Short-term evolution of forward curves and volatility in illiquid power markets

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February 2008

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First version: February 2008
This version: May 2008

We propose in this paper a model for the description of electricity spot prices, which we use to describe the dynamics of forward curves. The spot price model is based on a long-term/short-term decomposition, where the price is thought of as made up of two factors: A long-term equilibrium level and short-term movements around the equilibrium. We use a non-parametric approach to model the equilibrium level of power prices, and a mean-reverting process with GARCH volatility to describe the dynamics of the short-term component. Then, the model is used to derive the expression of the short-term dynamics of the forward curve implicit in spot prices. The rationale for the approach is that information concerning forward prices is not available in most of power markets, and the direct modeling of the forward curve is a difficult task. Moreover, power derivatives are typically written on forward contracts, and usually based on average prices of forward contracts. Then, it is difficult to obtain analytical expressions for the forward curves. The model of forward prices allows for the valuation of power derivatives, as well as the calculation of the volatilities and correlations required in risk management activities. Finally, the methodology is proven in the context of the Spanish wholesale market.

1. Introduction

One of the main consequences of the liberalization process that has taken place in the electricity industry during last decades, and probably one of its main motivations, is that market agents are responsible for the risks involved in their economic activity. In power systems under the cost-of-service regulation, electricity producers -vertically integrated utilities- were quite insensitive to the risk related to the operation of power plants, charging the consumers with any deviation from the expected result. The deregulation of the power industry has forced the suppliers to take into account the uncertainty related to the production of electricity, and consequently has promoted a considerable amount of trading activity. In this context, the description of the market behavior has attracted considerable efforts.

One can distinguish two main motivations for most of the models proposed in the literature to deal with power markets. Nowadays, it is widely agreed that most power markets cannot be adequately described through the perfect competition assumption. Therefore, the microeconomic analysis of the effects of horizontal concentration has become central in most investigations concerning power markets –see for instance Ventosa, Baíllo, Rivier and Ramos (2005) for a revision–. In general, these models focus on the analysis of the strategic interaction between market participants, and consider the uncertainty in a simplified manner. On the other hand, the need for pricing and hedging the particular risks of power markets has motivated an unprecedented amount of investigations concerned with the role of uncertainty in the production of electricity. However, they are based on the results provided by the financial theory, and consequently disregards the strategic interaction between power producers.

From this standpoint, the two streams of the literature analyzing power markets can be thought of as a decomposition of the problem. In the short-term, the most critical effects concern operational issues. The short-term behavior of the market will be highly influenced by the technical operation of power plants, as well as short-term uncertainty. However, the strategic interaction between market players may be described in a simplified way, assuming that it is close to constant in the short-term, and may be estimated through past market data. In longer terms, however, the role of strategic issuesbecomes central, because the behavior of market players may change, and consequently should be anticipated.

We are concerned, in this paper, with the uncertainty associated to the production of electricity. Therefore, an adequate modeling of the dynamics of the market prices is an important subject of our investigation. Most of the models proposed in the literature are based on the statistical description of spot power prices, describing the seasonality as a certain deterministic function, which is not formally defined –see for instance Deng (1999), Lucia and Schwartz (2002), Escribano, Peña and Villaplana (2002), Geman and Roncoroni (2002)–. However, we propose the use of a non-parametric approach, based on the model in Sánchez-Úbeda (1999), for the description of the seasonal component. Furthermore, as the seasonality is considered predictable, it is important to understand the dynamics that we are not considering. In this context, one can think of the logarithm of power prices as made up of two factors: A long-term component that describes some equilibrium level and a short-term component aimed to capture temporary deviations from the equilibrium price. The description of prices as a combination of a long-term price level and short-term stochastic movements around the long-term factor can be found, for instance, in Pilipovic (1997). Schwartz and Smith (2000) interpret these factors as the combination of an equilibrium price and a short-term factor. The role that the strategic behavior of market participants plays in the description of the long-term equilibrium price is investigated in Vázquez and Barquín (2007). From this viewpoint, the model proposed in this paper for the price evolution takes into account just short-term dynamics.

However, most power derivatives are written on forward or futures contracts. As in any other market, this kind of contract is usually more liquidly traded than the underlying asset. However, the non-storability of electricity makes forward trading especially adequate in power markets. Actually, since electricity cannot be stored, power delivered at any particular period, e. g. a day, represents a different commodity than the power delivered at any other period. By contrast, a forward contract expiring the next month is the same today and tomorrow, although its price changes, which makes more natural the study of the forward curve than the electricity itself. In addition, exercising a forward contract written on electricity does not imply the delivery of the underlying. On the other hand, the direct modeling of the forward curve dynamics –see for instance Eydeland and Wolyniec (2003) for a description of the methodology– is a difficult task in power markets. Forward trading is quite illiquid in most electricity markets, and consequently the market data required for the estimation of the model is not guaranteed.

In this paper, we propose an alternative to describe the short-term evolution of forward prices. We first propose a novel model for the description of electricity spot prices. In particular, we state a spot price model based on the long-term/short-term decomposition mentioned above. However, as we are concerned just with short-term uncertainty, we will consider the long-term component predictable from the initial date. Then, we derive the forward curve dynamics consistent with the risk-neutral version of the spot price model. The risk neutral transformation of the price process is based on the introduction of a price of risk. Ideally, in complete markets, the price of risk is the aggregation of the risk preferences of market agents at the equilibrium, and can be obtained through actual market data. However, the price of risk is difficult to obtain when markets are incomplete –it is clearly the case of power markets–. Actually, it is difficult to define how to obtain the risk preferences used to price contracts in markets that do not exist. In this sense, the risk-neutral price process can be interpreted as the valuation of the spot price of a single agent. In addition, most of methodologies proposed in the literature to describe the forward prices rely on the ability of market agents to replicate the payoff of a forward contract –see Clewlow and Strickland (1999) or Schwartz (1997)–. This is not always possible in power markets, and risk premia should be considered. By contrast, the methodology developed in this paper may be thought of as valuation of the forward price evolution that does not necessarily rely on replication arguments.

We begin in section 2 by stating the spot price model that we will use to characterize the dynamics of the forward curve. In section 3, we derive the forward curve dynamics implicit in the process of the shot-term component of spot prices. In section 4, we show how the methodology may be used for the calculation of volatilities and correlations, valuated at any future date, in the context of risk management. In section 5, we prove the methodology in the context of the Spanish wholesale market, and finally we conclude in section 6.

2. The model for the evolution of power prices

Let p_t be the spot price of the electricity at time t. We express the price by the following equation

(2.1) $$\log p_t = \pi_t + y_t$$

The long-term factor is given by π_t, whereas y_t represents short-term perturbations around the long-term component. This model is close to the general framework of Pilipovic (1997). In addition, we will describe the prices from a statistical point of view, so that no strategic behavior is further considered. In this context, the model should be able to capture the empirical characteristics of power prices. First, power prices are seasonal. From the standpoint of the short-term/long-term decomposition approach, seasonal movements are driven by seasonal properties of the equilibrium level in the market, motivated by the lack of storability of electricity. Typically, the power demanded in winter or summer is higher that the power demanded in autumn or spring. Then, electricity in summer (winter) will be more expensive, as it must be produced with the most expensive plants. The seasonality of the price depends strongly on the characteristics of the generation mix of the power system. For example, most of the production capacity of the Brazilian system is due to hydro plants, and consequently the level of the reservoirs and the probability of inflows determine the price of electricity. Therefore, the long-term component π_t should capture the seasonality of power prices.

The only dynamics considered in the model are related to short-term movements of prices. That is, we will not consider the dynamics of the seasonal component, in the sense that its estimation does not change from the initial date until the end of the horizon under study. Therefore, the dynamics of power prices will be represented in the proposed model by the short-term component y_t.

2.1. The long-term component

We will study the long-term evolution of power prices in the context of a regression problem, as it is appropriate for seasonal description purposes. The seasonal behavior of power prices may be identified with a yearly evolution. For example, the hydrothermal coordination of a generation portfolio is typically a yearly problem. Consequently, the problem is providing a function π_t that describes the level of electricity prices in a particular day of the year, (e. g. September 14). It is possible to consider a random variable D that takes values in the set of days in a year $\{d_1, d_2, ...\}$, where the elements d_i represent every day in a year (January 5, March 3...). Finally, one may identify the values of that the random variable D takes with the parameter t, and

calculate a function $f(D)$ minimizing the quadratic error $Err = (\pi_t - f(D))^2$. This is the general statement of a regression problem. The function minimizing the quadratic error is $E\left(\pi_t / D = t\right)$. Let us consider a sample made up of pairs of prices and days $\{\Pi_1, ..., \Pi_N, D_1, ..., D_N\}$. Then, the problem is obtaining the regression function

(2.2) $$f(t) = E\left(\Pi / D = t\right)$$

Finally, we identify the estimate and the long-term component $\pi_t = f(t)$.

There are two main approaches in the literature to tackle the above problem. On the one hand, the parametric regression is based on assuming a certain probability distribution of Π, e. g. a normal distribution. However, it is in general difficult to give an *a priori* assumption of the shape of power price distributions. In such a case, the nonparametric regression represents an interesting alternative, since it does not assume any distribution. A good revision of nonparametric methods can be found in Hastie, Tibshirani and Friedman (2001). The model for the long-term component π_t that we use in the proposed model belongs to the latter family.

In particular, the seasonal evolution of power prices will be given by the Linear Hinges Model, Sánchez-Úbeda (1999), Sánchez-Úbeda and Wehenkel (1998). The model is defined by a set of knots –hinges–, and a linear function interpolating the prices in between. That is, the model calculates a set of hinges given by $\left\{\hat{x}_i, \hat{y}_i\right\}$, $i = 1,...,K$, where K is the number of hinges in the model. The estimation of π_t consist of linear functions between each pair of hinges. This model is particularly interesting for power studies because it combines the qualities of local estimators –as kernel estimators– with a good performance when dealing with very volatile data –see Sánchez-Úbeda (1999) for a detailed investigation–.

2.2. The short-term component

The short-term component y_t accounts for the dynamics of the power price. We will use for this purpose a discrete time model. Most of the theoretical results in the financial literature may be obtained both in a discrete or continuous time setting –see for instance Garcia, Ghysels and Renault (2004)–, and we think that the discrete time setting is more convenient to describe power markets, where the trading activity is not usually performed in continuous time.

First, it is widely agreed that mean reversion is a major characteristic of power prices: After an upward deviation from some long-run mean, it is more probable that there is a downward movement. In the framework stated in this paper, this long-term mean is the seasonal component proposed in the above section. In other words, after a deviation from the equilibrium price, the system will react and the price will tend to normal levels. Therefore, the model for y_t should capture the mean-reversion property. Let us consider a certain delivery date, represented by T . We will state the following model:

(2.3) $$y_T = \sum_{t=1}^{T} h_{T-t+1} u_t$$

where u_t are random shocks, and h_{T-t+1} are the series of parameters. This quite general model can represent many of the models proposed to describe energy prices. In particular, we propose to fit an autoregressive model of the form

(2.4) $$H\left(q^{-1}\right) y_t = u_t$$

where q^{-1} is the lag operator and $H\left(q^{-1}\right)$ is an autoregressive polynomial. This model is fit using the Akaike criterion –see for instance Lutkepohl (1993)–. Then, we obtain the truncated polynomial $H^{-1}\left(q^{-1}\right)$ to get the coefficients $\left\{h_{T-t+1}\right\}$.

Turning to de model for the random shocks u_t , it should be taken into account that power prices usually are characterized by stochastic volatility. That is, the volatility of the price is higher during some periods, but then they revert to lower volatility levels –volatility clustering–. We will model the innovations u_t to have GARCH dynamics (Bollerslev (1986)), in order to take into account the stochastic volatility:

(2.5) $$u_t = k_u + \eta_t$$

(2.6) $$\sigma_t^2 = k_\sigma + \sum_{i=1}^{P} \alpha_i \sigma_{t-i}^2 + \sum_{j=1}^{Q} \beta_j \varepsilon_{t-j}^2$$

(2.7) $$\eta_t : N(0,\sigma_t), \quad \varepsilon_t : N(0,1)$$

where k_u is the mean of each u_t and k_σ is the constant of the GARCH process.

Another feature of some power markets is the spiky behavior of prices, which is in turn absent in other power markets. The rationale behind the price spikes is a period of scarcity in the system capacity, because of temporary outages, network failures... that creates an opportunity cost higher than the marginal power plant's. However, this behavior depends on the particular system. For example, the existence of price caps makes the price not to reflect the opportunity cost, or a high amount of hydro power prevents the scarcity periods of the system. We will not include spikes in the description of the spot prices –see for instance Deng (1999) or Geman and Roncoroni (2002) for models of electricity prices describing spiky behavior–.

2.3. The risk-neutral process

The previous model accounts for the dynamics of the spot price with respect to the real probability measure. However, we will need the risk-neutral process, because we will use the model to price forward contracts. With this aim, we will make use of a price of risk. The price of risk λ_t is in a loose sense an aggregate measure of the risk preferences of market players. When markets are complete, they are uniquely defined by equilibrium arguments by considering that the prices of the moving-average components of y_t are determined by the equilibrium in the financial market –see Duffie (2001) for a rigorous treatment of the problem–. However, power markets are not usually complete. In the incomplete setting, the price of risk is not unique. In such a case, the price of risk λ_t should be interpreted as the valuation of risk for, in general, every single agent. In this context, the risk-neutral valuation is the representation of the price that maximizes the utility of a certain participant, including her risk preferences. Although this representation accounts for the case where a unique price of risk that can be obtained from market data exists, it also describes the risk premium of single agents. In this sense, it would be a reflection of the market participants' policies. Either in the case of a market price of risk or in the case of single preferences, the risk-neutral version of the price process is given by the following expression:

(2.8) $$y_T^N = \sum_{t=1}^{T} h_{T-t+1}\left(u_t - \lambda_t\right)$$

The next section develops a methodology to describe the evolution of the forward prices. The description is based on the previous model, so the forward prices obtained therein must be interpreted consistently with the definition of the price of risk. That is, if the price of risk aims to describe the market equilibrium, forward prices should be understood as equilibrium prices. On the other hand, if the price of risk is the corresponding to a single agent, the forward prices are personal valuations of the contracts, in general different from the rest of valuations in the market.

Therefore, the model for spot prices is built in three steps:

- Estimation of the equilibrium component π_t from market data: Given the historical data of power spot prices, we use the model in 2.1 to estimate π_t.
- Estimation of the short-term component from the series $\log p_t - \pi_t$ by means of the model described in 2.2.
- Final transformation of the model in the risk-neutral process by considering the price of risk λ_t.

3. The short-term dynamics of forward curves

One of the results of this paper is the description of the forward curve process from the definition of the spot price evolution. Although there are several arguments in favor of forward curve models, they have no direct application, in general, when dealing with power markets. One of the main advantages of forward curve models cited in the literature –see for instance Clewlow and Strickland (1999)– is that these models allow for avoiding the estimation of the convenience yield, through extracting this information from actual market data. However, most electricity markets are quite immature at present, and the forward trading is still illiquid. Furthermore, forward curve models are technically more complex than spot price models. In fact, most of the models proposed in the literature –e. g. Schwartz (1997), Clewlow and Strickland (1999), Borovkova (2006)– share the assumption of log-normal distributions for forward prices. However, log-normality is not the most standard feature of power prices, and thus disregarding higher order effects is not always suitable.

However, as the underlying of most power derivatives is a forward contract, it is interesting to describe the forward curve evolution. Therefore, the procedure in this section is based on obtaining the forward curve dynamics embedded in the spot price model.

3.1. The forward curve evolution

The risk-neutral valuation states that the price of a forward contract is the risk-neutral expectation of the price process. Therefore, according to equations equations (2.8) and (2.5) to (2.7) , the risk-neutral price process is

(3.1) $$\log p_T^N = \pi_T + \sum_{t=1}^{T} h_{T-t+1}\left(u_t - \lambda_t\right)$$

We additionally assume that interest rates are deterministic. In such a case –in general, independent of spot prices, see Duffie (2001)– the price of a forward contract is

(3.2) $$F_{t,T} = E_t\left[p_T^N \right]$$

$F_{t,T}$ is the price of a forward contract, valuated at date t, which will be delivered at date T. The curve defined by the set of prices for forward contracts valuated at the same date t, with different delivery dates, is the forward curve. Therefore,

(3.3) $$F_{0,T} = E\left[\exp\left(\pi_T + \sum_{t=1}^{T} h_{T-t+1}\left(u_t - \lambda_t\right) \right) \right]$$

In this section, we additionally assume that the seasonal component is a deterministic variable. This assumption may be generalized, without major changes in the methodology, to consider the volatility of the long-term factor. To do so, we should still consider that the expectation of this function does not depend on time, or in other words, the conditional expectation at the initial date is the same as in any other date in the future. Furthermore, the long term component must be independent of the short-term innovations. The relaxation of these two conditions complicates the methodology developed below.

However, as we are investigating short-term issues, it is convenient to consider π_T as a deterministic variable, in the sense that it is predictable at the initial date. In addition, since the innovations u_t are independent, we can re-write the above expression as

(3.4) $$F_{0,T} = \exp\left(\pi_T\right) \prod_{t=1}^{T} E\left[\exp\left(h_{T-t+1}\left(u_t - \lambda_t\right) \right) \right]$$

Now, consider the value of the forward contract after one day. As the short-term process is observable, the innovation u_1 is no longer stochastic. Therefore, (3.5) $$F_{1,T} = \exp\left(\pi_T + h_T u_1\right) \prod_{t=2}^{T} E\left[\exp\left(h_{T-t+1}\left(u_t - \lambda_t\right)\right)\right]$$

Note that the relationship between the two values for the forward contract is

(3.6) $$\frac{F_{1,T}}{F_{0,T}} = \frac{\exp\left(\pi_T + h_T u_1\right)}{\exp\left(\pi_T\right)} \frac{\prod_{t=2}^{T} E\left[\exp\left(h_{T-t+1}\left(u_t - \lambda_t\right)\right)\right]}{\prod_{t=1}^{T} E\left[\exp\left(h_{T-t+1}\left(u_t - \lambda_t\right)\right)\right]}$$

being possible to recast the equation as

(3.7) $$\frac{F_{1,T}}{F_{0,T}} = \frac{\exp\left(\pi_T\right) \prod_{t=2}^{T} E\left[\exp\left(h_{T-t+1}\left(u_t - \lambda_t\right)\right)\right]}{\exp\left(\pi_T\right) \prod_{t=2}^{T} E\left[\exp\left(h_{T-t+1}\left(u_t - \lambda_t\right)\right)\right]} \frac{\exp\left(h_T u_1\right)}{E\left[\exp\left(h_T\left(u_1 - \lambda_1\right)\right)\right]}$$

and thus

(3.8) $$\frac{F_{1,T}}{F_{0,T}} = \exp\left(h_T u_1\right) \left\{E\left[\exp\left(h_T\left(u_1 - \lambda_1\right)\right)\right]\right\}^{-1}$$

It is interesting to highlight that the long-term component π_T does not appear in the equation. This is because, after the initial date, the seasonal component is known and the evolution of the forward follows from the short-term dynamics of the price. From this point of view, the equation above describes the evolution of the initial forward curve, determined by both the seasonal and the short-term component. Once the initial forward curve is given, the future evolution is driven by the evolution of the short-term component alone. Therefore, the methodology proposed in this paper does not account for long-term dynamics, and consequently for long-term risks.

Finally, let us transform the innovation u_1 to eliminate its mean, so that $\hat{u}_1 = u_1 - k_u$ and $\hat{u}_1 \sim N\left(0, \sigma_1\right)$. Therefore,

$$\left\{ E\left[\exp\left(h_T \left(u_1 - \lambda_1 \right) \right) \right] \right\}^{-1} = \left\{ E\left[\exp\left(h_T \hat{u}_1 + h_T \mu_1 - h_T \lambda_1 \right) \right] \right\}^{-1} \tag{3.9}$$

and

$$\left\{ E\left[\exp\left(h_T \left(u_1 - \lambda_1 \right) \right) \right] \right\}^{-1} = \exp\left(h_T \lambda_1 - h_T \mu_1 \right) \left\{ E\left[\exp\left(h_T \hat{u}_1 \right) \right] \right\}^{-1} \tag{3.10}$$

Since $\hat{u}_1$ is normally distributed,

$$\left\{ E\left[\exp\left(h_T \left(u_1 - \lambda_1 \right) \right) \right] \right\}^{-1} = \exp\left(h_T \lambda_1 - h_T \mu_1 \right) \left\{ \exp\left(\frac{1}{2} h_T^2 \sigma_1^2 \right) \right\}^{-1} \tag{3.11}$$

And finally, putting all the results together

$$\exp\left(h_T u_1 \right) \left\{ E\left[\exp\left(h_T \left(u_1 - \lambda_1 \right) \right) \right] \right\}^{-1} = \exp\left(h_T u_1 \right) \exp\left(h_T \lambda_1 - h_T \mu_1 \right) \left\{ \exp\left(\frac{1}{2} h_T^2 \sigma_1^2 \right) \right\}^{-1} \tag{3.12}$$

Then,

$$\frac{F_{1,T}}{F_{0,T}} = \exp\left(h_T \lambda_1 - \frac{1}{2} h_T^2 \sigma_1^2 \right) \exp\left(h_T \left(u_1 - \mu_1 \right) \right) = \exp\left(h_T \lambda_1 - \frac{1}{2} h_T^2 \sigma_1^2 \right) \exp\left(h_T \hat{u}_1 \right) \tag{3.13}$$

Thus, the equation for the evolution of the forward curve is given by

$$F_{1,T} = F_{0,T} \exp\left(h_T \lambda_1 - \frac{1}{2} h_T^2 \sigma_1^2 \right) \exp\left(h_T \hat{u}_1 \right) \tag{3.14}$$

The next section will study equation (3.14) in further detail.

3.2. Related models for the forward curve evolution

It is useful, to compare the above model to alternative proposals to state its continuous limit:

(3.15) $$F_{t,T} = F_{0,T} \exp\left(\int_0^t \left(h_{T-s}\lambda_s - \frac{1}{2} h_{T-s}^2 \sigma_s^2 \right) ds + \int_0^t h_{T-s}\sigma_s dW_s \right)$$

where dW_t represents the increments of a Brownian motion. This equation implies that the changes in the forward price are driven by

(3.16) $$d \ln F_{t,T} = \left(h_{T-t}\lambda_t - \frac{1}{2} h_{T-t}^2 \sigma_t^2 \right) dt + h_{T-t}\sigma_t dW_t$$

Then, applying the Ito's lemma, we have

(3.17) $$\frac{dF_{t,T}}{F_{t,T}} = h_{T-t}\lambda_t dt + h_{T-t}\sigma_t dW_t$$

where σ_t follows a GARCH process. First, it is important to note that the forward price evolution has a drift different to zero. However, the zero-drift condition follows ultimately from the convergence of forward price to the spot price at expiration, which is based on the ability of market participants to replicate the payoff of a forward contract by trading in the underlying. It is not usual that this assumption holds in power markets, so risk premia should explicitly be incorporated in the forward curve modeling. However, our description of the forward price process can describe the risk preferences of market agents.

By contrast, most of forward curve models proposed in the literature –see for instance Eydeland and Wolyniec (2003)– assume the zero-drift condition. For the sake of comparison, let us rewrite the process in (3.17), but without considering the risk aversion:

(3.18) $$\frac{dF_{t,T}}{F_{t,T}} = h_{T-t}\sigma_t dW_t$$

Therefore, since σ_t is given by a GARCH process, the forward price is not log-normally distributed. Most of models proposed to describe the movements of forward prices assume a log-normal distribution for the process. For example, Black (1976) consider the model:

(3.19) $$\frac{dF_{t,T}}{F_{t,T}} = \sigma dW_t$$

More recently, some authors –see for instance Clewlow and Strickland (1999)– propose a methodology based on the following model:

(3.20) $$\frac{dF_{t,T}}{F_{t,T}} = \sum_{i=1}^{N} \phi_{t,T}^{i} dW_t^i$$

where the changes in the forward curve are explained by means of N random shocks, usually determined through a principal components analysis. However, this model requires that forward prices are jointly log-normal, and this assumption is not usually satisfied in power markets. Our model may be interpreted as a one-factor version of the previous model, but considering the fat tails of the forward price distributions. The factor in the process proposed in this paper represents a decrease in the influence of shocks with the maturity time. This is natural taking into account that the spot price model just describes the short-term uncertainty. The study of a long-term factor, among others, would provide long-term movements in the forward curve, along the lines of the model in Clewlow and Strickland (1999). It is important to note that the extension of the methodology developed in this section can be easily generalized to the case where the spot price is described by several factors.

4. An application to risk management

The model above allows for the pricing of power derivatives that are written on forward contracts. For instance, Heston and Nandi (1997) proposes a closed-form solution to the valuation of European options accounting for GARCH effects. However, we are interested in this section in the calculation of the volatilities of different forward contracts and the correlation structure of the forward curve. This can be thought of within the framework of calculating the risk exposure of a certain portfolio.

Many of the products traded in power markets are based on some average of the underlying price. For instance, there are products based on the average price during a fixed period $[T_1, T_2]$. Consider a contract to sell electricity, with a fixed price, in every hour of the period $[T_1, T_2]$ –a base-load contract–. Therefore, the previous base-load contract is equivalent to a set of forwards with maturities at $\{T_1, ..., T_i, ..., T_2\}$, and its volatility will be given by the volatility of the set of forwards. However, even if we disregard GARCH effects, so that the distribution of each forward is log-normal, the contract for $[T_1, T_2]$ is not log-normally distributed, because it is the mean of log-normal distributions. Therefore, even without considering GARCH effects, finding analytical expressions for the volatilities in $[T_1, T_2]$ is a difficult task.

The purpose of this section is to show how the volatilities and correlations can be obtained.

Step 1. The initial forward curve

First, it is necessary to calculate the forward curve at the initial date $F_{0,T}$. Ideally, this should be done simply by matching the actual forward curve of the market. Unfortunately, such a forward curve is not always available. Hence, the procedure proposed in this section consist in finding the curve as the sample mean of a set of spot price scenarios –in the risk-neutral sense, i. e. discounting the risk aversion–. Then, from the spot price model given by equations (2.8) and (2.5) to (2.7) we obtain a set of scenarios.

- First, we generate a set of scenarios $\{\varepsilon_1, ..., \varepsilon_t, ..., \varepsilon_T\}$.
- Then the variance scenarios $\{\sigma_1^2, ..., \sigma_t^2, ..., \sigma_T^2\}$ are obtained, using (2.6).
- With the variance scenarios we generate next the sample $\{\eta_1, ..., \eta_t, ..., \eta_T\}$ and $\{u_1, ..., u_t, ..., u_T\}$.

- Finally, from the previous sample of innovations, we find the scenarios for the short-term component $\{y_1, ..., y_t, ..., y_T\}$. Adding the long-term component we obtain the sample of spot prices for the horizon $[1, T]$.

Let us denote the sample of initial forward curves as $\{\{F_{0,1}^1, ..., F_{0,T}^1\}, ..., \{F_{0,T}^n, ... F_{0,T}^n\}\}$, where n is the number of scenarios of the sample. The sample mean is the initial forward curve. The standard deviation of the sample is the volatility of the contracts, valuated at the initial date. Analogously, for average products, we obtain a sample $\{F_{0,T_1 \to T_2}^1, ..., F_{0,T_1 \to T_2}^n\}$, where $F_{0,T_1 \to T_2}^1$ is the first scenario of the product with maturity at $[T_1, T_2]$ calculated as the mean of the forwards $\{F_{0,T_1}^1, ..., F_{0,T_2}^1\}$.

Step 2. Volatilities and correlation

However, our main concern is related to the volatility of the contract valuated in some future date. Consider that we are interested in the volatility of a forward contract, with delivery at T, valuated at $T_0 < T$. This is a common situation where T_0 is a future date when the risk exposure of the portfolio must be evaluated. Therefore, there is a need for the calculation of the volatility of $F_{T_0,T}$. To do so, we use the forward curve evolution

(4.1) $$\ln F_{t+1,T} - \ln F_{t,T} = h_{T-t+1}\lambda_t - \frac{1}{2}h_{T-t+1}^2\sigma_t^2 + h_{T-t+1}\hat{u}_t$$

to obtain the corresponding set of scenarios. The volatility of the contract valuated at T_0 is the standard deviation of the previous sample mean. Analogously, the correlation between forward prices with different maturities are calculated as the sample correlation.

5. Case study: The Spanish wholesale market

One of the most important features of the Spanish wholesale market is that, at present, it is quite illiquid. Since July 2007, there is an organized exchange (OMIP[1]), although there is still little trading done in the exchange. Most of the trading activity is done over-the-counter. Additionally, the Regulator has imposed the obligation, since 2007, for the two major power producers of the market –Endesa and Iberdrola– to negotiate a percentage of their production capacity (Virtual Power Plants). They have the obligation to sell European call options, which give the right to the owner of buying a certain amount of electricity for each hour of a period of three months. The regulator fixes the strike price, and the price is settled in a public auction. In addition, a small amount of the demand (10% approximately) is allocated through quarterly products: Approximately ten days before the beginning of each quarter –e. g. the first quarter, from January to March– there is a public auction (CESUR auction[2]) where the demand is sold through quarterly products with delivery each hour of the quarter.

In this context, the methodology proposed in this paper is especially useful, since power producers must price different contracts and the availability of market is limited. Actually, the fundamental product traded in the Spanish market is the forward contract, but as the OTC is quite illiquid, the ability of market participants to use the information contained in the OTC prices is limited. In this context, we study the year 2008. The input data of the model proposed in this paper is the spot price data from 2004 to 2007. Previous years are not considered, because the regulation of the market has changed markedly, and consequently the prices are not comparable.

[1] http://www.omip.pt/.
[2] http://www.subasta-cesur.eu.

Step 1. The initial forward curve

The price data set allows us for fitting the spot price model. Then, we follow the procedure described in section 4, so that, from the spot price model, we obtain a sample of price scenarios and then the forward curve as the sample mean.

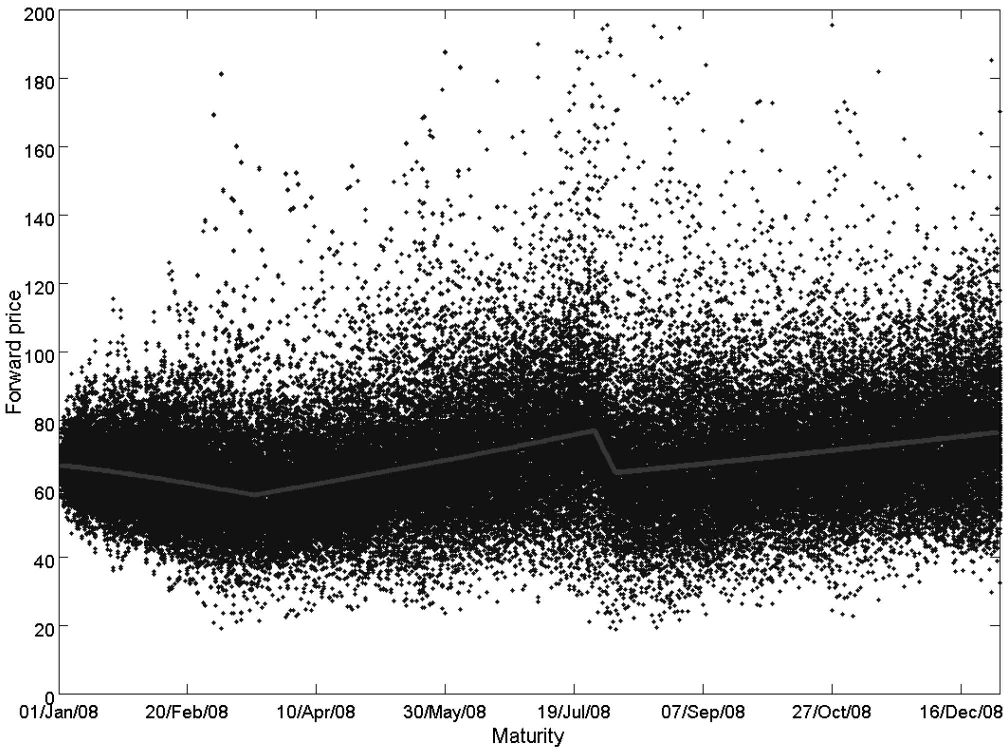

Figure 1.- The initial forward curve obtained from a set of spot price scenarios.

Figure 1 shows the initial forward curve for the daily product, whereas Figure 2 compares the curve for the daily and quarterly products.

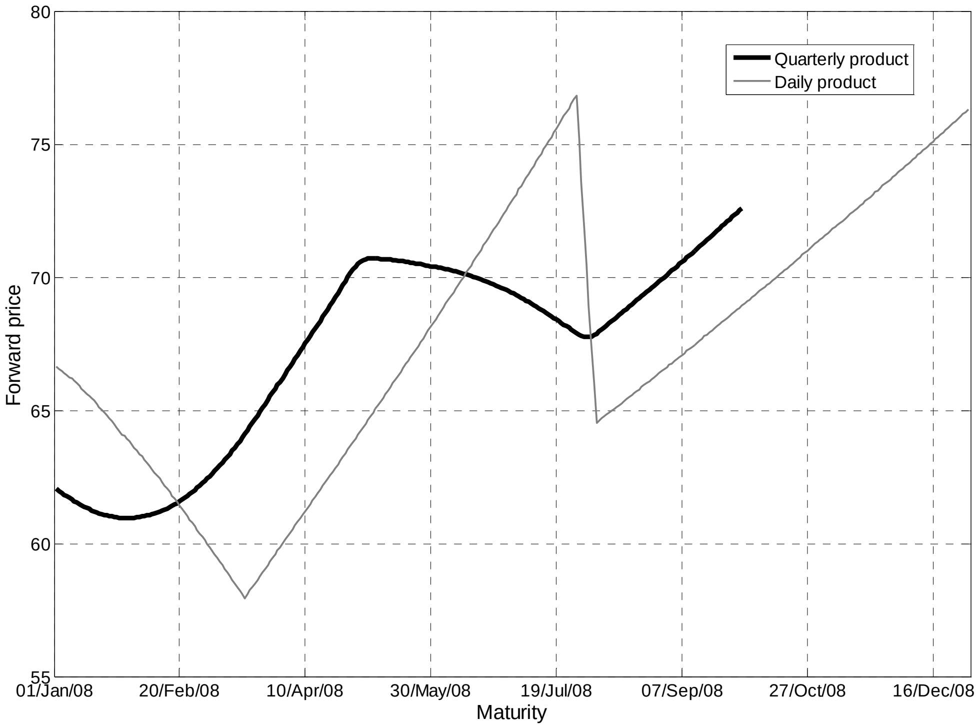

Figure 2.- Initial forward curves for different products. The dashed line is the forward curve of the daily product, and the black one represents the forward curve of the quarterly product.

The first step is needed both for derivative valuation and for risk position calculations, because the initial forward curve is a boundary condition of the evolution of the forward prices. However, this step is not always necessary. If the initial forward curve is available in the market, then it may be use to calibrate the results of the spot price model simulations. Unfortunately, it is not the case of the Spanish market. In addition, the seasonality of the quarterly product is not only smoother than the daily product seasonality, but also it is shifted closer to the beginning of the year: As the quarterly forward expiring in March depends on the power price in April, May and June.

Step 2. Volatilities and correlations

In this step, we obtain the sample of forward curves valuated at different dates, as described in section 4: From the process of the evolution of forward prices, we obtain a sample of scenarios of different evolutions of the forward curve.

We show in Figure 3 a scenario of the evolution of the initial forward curve, where the mean-reverting property of the process can be noted. After few days of evolution, the prices of forward contracts close to expiration have changed, but the contracts with maturity at the final of the year will not be affected. This is a consequence of our short-term setting, in the sense that we are not considering in the model the long-term dynamics of power prices, and consequently of forward prices. This set of scenarios allows for the valuation of any European derivative, even if there is no closed-form expression for its value.

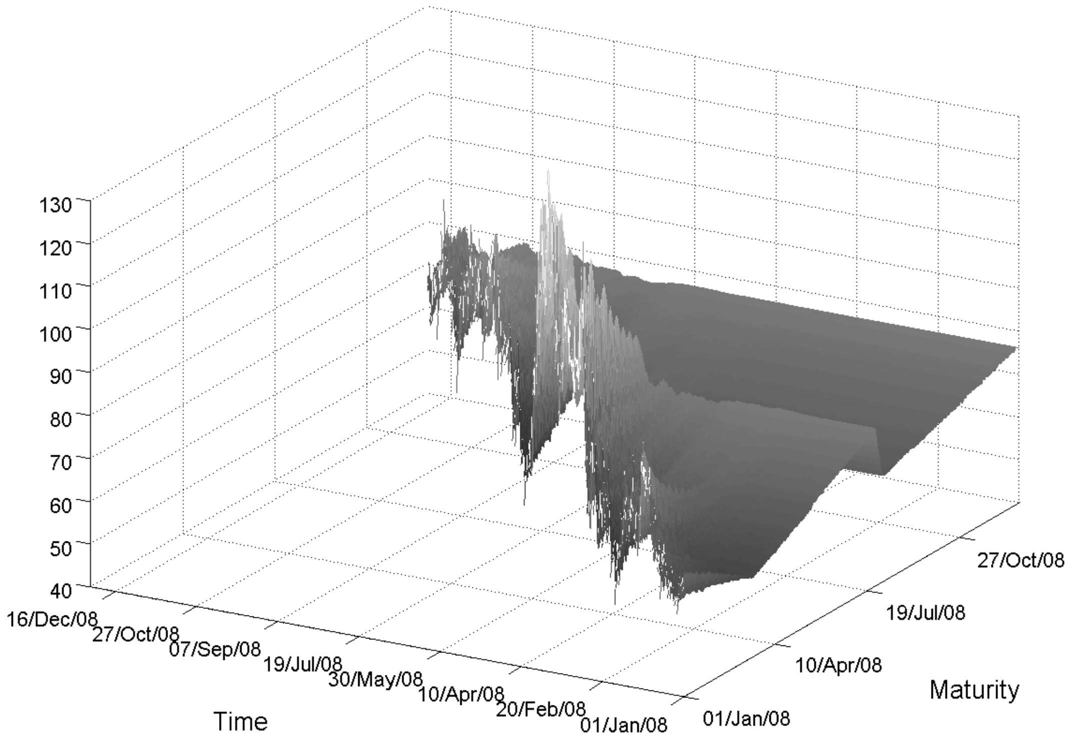

Figure 3.- A scenario of the evolution of forward prices.

The next two figures shows several curves of the surface in Figure 3. Thus, Figure 4 shows some instances of one scenario for the forward curve evolution. The first figure represents the forward curve ten days after the beginning of the simulation. It can be observed that most of the forward curve has not changed, because we are representing just the short-term dynamics. The last figure shows the forward curve considering the changes from the beginning of the simulation to ten months, and consequently it may be quite different of the initial curve.

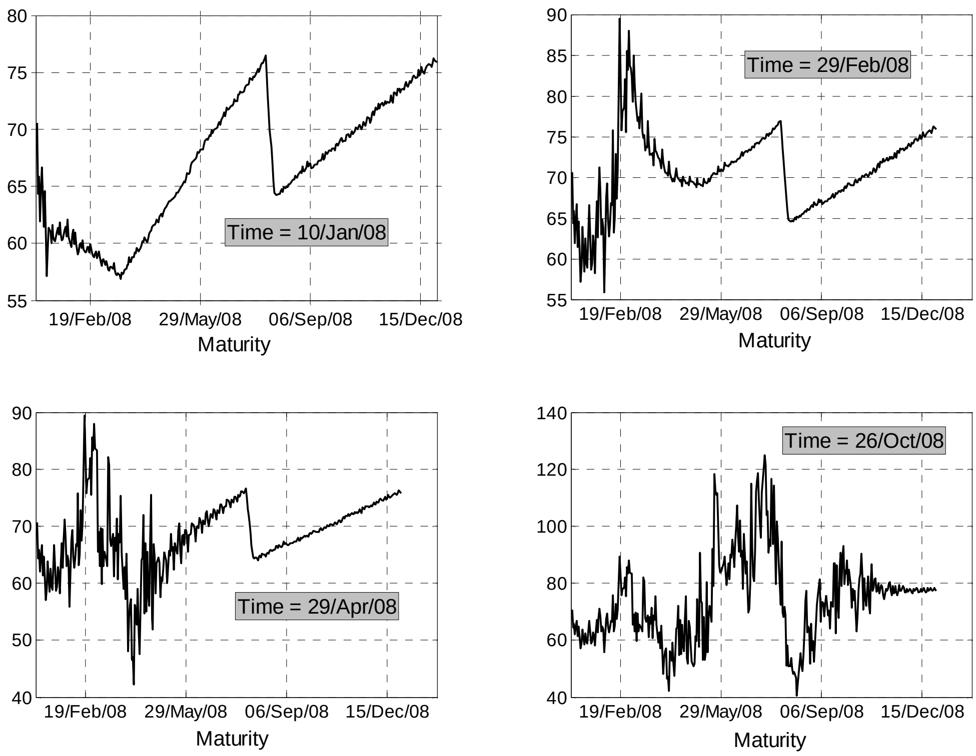

Figure 4.- An example of the evolution of the forward curve across time.

Figure 5 shows the evolution of the price of some forward contracts. The top left panel is the price of the contract expiring in February valuated at different dates. The panel represents with a dashed line that the price does not change after the contract expires. It is interesting to note that the forward contract expiring at the end of December, represented in the right bottom panel, does not change its initial value until the end of May.

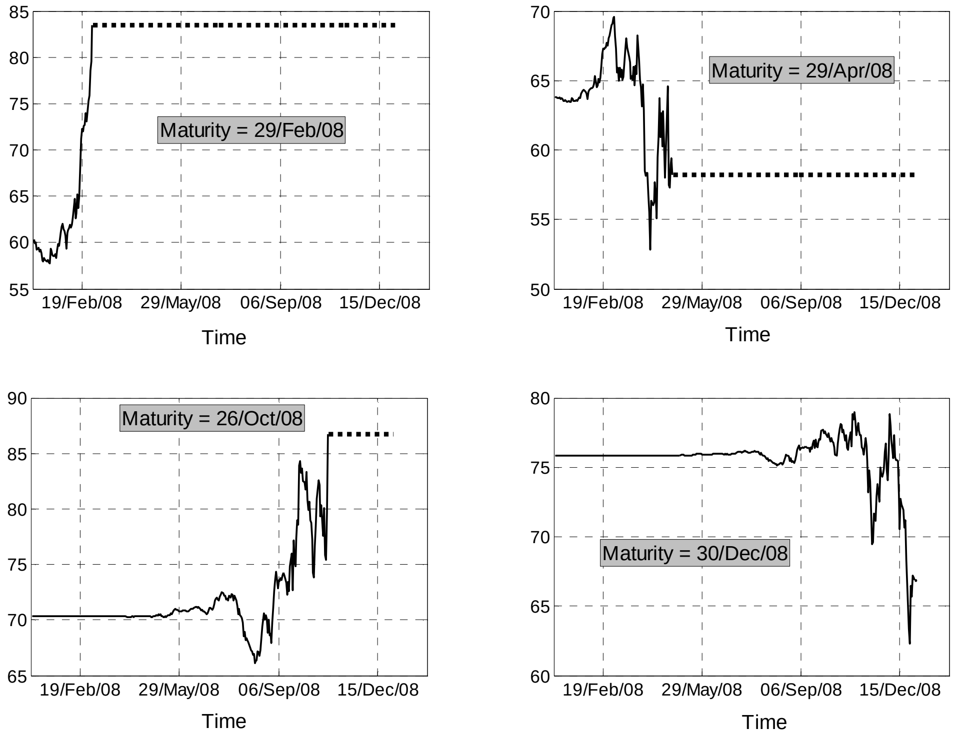

Figure 5.- The evolution of different forward prices with respect to the delivery date.

The last step concerns the risk management strategy of a power market player. The volatility of a certain portfolio should be evaluated in order to evaluate its risk exposure. In other words, the selection of the portfolio is highly affected by its risk properties, and therefore the volatility must be calculated –for instance, with the aim of calculating the VaR–. The previous step provides a sample of forward prices valuated at different dates. According to section 4, the standard deviation of the sample may be interpreted as the volatility.

Figure 6 shows the annualized volatility curves ($\overset{\circ}{\sigma} = 100\sigma_t\sqrt{365/T}$) of the monthly and quarterly products, valuated at the end of January. This may be interesting from the point of view of a power producer who must evaluate the risk exposure of their portfolio at the end of January. In addition, when deciding the value of a contract with maturity at the end of January, the producer is concerned with the risk associated to sell the contract. In other words, the contract faces higher risk if its price is very variable from the moment of the contract to the maturity date, than if the price is close to constant. Figure 6 shows that the volatility of the contracts is not persistent, as it reverts to zero according to the short-term model assumed.

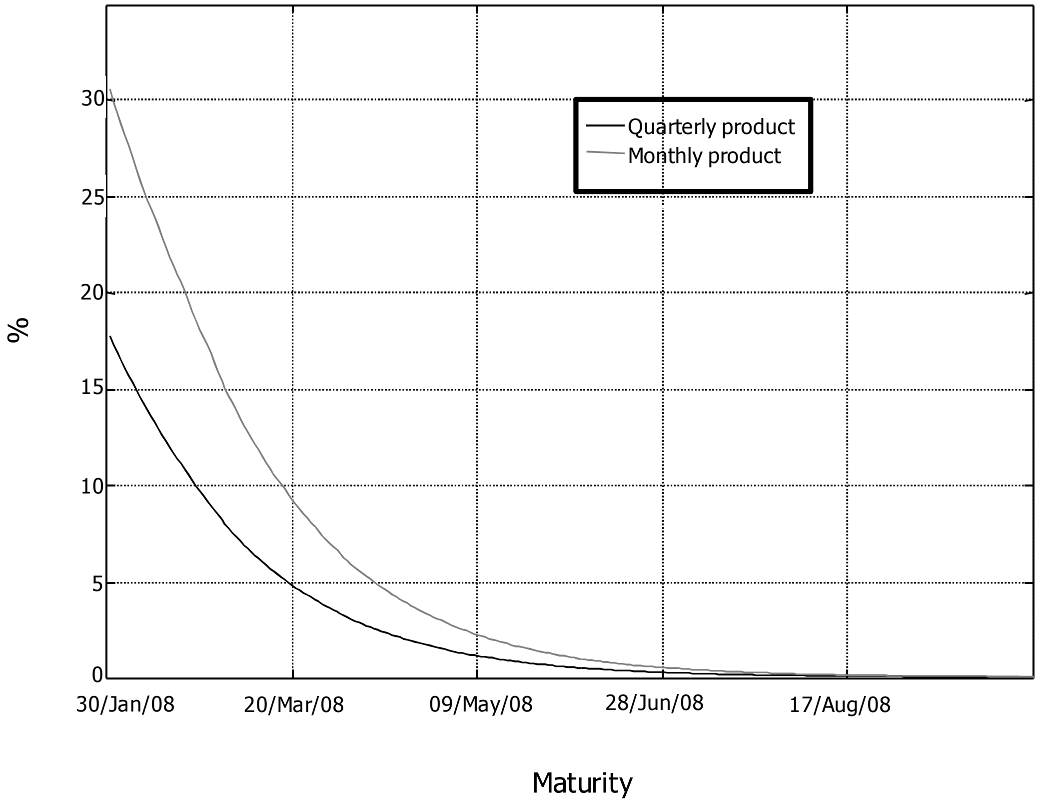

Figure 6.- Annualized volatility curves valuated at the end of January, for the monthly and the quarterly products.

In addition, the correlation between different products is important from the point of view of risk management. Figure 7 shows the correlations between daily and quarterly forwards with different maturities.

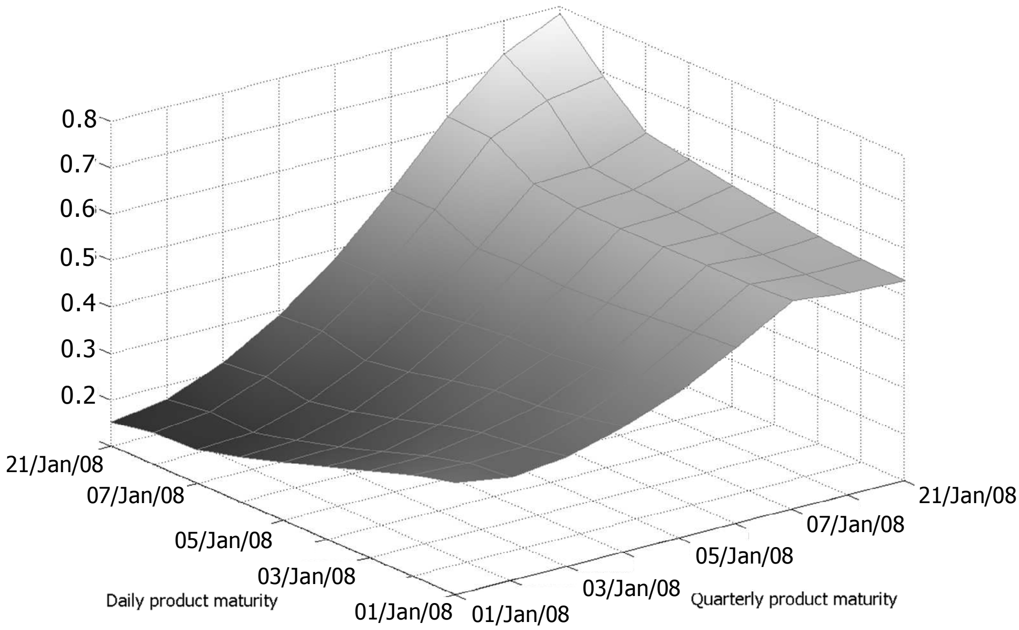

Figure 7.- Correlations between the daily and quarterly products.

6. Conclusion

The paper develops a new methodology to study the short-term dynamics of forward prices in electricity markets. First, we propose a novel model to describe the spot power prices. It is based on the decomposition of the price in two factors. The first component aims to capture the equilibrium level of the spot price, and hence it models the seasonality of power prices. However, the dynamics of this component is not considered in this paper, although we do think that a suitable representation of long-term issues affects the results of this paper, and therefore points out the need for further research on the topic.

The short-term component describes stochastic perturbations around the equilibrium level. This decomposition, found also in Schwartz and Smith (2000), allows for the separation of short- and long-term dynamics of spot prices. One important advantage of this approach is that the oligopolistic behavior of market players can be simplified in the short term.

The spot price model allows for the study of the forward prices evolution. The need for the forward curve evolution comes from, mainly, the fact that most of power derivatives are written on forward contracts, and, from the trading point of view, they represent the true underlying. This indirect modeling –instead of the description of the forward curve movements– is usually necessary in power markets, because forward trading is not liquid enough to provide reliable market data. In addition, the approach can represent risk premia in the forward price process. The zero-drift condition relies on the assumption that it is possible to replicate the payoff of a forward by trading in the underlying. However, this assumption is not satisfied in most power markets, so the risk premium should be explicitly considered. Finally, the approach is proven to be useful not only for power derivative pricing, but in addition to calculate the risk exposure of a portfolio. The evaluation of the prices, volatilities and correlations of average products –such as a monthly forward–, valuated at different dates, is not an easy task, since they are not log-normally distributed. The framework developed in this paper allows for the calculation of these parameters through the forward curve process. The model has been proved in the context of the Spanish wholesale market. The case study shows how the model allows power producers to value changes in forward prices and the risk associated with the contracts.

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