Mechanism design with farsighted agents

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Abstract

Agents are farsighted when they consider the ultimate results to which their own actions may lead to. We re-examine the classical questions of implementation theory under complete information in a setting with transfers where farsighted coalitions are regarded as fundamental behavioral units and the equilibrium outcomes of their interactions are predicted via the stability notion of the largest consistent set. The designer's exercise consists of designing a rights structure, which formalizes the idea of power distribution in society. His or her challenge lies in designing a rights structure in which the equilibrium behavior of agents always coincides with the recommendation given by a social choice function. We show that (Maskin) monotonicity *fully* identifies the class of social choice functions that are implementable by a rights structure. We also discuss how this result changes when social choice correspondences are considered.

Keywords: Farsightedness, implementation, transfers, rights structures, coalitions, largest consistent set, monotonicity.
JEL: C71, D71, D82.

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1 Introduction

The challenge of implementation lies in designing a mechanism (i.e., game form) in which the equilibrium behavior of agents always coincides with the recommendation given by a (single-valued) *social choice function* (SCF). If such a mechanism exists, the SCF is implementable.

As such, the key question is how to design an implementing mechanism so that its outcomes are predicted through the application of game theoretic concepts. Most early studies of implementation focused on noncooperative solution concepts such as the Nash equilibrium and its refinements. As demonstrated in the seminal paper by Koray and Yildiz (2018), an alternative to the noncooperative approach is to allow groups of agents to coordinate their behaviors in a mutually beneficial way. To move away from noncooperative modeling, the details of coalition formation are not modeled. Then, coalitions—not individuals—become the basic decision-making units. Here, the role of the solution concept is to explain why, when, and which coalition forms and what it can achieve.

More importantly, the chosen coalitional solution concept is independent of the physical structure under which coalition formation takes place (e.g., Chwe, 1994). This structure, often defined by an effectivity relationship, specifies which coalitions can form given a status quo outcome, and what they can achieve when they form (i.e., what new status quo outcomes they can induce). From the implementation viewpoint, the effectivity relationship is the design variable of the designer, playing the role of the mechanism.

Koray and Yildiz (2018) formalize this idea and study its implications. In their framework, the implementation of an SCF is achieved by designing a generalization of the effectivity relationship, introduced by Sertel (2001), called a *rights structure*.[1] A rights structure Γ consists of a state space S, an outcome function h that associates every state with an outcome, and a code of rights γ. A code of rights specifies, for each pair of states (s, t), a collection of coalitions $\gamma(s, t)$ effective at moving from s to t. The rights structure is more flexible than the effectivity function, as it allows the

[1]McQuillin and Sugden (2011) have more recently proposed a similar notion named *game in transition function form* as a generalization of effectivity functions.

strategic options of coalitions to depend on how the status quo outcome is reached (i.e., on the current state).

As a coalitional solution, Koray and Yildiz (2018) adopt a version of the *core*.[2] A state t *directly* dominates a state s if a coalition K exists that is effective at moving from s to t and each member of K has under t a payoff larger than the one he receives under s. A state s is a core state under a given rights structure and agents' preferences if no state that directly dominates it exists.

This classical solution is based on a myopic notion of dominance, which creates problems that cannot be ignored. Ray and Vohra (2014) illustrate this point clearly by means of the following example, using two agents and three states. Suppose that only agent 1 is effective at moving from s to t, that is, $s \to^{\{1\}} t$, and only agent 2 is effective at moving from t to s', that is, $t \to^{\{2\}} s'$. Figure 1 depicts this example, where the payoffs to the agents in each of the states are in parentheses.

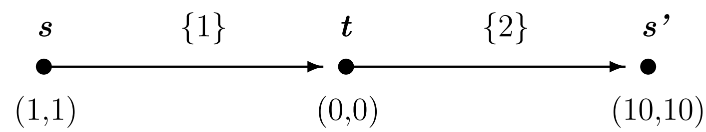

Figure 1.

The core consists of the states s and s'. Although agent 1 has the power to move from s to t, he has no incentive to do so: t does not directly dominate s. However, the stability of s is based on myopic reasoning. Indeed, if agent 1 was farsighted, he should move to t because agent 2 (who is rational) will in turn move to s'. Thus, a farsighted agent moves not necessarily because he has a direct objection, but because his moves can trigger further changes, eventually leading to a *better* outcome. Clearly, the classic notion of core does not incorporate any farsightedness.

Harsanyi (1974), in his critique of the vN-M stable set (von Neumann and Morgenstern, 1947), suggests replacing the notion of direct dominance with "indirect

[2]Korpela et al (2018) study implementation of the core points by a rights structure $\Gamma = (S, h, \gamma)$ where S is the set of outcomes and h is the identity map.

dominance". In defining his *largest consistent set* (LCS), Chwe (1994) formalizes a version of Harsanyi's indirect dominance. A state t *indirectly* dominates s if t can replace s via a sequence of "moves" such that, at each move, the effective moving coalition prefers the outcome associated with t (the final state) to the outcome it would obtain if it decided not to move (for a formal definition, see Definition 3). As shown in Figure 1, s' indirectly dominates s. This is so because agent 1 can move from s to t and his payoff at s' is larger than his payoff at t, and agent 2 can move from t to s' and his payoff at s' is larger than his payoff at t. Thus, indirect dominance captures the fact that farsighted agents consider the final states to which their moves may lead to.

Based on this notion of indirect dominance, Chwe (1994) suggests a new concept of stability, namely, the LCS, which has the advantages of "ruling out with confidence" and being non-empty under weak conditions.[3] However, as Chwe (1994) points out, it may be too inclusive. To check whether a state s is stable, suppose that a coalition K deviates to a state t. Further deviations from t may occur, which end up at s', where s' indirectly dominates t. Alternatively, no further deviations from t may occur, and so $t = s'$ is the final state. In either case, the final state s' should itself be stable. If a member of the deviating coalition does not prefer s' to the original state s, then the *deviation is deterred*. A state s is *stable* if all deviations are deterred. Since whether a state is stable depends on whether other stable states exist, a set of stable states is called a *consistent set*. Although many consistent sets may exist, there *uniquely* exists the LCS, that is, a consistent set that includes all others. If a state s is not contained in the LCS, the interpretation is that s cannot be stable: there is no consistent story behind s.

In this paper, we adopt the LCS as a coalitional solution. The implementation problem consists of designing a rights structure Γ with the property that, for each profile of agents' preferences, the outcome associated with the LCS always coincides

[3] There is a growing literature that studies farsighted stability in coalitional games, which includes Aumann and Myerson (1988), Xue (1998), Diamantoudi and Xue (2003), Herings et al (2004), Jordan (2006), Ray (2007), Mauleon et al (2011), Vartiainen (2011), Kimya (2015), Ray and Vohra (2015), Bloch and van den Nouweland (2017), Dutta and Vohra (2017), Dutta and Vartiainen (2018), and Vohra and Ray (2019).

with the recommendation of the given SCF. If such a rights structure exists, the SCF is LCS-implementable by a rights structure.

We investigate the LCS-implementation of SCFs in environments with transfers. Agents' preferences are continuous and money-monotonic (Morimoto and Serizawa, 2015). For this class of implementation problems, an SCF is LCS-implementable by a rights structure if and only if it is (Maskin) monotonic—monotonicity is the central condition for implementation in Nash equilibria (Maskin, 1999). Further, any SCF that is LCS-implementable via a rights structure is also LCS-implementable via an individual-based rights structure. This means that to LCS-implement a monotonic SCF it is sufficient to allocate power only to coalitions of size one.

The characterization result above leaves unspecified how the initial state is determined. However, there are economic situations in which the initial state is naturally determined. For instance, the initial state is no production in a Cournot oligopoly market. We thus analyze implementation problems in which the initial state is predetermined and show that every monotonic SCF is LCS-implementable by a rights structure satisfying the following *convergence property*: every stable state directly dominates the initial state. Therefore, we establish a direct convergence from the initial unstable state to stable states, which is particularly important in our design framework. This result relies on the domain assumption that each agent considers the outcome associated with the initial state to be worse than any outcome in the range of the SCF. As an application of this result, we consider oligopoly markets with farsighted firms in which the initial state is characterized by no production. We show that the Cournot oligopoly equilibrium output is LCS-implementable by a rights structure satisfying the property above.

The remainder of the paper is divided into four sections. Section 2 sets out the theoretical framework and outlines the basic model. Section 3 provides a characterization of the class of SCFs that are LCS-implementable by rights structures. Section 4 characterizes a class of monotonic SCFs that are LCS-implementable by a rights structure satisfying the convergence property. Section 5 concludes the paper.

2 Setup

The *environment* consists of a collection of n agents (we write N for the set of agents), a set of possible types Θ, and a (nonempty) set of outcomes Z. We consider an environment with transfers. Specifically, we assume that the set of outcomes is $Z \equiv D \times \mathbb{R}^n$. D is the set of potential social decisions, with $d \in D$ as typical element. $\mathbb{R}^n$ is the set of transfers to the agents, with $t = (t_1, ..., t_n) \in \mathbb{R}^n$ as a typical transfer profile.

To each type in Θ, we associate for each agent i a utility function $u_i : Z \times \Theta \to \mathbb{R}$. Given a type θ and an outcome x, let the upper contour set and lower contour set of $u_i(\cdot, \theta)$ at x be defined by $U_i(x, \theta) \equiv \{x' \in Z | u_i(x', \theta) \geq u_i(x, \theta)\}$ and $L_i(x, \theta) \equiv \{x' \in Z | u_i(x, \theta) \geq u_i(x', \theta)\}$, respectively. For each agent $i \in N$, agent i's utility function $u_i : Z \times \Theta \to \mathbb{R}$ is assumed to satisfy the following properties.

Definition 1 Agent i's utility function $u_i : Z \times \Theta \to \mathbb{R}$ is *money-monotonic* provided that for each $\theta \in \Theta$, each $d \in D$, each $t_{-i} \in \mathbb{R}^{n-1}$ and each $t_i, t_i' \in \mathbb{R}$, if $t_i < t_i'$, then $u_i(d, (t_{-i}, t_i'), \theta) > u_i(d, (t_{-i}, t_i), \theta)$.

Definition 2 Agent i's utility function $u_i : Z \times \Theta \to \mathbb{R}$ is *continuous* provided that for each $\theta \in \Theta$ and each $x \in Z$, the sets $L_i(x, \theta)$ and $U_i(x, \theta)$ are closed.

We focus on complete information environments in which the true type is common knowledge among agents but unknown to the designer. The power set of N is denoted by $\mathcal{N}$, and $\mathcal{N}_0 \equiv \mathcal{N} - \{\varnothing\}$ is the set of all nonempty subsets of N. Each group of agents K (in $\mathcal{N}_0$) is a coalition.

The goal of the designer is to implement an SCF $f : \Theta \to Z$ defined by $f(\theta) \in Z$ for every $\theta \in \Theta$. We refer to $f(\theta)$ as the f-optimal outcome at θ.

To implement his goal, the designer devises a rights structure Γ, which is a triplet (S, h, γ), where:

- S is the state space;
- $h : S \to Z$ the outcome function; and
- γ a code of rights, which is a (possibly empty) correspondence $\gamma : S \times S \twoheadrightarrow \mathcal{N}$.

In other words, a code of rights γ specifies, for each pair of states (s,t), a family of coalitions $\gamma(s,t)$ entitled to approve a change from state s to t. A rights structure Γ is said to be an individual-based rights structure if, for each pair of distinct states (s,t), $\gamma(s,t)$ contains only coalitions of size one if it is non-empty.

To capture farsightedness, Chwe (1994) formalizes the following notion of "indirect dominance" relation—a notion informally introduced by Harsanyi (1974) in his criticism of the vN-M stable set (von Neumann and Morgenstern, 1947), which is based on "direct dominance." For all $\theta \in \Theta$ and $K \in \mathcal{N}_0$, let $x \ u_K^\theta \ y$ denote $u_i(x,\theta) > u_i(y,\theta)$ for all $i \in K$.

Definition 3 *A* state s is *indirectly dominated* by s' at (Γ,θ), or $s' \gg^{(\Gamma,\theta)} s$, if there exist $s_0, s_1, ..., s_J$ in S (where $s_0 = s$ and $s_J = s'$) and $K_0, K_1, ..., K_{J-1}$ in $\mathcal{N}_0$ such that $K_{j-1} \in \gamma(s_{j-1}, s_j)$ and $h(s') \ u_{K_{j-1}}^\theta \ h(s_{j-1})$ for $j = 1, ..., J$. A state s is *directly dominated* by s' at (Γ,θ) if $J = 1$.

Based on this indirect dominance, the LCS of Chwe (1994) can be defined as follows.[4]

Definition 4 (Chwe, 1994) For any Γ and any $\theta \in \Theta$, a set $T \subseteq S$ is a *consistent set* at (Γ,θ) if $s \in T$ if and only if for all $t \in S$ and all $K \in \mathcal{N}_0$ such that $K \in \gamma(s,t)$, there exists $s' \in T$, where $s' = t$ or $s' \gg^{(\Gamma,\theta)} t$, such that not $h(s') \ u_K^\theta \ h(s)$. The *LCS* at (Γ,θ), denoted by $LCS(\Gamma,\theta)$, is the unique maximal consistent set at (Γ,θ) with respect to set inclusion. We refer to $s \in LCS(\Gamma,\theta)$ as a *stable state* (at (Γ,θ)).

Our notions of implementation can be stated as follows.

Definition 5 A rights structure Γ *implements f in the LCS*, or simply *LCS-implements* f, if and only if $f(\theta) = h \circ LCS(\Gamma,\theta)$ for all $\theta \in \Theta$. If such a Γ exists, then f is *LCS-implementable by a rights structure.*

[4]Given a game (Γ,θ) where Γ is such that S is the set of outcomes and h is the identity map, Chew shows that if S is countable and contains no infinite sequence $s_1, s_2, ...$ such that $j > i$ implies that $s_j \gg^{(\Gamma,\theta)} s_i$, then $LCS(\Gamma,\theta)$ is nonempty. This result has been extended by Xue (1997) by removing the countability requirement.

Definition 6 The SCF f is *LCS-implementable by an individual-based rights structure* if there exists an individual-based rights structure Γ such that it LCS-implements f.

3 A characterization result

A well-known condition in mechanism design is (Maskin) *monotonicity* (Maskin, 1999). This condition states that if the f-optimal outcome at type θ does not strictly fall in the preference for anyone when the type is changed to θ', then $f(\theta)$ must remain the f-optimal outcome at θ'.

Definition 7 The SCF f is *monotonic* provided that for all $\theta, \theta' \in \Theta$, if

$$L_i(f(\theta), \theta) \subseteq L_i(f(\theta), \theta') \text{ for all } i \in N,$$

then $f(\theta) = f(\theta')$.

In our environment, monotonicity is equivalent to quasimonotonicity, which appears in Cabrales and Serrano (2011). As the type changes from θ to θ', quasimonotonicity requires that the f-optimal outcomes coincide whenever, for each agent, $f(\theta)$ does not move down in the agent's strict ranking. To introduce this condition, we need the following additional notation. For each agent $i \in N$, each type $\theta \in \Theta$, and each outcome $x \in Z$, agent i's strict lower contour set of $u_i(\cdot, \theta)$ at x is defined by $SL_i(x, \theta) = \{y \in Z | u_i(x, \theta) > u_i(y, \theta)\}$.

Definition 8 The SCF f is *quasimonotonic* provided that for all $\theta, \theta' \in \Theta$, if

$$SL_i(f(\theta), \theta) \subseteq SL_i(f(\theta), \theta') \text{ for all } i \in N,$$

then $f(\theta) = f(\theta')$.

Lemma 1 f is *monotonic* if and only if f is *quasimonotonic.*

Proof. Suppose that f is quasimonotonic. Fix any $\theta, \theta' \in \Theta$. Suppose that $L_i(f(\theta), \theta) \subseteq L_i(f(\theta), \theta')$ for all $i \in N$. Take any $x = (d, t) \in SL_i(f(\theta), \theta) \subseteq$

$L_i(f(\theta),\theta)$ for some $i \in N$. We show that $x \in SL_i(f(\theta),\theta')$. Assume, on the contrary, that $x \notin SL_i(f(\theta),\theta')$, so that $u_i(x,\theta') \geq u_i(f(\theta),\theta')$. Since $SL_i(f(\theta),\theta) \subseteq L_i(f(\theta),\theta')$, it follows that $u_i(x,\theta') = u_i(f(\theta),\theta')$. Because $u_i(x,\theta) < u_i(f(\theta),\theta)$ and because u_i is continuous and money-monotonic, there exists a transfer profile $\epsilon \in \mathbb{R}^n_{++}$ such that

$$\begin{aligned} u_i((d,t+\epsilon),\theta') &> u_i(x,\theta') = u_i(f(\theta),\theta') \text{ and} \\ u_i(f(\theta),\theta) &> u_i((d,t+\epsilon),\theta), \end{aligned}$$

which contradicts our initial supposition that $L_i(f(\theta),\theta) \subseteq L_i(f(\theta),\theta')$. Since the choice of the outcome x as well as that of agent i was arbitrary, we conclude that $SL_i(f(\theta),\theta) \subseteq SL_i(f(\theta),\theta')$ for all $i \in N$. Quasimonotonicity implies that $f(\theta) = f(\theta')$. Thus, f is monotonic.

Suppose that f is monotonic. Fix any $\theta, \theta' \in \Theta$. Suppose that $SL_i(f(\theta),\theta) \subseteq SL_i(f(\theta),\theta')$ for all $i \in N$. Take any $x = (d,t) \in L_i(f(\theta),\theta)$ for some $i \in N$. We show that $x \in L_i(f(\theta),\theta')$. Assume, on the contrary, that $x \notin L_i(f(\theta),\theta')$, so that $u_i(x,\theta') > u_i(f(\theta),\theta')$. An immediate contradiction is obtained when $x \in SL_i(f(\theta),\theta)$. Thus, let us consider the case in which $u_i(x,\theta) = u_i(f(\theta),\theta)$. Since $u_i(x,\theta') > u_i(f(\theta),\theta')$ and since u_i is continuous and money-monotonic, there exists a transfer profile $\epsilon \in \mathbb{R}^n_{++}$ such that

$$\begin{aligned} u_i((d,t-\epsilon),\theta') &> u_i(f(\theta),\theta') \text{ and} \\ u_i(f(\theta),\theta) &= u_i(x,\theta) > u_i((d,t-\epsilon),\theta), \end{aligned}$$

which contradicts our initial supposition that $SL_i(f(\theta),\theta) \subseteq SL_i(f(\theta),\theta')$. Since the choice of the outcome x, as well as that of agent i, was arbitrary, we conclude that $L_i(f(\theta),\theta) \subseteq L_i(f(\theta),\theta')$ for all $i \in N$. Monotonicity implies that $f(\theta) = f(\theta')$. Thus, f is quasimonotonic. ■

The following lemma shows that quasimonotonicity is necessary and sufficient for LCS-implementation by a rights structure. The rights structure Γ we construct works as follows. We define it as the union of *disconnected* rights structures $\Gamma^\theta = \left(S^\theta, h^\theta, \gamma^\theta\right)$, for $\theta \in \Theta$. They are disconnected in the sense that for any two θ and

θ', no coalition is effective at moving from one state of Γ^θ to a state of $\Gamma^{\theta'}$. Figure 2 depicts Γ for the case in which $\Theta = \{\theta, \theta'\}$ and $i, j \in N$.

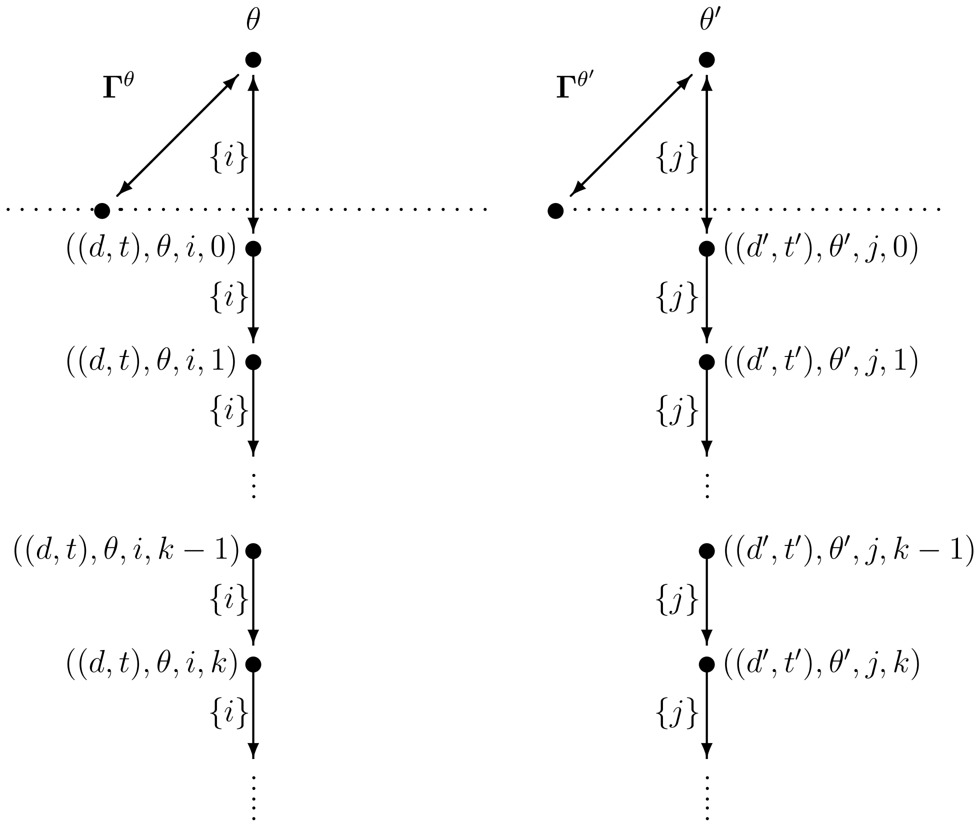

Figure 2. Construction of the rights structure Γ where $\Theta = \{\theta,\ \theta'\}$, $i, j \in N$, $(d,\ t) \in SL_i(f(\theta),\ \theta)$, $(d',\ t') \in SL_j(f(\theta'),\ \theta')$, $h(\bar{\theta}) = f(\bar{\theta})$ for both $\bar{\theta} \in \Theta$, and $h\left((d,t),\theta,i,k)\right) = \left(d, \left(t_{-i}, t_i + \frac{k}{k+1}\hat{t}_i\right)\right)$, where $\hat{t}_i$ is a small transfer that satisfies the strict inequality in (2).

The state space S^θ is $S^\theta = \theta \cup T^\theta$, where T^θ is defined by

$$T^\theta = \left\{\left(\left(d,t\right),\theta,i,k\right) \middle| \left(d,t\right) \in SL_i\left(f\left(\theta\right),\theta\right) \text{ for } i \in N \text{ and } k \in \mathbb{Z}_+\right\}, \qquad (1)$$

where $\mathbb{Z}_+$ denotes the set of non-negative integers. The outcome corresponding to θ is $f(\theta)$. To define the outcome corresponding to the state $((d,t),\theta,i,k)$, we fix an arbitrarily small transfer $\hat{t}_i$ such that[5]

$$u_i(f(\theta),\theta) > u_i\left((d,t) + \left(0_{-i},\hat{t}_i\right),\theta\right) > u_i((d,t),\theta). \tag{2}$$

This transfer exists because u_i is continuous. The outcome corresponding to $((d,t),\theta,i,k)$ is $h^{\theta}((d,t),\theta,i,k) = \left(d, t + \left(0_{-i}, \frac{k}{k+1}\hat{t}_i\right)\right)$, so that agent i's outcome is $\left(d, t_i + \frac{k}{k+1}\hat{t}_i\right)$. This definition is important because it rules out the state $((d,t),\theta,i,k)$ as a stable state, irrespective of the true type. To see this, we first need to define the code of rights γ^{θ}. Since $(d,t) \in SL_i(f(\theta),\theta)$, we allow only agent i to be effective at moving from θ to $((d,t),\theta,i,0)$, from $((d,t),\theta,i,0)$ back to θ, and from $((d,t),\theta,i,k)$ to $((d,t),\theta,i,k+1)$. In all other cases, no coalition is effective (see Figure 2). To see that no state of the form $((d,t),\theta,i,k)$ can be a stable state, it suffices to observe that the money-monotonicity of agent i's utility function assures that

$$u_i\left(d, t_i + \frac{k+1}{k+2}\hat{t}_i, \theta'\right) > u_i\left(d, t_i + \frac{k}{k+1}\hat{t}_i, \theta'\right)$$

for every non-negative integer $k \geq 0$ and every type $\theta' \in \Theta$, so that agent i always has the power as well as the incentive to move from $((d,t),\theta,i,k)$ to $((d,t),\theta,i,k+1)$.

An important consequence of this construction is that if $SL_i(f(\theta),\theta) \subseteq SL_i(f(\theta),\theta')$ for each agent $i \in N$, then the LCS of the game $\left(\Gamma^{\theta},\theta'\right)$ is equal to $\{\theta\}$. In all other cases, namely, when there is a preference reversal, it is empty.

To see that $LCS\left(\Gamma^{\theta},\theta'\right) = \{\theta\}$, note that no state of the form $((d,t),\theta,i,k)$ can be a stable state, as we have already noted. Thus, $LCS\left(\Gamma^{\theta},\theta'\right) = \{\theta\}$ if we show that the set $\{\theta\}$ is a consistent set of $\left(\Gamma^{\theta},\theta'\right)$ when $SL_i(f(\theta),\theta) \subseteq SL_i(f(\theta),\theta')$ for each $i \in N$. By construction, agent i is effective at moving from θ to $((d,t),\theta,i,0)$ and from $((d,t),\theta,i,0)$ back to θ. Suppose that θ is the status quo and agent i moves the state to $((d,t),\theta,i,0)$. Agent i has the incentive, as well as the power, to go back to θ since the inequality in (2) holds and since $SL_i(f(\theta),\theta) \subseteq SL_i(f(\theta),\theta')$. Because this reasoning holds for any state of the form $((d,t),\theta,i,0)$, one can see that θ is a consistent set of $\left(\Gamma^{\theta},\theta'\right)$.

[5] $(0_{-i},t_i)$ denotes a transfer profile which assigns t_i to agent i and zero to everyone else.

Let us discuss why $LCS\left(\Gamma^{\theta}, \theta'\right)$ is empty when there is a preference reversal. Suppose that agent i has a preference reversal around $f(\theta)$ when θ moves to θ'; that is,

$$u_i\left(f(\theta), \theta\right) > u_i\left((d,t), \theta\right) \text{ and } u_i\left(f(\theta), \theta'\right) \leq u_i\left((d,t), \theta'\right). \tag{3}$$

As we have already noted, no state of the form $((d,t), \theta, i, k)$ can be a stable outcome of $\left(\Gamma^{\theta}, \theta'\right)$. If $LCS\left(\Gamma^{\theta}, \theta'\right)$ were non-empty, it should hold that $LCS\left(\Gamma^{\theta}, \theta'\right) = \{\theta\}$. However, $\{\theta\}$ is not a consistent set. To see this, suppose that agent i moves from θ to $((d,t), \theta, i, 0)$. This agent also has the power to move the state back to θ. However, he has no incentive to do so since the weak inequality in (3) holds. Thus, when there is a preference reversal, the LCS of $\left(\Gamma^{\theta}, \theta'\right)$ is empty.

The rest of the proof is based on this consequence. Suppose that θ' is the true type. We want to argue that $f(\theta') = h\left(LCS\left(\Gamma, \theta'\right)\right)$. As we have already noted in the preceding paragraphs, one can easily see that θ' is a consistent set of the game $\left(\Gamma^{\theta'}, \theta'\right)$, and so $\theta' \in LCS\left(\Gamma, \theta'\right)$. For the converse, suppose that $\theta \in LCS\left(\Gamma, \theta'\right)$. By construction, one can see that $LCS\left(\Gamma, \theta'\right)$ is the union of LCSs of games of the type $\left(\Gamma^{\bar{\theta}}, \theta'\right)$, one for each $\bar{\theta} \in \Theta$. Therefore, it follows that $\theta \in LCS\left(\Gamma^{\theta}, \theta'\right)$ and that preferences change when we move from θ to θ' in such a way that it is true for no agent that $f(\theta)$ has fallen with respect to any other outcome in his personal ranking; that is, the strict lower contour sets at $f(\theta)$ are nested. Quasimonotonicity implies that $f(\theta) = f(\theta')$.

We now turn to the formal argument.

Lemma 2 *The following statements are equivalent:*

(i) *F is* LCS-implementable by a rights structure*;*

(ii) *F is* quasimonotonic*;*

(iii) *F is* LCS-implementable by an individual-based rights structure*.*

Proof.

(i) $\Longrightarrow$ ***(ii)*** Suppose that Γ LCS-implements f. Fix any $\theta, \theta' \in \Theta$. Suppose that $SL_i(f(\theta), \theta) \subseteq SL_i(f(\theta), \theta')$ for all $i \in N$. We show that $f(\theta) = f(\theta')$.

By the LCS-implementability of f, we have that $h \circ LCS\,(\Gamma, \theta) = f\,(\theta)$, and so, there exists $s \in LCS\,(\Gamma, \theta)$ such that $h\,(s) = f\,(\theta)$. Let us first show that the set $LCS\,(\Gamma, \theta)$ is a consistent set at (Γ, θ').

Fix any $t \in S$ and any $s \in LCS\,(\Gamma, \theta)$ such that $h\,(s) = f\,(\theta)$. Nothing has to be proved if $\gamma\,(s,t) = \varnothing$. Then, suppose that $K \in \gamma\,(s,t)$ for some $K \in \mathcal{N}_0$. Since $h \circ LCS\,(\Gamma, \theta) = f\,(\theta)$ and $K \in \gamma\,(s,t)$, and since $LCS\,(\Gamma, \theta)$ is a consistent set at (Γ, θ), it follows that there exists $s' \in LCS\,(\Gamma, \theta)$ such that either $s' = t$ or $s' \gg^{(\Gamma,\theta)} t$, and not $h\,(s')\ u^{\theta}_K\ h\,(s)$. Note that $h\,(s') = h\,(s) = f\,(\theta)$.

Suppose that $s' = t$. Then, there exists $s' \in LCS\,(\Gamma, \theta)$ such that not $h\,(s')\ u^{\theta'}_K$ $h\,(s)$.

Suppose that $s' \neq t$, and so $s' \gg^{(\Gamma,\theta)} t$. Since $SL_i(h\,(s')\,, \theta) \subseteq SL_i(h\,(s')\,, \theta')$ for all $i \in N$, it follows that $s' \gg^{(\Gamma,\theta')} t$. Thus, we have established that there exists $s' \in LCS\,(\Gamma, \theta)$, where $s' \gg^{(\Gamma,\theta')} t$, such that not $h\,(s')\ u^{\theta'}_K\ h\,(s)$.

Since $t \in S$, $K \in \mathcal{N}_0$ and $s \in LCS\,(\Gamma, \theta)$ have been chosen arbitrarily, we have proved that $LCS\,(\Gamma, \theta)$ is a consistent set at (Γ, θ').

Since $LCS\,(\Gamma, \theta')$ is the LCS at (Γ, θ') with respect to set inclusion and $LCS\,(\Gamma, \theta)$ is a consistent set at (Γ, θ'), it follows that $LCS\,(\Gamma, \theta) \subseteq LCS\,(\Gamma, \theta')$, and so $f\,(\theta) = f\,(\theta')$, by the LCS-implementability of f. Thus, f is quasimonotonic

(ii) $\Longrightarrow$ ***(iii)*** Suppose that f is quasimonotonic. For any $\theta \in \Theta$, let $\Gamma^{\theta} = \left(S^{\theta}, h^{\theta}, \gamma^{\theta}\right)$ be defined as follows. Define the set T^{θ} as in (1). Then, define the set S^{θ} by $S^{\theta} = \theta \cup T^{\theta}$.

Fix any $i \in N$. Suppose that $y = (d,t) \in SL_i\,(f\,(\theta)\,, \theta)$. Fix any arbitrarily small positive transfer $\hat{t}_i$ such that the inequality in (2) is satisfied. Define the outcome function $h^{\theta} : S^{\theta} \to Z$ by

$$h^{\theta}\,(\theta) = f\,(\theta) \text{ and } h^{\theta}\,((d,t)\,, \theta, i, k) = \left(d, t + \left(0_{-i}, \frac{k}{k+1}\hat{t}_i\right)\right).$$

Let us define $\gamma^{\theta} : S^{\theta} \times S^{\theta} \twoheadrightarrow \mathcal{N}$ as follows.

(1) For all $(y, \theta, i, 0) \in S^{\theta}$, $\gamma^{\theta}\,(\theta, (y, \theta, i, 0)) = \gamma^{\theta}\,(\theta, (y, \theta, i, 0)) = \{i\}$.

(2) For all $(y, \theta, i, k)\,, (y, \theta, i, k+1) \in T^{\theta}$, $\{i\} = \gamma\,((y, \theta, i, k)\,, (y, \theta, i, k+1))$.

(3) Otherwise, it is empty.

Let us define the individual-based rights structure $\Gamma = (S, h, \gamma)$ as follows. Define the state space S by

$$S = \cup_{\theta \in \Theta} S^{\theta}.$$

Define the outcome function $h : S \to Z$ by $h(s) = h^{\theta}(s)$ for all $s \in S^{\theta}$ and all $\theta \in \Theta$. Define the code of rights $\gamma : S \times S \twoheadrightarrow \mathcal{N}$ as follows. For all $s, s' \in S$,

(A) If $s, s' \in S^{\theta}$ for some $\theta \in \Theta$, then $\gamma(s, s') = \gamma^{\theta}(s, s')$.

(B) Otherwise, $\gamma(s, s')$ is empty.

Let us show that Γ LCS-implements f. We prove this statement shortly, but first we need the following key result.

Claim 1 Let $\theta, \theta' \in \Theta$ be given. Then, (i) if $SL_i(f(\theta), \theta) \subseteq SL_i(f(\theta), \theta')$ for all $i \in N$, then $LCS\left(\Gamma^{\theta}, \theta'\right) = \{\theta\}$; (ii) otherwise, $LCS\left(\Gamma^{\theta}, \theta'\right)$ is empty.

Proof. Take any $\theta, \theta' \in \Theta$. Let us first show part (i) of the statement. To this end, suppose that $SL_i(f(\theta), \theta) \subseteq SL_i(f(\theta), \theta')$ for all $i \in N$. We show that $LCS\left(\Gamma^{\theta}, \theta'\right) = \{\theta\}$.

Take any $(y, \theta, i, 0) \in T^{\theta}$ and suppose that θ is the status quo. Then, from part (1) of the definition of γ^{θ}, $\{i\} = \gamma^{\theta}(\theta, (y, \theta, i, 0))$. Since $y \in SL_i(f(\theta), \theta)$, from the definition of T^{θ}, and since $SL_i(f(\theta), \theta) \subseteq SL_i(f(\theta), \theta')$, it follows that $y \in SL_i(f(\theta), \theta')$. Since, from part (1) of the definition of γ^{θ}, $\{i\} = \gamma^{\theta}((y, \theta, i, 0), \theta)$, agent i has the power and incentive to go back to the status quo θ.

Since the choice of the state $(y, \theta, i, 0) \in T^{\theta}$ is arbitrary and agents can only induce states of the form $(y, \theta, i, 0)$ when θ is the status quo, it follows that $\{\theta\}$ is a consistent set of $\left(\Gamma^{\theta}, \theta'\right)$.

To see that $LCS\left(\Gamma^{\theta}, \theta'\right) = \{\theta\}$, let us take any state of the form (y, θ, i, k) and suppose that $(y, \theta, i, k) \in LCS\left(\Gamma^{\theta}, \theta'\right)$. By construction, agent i has the power to move to $(y, \theta, i, k+1)$, from part (2) of the definition of γ^{θ}. Since, by construction, agent i has the power as well as the incentive to move the state from $(y, \theta, k + k')$ to $(y, \theta, k + k' + 1)$ for every $k' \in \mathbb{Z}_{+}$, it follows that (y, θ, i, k) is not a stable outcome.

Since the choice of the state (y, θ, i, k) is arbitrary, we find that no state of the form $(y, \theta, i, k) \in T^{\theta}$ is stable, and so $LCS\left(\Gamma^{\theta}, \theta'\right) = \{\theta\}$. This completes the proof of part (i) of the statement.

Let us now show part (ii). Suppose that there exist an agent i and outcome $y \in SL_i(f(\theta), \theta)$ such that $y \notin SL_i(f(\theta), \theta')$. Since $y \in SL_i(f(\theta), \theta)$, it follows that $(y, \theta, i, 0) \in T^{\theta}$. We show that $LCS\left(\Gamma^{\theta}, \theta'\right)$ is empty. Assume, on the contrary, that $LCS\left(\Gamma^{\theta}, \theta'\right)$ is non-empty. Since no state in T^{θ} can be stable, from the construction of Γ^{θ} and the assumption that agents' preferences are money-monotonic, it follows that $LCS\left(\Gamma^{\theta}, \theta'\right) = \{\theta\}$. As in the preceding paragraphs, from part (1) of the definition of γ^{θ}, we find that $\{i\} = \gamma^{\theta}(\theta, (y, \theta, i, 0)) = \gamma^{\theta}((y, \theta, i, 0), \theta)$. Since $y \notin SL_i(f(\theta), \theta')$, it follows that $u_i(y, \theta') \geq u_i(f(\theta), \theta')$, and so agent i has no incentive to return to θ, which is a contradiction. ■

Suppose that θ' is the true type. Let us show that $\theta' \in LCS(\Gamma, \theta')$. To this end, it suffices to show that $\{\theta'\}$ is a consistent set of (Γ, θ'). Suppose that θ' is the status quo. Then, from the definition of γ, the only possible way to move away from θ' is to move to states of the form $(y, \theta', i, 0)$. Since $\{i\} = \gamma(\theta', (y, \theta', i, 0)) = \gamma^{\theta'}(\theta', (y, \theta', i, 0))$ and since $y \in SL_i(f(\theta'), \theta')$, from the construction of $T^{\theta'}$, it follows that agent i has the power as well as the incentive to go back to state θ'. Since the choice of $(y, \theta', i, 0)$ was arbitrary, it follows that $\{\theta'\}$ is a consistent set of (Γ, θ').

To complete the proof, let us show that $h \circ LCS(\Gamma, \theta') = f(\theta')$. To this end, note that

$$LCS(\Gamma, \theta') = \cup_{\theta \in \Theta} LCS\left(\Gamma^{\theta}, \theta'\right) \subseteq \Theta,$$

from the construction of Γ. Thus, take any $\theta \in LCS(\Gamma, \theta')$. From the construction of Γ, it follows that $\theta \in LCS\left(\Gamma^{\theta}, \theta'\right)$. Since part (ii) of Claim 1 cannot hold, it follows that $SL_i(f(\theta), \theta) \subseteq SL_i(f(\theta), \theta')$ for all $i \in N$. Quasimonotonicity implies that $f(\theta) = f(\theta')$. Since the choice of θ' was arbitrary, we find that $h \circ LCS(\Gamma, \theta') = f(\theta')$ for all $\theta' \in \Theta$. Thus, the individual-based rights structure Γ LCS-implements f.

(iii)* $\Longrightarrow$ *(i)

Clearly, f is LCS-implementable by a rights structure if it is also LCS-implementable by an individual-based rights structure. ■

Remark 1 Quasimonotonicity remains a necessary condition for LCS-implementation even if we consider environments without transfers. Indeed, the same arguments of the proof of Lemma 2 show that quasimonotonicity is a necessary condition for the implementation of SCFs in any environment.

Remark 2 We could also impose the property of balancedness (i.e., transfers sum to zero). The devised rights structure is balanced on the equilibrium path. Indeed, arbitrarily small transfers are used off the equilibrium path.

When Lemma 2 is combined with Lemma 1, it shows that monotonicity is necessary and sufficient for LCS-implementation by a rights structure of SCFs in environments with transfers in which agents have continuous and money-monotonic preferences.

Theorem 1 *The following statements are equivalent:*

(i) F *is* LCS-implementable by a rights structure*;*

(ii) F *is* monotonic*;*

(iii) F *is* LCS-implementable by an individual-based rights structure*.*

Proof. This follows from Lemma 1 and Lemma 2. ■

Rather than providing direct applications of the theorem above, we provide them indirectly by means of the following corollary.

Corollary 1 f *is* implementable in (pure) Nash equilibrium strategies *if and only if* f *is* LCS-implementable by an individual-based rights structure*.*

Proof. This follows from Theorem 1 and Maskin's sufficiency result (Maskin, 1999; Theorem 3). Recall that according to Maskin's sufficiency result, monotonicity is necessary and sufficient for implementation in Nash equilibrium strategies when agents' preferences are money-monotonic—no veto power is always satisfied in our framework. ■

4 Convergence to stable states

In the analysis above, an important detail that has been left unspecified is how the initial state is determined. There are, however, environments in which the initial state can be identified naturally. For instance, the initial state is the no trade allocation in a house allocation problem, no production in a Cournot oligopoly market or when agents can together produce one unit of output but no agent by himself can produce any output, and so on. Let us denote the outcome corresponding to this initial state by $\sigma \in Z$.

When this initial state is not a stable state, the implementing rights structure devised in the constructive proof of Lemma 2 does not assure that a stable outcome is reached via a sequence of states such that the passage from each state of the sequence to the next one is justified in terms of rights as well as incentives.

In this section, we modify the rights structure devised above in a way that every stable outcome is reached from the initial unstable state. We achieve this result under the following assumption:

(A1) The outcome $\sigma \in Z$ corresponding to the initial situation is such that $u_i\left(f\left(\theta\right),\theta\right) > u_i\left(\sigma,\theta\right)$ for all $i \in N$ and all $\theta \in \Theta$.

(A1) is a requirement that the outcome σ is such that each agent considers it to be worse than any outcome in the range of f. For example, in a house allocation problem, this would be satisfied by requiring that there are high gains from trade.

Let us consider the implementing rights structure $\Gamma = (S, h, \gamma)$ devised in the proof of Lemma 2. A variant $\bar{\Gamma}$ of this Γ that LCS-implements f such that the initial state is directly dominated by stable outcomes can be defined as follows:

- The set of states is $\bar{S} = S \cup \{\sigma\}$.
- The outcome function $\bar{h} : \bar{S} \to Z$ is defined by $\bar{h}\left(s\right) = h\left(s\right)$ for all $s \in S$ and $\bar{h}\left(\sigma\right) = \sigma$.
- The code of rights $\bar{\gamma} : \bar{S} \times \bar{S} \twoheadrightarrow \mathcal{N}$ is defined by (a) $\bar{\gamma}\left(s,t\right) = \gamma\left(s,t\right)$ for all $s,t \in S$; (b) $\bar{\gamma}\left(\sigma,\theta\right) = \bar{\gamma}\left(\theta,\sigma\right) = N$ for all $\theta \in \Theta$; (c) $\bar{\gamma}\left(\sigma,s\right) = \bar{\gamma}\left(s,\sigma\right) = \varnothing$ for all $s \in S - \Theta$.

In other words, Γ is modified by adding a new state σ to the set S, which represents the initial state, and by allowing only the (grand) coalition N to approve the change from the initial state σ to a state $\{\theta\}$, and from $\{\theta\}$ back to σ (see Figure 3).

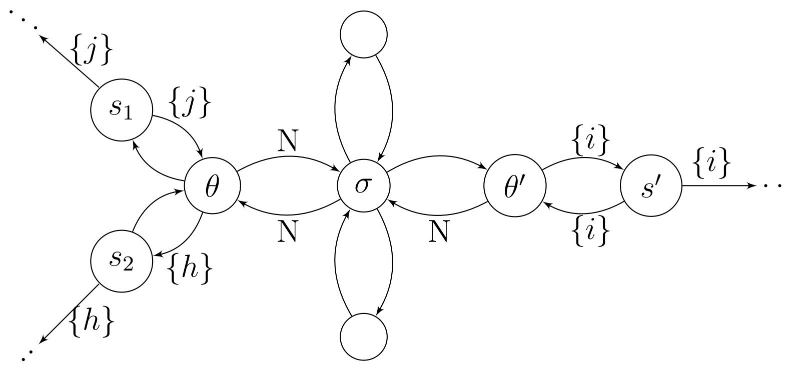

Figure 3. A schematic picture of right structure $\bar{\Gamma}$ where $s_1, s_2 \in T^{\theta}$ and $s' \in T^{\theta'}$.

The next result shows that $\bar{\Gamma}$ LCS-implements f when f is monotonic and it satisfies the property that the initial state σ is directly connected to every stable state.

Theorem 2 Let assumption A1 hold. If f is *monotonic*, then $\bar{\Gamma}$ *LCS-implements* f. For each $\theta \in \Theta$, the initial state σ is *directly dominated* by each $s \in LCS\left(\bar{\Gamma}, \theta\right)$.

Proof. Let the premises hold. Suppose that θ is the true type. Let us show that $\bar{\Gamma}$ LCS-implements f. To this end, let us first show that $\theta \in LCS\left(\bar{\Gamma}, \theta\right)$. From the definition of $\bar{\gamma}$, there are only two possible ways to move away from θ.

First, suppose that $\{i\} = \bar{\gamma}\left(\theta, (y, \theta, i, 0)\right) = \bar{\gamma}^{\theta}\left((y, \theta, i, 0), \theta\right)$. Since $y \in SL_i(f(\theta), \theta)$, from the construction of T^{θ}, we can go back to the state θ.

Second, let us consider $N \in \bar{\gamma}^{\theta}(\theta, \sigma)$. Since $N \in \bar{\gamma}^{\theta}(\sigma, \theta)$, from the definition of $\bar{\gamma}$, and since $u_i\left(f(\theta), \theta\right) > u_i(\sigma, \theta)$ for all $i \in N$, from requirement (A1), it follows that we are back to the state θ.

Since the choice of $(y, \theta, i, 0)$ was arbitrary, it follows that $\{\theta\}$ is a consistent set of (Γ, θ).

Let us show that $\bar{h} \circ LCS\left(\bar{\Gamma}, \theta\right) \subseteq f(\theta)$. We already know that $\theta \in LCS\left(\bar{\Gamma}, \theta\right)$. Moreover, following the same reasoning used in the proof of Lemma 2, we can see that no state $t \in \cup_{\bar{\theta} \in \Theta} T^{\bar{\theta}}$ can be a stable state at $\left(\bar{\Gamma}, \theta\right)$. Then, it follows that $LCS\left(\bar{\Gamma}, \theta\right) \subseteq \Theta \cup \{\sigma\}$.

Let us show that $\sigma \notin LCS\left(\bar{\Gamma}, \theta\right)$. Assume, on the contrary, that $\sigma \in LCS\left(\bar{\Gamma}, \theta\right)$. Since $N \in \bar{\gamma}^{\theta}(\sigma, \theta)$, by construction, it follows from the definition of the LCS that $s \in LCS\left(\bar{\Gamma}, \theta\right)$ exists, where $s = \theta$ or $s \gg^{\left(\bar{\Gamma}, \theta\right)} \theta$, such that not $\bar{h}(s)\ u_N^{\theta}\ \bar{h}(\sigma)$. This means that $\sigma \neq s$, and so $s \in \Theta$. An immediate contradiction of requirement (A1) is obtained if $s = \theta$. Then, it must be the case that $s \gg^{\left(\bar{\Gamma}, \theta\right)} \theta$. By definition of $\bar{\gamma}$ and the fact that $s \gg^{\left(\bar{\Gamma}, \theta\right)} \theta$, it holds that $N \in \bar{\gamma}(\theta, \sigma)$ and $\bar{h}(s)\ u_N^{\theta}\ f(\theta)$. Since $f(\theta)\ u_N^{\theta}\ \bar{h}(\sigma)$, by requirement (A1), and since $\bar{h}(s)\ u_N^{\theta}\ f(\theta)$, it follows by transitivity that $\bar{h}(s)\ u_N^{\theta}\ \bar{h}(\sigma)$, which is a contradiction. Thus, $\sigma \notin LCS\left(\bar{\Gamma}, \theta\right)$, and so $LCS\left(\bar{\Gamma}, \theta\right) \subseteq \Theta$.

Finally, let us show that $f(\theta') = f(\theta)$ for all $\theta' \in LCS\left(\bar{\Gamma}, \theta\right)$. Assume, on the contrary, that $f(\theta') \neq f(\theta)$ for some $\theta' \in LCS\left(\bar{\Gamma}, \theta\right)$. Since f is monotonic, it is quasimonotonic, following Lemma 1. Since $f(\theta') \neq f(\theta)$, it follows that there exist $i \in N$ and $y \in SL_i(f(\theta'), \theta')$ such that $u_i(y, \theta) \geq u_i(f(\theta'), \theta)$. Then, $(y, \theta', i, 0) \in T^{\theta'}$, from the definition of $T^{\theta'}$ given in (1). Note that $\{i\} = \bar{\gamma}(\theta', (y, \theta', i, 0))$, from the definition of $\bar{\gamma}$. Since $\theta' \in LCS\left(\bar{\Gamma}, \theta\right)$, there exists $s \in LCS\left(\bar{\Gamma}, \theta\right)$, where $s = (y, \theta', i, 0)$ or $s \gg^{\left(\bar{\Gamma}, \theta\right)} (y, \theta', i, 0)$, such that not $h(s)\ u_i^{\theta}\ f(\theta')$.

From the definition of $s \gg^{\left(\bar{\Gamma}, \theta\right)} (y, \theta', i, 0)$, it follows that there exist $s_0, s_1, ..., s_J$ in S (where $s_0 = (y, \theta', 0)$ and $s_J = s$) and $K_0, K_1, ..., K_{J-1}$ in $\mathcal{N}_0$ such that $K_{j-1} \in \bar{\gamma}(s_{j-1}, s_j)$ and $h(s)\ u_{K_{j-1}}^{\theta}\ h(s_{j-1})$ for $j = 1, ..., J$. From the construction of $\bar{\gamma}$, it follows that for some $j = 1, ..., J$, it holds that $K_{j-1} = N$, $s_{j-1} = \theta'$ and $s_j = \sigma$. Therefore, we have established that $h(s)\ u_N^{\theta}\ f(\theta')$, which contradicts not $h(s)\ u_i^{\theta}$ $f(\theta')$. We conclude that $f(\theta') = f(\theta)$ for all $\theta' \in LCS\left(\bar{\Gamma}, \theta\right)$. Finally, we observe that σ is directly dominated by each $\theta' \in LCS\left(\bar{\Gamma}, \theta\right)$, from requirement (A1) and the definition of $\bar{\gamma}$. ■

4.1 Cournot oligopoly

In this subsection, we apply the theorem above to Cournot oligopoly markets in which it is natural to assume that the initial state σ is characterized by no production.

A single (homogeneous) product is produced by $n \geq 2$ firms. A *Cournot oligopoly problem* is described by a type θ satisfying the following properties. The cost for firm i of producing q_i units of the good in type θ is $C_i(q_i, \theta)$. Firm i's cost function is defined for all $q_i \geq 0$. The firms' total output is $Q = (q_1 + ... + q_n)$ and the inverse demand function for the good is $P(Q, \theta)$ at θ. Each firm's preferences at θ are represented by its profits, that is, $\pi_i(Q_{-i}, q_i, \theta) = q_i P(Q, \theta) - C_i(q_i, \theta)$, where $Q_{-i} == (q_1 + ... + q_{i-1} + q_{i+1} + ... + q_n)$. Each firm i picks its output q_i to solve $\max_{q_i} \pi_i(Q_{-i}, q_i, \theta)$. Following Gaudet and Salant (1991), we assume that the Cournot oligopoly problem θ satisfies the following assumptions as well.

Assumption 1 There exists a $\xi \in (0, \infty)$ such that $P(Q, \theta) > 0$ for $Q \in [0, \xi)$ and $P(Q, \theta) = 0$ for all $Q \in [\xi, \infty)$.

Assumption 2 $P(Q, \theta)$ is twice-continuously differentiable, and decreasing when it is strictly positive.

Assumption 3 For any firm $i \in N$, $C_i(q_i, \theta)$ is twice-continuously differentiable and, for any $q_i > 0$, $C_i'(q_i, \theta) > 0$.[6]

Assumption 4 For all $Q \in [0, \xi)$ and all $i \in N$, there exists $\alpha < 0$ (possibly dependent on Q and i) such that $P'(Q, \theta) - C_i''(q_i, \theta) \leq \alpha < 0$ for every $q_i \geq 0$.

Assumption 5 For all $Q \in [0, \xi)$, all $i \in N$ and all $q_i \in [0, Q]$, $P'(Q, \theta) + q_i P''(Q, \theta) \leq 0$.

We refer to Θ as a *class of Cournot oligopoly problems*, with θ as a typical problem. Given Assumptions 1–5, Gaudet and Salant (1991) show that these assumptions are sufficient for the existence of a unique Cournot equilibrium for each Cournot oligopoly problem $\theta \in \Theta$. The Cournot solution to the Cournot oligopoly problem $\theta \in \Theta$,

[6] $C_i'(q_i, \theta)$ denotes $\frac{dC_i(q_i,\theta)}{dq_i}$.

denoted by $f_C(\theta)$, specifies the Cournot equilibrium quantities. The following result shows that this solution is LCS-implementable when the designer does not know the true type.

Theorem 3 Let Assumption A1 hold. Consider the rights structure $\bar{\Gamma}$, where $\bar{h}(\sigma) = (0, ..., 0)$. The *Cournot solution* f_C, defined over Θ, is *LCS-implementable by* $\bar{\Gamma}$. Moreover, for each $\theta \in \Theta$, the initial state σ is directly dominated by each $s \in LCS\left(\bar{\Gamma}, \theta\right)$.

Proof. In light of Theorem 1, it suffices to show that the Cournot solution f_C is monotonic. This follows directly from the fact that f_C is implementable in Nash equilibrium strategies for each type $\theta \in \Theta$. To complete the proof, we observe that, by construction, for each type $\theta \in \Theta$, it holds that $f_C(\theta) > (0, ..., 0)$, so that assumption A1 is satisfied. ■

5 Concluding remarks

So far, we have focussed on SCFs. Let us now discuss the extension of our result to social choice correspondences (SCCs). An SCC is a mapping F which associates each type $\theta \in \Theta$ with a nonempty subset of Z, denoted by $F(\theta)$. The outcomes in $F(\theta)$ are the *F-optimal outcomes* for the type θ. When our attention shifts to SCCs, the analysis would change substantially if we sought to derive necessary or sufficient conditions for implementation for LCS-implementation by a rights structure of the SCCs.

Monotonicity is not necessary for LCS-implementation by a rights structure of the SCCs. To illustrate this point, let us consider the following example. There are two agents; the set of pure social decisions is $D = \{v, w, x, y, z\}$; and $\Theta = \{\theta, \theta'\}$. To each type, we associate each agent i with a quasilinear utility function $u_i : Z \times \Theta \to \mathbb{R}$, that is, his utility function takes the form

$$u_i((d, t), \psi) = v_i(d, \psi) + t_i \text{ for each } \psi \in \Theta,$$

where

	$v_1(\cdot,\theta)$	$v_2(\cdot,\theta)$	$v_2(\cdot,\theta')$
v	2	1	3
w	0	3	2
x	1	-2	0
y	3	2	1
z	4	4	4

and $v_1(\cdot,\theta) = v_1(\cdot,\theta')$. Define F on Θ by $F(\theta) = \{(v,0),(w,0),(x,0),(z,0)\}$ and $F(\theta') = \{(v,0),(z,0)\}$. With an abuse of notation, we denote $(d,0) \in Z \equiv D \times \mathbb{R}^n$ by d. F is not monotonic because $x \in F(\theta)$, $L_i(x,\theta) \subseteq L_i(x,\theta')$ for all $i \in N$ but $x \notin F(\theta')$.[7] However, F is LCS-implementable by a rights structure.

To see this, let us consider the following rights structure $\Gamma = (S,h,\gamma)$, where the state space is $S = \{a,b,c,d,e\}$ and where the outcome function h and the code of rights are depicted in Figure 4, where $s \rightarrow^K t$ means that only K is effective at moving from s to t; the outcome corresponding to each state is in parentheses,

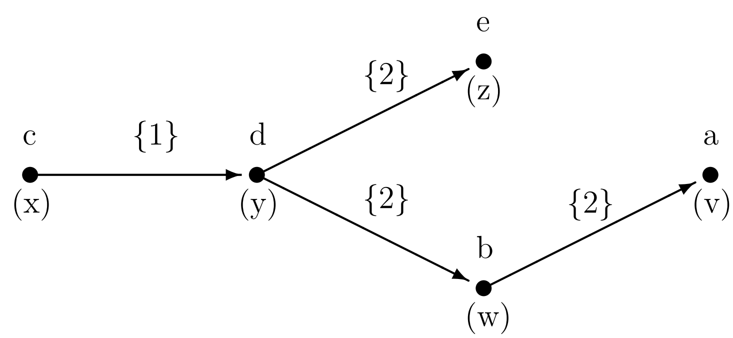

Figure 4.

We can check by using Figure 4 that $LCS(\Gamma,\theta) = S - \{d\}$ and $LCS(\Gamma,\theta') = \{a,e\}$.

[7]The set inclusion for agent 2 also holds since for all $d \in D$, it holds that

$$v_2(d,\theta) - v_2(x,\theta) \geq v_2(d,\theta') - v_2(x,\theta').$$

Set-quasimonotonicity. We have already noted that monotonicity is equivalent to quasimonotonicity in environments with transfers in which agents' utility functions are continuous and money-monotonic. Moreover, we have also noted in Remark 1 that quasimonotonicity is a necessary condition for LCS-implementation of SCFs in any environment. However, quasimonotonicity is not a necessary condition for LCS-implementation when the designer's goal is represented by an SCC. Indeed, the preceding example also illustrates this point when the set of outcomes coincides with the set of pure social decisions, that is, $Z = D = \{v, w, x, y, z\}$, and agent i's utility for outcome d at type profile ψ coincides with $v_i(d, \psi)$, that is, $u_i(d, \psi) = v_i(d, \psi)$ for each agent $i \in N$, each outcome $d \in Z$ and each type profile $\psi \in \Theta$.

We conclude this section by presenting a monotonicity notion, which can be shown to be a necessary condition for the LCS-implementation of SCCs—its proof is similar to that of Lemma 2.

Definition 9 A SCC F is *set-quasimonotonic* provided that for all $\theta, \theta' \in \Theta$, if

$$SL_i(x,\theta) \subseteq SL_i(x,\theta') \text{ and } L_i(x,\theta) \cap F(\theta) \subseteq L_i(x,\theta')$$

for all $x \in F(\theta)$ and all $i \in N$, then $F(\theta) \subseteq F(\theta')$.

Clearly, set-quasimonotonicity coincides with quasimonotonicity when F is a single-valued SCC. In words, the condition requires that if for each agent i, his strict lower contour set at each optimal outcome at θ does not shrink when the type changes from θ to θ', as well as his set $L_i(x,\theta) \bigcap F(\theta)$ at each optimal outcome $x \in F(\theta)$ does not shrink, then the set $F(\theta')$ of optimal outcomes at θ' is a superset of the set $F(\theta)$ of optimal outcomes at θ.

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