mostly designed for both practical and aesthetic reasons and the designs are normally so protected. Each version that is protected is a quasi-monopoly of the producer. None of the different versions of a good need be unambiguously better or worse than any other, for one may be preferable to another in some respects but not in others. Or one may be better than another but costlier. Consumers buy according to income, taste and convenience. Competition among producers of differentiated goods of which the designs of the various versions are protected consists of designing versions with combinations of characteristics that do not infringe on the intellectual property rights of others and, as quasi-monopolies, fetch high enough prices to be profitable.

This diversity of competing versions at any time does not, in theory, carry over to capital goods. Different versions of a consumption good, including more expensive, better versions and cheaper, inferior versions, can be sold at the same time because consumers differ in their preferences and incomes. In contrast, firms all have the same preferences, profit maximisation, and their incomes do not matter if returns to scale are constant. So, if firms with the same wage and profit rates choose different versions of the same capital or intermediate goods to produce the same good, they do so because the different versions of the good they produce require different types of equipment. In reality the capital goods for producing the same good are differentiated for other reasons as well, among them returns to scale not being constant, externalities and physical circumstances, including costs of untradables, considerations that have to be mentioned but are not pursued here.

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[31] Lancaster, "A New Approach to Consumer Theory."

Technical progress is obviously a source of differentiation when it results in new goods or new versions of old ones. It can result in a new version being better than an old one because it increases some desirable characteristics and lessens some undesirable ones, or because it adds new, desirable characteristics and eliminates old, undesirable ones. Motor cars illustrate this; cars can differ in thousands of ways and technical progress constantly results in improvements leading to new versions, though they are all versions of the same good. But the distinction between a new good and a new version of an existing good is to a great extent arbitrary. Some components of a car may be considered new goods, as, for example the automatic transmission and anti-blocking system in their times, though the cars that included them were not considered new goods. Most people would think of the locomotive, which was an adaptation of the principle of the steam engine from mining, to have been a new good. But did the use of a similar engine in a ship mean the steamship was a new good or a new version of a ship? If the steamship was a new good, was it partly because it used a propeller or a paddle wheel, which sailing vessels did not? The incandescent lightbulb has been followed by fluorescent lights, mercury bulbs and light emitting diodes, all of which work on different principles. But are they different versions of the same good or different goods?

For the purposes of the discussion here there is no need for definite criteria. But the distinction has to be made because in the endogenous growth models discussed later there is a distinction in the utility and production functions. In these models a new consumption good is added to the other goods in a utility function, so that the individual consumer may be consuming that good at the same time as the older ones, whereas the newer, better version of an existing consumption good is chosen to the exclusion of the older version. Similarly for capital and intermediate goods used in production; new ones are added to the production functions and new versions displace old versions.

Types of technical progress.

However the distinction between new versions and new goods is made, technical progress takes four forms. Two are the improvement of existing goods and the invention of new goods, whether for consumption or production. A third is the improvement of processes of production of the same goods using the same labour, goods and factors, provided it is independent of the experience of the workers operating the processes. If it is not, it is Smith's acquisition of skill, which can be supposed to have a limit. A good that is improved is necessarily differentiated, for, at the least, there are a new and old version. A new production process that has new or improved goods as inputs or output is subsumed under the first two forms. The fourth form is an increase of utility from the same consumption using knowledge generated by R&D and for which people are willing to pay. It is only mentioned for the sake of completeness since there seem to be no models of the kind and it would not be manifested in the national accounts. For brevity, most of the discussion in the following is confined to the first two forms. The third form, being less important, is referred to only when necessary and the fourth can be ignored.

It can be supposed that technical progress increases the welfare attainable. More technical progress is better than less, other things being equal, but not all of it is relevant to any particular economy. Technical progress in the general sense used here is progress of technical possibilities and is to be distinguished from the application to a specific economy. A new production technique may be used in one economy but not in another because it is unsuited to the wage and profit rates of the second. It is, in principle, possible for a new technique not to be suitable to any economy because of the existing set of wage and profit rates, though it would be for some other set and, therefore, constitutes technical progress, though not relevant under the circumstances.

The application of technical progress in an economy is not instant. Durable goods are often not displaced as soon as there are new versions and every economy has stocks of durable goods of various ages. Families do not as a rule change their television sets the moment an improved set is available, better ways of conserving energy in homes spread gradually and most of the housing stock in any country is technically out of date. Improvements of production techniques usually involve new types of capital equipment and the rates at which they replace equipment in use depend on, among other things, how much of an improvement they are and the ages of what is in use. As the new versions and new goods are adopted yet newer ones come into being. Hence, the rate at which technical progress actually takes place in an economy depends partly on the rates at which improvements and inventions occur and partly on how fast they displace old goods, both consumption and capital goods, and differs from one economy to another according to the composition of its production.

Several questions arise; among them, what are the causes or explanations of technical progress, can they be influenced and, if so, how, what are the benefits to other countries of technical progress in a given country, what are the gains and costs of faster adoption of newer versions of consumption and capital goods, how can the effects of technical progress on welfare be assessed and can the technical progress of an economy be measured? Only the last question can be answered here; the technical progress of an economy as a whole cannot be quantified. For the rest, it is the purpose of the following to show that the questions cannot be realistically discussed, let alone answered, if production is represented by functions of labour and capital with, perhaps, other factors, or of labour alone.

Using production functions to represent the aggregate production of an economy eliminates the first two forms of technical progress for consumption goods. Output is represented as a single quantity, which means its identity and composition do not change and, therefore, that technical progress in the form of new and improved goods cannot be allowed for. There is as yet no objective way of allowing for these types of technical progress in the usual measures of aggregate output, such as GDP. Sometimes notional adjustments are made to allow for improvements of consumption goods, but they are not objective. It would make no difference if output were to consist of several goods, since each would be a single quantity. The same is true for capital in models with one good or with a fixed set of types of capital all of which are equally available. Pasinetti's point referred to earlier is that capital goods also change and, even if a capital good does not change, the capital goods for making it may. To take the capital in a production function as given is to leave out this part of technical progress.

Technical progress as a residual.

If neither the technical progress in consumption goods nor that in capital goods is to be considered, only the third form is left; there is no alternative to arguing that, apart from technical progress "embodied" in goods, that is the first two forms, there is "disembodied" technical progress. That is how the attempts to estimate technical progress of Solow, Denison, Jorgenson and Griliches and others must be understood. Even if it is conceded that output per head can increase without change of physical equipment and do so without limit, the assertion that disembodied forms of technical progress are so much greater than the embodied forms that the latter can be neglected, apart from being hard to imagine, requires both empirical evidence and some description of how that technical progress takes place.

As the following brief survey shows, these attempts to estimate technical progress are devoid of any explanation or description of technical progress but identify it with that part of the increase of output that is not explained by increases of factors. In other words, it is a residual. Then what is the evidence that it is not, as Abramovitz stated regarding Denison's 1962 estimate of the residual, termed the Advance of Knowledge, '…the grand legatee of all the errors of estimate embodied in the measures of national product, of inputs conventional and otherwise, and of the economies of scale and other factors classified under productivity growth.'[32]? If different methods had yielded similar results there could have been some grounds for supposing that they were measures of the same thing. Instead, the results of the estimates discussed below were sufficiently far apart to cause disagreement over how the statistical data should have been used, although the method was the same, namely using prices as indicators of marginal products.

Solow was the first to try to estimate technical progress from an aggregate production function by equating it to the residual. In his article published in 1957[33] he excluded agriculture from his estimate and assumed labour and capital to be the only factors, so that their incomes as given by the national accounts add to the value of the output. He assumed factors were paid their marginal products, which allowed the further assumption that the production function was linearly homogeneous in the two factors. Technical progress was assumed to be Harrod neutral, meaning that the marginal product of capital does not change if the ratio of capital to output does not change. Solow claimed that the data allowed this assumption and assumed a production function in the form of a static production function multiplied by a factor representing technical progress that grew with time, which gives Harrod neutral technical progress. It follows that the rate of technical progress, which is the rate of change of its factor, is obtained by deducting the increase of capital per head multiplied by the share of capital in income from the increase of output per head.

From this Solow estimated that technical progress accounted for seven eighths of economic growth from 1909 to 1949. The scatter diagram showed a positive, almost linear relation between output per head divided by the factor for technical progress and capital per head, except for the last seven years, which lay on a parallel but separate line and had to be left out for lack of explanation for the difference. In other words, if output per head is adjusted to allow for technical progress, it increases in proportion to capital per head. Solow fitted five formulae for production functions to the data and obtained high correlations for all of them, the highest (0.9996) being obtained by a Cobb-Douglas function.

Later Solow did introduce embodied technical progress in two models, but their purposes not being to estimate or explain it, they need only be briefly described. In the one the capital created in one period is more productive than the capital of the previous period in the sense that the production function of the former is equal to that of the latter multiplied by a factor representing technical progress.[34] Total output at any time is the sum of the outputs of the stocks of capital of that and each earlier period, with due allowance for depreciation. Solow's purpose was to discuss the relationship between unemployment and output assuming various rates of technical progress, which he did not try to estimate, and taking the capital of each period as given. His other model was intended to show that the marginal products of

[32] Abramovitz, "Economic Growth in the United States," 775.
[33] Solow, "Technical Change and the Aggregate Production Function."
[34] Solow, "Technical Progress, Capital Formation, and Economic Growth."

labour and capital can be defined and are equal to the wage and profit rates even when capital goods are durable and heterogeneous.[35] For this he assumed that machines for making the consumption good are handmade and that there are different types, those using less labour to produce a unit of the consumption good requiring more labour to be made. Assuming that machines are handmade allowed him to avoid the complications of machines making machines, etc., but Solow added, '… I do not believe that any matter of principle is involved.'[36]. He did not mention that he also avoided the complication of profit entering into the cost of machines and that his conclusions depended on that.

Jorgenson and Griliches used the same theory as Solow, but in their article of 1967 they came to virtually the opposite of Solow's result, namely that little of the increase of output per head could be attributed to the residual.[37] Where Solow had concluded that seven eighths of the increase of output over the years 1909 to 1942 came from technical progress, they estimated that 79.8 per cent of the increase of output from 1953 to 1965 was explained by the growth of inputs and that the residual had grown at 0.72 per cent a year.[38] They attributed the differences to different statistical procedures. Also, they did not, as did Solow, estimate an explicit production function, but assumed, instead, that the relation between outputs and inputs could be represented by an implicit function without needing to specify its form. Then, when competition is perfect and returns to scale are constant, so that the prices of inputs and outputs are inversely related to the relevant partial derivatives of the production function, the difference between the price weighted indices of outputs and inputs is zero. If, therefore, the data show that the output index has increased more than the index for inputs from one period to the next, the implicit function must have changed and the difference between the increases is the residual. Jorgenson and Griliches used Divisia index numbers over several years and called the residual "total factor productivity".

In turn Denison[39] criticised their procedures, though the differences of procedure need not be described here. Jorgenson and Griliches revised their calculations and their final result was that growth from the residual had been 1.03 per cent per year over 1950-62, as compared to Denison's figure of 1.37 per cent.[40] There was no dispute about theory; all agreed that marginal products could be known from prices and that the inputs, or factors, should be limited to labour and capital, though there could be several types of either.

None of these procedures takes into account Pasinetti's point, that capital is a product as well as an input, so they neglect the effect of technical progress on its production. Abramovitz made much the same point in his review of Denison's estimates, where he questioned whether Denison was '… right in excluding the effect of quality change from his index of capital input …'.[41] Using numerical examples, Read showed that, because it neglects this point, Solow's residual is misleading; if, for example, the technical progress

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[35] Solow, "Substitution and Fixed Proportions in the Theory of Capital."

[36] Solow, 207.

[37] Jorgenson and Griliches, "The Explanation of Productivity Change."

[38] Jorgenson and Griliches, 272.

[39] Denison, "Some Major Issues in Productivity Analysis: An Examination of Estimates by Jorgenson and Griliches."

[40] Jorgenson and Griliches, "Issues in Growth Accounting: A Reply to Edward F. Denison," 89. Table 25.

[41] Abramovitz, "Economic Growth in the United States," 772.

were to be confined to the production of capital goods, it would show no technical progress.[42] Taking up Read's point, Rymes showed, what was implicit in Solow's article, that identifying technical progress with the residual after deducting the growth of output attributable to the increases of factors means that it depends on the factor shares.[43] Hence, two economies in steady states with the same production functions, but not Cobb-Douglas, and the same rates of growth of the labour force, capital per head and output will still have different rates of technical progress if their wage and profit rates are different.

Rymes proposed that, just as the part of output per head attributable to factor increases is obtained by deducting the technical progress, the growth of capital per head should be adjusted by deducting the technical progress in the production of capital and he showed that if the two rates of technical progress are the same and constant, i.e. Harrod neutral technical progress in a steady state, that rate is independent of factor shares. In Solow's model output and capital are each treated as the same good, but Rymes demonstrated that his adjustment gives the same independence of factor shares whether there are one or several capital goods in the production of one consumption good, also when goods enter into their own production. Similar adjustments can be made to the estimates of Denison and Jorgenson and Griliches. That would make them more logical on the assumption of the same rates of technical progress for capital and final goods, but does not go so far as to treat capital goods as durable and heterogeneous.

Apart from the objections to the theory, there are objections to the statistical procedures of both Denison and of Jorgenson and Griliches that are unrelated to their disagreements. What these procedures have in common is that they go from assuming that the marginal relations between prices, inputs and outputs that apply to individual goods and factors also apply to aggregates. Thus, it is assumed that dividing the value of a set of goods by a price index to obtain an index of quantity is the same as dividing the value of a single good by the price to give its quantity, and, the price being the marginal cost of producing that good, the price index can be presumed to be the marginal cost of increasing output as measured by the quantity index. This is something that would have to be demonstrated case by case. For example, if motor vehicles were to be aggregated, the value of the output divided by the price index should not be supposed to indicate the marginal cost of the increase of output of motor vehicles from one period to the next, for that cost would depend on the composition of the increase, whereas the price index would for practical reasons include vehicles from previous years with weights that may not correspond to recent sales. Such an error may be thought small, but it can accumulate over time even if it is. Moreover, the fewer the goods in any aggregation the more the composition is likely to change each period and the bigger the error to be expected. Hence, attempts to improve the estimates by having more aggregates with fewer components and more index numbers increase the likelihood of error in the estimation of marginal costs. No attempt seems to have been made to estimate the error, presumably because it is considered small, but the residual being estimated is also small and must be shown to be bigger than the errors and not part of Abramovitz's "grand legatee".

R&D as a cause of technical progress.

When their first estimates seemed to show that technical progress accounted for little of economic growth Jorgenson and Griliches were led to make a point that seems not to have

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[42] Read, "The Measure of Total Factor Productivity Appropriate to Wage-Price Guidelines."

[43] Rymes, *On Concepts of Capital and Technical Change*.

been made before, namely that there had been progress in production techniques and the types of consumption goods available, but it was the consequence of investment in research and the technical development of goods (R&D) fetching a normal return, hence not the residual with which technical progress had been identified. They remarked that there were sceptics who preferred to call the residual a measure of ignorance and they concluded '… not that advances in knowledge are negligible, but that the accumulation of knowledge is governed by the same economic laws as any other process of capital accumulation.'[44] Taken literally the statement makes three assertions relevant here. One is that the production of knowledge should be treated as a type of capital accumulation, the second that knowledge can be treated as though it were an ordinary good and the third that, since the residual is the only measure of technical progress, part of what has normally counted as technical progress cannot be quantified. To the extent that knowledge is produced, does accumulate and is generated by R&D at a cost, the first assertion is valid. But the second is misleading. New knowledge is not lost and the usual assumption, with which Jorgenson and Griliches agree, is that, once generated, it is available to all, unless there are arrangements that can prevent that. Knowledge may resemble capital in being produced but is not a factor in the sense of Jorgenson and Griliches; reducing the quantities of factors reduces output, whereas the increase knowledge brings about of the quantity of output possible or of the ability to improve goods is permanent. Jorgenson and Griliches did not take up the questions raised by the third assertion, namely if technical progress results from the knowledge generated by R&D, how it is to be measured. Neither did Denison, who seemed not to accept technical progress as the outcome of investment in R&D.[45] By the definition of the present work, technical progress can, in principle, be entirely the result of R&D and the residuals just described no more than the consequences of errors of theory and statistical procedure.

R&D is so obviously the main source of technical progress in reality that, for present purposes at least, other sources can be ignored. It is also obvious that much of it consists of improving existing goods and is, therefore, a source of competition. Firms that make goods that are improved by R&D compete to produce new, improved versions knowing that their competitors are doing the same and that, as new versions come out, older versions, being inferior, will have to be sold for less and will in time become unprofitable, by when their producers will have stopped them to free production capacity for newer versions. Virtually all such goods have numerous characteristics that users like or dislike and which vary from one version to another. For example, different types of bicycle are used for ordinary conveyance, for long tours, for racing on smooth surfaces and for rough terrain, and each firm making bicycles may make several versions of each type, each version with its own technical characteristics.

Now technical progress must be distinguished from the residual of Solow, Denison and Jorgenson and Griliches and from Arrow's "learning by doing" in that those who undertake R&D do so deliberately to generate knowledge useful to them at a cost. Its cost, being deliberately incurred, is an investment made by firms for profit, made possible by intellectual property rights, i.e. patents, copyright and trademarks, which gives them the means of preventing others from using knowledge obtained from their R&D. Every time a firm's R&D results in an improvement of some characteristic that firm has a quasi-monopoly. It is not a full monopoly because its competitors produce the same good, but versions with different combinations of characteristics, and each firm, as long as it keeps on producing new and

[44] Jorgenson and Griliches, "The Explanation of Productivity Change," 274.

[45] Denison, "Some Major Issues in Productivity Analysis: An Examination of Estimates by Jorgenson and Griliches," 62–63.

better versions, can obtain a profit on its quasi-monopoly before the knowledge is superseded. But for patents, firms that have incurred the costs of R&D would be undercut by those that have not, for much knowledge generated by R&D cannot be kept secret for long. A good may reach a stage when the gains from R&D are too small to be worthwhile for producers. At this stage, as patents and intellectual property rights lapse with time, the good can be produced competitively by any firm. Or the costs of improvement may be so great that they can only be profitable if the firms producing the good are big enough. Since the improvements brought about by R&D are likely to be independent of the size of the firms making them, returns to scale are not constant when the cost of R&D is included.

R&D without durable, heterogeneous capital: endogenous growth models.

Competition as described above among firms producing differentiated products must be confined to consumption goods if production is represented by functions of labour and undifferentiated, malleable capital. Then each version of a good has a demand curve, determined in part by the versions competitors are producing and by income distribution, and the firm producing it, as a quasi-monopolist with a given stock of capital, chooses the output that maximises profit. If the demand curve is known it is, in principle, a straightforward optimisation problem. To pay for the R&D the profit must exceed the marginal product of capital. (*consumption goods models*)

Rymes's method of adjusting the quantity of capital for technical progress in its production cannot be used, for there is no residual from the R&D, except with the third type of technical progress. If there is to be technical progress arising from R&D embodied in capital and the capital is to be malleable, the capital has to be differentiated in some way, which must be by supposing that R&D results in new types of capital and that there is no technical progress with the capital in existence. Romer devised a model like this.[46] He assumed a single final good of which the part not consumed was capital and was transformed into producer durables in a fixed proportion. New types of these durables come into being through R&D carried out by trained workers (human capital). Only the newest durable is produced each period and the output, raised to a power less than one, say zeta, is added to the previous outputs of durables, each raised to the same power, to give the total capital, which, along with labour, enters into the production function of the final good. As new durables are added total capital increases and so does output. Depreciation can be allowed for, but is left out for simplicity.

Romer's distinctions between the consumption good and capital and between capital and producer durables are illusory, for there are no costs of going from the one to the other. He refers to capital as trucks, trains, computers and communication networks.[47] Elsewhere he refers to "blast furnaces, lathes, fork lift trucks, looms, etc."[48]. But he does not explain the meaning of a fork lift truck raised to the power of zeta. There can only be a meaning if that object is a quantity of some substance. Then, would a better fork lift truck be more capital? Even if technical progress reduced its cost of production? And, since the object is added to a blast furnace raised to the same power, both objects must be of the same substance, a special form of malleable capital that can be cut into segments that, after having their quantities raised to the power zeta, can be added to give a new quantity of capital, which, in the model is produced by the same process as the consumption good.

[46] Romer, "Endogenous Technological Change."
[47] Romer, 81.
[48] Romer, "New Goods, Old Theory, and the Welfare Costs of Trade Restrictions," 31.

Equally illusory is Romer's technical progress; his model is really one of accumulation. There is no technical progress in the durables, themselves; each type is like the other, except for the date it is added. As remarked by Aghion and Howitt, there is '… no obsolescence; new products are no better than existing ones.'[49] The production function is independent of time and is symmetric with respect to the types of durables; if the quantities of different types are the same, the order in which they were added makes no difference, whereas the notion of technical progress implies that the later types should be the more productive. Zeta being less than one, the output of the final good is made greater if the output of each new durable is divided into smaller batches. Alternatively, instead of adding new types, one durable can be added repeatedly. The R&D is redundant.

Romer creates the illusion of technical progress by assuming that zeta is less than one and that the rate at which new durables are invented is proportional both to the numbers of trained workers in R&D and to the measure of the number of different durables already in existence. He contends that R&D becomes more productive with the accumulation of the knowledge it generates, which only means that the rate at which new durables come into being rises. Since the new durables are no different to the old ones, the rate at which they are invented makes a difference only because, the later the period a given quantity of durables is produced, the more different types it is divided into and, zeta being less than one, the greater the addition to the production function.

This can be illustrated by taking time as discrete intervals, which is how Romer begins his exposition. From some point on several new type of durables are invented in a single period and, then, comparing two periods, assuming the quantity of the final good made into durables is the same in both and that in each period the amounts of the various types are equal, the later period adds a bigger quantity to the production function because the same quantity of the final good is divided into smaller packets and the packets are all raised to the power zeta. If zeta is one the two periods add the same amount to the production function and if it is zero the amounts added are the number of packets, which is greater in the later period. Romer's use of a Cobb-Douglas production function adds to the oddity, for zeta represents the share of capital and that implies that the greatest technical progress occurs when the share of capital is zero and the least when it is the only factor. Changing to continuous time alters none of this.

Several models were devised after Romer's with the same intention, namely that of showing that, in free, competitive markets with rational consumers, investment in R&D is endogenously determined by firms maximising profits and, therefore, so are the technical progress arising from the knowledge it generates and the consequent economic growth. Rational behaviour in these endogenous growth models means that consumers have utility functions, which they maximise over time. Producers and consumers are assumed to know all they need to know about the future to make correct decisions, which, if R&D is a stochastic process, includes the extraneously given probabilities.

Endogenous growth models are formulated mathematically and must, therefore, be simple enough for the mathematics not to be intractable. One simplification is to confine the scope of technical progress to consumption goods, or to their production or to the process of producing the goods used in their production, but to only one of these forms in any model. Another is to leave out consumer durables; every consumption good is used once. Consumers are assumed to be alike; apart from having the same preferences and the same income, they must live equally long, best brought about by assuming they are immortal. Production, too, must be kept simple. Firstly, the prices of goods used in production must

[49] Aghion and Howitt, "A Model of Growth Through Creative Destruction," 392.

depend on labour costs and not include profit. Durable capital goods, unless malleable, are excluded because the profit rate enters into their prices. So, too, are intermediate goods, inputs that are used only once, if they are produced by means of labour and manufactured goods. Hence, when production is not represented as a function of malleable capital and labour, it is represented as a function of labour alone, or of labour using intermediate goods made by labour alone, or of intermediate goods without labour. In addition, when a good has several versions all are assumed to have the same factor and R&D costs. Secondly, if the technical progress occurs in production, there is only one consumption or final good. This avoids having several different forms of capital or intermediate goods at the same time. No endogenous growth model does without these simplifications.

Another source of unmanageable mathematics is consumer choice of the kind described earlier, with several versions of the same good catering to different preferences and incomes. All endogenous growth models in which consumption goods are improved through R&D simplify with the assumption that different versions of a good can be ranked unambiguously as better or worse, allowing a single index as identifier. Then, when there are several versions of a good, each consumer chooses only one and, since consumers are alike, all choose the same one. Lancaster's proposal of attributing several characteristics to each good is ruled out, except when so restricted that goods can be ranked, an example being Stokey's model of learning by doing.[50] In that model each good has associated desirable characteristics and each new version is the same as the previous one with the addition of one more characteristic.

Endogenous growth models and reality.

This simplification leads to two unrealistic conclusions; first, that all goods are produced by monopolists and, second, that, if all firms are alike, no firm actually producing the latest version of a good invests in R&D. The first holds for consumption goods because of the restrictive assumptions that only one version is the latest and that consumers are alike, for the producer then maximises profit by pricing the latest version just low enough for previous ones to become obsolete, i.e. Bertrand competition resulting in a monopoly. It holds normally for capital and intermediate goods with the usual assumption of constant returns to scale, for producers are necessarily alike since they all maximise profit. Second, if all firms that carry out R&D are equally likely to devise the next version, regardless of whether they actually produce the good or not, the one that devised the version being made at the time does not invest in further improvement of that good while its monopoly lasts, for it would thereby shorten the period of monopoly and incur a cost. Alternatively, if it does develop what would be the latest version at the time and postpones production until its current monopoly ends, the R&D of its competitors in the meantime reduces the expected duration of that version. Each new version is, therefore, made by a different monopolist. Yet, in reality monopolies are rare and almost all new versions of existing goods are the outcome of the R&D of firms that have been producing those goods for some time and intend to continue their R&D to produce newer versions.

Segerstrom and Zolnierek remarked of the second conclusion that it is 'strongly counterfactual'.[51] They devised a model in which firms are not alike; a firm that has been producing a good is assumed to be more likely, for given expenditure on R&D, to find the next version than other firms. In this case, in place of a set of consumer goods, the goods in

[50] Stokey, "Learning by Doing and the Introduction of New Goods."
[51] Segerstrom and Zolnierek, "The R&D Incentives of Industry Leaders," 745.

question are the intermediate goods, which are used with labour to make the single final good according to a production function. Both the intermediate goods and R&D, which increases their productivity, consist solely of the final good. With a suitable choice of coefficients, Segerstrom and Zolnierek obtained the desired conclusion, that the firm that has been producing the latest version invests in R&D and is more likely than other firms to produce the next version. Nevertheless, eventually every firm producing a good is, by the laws of probability, replaced by a newcomer; instead of it happening immediately, there is a stochastically determined delay.

Choosing coefficients in this way to reach the result wanted may seem a minor modification of the mathematics of the model but, as a question of method, it is arbitrary and can be expected to create new problems. In this case a new problem is that it has an economic meaning; it implies a second type of knowledge, in-house knowledge of which each firm has its own, is acquired instantly at no cost, cannot be transferred and is extinguished when another firm produces the newest version. It is a departure from the principle common to all these models, that knowledge once generated is known to all. In-house knowledge does exist in reality and acquiring that of other firms is sometimes a reason for take-overs and mergers, but its characteristics are not as in the model. Moreover, it is rare in reality that a firm producing a good is replaced by a newcomer. Firms and industries do cease to exist as new goods come into existence, and firms close when they fail to make goods that are competitive with those of established competitors, but seldom are established firms replaced by new firms making new versions of the same goods. Finally, producing firms are still monopolies in the model.

If technical progress is the cause of increases of income or output per head and is faster the more knowledge R&D generates, it is to be expected that bigger economies with more R&D should grow faster than smaller ones and that growth should accelerate. Jones pointed out that such scale effects are what a number of endogenous growth models lead to and that they are contrary to what has been observed.[52] Since 1950 the numbers of scientists and engineers in the high wage countries have grown faster than the economies and workforces of these countries, yet growth rates have not increased. Jones also concluded from his calculations of total factor productivity in the US that the rate of increase of productivity did not seem to rise with time. It is also not true that bigger economies grow faster than smaller ones.

Jones's own explanation of why greater amounts of R&D had not led to faster growth was, in contrast to Romer, that, as knowledge accumulates, additions to it become harder, mainly because 'the most obvious ideas are discovered first', but also because of 'duplication and overlap of research'.[53] His model is like that of Romer, with a single final good produced according to a function of labour and durables, each of which is made from the same amount of the final good.[54] The durables are indexed according to the order of their invention and R&D, which is done solely by labour, adds new ones so that the index of the latest one is the measure of knowledge or technical progress. By choosing the coefficients relating the labour used for R&D to the change of knowledge suitably the rate of increase of productivity for a given amount of R&D can be made to decline as productivity increases and can be made to be less than proportional to the R&D.

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[52] Jones, "R & D-Based Models of Economic Growth."
[53] Jones, 765.
[54] Jones, Appendix.

But, as Jones pointed out, his model's growth in the steady state is that of the population. People optimise consumption over time and it takes ever more R&D to obtain the same increase of output, so they gradually reduce the amount of consumption they sacrifice to R&D. Like all proponents of endogenous growth models, Jones refers to steady states for his conclusions, so his model replaces the faster growth rates of bigger economies by stagnation.

Young proposed another explanation, namely that a bigger economy uses a wider range of goods for production and that its R&D, because it is spread over more goods, improves each good less.[55] His model has a single final good produced by intermediate goods according to a symmetric function of these goods, their quantities multiplied by their productivity parameters. The intermediate goods make a continuum ranging to infinity, but only a finite part of that range is used, because there is a fixed cost to the use of each intermediate each period. Since the cost of improving productivity is assumed to increase more than proportionately to the improvement, production is spread over a range of intermediate goods. In the steady state the rate of growth of production is independent of the size of the economy or labour force. Young points out, however, that output per head is greater for a bigger economy, i.e. that the scale effect applies to the level and not to the rate.

Howitt devised a model in which goods are improved and new ones are invented by modifying Young's model. He assumed that intermediate goods that have not already been used have to be invented, so that R&D is needed to add to the intermediate goods in use.[56] His model has one final good, also produced according to a symmetric function of quantities of intermediate inputs multiplied by productivity parameters, and these inputs are, again, a continuum and are all made by the same amount of labour per unit. Both consumption and R&D consist of the final good, some of the R&D going to improving the productivity of the intermediate goods in use and the rest to inventing new ones. Howitt assumes that the improvements are stochastic and that the highest level of productivity increases in proportion to the ratio of R&D for productivity to the range of intermediate goods in use; the proportionate growth rate of this level of productivity is being assumed to be in inverse proportion to its level and in proportion to the R&D per unit of intermediate good in use. In the steady state in which the growth rate of the highest level of productivity, the ratio of labour to the range of intermediate goods in use and the proportion of the final good used as R&D to add new intermediate goods are all constant neither the growth rate nor consumption per head depends on the scale of the economy.

Quantifying knowledge and R&D.

Each of these three models is an attempt to formulate an intuitively plausible idea as an endogenous growth model to obtain a specific desired result. But are these ideas well founded and what do the models add to them?

Jones's assertion that new knowledge becomes harder to obtain as knowledge accumulates seems reasonable on the analogy to the classical theory of economic rent from the extensive margin of cultivation of land; those parts of the land that yield the most for the effort are the first to be cultivated. But the analogy is false. When a patch of land is chosen for cultivation the choice is made with knowledge of the relation between yield and effort for all the different patches available. No such knowledge can be assumed of a relation between future research effort and its results. Some idea of what can be expected of research in the

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[55] Young, "Growth Without Scale Effects."

[56] Howitt, "Steady Endogenous Growth with Population and R. & D. Inputs Growing."

present or the near future is normal, for research is not undertaken blindly, especially since it has a cost. The right analogy would be to say that the yield of a known patch of land gives some expectation of the yield of the land next to it; the expectation would be reasonable, though it could be wrong and, barring other information, would be irrelevant to land further away.

Young and Howitt assumed that productivity increases with the amount of R&D and here the analogy is with the classical theory of economic rent from the intensive margin of cultivation; on any piece of land the yield increases with the effort with diminishing returns. This analogy is false for the same reason. Not only do the models assume that the relation through the future between R&D and the productivity is known for the various capital and intermediate good inputs into production in use, but it is also assumed that the relation is known for inputs that have not yet been used or have still to be invented.

In one respect the two analogies have a use; they draw attention to how the objections to the aggregation of capital apply to R&D in endogenous growth models. The term effort used above in describing the classical theory of rent implies that the various inputs into cultivation of land can be unambiguously represented as a single quantity, although they can consist of, among other things, labour, various kinds of machinery, fertilisers, pesticides and products of infrastructure like irrigation. Effort is, then, like capital, not the simple quantity it was taken to be by classical economists, such as West, Ricardo and J. S. Mill. In all endogenous growth models R&D is a single quantity, either an amount of labour or of the final good, although, as Jones, Romer and the other proponents of endogenous growth models have pointed out, the ability to do research has always depended on the products of earlier research. It is not simply that new knowledge is added to existing knowledge, what is termed “spillover” in these models. All scientific and engineering research depends on apparatuses and materials, which, themselves, were made possible by scientific research and which may be the main element of cost of any research. In other words, R&D is conducted by workers with various types of training using apparatuses and materials and cannot be represented as a single quantity independent of wages and profit for the same reason as capital.

The observation that firms compete in the production of goods with a variety of characteristics and invest in R&D to improve on them yields the evident explanation of why the numbers of scientists and engineers in the industrial countries have risen as proportions of the populations, which is what Jones wanted to explain, and reconciles those numbers with the estimates of investment in research in the US used by Howitt, which do not show such a tendency. Continual investment in R&D results in more types of complex goods and higher degrees of complexity, and the manufacture and maintenance, especially of the manufacturing equipment, accordingly require more suitably trained workers. Jones was right in drawing attention to the numbers, but wrong in associating them primarily with R&D. Howitt’s assumptions are contrived to give a steady level of R&D, but do not allow conclusions about reality and the degree to which the level of R&D has fluctuated in the US.

Endogenous growth models, interconnectedness and durability of capital goods.

Production as represented in these models does not take account of the interconnectedness of the various parts of an industrial economy. In a developed industrial economy a good produced in one part of the economy or sector can enter, directly or indirectly, into the production of a good in another part or sector. Steel, for example, is part of practically all machinery and means of transport. Electric power, apart from being the energy source of many forms of transport, enters into the production of petrol and diesel engines, and all industrial products are transported at some stage. Advances in computational ability make for better design of machines and their components and better organisation of production and transport. So, directly or indirectly, steel, electric power and computers enter into the production of virtually all manufactures and can be called basic. Some goods enter into the production of some goods and not others and some, like most consumption goods, do not enter into production at all. Hence, technical progress in metallurgy, in the generation and transmission of power and in computers and computational techniques have influence throughout the economy, whereas a new soft drink only affects some consumers.

An implication is that the consequences of a technical advance with regard to a good depend on how the good enters into the production of other goods. An advance in steel metallurgy may result in cost reductions or quality improvements for many other goods, more so than an advance in the production of leather. A good may not be improved, but its production may be because of advances in what goes into its production. Representing production as a function of handmade intermediate goods does not allow sectors to be interconnected. Technical advances with respect to some intermediate goods have no effect on others and where R&D has not resulted in advances the goods remain backward. If one of these backward goods were to be chosen, as happens in endogenous growth models, it would have to become as advanced as the others instantly. In these models, to avoid having R&D taking a good through several periods of technical improvement, it is assumed that the technical level of all goods is automatically advanced to the most advanced level of other goods or somewhere near it when they are produced even if it is for the first time. Thus, in place of the pervasiveness of technical progress that occurs necessarily in an industrial economy there is an assumption, one that depends on another assumption, that the levels of technical advancement of different goods are quantified on the same scale, as though improvements of steel, leather and computer algorithms can be compared quantitatively. This is merely an elaboration of Pasinetti's point.

The alternative to representing production by a function of handmade intermediate goods is to represent it by functions of labour and malleable capital. Since technical progress does not change the substance of the capital, but is represented by multiplication of the labour and capital by a parameter, there is no need to discuss this alternative further.

Another consequence of representing production by handmade intermediate goods and no fixed capital is to obscure the difference between the results of R&D and the rate at which they are adopted through investment in productive capacity and infrastructure. Malleable capital, intermediate goods and labour adapt immediately to the new production and a new firm that starts producing a new or improved good produces instantly. No time and money are spent on creating or adapting productive capacity and firms only risk their investments in R&D, unlike reality, in which the cost of the capacity to produce a new or improved good may be several times the cost of the R&D and the main determinant of if and when the good is made. If fixed capital is assumed away old goods are replaced by new ones whenever the cost of the R&D can be profitably regained. Old equipment does not exist alongside new equipment. If a firm's plant is older than a competitor's it must be scrapped, and old bridges are instantly replaced by better new ones.

The purpose of endogenous growth models.

Perhaps some proponents of endogenous growth models, rather than dispute the criticisms, would put the purpose of their models as to show how, if markets are free and consumers rational, firms will invest in R&D and bring about economic growth. If that is the purpose, it is inconsistent with the efforts that have been made to devise models that conform to some of the observed facts and is at the same time a justification of attempts to demonstrate something that is not true. For the state has for long been responsible for a large part of the R&D in every industrial economy and many technical advances, like radar, the jet engine, the computer and many medicines, have depended on research by state institutions or by institutions, including universities and private firms, working on behalf of the state.[57] It is unnecessary to argue this in detail, for the point being made here is that technical progress cannot be understood or satisfactorily described if production is not represented as labour using durable, heterogeneous capital goods.

Still, the academic question remains, is it possible to achieve that purpose by adding R&D to a model of intertemporal general equilibrium like that expounded by Arrow and Hahn? If so, it would avoid the objections to the kinds of models discussed here in the same way as the model of Arrow and Hahn avoids the objections to production functions, equally implausibly. But there is no reason to believe that R&D is compatible with the assumptions about production used in the proof that there is an equilibrium. For, though R&D is like capital in that its cost is composed of wage and profit, its effects on production are different and there is no assurance that they can be accommodated in the assumptions. Investment in capital goods allows more production, whereas R&D is the use of labour and goods to alter the goods available and the processes for making them, and does this irreversibly. In addition, if there are no laws on intellectual property, the benefit from the R&D does not accrue solely to whoever incurred the cost, and, if there, are the production possibilities are not the same for all. The durations of patents and copyright are determined by law and can change and do vary according to product. As pointed out earlier, because the representation of production in such models is a mathematical abstraction, it is not always possible to say if a particular form of production is compatible with the assumptions. For instance, there is no formula that tells to what extent departures from constant returns to scale and externalities are compatible with the model. The same is true of the compatibility of R&D. Perhaps mathematical research will one day supply the knowledge lacking, and it may show that the R&D compatible with the existence proof of general equilibrium is too small to be used for discussing economic growth. As a matter of economic theory, investment in R&D is obviously related to investment in productive capacity and is not determined by calculations of future profit with perfect foresight.

Finally, unrelated to the question of how production is represented, there is the oft made point of the illogicality of models of a distant future determined by technical progress, which is, by its nature, not predictable, except roughly for the near future. Proponents of endogenous growth models like to draw their conclusions from steady states, presumably in some indefinite future, and, therefore, simplify their depictions of technical progress to yield steady states. To be able to do this they confine technical progress to improvements and inventions that differ from those preceding them only by a factor, if at all, a sameness that is the antithesis of technical progress. They go so far as to call the replacement of old versions of goods by improved versions ‘Schumpeterian creative destruction’. Schumpeter, who insisted on the unpredictability of inventions and their disruption of the equilibrium into which the economy moved between inventions, would have objected. For him the steady state was stagnation and its disruption the occasion for entrepreneurship, profit and growth.

[57] Mazzucato, *The Entrepreneurial State : Debunking Public vs. Private Sector Myths*.

3. Capital and International Trade

This section is intended to show that international trade cannot be plausibly described or explained if capital goods are not taken as durable, heterogeneous products. There does not seem to be any theory or model of trade with tradable capital goods of the kind. Rather, what is nowadays the conventional theory, Heckscher-Ohlin, or factor endowments, theory presupposes quantities of factors, one or more usually being capital.

Among the assumptions that are explicit or implicit in the Heckscher-Ohlin theory and necessary for any theory of comparative advantage are the following: goods are either freely tradable without trading costs or are untradable, all production processes function equally well in all countries if the workers with the requisite training are available, all countries have equal access to all production processes and there is no technical progress. Apart from the last, the same assumptions are made here. Trade is assumed to be free of tariff and other administrative hindrances, unless otherwise stated. In addition, capital goods are assumed to be durable and heterogenous and made by workers using capital goods. Ohlin, himself, insisted on the necessity of the second assumption, although he mistakenly thought it followed from the fact that the laws of physics are the same everywhere, and his theory depends on it, for, if production processes differ from country to country, not only do factor endowments alone not explain trade, but almost any pattern of trade can be explained by convenient assumptions about differences of production processes.

Capital as tradable goods.

The argument against the Heckscher-Ohlin theory and against any theory or model that has quantities of capital is that where the various capital goods are made depends on the nominal wages and not the other way round. Wages compared at the going exchange rates differ between countries, often by multiples, and the relative differences have persisted for a century or more with few exceptions, notably Japan, South Korea and China. Most tradable capital goods take less than a year to make and not many are used for more than a decade, at least in the high wage countries, so it cannot be argued that capital goods take too long to make or last too long for it to matter where they are made.

When a tradable good is made in two different countries, the profit margin is higher in the country with the lower wage since the price is the same for both. If the good is made only in the country with the lower wage, the profit margin lies within a range; the price cannot be so high that it becomes profitable to make it in the high wage country and cannot be so low as to make some other tradable good more profitable. This holds obviously if the production techniques are the same in both countries, for the second assumption ensures that the costs from untradable inputs cannot be higher in the low wage country. With a choice of techniques, that used in the country with the lower wage has a lower capital investment and a still bigger profit margin. It is unnecessary to have a full model of trade to conclude that, since trade is free, where goods are produced and what their prices are depend, at least partly, on the nominal wage rates of the various countries. This is true also for tradable capital goods. There is a dual to this; each tradable capital good installed in a country is installed there because it is expected to yield more profit than one that has not been installed there, and that profit depends on nominal wages and the prices of goods.

Wage differences and the prices of goods cannot be the consequences of factor endowments, a phrase that gives the impression that factors are some primal elements with which a country starts. However measured, a country's stock of capital is the result of investment in capital goods produced in that and other countries over the past and, therefore, the consequence of wage differences of the past. And, if wage differences persist, the stocks in the future also depend on them.

As to be expected from a theory with untenable premises, the conclusions are at odds with reality. Several have been much discussed, notably Leontief's calculations that the exports of the US were less capital intensive than the imports, although the US was thought to have more capital per head than any other country; factor price equalisation, according to which all countries would have the same factor prices if the number of goods they exchanged was not smaller than the number of factors, which was surely the case if the factors were only labour and capital, and the tendency to protection, despite the conclusion of the theory that free trade resulted in the best outcome for all countries, barring monopoly powers[58]. Rather than go over the various attempts to avoid or explain these conclusions away, the following discussion is intended to show that there is another phenomenon that the Heckscher-Ohlin theory cannot explain cogently, namely that countries routinely import and export the same goods. It is not compatible with the unmodified theory and its causes show that the theory cannot be used to understand how most trade in manufactures occurs.

Modifying the theory with increasing returns.

According to the Heckscher-Ohlin theory the exchange of the same goods between different countries is impossible, except, as Vanek showed,[59] when all countries have the same factor prices. In reality wage differences are too great to be ignored, yet Helpman and Krugman tried to provide an explanation by assuming that countries do have the same factor prices, so that, because of increasing returns to scale, a firm can reduce its factor costs per unit by specialising in the production of one version of a good whilst firms in the same and other countries producing the same good specialise in the production of other versions.[60] Each version can, then, have a brand name. When returns to scale are increasing the value of the output is less than the payments to factors and monopolistic competition must be assumed to obtain profit. It is Sraffa's argument of 1926.

But its conclusion, that firms concentrate on a single version of the goods they produce, is the opposite to what is observed. Most producers of differentiated goods produce several versions with the same brand names to meet the requirements of different buyers. For example, makers of excavating equipment, printers, refrigerators or toothpaste have several versions on offer and change them as demand and technical possibilities change. As pointed out in the previous section, an evident and more realistic explanation is that most goods are differentiated because they have several characteristics that appeal differently to different buyers, whether consumers, who vary in taste and income, or firms, and because producers invest in R&D to compete in producing better new versions.

Differentiation, intellectual property rights, technical progress and competition.

Helpman and Krugman do allow for goods with several characteristics but must invoke increasing returns because they do not allow for intellectual property rights, i.e. patents,

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[58] Rodrik, *What Does the Political Economy Literature on Trade Policy (Not) Tell Us That We Ought to Know?*

[59] Vanek, "The Factor Proportions Theory: The N-Factor Case."

[60] Helpman and Krugman, *Market Structure and Foreign Trade.*, chap. 7.

copyright and trademarks, as the reasons why different producers do not make the same versions of goods. Yet intellectual property rights are needed for the R&D of firms to be profitable and much technical knowledge is consequently proprietary knowledge of the firms that generated it. These rights lapse with time and, as they do, all producers become free to produce any versions using that knowledge they like, which, assuming returns to scale are not too sharply increasing, some would do but for technical progress. When new and better versions are produced by the various producers old versions cease to be produced, except, perhaps, by producers in low wage countries able to price them low enough to offset their inferiority. What is presupposed in the description of Helpman and Krugman is that there has been no technical progress for so long that all intellectual property rights have lapsed.

Product differentiation and brand names are the rule, not the exception, for manufactured consumption goods in any industrial economy, so that the argument of Helpman and Krugman implies that increasing returns apply to the greater part of the production of these goods. They presumably apply to the production of capital goods as well, with the difference that, whereas consumers vary in preferences and income, firms have the same objective of maximising profit and choose the same capital goods to make the same products if returns to scale are constant. Leaving aside its occurrence because of circumstances specific to individual firms, product differentiation occurs with capital goods either because different versions of a good are made using different versions of the same capital good, or because returns to scale are not constant. Countries can, then, export and import the same capital goods, but that trade depends on the details of product differentiation rather than factor endowments. Thus, it makes a difference to the volume of a country's exports and imports whether the version of a capital good for making a particular version of a consumption good or another capital good is made in the country or abroad, though it could equally well be either. This indeterminacy is similar to that of the Vanek theorem. But the theorem presupposes constant returns and given endowments of factors, whereas Helpman and Krugman assume increasing returns to be prevalent and the stock of the factor capital in each country to be the consequence of how product differentiation occurs. In contrast, Sraffa's argument is not affected by increasing returns in the production of capital goods.

When countries can exchange the same goods and firms can own the designs of versions of goods with the right to prevent others from making or copying them, much of production and competition in foreign markets depends on the prowess of the individual firm, how well the characteristics and prices of the firm's versions compare with those of its competitors. This may seem not to matter if the firms all have the same factor prices, the assumption needed by Helpman and Krugman, but such trade occurs between countries with different factor prices. It is because all but the simplest manufactures are made by firms that have accumulated specialised knowledge that the firms that make these goods are especially good at what they do and have brand names. For instance, Denmark exports high quality stereo music systems, Germany bathroom fixtures, the US bicycles, Brazil passenger aircraft and South Korea steel.

The many different ways by which individual firms came to have their particular capabilities are beyond the scope of this work. It is enough to note that books and articles on the origins and growth of individual firms are a commonplace and show how diverse the origins have been. Some of the biggest firms of the twentieth century came into being because of inventions in the nineteenth; the separation of nickel, a process for making soda, the electric generator, soap packaged with pretty pictures and so on, and then grew through new and improved products and combination with other firms. Others came into being because, as with the motor car, entry was at first practically free but competition and the costs of technical progress in time allowed only a few brands to remain from the many at the start. Yet others had their origins in commerce or were fostered by the state. Of the biggest firms at present, several grew from a single product, perhaps originally developed by state institutions, and have elaborated on that.

Intellectual property rights and access to technology.

Nowadays the capabilities of firms are mostly protected by intellectual property rights generated by technical progress and by in-house knowledge and take form as product differentiation, with the consequence that the third assumption made here, that all countries have equal access to all production techniques, cannot hold. Each firm has its own versions of the goods it produces and others are prevented from making or copying them without permission, at least until the intellectual property rights expire. And, if R&D results in new versions, older versions are discarded, the theoretically possible exception being that a firm acquires the right to make an old version and makes it profitably because it pays lower wages. This exception is not an opportunity to catch up, for the firm granting the right does not want to create another competitor, especially not one making goods like its own, and sells it with conditions, such as restricting the sale to the domestic market of the producer and the exclusive right to supply components and inputs.

When a firm in a low wage country starts producing a differentiated good that is already being produced by firms of advanced industrial economies and is being frequently improved by R&D, it will be technically comparatively backward and unable to produce versions at the same technical level as the established producers. It lacks the proprietary scientific and engineering knowledge that the high wage country firms have acquired over the years through their own R&D. It does have access to the knowledge that is generally available because it has been published or becomes available as patents expire, but it cannot expect to start with that knowledge and catch up with the firms of the high wage countries, which continue advancing through their R&D and also usually have better R&D capabilities. At best, it can hope to produce versions that are out of date compared to the versions of the established firms but can be sold cheaply.

Competitiveness and viability.

A low wage country can, nevertheless, be viable using only scientific and engineering knowledge that is generally available, provided it protects its industries against the competition of the superior products of high wage country firms. It would be using the knowledge that the high wage countries had used some decades earlier, with the addition of such advances as were made in the interval and are generally available. By itself, its economy would be viable, for the high wage countries were viable economies with that knowledge. But its firms would be unable to produce goods as technically advanced as those of the firms of high wage countries and would, therefore, be unable to export and would need protection in the domestic market, except when out of date versions find buyers because they are cheap. If the low wage country did not have to import raw materials it could be autarkic, otherwise it would have to export primary products, manufactures not affected by R&D and some inferior versions of goods to compete with those of the high wage countries. These exports would have low value added compared to the value added they would have if the wage rate was that of the high wage countries, what is called "low wage goods". But they could allow the country's firms to import some technically advanced goods as well while remaining protected. This raises a question of how to choose the combination of, on the one hand, imports of capital equipment technically more advanced than what the domestic industry can produce and, on the other, autarky in other production so as to create more income over the long run. It is a question of economic planning that goes beyond the scope of this work.

Because the third assumption does not hold, the viability of an economy and competitiveness are distinct; the former does not imply the latter. If the assumption did hold, it would follow from the properties of positive matrices that an economy of which no activity is profitable at a set of unique international prices would not be viable. Up to date manufacturing techniques usually require proprietary knowledge and are unavailable to firms that do not have it, even though all production techniques are here assumed to function equally well in all countries. Economies are only competitive in manufacturing activities to the extent that their firms have up to date knowledge or can offset a lack by lower wages. This is nothing new. Among the well known examples, England encouraged the immigration of Flemish weavers, Japan after the Meiji Restoration sent its scientists and engineers to work in the industries of the West to learn from them, the Swiss pharmaceutical industry spied on German chemical companies and China has for some time been requiring foreign firms to agree to transfer proprietary knowledge in exchange for access to its markets. Countries also have the alternative of protecting their domestic industries and advancing technically as generally available knowledge advances. Their firms will have little reason to do their own R&D since it would only generate knowledge equivalent to what the high wage country firms have already generated. This is roughly how India proceeded when it began to industrialise with almost total protection of its industries, and the result has been a big industrial sector, though not up to date or internationally competitive.

Apart from these two alternatives, a country can be open to free trade or can protect a few consumption goods industries, but the tradable manufactures it will produce will be goods or versions of goods the high wage countries do not produce because they are too cheap to be profitable at high wage rates. These include out of date versions of goods the high wage countries do produce. In either case its wage rate must be low if its industries are to be profitable. To the extent, therefore, that the technical knowledge of its firms is behind that of the firms of the high wage countries, a country competes with other countries producing similarly cheap goods and can only compete in the production of goods produced in low wage countries if they have low wages themselves.

Comparative advantage.

These are not conclusions that can be drawn from the notion of comparative advantage. Comparative advantage is taken here to be the notion that the pattern of trade, that is which goods each country exports and imports, and prices are determined by how specific characteristics of countries compare. In Ricardo’s theory the characteristics are the amounts of labour need to make the various goods and in the Heckscher-Ohlin, or factor endowments, theory they are the quantities of factors in each country. Ricardo’s theory has been left aside because it does not have explicit costs associated with capital. This section began with a refutation of the Heckscher-Ohlin theory’s notion of capital as a factor and went on to argue that, as is obvious to any observer, the main determinants of international trade are the nominal wage rates of the various countries, product differentiation and technical progress. Its implication for economic theory is that comparative advantage is a notion that cannot be logically justified and cannot be used to describe or explain international trade.

4. Summing Up

At issue is whether or not representing economic production as functions of quantities of capital can be used to describe or explain the functioning of real economies, and the answer is that any assertion about economic reality that depends on functions of quantities of capital is wrong. Three reasons have been given. One is that of Sraffa, that the value of the stock of capital is not a quantity independent of wages and profits but is determined by them. A second reason derives from technical progress and was first referred to by Pasinetti. The third derives from international trade. They are logically distinct, though the second and the third only became apparent because of the first.

It follows from the first reason that production relations alone do not determine wage and profit rates. This indeterminacy can be removed by introducing demand. For instance, wage and profit rates in steady states are determined by the saving and consumption rates. One degree of indeterminacy may remain, for employment is not necessarily determined. Some of the marginal relations derived from production functions can hold for comparisons between steady states, which means they are mathematical relations, not relations of cause and effect. And they cannot be used to describe change over time outside steady states, for wages and profits change and cause the plant and machinery that are fixed capital and relative prices to change, too. Such changes are too complicated to describe or for households and firms to calculate optimal expenditure and production, and, as is evident, real economies are not in equilibrium, for expectations are not all realised. Intertemporal general equilibrium models can show, with some assumptions that seem general and others impossible, that there is equilibrium with full employment apart from steady states, but these models are just mathematical existence theorems unrelated to reality. Their very generality and abstractness obscure the extent to which the assumptions can allow for the complications of reality. It is not known, for instance, to what extent their assumptions can accommodate R&D and the technical progress it results in. These models are also illogical in the sense that they require perfect foresight, though technical progress is inherently unpredictable, certainly over the long run.

There are also the models and estimates of technical progress using aggregates which equate technical progress to the part of increased output or GDP not attributable to increased use of factors. There is, as yet, no objective way of quantifying technical progress in the form of improvements of consumption and capital goods and the invention of new ones, nor the R&D of which it is the result. Notional adjustments of output and GDP can be made and may be necessary, but they are not objective. Hence, these models and estimates either omit some technical progress or, if the notional adjustments are made, rely on supposition. Technical progress defined like this, whether adjusted for technical progress in the production of capital or not, is only what the models cannot explain, besides which there is no independent check that its estimates are not merely the outcome of the errors and approximations of the data and their treatment. R&D should be allowed for in any residual by deducting its cost, which can leave little or nothing, and whatever may be left can still be technical progress from the same source. Then there are the endogenous growth models, which do not provide means for estimating technical progress but have as their purpose to show that firms in competitive free markets will invest in R&D and reach a steady state. They need the assumptions that R&D can be quantified, that goods are made by hand or are same as the final good and that firms and consumers foresee the future goods, which are just the present goods multiplied by a known parameter, sometimes unity.

Since most international trade is in manufactures, it is to be expected that technical progress should be one of the main determinants of the pattern of trade. Most tradable manufactures are goods made by firms competing among themselves by using R&D to design improved versions of the goods they make. The competition is international and countries export versions made by their firms and import versions made abroad. Firms are prevented from copying the versions of competitors by the laws on intellectual property, besides which much of the technical knowledge of firms is in-house and not available to others. A firm starting to produce a good of this kind will not have technical knowledge equivalent to the patented and in-house knowledge of established producers, which continue their R&D and keep their lead. Low wage country firms that do not have up to date technical knowledge may still be able to compete by producing cheap versions inferior to those of high wage country firms. But, to reach the level of these firms, they must give the established producers enough inducement for them to impart their knowledge despite their aversion to creating new competitors, for instance by allowing access to promising markets. If they cannot do that, low wage country firms are confined to competing among themselves with the same simple goods and, perhaps, cheap versions of goods produced in high wage countries, i.e. low wage goods with little value added.

Much of manufacture is accounted for by firms with proprietary specialised knowledge, be they big multinational firms or small, and they persist because of their R&D, contrary to the endogenous growth models. A function of brand names is to signal this. In addition, much R&D and scientific research are carried out by state and other organisations. One consequence is that the firms, especially the biggest, that constitute much of the industry of a country came into existence under a variety of circumstances, not because of factor endowments. A firm that began with one good may have changed out of recognition through diversification, acquisitions, mergers and changes of its original products. Another consequence is that the relation between the degree of advancement of an economy and its competitiveness is loose, for competing firms almost certainly do similar R&D. In rough terms, the more different versions of a particular good there are, the greater the amount of R&D. It is also possible for the firms of two countries producing the same good to be equally advanced, but for those of the one to produce more versions than those of the other. In that case the first country is likely to be more competitive in the sense that it will sell more of that good, other things being equal. Such assertions are necessarily imprecise because they concern phenomena, notably technical progress and R&D, that cannot be quantified and that make dependable mathematical formulation of economies impossible, at least at present. This may seem unfortunate, but such assertions are not the less useful for being imprecise. Certainly more useful than models that provide formulae for equilibria in some indefinite futureå when it is hard to say what the state of an economy is in the present, and seem just to be ends in themselves.

Far from being about theoretical abstractions, the disputes about capital were about the basic and practical questions of understanding how economies function and formulating policies. Ignoring the conclusions to be drawn from the disputes as mere abstractions on the grounds of being practical or pragmatic has had the perverse effect, that it is the models that have resulted that are abstractions remote from reality. Being practical and pragmatic is also often given as justification for simplifications that make the mathematics of models easier and more manageable, one aspect being that the results are those desired or intended, and the consequence has been that it has become normal to devise mutually irreconcilable models purporting to explain different aspects of the same phenomena and ignore the inconsistencies. Yet it is sometimes asserted that mathematically formulated models are rigorous and verbal arguments are impressionistic. The mathematics may be rigorous, but the economics is not.

Things that are not quantifiable or too complicated to be amenable to mathematics can be discussed verbally with rigour, assertions need not be quantifiable to be precise and statements that cannot be proved can be made explicitly as conjecture. Assertions only obtainable from simplifications made to formulate such things mathematically are tautologies of the simplifications, conjecture at best, otherwise unrelated to things that are.

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