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Abstract

Contrary to the current practice that mainly considers stand-alone statistical loss functions, the aim of the paper is to assess oil price volatility forecasts based on objective-based evaluation criteria, given that different forecasting models may exhibit superior performance at different applications. To do so, we forecast implied and several intraday volatilities and we evaluate them based on financial decisions for which these forecasts are used. In this study we confine our interest on the use of such forecasts from financial investors. More specifically, we consider four well established trading strategies, which are based on volatility forecasts, namely (i) trading the implied volatility based on the implied volatility forecasts, (ii) trading implied volatility based on intraday volatility forecasts, (iii) trading straddles in the United States Oil Fund ETF and finally (iv) trading the United States Oil Fund ETF based on implied and intraday volatility forecasts. We evaluate the after-cost profitability of each forecasting model for 1-day up to 66-days ahead. Our results convincingly show that our forecasting framework is economically useful, since different models provide superior after-cost profits depending on the economic use of the volatility forecasts. Should investors evaluate the forecasting models based on statistical loss functions, then their financial decisions would be sub-optimal. Thus, we maintain that volatility forecasts should be evaluated based on their economic use, rather than statistical loss functions. Several robustness tests confirm these findings.

Keywords: Volatility forecasting, implied volatility, intraday volatility, WTI crude oil futures, objective-based evaluation criteria.

JEL: C22, C53, G11, G17, Q47.

1. Introduction

The evidenced financialisation of the crude oil market over recent years[1] has led to an increased interest in oil price volatility forecasting. This interest primarily stems from the fact that oil price volatility is important for a number of stakeholders, including policy makers, industrial sectors, as well as, investors. As far as the latter stakeholders are concerned, oil price volatility constitutes important information for energy commodities trading, portfolio optimization, financial risk management, option pricing and speculative strategies.

Early efforts in this line of research include the works of Sadorsky (2006) and Sadorsky and McKenzie (2008) who focus on GARCH-family forecasting models based on daily sampling frequency. Since then, a number of other studies emerged in an effort to develop modelling frameworks for accurate oil price volatility forecasts[2]. It falls beyond the scope of the paper to provide an extensive review of the related literature[3]. Rather, it is more constructive to summarise the key ingredients of the existing studies so to identify the contribution of this paper in this exciting line of research.

First, the bulk of the recent studies tend to forecast the realized volatility of oil prices using ultra-high frequency data (intraday), as opposed to the earlier studies that showed preference in the forecast of conditional volatility based on low sampling frequency (daily, weekly or monthly). The advantages of using ultra-high frequency data is well documented in the financial literature and rests primarily on the fact that they are more information-rich and thus, they can produce more accurate forecasts (Andersen and Bollerslev 1998; Andersen *et al*., 2003, 2005; McAleer and Medeiros, 2008; Tay *et al*., 2009).

Second, the model that has received great attention in the recent years is Corsi's (2009) Heterogeneous Autoregressive (HAR) model, as opposed to GARCH-type models, given its ability to capture some stylized facts in volatility (e.g. long-memory), its parsimony, as well as, the fact that accommodates the heterogeneous

[1] See the evidence provided by Tang and Xiong (2012), Büyüksahin and Robe (2014) and more recently by Le Pen and Sevi (2017), Degiannakis and Filis (2017, 2018).

[2] Some notable contributions include, Kang *et al*. (2009), Nomikos and Pouiasis (2011), Chkili *et al*. (2014), Haugom *et al*. (2014), Sevi (2014), Prokopczuk *et al*. (2016), Degiannakis and Filis (2017).

[3] The reader is directed to the work of Degiannakis and Filis (2017), as well as, to some more recent studies by Ma *et al*. (2017), Gong and Lin (2018), Liu *et al*. (2018) and Ma *et al*. (2018).

beliefs of investors (Andersen *et al*., 2007; Corsi, 2009; Busch *et al*., 2011; Fernandes *et al*., 2014).

Third, existing studies evaluate oil price volatility point forecasts using a battery of statistical loss functions, such as the Root Mean Squared Error (RMSE), the Mean Absolute Error (MAE) and the quasi-likelihood (QLIKE), as well as, the success ratio, which evaluates the forecasts' directional accuracy.

Despite the increasing interest in oil price volatility forecast, there are still several issues that must be addressed. The most obvious gap is the fact that studies do not seem to forecast other types of intraday volatility measures apart from realized volatility (e.g. Barndorff-Nielsen and Shephard's (2004 and 2006) bipower variation, Barndorff-Nielsen's *et al.* (2010) realized semi-variance, Andersen's *et al.* (2012) minimum and median realized volatility) or the implied volatility. Rather, authors tend to use these different volatility measures as predictors of the realized volatility. Even more, the literature has disregarded the use of other external predictors of oil price volatility, despite the fact that Degiannakis and Filis (2017) convincingly show that the use of realized volatilities from different asset classes improves the forecasts of oil price realized volatility. More importantly, though, studies have ignored to take into consideration the purpose of the oil price volatility forecasts when evaluating them. This is rather important given that oil price volatility forecasts can be employed for different applications and thus, they could have different economic uses.

In short, the aforementioned gaps can be summarised succinctly, as follows. Economic agents are faced with multiple oil price volatility measures and use their forecasts for multiple purposes. Even more, oil price volatility forecasts could be improved by the use of external predictors. It is therefore necessary for the literature to provide a framework which will consider the range of volatility measures and will allow oil price volatility users to be able to choose the most appropriate measure according to the economic decision for which the forecast will be used.

This study addresses all these important issues and provides such framework. To do so, we first produce oil price volatility forecasts for several oil price intraday volatility measures, as well as, the oil price implied volatility index, the OVX[4], using the HAR model. Next, we use the different intraday volatility measures as potential predictors of the OVX. However, we further consider the intraday volatility measures

[4] The OVX is the 30-day volatility of the United States Oil Fund (USO), which trades WTI futures contracts.

of other asset classes as additional potential predictors of oil price volatility (either of the OVX or the intraday volatility). Finally, we employ evaluation criteria that reflect the purpose of the oil price volatility forecasts, i.e. objective-based evaluation criteria. Given the complexity of the development of such framework, we limit our focus to the different investors' financial decisions.

Thus, our forecasts are evaluated based on the after-cost profitability of four common trading decisions, namely (i) trading OVX based on the OVX forecasts, (ii) trading OVX based on oil price intraday volatility forecasts, (iii) trading straddles in United States Oil Fund (USO) underlying price based on OVX forecasts and finally (iv) trading the USO underlying price based on oil price volatility forecasts (either of the OVX or the intraday volatility). We evaluate the after-cost profitability of each forecasting model for 1-trading-day up to 66-trading-days ahead.

In a nutshell, our real out-of-sample forecasts convincingly show that evaluating oil price volatility forecasts based on the economic use that they serve produce superior benefits to financial traders. In particular, we show that different models provide superior after-cost profits depending on the different trading strategies employed. Thus, we maintain that volatility forecasts should be evaluated based on objective-based criteria, rather than statistical loss functions. Our results remain robust against forecast averaging and several robustness tests.

The remaining of the paper is structured as follows. Section 2 describes the data used in the study; Section 3 provides the details of the modelling framework and Section 4 presents the forecasting procedure. Section 5 analyses the findings of the study and finally Section 6 concludes the study.

2. Data description

This study concerns with the development of forecasting frameworks for both the intraday and implied volatility of crude oil prices. In this study a variety of intraday volatility measures is used, which are presented in the following section. Following Andersen *et al*. (2003, 2007) and Sevi (2014) we construct the time-series intraday volatility measures using tick-by-tick transaction data of the front-month futures contracts for the WTI crude oil (WT). The implied volatility of the crude oil prices is approximated by the OVX index and the data are readily available at a daily frequency. We use WTI rather than Brent crude oil prices, given that the OVX is the implied volatility index of the former oil benchmark. Our sample period spans from 4th January 2010 until 30th October 2017 (1971 trading days) and it is dictated by the availability of data.

Motivated by Yang and Zhou (2017) and Degiannakis and Filis (2017) we also consider representative assets/indices of four different asset classes as potential predictors of the WTI intraday volatilities and OVX. In particular, we consider tick-by-tick data of the front-month futures contracts for (i) the Brent crude oil (CO) and the DJ UBS commodity index (AI) as approximations of the commodities asset class, (ii) the US Dollar index (DX), which represents the foreign exchange market, (iii) the S&P500 index (SP) as a proxy of global stock market asset class and (iv) the US T-bills (TY) as a representative asset of the global macroeconomic conditions. The literature has shown that these four asset classes exhibit cross-linkages with the oil market (see, Degiannakis and Filis, 2017 and references therein). Given that our evaluation criteria are also based on trading profits from the United States Oil Fund (USO), we also obtain daily prices for this exchange traded fund.

All tick-by-tick data are obtained from TickData, whereas the data for the OVX and USO are retrieved from CBOE and Nasdaq, respectively.

2.1. Intraday volatility measures

The most known estimator of realized volatility on a daily sampling frequency is the realized variance, which is proposed by Andersen and Bollerslev (1998) and it can be computed as:

$$RV_t = \sum_{i=1}^{\tau} r_{t,i}^2 , \quad (1)$$

where $r_{t,i} = log(P_{t,i}) - log(P_{t,i-1})$ is the i^{th} intraday return (for i=1,…,τ) at day t, τ is the number of intervals in the trading day and $P_{t,i}$ is the i^{th} intraday asset price at day t. As $\tau \to \infty$, the accuracy improves, but at a high sampling frequency the market frictions is a source of additional noise. Hence, the intraday points in time must be as many as the market microstructure features do not induce bias to the estimation of variance.

The volatility signature plot, which provides a graphical representation of the average realized volatility against the sampling frequency, has been widely used for finding the trade-off between accuracy and potential bias due to microstructure frictions. In the signature plot we look for the highest frequency where the average realized volatility stabilizes or the autocovariance bias term approaches zero. The intraday autocovariance is calculated as:

$$Cov(r_{t,i}, r_{t,i-j}) = \sum_{j=1}^{\tau-1} \sum_{i=j+1}^{\tau} r_{t,i} r_{t,i-j}. \quad (2)$$

The majority of the studies have proposed the use of minute-by-minute data to obtain the optimal sampling frequency (see, for instance, Andersen and Bollerslev, 1998; Andersen *et al.*, 1999; Andersen *et al.*, 2000; and Andersen *et al.*, 2001). However, Chaboud *et al.* (2010) proposed the use of richer intraday information (i.e. second-by-second) so to identify the optimal sampling frequency. We subscribe to the approach suggested by Chaboud *et al.* (2010) given the use of such richer intraday information. In our case, we consider 5-seconds intervals to compute the optimal sampling frequencies for our dataset. Hence, the sampling frequencies that minimize the intraday autocovariance for WT, CO, AI, DX, SP and TY are 20-minutes, 10-minutes, 30-minutes, 10-minutes, 30-minutes and 15-minutes, respectively.

A number of studies have proposed various adjustments in order to account for overnight volatility. Among others, Hansen and Lunde (2005) have introduced the combination of intraday volatility during the open-to-closed period with the closed-to-open inter-day volatility, such as that:

$$ScaledRV_t = \omega_1(logP_{t,1} - logP_{t-1,\tau}) + \omega_2 \sum_{i=1}^{\tau} r_{t,i}^2. \quad (3)$$

The parameters ω_1 and ω_2 are estimated such as $\min_{(\omega_1,\omega_2)} V(ScaledRV_t)$.[5] According to Hansen and Lunde (2005) the $\omega_1 = \left(1 - \frac{\mu_2^2\eta_1 - \mu_1\mu_2\eta_{12}}{\mu_2^2\eta_1 + \mu_1^2\eta_2 - 2\mu_1\mu_2\eta_{12}}\right)\frac{\mu_0}{\mu_1}$ and $\omega_2 = \frac{\mu_2^2\eta_1 - \mu_1\mu_2\eta_{12}}{\mu_2^2\eta_1 + \mu_1^2\eta_2 - 2\mu_1\mu_2\eta_{12}}\frac{\mu_0}{\mu_2}$ are consistent estimators[6].

Literature assumes that the logarithmic price within a trading day follows a standard jump-diffusion process:

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[5] Note that $\arg\min_{(\omega_1,\omega_2)} E(ScaledRV_t - IV_t) = \arg\min_{(\omega_1,\omega_2)} V(ScaledRV_t)$.

[6] The $\mu_1 = T^{-1}\sum_{t=1}^{T}(logP_{t,1} - logP_{t-1,\tau})^2$ is the sample average of the squared closed-to-open log-returns. The $\eta_1 = T^{-1}\sum_{t=1}^{T}\left((logP_{t,1} - logP_{t-1,\tau})^2 - \mu_1\right)^2$ is the estimate of the variance of the squared closed-to-open log-returns. The $\mu_2 = T^{-1}\sum_{t=1}^{T}\sum_{i=1}^{\tau}(logP_{t,i} - logP_{t,i-1})^2$ is the sample average of the daily realized variances. The $\eta_2 = T^{-1}\sum_{t=1}^{T}\left(\sum_{i=1}^{\tau}(logP_{t,i} - logP_{t,i-1})^2 - \mu_2\right)^2$ is the variance of the daily realized variances. The $\eta_{12} \equiv Cov\left((logP_{t,1} - logP_{t-1,\tau})^2, RV_t\right)$ is estimated as $\eta_{12} = T^{-1}\sum_{t=1}^{T}\left(\left((logP_{t,1} - logP_{t-1,\tau})^2 - \mu_1\right)\left(\sum_{i=1}^{\tau}(logP_{t,i} - logP_{t,i-1})^2 - \mu_2\right)\right)$. Finally, $\mu_0 = \mu_1 + \mu_2$.

$$dlog\big(P(t')\big) = \mu(t')dt' + \sigma(t')dW(t') + \kappa(t')dq(t'), \tag{4}$$

for $0 \leq t' \leq T$, where $\mu(t')$ is the drift term with a continuous sample path variation, $\sigma(t')$ denotes a strictly positive stochastic volatility process, $W(t')$ denotes a standard Brownian motion and $\kappa(t')dq(t')$ is the pure jump component.

For the discrete price process, the log-return volatility at time *t* includes the jump volatility and it is not an unbiased estimator of integrated volatility. Hence, the quadratic variation is denoted as:

$$QV_t = \int_{t-1}^{t} \sigma_s^2 ds + \sum_{t-1<s\leq t} \kappa_s^2, \tag{5}$$

where $\int_{t-1}^{t} \sigma_s^2 ds < \infty$ is the integrated variation and denotes the continuous component of the total variation, whereas the $\sum_{t-1<s\leq t} \kappa_s^2$ is the cumulative jump variation in [*t*-1,*t*].

Barndorff-Nielsen and Shephard (2004 and 2006) showed that the realized bipower variation BPV_t is an estimator of the integrated volatility IV_t in the presence of jumps. The BPV_t is calculated as:

$$BPV_t = (2/\pi)^{-1}\left(\frac{\tau}{\tau-1}\right)\sum_{i=1}^{\tau-1}|r_{t,i}|\,|r_{t,i+1}|\,, \tag{6}$$

where $Z \sim N(0,1),\ \ a > 0$. If $r_{t,i} \sim i.i.d. N(0,\frac{\sigma^2}{\tau})$ then $E\big[|r_{t,i}||r_{t,i+1}|\big] = \frac{2}{\pi}\frac{\sigma^2}{\tau}$ and $\tau/(\tau$-1) is considered a required finite sample correction factor. According to Barndorff-Nielsen and Shephard (2004 and 2006) and Huang and Tauchen (2005), we should use the following statistic in order to identify the discontinuous jump variation:

$$ZJ_t^{(BV)} = \frac{(RV_t - BPV_t)RV_t^{-1}}{\left(\left(\sqrt{2/\pi}\right)^{-4} + 2\left(\sqrt{2/\pi}\right)^{-2} - 5\right)\frac{1}{\tau}\max\left(1, \frac{TQ_t}{BPV_t^2}\right)} \to N(0,1), \tag{7}$$

where TQ_t is the tri-power quarticity:

$$TQ_t = \tau\mu_{4/3}^{-3}\left(\frac{\tau}{\tau-2}\right)\sum_{i=1}^{\tau-2}|r_{t,i}|^{4/3}|r_{t,i+1}|^{4/3}|r_{t,i+2}|^{4/3}, \tag{8}$$

where $\mu_{4/3} = E\left(|Z_T|^{\frac{4}{3}}\right) = 2^{\frac{2}{3}}\Gamma\left(\frac{7}{6}\right)\Gamma(1/2)^{-1}$. The daily discontinuous jump variation BVJ_t^d can be defined by:

$$BVJ_t^d = I(ZJ_t^{(BV)} > \varphi_\alpha)(RV_t - BPV_t). \tag{9}$$

Additionally, the continuous sample path variation BVC_t^d can be calculated by:

$$BVC_t^d = I\left(ZJ_t^{(BV)} \leq \varphi_\alpha\right)RV_t + I\left(ZJ_t^{(BV)} > \varphi_\alpha\right)BPV_t, \tag{10}$$

where I(.) is an indicator function and α equals 0.99.

In order to estimate the integrated volatility in the presence of jumps, Andersen *et al.* (2012) have proposed a set of estimators for integrated variance in the presence of jumps. These estimators are based on the median and minimum of a number of consecutive absolute intraday returns and they are defined as:

$$MedRV_t = \frac{\pi}{6-4\sqrt{3}+\pi}\left(\frac{\tau}{\tau-2}\right)\sum_{i=2}^{\tau-1} med(|r_{t,i-1}|, |r_{t,i}|, |r_{t,i+1}|)^2, \quad (11)$$

and

$$MinRV_t = \frac{\pi}{\pi-2}\left(\frac{\tau}{\tau-1}\right)\sum_{i=1}^{\tau-1} min(|r_{t,i}|, |r_{t,i+1}|)^2. \quad (12)$$

According to Theodosiou and Zikes (2011), these two estimators are more robust than the multipower variations due to the fact that large absolute returns associated with jumps tend to be eliminated from the calculation of the median and minimum operators. Moreover, the median realized volatility offers a number of advantages over alternative measures of integrated variance in the presence of infrequent jumps and it is less sensitive to the presence of occasional zero intraday returns. Thus, we could implement an extension of the adjusted jump ratio statistic to the MedRV and MinRV estimators. According to Andersen *et al.* (2012), the test could be adapted to the MedRV and MinRV estimators as:

$$ZJ_t^{(MedRV)} = \frac{(RV_t - MedRV_t)RV_t^{-1}}{0.96\frac{1}{\tau}\max\left(1, \frac{MedRQ_t}{MedRV_t^2}\right)} \to N(0,1), \quad (13)$$

and

$$ZJ_t^{(MinRV)} = \frac{(RV_t - MinRV_t)RV_t^{-1}}{1.81\frac{1}{\tau}\max\left(1, \frac{MinRQ_t}{MinRV_t^2}\right)} \to N(0,1), \quad (14)$$

where $MedRQ_t$ and $MinRQ_t$ denote the estimates of the integrated quarticity, respectively. Particularly, $MedRQ_t$ and $MinRQ_t$ are written as:

$$MedRQ_t = \frac{3\pi}{9\pi+72-52\sqrt{3}}\left(\frac{\tau}{\tau-2}\right)\sum_{i=2}^{\tau-1} med(|r_{t,i-1}|, |r_{t,i}|, |r_{t,i+1}|)^4 \quad (15)$$

and

$$MinRQ_t = \frac{\pi}{3\pi-8}\left(\frac{\tau}{\tau-1}\right)\sum_{i=1}^{\tau-1} min(|r_{t,i}|, |r_{t,i+1}|)^4. \quad (16)$$

Implementing the same framework in order to estimate the daily discontinuous jump variation, we define the $MedJ_t^d$ and $MinJ_t^d$ as:

$$MedJ_t^d = I\left(ZJ_t^{(MedRV)} > \varphi_\alpha\right)(RV_t - MedRV_t), \quad (17)$$

and

$$MinJ_t^d = I(ZJ_t^{(MinRV)} > \varphi_\alpha)(RV_t - MinRV_t), \tag{18}$$

respectively. Additionally, the continuous sample path variations $MedRVC_t^d$ and $MinRVC_t^d$ can be calculated as:

$$MedRVC_t^d = I\left(ZJ_t^{(MedRV)} \leq \varphi_\alpha\right)RV_t + I\left(ZJ_t^{(MedRV)} > \varphi_\alpha\right)MedRV_t, \tag{19}$$

and

$$MinRVC_t^d = I\left(ZJ_t^{(MinRV)} \leq \varphi_\alpha\right)RV_t + I\left(ZJ_t^{(MinRV)} > \varphi_\alpha\right)MinRV_t, \tag{20}$$

where I(.) is an indicator function and α equals 0.99.

Barndorff-Nielsen *et al.* (2010) proposed the daily realized semi variance, which can capture the variation solely from negative or positive returns. The daily positive realized semi variance estimator is written as:

$$RSV_t^+ = \sum_{i=1}^{\tau} I\{r_{t,i} \geq 0\} r_{t,j}^2 \,. \tag{21}$$

Similarly, the daily negative realized semi variance estimator is defined as:

$$RSV_t^- = \sum_{i=1}^{\tau} I\{r_{t,i} < 0\} r_{t,j}^2 \,. \tag{22}$$

Due to the fact that we can decompose quadratic variation into the contribution from the continuous term of prices and the impact of jumps, we can check the contribution of jumps and its significance. The first radical alternative to the commonly used realized variance estimator was defined by Barndorff-Nielsen and Shephard (2004), who produced an estimator of integrated variance in the presence of jumps, namely the bipower variation. They suggested this kind of estimator because of its robustness to the presence of jumps. Moreover, when they added jumps to the SV model the probability limit of the bipower estimator did not change, which is important due to the fact that we can combine realized variance with realized bipower variation to estimate jump variation component. Barndorff-Nielsen and Shephard (2006) drew the conclusion that the large jumps in their dataset were mainly caused by macroeconomic news announcements. The assumption that exchange rates have continuous sample paths seems at odds under their findings.

Barndorff-Nielsen *et al.* (2010) proposed the realized semi-variance. Patton and Sheppard (2015), based on the realized semi-variance, have found that for equity data, the negative realized semi-variance is much more important for forecasting future volatility than the positive realized semi-variance. They also proposed realized semi-variances in order to obtain a measure of signed jump variation, which has the potential to decompose the realized variance into a component that relates only to positive high-frequency returns and a component that relates only to negative high-frequency returns.

As far as the impact of jumps on oil price volatility modelling and forecasting, this has been investigated by Tseng *et al*. (2009), Sevi (2014) and Prokopczuk *et al*. (2015), among others. Their findings show that jumps do not offer any incremental predictive ability.

Table 1 and Figure 1 depict the descriptive statistics of the different oil price volatility measures, as well as, their plots[7].

[TABLE 1 HERE]

[FIGURE 1 HERE]

From Table 1 we notice that the implied volatility index (OVX) exhibits the highest average value, whereas at the other side of the spectrum, the lowest volatilities are shown to belong to the semi variances (either the positive or the negative). Interestingly enough, we do not observe any material difference between the average value of the positive and negative semi variance. Even more, these two measures of intraday volatility also exhibit the highest variability, as shown by the coefficient of variation. None of the volatility measures are normally distributed, but rather they are positively skewed, as expected, and leptokurtic, suggesting fat tails in the distribution due to extreme volatility movements. Finally, we observe that all measures are stationary. Table 1 also presents the static correlations between each measure of intraday volatility and the OVX. It is clear that all intraday volatilities are highly correlated with OXV, although the lowest numbers are observed for the cases of the positive and negative semi variances. This is rather expected since these two volatility measures consider only the oil price volatility during positive or negative oil price changes.

From Figure 1 it is evident that all oil price volatility measures exhibit similar peaks and troughs. There are two distinct peaks in the volatility. The first peak is observed during 2011, when oil prices lost about 35% of their value within a 6 month period. Furthermore, the increased volatility during 2014-2016 is related to the oil price slump of that period, when WTI oil prices fell from about $108 (June 2014) to $29 (February 2016).

[7] For brevity, we do not show the descriptive statistics and plots of the remaining asset classes considered in this study. These are available upon request, though.

To motivate further the use of the different intraday volatility measures as possible predictors of the OVX, as well as, to show that their forecasts could exhibit different information for oil volatility traders we “zoom-in” to Figure 1 so to present the behaviour of the volatility measures for a random month. The volatility plots for this random month are shown in Figure 2. It is clear that the different volatilities exhibit different patterns and that the OVX is materially less volatile. Furthermore, even though the RV, BPV and $MinRV$ exhibit two peaks during this particular month, this is not the case for the remaining intraday volatility measures. Similar observations can be made for any other month. Overall, Figure 2 strengthens our choice to consider alternative intraday volatility measures within our modelling framework.

[FIGURE 2 HERE]

3. Modelling framework

Section 3.1 presents the models for the estimation of the implied volatility (OVX_t), whereas Section 3.2 illustrates the models for the different intraday volatility measures, i.e. IRV_t: $\{RV_t, ScaledRV_t, BPV_t, MedRV_t, MinRV_t, RSV_t^+, RSV_t^-\}$.

3.1. Modelling the Implied Volatility

Naïve models for OVX

Following the current literature we define as naïve frameworks the Random Walk model with a drift, as well as, the 1st order Autoregressive model for the logarithmic transformation of volatility.

Random Walk model:

$$log(OVX_t) = w_0^{(t)} + \varepsilon_t, \tag{23}$$

AR(1) model:

$$log(OVX_t) = w_0^{(t)}\left(1 - \varphi_1^{(t)}\right) + w_1^{(t)} log(OVX_{t-1}) + \varepsilon_t, \tag{24}$$

where $w_0^{(t)}$, $w_1^{(t)}$ are the rolling estimated coefficients and ε_t denotes the white noise.

Heterogeneous Autoregressive model (HAR-OVX) for OVX

The HAR model is being considered by the literature (i.e. Haugom *et al.*, 2014, Sévi, 2014) as a prominent framework in predicting volatility accurately . We estimate the HAR-OVX model as it is estimated in its original version by Corsi (2009):

$$log(OVX_t) = w_0^{(t)} + w_1^{(t)} log(OVX_{t-1}) + w_2^{(t)}\left(5^{-1} \sum_{k=1}^{5} log(OVX_{t-k})\right) + w_3^{(t)}(22^{-1} \sum_{k=1}^{22} log(OVX_{t-k})) + \varepsilon_t, \quad (25)$$

where $w_0^{(t)}, w_1^{(t)}, w_2^{(t)}$, and $w_3^{(t)}$ denote the rolling parameters to be estimated.

Heterogeneous Autoregressive model with exogenous predictors (HAR-OVX-x) for OVX

Next, we estimate a HAR model including each of the intraday volatility measures (IRV) of WTI, as exogenous predictors. The model entitled HAR-OVX-WT is estimated in the form:

$$log(OVX_t) = w_0^{(t)} + w_1^{(t)} log(OVX_{t-1}) + w_2^{(t)}\left(5^{-1} \sum_{k=1}^{5} log(OVX_{t-k})\right) + w_3^{(t)}(22^{-1} \sum_{k=1}^{22} log(OVX_{t-k})) + w_4^{(t)} log\left(IRV_{(WT),t-1}\right) + w_5^{(t)}\left(5^{-1} \sum_{k=1}^{5} log\left(IRV_{(WT),t-k}\right)\right) + w_6^{(t)}\left(22^{-1} \sum_{k=1}^{22} log\left(IRV_{(WT),t-k}\right)\right) + \varepsilon_t, \quad (26)$$

where $IRV_{(WT),t}$ expresses the various intraday volatility measures of WTI. Intuitively, the HAR-OVX-WT is built to capture the incremental predictive information that may be extracted from the current looking volatility measures of WTI oil. The HAR-OVX-WT is estimated for the

$$IRV_{(WT),t}: \left\{\begin{matrix} RV_{(WT),t}, ScaledRV_{(WT),t}, BPV_{(WT),t}, MedRV_{(WT),t}, \\ MinRV_{(WT),t}, RSV^{+}_{(WT),t}, RSV^{-}_{(WT),t} \end{matrix}\right\}$$

intraday volatility measures of WTI oil. However, we have to explore whether the current looking volatility measures of other assets classes are able to provide any predictive ability on OVX, as well. Hence, the HAR-OVX-x model is also estimated for the intraday volatility of Brent (CO), commodity index (AI), Dollar index (DX), S&P500 (SP) and T-bills (TY), where $(x): \{CO, AI, DX, SP, TY\}$:

$$log(OVX_t) = w_0^{(t)} + w_1^{(t)} log(OVX_{t-1}) + w_2^{(t)}\left(5^{-1} \sum_{k=1}^{5} log(OVX_{t-k})\right) + w_3^{(t)}(22^{-1} \sum_{k=1}^{22} log(OVX_{t-k})) + w_4^{(t)} log\left(IRV_{(x),t-1}\right) + w_5^{(t)}\left(5^{-1} \sum_{k=1}^{5} log\left(IRV_{(x),t-k}\right)\right) + w_6^{(t)}\left(22^{-1} \sum_{k=1}^{22} log\left(IRV_{(x),t-k}\right)\right) + \varepsilon_t. \quad (27)$$

The $IRV_{(x),t}$ denotes the different intraday volatility measures of the different asset classes, i.e. $IRV_{(x),t}: \left\{\begin{matrix} RV_{(x),t}, ScaledRV_{(x),t}, BPV_{(x),t}, MedRV_{(x),t}, \\ MinRV_{(x),t}, RSV^{+}_{(x),t}, RSV^{-}_{(x),t} \end{matrix}\right\}$.

Going one step beyond, we augment the HAR-OVX-WT framework adding one more exogenous predictor; the intraday volatility of one more asset class among the CO, AI, DX, SP and TY. The HAR-OVX-WT-x model is estimated in the form:

$$log(OVX_t) = w_0^{(t)} + w_1^{(t)} log(OVX_{t-1}) + w_2^{(t)}\left(5^{-1}\sum_{k=1}^{5} log(OVX_{t-k})\right) + w_3^{(t)}(22^{-1}\sum_{k=1}^{22} log(OVX_{t-k})) + w_4^{(t)} log\left(IRV_{(WT),t-1}\right) + w_5^{(t)}\left(5^{-1}\sum_{k=1}^{5} log\left(IRV_{(WT),t-k}\right)\right) + w_6^{(t)}\left(22^{-1}\sum_{k=1}^{22} log\left(IRV_{(WT),t-k}\right)\right) + w_7^{(t)} log\left(IRV_{(x),t-1}\right) + w_8^{(t)}\left(5^{-1}\sum_{k=1}^{5} log\left(IRV_{(x),t-k}\right)\right) + w_9^{(t)}\left(22^{-1}\sum_{k=1}^{22} log\left(IRV_{(x),t-k}\right)\right) + \varepsilon_t, \quad (28)$$

where $(x)\colon \{CO, AI, DX, SP, TY\}$.

We should highlight here that we are aware of the possibility of including implied volatility measures as potential predictors of OVX. Nevertheless, implied volatility measures for all selected asset classes, so to allow direct comparisons with the intraday volatility measures, are not available and thus we opt out of using a selection of these indices in our framework. Furthermore, various modifications of the HAR model have been proposed; for example the inclusion of jump components, the use of alternative volatility measures as predictive variables of the realized volatility or the adjustment of the HAR's lagged orders. We have tested various alterations of the HAR model and we conclude that the model in its original version provides better forecasts.

3.2. Modelling the Intraday Realized Volatility

Naïve models for IRV

The rolling coefficients for the naïve models are estimated from the Random Walk model:

$$log\left(IRV_{(WT),t}\right) = w_0^{(t)} + \varepsilon_t, \quad (29)$$

and the AR(1) model:

$$log\left(IRV_{(WT),t}\right) = w_0^{(t)}\left(1 - \varphi_1^{(t)}\right) + w_1^{(t)} log\left(IRV_{(WT),t-1}\right) + \varepsilon_t. \quad (30)$$

The $IRV_{(WT),t}$ expresses the various intraday volatility measures of WTI, or $IRV_{(WT),t}\colon \left\{ \begin{matrix} RV_{(WT),t}, ScaledRV_{(WT),t}, BPV_{(WT),t}, MedRV_{(WT),t}, \\ MinRV_{(WT),t}, RSV^{+}_{(WT),t}, RSV^{-}_{(WT),t} \end{matrix} \right\}$.

Heterogeneous Autoregressive model (HAR-WT) for IRV of the WTI

Similarly to HAR-OVX, we estimate the HAR-WT model:

$$log\left(IRV_{(WT),t}\right) = w_0^{(t)} + w_1^{(t)} log\left(IRV_{(WT),t-1}\right) + w_2^{(t)}\left(5^{-1} \sum_{k=1}^{5} log\left(IRV_{(WT),t-k}\right)\right) + w_3^{(t)}\left(22^{-1} \sum_{k=1}^{22} log\left(IRV_{(WT),t-k}\right)\right) + \varepsilon_t. \quad (31)$$

Heterogeneous Autoregressive model with exogenous predictors (HAR-WT-x) for IRV of the WTI

The HAR-WT-x model with the intraday volatility measures of Brent (CO), commodity index (AI), Dollar index (DX), S&P500 (SP) and T-bills (TY), as exogenous predictors, is estimated in the form:

$$log\left(IRV_{(WT),t}\right) = w_0^{(t)} + w_1^{(t)} log\left(IRV_{(WT),t-1}\right) + w_2^{(t)}\left(5^{-1} \sum_{k=1}^{5} log\left(IRV_{(WT),t-k}\right)\right) + w_3^{(t)}\left(22^{-1} \sum_{k=1}^{22} log\left(IRV_{(WT),t-k}\right)\right) + w_4^{(t)} log\left(IRV_{(x),t-1}\right) + w_5^{(t)}\left(5^{-1} \sum_{k=1}^{5} log\left(IRV_{(x),t-k}\right)\right) + w_6^{(t)}\left(22^{-1} \sum_{k=1}^{22} log\left(IRV_{(x),t-k}\right)\right) + \varepsilon_t, \quad (32)$$

where (x): $\{CO, AI, DX, SP, TY\}$.

The aforementioned HAR-WT-x model captures the predictive gains that may be extracted from the intraday volatility measures of the other assets.

4. Forecasting framework

4.1. Forecasting models

In order to evaluate a forecasting framework is crucial to secure the validity of the produced forecasts. For instance, it is typical in forecasting exercises to commit *looking ahead bias*. The present study avoids using future actual information either in selecting the predictors or in estimating the models.

Concerning the model with autoregressive structure; i.e. AR and HAR, the forecasts are computed based on data that belong to the information set at time *t* and thus, they are known to the forecaster at the time of the forecasting exercise.

However, the HAR-OVX-WT, HAR-OVX-WT-x or HAR-WT-x models require the use of future data that do not belong to the information set at time *t*. Based on Degiannakis and Filis (2017), we predict the futures values of the exogenous predictors based on satellite HAR models. In other words, the exogenous volatilities data that are required for the estimation of the $t + 2, \ldots, t + 66$ forecasts of the WTI oil volatility (which are not available to the forecaster at time *t*), are computed from

HAR models for the intraday volatility measures of Brent, commodity index, Dollar index, S&P500 and US T-bills. Hence, we produce real out-of-sample forecasts at all times.

Regarding the next day's forecast, all the required information belongs to the information set at time *t*. For instance, the one-day-ahead forecasts of the HAR-WT-x model is:

$$IRV_{(WT),t+1|t} = exp\Big(\hat{w}_0^{(t)} + \hat{w}_1^{(t)} log\big(IRV_{(WT),t}\big) + \hat{w}_2^{(t)}\big(5^{-1}\textstyle\sum_{k=1}^{5} log\big(IRV_{(WT),t-k+1}\big)\big) + \hat{w}_3^{(t)}\big(22^{-1}\textstyle\sum_{k=1}^{22} log\big(IRV_{(WT),t-k+1}\big)\big) + \hat{w}_4^{(t)} log\big(IRV_{(x),t}\big) + \hat{w}_5^{(t)}\big(5^{-1}\textstyle\sum_{k=1}^{5} log\big(IRV_{(x),t-k+1}\big)\big) + \hat{w}_6^{(t)}\big(22^{-1}\textstyle\sum_{k=1}^{22} log\big(IRV_{(x),t-k+1}\big)\big) + {}^{1}\!/_{2}\,\hat{\sigma}_{\varepsilon}^{2}\Big). \tag{33}$$

But, regarding the s-days-ahead forecast of the HAR-WT-x models, for example, the predictions for $s \geq 2$ are computed as:

$$\begin{aligned} IRV_{(WT),t+s|t} = exp\Bigg(&\hat{w}_0^{(t)} + \hat{w}_1^{(t)} log\big(IRV_{(WT),t+s-1|t}\big) \\ &+ \hat{w}_2^{(t)}\left(s^{-1}\sum_{k=1}^{s-1} log\big(IRV_{(WT),t-k+s|t}\big)\right. \\ &\left.+ (5-s)^{-1}\sum_{k=s}^{5} log\big(IRV_{(WT),t-k+s}\big)\right) \\ &+ \hat{w}_3^{(t)}\left(s^{-1}\sum_{k=1}^{s-1} log\big(IRV_{(WT),t-k+s|t}\big)\right. \\ &\left.+ (22-s)^{-1}\sum_{k=s}^{22} log\big(IRV_{(WT),t-k+s}\big)\right) \\ &+ \hat{w}_4^{(t)} log\big(IRV_{(x),t+s-1|t}\big) \\ &+ \hat{w}_5^{(t)}\left(s^{-1}\sum_{k=1}^{s-1} log\big(IRV_{(x),t-k+s|t}\big)\right. \\ &\left.+ (5-s)^{-1}\sum_{k=s}^{5} log\big(IRV_{(x),t-k+s}\big)\right) \\ &+ \hat{w}_6^{(t)}\left(s^{-1}\sum_{k=1}^{s-1} log\big(IRV_{(x),t-k+s|t}\big)\right. \\ &\left.+ (22-s)^{-1}\sum_{k=s}^{22} log\big(IRV_{(x),t-k+s}\big)\right) + {}^{1}\!/_{2}\,\hat{\sigma}_{\varepsilon}^{2}\Bigg). \end{aligned} \tag{34}$$

The $IRV_{(WT),t-k+s|t}$ and $IRV_{(x),t-k+s|t}$ terms represent the predictions for the intraday volatilities, whereas the $IRV_{(x),t-k+s}$ indicate the actual values of the exogenous variables. The same procedure is followed for all HAR-WT-x, HAR-OVX-WT and HAR-OVX-WT-x models.

The estimation of a multivariate HAR model could have captured the multi-directional effects among variables. However, the employed technique already captures those effects (as it considers the predicted values of the exogenous variables, where needed), and removes part of the complexity of the estimation of the models.

4.2. Forecasting strategy

We use an initial sample period of $\tilde{T} = 1000$ trading days. The remaining $\breve{T} = 971$ trading days are used for the real out-of-sample forecasting period. The choice of the initial sample period, which stops at the end of 2013, is justified by the fact that a large enough sample size is required for the estimation of the forecasting models but also due to the fact we intentionally need the post-2014 period to be part of the out-of-sample period. Figure 1 demonstrates that oil price volatility behaviour changes radically in the post-2014 period, reflecting primarily the oil price slump of 2014-2016. Thus, we intentionally require the change in this oil price volatility behaviour to be part of the out-of-sample period, since it allows the even better performance evaluation of our forecasting models. We produce forecasts from 1-day up to 66-days ahead. Hence, for the first set of real-out-of-sample forecasts for 1-day to 66-days ahead, we use the initial sample period $\tilde{T} = 1000$. For the remaining forecasts we employ a rolling window approach with a fixed window length of 1000 daily observations.

4.3. Forecast evaluation criteria

As discussed in the Introduction, the aim of this study is to move beyond the statistical loss functions, which are commonly used in the existing literature. Rather, we maintain that oil price volatility forecasts should be assessed based on the economic use that they serve. Hence, we develop a series of objective-based evaluation criteria based on four well established trading strategies[8] based on volatility forecasts, which are the following:

[8] We note that the chosen trading strategies are used indicatively so to provide the evidence that objective-based evaluation criteria should be used on oil price volatility forecasting evaluation. Other trading strategies, such as risk management, could be also employed.

Trading strategy 1 (TS1): Trade the implied volatility index based on implied volatility forecasts.

Trading strategy 2 (TS2): Trade the implied volatility index based on actual volatility forecasts (i.e. intraday volatility forecasts).

Trading strategy 3 (TS3): Trade a straddle based on implied volatility forecasts.

Trading strategy 4 (TS4): Trade the underlying price (net asset value - *NAV*) of the oil exchange traded fund based on either implied or actual (intraday) volatility forecasts.

The full list of the forecasting models for each trading strategy is shown in Table A1 in the appendix.

Given the aforementioned trading strategies, we develop the following trading rules:

Trading strategy 1 (TS1):

- Trading rule 1a (TR1a): If $OVX_{(j),t+s|t} > (<) OVX_t$, for $s = 1, \dots, 66$ days ahead, then the trader takes long (short) position in the OVX index at time t. The argument for this trading rule stems from the fact that if an investor believes that the implied volatility index will increase (decrease) in the future, then the long (short) position will be the profitable choice. For each one of the j forecasting models, for j=1,…80, the cumulative returns from TR1a are being computed as:

$$CR_{(j)}^{(s)} = \sum_{t=1}^{\check{T}} \left(I_{OVX,(j),t} \times \frac{(OVX_{t+s} - OVX_t)}{OVX_t} \right) \tag{35}$$

where the indicator function is defined as $I_{OVX,(j),t} = \begin{cases} 1 & \text{if} \quad OVX_{(j),t+s|t} > OVX_t \\ -1 & \text{if} \quad OVX_{(j),t+s|t} \leq OVX_t \end{cases}$

and (s) denotes the trading days ahead.

- Trading rule 1b (TR1b): If $OVX|x(RSV+)_{(j),t+s|t} < (>) OVX|x(RSV-)_{(j),t+s|t}$, for $s = 1, \dots, 66$ days ahead, then the trader takes long (short) position in the OVX index at time t. We shall remind the reader that $RSV+$ and $RSV-$ denote the positive and negative semi variance of the exogenous variables. This rule is justified by the fact that if the s-days ahead forecast of the positive semi variance is lower (higher) than the corresponding negative semi variance, it is expected that the negative (positive) returns prevail and thus an increase (decrease) in the OVX index is anticipated. We denote this rule as *SemiRV* and it based on Patton and Sheppard's (2015) finding that $RSV-$ is more informative for forecasting future volatility compared to $RSV+$. For each one of the j forecasting models, for j=1,…11, the cumulative returns from TR1b are:

$$CR_{(j)}^{(s)} = \sum_{t=1}^{\check{T}} \left(I_{OVX,(j),t}^{Semi} \times \frac{(OVX_{t+s} - OVX_t)}{OVX_t} \right) \tag{36}$$

where $I_{OVX,(j),t}^{Semi} = \begin{cases} 1 & \text{if} \quad OVX|x(RSV+)_{(j),t+s|t} < OVX|x(RSV-)_{(j),t+s|t} \\ -1 & \text{if} \quad OVX|x(RSV+)_{(j),t+s|t} \geq OVX|x(RSV-)_{(j),t+s|t} \end{cases}.$

Trading strategy 2 (TS2):

Similarly to the trading rule of the first trading strategy, we construct the rules for the second trading strategy, as follows:

- Trading rule 2a (TR2a): If $WT(IRV)_{(j),t+s|t} > (<) WT(IRV)_t$, for $s = 1, \dots, 66$ days ahead, then the trader takes long (short) position in the OVX index at time t. The TR2a cumulative returns for each j forecasting models, for j=1,…56, are computed as:

$$CR_{(j)}^{(s)} = \sum_{t=1}^{\check{T}} \left(I_{IRV,(j),t} \times \frac{(OVX_{t+s} - OVX_t)}{OVX_t} \right), \tag{37}$$

where $I_{IRV,(j),t} = \begin{cases} 1 & \text{if} \quad WT(IRV)_{(j),t+s|t} > WT(IRV)_t \\ -1 & \text{if} \quad WT(IRV)_{(j),t+s|t} \leq WT(IRV)_t \end{cases}.$

- Trading rule 2b (TR2b): If $WT(RSV+)_{(j),t+s|t} < (>) WT(RSV-)_{(j),t+s|t}$, for $s = 1, \dots, 66$ days ahead, then the trader takes long (short) position in the OVX index at time t. The TR2b cumulative returns for each j forecasting models, for j=1,…8, are computed as:

$$CR_{(j)}^{(s)} = \sum_{t=1}^{\check{T}} \left(I_{IRV,(j),t}^{Semi} \times \frac{(OVX_{t+s} - OVX_t)}{OVX_t} \right), \tag{38}$$

where $I_{IRV,(j),t}^{Semi} = \begin{cases} 1 & \text{if} \quad WT(RSV+)_{(j),t+s|t} < WT(RSV-)_{(j),t+s|t} \\ -1 & \text{if} \quad WT(RSV+)_{(j),t+s|t} \geq WT(RSV-)_{(j),t+s|t} \end{cases}.$

Trading strategy 3 (TS3):

Equivalently to trading rule of TS1, the following rule applies for trading in straddles:

- Trading rule 3 (TR3): If $OVX_{(j),t+s|t} > (<) OVX_t$, for $s = 1, \dots, 66$ days ahead, then the trader takes long (short) position in the straddle at time t.

A straddle is an options' strategy in which the investor holds a position in both a call and put with the same strike price and expiration date. The holder's rate of return is affected from any changes in volatility and large changes in the price of the underlying asset. Naturally, for a short holding period, the profits from straddles' trading are not expected to be related to the increase or decrease of the underlying asset's price, but with the changes in implied volatility.

Inspired by Engle *et al.* (1993) and Angelidis and Degiannakis (2008), we construct a trading platform with 80 participants (one for each forecasting model). Each participant is an investor who trades her beliefs regarding the future price of the straddles based on her implied volatility forecasts. Each trader follows a model and goes long (short) in a straddle when the forecasted volatility at time $t+s$ is higher (lower) than the actual volatility at the present time t.

The next trading day's straddle price on a \$1 share of the underlying asset; i.e. in our case the USO index, with s days to expiration and \$1 exercise price is computed as:

$$S_{t+1|t}^{(j)} = 2N\left(\frac{rf_t\sqrt{s}}{\overline{OVX}_{(j),t+s|t}} + \frac{\overline{OVX}_{(j),t+s|t}\sqrt{s}}{2}\right) - 2e^{-rf_t s}N\left(\frac{rf_t\sqrt{s}}{\overline{OVX}_{(j),t+s|t}} - \frac{\overline{OVX}_{(j),t+s|t}\sqrt{s}}{2}\right) + e^{-rf_t s} - 1,, \quad (39)$$

where $N(.)$ denotes the cumulative normal distribution function,

$$\overline{OVX}_{(j),t+s|t} = \frac{1}{s-1}\sum_{i=1}^{s}\left(\frac{OVX_{(j),t+s|t}}{\sqrt{252}}\right) \quad (40)$$

is the average volatility forecast during the life of the option, and rf_t is the risk free interest rate. The daily profit from holding the straddle equals to

$$\pi_t = max(e^{y_t} - e^{rf_t}, e^{rf_t} - e^{y_t}), \quad (41)$$

for y_t denoting the USO index log-returns and rf_t being the risk-free interest rate.

Each one of the 80 participants prices the straddles on a daily basis. A trade between any two participants, j and j^*, is executed at the average of their predicted prices, yielding to investor j a profit of:

$$\pi_{t+1}^{(j,j^*)} = \begin{cases} \pi_{t+1} - \left(S_{t+1|t}^{(j)} + S_{t+1|t}^{(j^*)}\right) & \text{if } S_{t+1|t}^{(j)} > S_{t+1|t}^{(j^*)} \\ \left(S_{t+1|t}^{(j)} + S_{t+1|t}^{(j^*)}\right) - \pi_{t+1} & \text{if } S_{t+1|t}^{(j)} \leq S_{t+1|t}^{(j^*)} \end{cases}. \quad (42)$$

As an economic evaluation criterion, we define the cumulative returns computed as $\pi^{(j)} = \frac{1}{2}\sum_{t=1}^{\tilde{T}}\sum_{j^*=1}^{2}\pi_t^{(j,j^*)}$.

Trading strategy 4 (TS4):

For this trading strategy the rules are constructed based on the premise that the volatility is considered a fear index. Hence, when investors expect volatility to increase (decrease) then they anticipate that the underlying asset price will decline (increase) returns. Under this premise:

- Trading rule 4a (TR4a): If $OVX_{(j),t+s|t} < (>) OVX_t$, for $s = 1, \ldots, 66$ days ahead, then the trader takes long (short) position in the NAV of the USO ETF at time t. For each one of the j forecasting models for j=1,…80, the cumulative returns from TR4a are computed as:

$$CR_{(j)}^{(s)} = \sum_{t=1}^{\tilde{T}} \left(I_{OVX,(j),t} \times \frac{(USO_{t+s} - USO_t)}{USO_t} \right) \qquad (43)$$

where $I_{OVX,(j),t} = \begin{cases} 1 & \text{if} \quad OVX_{(j),t+s|t} < OVX_t \\ -1 & \text{if} \quad OVX_{(j),t+s|t} \leq OVX_t \end{cases}$ and (s) denotes the trading days ahead. To avoid replication, we use the same estimation procedure for the cumulative returns of the remaining trading rules.

- Trading rule 4b (TR4b): If $WT(IRV)_{(j),t+s|t} < (>) WT(IRV)_t$, for $s = 1, \ldots, 66$ days ahead, then the trader takes long (short) position in the NAV of the USO ETF index at time t.
- Trading rule 4c (TR4c): If $OVX|x(RSV+)_{(j),t+s|t} > (<) OVX|x(RSV-)_{(j),t+s|t}$, for $s = 1, \ldots, 66$ days ahead, then the trader takes long (short) position in the NAV of the USO ETF at time t.
- Trading rule 4d (TR4d): If $WT(RSV+)_{(j),t+s|t} > (<) WT(RSV-)_{(j),t+s|t}$, for $s = 1, \ldots, 66$ days ahead, then the trader takes long (short) position in the NAV of the USO ETF index at time t.
- Trading rule 4e (TR4e): For this strategy, we also consider the Buy-and-hold, as well as, the Sell-and-hold strategy for benchmarking purposes.

Once all aforementioned steps are followed, we evaluate the forecasting models based on the after-cost profits of the four trading strategies. Based on Jung (2016), we estimate the transaction costs per trade to be between 0.6%-1.2%. As far as the USO trading, we consider an annual expense ratio of 0.76% and an annual management fee of 0.45%[9]. We note that transaction costs are incurred every time a trader changes her position (from long to short and vice versa), as dictated by the OVX or WT(IRV) forecast for each of the s-steps-ahead horizons.

To assess the models which yield the highest after-cost trading profits we employ the Model Confidence Set (MCS) by Hansen *et al.* (2011), based on two profitability criteria, namely the Mean Squared Distance (MSD) and Mean Absolute Distance (MAD), where the distance expresses the difference between the profits of each individual forecasting model (for instance, for trading rule 1a (TR1a), we have $I_{OVX,(j),t} \times \frac{(OVX_{t+s} - OVX_t)}{OVX_t}$ where $I_{OVX,(j),t} = 1$ if $OVX_{(j),t+s|t} > OVX_t$ and zero otherwise) and the profits of the best performing model $\left(\text{i.e., } \max_{(j)} \left(I_{OVX,(j),t} \times \frac{(OVX_{t+s} - OVX_t)}{OVX_t}\right)\right)$. The MCS test determines the set of models that consists of the best models, where best is defined in terms of a predefined evaluation criterion; in our case the MSD and MAD trading profits evaluation criteria. Let us define as $\Psi_{(j),t}$ the trading profits' criterion[10] of model j at trading day t, and $d_{(j),(j^*),t} = \Psi_{(j),t} - \Psi_{(j^*),t}$ as the evaluation differential for $\forall\, j, j^* \in M^0$.[11]

The null hypothesis

$$H_{0,M}: E\left(d_{(j),(j^*),t}\right) = 0, \text{ for } \forall\ j, j^* \in M, M \subset M^0, \tag{44}$$

is tested against the alternative one

$$H_{1,M}: E\left(d_{(j),(j^*),t}\right) \neq 0, \tag{45}$$

for some $j, j^* \in M$. The MCS test employs an elimination rule sequentially for $\forall\, M \subset M^0$ and, at each iteration, it identifies the model j to be removed from M in the case that $H_{0,M}$ is rejected. For brevity all tables that present the MCS tests are based on the MAD. The results of the MCS tests based on the MSD are qualitatively similar and available upon request.

5. Empirical results

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[9] These costs are based on the figures provided by the USCF investments, which launched the USO exchange traded fund (http://www.uscfinvestments.com/uso).

[10] I.e. for the MSD, we have $\Psi_{(j),t} = \left(\left(I_{OVX,(j),t} \times \frac{(OVX_{t+s} - OVX_t)}{OVX_t}\right) - \max_{(j)} \left(I_{OVX,(j),t} \times \frac{(OVX_{t+s} - OVX_t)}{OVX_t}\right)\right)^2$, where $I_{OVX,(j),t} = 1$ if $OVX_{(j),t+s|t} > OVX_t$ and zero otherwise.

[11] M^0 denotes the initial set of models under investigation.

Before we proceed with the forecasts' evaluations we should reiterate that our forecasting frameworks have been also extended to consider the incremental predictive gains from the jump components. In line with Tseng *et al*. (2009), Sevi (2014) and Prokopczuk *et al*. (2015), we also find that the jump component does not provide any incremental predictive gains and thus they are excluded from the remaining analysis. Nevertheless, the results are available upon request.

5.1. MCS results based on the four trading strategies

Our analysis is based on the evaluation of the after-cost trading profits of the four trading strategies presented in Section 4.3. The results are presented in Tables 2 – 5.

[TABLES 2-5 HERE]

A common feature across all strategies is that all competing models are capable of providing insignificantly different after-cost trading profits for the first 10-days ahead. Hence, we do not analyse the results for the 1-day up to 10-days ahead. Following this result, the remaining of our analysis focuses on the forecasting horizons of 22-, 44- and 66-days ahead[12].

Trading the OVX index based on TS1 and TS2

Starting with the results of the first trading strategy, as shown in Table 2, we notice that the models with the highest equivalent profitability are those that forecast the OVX taking into consideration the incremental predictive information of the WT volatility, as well as, the volatility of one additional asset class. Even more, we show that the best performing models are primarily those based on the trading rule TR1b, which utilises simultaneously the information from the positive and negative realized semi-variance of the WT and of one more asset class. Nevertheless, there are still two models that remain in the set of the best performing models, based on trading rule TR1a (although the HAR-OVX-WT(RSV-)-TY(RSV-) is only among the best model for the 22-days ahead horizon). Importantly, both models are based on the incremental predictive information provided by the negative realized semi-variance of WT and of one more asset class (AI and TY).

Interestingly enough, we cannot find evidence to suggest that there is also a particular asset class that also provides an edge to investors when trading the OVX

[12] The results for all the s-days ahead forecasts, *s*=1,…, 66 are available upon request.

index based on OVX forecasts. The only exception is the Brent crude oil price volatility, since none of the models which incorporate information from this asset's volatility is included in the set of the best performing models.

Overall, we can summarise the findings for the first trading strategy as follows. Traders who engage in such trading strategy should focus on OVX forecasts that incorporate information from the realized semi-variance of the WTI crude oil prices, as well as, the realized semi-variance of any of the remaining asset classes.

Turning our attention to the second trading strategy, Table 3 presents the results of the best performing models for traders who take positions (long or short) in the OVX based on forecasts of the WTI intraday volatility. The findings show strong evidence that the best performing models for the 22-, 44- and 66-days ahead horizons are those that include the incremental predictive information of the Brent crude oil intraday volatility, irrespectively of the volatility measure. This is a very important finding as it already shows that depending on the economic use of the forecasts, different models seems to be deemed more appropriate.

However, we should note that the first two trading strategies focus on the same asset (i.e. the OVX index) based on different forecasts. Hence, even though Table 3 presents the best performing models based on the WTI intraday volatility forecasts, we evaluate them against the performance of the OVX forecasts. The underlying purpose for such exercise is to provide additional evidence to investors who are interested in trading the OVX index. This evidence is related to the identification of which strategy between TS1 and TS2 is the most preferred. Under this comparison, we notice that none of the WTI intraday volatility forecasts can provide after-cost profits on OVX trading, which are equivalent (or insignificantly different) to the best performing models shown in Table 1. Hence, TS2 is a rather redundant strategy when it comes to OVX trading decisions.

Trading straddles based on TS3

Next, we consider the performance of the OVX forecasts under the third trading strategy (see Table 4). It is clear that investors who price their straddles based on OVX forecasts that are extended to incorporate the incremental information from the US Dollar index intraday volatility are able to generate superior profits, against all other traders. It is important to note that, unlike the evidence shown in Tables 2 and 3, forecasting models that incorporate the information of the WTI intraday volatility measures are not included in the set of the best performing models.

Once again, this finding further strengthens our initial thesis that objective-based evaluation criteria provide economically useful information to investors, as they show that different forecasting models are capable of exhibiting superior performance at different trading strategies.

Trading the USO ETF based on TS4

All aforementioned strategies are based on oil volatility trading. However, it is rather common for traders to take long or short positions in the underlying asset based on the asset's volatility expectations. Thus, in the final trading strategy investors trade the USO ETF underlying price based on forecasts of the OVX and WTI intraday volatility measures. The findings presented in Table 5 suggest that, on one hand, the best performing models are primarily those that forecast the OVX (rather than WTI intraday volatility), which is a similar finding with trading strategy TR1; however, on the other hand, these models are primarily based on the incremental information of the WT and CO realized semi-variance. We shall remind the reader that none of the models which incorporated the CO intraday volatility was included in the set of the best models in TS1. Finally, we shall also mention that the HAR-OVX-WT(SemiRV)-TY(SemiRV) model is also included in the set of the best performing models at all horizons for TS4.

Overall, the findings reported in Tables 2-5 clearly show that depending on the trading strategy that is followed, different set of models exhibit superior performance in terms of after-cost trading profitability.

5.2. Forecast average results

Despite the large number of forecasting models, we should not disregard the possibility that forecast averaging could provide significantly increased after-cost trading profits from the above strategies. Thus, we next proceed to the estimation of the model-averaged forecasts for both the implied and intraday volatility measures.

Starting from the implied volatility, we develop model-averaged forecasts based the forecasts from all HAR-OVX-WT, HAR-OVX-x and HAR-OVX-WT-x models (FA_OVX_WT_ALL). However, we would like to assess further whether there are certain intraday volatility measures that provide superior profits. Hence, we develop separate model-averaged forecasts based on the different intraday volatility measures (i.e. FA_OVX_WT_BPV, FA_OVX_WT_MedRV, FA_OVX_WT_MinRV, FA_OVX_WT_RSV-, FA_OVX_WT_RSV+, FA_OVX_WT_RV, FA_OVX_WT_ScaledRV). Finally, we average all forecasting models of the OVX index, along with the three naive models (i.e. RW-OVX, AR-OVX and HAR-OVX), which is denoted as FA_OVX_ALL. Similarly, we develop model-averaged forecasts for the intraday volatility measures[13]. Table 6 presents the results for TS1 and TS2, whereas Tables 7 and 8 present the results for TS3 and TS4, respectively.

[TABELS 6 - 8 HERE]

As far as the first two trading strategies are concerned, we evaluate the after-cost trading profits by including in the MCS test all individual models of TS1 and TS2, as well as, the model-averaged models. Similarly with the evidence provided by Tables 2 and 3, all model-averaged models (with few exceptions) are included in the confidence set with the models that provide superior profits for the 1-day up to 10-days ahead horizons (see Table 6). By contrast, none of these models are among those with significantly equal superior profits for any of the remaining horizons. Hence, the models in Table 2 remain the only ones that can provide the highest after-cost profitability for traders interested in investing in the OVX index.

A similar procedure is followed for the model-averaged forecasts performance in relation to TS3 and TS4. Thus, the MCS test includes all models from Tables 4 and 5 (for the TS3 and TS4, respectively), as well as, the equivalent model-averaged forecasts. Interestingly, the model-averaged forecasts are included among the best performing models only for the 1-day ahead horizon. For all other forecasting horizons, none of the model-averaged forecasts are capable of providing equal or better trading profits compared to the models on Tables 4 and 5.

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[13] FA_WT = WT forecast average based on all HAR-WT(IRV)-x(IRV) models. FA_WT_BPV = WT forecast average based on all HAR-WT(BPV)-x(BPV) models. FA_WT_MedRV = WT forecast average based on all HAR-WT(MedRV)-x(MedRV) models. FA_WT_MinRV = WT forecast average based on all HAR-WT(MinRV)-x(MinRV) models. FA_WT_RSV- = WT forecast average based on all HAR-WT(RSV-)-x(RSV-)models. FA_WT_RSV+ = WT forecast average based on all HAR-WT(RSV+)-x(RSV+) models. FA_WT_RV = WT forecast average based on all HAR-WT(RV)-x(RV) models. FA_WT_ScaledRV = WT forecast average based on all HAR-WT(ScaledRV)-x(ScaledRV) models. FA_WT_ALL = WT forecast average based on all models, i.e. naive and HAR-WT(IRV)-x(IRV) models.

Overall, the evidence convincingly shows that forecasting averages are not economically useful for any of the abovementioned trading strategies.

5.3. After-cost trading profits over time

In this section, we assess the profitability provided by the competing forecasting models, over time. We do so, since our results could be impacted by changes in the oil market conditions. For instance, we notice from Figure 1 an increase in the oil price volatility during the period 2014-2016, which is followed by a constant decline until the end of our sample period.

Hence, we proceed with the calculation of the cumulative after-cost trading profits, focusing only on the forecasting models that are included in the confidence set for each of the trading strategies, as shown in Tables 2-5. We should highlight that in this section we concentrate on the 22-, 44- and 66-days ahead horizons, since in the shorter-run horizons all forecasting models generate insignificantly different profits.

Figure 3 exhibits the cumulative profits of the best performing forecasting models from TS1 and TS2, as well as, the performance of the three naive models (i.e. RW, AR and HAR). Recall, that traders who engage in either TS1 or TS2 are taking positions in the OVX based on OVX or intraday volatility forecasts, respectively. In addition, we re-emphasize that none of the intraday volatility forecasting models are capable of generating superior or equal profits compared to the best performing OVX forecasting models; hence these models are excluded from Figure 3.

[FIGURE 3 HERE]

It is evident from Figure 3 that apart from a short period at the start of our out-of-sample period, the cumulative profits of the best performing models are constantly increasing. By contrast, the three naive models are not able to provide positive profitability for the largest part of our sample period, at least for the 22- and 44-days ahead forecasts.

Turning our attention to the straddles (see Figure 4), we can reach to similar conclusions as in the case of Figure 3. In particular, the naive models generate negative profits for a fairly large part at the start of the out-of-sample period, followed by a clear underperformance (compared to the best models of TS3) during the remaining time. On the other hand, the best performing models generate positive profits during the whole out-of-sample period, with the only exception being the HAR-OVX-AI(RV).

[FIGURE 4 HERE]

Finally, Figure 5 reveals the performance of the forecasting models for TS4. The findings for this trading strategy demonstrates that the HAR-OVX-WT-x models generate positive profits throughout the out-of-sample period, with the highest increase to be observed during the middle part of the period, which is characterised by the very volatile environment of the oil market. Clearly, the naive models are not even able to provide cumulative positive profits for the traders.

[FIGURE 5 HERE]

Overall, the findings from Figures 3-5 suggest that irrespectively of the market conditions and trading strategy, our modelling framework provides economically useful forecasts for traders who are interested in the OVX and USO assets.

5.4. Robustness tests

In this section we proceed with the estimation of several robustness tests so to verify the aforementioned findings.

First, we look at the maximum possible losses or profits that a trader could ever encounter when investing in either the OVX or USO, using any of the individual forecasting models or any of the forecast averages. We should highlight that our calculation does not assume that a trader should start investing at the first day of our out-of-sample period and liquate her investment at the last day of the out-of-sample period. Rather, we calculate the maximum possible losses or profits, considering that a trader can initiate or liquidate her investment at any point and for any length during the out-of-sample period.

Considering the first two trading strategies (TS1 and TS2), the maximum possible loss, for each one of the j forecasting models, for j=1,…,173 and for s-day ahead horizons, where s=1,…,66, is computed as:

$$maxL_{(j)}^{(s)} = \min_{\substack{\forall 1\leq m\leq n \\ \forall m\leq n\leq \check{T}}} \left(\sum_{t=m}^{n} \left(I_{OVX,(j),t} \times \frac{(OVX_{t+s} - OVX_t)}{OVX_t} \right) \right) \quad (46)$$

where $I_{OVX,(j),t} = \begin{cases} 1 & \text{if} \quad OVX_{(j),t+s|t} > OVX_t \\ -1 & \text{if} \quad OVX_{(j),t+s|t} \leq OVX_t \end{cases}$.

Equivalently, the maximum possible profit ever is computed as:

$$maxP_{(j)}^{(s)} = \max_{\substack{\forall 1 \leq m \leq n \\ \forall m \leq n \leq \check{T}}} \left(\sum_{t=m}^{n} \left(I_{OVX,(j),t} \frac{(OVX_{t+s} - OVX_t)}{OVX_t} \right) \right) \tag{47}$$

where $I_{OVX,(j),t}$ is an indicator function; i.e. $I_{OVX,(j),t} = \begin{cases} 1 & \text{if} \quad OVX_{(j),t+s|t} > OVX_t \\ -1 & \text{if} \quad OVX_{(j),t+s|t} \leq OVX_t \end{cases}$. Similarly, we compute the maximum possible losses and profits for the Straddles (TS3, for 90 forecasting models) and the USO trading (TS4, for 173 forecasting models).

Second, we compute the maximum number of sequential losses or gains of any of the 173 forecasting models for TS1, TS2, TS4 and the 90 forecasting models for TS3. The maximum number of sequential trading profits (SP) is computed as:

$$maxSP_{(j)}^{(s)} = \max_{\substack{\forall 1 \leq m \leq n \\ \forall m \leq n \leq \check{T}}} \left(\left(\sum_{t=m}^{n} I_{CF,(j),t} \right) \times I_{CF,(j),n} \right), \tag{48}$$

where $I_{CF,(j),t}$ is the indicator function of correct directional forecast; i.e. $I_{CF,(j),t} = 1$ if the directional accuracy is correctly predicted, and $I_{CF,(j),t} = 0$ otherwise. The maximum number of sequential trading losses (SL) is computed as:

$$maxSL_{(j)}^{(s)} = \max_{\substack{\forall 1 \leq m \leq n \\ \forall m \leq n \leq \check{T}}} \left(\left(\sum_{t=m}^{n} I_{WF,(j),t} \right) \times I_{WF,(j),n} \right), \tag{49}$$

where $I_{WF,(j),t}$ is the indicator function of incorrect directional forecasts; i.e. $I_{WF,(j),t} = 1 - I_{CF,(j),t}$.

The results for the aforementioned calculations are shown in Tables 9, 10 and 11, as well as, in Figure 6.

[TABLES 9, 10 and 11 HERE]

[FIGURE 6 HERE]

Table 9 shows the maximum possible trading losses/profits and maximum sequential trades with losses/gains for the OVX trading, i.e. TS1 and TS2. For brevity the table reports only the values for the best performing models, as suggested by the MCS test from Tables 2, 3 and 6. Similarly to Table 9, Tables 10 and 11 show the maximum possible trading losses/profits and maximum sequential trades with losses/gains for the Straddles and USO trading, i.e. TS3 and TS4. Once again, only the best performing models, according to the MCS tests of Tables 4, 5, 7 and 8 are reported on the table.

Figure 6, on the other hand, indicatively shows the distribution of the maximum possible profits and losses of the 44-days ahead horizon, for all forecasting models of all four trading strategies. The respective distributions for the remaining forecasting horizons are available upon request. The shaded area in Figure 6 denotes where the best performing models from MCS tests on Tables 2 – 8 can be located.

Overall, the results clearly suggest that our models are included in the top part of the distribution, which is suggestive of the fact that they offer both very low possible losses, as well as, the highest possible trading profits. These results further confirm the superiority of the chosen models and offer support to our reported findings from Sections 5.1-5.3.

Finally, the small possible losses of all best models of OVX, Straddles or USO trading further suggest that an investor requires lower leverage to protect her investment.

5.5. Statistical loss functions: A note

We should highlight here that the standard statistical loss functions (MSPE and MAE) have been also considered in the study. However, we have clearly shown that the objective-based evaluations criteria are more adequate to capture the forecasting performance of our models, since different models are shown to generate superior after-cost profits at the different strategies. Intuitively, if investors base their choice of the best forecasting models given the information extracted from the statistical loss functions, then their choice would be the same irrespectively of the trading strategy, which would clearly lead to a sub-optimal position, at least for some of these strategies.

To provide further evidence, Table 12 presents the models that are consistently included in the set of the best performing models based on the mean squared predictive error (MSPE).

[TABLE 12 HERE]

The results in Table 12 corroborate our argument that there is not a single model that is able to provide superior profitability for all four trading strategies. More importantly, there is a model (HAR-OVX-WT(ScaledRV)-AI(ScaledRV)) that, even though, it is included in the set of the best performing models for the 22-days ahead horizon, based on the statistical loss function, it never appears to the best performing models in any of the four trading strategies, according to the objective-based evaluation criteria. In addition, Figure 7 indicatively shows how investors would be led to sub-optimal investment choices at different forecasting horizons and different strategies. For instance, if a trader is interested in engaging in TS1 or TS2, employing the HAR-OVX-CO(SemiRV), then her after-cost profitability would be significantly lower compared to the best forecasting models, as these have been identified by the objective-based evaluation criteria. Even more, if an investor who is trading straddles (TS3) preferred to follow the predictions by HAR-OVX-WT(RSV-)-AI(RSV-), which is one of the best performing models based on the MSPE, then she would suffer cumulative losses during the out-of-sample period. Similar observations hold for the TS4.

[FIGURE 7 HERE]

We should mention that Gargano *et al.* (2017) also express the view that there is disparity between evaluations of forecasts based on statistical loss functions against the potential economic gains of such forecasts.

6. Conclusion

The aim of this paper is to contribute to the growing literature on oil price volatility forecasting. Contrary, though, to the current practice that mainly considers stand-alone statistical loss functions, we assess our forecasts using objective-based evaluations criteria. More specifically, we consider four well established trading strategies, which are based on volatility forecasts, namely (i) trading the implied volatility based on the implied volatility forecasts, (ii) trading implied volatility based on intraday volatility forecasts, (iii) trading straddles in the United States Oil Fund ETF and finally (iv) trading the United States Oil Fund ETF based on implied and intraday volatility forecasts. We then evaluate our forecasts based on the after-cost profitability for each of the four trading strategies. Our forecasting horizons range from 1-day up to 66-days ahead.

To do so, we forecast both the implied and (seven) intraday volatilities. For the latter, we use tick by tick data of the front-month WTI futures contracts. The period of the study spans from 4th January 2010 until 30th October 2017 (1971 trading days).

Motivated by the current state-of-the-art, we use HAR-type models for both the implied and intraday volatility forecasts, which are extended to accommodate various predictive components. More specifically, we extend the HAR models for OVX so to assess the incremental predictive ability of the intraday volatility measures from the WTI, as well as, four other asset classes (commodities, stocks, forex and macro). Similarly, the HAR models for each of the WTI intraday volatility measure are extended to accommodate the potential incremental gains from the intraday volatilities of the different asset classes.

Our real out-of-sample forecasts convincingly show our forecasting framework, as well as, the objective-based evaluation criteria are economically useful for financial traders, since different models provide superior after-cost profits depending on the economic use of the volatility forecast (i.e. the different trading strategies). Thus, we maintain that volatility forecasts should be evaluated based on their economic use, rather than statistical loss functions. Our results remain robust against forecast averaging and several robustness tests. Future research could develop alternative trading strategies or even hedging strategies to further investigate the issue at hand.

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TABLES

Table 1: Descriptive statistics of the WTI crude oil price volatility measures

	OVX	WTI(RV)	WTI(ScaledRV)	WTI(BPV)	WTI(MinRV)	WTI(MedRV)	WTI(RSV+)	WTI(RSV-)
Mean	33.754	27.770	28.384	27.105	26.488	26.505	19.189	19.447
Median	32.450	25.250	25.780	24.450	23.880	23.880	16.850	17.360
Maximum	78.970	101.820	103.430	101.820	101.820	101.820	87.100	70.380
Minimum	14.500	5.900	5.990	5.320	4.470	4.850	4.720	3.530
Std. Dev.	10.699	13.107	13.445	13.014	12.932	12.792	10.022	9.802
Coeff. of Var.	0.3170	0.4720	0.4737	0.4801	0.4882	0.4826	0.5223	0.5040
Skewness	0.710	1.665	1.682	1.726	1.714	1.711	2.018	1.456
Kurtosis	3.590	7.074	7.146	7.430	7.357	7.414	9.403	6.088
Jarque-Bera	193.920***	2273.169***	2341.191***	2589.927***	2523.946***	2561.711***	4705.045***	1479.614***
ADF	-3.854***	-5.899***	-5.894***	-5.847***	-5.730***	-5.558***	-5.288***	-5.505***
Correlation (*OVX* vs *IRV*)		0.832	0.832	0.832	0.823	0.831	0.757	0.776

Note: RV = realized volatility, ScaledRV = realized volatility adjusted for the close-to-open volatility, BPV = bipower volatility, MinRV = minimum realized volatility, MedRV = Median realized volatility, RSV+ = positive realized semi variance, RSV- = negative realized semi variance.
*** denotes significance at 1% level.

Table 2: Evaluation of trading strategy 1 (TS1): MCS p-values						
	Forecasting horizons					
Forecasting model:	1-day	5-days	10-days	22-days	44-days	66-days
Trading rule 1a for 80 models: *if* $OVX_{t+s\|t} > (<)\ OVX_t$ *then go long(short) on OVX*						
HAR-OVX-WT(RSV-)-AI(RSV-)	1.0000*	0.9960*	0.1521*	0.7004*	0.0374	0.1884*
HAR-OVX-WT(RSV-)-TY(RSV-)	0.9265*	0.9836*	0.2269*	0.2190*	0.0374	0.0323
Trading rule 1b for 11 models: *if* $OVX\|x(RSV+)_{t+s\|t} < (>)\ OVX\|x(RSV-)_{t+s\|t}$ *then go long(short) on OVX*						
HAR-OVX-WT(SemiRV)	0.9914*	0.9960*	0.7408*	0.7004*	0.0634	0.0185
HAR-OVX-WT(SemiRV)-AI(SemiRV)	1.0000*	0.9836*	0.3516*	1.0000*	1.0000*	0.1176*
HAR-OVX-WT(SemiRV)-DX(SemiRV)	0.9861*	0.9960*	1.0000*	0.6093*	0.0134	0.0465
HAR-OVX-WT(SemiRV)-SP(SemiRV)	1.0000*	1.0000*	0.6311*	0.7004*	0.0634	1.0000*
HAR-OVX-WT(SemiRV)-TY(SemiRV)	0.9707*	0.9863*	0.7306*	0.7004*	0.0374	0.1884*

Note: * denotes that the model is included in the confidence set of the models with the highest trading profits for TS1, according to the MCS test. We run the MCS test based on all 91 models. We note that the vast majority of the 91 models present statistically insignificant different profits for the 1-day up to 10-days ahead forecasting horizons. Thus, to reduce the dimension of the table we only present the models that maintain their significantly higher trading profits beyond the 10-days ahead horizon.

Table 3: Evaluation of trading strategy 2 (TS2): MCS p-values						
	Forecasting horizon					
Forecasting model:	1-day	5-days	10-days	22-days	44-days	66-days
Trading rule 2a for 56 models: *if* $WT(IRV)_{t+s\|t} > (<)\ WT(IRV)_t$ *then go long(short) on OVX*						
HAR-WT(BPV)-CO(BPV)	0.9999*	0.9707*	0.7200*	0.0984	0.0098	0.0225
HAR-WT(MedRV)-CO(MedRV)	0.9998*	0.9912*	0.6383*	0.1106*	0.0177	0.0212
HAR-WT(MinRV)-CO(MinRV)	0.9998*	0.9912*	0.6591*	0.0984	0.0115	0.0255
HAR-WT(RV)-CO(RV)	0.9998*	0.9817*	0.8273*	0.0641	0.0077	0.0212
HAR-WT(ScaledRV)-CO(ScaledRV)	0.9998*	0.9910*	0.8837*	0.0804	0.0069	0.0181
Trading rule 2b for 8 models: *if* $WT(RSV+)_{t+s\|t} < (>)\ WT(RSV-)_{t+s\|t}$ *then go long(short) on OVX*						
HAR-WT(SemiRV)-AI(SemiRV)	0.9999*	0.9226*	0.6591*	0.0113	0.0011	0.0051
HAR-WT(SemiRV)-CO(SemiRV)	0.9998*	0.9610*	0.9974*	0.0781	0.0024	0.0003

Note: * denotes that the model is included in the confidence set of the models with the highest trading profits for both TS1 and TS2, according to the MCS test. Thus, we run the MCS test based on the 91 models from TS1 and the 64 models from TS2 jointly. We note that the majority of the 64 models of TS2 present statistically insignificant different profits for the 1-day up to 10-days ahead forecasting horizons. Thus, to reduce the dimension of the table we only present the models that maintain their significantly higher trading profits beyond the 10-days ahead horizon for TS2.

Table 4: Evaluation of trading strategy 3 (TS3): MCS p-values

Forecasting model:	1-day	5-days	10-days	22-days	44-days	66-days
	Forecasting horizon					
Trading rule 3 for 80 models: $if\ OVX_{t+s\|t} > (<)\ OVX_t$ *then go long(short) on straddle*						
HAR-OVX-AI(RV)	0.5483*	0.2876*	0.1574*	0.4547*	1.0000*	1.0000*
HAR-OVX-DX(RSV+)	0.7029*	0.7301*	1.0000*	1.0000*	0.6032*	0.6156*
HAR-OVX-DX(MinRV)	0.8061*	0.7301*	0.4287*	0.0365	0.0000	0.0003
HAR-OVX-DX(MedRV)	0.8061*	1.0000*	0.8742*	0.4547*	0.1463*	0.4718*
HAR-OVX-DX(BPV)	0.8061*	0.2876*	0.1650*	0.0016	0.0000	0.0000

Note: * denotes that the model is included in the confidence set of the models with the highest trading profits in TS3, according to the MCS test. We run the MCS test based on 80 models. We note that the majority of the 80 models present statistically insignificant different profits for the 1-day up to 10-days ahead forecasting horizons. Thus, to reduce the dimension of the table we only present the models that maintain their significantly higher trading profits beyond the 10-days ahead horizon.

Table 5: Evaluation of trading strategy 4 (TS4): MCS p-values

Forecasting model:	1-day	5-days	10-days	22-days	44-days	66-days
	Forecasting horizons					
Trading rule 4a and 4b for 136 models: $if\ OVX_{t+s\|t} > (<)\ OVX_t$ *then go short (long) on USO*; $if\ WT(IRV)_{t+s\|t} > (<)\ WT(IRV)_t$ *then go short (long) on USO*						
HAR-OVX-CO(RSV-)	1.0000*	0.2233*	0.6577*	0.1084*	0.0127	0.0014
HAR-OVX-WT(RSV-)-CO(RSV-)	1.0000*	0.2598*	0.6218*	0.1400*	0.0127	0.0014
HAR-OVX-WT(RSV-)-TY(RSV-)	1.0000*	0.1503*	0.6218*	0.4177*	0.0258	0.0014
Trading rule 4c and 4d for 19 models: $if\ OVX\|x(RSV+)_{t+s\|t} > (<)\ OVX\|x(RSV-)_{t+s\|t}$ *then go long (short) on USO*; $if\ WT(RSV+)_{t+s\|t} > (<)\ WT(RSV-)_{t+s\|t}$ *then go long (short) on USO*						
HAR-OVX-AI(SemiRV)	1.0000*	0.6087*	1.0000*	0.4177*	0.0000	0.0000
HAR-OVX-CO(SemiRV)	1.0000*	1.0000*	0.9981*	0.4177*	0.1168*	0.7807*
HAR-OVX-WT(SemiRV)	1.0000*	0.4195*	0.9589*	0.8472*	0.1420*	0.3157*
HAR-OVX-WT(SemiRV)-AI(SemiRV)	1.0000*	0.5606*	0.9681*	1.0000*	1.0000*	0.0014
HAR-OVX-WT(SemiRV)-CO(SemiRV)	1.0000*	0.4133*	0.9559*	0.4177*	0.0258	0.0071
HAR-OVX-WT(SemiRV)-SP(SemiRV)	1.0000*	0.4200*	0.9681*	0.8583*	0.0133	0.0028
HAR-OVX-WT(SemiRV)-TY(SemiRV)	0.9956*	0.1746*	0.9063*	0.8583*	0.1168*	1.0000*

Note: * denotes that the model is included in the confidence set of the models with the highest trading profits in TS4, according to the MCS test. We run the MCS test based on all 155 models. We note that the vast majority of the 155 models present statistically insignificant different profits for the 1-day up to 10-days ahead forecasting horizons. Thus, to reduce the dimension of the table we only present the models that maintain their significantly higher trading profits beyond the 10-days ahead horizon.

Table 6: Evaluation of trading strategies 1 and 2 (TS1 and TS2): Forecast averages - MCS p-values

	Forecasting horizons					
Forecasting model:	1-day	5-days	10-days	22-days	44-days	66-days
Trading rule 1a and 2a for 18 forecasting average models: *if* $OVX_{t+s\mid t} > (<) OVX_t$ *then go long(short) on OVX*; *if* $WT(IRV)_{t+s\mid t} > (<) WT(IRV)_t$ *then go long(short) on OVX*						
FA_OVX_WT_ALL	1.0000*	0.9986*	0.2467*	0.0007	0.0004	0.0000
FA_OVX_WT_BPV	1.0000*	0.9912*	0.3458*	0.0006	0.0002	0.0000
FA_OVX_WT_MedRV	1.0000*	0.9923*	0.2776*	0.0003	0.0002	0.0000
FA_OVX_WT_MinRV	1.0000*	0.9972*	0.2767*	0.0003	0.0003	0.0000
FA_OVX_WT_RSV-	1.0000*	0.9986*	0.3458*	0.0046	0.0007	0.0025
FA_OVX_WT_RSV+	1.0000*	0.9972*	0.2519*	0.0006	0.0000	0.0000
FA_OVX_WT_RV	1.0000*	0.9986*	0.2519*	0.0006	0.0005	0.0003
FA_OVX_WT_ScaledRV	1.0000*	0.9986*	0.4019*	0.0008	0.0005	0.0000
FA_OVX _ALL	1.0000*	0.9986*	0.2904*	0.0015	0.0005	0.0000
FA _WT	0.9984*	0.1922*	0.1084*	0.0000	0.0000	0.0000
FA_ WT_BPV	1.0000*	0.0864	0.0376	0.0000	0.0000	0.0000
FA_ WT_MedRV	1.0000*	0.1290*	0.0550	0.0000	0.0000	0.0000
FA_ WT_MinRV	0.9985*	0.1236*	0.0537	0.0000	0.0000	0.0000
FA_ WT_RSV-	0.9997*	0.1973*	0.1084*	0.0000	0.0000	0.0000
FA_ WT_RSV+	0.9997*	0.1973*	0.1084*	0.0000	0.0000	0.0000
FA_ WT_RV	1.0000*	0.0727	0.0118	0.0000	0.0000	0.0000
FA_ WT_ScaledRV	0.9995*	0.0323	0.0057	0.0000	0.0000	0.0000
FA_WT_ALL	0.9997*	0.1973*	0.1084*	0.0000	0.0000	0.0000

Note: * denotes that the model is included in the confidence set of the models with the highest trading profits according to the MCS test. We run the MCS test based on the 91 models from TS1, 64 models from TS2 and the 18 forecast averages (a total of 173 models).

FA_OVX_WT_ALL = OVX forecast average based on all HAR-OVX-WT, HAR-OVX-x and HAR-OVX-WT-x models. FA_OVX_WT_BPV = OVX forecast average based on all HAR-OVX-WT(BPV), HAR-OVX-x(BPV) and HAR-OVX-WT(BPV)-x(BPV) models. FA_OVX_WT_MedRV = OVX forecast average based on all HAR-OVX-WT(MedRV), HAR-OVX-x(MedRV) and HAR-OVX-WT(MedRV)-x(MedRV) models. FA_OVX_WT_MinRV = OVX forecast average based on all HAR-OVX-WT(MinRV), HAR-OVX-x(MinRV) and HAR-OVX-WT(MinRV)-x(MinRV) models. FA_OVX_WT_RSV- = OVX forecast average based on all HAR-OVX-WT(RSV-), HAR-OVX-x(RSV-) and HAR-OVX-WT(RSV-)-x(RSV-) models. FA_OVX_WT_RSV+ = OVX forecast average based on all HAR-OVX-WT(RSV+), HAR-OVX-x(RSV+) and HAR-OVX-WT(RSV+)-x(RSV+) models. FA_OVX_WT_RV = OVX forecast average based on all HAR-OVX-WT(RV), HAR-OVX-x(RV) and HAR-OVX-WT(RV)-x(RV) models. FA_OVX_WT_ScaledRV = OVX forecast average based on all HAR-OVX-WT(ScaledRV), HAR-OVX-x(ScaledRV) and HAR-OVX-WT(ScaledRV)-x(ScaledRV) models. FA_OVX_ALL = OVX forecast average based on all models, i.e. naive, HAR-OVX-WT, HAR-OVX-x and HAR-OVX-WT-x models.

FA_WT = WT forecast average based on all HAR-WT(IRV)-x(IRV) models. FA_WT_BPV = WT forecast average based on all HAR-WT(BPV)-x(BPV) models. FA_WT_MedRV = WT forecast average based on all HAR-WT(MedRV)-x(MedRV) models. FA_WT_MinRV = WT forecast average based on all HAR-WT(MinRV)-x(MinRV) models. FA_WT_RSV- = WT forecast average based on all HAR-WT(RSV-)-x(RSV-)models. FA_WT_RSV+ = WT forecast average based on all HAR-WT(RSV+)-x(RSV+) models. FA_WT_RV = WT forecast average based on all HAR-WT(RV)-x(RV) models. FA_WT_ScaledRV = WT forecast average based on all HAR-WT(ScaledRV)-x(ScaledRV) models. FA_WT_ALL = WT forecast average based on all models, i.e. naive and HAR-WT(IRV)-x(IRV) models.

Table 7: Evaluation of trading strategy 3 (TS3): Forecast averages - MCS p-values						
	Forecasting horizons					
Forecasting model:	1-day	5-days	10-days	22-days	44-days	66-days
Trading rule 3 for 9 forecasting average models: *if* $OVX_{t+s|t} > (<) OVX_t$ *then go long(short) on straddle*						
FA_OVX_WT_ALL	0.2213*	0.0000	0.0000	0.0000	0.0000	0.0000
FA_OVX_WT_BPV	0.2310*	0.0000	0.0000	0.0000	0.0000	0.0000
FA_OVX_WT_MedRV	0.1968*	0.0000	0.0000	0.0000	0.0000	0.0000
FA_OVX_WT_MinRV	0.1422*	0.0000	0.0000	0.0000	0.0000	0.0000
FA_OVX_WT_RSV-	0.2213*	0.0000	0.0000	0.0000	0.0000	0.0000
FA_OVX_WT_RSV+	0.1582*	0.0000	0.0000	0.0000	0.0000	0.0000
FA_OVX_WT_RV	0.3373*	0.0000	0.0000	0.0000	0.0000	0.0000
FA_OVX_WT_ScaledRV	0.5894*	0.0000	0.0000	0.0000	0.0000	0.0000
FA_OVX _ALL	0.6002*	0.0000	0.0000	0.0000	0.0000	0.0000

Note: * denotes that the model is included in the confidence set of the models with the highest trading profits according to the MCS test. We run the MCS test based on the 80 models from TS3 and the 9 forecast averages (a total of 89 models).

FA_OVX_WT_ALL = OVX forecast average based on all HAR-OVX-WT, HAR-OVX-x and HAR-OVX-WT-x models. FA_OVX_WT_BPV = OVX forecast average based on all HAR-OVX-WT(BPV), HAR-OVX-x(BPV) and HAR-OVX-WT(BPV)-x(BPV) models. FA_OVX_WT_MedRV = OVX forecast average based on all HAR-OVX-WT(MedRV), HAR-OVX-x(MedRV) and HAR-OVX-WT(MedRV)-x(MedRV) models. FA_OVX_WT_MinRV = OVX forecast average based on all HAR-OVX-WT(MinRV), HAR-OVX-x(MinRV) and HAR-OVX-WT(MinRV)-x(MinRV) models. FA_OVX_WT_RSV- = OVX forecast average based on all HAR-OVX-WT(RSV-), HAR-OVX-x(RSV-) and HAR-OVX-WT(RSV-)-x(RSV-) models. FA_OVX_WT_RSV+ = OVX forecast average based on all HAR-OVX-WT(RSV+), HAR-OVX-x(RSV+) and HAR-OVX-WT(RSV+)-x(RSV+) models. FA_OVX_WT_RV = OVX forecast average based on all HAR-OVX-WT(RV), HAR-OVX-x(RV) and HAR-OVX-WT(RV)-x(RV) models. FA_OVX_WT_ScaledRV = OVX forecast average based on all HAR-OVX-WT(ScaledRV), HAR-OVX-x(ScaledRV) and HAR-OVX-WT(ScaledRV)-x(ScaledRV) models. FA_OVX_ALL = OVX forecast average based on all models, i.e. naive, HAR-OVX-WT, HAR-OVX-x and HAR-OVX-WT-x models.

Table 8: Evaluation of trading strategy 4 (TS4): Forecast averages - MCS p-values

	Forecasting horizons					
Forecasting model:	1-day	5-days	10-days	22-days	44-days	66-days
Trading rule 4a and 4b for 18 forecasting average models: *if* $OVX_{t+s\|t} > (<) OVX_t$ *then go short (long) on USO* *if* $WT(IRV)_{t+s\|t} > (<) WT(IRV)_t$ *then go short (long) on USO*						
FA_OVX_WT_ALL	1.0000*	0.0603	0.0006	0.0000	0.0000	0.0000
FA_OVX_WT_BPV	1.0000*	0.0489	0.0006	0.0000	0.0000	0.0000
FA_OVX_WT_MedRV	1.0000*	0.0399	0.0004	0.0000	0.0000	0.0000
FA_OVX_WT_MinRV	1.0000*	0.0589	0.0006	0.0000	0.0000	0.0000
FA_OVX_WT_RSV-	1.0000*	0.0677	0.0018	0.0000	0.0000	0.0000
FA_OVX_WT_RSV+	0.9999*	0.0587	0.0006	0.0000	0.0000	0.0000
FA_OVX_WT_RV	1.0000*	0.0550	0.0003	0.0000	0.0000	0.0000
FA_OVX_WT_ScaledRV	1.0000*	0.0662	0.0004	0.0000	0.0000	0.0000
FA_OVX _ALL	1.0000*	0.0320	0.0003	0.0000	0.0000	0.0000
FA _WT	1.0000*	0.0163	0.0099	0.0000	0.0000	0.0000
FA_ WT_BPV	1.0000*	0.0089	0.0018	0.0000	0.0000	0.0000
FA_ WT_MedRV	1.0000*	0.0091	0.0040	0.0000	0.0000	0.0000
FA_ WT_MinRV	1.0000*	0.0089	0.0040	0.0000	0.0000	0.0000
FA_ WT_RSV-	1.0000*	0.0195	0.0099	0.0000	0.0000	0.0000
FA_ WT_RSV+	1.0000*	0.0195	0.0099	0.0000	0.0000	0.0000
FA_ WT_RV	1.0000*	0.0059	0.0009	0.0000	0.0000	0.0000
FA_ WT_ScaledRV	1.0000*	0.0018	0.0005	0.0000	0.0000	0.0000
FA_WT_ALL	1.0000*	0.0195	0.0099	0.0000	0.0000	0.0000

Note: * denotes that the model is included in the confidence set of the models with the highest trading profits according to the MCS test. We run the MCS test based on the 91 models from TS1, 64 models from TS2 and the 18 forecast averages (a total of 173 models).

FA_OVX_WT_ALL = OVX forecast average based on all HAR-OVX-WT, HAR-OVX-x and HAR-OVX-WT-x models. FA_OVX_WT_BPV = OVX forecast average based on all HAR-OVX-WT(BPV), HAR-OVX-x(BPV) and HAR-OVX-WT(BPV)-x(BPV) models. FA_OVX_WT_MedRV = OVX forecast average based on all HAR-OVX-WT(MedRV), HAR-OVX-x(MedRV) and HAR-OVX-WT(MedRV)-x(MedRV) models. FA_OVX_WT_MinRV = OVX forecast average based on all HAR-OVX-WT(MinRV), HAR-OVX-x(MinRV) and HAR-OVX-WT(MinRV)-x(MinRV) models. FA_OVX_WT_RSV- = OVX forecast average based on all HAR-OVX-WT(RSV-), HAR-OVX-x(RSV-) and HAR-OVX-WT(RSV-)-x(RSV-) models. FA_OVX_WT_RSV+ = OVX forecast average based on all HAR-OVX-WT(RSV+), HAR-OVX-x(RSV+) and HAR-OVX-WT(RSV+)-x(RSV+) models. FA_OVX_WT_RV = OVX forecast average based on all HAR-OVX-WT(RV), HAR-OVX-x(RV) and HAR-OVX-WT(RV)-x(RV) models. FA_OVX_WT_ScaledRV = OVX forecast average based on all HAR-OVX-WT(ScaledRV), HAR-OVX-x(ScaledRV) and HAR-OVX-WT(ScaledRV)-x(ScaledRV) models. FA_OVX_ALL = OVX forecast average based on all models, i.e. naive, HAR-OVX-WT, HAR-OVX-x and HAR-OVX-WT-x models.

FA_WT = WT forecast average based on all HAR-WT(IRV)-x(IRV) models. FA_WT_BPV = WT forecast average based on all HAR-WT(BPV)-x(BPV) models. FA_WT_MedRV = WT forecast average based on all HAR-WT(MedRV)-x(MedRV) models. FA_WT_MinRV = WT forecast average based on all HAR-WT(MinRV)-x(MinRV) models. FA_WT_RSV- = WT forecast average based on all HAR-WT(RSV-)-x(RSV-)models. FA_WT_RSV+ = WT forecast average based on all HAR-WT(RSV+)-x(RSV+) models. FA_WT_RV = WT forecast average based on all HAR-WT(RV)-x(RV) models. FA_WT_ScaledRV = WT forecast average based on all HAR-WT(ScaledRV)-x(ScaledRV) models. FA_WT_ALL = WT forecast average based on all models, i.e. naive and HAR-WT(IRV)-x(IRV) models.

Table 9: Evaluation of best performing models from trading strategies 1 and 2 (TS1 and TS2) based on maximum possible trading losses/profits and maximum number of sequential trading losses/profits.

	Forecasting horizons					
Forecasting model:	1-day	5-days	10-days	22-days	44-days	66-days
Trading rule 1a and 2a for 156 models: $if\ OVX_{t+s\|t} > (<)\ OVX_t$ *then go long(short) on OVX*; $if\ WT(IRV)_{t+s\|t} > (<)\ WT(IRV)_t$ *then go long(short) on OVX*						
Trading rule 1b and 2b for 19 models: $if\ OVX\|x(RSV+)_{t+s\|t} < (>)\ OVX\|x(RSV-)_{t+s\|t}$ *then go long(short) on OVX*; $if\ WT(RSV+)_{t+s\|t} < (>)\ WT(RSV-)_{t+s\|t}$ *then go long(short) on OVX*						
HAR-OVX-WT(RSV-)-AI(RSV-)	-1.17, 0.95, 7, 6	-2.76, 9.23, 21, 17	-5.55, 15.03, 21, 20	-5.69, 53.10, 21, 48	-4.34, 145.80, 22, 75	-3.70, 229.16, 17, 158
HAR-OVX-WT(RSV-)-TY(RSV-)	-2.13, 0.80, 7, 7	-2.87, 7.57, 21, 17	-5.28, 17.07, 23, 20	-3.39, 64.83, 17, 40	-4.96, 147.87, 22, 75	-8.04, 221.55, 60, 154
HAR-OVX-WT(SemiRV)	-3.63, 0.85, 10, 9	-4.45, 16.07, 25, 13	-8.66, 31.99, 26, 33	-5.27, 68.54, 20, 41	-5.80, 147.04, 23, 118	-7.08, 209.38, 20, 117
HAR-OVX-WT(SemiRV)-AI(SemiRV)	-3.04, 1.07, 9, 9	-5.57, 14.29, 21, 17	-9.91, 27.13, 23, 33	-4.40, 69.13, 22, 41	-5.91, 152.17, 24, 124	-5.88, 219.37, 17, 126
HAR-OVX-WT(SemiRV)-DX(SemiRV)	-3.86, 0.70, 9 , 8	-3.15, 17.27, 21, 14	-7.21, 33.32, 28, 33	-5.53, 68.89, 22, 41	-4.98, 134.34, 25, 91	-5.58, 209.38, 20, 117
HAR-OVX-WT(SemiRV)-SP(SemiRV)	-2.49, 1.35, 8, 8	-3.95, 19.22, 18, 29	-7.98, 28.98, 28, 33	-7.83, 64.49, 24, 53	-4.95, 145.97, 27, 117	-4.36, 226.52, 18, 121
HAR-OVX-WT(SemiRV)-TY(SemiRV)	-4.10, 0.74, 8, 8	-4.83, 12.91, 21, 18	-9.12, 31.47, 23, 33	-6.17, 66.13, 22, 41	-4.39, 144.09, 25, 118	-5.79, 222.76, 18, 126
Optimal results						
Lowest losses	-0.86	-1.48	-3.09	-1.58	-2.94	-1.88
Highest profits	2.16	19.22	33.32	71.17	152.17	229.16
Lowest number of sequential losses	6	8	33	8	7	10
Highest number of sequential gains	12	29	9	53	124	158

Note: The sequence of the numbers refer to the (i) maximum possible losses (in times), (ii) maximum possible profits (in times), (iii) maximum number of sequential trading losses and (iv) maximum number of sequential trading profits, for each forecasting model at each forecasting horizon.

The optimal results do not refer to the same model. Different models generate the lowest losses, highest profits, lowest number of sequential losses and highest number of sequential profits within each of the forecasting horizon. We present these results for comparative reasons.

Table 10: Evaluation of best performing models from trading strategy 3 (TS3) based on maximum possible trading losses/profits and maximum number of sequential trading losses/profits.

	Forecasting horizons					
Forecasting model:	1-day	5-days	10-days	22-days	44-days	66-days
	Trading rule 3 for 89 models: *if* $OVX_{t+s\|t} > (<) OVX_t$ *then go long(short) on straddle*					
HAR-OVX-AI(RV)	-0.19, 0.62, 10, 10	-0.65, 9.44, 45, 35	-1.24, 17.14, 61, 148	-2.11, 31.75, 90, 218	-3.04, 51.26, 100, 428	-3.68, 61.49, 100, 436
HAR-OVX-DX(RSV+)	-0.14, 0.65, 7, 9	-0.67, 9.92, 34, 47	-1.25, 17.99, 45, 109	-2.37, 31.61, 51, 330	-3.80, 46.61, 41, 319	-5.51, 53.98, 75, 307
HAR-OVX-DX(MinRV)	-0.15, 0.79, 10, 13	-0.57, 10.03, 31, 79	-1.06, 17.71, 43, 188	-2.07, 30.48, 47, 232	-3.79, 43.65, 44, 319	-6.05, 51.02, 75, 308
HAR-OVX-DX(MedRV)	-0.14, 0.78, 12, 13	-0.68, 10.06, 32, 79	-1.28, 17.88, 43, 199	-2.48, 31.05, 48, 232	-3.98, 45.16, 44, 324	-5.86, 52.93, 75, 309
HAR-OVX-DX(BPV)	-0.12, 0.83, 10, 13	-0.55, 9.81, 34, 79	-1.08, 17.34, 39, 198	-2.21, 29.95, 50, 234	-3.55, 42.75, 40, 319	-5.50, 49.93, 75, 308
	Optimal results					
Lowest losses	-0.03	-0.21	-0.40	-0.92	-1.55	-2.49
Highest profits	1.28	11.22	20.60	36.13	55.81	70.32
Lowest number of sequential losses	6	16	25	44	38	46
Highest number of sequential gains	17	100	245	447	554	554

Note: The sequence of the numbers refer to the (i) maximum possible losses (in times), (ii) maximum possible profits (in times), (iii) maximum number of sequential trading losses and (iv) maximum number of sequential trading profits, for each forecasting model at each forecasting horizon.
The optimal results do not refer to the same model. Different models generate the lowest losses, highest profits, lowest number of sequential losses and highest number of sequential profits within each of the forecasting horizon. We present these results for comparative reasons.

Table 11: Evaluation of best performing models from trading strategies 4 (TS4) based on maximum possible trading losses/profits and maximum number of sequential trading losses/profits.

	Forecasting horizons					
Forecasting model:	1-day	5-days	10-days	22-days	44-days	66-days
Trading rule 4a and 4b for 156 models: *if* $OVX_{t+s\|t} > (<) OVX_t$ *then go short (long) on USO* *if* $WT(IRV)_{t+s\|t} > (<) WT(IRV)_t$ *then go short (long) on USO* Trading rule 4c and 4d for 19 models: *if* $OVX\|x(RSV+)_{t+s\|t} > (<) OVX\|x(RSV-)_{t+s\|t}$ *then go long (short) on USO* *if* $WT(RSV+)_{t+s\|t} > (<) WT(RSV-)_{t+s\|t}$ *then go long (short) on USO*						
HAR-OVX-CO(RSV-)	-1.54, 0.66, 9, 9	-1.47, 3.84, 21, 14	-2.96, 5.91, 23, 25	-3.40, 23.71, 28, 40	-3.76, 70.83, 41, 79	-4.21, 94.70, 53, 140
HAR-OVX-WT(RSV-)-CO(RSV-)	-1.51, 0.37, 9, 9	-2.03, 4.22, 16, 17	-2.54, 5.13, 27, 21	-4.30, 24.67, 28, 40	-2.77, 71.15, 41, 79	-4.21, 95.80, 40, 140
HAR-OVX-WT(RSV-)-TY(RSV-)	-1.83, 0.38, 9, 9	-2.16, 2.59, 15, 22	-2.73, 5.21, 31, 25	-2.75, 27.53, 28, 40	-2.53, 73.12, 41, 79	-4.00, 97.38, 35, 126
HAR-OVX-AI(SemiRV)	-1.03, 0.78, 11, 6	-1.83, 7.30, 20, 22	-3.39, 12.69, 22, 26	-5.91, 29.65, 40, 68	-2.96, 34.99, 106, 77	-5.31, 40.12, 169, 92
HAR-OVX-CO(SemiRV)	-2.77, 0.47, 15, 9	-1.61, 9.57, 15, 29	-4.47, 15.15, 24, 62	-6.83, 33.59, 28, 88	-2.16, 79.32, 18, 154	-5.03, 113.16, 50, 162
HAR-OVX-WT(SemiRV)	-2.79, 0.42, 16, 7	-2.39, 8.01, 17, 29	-4.01, 14.35, 20, 62	-3.42, 35.28, 26, 88	-4.03, 81.22, 34, 155	-5.03, 110.44, 54, 161
HAR-OVX-WT(SemiRV)-AI(SemiRV)	-2.89, 0.47, 15, 7	-3.04, 6.86, 13, 17	-4.90, 11.57, 23, 62	-2.64, 35.35, 23, 88	-2.10, 83.64, 27, 128	-4.49, 95.67, 48, 125
HAR-OVX-WT(SemiRV)-CO(SemiRV)	-3.11, 0.45, 14, 6	-1.66, 7.01, 25, 18	-4.75, 12.91, 23, 62	-5.17, 32.48, 28, 88	-2.24, 75.78, 26, 141	-5.25, 106.79, 52, 154
HAR-OVX-WT(SemiRV)-SP(SemiRV)	-3.08, 0.33, 12, 7	-2.12, 6.51, 17, 16	-3.29, 12.29, 20, 40	-3.30, 35.00, 22, 88	-4.65, 75.30, 57, 121	-5.29, 99.60, 54, 138
HAR-OVX-WT(SemiRV)-TY(SemiRV)	-.389, 0.49, 16, 9	-3.72, 6.18, 15, 28	-5.66, 14.32, 20, 61	-2.61, 35.91, 28, 88	-2.11, 80.98, 27, 153	-4.11, 113.60, 23, 157
Optimal results						
Lowest losses	-0.74	-1.47	-2.09	-2.27	-1.51	-2.14
Highest profits	0.77	9.57	15.15	35.91	83.64	113.60
Lowest number of sequential losses	7	10	9	9	9	9
Highest number of sequential gains	10	29	62	88	155	162

Note: The sequence of the numbers refer to the (i) maximum possible losses (in times), (ii) maximum possible profits (in times), (iii) maximum number of sequential trading losses and (iv) maximum number of sequential trading profits, for each forecasting model at each forecasting horizon.
The optimal results do not refer to the same model. Different models generate the lowest losses, highest profits, lowest number of sequential losses and highest number of sequential profits within

each of the forecasting horizon. We present these results for comparative reasons.

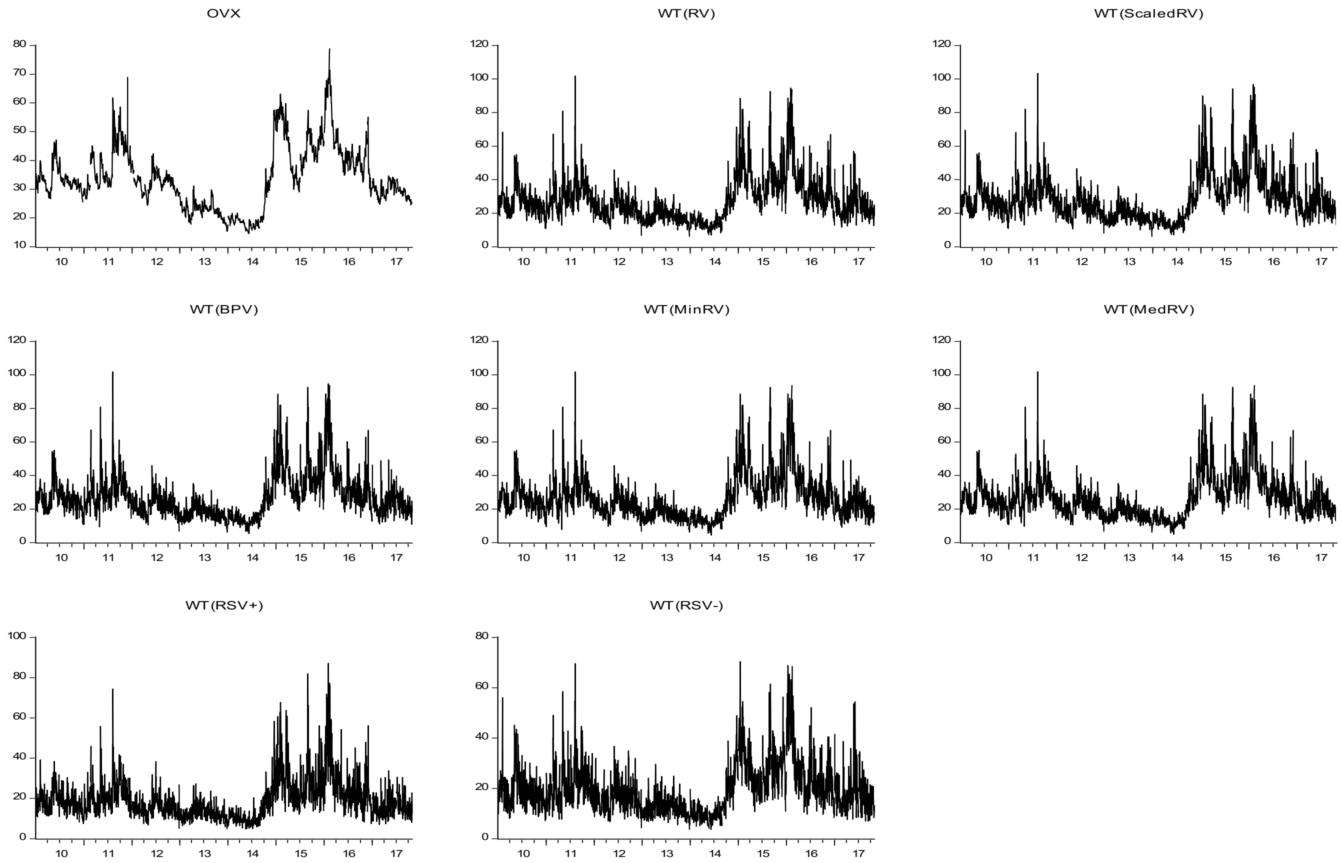
Table 12: Statistical evaluation of competing models: MCS p-values

	Forecasting horizons					
Forecasting model:	1-day	5-days	10-days	22-days	44-days	66-days
MSPE						
HAR-OVX-WT(RSV-)-AI(RSV-)	0.9990*	0.9990*	0.1430*	0.0063	0.1013*	1.0000*
HAR-OVX-WT(ScaledRV)-AI(ScaledRV)	0.9990*	0.9990*	0.9029*	0.6314*	0.0004	0.0000
HAR-OVX-CO(SemiRV)	0.9990*	0.9990*	1.0000*	1.0000*	0.0549	0.0000
HAR-OVX-WT(SemiRV)	0.9990*	0.9990*	0.5796*	0.6314*	1.0000*	0.1003*
Trading strategies for which the models are included in the set of the best performing models.						
HAR-OVX-WT(RSV-)-AI(RSV-)	TS1,TS4	TS1	TS1	TS1	TS1	TS1
HAR-OVX-WT(ScaledRV)-AI(ScaledRV)	TS1,TS4	TS1,TS4	TS1	-	-	-
HAR-OVX-CO(SemiRV)	TS1,TS4	TS1,TS4	TS1,TS4	TS4	TS4	TS4
HAR-OVX-WT(SemiRV)	TS1,TS4	TS1,TS4	TS1,TS4	TS1,TS4	TS4	TS4

Note: The upper panel shows the MCS p-values. * denotes that the model is included in the confidence set of the models with the highest trading profits according to the MCS test. We run the MCS test based on all 175 models. For brevity we only show the models that are consistently included in the set of the best performing models for the 22-days up to 66-days ahead forecasting horizons. The lower panel shows for which trading strategies the models are also included in the best performing models, according to the objective-based evaluation criteria, for each of the four trading strategies.

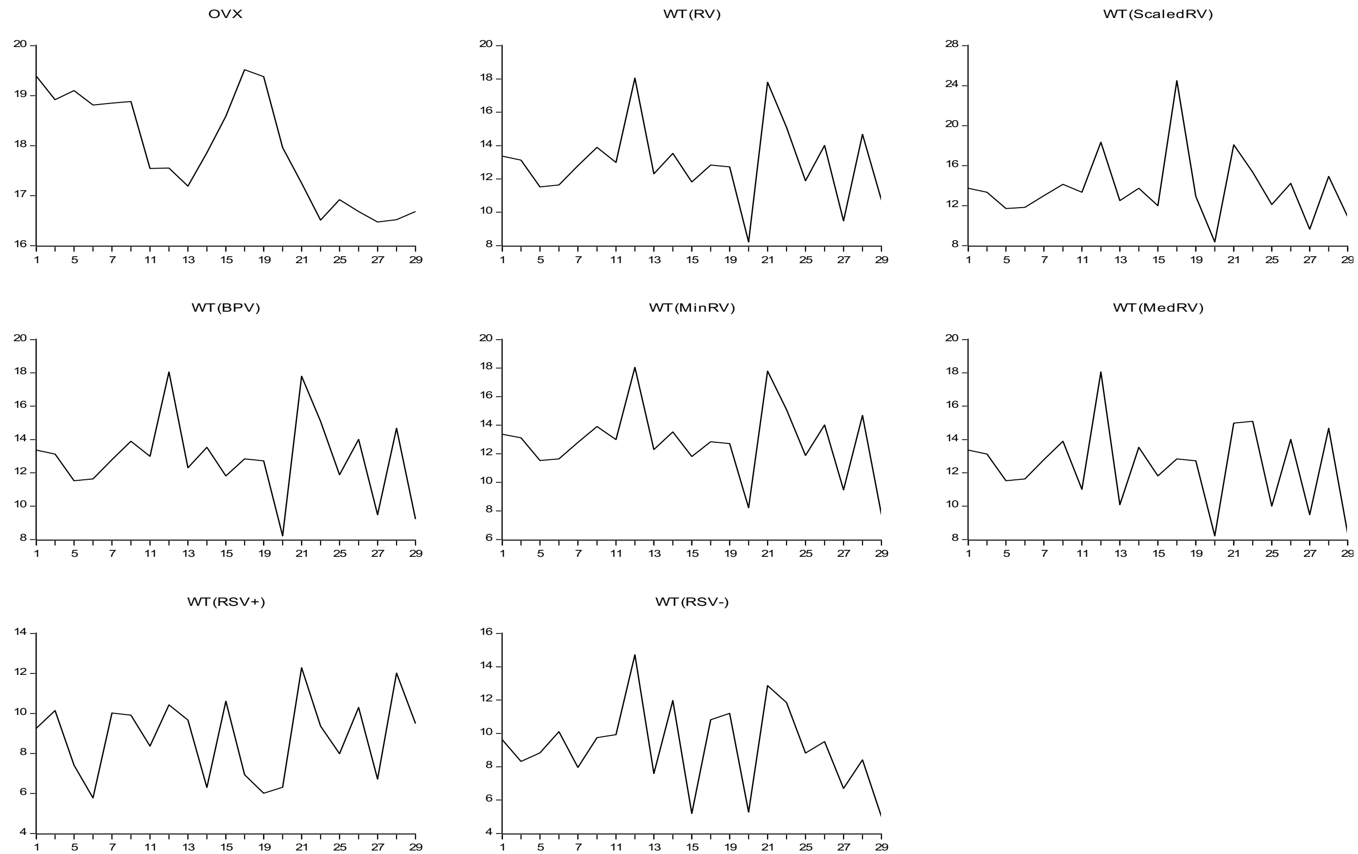
FIGURES

Figure 1: WTI crude oil price volatility plots



Note: RV = realized volatility, ScaledRV = realized volatility adjusted for the close-to-open volatility, BPV = bipower volatility, MinRV = minimum realized volatility, MedRV = Median realized volatility, RSV+ = positive realized semi variance, RSV- = negative realized semi variance.

Figure 2: WTI crude oil price volatility plots over a single random month



Note: RV = realized volatility, ScaledRV = realized volatility adjusted for the close-to-open volatility, BPV = bipower volatility, MinRV = minimum realized volatility, MedRV = Median realized volatility, RSV+ = positive realized semi variance, RSV- = negative realized semi variance.

Figure 3: After-cost profits for the best performing models from TS1 and TS2, along with the three naive models, for 22-, 44- and 66-days ahead horizons.

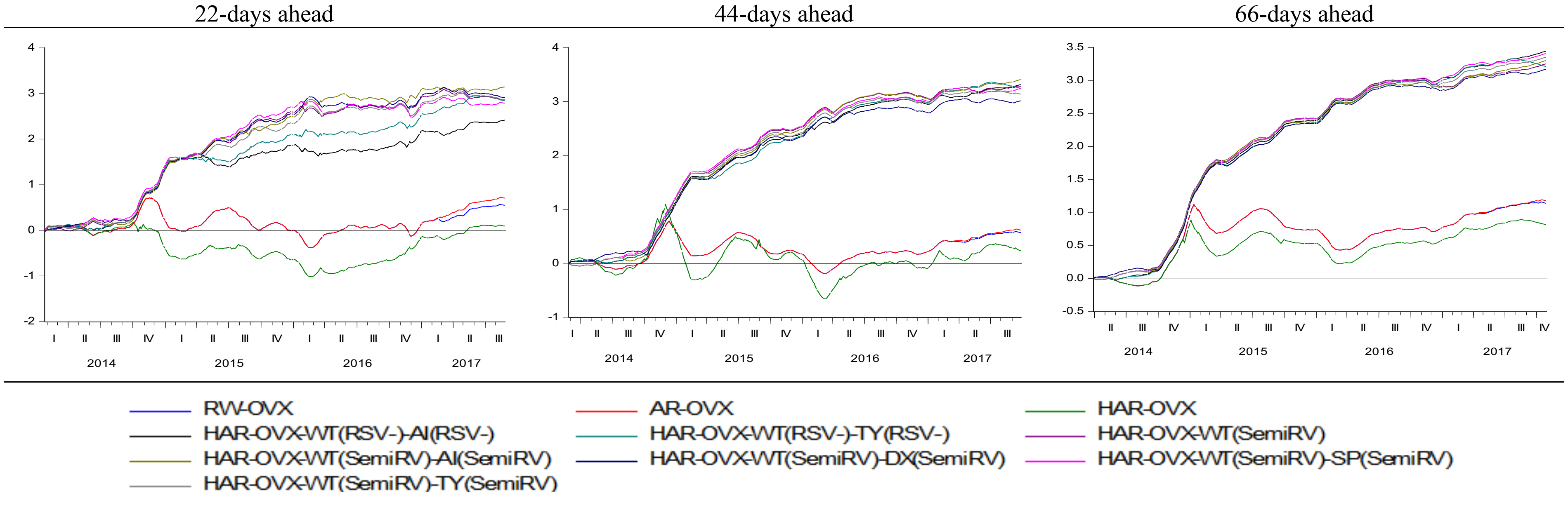


Figure 4: After-cost profits for the best performing models from TS3, along with the three naive models, for 22-, 44- and 66-days ahead horizons.

22-days ahead	44-days ahead	66-days ahead

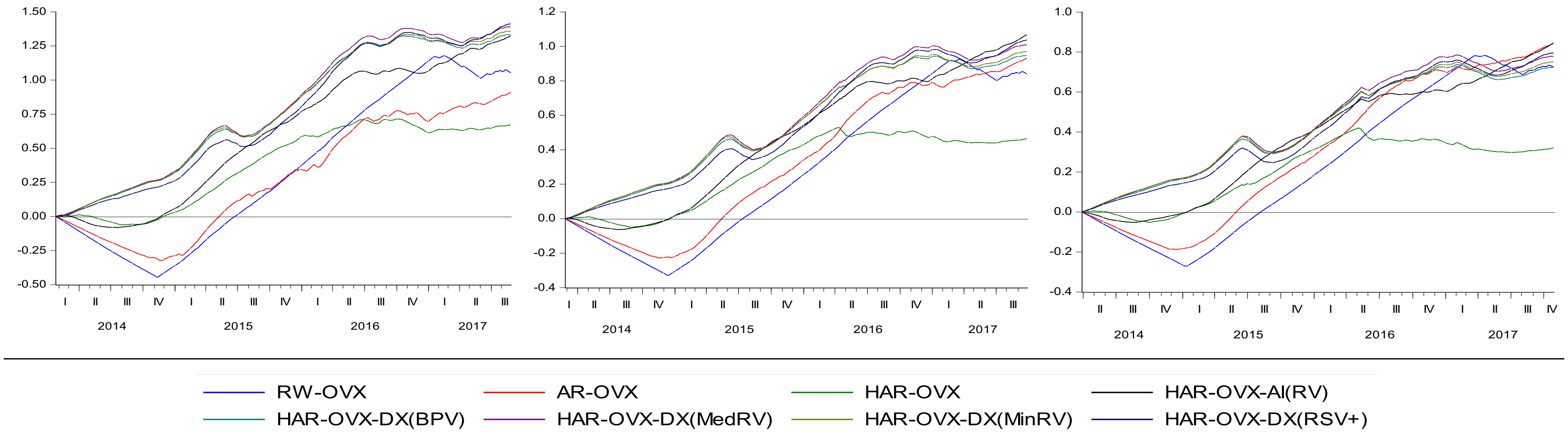


Figure 5: After-cost profits for the best performing models from TS4, along with the three naive models, for 22-, 44- and 66-days ahead horizons.

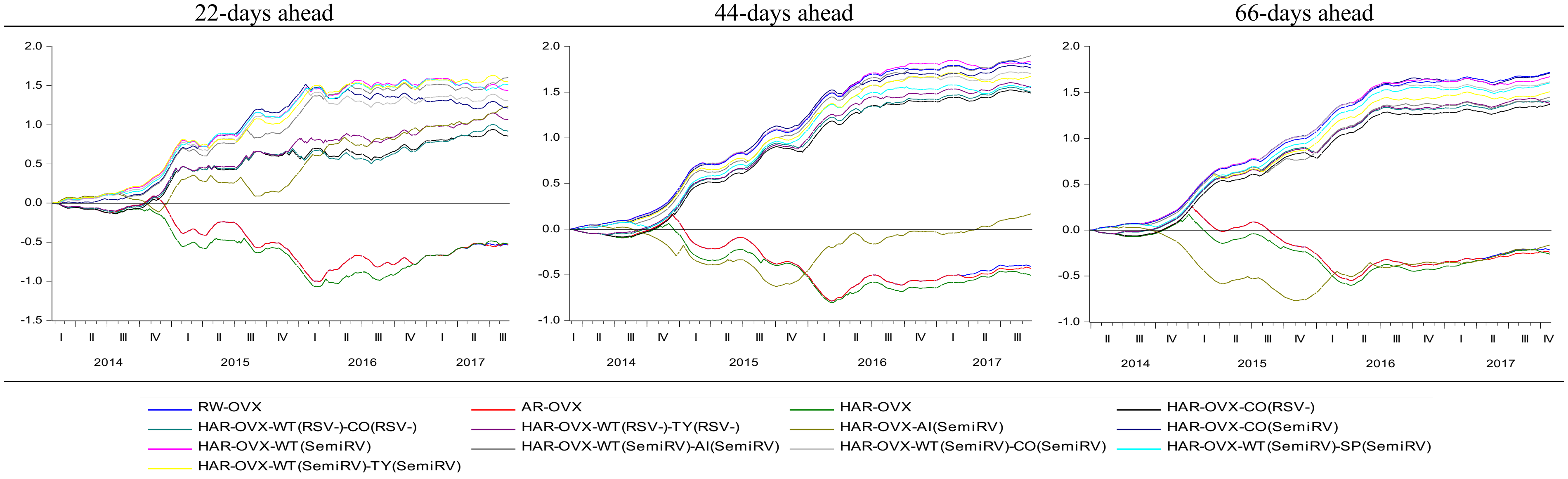
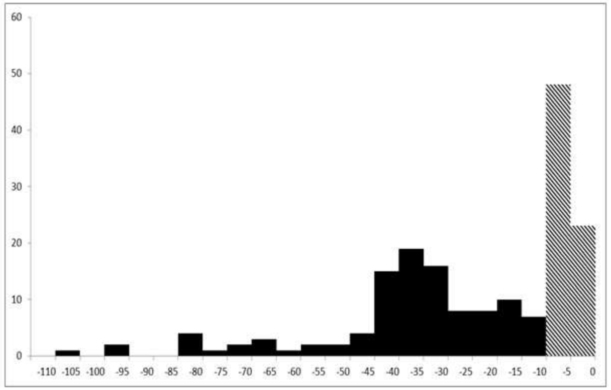
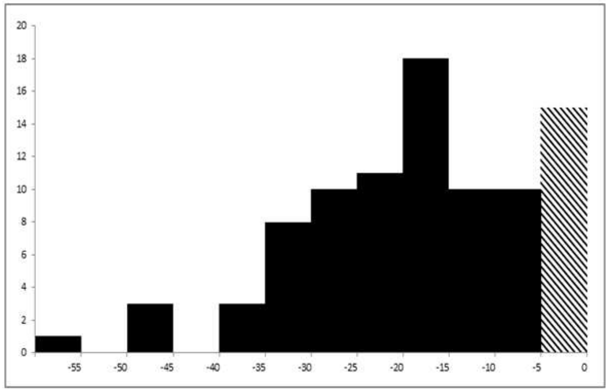


Figure 6: Distribution of maximum possible trading losses and profits for the OVX and the USO, for the 44-days ahead horizon.

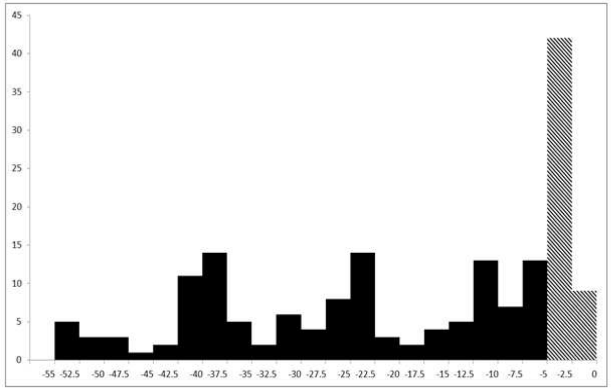
Maximum possible trading losses for OVX trading (TS1 and TS2)



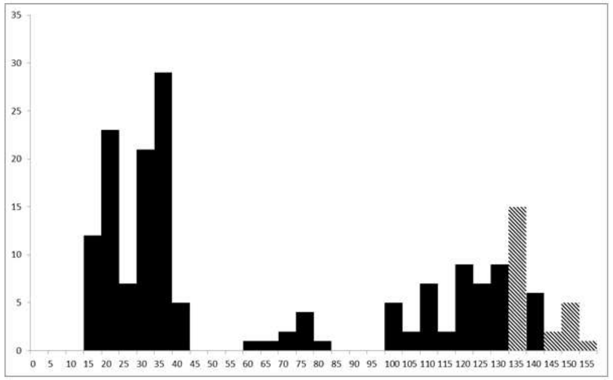
Maximum possible trading losses for straddles trading (TS3)



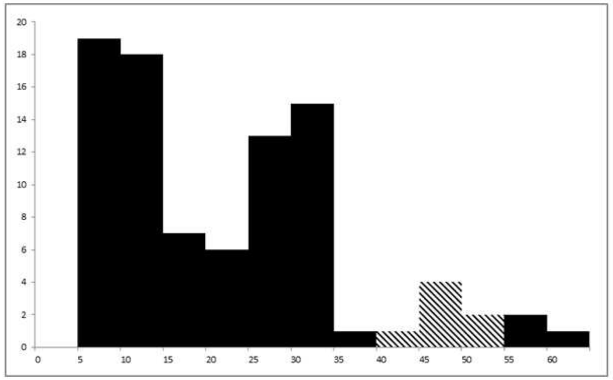
Maximum possible trading losses for USO trading (TS4)



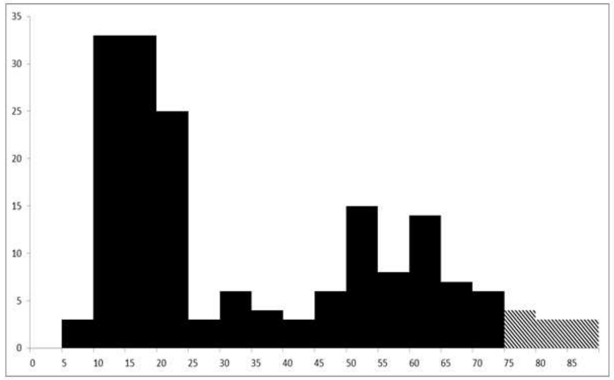
Maximum possible trading profits for OVX trading (TS1 and TS2)



Maximum possible trading profits for straddles trading (TS3)



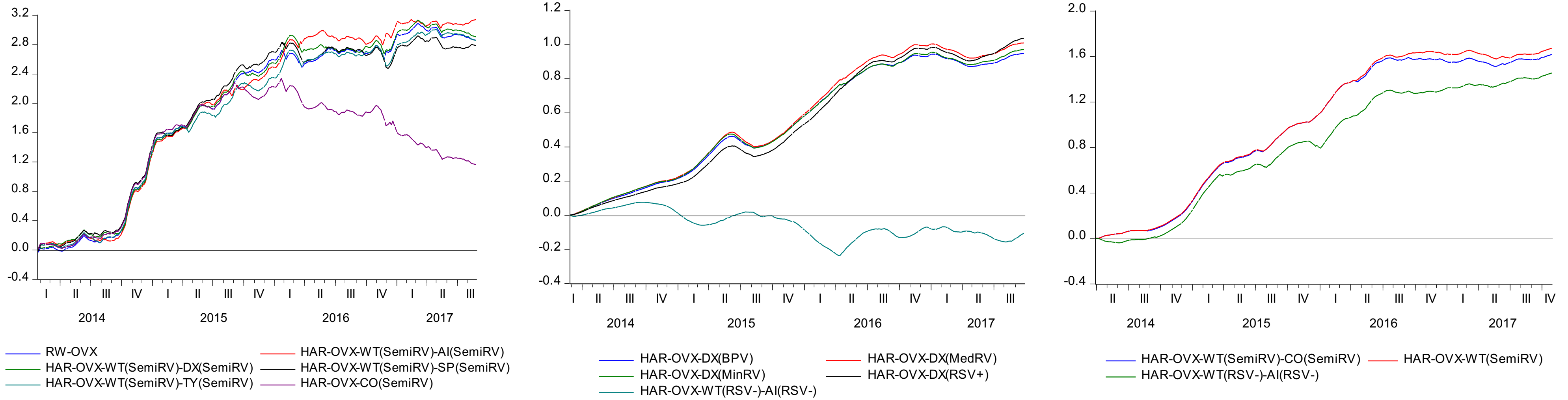
Maximum possible trading profits for USO trading (TS4)



Note: The distribution is based on 173 forecasting models for TS1, TS2 and TS4, whereas for TS3 the forecasting models are 89. The shaded area denotes the point of the distribution that the best forecasting models can be found based on Tables 2-5.

Figure 7: After-cost profits for the best performing models based on the objective-based evaluation criteria vs the statistical loss functions for 22-, 44- and 66-days ahead horizons.

22-days ahead for TS1 and TS2	44-days ahead for TS3	66-days ahead for TS4



Appendix A.1

Table A1: List of models included in our forecasting framework

	TS1		*TS2*		*TS3*	*TS4*				
	TR1a	TR1b	TR2a	TR2b	TR3	TR4a	TR4b	TR4c	TR4d	TR4e
Forecasting models for OVX										
RW-OVX	●				●	●				
AR-OVX	●				●	●				
HAR-OVX	●				●	●				
HAR-OVX-AI(BPV)	●				●	●				
HAR-OVX-AI(MedRV)	●				●	●				
HAR-OVX-AI(MinRV)	●				●	●				
HAR-OVX-AI(RSV-)	●				●	●				
HAR-OVX-AI(RSV+)	●				●	●				
HAR-OVX-AI(RV)	●				●	●				
HAR-OVX-AI(ScaledRV)	●				●	●				
HAR-OVX-CO(BPV)	●				●	●				
HAR-OVX-CO(MedRV)	●				●	●				
HAR-OVX-CO(MinRV)	●				●	●				
HAR-OVX-CO(RSV-)	●				●	●				
HAR-OVX-CO(RSV+)	●				●	●				
HAR-OVX-CO(RV)	●				●	●				
HAR-OVX-CO(ScaledRV)	●				●	●				
HAR-OVX-DX(BPV)	●				●	●				
HAR-OVX-DX(MedRV)	●				●	●				
HAR-OVX-DX(MinRV)	●				●	●				
HAR-OVX-DX(RSV-)	●				●	●				
HAR-OVX-DX(RSV+)	●				●	●				

HAR-OVX-DX(RV)	●	●	●
HAR-OVX-DX(ScaledRV)	●	●	●
HAR-OVX-SP(BPV)	●	●	●
HAR-OVX-SP(MedRV)	●	●	●
HAR-OVX-SP(MinRV)	●	●	●
HAR-OVX-SP(RSV-)	●	●	●
HAR-OVX-SP(RSV+)	●	●	●
HAR-OVX-SP(RV)	●	●	●
HAR-OVX-SP(ScaledRV)	●	●	●
HAR-OVX-TY(BPV)	●	●	●
HAR-OVX-TY(MedRV)	●	●	●
HAR-OVX-TY(MinRV)	●	●	●
HAR-OVX-TY(RSV-)	●	●	●
HAR-OVX-TY(RSV+)	●	●	●
HAR-OVX-TY(RV)	●	●	●
HAR-OVX-TY(ScaledRV)	●	●	●
HAR-OVX-WT(BPV)	●	●	●
HAR-OVX-WT(MedRV)	●	●	●
HAR-OVX-WT(MinRV)	●	●	●
HAR-OVX-WT(RSV-)	●	●	●
HAR-OVX-WT(RSV+)	●	●	●
HAR-OVX-WT(RV)	●	●	●
HAR-OVX-WT(ScaledRV)	●	●	●
HAR-OVX-WT(BPV)-AI(BPV)	●	●	●
HAR-OVX-WT(BPV)-CO(BPV)	●	●	●
HAR-OVX-WT(BPV)-DX(BPV)	●	●	●
HAR-OVX-WT(BPV)-SP(BPV)	●	●	●
HAR-OVX-WT(BPV)-TY(BPV)	●	●	●

HAR-OVX-WT(MedRV)-AI(MedRV)	●				●	●
HAR-OVX-WT(MedRV)-CO(MedRV)	●				●	●
HAR-OVX-WT(MedRV)-DX(MedRV)	●				●	●
HAR-OVX-WT(MedRV)-SP(MedRV)	●				●	●
HAR-OVX-WT(MedRV)-TY(MedRV)	●				●	●
HAR-OVX-WT(MinRV)-AI(MinRV)	●				●	●
HAR-OVX-WT(MinRV)-CO(MinRV)	●				●	●
HAR-OVX-WT(MinRV)-DX(MinRV)	●				●	●
HAR-OVX-WT(MinRV)-SP(MinRV)	●				●	●
HAR-OVX-WT(MinRV)-TY(MinRV)	●				●	●
HAR-OVX-WT(RSV-)-AI(RSV-)	●				●	●
HAR-OVX-WT(RSV-)-CO(RSV-)	●				●	●
HAR-OVX-WT(RSV-)-DX(RSV-)	●				●	●
HAR-OVX-WT(RSV-)-SP(RSV-)	●				●	●
HAR-OVX-WT(RSV-)-TY(RSV-)	●				●	●
HAR-OVX-WT(RSV+)-AI(RSV+)	●				●	●
HAR-OVX-WT(RSV+)-CO(RSV+)	●				●	●
HAR-OVX-WT(RSV+)-DX(RSV+)	●				●	●
HAR-OVX-WT(RSV+)-SP(RSV+)	●				●	●
HAR-OVX-WT(RSV+)-TY(RSV+)	●				●	●
HAR-OVX-WT(RV)-AI(RV)	●				●	●
HAR-OVX-WT(RV)-CO(RV)	●				●	●
HAR-OVX-WT(RV)-DX(RV)	●				●	●
HAR-OVX-WT(RV)-SP(RV)	●				●	●
HAR-OVX-WT(RV)-TY(RV)	●				●	●
HAR-OVX-WT(ScaledRV)-AI(ScaledRV)	●				●	●
HAR-OVX-WT(ScaledRV)-CO(ScaledRV)	●				●	●
HAR-OVX-WT(ScaledRV)-DX(ScaledRV)	●				●	●

HAR-OVX-WT(ScaledRV)-SP(ScaledRV)	●			●	●		
HAR-OVX-WT(ScaledRV)-TY(ScaledRV)	●			●	●		
HAR-OVX-AI(SemiRV)		●					●
HAR-OVX-CO(SemiRV)		●					●
HAR-OVX-DX(SemiRV)		●					●
HAR-OVX-SP(SemiRV)		●					●
HAR-OVX-TY(SemiRV)		●					●
HAR-OVX-WT(SemiRV)		●					●
HAR-OVX-WT(SemiRV)-AI(SemiRV)		●					●
HAR-OVX-WT(SemiRV)-CO(SemiRV)		●					●
HAR-OVX-WT(SemiRV)-DX(SemiRV)		●					●
HAR-OVX-WT(SemiRV)-SP(SemiRV)		●					●
HAR-OVX-WT(SemiRV)-TY(SemiRV)		●					●
Forecasting models for WT							
RW-WT(BPV)			●			●	
RW-WT(MedRV)			●			●	
RW-WT(MinRV)			●			●	
RW-WT(RSV-)			●			●	
RW-WT(RSV+)			●			●	
RW-WT(RV)			●			●	
RW-WT(ScaledRV)			●			●	
AR-WT(BPV)			●			●	
AR-WT(MedRV)			●			●	
AR-WT(MinRV)			●			●	
AR-WT(RSV-)			●			●	
AR-WT(RSV+)			●			●	
AR-WT(RV)			●			●	
AR-WT(ScaledRV)			●			●	

HAR-WT(BPV)	●	●
HAR-WT(MedRV)	●	●
HAR-WT(MinRV)	●	●
HAR-WT(RSV-)	●	●
HAR-WT(RSV+)	●	●
HAR-WT(RV)	●	●
HAR-WT(ScaledRV)	●	●
HAR-WT(BPV)-AI(BPV)	●	●
HAR-WT(BPV)-CO(BPV)	●	●
HAR-WT(BPV)-DX(BPV)	●	●
HAR-WT(BPV)-SP(BPV)	●	●
HAR-WT(BPV)-TY(BPV)	●	●
HAR-WT(MedRV)-AI(MedRV)	●	●
HAR-WT(MedRV)-CO(MedRV)	●	●
HAR-WT(MedRV)-DX(MedRV)	●	●
HAR-WT(MedRV)-SP(MedRV)	●	●
HAR-WT(MedRV)-TY(MedRV)	●	●
HAR-WT(MinRV)-AI(MinRV)	●	●
HAR-WT(MinRV)-CO(MinRV)	●	●
HAR-WT(MinRV)-DX(MinRV)	●	●
HAR-WT(MinRV)-SP(MinRV)	●	●
HAR-WT(MinRV)-TY(MinRV)	●	●
HAR-WT(RSV-)-AI(RSV-)	●	●
HAR-WT(RSV-)-CO(RSV-)	●	●
HAR-WT(RSV-)-DX(RSV-)	●	●
HAR-WT(RSV-)-SP(RSV-)	●	●
HAR-WT(RSV-)-TY(RSV-)	●	●
HAR-WT(RSV+)-AI(RSV+)	●	●

HAR-WT(RSV+)-CO(RSV+)	●		●		
HAR-WT(RSV+)-DX(RSV+)	●		●		
HAR-WT(RSV+)-SP(RSV+)	●		●		
HAR-WT(RSV+)-TY(RSV+)	●		●		
HAR-WT(RV)-AI(RV)	●		●		
HAR-WT(RV)-CO(RV)	●		●		
HAR-WT(RV)-DX(RV)	●		●		
HAR-WT(RV)-SP(RV)	●		●		
HAR-WT(RV)-TY(RV)	●		●		
HAR-WT(ScaledRV)-AI(ScaledRV)	●		●		
HAR-WT(ScaledRV)-CO(ScaledRV)	●		●		
HAR-WT(ScaledRV)-DX(ScaledRV)	●		●		
HAR-WT(ScaledRV)-SP(ScaledRV)	●		●		
HAR-WT(ScaledRV)-TY(ScaledRV)	●		●		
RW-WT(SemiRV)		●		●	
AR-WT(SemiRV)		●		●	
HAR-WT(SemiRV)		●		●	
HAR-WT(SemiRV)-AI(SemiRV)		●		●	
HAR-WT(SemiRV)-CO(SemiRV)		●		●	
HAR-WT(SemiRV)-DX(SemiRV)		●		●	
HAR-WT(SemiRV)-SP(SemiRV)		●		●	
HAR-WT(SemiRV)-TY(SemiRV)		●		●	
Naive strategies for USO					
Buy-and-hold					●
Sell-and-hold					●