Abuse of Dominance and Licensing of Intellectual Property

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Abstract

This paper examines the impact of the licensing policies of one or more upstream owners of *essential* intellectual property (*IP* hereafter) on the downstream firms that require access to that IP, as well as on consumers and social welfare. The paper considers a model in which there is product differentiation downstream. License fees and fixed entry costs determine the number of downstream competitors and thus variety.

We first consider the case where there is a single upstream owner of essential IP. Increasing the number of licenses enhances product variety, which creates added value, but it also intensifies downstream competition, which dissipates profits. We derive conditions under which the upstream IP monopoly will then want to provide an excessive or insufficient number of licenses, relative to the number that maximizes consumer surplus or social welfare.

When there are multiple owners of essential IP, royalty stacking can reduce the number of the downstream licensees, but also the downstream equilibrium prices the consumers face. The paper derives conditions determining whether this reduction in downstream price and variety is beneficial to consumers or society.

Finally, the paper explores the impact of alternative licensing policies. With fixed license fees or royalties expressed as a percentage of the price, an upstream IP owner cannot control the intensity of downstream competition. In contrast, volume-based license fees (i.e., per-unit access fees), do permit an upstream owner to control downstream competition and to replicate the outcome of complete integration. The paper also shows that vertical integration can have little impact on downstream competition and licensing terms when IP owners charge fixed or volume-based access fees.

1 Introduction

Patent thickets, layers of licenses a firm needs to be able to offer products that embody technologies owned by multiple firms, and licensing policies have drawn increasing scrutiny from policy makers. Patent thickets involve complementary products, which gives rise to double marginalization – the so-called royalty stacking problem – and has the potential to retard diffusion of new technologies and reduce consumer welfare.[1] This paper examines the impact of licensing policies of one or more upstream owners of *essential* intellectual property (*IP* hereafter) on the downstream firms that require access to that IP. The terms under which downstream firms can access this IP affects entry decisions, product diversity, prices and welfare. We consider both the case in which a single party controls the essential IP and the case in which different parties control complementary pieces of essential IP. We compare the outcome of several alternative standard licensing arrangements, such as flat rate access fees, royalty percentages, per unit fees, patent pools and cross-licensing arrangements, with or without vertical integration.

We first consider the case where there is a single owner of essential IP. The IP owner faces a trade-off between two conflicting forces. Increasing the number of licenses enhances product variety, which allows downstream firms to better meet consumer demand, thus creating added value. However, it also intensifies downstream competition, which dissipates profits. We adopt a framework that reflects this trade-off, in which the IP owner can have an incentive to sell a larger or a smaller number of licenses than is socially desirable.

Specifically, we suppose that downstream firms compete in price and other non-price attributes. The non-price attributes are firm locations on a circular market, as in Salop (1979). Consumers are uniformly distributed on the circle and transportation costs are proportional to the distance between the firm and the consumer. Consumers buy from the firm offering the lowest delivered price, as long as this price does not exceed the consumer's reservation price. The number of downstream competitors is endogenous: entrants must each incur a fixed cost, and the license fees together with

[1] See for example SCM v Xerox: Paper Blizzard for $1.8 Billion," New York Times, June 27, 1977. As technology has become increasingly complex, this concern has drawn both judicial and legislative scrutiny – see Business Week Online http://www.businessweek.com/magazine/content/07_20/b4034049.htm May 14, 2007, From Business Week, May 23, 2007

http://www.businessweek.com/smallbiz/content/may2007/sb20070523_462426.htm, http://www.house.gov/apps/list/press/ca28_berman/berman_patent_bill.pdf and http://www.ip-watch.org/weblog/index.php?p=427.

this fixed cost contribute to determine the number of competitors.

As pointed out by Spence (1975), a key factor determining the number of licenses downstream is the effect of the number of licenses on the downstream market price, rather than on the incremental surplus derived by all downstream consumers. This market price, in turn, depends on the value of the marginal consumer served by each downstream firm. A higher density of firms means lower transportation costs on average, and even more so for the marginal consumer. Since marginal consumers thus benefit most from increased variety, an integrated monopolist would typically wish to have too many downstream outlets. An unintegrated IP owner will however fear profit dissipation through downstream competition, and may thus wish to have too many or too few downstream firms competing against each other.

We assume that the upstream firm sets the terms under which downstream firms can obtain access to its essential IP; we consider alternative common licensing arrangements: fixed access fees, per-unit access fees and royalty percentages.

We contrast the effect of volume based access fees with unit fees and flat rate access charges. We find that for essential IP, a single upstream IP owner can better control the intensity of downstream competition with per unit fees than with either royalty percentages or fixed access fees. As a result, volume-based access fees encourage the IP owner to issue more (and possibly too many) licenses; in our framework, per unit fees actually allow the IP owner to replicate the fully integrated outcome, while flat rate access fees and percentage royalties yield the same outcome and may result in reduced variety, higher consumer prices and lower profits. Vertical integration appears to have less of an impact on the IP owner's ability to control competition; in particular, it has no impact on the equilibrium prices, profits and variety in the case of flat rate and per unit access fees.

The paper also studies regulatory intervention that ignores or supersedes patent protection. In the model, oversight of the downstream market structure, intended to increase social welfare under abuse of dominance doctrine, is equivalent to regulating the licensing fees when there is no uncertainty about demand and costs. When there is uncertainty about demand or costs, we characterize conditions that determine whether regulation of price or market structure results in greater welfare.

We extend the basic model to the case in which there are two independent owners of complementary and essential IP. We find that the "patent thicket" can reduce variety downstream relative to the case of monopoly. More precisely, by relying on per unit license fees, the IP owners can still replicate the fully integrated outcome; however, when they rely instead on flat access fees or percentage royalties, double marginalization leads to higher access charges and fewer downstream firms than does monopoly or joint licensing. This reduction in variety is accompanied by a reduction in consumer prices, and the net effect benefits consumers and may or may not increase social welfare when an IP monopolist (or a patent pool) would sell too many licenses. Vertical integration does not appear to have more impact than in the case of a single owner of IP, while patent pools and cross-licensing agreements allow the IP owners to replicate the same outcome as an upstream monopoly controlling all the IP.

The literature on IP licensing initially focused on the case of a single owner of (inessential) innovation that allows a reduction in cost in a downstream market. Arrow (1962) studied the impact of competition in that downstream market on the incentives to innovate, while most of the other pioneering work focused on specific modes of licensing such as the auctioning of a given number of licenses, flat rate licensing or per unit fees. Katz and Shapiro (1985,1986) focus for example on the use of flat rate licensing and study the incentive to share or auction an innovation, while Kamien and Tauman (1986) show that flat rate licensing is indeed more profitable (for non-drastic, and thus inessential IP) than volume-based royalties in the case of a homogenous Cournot oligopoly.[2] This is partly because the licensing agreement offered to one firm affects its rivals' profits if they do not buy a license, and thus their bargaining position vis-à-vis the IP owner; such strategic effects do not arise in the case of essential (or, in their context, of drastic) innovation, since firms get no profit if they do not buy a license - whatever the agreements offered to their rivals. This optimality of flat rate licensing is somewhat at odds with what is observed in practice, which triggered many authors to identify reasons justifying the use of royalties. Muto (1993) shows for example that per unit fees can be more profitable in the case of Bertrand oligopoly with differentiated products;[3] Wang (1998) obtains a similar result in the original context of a Cournot oligopoly when the IP owner is one of the downstream firms, while Kishimoto and Muto (2008) extend this insight to Nash Bargaining between an upstream IP owner and downstream firms; and Sen (2005) shows that lumpiness, too, can provide a basis for the optimality of volume-based royalties.[4]

[2]See Kamien (1992) for an overview of this early literature.

[3]Hernandez-Murillo and Llobet (2006) consider monopolistic competition with differentiated products and introduce private information on the value of the innovation for the downstream firms.

[4]Faulli-Oller and Sandonis (2002) and Erutku and Richelle (2006) look at two part licensing policy when there is a differentiated product downstream duopoly and the upstream IP owner is vertically integrated with one of the downstream firms.

In practice, new technologies involve multiple components of complementary IP; this has triggered an abundant literature, which has identified two offsetting factors in ascertaining the effect on the number of licensors. One is the patent thicket problem, which is an extension of Cournot's 1838 complementary product oligopoly model to IP.[5] When there are two or more owners of essential IP, each fails to take into account the impact of its licensing policy on the owners of complementary IP; this results in double-marginalization (possibly in addition to the double-marginalization that occurs when an upstream monopoly sells an input to downstream firms with market power), which restricts the number of licensees relative to a welfare optimum, or even to the number of licensees that would be allocated by an integrated monopolist. However, when IP users are not final consumers but rather, (differentiated) intermediaries that compete in a downstream market, "business stealing" effects may also generate excessive entry: as some of the customers buying from a new entrant are switching away from rivals, the revenue they generate may exceed the social value created by entry.[6] Excessive entry can result in inefficient duplication of fixed costs, in which case royalty stacking can have beneficial effects: welfare can increase, and the downstream market price decrease, when the number of downstream competitors decreases from the level that would arise when there is a single, integrated owner of IP.[7] In contrast, Scotchmer (1991), Green and Scotchmer (1995), and Scotchmer and Menell (2005) indicate that when investment is sequential, early investors may not be able to capture the benefits accruing to subsequent investors. Their analysis supports stronger patent protection for complementary technologies.[8] This paper identifies a different factor that explains how licensing fees affect downstream competition and variety, in determining the effects of complementary IP.

[5] See Shapiro (2001,2006), Lichtman (2006), and Lemley and Shapiro (2006). See also Geradin, Layne-Farrar and Padilla (2006) for further discussion.

[6] See Spence (1976), Dixit and Stiglitz (1977), Salop (1979), and Mankiw and Whinston (1986) for detailed analyses of this issue. Tirole (1988, chapter 7) provides a good overview of this literature.

[7] See Rey and Tirole (2007) for a related discussion of potential defences for exclusionary conduct and Vickers (1995) for an analysis of the cost and benefits of vertical integration and foreclosure in the context of a partially regulated upstream monopolist.

[8] Studies of the effects of patent thickets include Heller and Eisenberg (1998), Kiley (1992) and Kitch (2993) in bio-medical research, and Gerarin, Layne-Farrar, and Padilla (2006), Schankerman and Noel (2006), Walsh, Arora and Cohen (2003) and Ziedonis (2003) in technology intensive industries.

2 General framework

Upstream firms own a technology, protected by an IP right, which is a key input to be active in a downstream market. Initially, a single IP owner does not use the technology itself but licences it to downstream competitors (we discuss the impact of vertical integration and of multiple complementary IP firms and rights later on).

Entry in the downstream market can affect consumers in two ways: directly, through enhanced product variety, and indirectly, through increased competitive pressure on prices. Suppose first that downstream competitors produce the same product at the same cost. Entry has then no intrinsic value and, if there is any set-up cost, it would clearly be socially as well as privately optimal to have the market served by a single downstream firm.[9] Yet a regulator might wish to stimulate entry in order to encourage downstream competition. Suppose for example that the IP owner charges a fixed access fee and the downstream firms compete imperfectly in a Cournot fashion. The IP owner would then maximize and appropriate all the industry profit by charging a fee equal to the downstream monopoly profit, whereas a regulator might want to impose a cap on the access fee, in order to reduce consumer prices and allocative inefficiency, even if this inefficiently duplicates entry costs.[10]

When instead variety is valuable, increasing the number of firms can have an ambiguous impact on consumer surplus: enhancing product variety tends to benefit consumers, but it may also lead to higher prices, since firms' offerings then better respond to consumer needs. As a result, the IP holder may want to issue either too many or too few licenses.

To see this, suppose that there is an infinite number of potential entrants in the downstream market. However, to enter the market a downstream firm must have access to the technology and pay a license fee ϕ to the IP holder. The timing is as follows:

[9]Suppose for example that the downstream firms have the same cost function $C(q) = f + cq$, and let $U(q)$ denote consumers' gross surplus, and $P(q) = U'(q)$ the associated inverse demand function. The social optimum maximizes $U(q) + (P(q) - c)q - nf$, possibly subject to a budget constraint $(P(q) - c)q \geq nf$, whereas the private optimum maximizes $(P(q) - c)q - nf$. The social and private interests then lead to different pricing rules (marginal or average cost versus monopoly price) but agree on the optimal number of firms, $n = 1$.

[10]Consider the example described in the previous footnote and let $q^C(n)$ denote the aggregate quantity produced when n downstream firms compete à la Cournot. A regulator would seek to maximize

$$U\left(q^C(n)\right) - cq^C(n) - nf$$

and would thus choose $n > 1$ whenever (ignoring integer problems) $P\left(q^C(1)\right) > c + f/\left(q^C\right)'(1)$.

- First, the IP owner sets the license fee, ϕ.
- Second, potential entrants decide whether to buy a license or not.
- Third, downstream competition takes place among the licensees.

We will further assume that downstream competition leads to a symmetric equilibrium. It is natural to posit that the per-firm profit, $\pi^*(n)$, decreases as the number of licensees, n, increases. Given the licensing fee ϕ set in the first stage, in the second stage firms want to enter as long as $\pi^*(n) > \phi$, and prefer to stay out if $\pi^*(n) < \phi$. The number of downstream competitors, $n = n^*(\phi)$, is thus characterized by the free entry condition

$$\pi^*(n) = \phi.$$

Therefore, in the first stage, the IP owner can "choose" the number of firms n by setting the licensing to $\phi^*(n) = \pi^*(n)$, and moreover, this fee allows the IP owner to extract all downstream profits. The IP owner will thus seek to maximize aggregate profit

$$n\phi^*(n) = \Pi^*(n) \equiv n\pi^*(n).$$

We will denote by n^{Π} the optimal number of firms for the IP owner and suppose that the market is viable: $\Pi^*\left(n^{\Pi}\right) > 0$.

In contrast, let n^S and n^W respectively denote the number of firms that maximizes (subject to a budget constraint $\Pi^*(n) \geq 0$) consumer surplus, $S^*(n)$, and total welfare,

$$W^*(n) \equiv S^*(n) + \Pi^*(n).$$

By construction:

$$S^*\left(n^S\right) \geq S^*\left(n^{\Pi}\right).$$

Similarly, a revealed preference argument yields:

$$\begin{aligned} \Pi^*\left(n^{\Pi}\right) &\geq \Pi^*\left(n^W\right), \\ S^*\left(n^W\right) + \Pi^*\left(n^W\right) &\geq S^*\left(n^{\Pi}\right) + \Pi^*\left(n^{\Pi}\right), \end{aligned}$$

and thus (summing-up these two inequalities):

$$S^*\left(n^W\right) \geq S^*\left(n^{\Pi}\right).$$

Therefore, when entry in the downstream market overall benefits consumers (i.e., $S^*(.)$ increases with n), the IP owner will tend to restrict entry, compared to what would be desirable for consumers or society: $n^{\Pi} \leq n^W, n^S$. This is the case in the situation discussed above, where consumers do not care about variety; when consumers enjoy variety, this remains the case as long as increasing the number of downstream firms still yields lower prices or generates only moderate price increases, despite the positive impact of variety on demand. A regulator would then wish to foster entry, e.g., by imposing a cap on the licensing fee. In contrast, when additional entry generates price increases that dominate the direct impact on demand (i.e., $S^*(.)$ decreases as n increases), then the IP owner will tend to issue too many licenses, by setting the licensing fee too low.

The overall impact of downstream competition on consumer surplus also drives the analysis of royalty stacking when there are complementary IP technologies. Suppose for example that there are not one but two upstream firms, each owning an essential technology: that is, combined together, the two technologies allow firms to be active in the downstream market, but each of them is necessary to be active in that market. If the two IP holders were to join forces for the licensing of their rights, they would as above choose to charge a total fee equal to $\phi^{\Pi} \equiv \pi^*\left(n^{\Pi}\right)$ and share the corresponding profit. If instead they independently set their own license fees ϕ_1 and ϕ_2, each downstream firm will earn

$$\pi^*(n) - (\phi_1 + \phi_2),$$

and the number of entrants will thus be equal to

$$n = n^*(\phi_1 + \phi_2).$$

Each IP holder i will therefore maximize

$$\phi_i n^*(\phi_1 + \phi_2),$$

which leads to a standard double-marginalization problem. When the IP owners would already issue too few licences under joint licensing, double marginalization makes things worse by further restricting the number of licensees. However, when joint licensing would instead lead the IP holders to issue too many licenses, double marginalization may come as a blessing since it counterbalances the bias towards excessive entry – provided there is no "overshooting": double marginalization may also lead to a number of licensees that is lower than socially desirable, to an extent such that social welfare is reduced.

This discussion can be summarized as follows.

Proposition 1 *Suppose that downstream competition generates a symmetric equilibrium in which per-firm profit $\pi^*(n)$ decreases with n; then:*

- *When $S^*(n)$ always increases with n, the owner of a single essential technology would issue too few licences, compared with what would be optimal for society or consumers; when there are multiple essential technologies, double marginalization leads to access prices which are even more excessive and entry which is even less sufficient in the downstream market;*

- *When instead $S^*(n)$ decreases as n increases, the owner of a single essential technology would instead issue too many licences; in the case of multiple essential technologies, double marginalization tends to counterbalance this bias and may thus lead to a more desirable outcome.*

We now explore in more detail these issues in the context of a standard model of downstream competition with horizontal product differentiation.

3 Downstream competition with product differentiation

3.1 The model

We adopt the model proposed by Salop (1979), which adapted the Hotelling model of horizontal differentiation to allow for any number of downstream competitors. There is a continuum of consumers of total mass 1, uniformly distributed along a circle of length one. A consumer buying from a firm "located" at a distance d gets a utility r but incurs a "transportation cost" td, reflecting the disutility from not having a unit corresponding to that consumer's ideal characteristics.

As before, there is an infinite number of potential entrants in the downstream market, and any firm with access to the technology can enter the market by incurring a fixed cost f; for expositional simplicity, we will suppose that downstream firms can then produce at no cost (introducing a constant marginal cost would not affect the analysis, rescaling the reservation and the equilibrium prices by the same amount). For the sake of exposition, we will also ignore integer problems and treat the number of entrants as a continuous variable.

This simple and well-known model, which relies on a standard discrete choice approach, moreover allows us to focus on variety (i.e., entry) since, as long as the market is served, prices do not affect total welfare directly (the terms of the licensing agreements may and will however have an indirect impact, through their effect on entry). It is also flexible enough to reflect the benefits of entry for consumers (directly through increased variety, but also possibly indirectly through more intense competition), as well as potential adverse effects (through increased local market power, as discussed below). As a result, increasing the number of competitors may have either a positive or a negative impact on consumers.

Before studying the impact of access terms on downstream competition, it is useful to characterize first the optimal degree of variety, both from the private standpoint of a fully integrated company, who would own and control the IP as well as the downstream firms, and from the social (i.e., total welfare) standpoint.

3.2 Fully integrated monopoly

If a fully integrated monopolist could determine both the number of downstream firms *and* their prices, it would choose to serve the entire market (or none) and to distribute the downstream outlets uniformly along the circle in order to minimize transportation costs and thus maximize demand. Therefore, for a given number of firms n, an integrated monopolist would opt for a price such that $p + t/2n = r$, that is, a price equal to

$$\hat{p}(n) \equiv r - \frac{t}{2n}.$$

Total industry profit would thus be equal to

$$\hat{\Pi}(n) \equiv r - \frac{t}{2n} - nf,$$

which, ignoring divisibility problems, is maximal for

$$n = n^M \equiv \sqrt{\frac{t}{2f}}.$$

The corresponding price and profit are respectively equal to

$$p^M \equiv \hat{p}\left(n^M\right) = r - \sqrt{\frac{tf}{2}}, \Pi^M \equiv \hat{\Pi}\left(n^M\right) = r - \sqrt{2tf}.$$

In what follows, we will assume that the industry is viable:

Assumption 1. *Consumers' reservation price is large enough, compared with production and transportation costs, to make the industry viable:* $\Pi^M > 0$*, or*

$$\frac{r^2}{tf} > 2.$$

3.3 Welfare optimum

For any given number of firms n, as long as all consumers are served total welfare is equal to the consumers' value for the good, r, net of entry costs as well as of transportation costs. Total transportation costs are again minimized when downstream outlets are uniformly distributed along the circle, in which case they are equal to:

$$T(n) \equiv 2n \int_0^{\frac{1}{2n}} tx dx = \frac{t}{4n}.$$

Total welfare is therefore equal to

$$W(n) \equiv r - \frac{t}{4n} - nf,$$

which, ignoring again divisibility problems, is maximal for:[11]

$$n^W \equiv \sqrt{\frac{t}{4f}}.$$

Assumption 1 implies

$$W\left(n^W\right) = r - \sqrt{tf} > 0.$$

It moreover implies that it is indeed socially desirable to cover the entire market. To see this, suppose that, by selling at (marginal) cost, n firms do not cover the entire market. The marginal consumers would then be located at a distance $\hat{x}$ from the nearest firm such that $t\hat{x} = r$; total welfare would thus be equal to:

$$n\left[2\int_0^{\hat{x}} (r - tx)\, dx - f\right] = n\left[2\hat{x}\left(r - t\frac{\hat{x}}{2}\right) - f\right] = n\left(\frac{r^2}{t} - f\right),$$

where the last term between parenthesis is positive under Assumption 1. Therefore, it would be optimal to increase the number of firms until the entire market is served.

Note that $n^M > n^W$. As already mentioned, when deciding whether to add a downstream outlet, an integrated monopolist – who fully internalizes the additional entry cost f – focuses on its impact on marginal consumers (since they are the ones that determine prices), which are the farthest away from the existing outlets and thus benefit most from the introduction of additional outlets. In contrast, total welfare takes into consideration the impact on all consumers, including inframarginal ones. As a result, a fully integrated monopolist has an incentive to introduce excessively many downstream subsidiaries.

[11] Imposing $t > 4f$ (which is compatible with the other relevant conditions on r versus tf) would ensure that all relevant numbers exceed 1.

4 Licensing arrangements and downstream competition

We now study in this framework the IP holder's optimal licensing policy, given its impact on the downstream market. We will again consider the following timing:

- First, the IP owner sets the terms for its licenses; these terms are non-discriminatory and available to any firm wishing to enter the downstream market.[12]
- Second, potential entrants decide whether to buy a license or not; for the sake of exposition, we will assume that firms entering the market locate themselves uniformly along the circle; that minimizes total transportation costs and it thus desirable for consumers as well as for the upstream firm.
- Third, downstream competition takes place among the licensees.

4.1 Fixed access fees

We first consider the case where the IP holder charges a fixed fee ϕ per license.

4.1.1 Downstream equilibrium

We now characterize the downstream competitive equilibrium price and profits. For any given number, n, of firms which are uniformly distributed along the circle, there exists a symmetric equilibrium $p^* (n)$ determined by

$$p^* = \arg\max_p pD (p, p^*; n),$$

where $D (p, \tilde{p}; n)$ denotes the demand facing a firm charging a price p when all others charge the (equilibrium) price $\tilde{p}$. When the number of downstream firms n is large enough, the firms are sufficiently close to each other as to fight actively for market share; in that case, the demand is given by:

$$D (p, \tilde{p}; n) = D^H (p, \tilde{p}; n) \equiv \frac{1}{n} - \frac{p - \tilde{p}}{t}.$$

[12] Allowing for secret, possibly discriminatory licensing terms might give the IP owner an incentive to behave opportunistically and issue more licenses than it would otherwise. See Hart and Tirole (1990), O'Brien and Shaffer (1992) and McAfee and Schwartz (1994), or Rey and Tirole (2007) for an overview of this literature.

Competition then drives the price down to the standard Hotelling competitive level:

$$p^H(n) \equiv \frac{t}{n}.$$

This competitive price characterizes the equilibrium when, at that price, all consumers strictly prefer to buy. This is the case when the "generalized price", taking into account the transportation cost, is lower than consumers' reservation value, r, even for the consumers that are the farthest away. Since the maximal distance between a consumer and the closest firm is equal to $t/2n$, the Hotelling price constitutes the equilibrium price when

$$p^H(n) + \frac{t}{2n} = \frac{3t}{2n} < r,$$

or

$$n > \overline{n} \equiv \frac{3}{2}\frac{t}{r}. \tag{1}$$

When this condition is satisfied, each downstream firm thus earns a profit (gross of the licensing fee) equal to

$$\pi^H(n) \equiv \frac{t}{n^2} - f,$$

and total industry profit is thus equal to:

$$\Pi^H(n) \equiv n\pi^H(n) = \frac{t}{n} - nf.$$

When the number of downstream is so low that condition (1) is violated, marginal consumers are indifferent between buying or not. Indeed, when t is very large, each downstream firm acts as a local monopoly: by setting a price p, it will serve all consumers located at a distance x such that

$$p + tx \leq r,$$

and will thus face a local monopoly demand:

$$D(p, p^*; n) = D^m(p) \equiv \frac{2(r-p)}{t}.$$

its profit (gross of the licensing fee) is then equal to

$$\frac{2p(r-p)}{t} - f$$

and is maximal for the monopoly price

$$p^m \equiv \frac{r}{2}.$$

This monopoly price constitutes indeed the equilibrium price when the local monopoly markets do not overlap, that is, when marginal consumers are located at no more than $1/2n$ from the firm; this is the case when

$$p^m + \frac{t}{2n} = \frac{r}{2} + \frac{t}{2n} \geq r,$$

that is, when

$$n \leq \underline{n} \equiv \frac{t}{r}. \tag{2}$$

The gross profit of a downstream firm is then equal to the local monopoly profit

$$\pi^m \equiv \frac{r^2}{2t} - f,$$

and total industry profit is thus equal to:

$$\Pi^m(n) \equiv n\pi^m = n\left(\frac{r^2}{2t} - f\right).$$

Finally, when the number of downstream firms, n, lies between $\underline{n}$ and $\overline{n}$, the whole market is served but marginal consumers are indifferent between buying or not; the equilibrium price then coincides with the industry optimal pricing policy (conditional on the number of firms n),

$$\hat{p}(n) = r - \frac{t}{2n}.$$

In contrast with the above "Hotelling" case, the equilibrium price here *increases* with the number of firms. This is due to the already mentioned fact that increasing the number of downstream firms increases variety, which enhances consumers' demand and allows here firms to take advantage of "niche" strategies. The downstream equilibrium moreover replicates the outcome of a fully integrated monopolist. Total industry profit is thus equal to $\hat{\Pi}(n)$ and each downstream firm earns a gross profit equal to

$$\hat{\pi}(n) \equiv \frac{\hat{\Pi}(n)}{n} = \frac{r}{n} - \frac{t}{2n^2} - f,$$

which, since $n > \underline{n}$, decreases as n increases:

$$\hat{\pi}'(n) = -\frac{r}{n^2} + \frac{t}{n^3} = \frac{r}{n^3}(\underline{n} - n) < 0$$

We can thus describe the downstream equilibrium price, $p^*(n)$, and aggregate profit, $\Pi^*(n)$, as follows:

$$\begin{cases} \text{for } n < \underline{n} = \frac{t}{r}, & p^*(n) = p^m = \frac{r}{2} & \text{and} \quad \Pi^*(n) = \Pi^m(n) = n\left(\frac{r^2}{2t} - f\right), \\ \text{for } \underline{n} \leq n \leq \overline{n} = \frac{3}{2}\frac{t}{r}, & p^*(n) = \hat{p}(n) = r - \frac{t}{2n} & \text{and} \quad \Pi^*(n) = \hat{\Pi}(n) = r - \frac{t}{2n} - nf, \\ \text{for } n > \overline{n}, & p^*(n) = p^H(n) = \frac{t}{n} & \text{and} \quad \Pi^*(n) = \Pi^H(n) = \frac{t}{n} - nf. \end{cases}$$

In particular, as n increases:

- The profit of a downstream firm (gross of the licensing fee ϕ),

$$\pi^*(n) = \frac{\Pi^*(n)}{n},$$

first remains constant at the local monopoly level, π^m (as long as n remains below $\underline{n}$) and then strictly decreases: $\hat{\pi}(n)$ decreases with n when $n > \underline{n}$, and $\pi^H(n)$ always decreases with n.

- Consumer surplus first increases proportionally to the number of firms, then decreases when n lies between $\underline{n}$ and $\overline{n}$, before increasing again.[13]

This model of horizontal differentiation thus reflects the various aspects discussed above: an increase in the number of competitors benefits consumers and dissipates profit when Hotelling-type competition prevails (that is, when $n > \overline{n}$), but it can also allow firms to extract a bigger share of consumers' benefit from variety, resulting in higher prices that reduce consumer surplus (when $n \in [\underline{n}, \overline{n}]$).[14]

4.1.2 Optimal and equilibrium access fees

We now characterize the privately optimal licensing fee, ϕ^Π, which maximizes the profit of the upstream firm. From the above analysis of downstream competition, each licensee's profit decreases monotonically from $\pi^m = r^2/2t - f > 0$ to 0 as the number of firms increases; the IP owner can thus determine the number of downstream firms by adjusting the licensing fee ϕ:

- if the upstream IP owner sets $\phi > \pi^m$, no firm enters the market;
- if instead the IP owner sets $\phi = \pi^m$, any $n \leq \underline{n}$ firms would be willing to enter; it is then optimal for the IP owner to let as many firms as possible enter the market (i.e., $n = \underline{n}$), since the IP owner's profit is proportional to the number of licenses issued (since $\pi^m > 0$) – the IP owner can moreover achieve this by offering if needed an arbitrarily small discount below π^m;

[13] Consumer surplus is equal to $2n \int_0^{x^m} txdx = nr^2/4t$ where x_m is the distance to the marginal consumer, i.e., $x^m = \frac{r}{2t}$, for $n < \underline{n}$, to $2n \int_0^{1/2n} txdx = t/4n$ for $\underline{n} \leq n \leq \overline{n}$ and to $r - t/4n - p^*(n) = r - 5t/4n$ for $n > \overline{n}$.

[14] Chen and Riordan (2007) provide another model of differentiated products competition with this feature.

- last, if the IP owner sets $\phi < \pi^m$, then there exists a unique n such that $\pi^*(n) = \phi$; this licensing fee thus triggers a unique continuation equilibrium where, at stages 2 and 3, n downstream firms enter (ignoring again integer problems) and set a price $p^*(n)$.

Thus, as before, the licensing fee ϕ allows the upstream IP owner to control the number n of downstream firms, by setting the licensing fee to $\phi^*(n) = \pi^*(n)$, and to recover all the downstream profits. The optimal licensing fee will thus induce the number of firms n that maximizes the industry profit:

$$\max_n n\phi^*(n) = \Pi^*(n).$$

Without loss of generality, we can furthermore restrict attention to $n \geq \underline{n}$. Conversely, intense downstream competition dissipates profit: $\Pi^H(n)$ decreases as n increases; the upstream firm will thus never choose $n > \overline{n}$. In the range $[\underline{n}, \overline{n}]$, the industry profit coincides with the integrated monopoly profit $\left(\Pi^*(n) = \hat{\Pi}(n)\right)$, which is concave and maximal for $n = n^M$. Therefore, the industry profit is globally quasi-concave and the upstream firm will thus find it optimal to induce the entry of n^Π downstream firms, where

$$n^\Pi \equiv \min\left\{n^M, \overline{n}\right\}.$$

Indeed, if it could control prices as well as the number of the downstream firms, the IP owner would choose to let n^M firms enter the market. However, having that many firms in the downstream market can trigger price competition and dissipate profits: this occurs when $n^M > \overline{n}$, in which case the IP owner prefers that only $\overline{n}$ firms enter the market.

It can be checked that $n^M \geq \overline{n}$ if and only if:

$$\frac{r^2}{tf} \geq \frac{9}{2}.$$

Therefore, the IP holder makes positive profits whenever the industry is viable (i.e., $r^2/tf > 2$):

- when $2 < r^2/tf \leq 9/2$, $n^\Pi = n^M \leq \overline{n}$ and thus $\Pi^*\left(n^\Pi\right) = \Pi^M > 0$;
- when instead $r^2/tf > 9/2$, $n^\Pi = \overline{n} < n^M$ and thus $\Pi^*\left(n^\Pi\right) < \Pi^M$ but:

$$\Pi^*\left(n^\Pi\right) = \Pi^*(\overline{n}) = \left(\frac{r^2}{tf} - \frac{9}{4}\right)\frac{2tf}{r} > 0.$$

In both cases, the IP owner will generate the (constrained) optimal number n^{Π} of downstream firms by setting

$$\phi^{\Pi} = \pi^* \left(n^{\Pi}\right).$$

We can now compare the privately optimal number of downstream firms, n^{Π}, with the socially desirable one, n^W, which could be achieved by setting the licensing fee to

$$\phi^W \equiv \pi^* \left(n^W\right).$$

As long as $\overline{n} > n^W$, the IP owner issues too many licenses: either n^M, if $\overline{n} > n^M > n^W$, or $\overline{n}$, if $n^M > \overline{n} > n^W$. If instead $n^W > \overline{n}$, downstream competition would dissipate profits not only with n^M competitors, but also with the (smaller) number of competitors that would be socially desirable, n^W; in that case, the IP owner excessively restricts entry, in order to limit downstream competition. Even in this case, though, a positive licensing fee is required to induce the socially desirable number of downstream firms, since:

$$\phi^W = \pi^* \left(n^W\right) = \pi^H \left(n^W\right) = \frac{t}{\left(n^W\right)^2} - f = 3f > 0.$$

This positive license fee is needed to prevent the "excessive entry" that would otherwise derive from a "business stealing" effect, each downstream firm failing to take into account that (some of) the customers it serves would otherwise be served anyway by other firms.

It can be checked that $n^W \geq \overline{n}$ if and only if:

$$\frac{r^2}{tf} \geq 9,$$

which leads to:[15]

Proposition 2 *Suppose that the market is viable: $r^2/tf \geq 2$. Then:*

- *if in addition $r^2/tf > 9$, the IP owner lets too few firms enter the downstream market;*
- *if instead $r^2/tf < 9$, the IP owner lets too many firms enter the downstream market.*

[15]In the second case ($2 < r^2/tf < 9$), the socially desirable number of downstream firms may yield negative industry profits; taking into account a budget constraint ($\Pi \geq 0$) would then call for a higher number of firms, $\hat{n}^W > n^W$, which would however remain larger than n^{Π}.

Thus, when variety is "cheap" (i.e., the fixed cost f is small) and/or "not highly regarded" (i.e., the transportation cost t is small, implying that variety is not very valuable) compared with the intrinsic value of the good (as measured by r), the upstream IP holder issues too few licences: it would be desirable in that situation to have more firms in the downstream market, but competition would then dissipate the profits that the IP owner can recover. When instead variety is costly and/or particularly viable (i.e., f and t are substantial), the IP holder encourages too many firms to enter the downstream market: despite competition, the increase in variety leads downstream firms to charge higher prices, in order to extract marginal consumers' gain from variety, which overall increases the industry profit.

This ambiguity in the comparison between the privately and socially desirable numbers of firms reflects a similar ambiguity for the licensing fees: the IP owner will seek to charge an excessively high fee when $r^2/tf > 9$, but will charge instead too low a fee when $r^2/tf < 9$.

Finally, it can be noted that the IP owner's inability to fully control the downstream firms' pricing policies limits the risk of excessive entry. In the present set-up, where a fully integrated industry would generate more variety than is socially desirable (i.e., $n^W < n^M$), the IP owner's inability to prevent profit dissipation through Hotelling-like product market competition leads it to somewhat limit the number of downstream firms, which, in turn, reduces the scope for excessive entry (e.g., when $n^\Pi < n^W < n^M$).

4.2 Alternative licensing arrangements

We have so far focussed on fixed licensing fees. We now briefly discuss alternative arrangements, such as volume-based access fees or royalty percentages.

4.2.1 Royalties

We first note that replacing fixed fees with *revenue-based royalties* does not affect the analysis. Suppose indeed that, instead of a fixed licensing fee ϕ, the IP holder asks for a percentage τ of downstream revenues. If n firms enter the downstream market and the other firms charge the same price $\widetilde{p}$, a downstream firm then maximizes

$$(1-\tau)\,pD\left(p,\widetilde{p};n\right)-f,$$

which leads to the same best response as before and thus, given n, to the same equilibrium price $p^*(n)$. Each firm thus gets:

$$(1-\tau)\left[\pi^*(n)+f\right]-f=(1-\tau)\,\pi^*(n)-\tau f,$$

which decreases as τ or n increases. Free entry yields the following relationship between the royalty rate τ and the equilibrium number of firms n:

$$\tau = \tau^* (n) \equiv \frac{\pi^* (n)}{\pi^* (n) + f}.$$

The expression $\tau^* (n)$ decreases as n increases.[16] The IP holder can thus fully control n by charging setting the rate to $\tau^* (n)$ and, since free-entry drive downstream profits to zero, the IP holder recovers as before the whole aggregate profit $\Pi^* (n)$. The IP holder will thus issue n^Π licenses, by charging a rate $\tau^\Pi \equiv \tau \left(n^\Pi\right)$. There will therefore be too many or too few licences, depending on whether $r^2 \lesseqgtr 9tf$. Likewise, since $\tau^* (n)$ decreases as n increases in the relevant range $[\underline{n}, \overline{n}]$, the IP owner will seek to charge too low (if $r^2 < 9tf$) or too high (if $r^2 > 9tf$) royalty rates.

4.2.2 Unit fees

In contrast, the IP holder can achieve the fully integrated monopoly outcome through the use of *volume-based access fees or royalties* where, say, a downstream firm pays a per-unit fee γ to the IP holder. We show this informally here, and provide a formal proof in the Appendix. Note first that the IP owner will always ensure that the market is served. Indeed, if the entire market was not served, each downstream firm would have a market share, say α, lower than $1/n$, and would thus act a local monopolist (given its "marginal cost" γ); the IP owner's profit would thus be equal to

$$\Pi_U = \gamma n \alpha.$$

But then, given γ and α, the IP owner would issue as many licenses as needed to "just" cover the market: doing so would not alter the downstream firms' "local monopoly" power, but would increase total coverage and thus profit.

Conversely, as long as the entire market is served, the IP owner obtains a profit equal to γ, and thus wishes to increase γ as much as possible. And since downstream firms' profits are greater in the monopoly regime, the IP owner will thus choose γ so that a local downstream monopolist would make zero profit. The local monopoly price and profit, based on a unit cost γ, are respectively equal to:

$$\begin{aligned} p^m (\gamma) &\equiv \arg\max_p (p - \gamma) D^m (p) = \frac{r + \gamma}{2}, \\ \pi^m (\gamma) &= \frac{(r - \gamma)^2}{2t} - f. \end{aligned}$$

[16]The rate τ^* increases with π^*, which in turn decreases as n increases.

The maximal unit fee that the IP owner can charge (for which $\pi^m(\gamma) = 0$) is thus equal to:

$$\bar{\gamma} \equiv r - \sqrt{2tf}.$$

But for this fee, the market price is at the fully integrated level:

$$p^m(\bar{\gamma}) = r - \sqrt{\frac{tf}{2}} = p^M.$$

This, in turn, implies that the local monopolists' market shares yield the optimal number of firms: marginal consumer are located at a distance $\hat{x}$ such that $p^M + t\hat{x} = r$, and thus each local monopolist's market share is equal to

$$2\hat{x} = 2\frac{r - p^M}{t} = \sqrt{\frac{2f}{t}};$$

covering the entire market thus requires a number of firms equal to:

$$n = \frac{1}{2\hat{x}} = \sqrt{\frac{t}{2f}} = n^M.$$

This leads to:

Proposition 3 *Offering revenue-based royalties yields the same outcome as fixed access fees. In contrast, offering volume-based royalties (i.e., per unit access fees) allows the IP owner to replicate the fully integrated outcome and thus issue excessively many licenses.*

Proof. See Appendix A. ■

Interestingly, a volume-based fee cannot be used to sustain the socially desirable outcome. It is shown in Appendix A that the equilibrium per-firm profit, $\pi^*(n;\gamma)$, decreases as n or γ increases. Therefore:

- any γ larger than $\bar{\gamma}$ triggers no entry, since then, for any n, $\pi^*(n;\gamma) \leq \pi^m(\gamma) < 0$;
- $\gamma = \bar{\gamma}$ triggers any $n \leq n^M$ firms, but the market is entirely served for $n = n^M$; therefore, while it is possible to sustain exactly n^W firms, only part of the market would then be served;
- and any $\gamma < \bar{\gamma}$ triggers either a continuation equilibrium in which all the market is served, since $\pi^m(\gamma) > (\pi^m(\bar{\gamma}) =) \, 0$, but in which more than n^M downstream firms enter the market, since then $\pi^*\left(n^M;\gamma\right) > 0$[17] and $\pi^*(n;\gamma)$ further increases (up to $\pi^m(\gamma)$) as n decreases below n^M.

[17]Either $\pi^*\left(n^M;\gamma\right) = \hat{\pi}\left(n^M;\gamma\right) > \pi^*\left(n^M;\gamma^M\right) = 0$, or $\pi^*\left(n^M;\gamma\right) = \pi^C\left(n^M\right) = f > 0$.

These observations moreover indicate that, while a volume-based fee cannot sustain the socially desirable outcome, its private interest however leads the IP owner to choose the "second-best" level for such a fee: *conditional on relying on volume-based fees*, the (second-)best fee γ^W coincides with $\bar{\gamma}$, since any higher level would generate no entry and any lower level would generate additional entry, from a point where there is already excessive entry (since $n^M > n^W$).

4.3 Vertical integration

We now suppose that the upstream monopolist is vertically integrated and thus owns one of the downstream firms; we thus consider a case of pure vertical integration, where the integrated subsidiary still competes with the other, non-integrated downstream firms. Clearly, vertical integration does not affect the behavior of non-integrated downstream firms. It turns out that it does not affect the behavior of the downstream subsidiary either, and thus has no effect on the final outcome, when the IP holder charges either fixed or per-unit access fees; as we will see, this does not carry over to the case of revenue royalties.

Suppose first that the IP holder charges a fixed licensing fee ϕ. Once it has sold n licenses, the variable profit of the vertically integrated firm coincides with that of its downstream subsidiary; therefore, vertical integration has no impact on its downstream behavior. For a given total number of downstream competitors, each unintegrated firm thus earns $\pi^* (n) - \phi$ and the number of firms is determined as before by $\pi^* (n) = \phi$. The integrated firm therefore earns:

$$\pi^* (n) + (n - 1) \phi = n\pi^* (n) ,$$

and chooses again to let n^Π firms (including its own subsidiary) enter the downstream market.[18]

Suppose now that the IP holder charges instead a unit fee γ. When setting its downstream market price p, the integrated firm takes into account that it loses γ on

[18]When the IP owner can deal secretly with downstream firms, vertical integration may help avoid opportunism by the IP owner (since issuing an additional license then hurts the integrated subsidiary as well as the other downstream competitors), in which case it may result in fewer licenses being issued.

any unit taken away from its rivals; more precisely, it will maximize:[19]

$$pD(p,\widetilde{p};n)+\gamma\left[1-D(p,\widetilde{p};n)\right],$$

which amounts to maximizing

$$(p-\gamma)\,D(p,\widetilde{p};n),$$

exactly as do the other, unintegrated competitors. Vertical integration thus again does not affect the pricing behavior of the downstream subsidiary. The downstream equilibrium still remains the same as before, and the integrated firm can thus obtain the monopoly profit Π^M by setting $\gamma=\gamma^M$.[20] We thus have:

Proposition 4 *Vertical integration does not affect the equilibrium outcome when the licensing terms consist of either fixed or per unit access fees.*

Remark: revenue-based royalties. Vertical integration affects the outcome when the IP owner relies on revenue-based royalty percentages: in that case, the integrated firm maximizes

$$pD(p,\widetilde{p};n)+\tau\widetilde{p}\left[1-D(p,\widetilde{p};n)\right]$$

and is thus less aggressive than the others. The downstream equilibrium is then a bit more complex to characterize, since the reduction in the competitive pressure is greater for the integrated subsidiary's immediate neighbors than for the other unintegrated firms, and is asymmetric, the integrated firm's downstream subsidiary having a lower market share than the unintegrated firms. Such royalty schemes thus lead to an inefficient allocation of consumers among the existing firms; they can moreover lead to an inefficient distribution of firms along the circle, since locations closer to the integrated firm downstream subsidiary are more profitable.

[19]This assumes that the entire market is served, which is the case in equilibrium (since the IP holder has always an incentive to issue sufficiently many licenses to cover the market). More generally, vertical integration could affect downstream pricing behavior when prices affect total demand as well as market shares.

[20]It is always optimal for the IP holder to let enough downstream firms enter to cover the entire market. For a given fee γ and associated number of firms n, the total profit of the integrated IP holder is then equal to (using the free-entry condition)

$$\frac{p^*(n)}{n}+\gamma\left(1-\frac{1}{n}\right)=\frac{p^*(n)-\gamma}{n}+\gamma=f+\gamma.$$

The integrated IP holder thus wishes to maximize γ, as when there is no vertical integration.

4.4 Recap

The above analysis can be summarized as follows:

- When the IP holder relies on fixed licensing fees, with or without vertical integration, or on royalty percentages without integration, it cannot fully "control" the intensity of downstream competition and, as a result, cannot achieve the fully integrated outcome; it may still issue too many licenses by charging too low fees but, when competition is a serious concern, it issues instead too few licenses or, equivalently, charges too high fees for these licenses.

- When instead the IP holder asks for volume-based royalties, with or without vertical integration, the per-unit access fee allows the IP holder to control the intensity of downstream competition and achieve the fully integrated outcome; it then issues more licenses than is socially desirable. In that case, however, altering the level of the unit fee can only decrease total welfare.

This model can also be used to address the following question: suppose that one cannot "regulate" the actual level of the licensing terms (i.e. the amount of the fee or the royalty rate), but still dictate the type of licensing arrangement (e.g., fixed fees versus revenue-based or volume-based royalties); which type of arrangement works best for society?

Insisting on fixed licensing fees or revenue-based royalties leads the IP owner (with or without vertical integration in the first case, and without integration in the second case) to issue $n^{\Pi} = \min\left\{\overline{n}, n^M\right\}$ licenses, whereas allowing for alternative (e.g., volume-based fees) and more profitable licensing schemes leads instead the IP owner (with or without vertical integration) to issue $n^M > n^W$ licenses. As a result, allowing for more flexible licensing schemes has no impact on the number of licensees and thus on welfare when $n^{\Pi} = n^M$ $(> \overline{n})$, and instead increases the number of licensees (from $\overline{n}$ to n^M) when $n^{\Pi} = \overline{n} < n^M$; in that case, this can decrease welfare (e. g., if $n^W < \overline{n}$, since in that case n^{Π} is already excessively high) but may increase it as well when $\overline{n}$ is low enough.[21]

5 Uncertainty and regulatory bias

We now consider whether uncertainty about upstream (innovation) or downstream costs and values may lead the regulator to jack up or down the level of the licensing

[21]This is for example the case for $t = r = 2f$.

fee.

Generally speaking, suppose that the welfare function depends on the licensing fee ϕ and on an uncertain variable θ, and that the regulator must choose ϕ before knowing the realization of the uncertainty about θ. The regulator will thus seek to solve (where $E[.]$ denotes to the expectation operator, for the distribution of θ):

$$\max_{\phi} E\left[\widetilde{W}(\phi, \theta)\right],$$

where

$$\widetilde{W}(\phi, \theta)$$

denotes the ex post level of welfare, conditional on the realization of the random variable θ.

Assuming that the welfare function $\widetilde{W}$ is continuously differentiable and concave with respect to ϕ, the optimal fee ϕ^* is then characterized by the first-order condition

$$E\left[\partial_\phi \widetilde{W}(\phi^*, \theta)\right] = 0.$$

In contrast, in the absence of uncertainty (i.e., if the variable θ was always equal to its mean value $E[\theta]$), the regulator would choose the licensing fee so as to maximize

$$\max_{\phi} \widetilde{W}(\phi, E[\theta]),$$

leading to a fee ϕ^{**} characterized by the first-order condition

$$\partial_\phi \widetilde{W}(\phi^{**}, E[\theta]) = 0.$$

Now, if $\partial_\phi \widetilde{W}$ is concave with respect to θ, we have:

$$\partial_\phi \widetilde{W}(\phi^{**}, E[\theta]) = 0 = E\left[\partial_\phi \widetilde{W}(\phi^*, \theta)\right] < \partial_\phi \widetilde{W}(\phi^*, E[\theta]),$$

and thus (since $\partial_\phi \widetilde{W}$ is decreasing with respect to ϕ, by the assumed concavity of $\widetilde{W}$ in ϕ):

$$\phi^* < \phi^{**},$$

implying that the introduction of uncertainty should introduce a "statistical bias" towards lower licensing fees. Conversely, if $\partial_\phi \widetilde{W}$ is convex with respect to θ, we have:

$$\phi^* > \phi^{**}.$$

5.1 Cost uncertainty

The socially optimal number of firms, n^W, depends here on supply side characteristics through the level of the fixed cost, f. We now study the implications of uncertainty about this fixed cost (i.e., $\theta \equiv f$). Suppose for example that f is distributed over some interval $\left[\underline{f}, \overline{f}\right]$ according to a cdf $F(.)$, and that:

- the regulator must choose the licensing fee ϕ before knowing the particular realization of the fixed cost f;
- but firms choose whether to enter once the uncertainty is resolved.

For the purpose of simplifying the exposition, we suppose that the socially desirable number of downstream firms is always large enough to ensure that Hotelling competition prevails (that is, $\overline{f} < r^2/9t$). We therefore focus here on the case when an unregulated IP holder would issue too few licenses.

Ex post, given the licensing fee ϕ and the realized cost of entry f, the number of firms, $n(\phi + f)$ is determined by

$$\pi^*(n; f) = \pi^H(n; f) = \phi$$

and is thus equal to:

$$n(\phi, f) = \sqrt{\frac{t}{f + \phi}}.$$

The welfare is thus equal to

$$\begin{aligned} \widetilde{W}(\phi, f) &= r - \frac{t}{4n(\phi, f)} - n(\phi, f) f \\ &= r - \frac{\phi + 5f}{4}\sqrt{\frac{t}{f + \phi}}. \end{aligned}$$

Therefore,

$$\partial_\phi \widetilde{W}(\phi, f) = -\frac{\phi - 3f}{8(f + \phi)}\sqrt{\frac{t}{f + \phi}}.$$

This function is concave around the optimal value in the absence of uncertainty (that is, for f around to $\phi/3$), but not for larger deviations:

$$\begin{aligned} \partial_f \left(\partial_\phi \widetilde{W}(\phi, f)\right) &= \frac{3}{16}\frac{3\phi - f}{(f + \phi)^2}\sqrt{\frac{t}{f + \phi}}, \\ \partial^2_{f^2} \left(\partial_\phi \widetilde{W}(\phi, f)\right) &= \frac{3}{32}\frac{3f - 17\phi}{(f + \phi)^3}\sqrt{\frac{t}{f + \phi}}. \end{aligned}$$

Thus, $\partial_\phi \widetilde{W}$ is concave in f as long as f remains below $17\phi/3$, but becomes convex beyond this threshold. The introduction of limited uncertainty about the downstream fixed cost f should thus lead the regulator to insist on a lower licensing fee than what would be optimal based on the "expected value" of this fixed cost, but the possibility of "really bad shocks" on the fixed cost f might instead encourage the regulator to consider higher licensing fees.

5.2 Demand uncertainty

In this simple Hotelling framework, the reservation price r does not influence the optimal number of firms n^W, which depends on the demand side only through the differentiation parameter t. Let us therefore introduce some uncertainty on t (i.e., $\theta \equiv t$). Since the transportation cost t enters the welfare function in a linear form, at first glance uncertainty about this parameter should not affect the optimal number of firms:

$$n^* = \max_n E_t \left[W(n,t)\right] = \max_n E_t \left[r - \frac{t}{4n} - nf\right] = \max_n \left(r - \frac{E_t[t]}{4n} - nf\right) = n^{**}.$$

However, in practice the regulator controls the licensing fee, which only indirectly determines the number of firms; moreover, this control depends also on the degree of differentiation t. As a result, expressed as a function of the licensing fee ϕ, the welfare function is no longer linear in t; using the same computation as before,[22] we have:

$$\widetilde{W}(\phi, t) = r - \frac{\phi + 5f}{4}\sqrt{\frac{t}{f+\phi}},$$

and

$$\partial_\phi \widetilde{W}(\phi, t) = -\frac{(\phi - 3f)}{8(f+\phi)}\sqrt{\frac{t}{f+\phi}},$$

which is convex in t. Thus, because it controls the number of firms only indirectly, and is sensitive to the degree of differentiation when exerting this control, the introduction of uncertainty over this parameter would lead the regulator to consider higher licensing fees, compared with what would be optimal in the absence of uncertainty.

[22]We thus assume again that the Hotelling competition regime always prevails for the optimal number of firms, which is the case when the upper bound on t is lower than $r^2/9f$.

5.3 Regulating licensing fees or the number of licenses?

Rather than setting the level of the licensing fee, the regulator could control the number of licenses. Since

$$W(n,t,f) = r - \frac{t}{4n} - nf$$

is linear in t and f, we have:

$$E_{t,f}\left[W(n,t,f)\right] = W\left(n,t^e,f^e\right),$$

where $t^e \equiv E_t[t]$ and $f^e \equiv E_f[f]$ denote respectively the expected values of the transportation cost t and the fixed cost f.

A regulator that would control the number of licenses would choose

$$n^W\left(t^e,f^e\right) = \sqrt{\frac{t^e}{4f^e}},$$

thus generating an expected welfare equal to

$$\begin{aligned} E_{t,f}\left[W\left(n^W\left(t^e,f^e\right),t,f\right)\right] &= W\left(n^W\left(t^e,f^e\right),t^e,f^e\right) \\ &= \max_n W\left(n,t^e,f^e\right). \end{aligned}$$

When instead the regulator sets the licensing fee, it seeks to maximize

$$E_{t,f}\left[\widetilde{W}(\phi,t,f)\right],$$

where, assuming as before that Hotelling competition prevails:[23]

$$\widetilde{W}(\phi,t,f) = r - \frac{\phi+5f}{4}\sqrt{\frac{t}{f+\phi}}$$

is convex in each of t and f; indeed, we have:

$$\frac{\partial\widetilde{W}}{\partial t} = -\frac{\phi+5f}{8\sqrt{t(f+\phi)}} \text{ and thus } \frac{\partial^2\widetilde{W}}{\partial t^2} = \frac{\phi+5f}{16t\sqrt{t(f+\phi)}} > 0,$$

and

$$\begin{aligned} \frac{\partial\widetilde{W}}{\partial f} &= \frac{1}{4}\left(\frac{\phi+5f}{2(f+\phi)} - 5\right)\sqrt{\frac{t}{f+\phi}} \\ &= -\frac{5f+9\phi}{8(f+\phi)}\sqrt{\frac{t}{f+\phi}}, \end{aligned}$$

[23] This is the case when the upper bounds on the transportation parameter and on the fixed cost, $\bar{t}$ and $\bar{f}$, satisfy $\bar{t}\bar{f} < r^2/9$.

so that

$$\begin{aligned}
\frac{\partial^2 \widetilde{W}}{\partial f^2} &= -\left(\frac{10\,(f+\phi)-2\,(5f+9\phi)-(5f+9\phi)}{8\,(f+\phi)^2}\right)\sqrt{\frac{t}{f+\phi}} \\
&= \frac{5f+17\phi}{16\,(f+\phi)^2}\sqrt{\frac{t}{f+\phi}}>0.
\end{aligned}$$

Therefore, for $\theta = t$ or f and $\widetilde{\phi}\,(\theta^e) \equiv \arg\max_\phi \widetilde{W}\,(\phi, \theta^e)$:

$$\begin{aligned}
\max_\phi E_\theta\left[\widetilde{W}\,(\phi,\theta)\right] &\geq E_\theta\left[\widetilde{W}\left(\widetilde{\phi}\,(\theta^e)\,,\theta\right)\right] \\
&> \widetilde{W}\left(\widetilde{\phi}\,(\theta^e)\,,\theta^e\right) \\
&\equiv \max_\phi \widetilde{W}\,(\phi,\theta^e) \\
&= \max_n W\,(n;\theta^e) \\
&= \max_n E_\theta\left[W\,(n;\theta)\right].
\end{aligned}$$

It is therefore preferable to regulate the licensing fee ϕ rather than the number of licenses n.

This discussion can be summarized as follows:

Proposition 5 *Suppose that random shocks affect either the fixed cost f or the transportation parameter t, and the upper bounds on these parameters, $\bar{t}$ and $\bar{f}$, satisfy $9\bar{t}\bar{f} < r^2$, so that an unregulated IP holder would issue fewer licenses than is socially desirable. Then:*

- *It is preferable to regulate the licensing fee ϕ rather than the number of licenses n.*
- *Compared with the situation with no random shocks:*
  - *uncertainty over t, and possibly a large uncertainty on f, increases the optimal licensing fee;*
  - *limited uncertainty about the fixed cost f instead decreases the optimal licensing fee.*

This discussion is reminiscent of the debate on price versus quantity regulation, pioneered by Weitzman (1974).[24] More generally, the optimal mode of intervention

[24] A related discussion concerns input versus output control. See Caillaud *et al.* (1988) for an early survey of the literature on utility regulation.

depends critically on the information available (or the information that could be made available, and at what cost) as well as on the regulatory toolbox (e.g., which transfers are allowed, whether discrimination is possible, to what extent can the regulator commit itself, and so forth).[25] A detailed analysis of these issues is however beyond the scope of the present paper.

6 Complementary technologies

We now consider a situation where two upstream firms, U_1 and U_2, which each control an essential technology. These two technologies are perfect complements: combined together, they allow firms to compete in the downstream market, and each of them is necessary to be active in that market. We first assume that the IP holders are not themselves present in the downstream market and consider different types of commercial arrangements.

6.1 Pool

A first possibility for the two IP holders is to "merge", e.g. by assigning their IP rights to a pool that sells the technology for them and retrocedes the profits, say on a fifty-fifty basis. The situation is formally the same as the one studied before, since the pool manager will behave exactly as does the single IP holder described in the previous section.

For example, if the pool manager relies on a fixed licensing fee ϕ, it can still control the number of firms n and recover downstream profits by setting the fee to $\phi^* (n) = \pi^* (n)$. The pool manager then seeks to maximize each owner's profit, equal to

$$n\frac{\phi^* (n)}{2} = \frac{\Pi^* (n)}{2},$$

and will thus again maximize total profit by selling n^Π licenses for a fee $\phi = \phi^\Pi$. Likewise, in the case of revenue-based royalties the pool manager would issue the same number of licenses, n^Π, by setting the royalty rate to $\tau = \tau^\Pi$. The pool manager could also replicate the fully integrated outcome by charging instead a unit fee $\gamma = \bar{\gamma}$.

[25] Laffont and Tirole (1993) provide a broad overview on regulation theory and incentives.

6.2 Independent IP licensing with fixed license fees

We now suppose instead that the two IP holders each market their own rights independently from each other. To fix ideas, we will consider the following timing, with the same structure as the one previously studied:

- First, each IP owner, $i = 1, 2$, simultaneously and independently sets a license fee, ϕ_i.
- Second, potential entrants decide whether or not to buy the licenses; as before, those that enter locate themselves uniformly along the circle.
- Third, downstream competitors set prices.

As already noted, independent licensing creates double marginalization problems and leads to higher total fees. It may even trigger a "coordination breakdown" where both IP owners charge prohibitively high fees, thereby discouraging any downstream firm from entering the market: any pair of fees satisfying $\phi_i \geq \pi^m$, for $i = 1, 2$, constitutes an equilibrium. Such equilibria rely on weakly dominated strategies; we will therefore focus our discussion on equilibria in which each IP owner charges a fee below the monopoly profit π^m.

Given its rival's equilibrium fee $\phi^e < \pi^m$, U_i can induce the entry of n_i firms by setting its own fee to $\phi_i^*(n_i)$, such that

$$\pi^*(n_i) = \phi_i + \phi^e. \tag{3}$$

That is, $\phi_i^*(n_i) = \pi^*(n_i) - \phi^e$. Each U_i will thus will want to choose n_i (or ϕ_i) so as to maximize:

$$\Pi_i = n_i \phi_i^*(n_i) = n_i (\pi^*(n_i) - \phi^e) = \Pi^*(n_i) - n_i \phi^e.$$

We show in the Appendix that the unique equilibrium (excluding weakly dominated strategies) is symmetric ($\phi_1 = \phi_2 = \phi^D$, where the superscript D stands for "Double marginalization") and leads to a number of firms equal to:

$$n^D \equiv \frac{r}{2f}\left(\sqrt{1 + 6\frac{tf}{r^2}} - 1\right),$$

which is such that $\underline{n} \leq n^D < n^\Pi = \min\left\{n^M, \overline{n}\right\}$. We thus have:

Proposition 6 *When the IP holders rely on fixed access fees, independent licensing leads to higher fees and fewer downstream firms than joint licensing.*

Proof. See Appendix. B ■

Because of double marginalization, independent licensing thus reduces the number of licenses that are eventually issued. This may enhance social welfare here, since joint licensing can lead to excessively many firms. Yet, independent licensing can also result in too few licenses. We show in the Appendix that, indeed, $n^D < n^W$ when:

$$\frac{r^2}{tf} > \frac{25}{4}.$$

This therefore only happens when variety is cheap (f small) and/or not very interesting (t small), compared with the intrinsic value of the good (as measured by r). More precisely:

- when $r^2/tf > 9$, joint licensing would already generate too few licenses $\left(n^\Pi = \overline{n} < n^W\right)$; independent licensing then reduces welfare, since double marginalization further reduces the number of licenses below the optimal level $\left(n^D < \overline{n} < n^W\right)$;
- in contrast, when $r^2/tf < 25/4$ (but $r^2/tf > 2$, to ensure the viability of the market), even independent licensing generates too many licenses; the associated double marginalization then brings the number of licensees closer to what is socially desirable and improves welfare $\left(n^W < n^D < n^\Pi\right)$;
- in the intermediate range where $9 > r^2/tf > 25/4$, double marginalization still reduces the number of licensees, but joint licensing would lead to too many licenses; independent licensing may thus improve welfare.[26]

Remark: cross-licensing. The IP holders could instead opt for cross-licensing agreements allowing them to issue "complete" licenses covering both technologies subject to paying the other IP holder a fee per license issued. We show in Appendix C that a *reciprocal* cross-licensing agreement allowing *both of them* to issue complete licenses by paying the other a fee equal to $\psi = \phi^\Pi/2$, leads them to issue n^M complete licenses at a fee $\Phi = 2\psi = \phi^\Pi$, thereby replicating the integrated monopoly outcome and sharing equally the associated profit. Indeed, when the reciprocal fee ψ is low enough, Bertrand competition between the two upstream firms leads them to set their fees to

$$\Phi_1 = \Phi_2 = \Phi \equiv 2\psi,$$

[26]By continuity, there is a threshold $\hat{\rho}$ for $\rho = r^2/tf$, such that $\hat{\rho} \in (25/4, 9)$, such that, compared with joint licensing, independent licensing and the associated double marginalization reduces welfare if $\rho > \hat{\rho}$, but instead enhances welfare if $\rho < \hat{\rho}$.

since each U_i is then indifferent between issuing a license and earning $\Phi - \psi = \psi$, or letting the other IP holder issue the license and learning ψ again. As a result, by adjusting the upstream cross-licensing fee ψ to $\phi^\Pi/2$, they can drive the downstream licensing fee $\Phi = 2\psi$ to ϕ^Π. The situation is then formally the same as when they merge or form a pool to market their IPs.

If instead each U_i independently sets its upstream fee ψ_i, then cross-licensing may again mitigate double marginalization problems and result in more downstream firms than n^D, but do not eliminate them entirely and still results in fewer firms than n^Π (see Appendix C).

Remark: Consumer surplus

An upstream monopoly IP owner will always choose n^Π, while a duopoly results in $n^D < n^\Pi$ firms. Double marginalization thus reduces variety, but it also results in lower downstream prices and greater consumer surplus. Indeed, whenever the market is viable (i.e., $\frac{r^2}{tf} > 2$), we have: $\underline{n} \leq n^D < n^\Pi \leq \overline{n}$ and, in this range, the consumer price is equal to $\hat{p}(n) = r - \frac{t}{2n}$ and increases with n, while consumer surplus is given by

$$CS\left(n\right) = r - (r - \frac{t}{2n}) - 2n \int_0^{\frac{1}{2n}} txdx = \frac{t}{4n},$$

and thus decreases with n. Therefore:

- Compared with the case of an upstream monopoly or an IP pool, an upstream IP duopoly results in fewer downstream firms and *lower* downstream prices facing consumers; the benefits of the lower prices more than offsets the effects of reduced variety, so consumer surplus is higher with an IP duopoly.

- This duopoly outcome may even be better than "free-entry" (i.e., the number of downstream firms obtained with free licenses), unless this free-entry equilibrium results in significantly more than $\overline{n}$ firms.[27]

[27] Consumer surplus decreases with n in the range $[\underline{n}, \overline{n}]$ and then increases with n for $n > \overline{n}$. Let denote by n^f the number of firms entering the downstream market when licenses are free (i.e., such that $\pi^*\left(n^f\right) = 0$) and by $\hat{n} > \overline{n}$ the number of firms that yields as much surplus as n^D. Then, as long as $n^f \leq \hat{n}$ (that is, when f is "large enough"), the outcome of IP duopoly and double marginalization is better for consumers than the free entry equilibrium – in that case, the number of firms that maximize consumer surplus, subject to non-negative profit constraint, is $\underline{n}$; when $n^f > \hat{n}$, however, consumers would prefer to have "as many firms" as possible and free-entry would work beter for them.

6.3 Alternative licensing arrangements

6.3.1 Royalties

Suppose now that the IP holders ask for *royalty percentages* τ_1 and τ_2 on downstream revenues, so that the total royalty rate is $\tau = \tau_1 + \tau_2$.

As long as the number of downstream firms exceeds $\overline{n}$, a downstream firm earns a profit equal to

$$(1-\tau)\frac{t}{n^2} - f,$$

so that the number of downstream firms is equal to

$$n(\tau) = \sqrt{(1-\tau)\frac{t}{f}},$$

which decreases as τ increases. Each U_i gets

$$\Pi_i = \tau_i \frac{t}{n(\tau)} = \tau_i \sqrt{\frac{tf}{1-\tau_1-\tau_2}},$$

which clearly increases with τ_i.

Therefore, each IP holder will seek to induce at most $\overline{n}$ downstream firms to enter. The downstream equilibrium price and individual profit will thus be equal to

$$\begin{aligned} p &= \hat{p}(n) = r - \frac{t}{2n}, \\ \pi &= (1-\tau)\frac{1}{n}\left(r - \frac{t}{2n}\right) - f = (1-\tau)\hat{\pi}(n) - \tau f. \end{aligned}$$

The number of downstream firms, $n(\tau)$, is thus such that

$$\hat{\pi}(n) = \frac{\tau f}{1-\tau}. \tag{4}$$

This number decreases as τ increases (i.e., $n'(\tau) < 0$), since $\hat{\pi}'(n) < 0$ while the right-hand side increases with τ. Each U_i gets (with $n = n(\tau)$):

$$\Pi_i = \tau_i \hat{p}(n) = \tau_i n [\hat{\pi}(n) + f]. \tag{5}$$

Using

$$0 = (1-\tau)\frac{1}{n}\left(r - \frac{t}{2n}\right) - f = (1-\tau_j)[\hat{\pi}(n) + f] - f - \tau_i [\hat{\pi}(n) + f],$$

U_i's profit can be rewritten as

$$\Pi_i = (1-\tau_j)\, n [\hat{\pi}(n) + f] - f = (1-\tau_j)\left[\hat{\Pi}(n) - n\frac{\tau_j f}{1-\tau_j}\right].$$

Therefore, there is again some double marginalization (reflected in the last term of the right-hand side in the above equation) which lead the IP owners to limit the of licenses. It it shown in Appendix B that this double marginalization is less severe here than with flat rate license fees; letting n^R denote the number of licenses generated by independent licensing and percentage royalties, we have:

Proposition 7 *When relying on percentage royalties, independent licensing creates again double marginalization problems, which are however less severe than in the case fixed access fees: we have*

$$n^D < n^R \leq n^\Pi,$$

with strict inequalities whenever $n^\Pi < n^M$.

Proof. See Appendix B. ■

6.3.2 Unit fees

Suppose now that the IP holders charge instead *unit fees* (i.e., volume-based royalties) γ_1 and γ_2, so that the total unit fee is $\gamma = \gamma_1 + \gamma_2$. As long as $\gamma < \overline{\gamma}$, the entire market market is served; therefore, each IP holder i gets

$$\Pi_i = \gamma_i,$$

which clearly increases with γ_i. In contrast, when $\gamma > \overline{\gamma}$, no entry occurs and thus $\Pi_1 = \Pi_2 = 0$; last, when $\gamma = \overline{\gamma}$, there are enough firms willing to enter to serve the entire market, and the total profits are

$$\Pi_1 + \Pi_2 = \Pi^M.$$

As a result, the equilibrium is such that $\gamma = \overline{\gamma}$, and the two IP holders share the integrated monopoly profit.[28] In order words, double marginalization does not preclude here the IP holders from maximizing their joint profits, and they issue as many licenses as is privately optimal $\left(n = n^M\right)$.

We thus have:

Proposition 8 *When relying on unit-fees, the IP holders can replicate the fully integrated outcome, whether they license their technologies jointly or independently.*

[28]There is actually an infinity of equilibria, which only differ in the way the profit Π^M is shared among the two IP holders: any couple of fees γ_1 and γ_2 adding-up to $\bar{\gamma}$ constitutes an equilibrium.

6.4 Vertical integration

Suppose now that the two IP holders are each vertically integrated. That is, they each have a downstream subsidiary, and the two subsidiaries compete with the other, unintegrated firms that are present in the downstream market.

6.4.1 Fixed licensing fees

If the IP holders sell their technologies for a fixed fee ϕ, then clearly vertical integration has again no impact on their behavior and thus on the final outcome. To see this, first note that each downstream affiliate behaves in the same way as unintegrated downstream competitors. Given the license fee ϕ set for example by the pool manager in the case of joint licensing, or the total fee $\phi = \phi_1 + \phi_2$ set by the two IP holders in the case of independent licensing, the total number n of downstream competitors thus remains characterized by $\pi^* (n) = \phi$ (with the caveat that, by assumption, at least two firms enter the market).

Therefore, if the IP holders jointly license their technology through a pool, which redistributes half of its profit to each of the two upstream firms, the pool manager will set ϕ, or equivalently pick the total number of firms n by setting $\phi = \pi^* (n)$, so as to maximize

$$\pi^* (n) + \frac{(n-2)\,\phi}{2} = \pi^* (n) + \frac{(n-2)\,\pi^* (n)}{2} = \frac{n\pi^* (n)}{2} = \frac{\Pi^* (n)}{2}.$$

The pool manager will thus again maximize total profits and chooses $n = n^{\Pi}$.

If instead the IP holders license their technology independently, given the rival's equilibrium license fee ϕ^e, each U_i can still "choose" a total number of firms n_i by charging a fee $\phi_i^* (n_i) = \pi^* (n_i) - \phi^e$. U_i thus maximizes:

$$\begin{aligned} \pi^* (n_i) - \phi^e + (n_i - 1)\,\phi_i^* (n_i) &= \pi^* (n_i) - \phi^e + (n_i - 1)\,(\pi^* (n_i) - \phi^e) \\ &= \Pi^* (n_i) - n_i \phi^e, \end{aligned}$$

as before. The licensing behavior of an IP holder is thus the same, whether it is integrated or not. As a result, the equilibrium outcome is the same, whether the IP holders are vertically integrated or not (the same observation carries over to the case where only one IP holder is vertically integrated).

6.4.2 Unit fees

Suppose now that the IP holders charge per-unit access fees. Vertical integration still has no impact on downstream competition. For example, in the case of independent licensing U_i will set its downstream market price p_i so as to maximize (with $j \neq i = 1, 2$, and as long as the entire market is served):

$$\left(p_i - \gamma_j\right) D\left(p_i, \widetilde{p}; n\right) + \gamma_i \left[1 - D\left(p_i, \widetilde{p}; n\right)\right],$$

which is the same as maximizing (letting $\gamma = \gamma_1 + \gamma_2$ denote the total unit fee):

$$\left(p_i - \gamma\right) D\left(p_i, \widetilde{p}; n\right),$$

as if the downstream subsidiary were an independent firm. Therefore, all firms, vertically integrated or not, behave in the same way in the downstream market. Similarly, in the case of joint licensing, and assuming for example that each subsidiary formally pays the same fee γ as the independent firms, U_i will set its downstream price so as to maximize:

$$\left(p_i - \gamma\right) D\left(p_i, \widetilde{p}; n\right) + \frac{\gamma}{2},$$

which again amounts to maximize $\left(p_i - \gamma\right) D\left(p_i, \widetilde{p}; n\right)$. Therefore, in both cases, vertical integration has no impact on downstream competition.

If the IP holders license their technologies jointly through a pool, the pool manager will set the fee γ so as to maximize each IP holder's total profit, equal to (using the free-entry condition $\left(p^* - \gamma\right)/n = f$):

$$\frac{p^* - \gamma}{n} + \frac{\gamma}{2} = f + \frac{\gamma}{2},$$

and will thus choose the maximal acceptable value for γ ($\gamma = \bar{\gamma}$).

If instead the IP holders license their technologies independently, each integrated IP holder will maximize

$$\frac{p^* - \gamma_j}{n} + \gamma_i \left(1 - \frac{1}{n}\right) = \frac{p^* - \gamma}{n} + \gamma_i = f + \gamma_i,$$

and will thus seek to increase its own fee, γ_i, as much as possible, as if it were not integrated. Therefore, vertical integration has no impact on the equilibrium, which remains such that $\gamma = \overline{\gamma}$ and the two IP holders share the fully integrated monopoly profit.

We thus have:

Proposition 9 *Vertical integration by one or both IP holders has again no impact on the equilibrium outcome when the licensing terms stipulate fixed or per unit access fees.*

7 Conclusion

Patent thickets have long been a concern due to the potential for delaying deployment of products and adversely affecting consumers. To examine the implications of such patent thickets, we consider a model in which the upstream IP owner or owners sell into a market in which there are differentiated products and positive fixed entry costs. It is well known that, in the absence of vertical licensing agreements, there can be excessive entry, due to business stealing effects, or insufficient entry, if firms entering the market appropriate only part of the surplus they generate. We revisit this issue, taking into account the upstream owner(s)' licensing policy.

When there is a single owner of essential IP, that owner can have an incentive to sell more licenses than is socially optimal. This does not occur when the downstream licensees offer quite homogeneous products, but can occur when products are significantly differentiated, in which case additional licensees can extract a substantial share of the surplus that consumers derive from enhanced variety. When the IP owner cannot control its licensees' pricing policies, however, the fear of profit dissipation through downstream competition tends to reduce the risk of excessive entry. When there are two or more upstream IP owners, royalty stacking also tends to reduce the number of downstream licensees. But when a single IP owner (or multiple IP owners jointly licensing their technologies) would issue too many licenses, the reduction in the number of downstream competitors and product variety can result in lower prices, and higher consumer surplus and social welfare. We also find that the IP owner(s) may sell fewer licenses than would be offered by a fully integrated monopolist when license fees assume the form of a fixed access fee or a revenue-based royalty percentage, but may replicate the fully integrated outcome by charging per-unit license fees. Last, when IP owners charge fixed or unit-based access fees, vertical integration does not alter the behavior of affiliated downstream subsidiaries, and as a result vertical integration has no effect on the equilibrium outcome.

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Appendix

A Proof of Proposition 3

We first study the impact of a unit fee γ on the downstream equilibrium. Charging a unit fee γ pushes the Hotelling price by the same amount:

$$p^H(n;\gamma) \equiv \gamma + \frac{t}{n};$$

as a result, downstream profits (net here of payments to the IP owner) are not affected by this fee:

$$\pi^H(n;\gamma) \equiv \pi^H(n) = \frac{t}{n^2} - f,$$

In contrast, when downstream firms act as local monopolists, they pass only part of the fee γ on to consumers; their prices and profits are then equal to:

$$\begin{aligned} p^m(\gamma) &\equiv \arg\max_p (p-\gamma) D^m(p) = \frac{r+\gamma}{2}, \\ \pi^m(\gamma) &\equiv \frac{(r-\gamma)^2}{2t} - f. \end{aligned}$$

The unit fee also affects the conditions under which the various competition regimes prevail. The Hotelling competitive regime now prevails when

$$p^H(n;\gamma) + \frac{t}{2n} = \gamma + \frac{3t}{2n} < r,$$

that is,

$$n > \overline{n}(\gamma) \equiv \frac{3}{2}\frac{t}{r-\gamma}, \tag{6}$$

whereas the local monopoly regime prevails when

$$p^m(\gamma) + \frac{t}{2n} = \frac{r+\gamma}{2} + \frac{t}{2n} \geq r,$$

or

$$n \leq \underline{n}(\gamma) \equiv \frac{t}{r-\gamma}. \tag{7}$$

In the intermediate range $[\underline{n}(\gamma), \overline{n}(\gamma)]$, the entire market is served at a price as before equal to $\hat{p}(n) = r - t/2n$, so that each downstream firm earns

$$\hat{\pi}(n;\gamma) \equiv \frac{1}{n}\left(r - \gamma - \frac{t}{2n}\right) - f,$$

which decreases in n in that range;[29] the downstream equilibrium is thus now such that (note again that profits are expressed here net of access fees):

- for $n < \underline{n}(\gamma) \equiv \dfrac{t}{r-\gamma}$:

$$p^*(n;\gamma) = p^m(\gamma) = \frac{r-\gamma}{2} \quad \text{and} \quad \pi^*(n;\gamma) = \pi^m(\gamma) = \frac{(r-\gamma)^2}{2t} - f.$$

- for $\underline{n}(\gamma) < n < \overline{n}(\gamma) \equiv \dfrac{3}{2}\dfrac{t}{r-\gamma}$:

$$p^*(n;\gamma) = \hat{p}(n) = r - \frac{t}{2n} \quad \text{and} \quad \pi^*(n;\gamma) = \hat{\pi}(n) = \frac{1}{n}\left(r - \gamma - \frac{t}{2n}\right) - f,$$

- for $n > \overline{n}(\gamma)$:

$$p^*(n;\gamma) = p^H(n;\gamma) = \gamma + \frac{t}{n} \quad \text{and} \quad \pi^*(n;\gamma) = \pi^H(n) = \frac{t}{n^2} - f,$$

We now show that the IP owner can replicate the fully integrated monopoly outcome by setting the maximal fee $\gamma = \bar{\gamma} = r - \sqrt{2tf}$. Indeed, charging $\gamma = \bar{\gamma}$ leads to $p^m(\gamma) = p^M = \hat{p}(n^M)$ and $\pi^m(\bar{\gamma}) = 0$, which ensures that $n = n^M$ and $p = p^M$ constitutes effectively a continuation equilibrium. Thus, setting the unit fee to $\gamma = \bar{\gamma}$ allows the IP holder to replicate the fully integrated industry outcome and earn the monopoly profit Π^M.

B Proof of Propositions 6 and 7

Consider first the case of flat rate access fees. Given the two IP owners' fees ϕ_1 and ϕ_2, the number of downstream firms entering the market is given by $n^*(\phi_1 + \phi_2)$, where

$$n^*(\phi) \equiv \begin{cases} (\pi^*)^{-1}(\phi) & \text{when} \quad \phi < \pi^m, \\ \text{any } n \leq \underline{n} & \text{when} \quad \phi = \pi^m, \\ 0 & \text{when} \quad \phi > \pi^m. \end{cases}$$

Each U_i then obtains a profit equal to

$$\Pi_i = n^*(\phi_1 + \phi_2)\,\phi_i.$$

[29]

$$\hat{\pi}'(n;\gamma) = -\frac{(r-\gamma)}{n^2} + \frac{t}{n^3} = \frac{r-\gamma}{n^3}\left(\underline{n}(\gamma) - n\right) < 0$$

as long as $n > \underline{n}(\gamma)$.

As already noted, independent licensing may trigger "coordination breakdown" where both IP owners charge fees higher than the monopoly profit π^m and no downstream firm enters the market, but these equilibria involve weakly dominated strategies. We now focus on equilibria which do not rely on such strategies, in which both upstream firms charge a fee lower than π^m.

Fix the rival's fee $\phi_j < \pi^m$ and suppose first that U_i chooses to induce a number n_i of downstream firms that is higher than $\overline{n}$, by setting a fee ϕ_i such that $\phi_i + \phi^e = \pi^* (n_i) = \pi^H (n_i)$; U_i would then rather increase ϕ_i in order to reduce n_i to $\overline{n}$: indeed, its profit is then given by

$$\Pi_i = n_i \phi_i = n_i \left(\pi^H (n_i) - \phi^e\right) = \Pi^H (n_i) - n_i \phi^e,$$

which decreases in n_i (since the total Hotelling-type profit $\Pi^H (n)$ decreases as n increases). Therefore, the upstream firms will never choose to have more than $\overline{n}$ downstream firms. Similarly, setting $\phi_i = \pi^m - \phi_j$ induces any $n \leq \underline{n}$ firms to enter and gives U_i a profit

$$\Pi_i = n_i (\pi^m - \phi_j),$$

which is positive and proportional to the number of firms; hence U_i will never choose to induce less than $\underline{n}$ downstream firms.[30]

Thus, without loss of generality, we can assume that U_i sets a fee ϕ_i such that $\phi_i + \phi_j \in [\overline{\pi}, \pi^m]$, where

$$\overline{\pi} = \pi^* (\overline{n}) = \frac{4r^2}{9t} - f,$$

so as to induce a number of firms $n_i \in [\underline{n}, \overline{n}]$, given by $\phi_i + \phi_j = \pi^* (n_i) = \hat{\pi} (n_i)$, that maximizes

$$\Pi_i = n_i \phi_i = n_i (\hat{\pi} (n_i) - \phi_j) = \hat{\Pi} (n_i) - n_i \phi_j = r - \frac{t}{2n_i} - n_i (f + \phi_j).$$

Ignoring the constraint $n_i \in [\underline{n}, \overline{n}]$ would lead U_i to choose

$$n_i = n^M (f + \phi_j) = \sqrt{\frac{t}{2 (f + \phi_j)}}, \qquad (8)$$

which is always larger than $\underline{n}$ and is smaller than $\overline{n}$ as long as

$$\frac{2r^2}{9t} - f \equiv \hat{\phi} \leq \phi^j \leq \frac{r^2}{2t} - f.$$

[30]Setting $\phi_i = \pi^m - \phi_j$ triggers any number $n_i \leq \overline{n}$; however, U_i can indeed "pick" $n = \overline{n}$ by charging a fee slightly below $\pi^m - \phi_j$.

Therefore, U_i's best response to ϕ_j is to induce a number of firms $n_i = n^R(\phi_j)$ characterized by

$$n^R(\phi) \equiv n^\Pi(f+\phi) = \begin{cases} \overline{n} = \dfrac{3}{2}\dfrac{t}{r} \leq n^M(f+\phi_j) & \text{when} \quad \phi \leq \hat{\phi}, \\ n^M(f+\phi_j) < \overline{n} & \text{when} \quad \hat{\phi} \leq \frac{r^2}{2t} - f \\ \underline{n} & \text{when} \quad \phi > \frac{r^2}{2t} - f \\ . \end{cases}$$

The corresponding fee is then $\phi_i = \phi^R(\phi_j)$, where

$$\phi^R(\phi) = \begin{cases} \overline{\pi} - \phi & \text{when} \quad \phi \leq \hat{\phi}, \\ \hat{\pi}\left(n^M(f+\phi)\right) - \phi & \text{when} \quad \frac{r^2}{2t} - f \geq \phi > \hat{\phi} \\ \pi^m - \phi & \text{when} \quad \phi \geq \frac{r^2}{2t} - f, \end{cases}$$

where

$$\hat{\pi}\left(n^M(f+\phi)\right) - \phi = \frac{r}{\sqrt{\frac{t}{2(f+\phi)}}} - \frac{t}{2\frac{t}{2(f+\phi)}} - (f+\phi) = r\sqrt{\frac{2(f+\phi)}{t}} - 2(f+\phi). \quad (9)$$

We now check that the best responses $\phi_i = \phi^R(\phi_j)$, for $i \neq j = 1, 2$, cross once in the range $\phi > \hat{\phi}$; the corresponding equilibrium then satisfies $n^R(\phi_1) = n^R(\phi_2)$ and is thus symmetric: $\phi_1 = \phi_2 = \phi^D$ and $n_1 = n_2 = n^D$, characterized by:

$$n^D = n^M\left(f + \phi^D\right) = \sqrt{\frac{t}{2\left(f+\phi^D\right)}} \text{ and } 2\phi^D = \hat{\pi}\left(n^D\right) = \frac{1}{n^D}\left(r - \frac{t}{2n^D}\right) - f.$$

These two conditions imply:

$$2\phi n^2 = t - 2fn^2 = rn - \frac{t}{2} - fn^2,$$

and thus:

$$fn^2 + rn - \frac{3t}{2} = 0, \quad (10)$$

which has a unique non-negative solution:

$$n^D \equiv \frac{r}{2f}\left(\sqrt{1 + 6\frac{tf}{r^2}} - 1\right).$$

It can be checked that $n^D \in (\underline{n}, \overline{n})$ (that is, $\hat{\phi} < \phi^D < \pi^m$) and thus constitutes indeed an equilibrium number of downstream firms:

$$\begin{aligned} n^D &> \underline{n} \iff \frac{r}{2f}\left(\sqrt{1+6\frac{tf}{r^2}}-1\right) > \frac{t}{r} \\ &\iff \sqrt{1+6\frac{tf}{r^2}} > 1+2\frac{tf}{r^2} \\ &\iff 1+6\frac{tf}{r^2} > 1+4\frac{tf}{r^2}+4\left(\frac{tf}{r^2}\right)^2 \\ &\iff 2\frac{tf}{r^2}\left(1-2\frac{tf}{r^2}\right) > 0, \end{aligned}$$

which boils down to $r^2 > 2tf$ and is thus satisfied whenever the industry is viable (assumption 1); and similarly

$$\begin{aligned} n^D &< \overline{n} \iff \frac{r}{2f}\left(\sqrt{1+6\frac{tf}{r^2}}-1\right) < \frac{3t}{2r} \\ &\iff \sqrt{1+6\frac{tf}{r^2}} > 1+3\frac{tf}{r^2}, \\ &\iff 1+6\frac{tf}{r^2} < 1+6\frac{tf}{r^2}+9\left(\frac{tf}{r^2}\right)^2, \end{aligned}$$

which is always satisfied.

We now show that this equilibrium is unique in the range $\phi < \pi^m$. Note first that, in this range, the best response function is uniquely defined and continuous. In addition, for $\phi > \hat{\phi}$ the slope of this reaction function is given by (using (9)

$$\frac{d\phi^R}{d\phi}(\phi) = \frac{r}{t\sqrt{\frac{2(f+\phi)}{t}}} - 2 = \frac{n^M(f+\phi)}{t/r} - 2,$$

where $n^M(f+\phi)$ decreases from $\frac{3t}{2r}$ to $\frac{t}{r}$ as ϕ increases from $\hat{\phi}$ to π^m; therefore, the slope of the best response is first constant and equal to -1 for $\phi < \hat{\phi}$ and then lies between $-1/2$ and -1 for $\hat{\phi} < \phi < \pi^m$. It follows that $\phi_1 = \phi_2 = \phi^D \in \left(\hat{\phi}, \pi^m\right)$ constitutes the only point where the two best responses intersect in the range $\phi_i < \pi^m$ (see Figure 1).

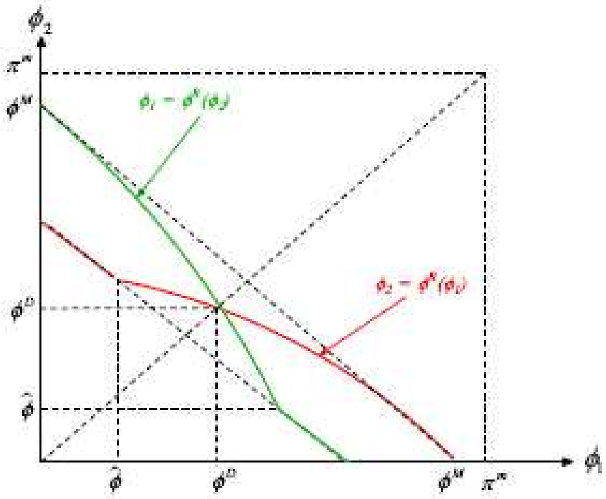

Figure 1: Best response fees for complementary technologies

Last, it is straightforward to confirm that double marginalization leads to fewer licenses being issued. This is obvious in the case of coordination breakdown, where no license is issued. But even if the upstream firms coordinate on the equilibrium $\phi_1 = \phi_2 = \phi^D$; indeed, $\phi^D > 0$ implies

$$n^D = n^M \left(f + \phi^D\right) < n^M = n^M \left(f\right),$$

which together with $n^D < \overline{n}$, leads to

$$n^D < \min\left\{n^M, \overline{n}\right\} = n^\Pi.$$

In addition, double marginalization can excessively reduce the number of licenses:

$$n^D < n^W \iff \frac{r}{2f}\left(\sqrt{1 + 6\frac{tf}{r^2}} - 1\right) < \sqrt{\frac{t}{4f}},$$

which boils down to

$$r^2 > \frac{25}{4}tf.$$

We now turn to the case of percentage royalties. We have seen that each U_i seeks to maximize

$$\Pi_i = (1 - \tau_j)\left[\hat{\Pi}\left(n\right) - n\frac{\tau_j f}{1 - \tau_j}\right].$$

In the absence of any restriction on n, it would therefore seek to induce a number of firms, n_i, such that:

$$\hat{\Pi}'\left(n_i\right) = \frac{\tau_j f}{1 - \tau_j}.$$

In equilibrium, the number of firms is therefore either $\overline{n}$ (if the number just defined, n_i, exceeds $\overline{n}$ for both IP holders) or both firms charge the same rate $\tau_1 = \tau_2 = \hat{\tau}^R$ and induce a number of firms $\hat{n}^R < \overline{n}$, which satisfy

$$\hat{\Pi}'\left(\hat{n}^R\right) = \frac{\hat{\tau}^R f}{1 - \hat{\tau}^R}. \tag{11}$$

Since $\hat{\Pi}\left(n\right)$ coincides with the industry profit and is quasi-concave, the number $\hat{n}^R$ is always strictly lower than π^M, characterized by $\hat{\Pi}\left(n\right) = 0$, but may exceed $\overline{n}$ (this may happen when n^M largely exceeds $\overline{n}$). The equilibrium number of firms therefore satisfies

$$n^R = \min\left\{\hat{n}^R, \overline{n}\right\} \leq n^\Pi = \min\left\{n^M, \overline{n}\right\},$$

with a strict inequality when and only when $n^\Pi = n^M$ (that is, when $n^M \leq \overline{n}$).

When $\hat{n}^R > \overline{n}$, then $n^R = \overline{n} > n^{\Pi}$. When instead $n^R \leq \overline{n}$, then $n^R = \hat{n}^R$. Combining (11) with condition (4) for $\tau = \tau_1 + \tau_2 = 2\tau^R$, i.e.

$$\hat{\pi}\left(n^R\right) = \frac{2\tau^R f}{1 - 2\tau^R},$$

then implies that n^R satisfies

$$\hat{\Pi}'(n) = \frac{1 - 2\tau^R}{2 - 2^R}\hat{\pi}(n). \tag{12}$$

Similarly, in the case of fixed access fees each U_i maximizes

$$\hat{\Pi}(n) - n\phi_j,$$

which leads to n^D and $\phi_1 = \phi_2 = \phi^D$, characterized by $\hat{\Pi}'\left(n^D\right) = \phi^D$.and $\hat{\pi}\left(n^D\right) = 2\phi^D$. Therefore, n^D satisfies

$$\hat{\Pi}'(n) = \frac{\hat{\pi}(n)}{2}. \tag{13}$$

The left-hand side is the same in (12) and (13) and, in both conditions, both sides decrease with n; moreover,

$$\begin{aligned}
\frac{d}{dn}\left[\hat{\Pi}'(n) - \frac{\hat{\pi}(n)}{2}\right] &= \hat{\Pi}''(n) - \frac{\hat{\pi}(n)}{2} \\
&= -\frac{t}{n^3} - \frac{1}{2}\left(-\frac{r}{n^2} + \frac{t}{n^3}\right) \\
&= -\frac{3t}{2n^3} + \frac{r}{2n^2} \\
&= \frac{r}{2n^2}\left(1 - \frac{3t}{rn}\right),
\end{aligned}$$

which, using (10), is negative for $n = n^D$; the left-hand side thus crosses the right-hand side "from above" in (13):

$$1 - \frac{3t}{rn} = 1 - \frac{3t}{2rn} - \frac{3t}{2rn} = -\frac{fn}{r} - \frac{3t}{2rn} < 0.$$

Finally, for any $\tau^R > 0$, we have

$$\frac{1 - 2\tau^R}{2 - 2^R} < \frac{1}{2},$$

implying that the right-hand side is smaller in (12) than (13); together with the above observations, this implies $n^D < n^R$.

C Cross licensing

We analyze here the situation where the upstream firms allow each other to license their own technology. We will denote by ψ_i the (upstream) fee that U_i charges to U_j for each license it issues, and by Φ_j the (downstream) fee charged by U_j for a "complete" license covering both technologies. The timing is as follows:

- first, the IP owners set the upstream fees ψ_1 and ψ_2 (more on this below);
- second, the IP owners set their downstream fees Φ_1 and Φ_2; the downstream firms then decide whether to buy a license and enter the market.

We will first characterize the continuation equilibria of the second stage, for given upstream fees ψ_1 and ψ_2. We will then consider two scenarios for the first stage: in the first scenario, the IP owners jointly agree on a reciprocal fee $\psi_1 = \psi_2 = \psi$; in the second scenario, the two IP owners sets their fees simultaneously and independently.

C.1 Downstream IP competition

We take here the upstream fees ψ_1 and ψ_2 and consider the second stage, where the two IP owners charge fees Φ_1 and Φ_2 for "complete" licenses; any downstream entrant then buys a license from the cheapest licensor and, given $\Phi = \min\{\Phi_1, \Phi_2\}$, the number of entrants is equal to $n^*(\Phi)$.

Note first that each U_i is unwilling to sell a complete license for a fee Φ_i lower than U_j's upstream fee ψ_j. Therefore, if $\min\{\psi_1, \psi_2\} > \pi^m$, then no license is issued and both IP owners get zero profit. If $\min\{\psi_1, \psi_2\} = \pi^m$, there are multiple continuation equilibria, in which the upstream firms set downstream fees exceeding π^m or serve up to $\underline{n}$ licences at a fee $\Phi = \pi^m$, thereby sharing up to $\underline{n}\pi^m$. If $\psi_i \geq \pi^m > \psi_j$ then, anticipating that U_j is unwilling to issue any license, U_i will set Φ_i so as to maximize

$$n^*(\Phi_i)(\Phi_i - \psi_j),$$

which using $\phi_i \equiv \Phi_i - \psi_j$ as the decision variable, amounts to maximize

$$n^*(\phi_i + \psi_j)\,\phi_i$$

and thus leads U_i to choose

$$\phi_i = \phi^R(\psi_j)$$

or, equivalently:$\Phi_i = \Phi^R(\psi_j)$, where

$$\Phi^R(\phi) \equiv \phi^R(\phi) - \phi,$$

which results in a number of downstream firms equal to $n^R(\psi_j)$. The two firms then obtain

$$\begin{aligned}\Pi_i &= n^R(\psi_j)\left(\Phi^R(\psi_j)-\psi_j\right) = n^R(\psi_j)\,\phi^R(\psi_j),\\ \Pi_j &= n^R(\psi_j)\,\psi_j.\end{aligned}$$

It is straightforward to check that U_j has indeed no incentive to undercut U_i, since this would require selling at a loss.

We now consider the case where both IP owners set fees lower than π^m.

Consider first a candidate equilibrium where $\Phi_1 = \Phi_2 = \Phi$. Each U_i can then obtain $n(\Phi)\,\psi_i$ by increasing its fee (and letting the other IP owner sell its license to all downstream entrants) and can also obtain $n(\Phi)(\Phi-\psi_j)$ by slightly undercutting its rival. Therefore, it must be the case that $\Phi = \psi_1+\psi_2$. Conversely, $\Phi_1 = \Phi_2 = \psi_1+\psi_2$ constitutes an equilibrium as long as no U_i benefits from undercutting its rival; this is the case when

$$\Phi_i < \psi_1+\psi_2 \implies n^*(\Phi_i)(\Phi_i-\psi_j) < n^*(\psi_1+\psi_2)\,\psi_i,$$

that is, using $\phi_i \equiv \Phi_i - \psi_j$, when

$$\phi_i < \psi_i \implies n^*(\phi_i+\psi_j)\,\phi_i < n^*(\psi_1+\psi_2)\,\psi_i. \tag{14}$$

Since the profit function $n(\psi_1+\psi_2)\,\psi_i = n(\Phi_i)(\Phi_i-\psi_j)$ is strictly quasi-concave in Φ_i,[31] (14) is equivalent to:

$$\psi_i \le \phi^R(\psi_j).$$

Consider now a candidate equilibrium in which $\Phi_i < \Phi_j$, implying that the two IP owners obtain respectively:(posing $\phi_i = \Phi_i - \psi_j$)

$$\begin{aligned}\Pi_i &= n^*(\Phi_i)(\Phi_i-\psi_j) = n^*(\phi_i+\psi_j)\,\phi_i,\\ \Pi_j &= n^*(\Phi_i)\,\psi_j = n^*(\phi_i+\psi_j)\,\psi_j.\end{aligned}$$

U_i should then not be able to gain from small deviations, which implies $\phi_i = \phi^R(\psi_j)$ (and thus $\Phi_i = \Phi^R(\psi_j)$, $n = n^R(\psi_j)$) and should not gain either from letting U_j sell at Φ_j, that is, Φ_j should be "large enough" (namely, such that $\Pi_i = n^R(\psi_j)\,\phi^R(\psi_j) \ge n^*(\Phi_j)\,\psi_i - \Phi_j > \pi^m$, implying $n^*(\Phi_j) = 0$, would do). In addition, U_j should not gain from undercutting U_i, that is:

$$\Pi_j = n^R(\psi_j)\,\psi_j \ge \max_{\Phi\le\Phi^R(\psi_j)} n^*(\Phi)(\Phi-\psi_i). \tag{15}$$

[31] It coincides with the industry profit, which is strictly concave, for $\Phi \in [\overline{\pi}, \pi^m]$, drops to zero for $\Phi > \pi^m$ (and lies anywhere between 0 and $\underline{n}\pi^m$ for $\Phi = \pi^m$), and is equal to $\Pi^H(n^*(\Phi))$ for $\Phi < \overline{\pi}$, in which case it strictly increases with Φ.

In particular, this implies (considering a deviation to just below $\Phi_i = \Phi^R(\psi_j)$)

$$\Pi_j = n^R(\psi_j)\psi_j \geq n^R(\psi_j)\left(\Phi^R(\psi_j) - \psi_i\right),$$

that is:

$$\psi_j \geq \Phi^R(\psi_j) - \psi_i$$

or

$$\psi_i \geq \Phi^R(\psi_j) - \psi_j = \phi^R(\psi_j).$$

Building on these insights, we have for $\psi_1, \psi_2 < \pi^m$:

- If $\psi_i \leq \phi^R(\psi_j)$ for $i \neq j = 1, 2$, there is a unique continuation equilibrium, which is such that $\Phi_1 = \Phi_2 = \psi_1 + \psi_2$. In this equilibrium, each U_i obtains

$$\Pi_i = n^*(\psi_1 + \psi_2)\psi_i.$$

- If $\psi_i > \phi^R(\psi_j)$ but $\psi_j \leq \phi^R(\psi_i)$, there is a unique continuation equilibrium, such that U_j charges a prohibitively high fee while U_i sells $n^R(\psi_j)$ complete licenses at a fee $\Phi^R(\psi_j)$; in this equilibrium the two IP owners obtain respectively

$$\begin{aligned} U_i &= n^R(\psi_j)\phi^R(\psi_j), \\ U_j &= n^R(\psi_j)\psi_j. \end{aligned}$$

Note that condition (15) is indeed satisfied, as $\psi_i > \phi^R(\psi_j)$ and $\psi_j \leq \phi^R(\psi_i)$ imply $\psi_i > \psi_j$ (see Figure 1) and thus $\Phi^R(\psi_i) \geq \Phi^R(\psi_j)$;[32] therefore:

$$\max_{\Phi \leq \Phi^R(\psi_j)} n(\Phi)(\Phi - \psi_i) = n^R(\psi_j)\left(\Phi^R(\psi_j) - \psi_i\right) > n^R(\psi_j)\psi_j,$$

where the last inequality follows from $\psi_i > \phi^R(\psi_j) = \Phi^R(\psi_j) - \psi_j$

- Finally, consider the case where $\psi_i > \phi^R(\psi_j)$ for $i \neq j = 1, 2$ and, without loss of generality, suppose that $\psi_i \geq \psi_j$. A similar reasoning then shows that there always exists an equilibrium in which U_j charges a prohibitively high fee while U_i sells $n^R(\psi_j)$ complete licenses at a fee $\Phi^R(\psi_j)$. In addition, there may exist an equilibrium in which U_i charges a prohibitively high fee while U_j sells $n^R(\psi_i)$ complete licenses at a fee $\Phi^R(\psi_i)$; for this to be an equilibrium, it must however be the case that

$$\Pi_i = n^R(\psi_i)\psi_i \geq \max_{\Phi \leq \Phi^R(\psi_i)} n^*(\Phi)(\Phi - \psi_j).$$

[32] $\Phi^R(\phi) = \pi^*\left(\min\left\{\overline{n}, n^M(f + \phi)\right\}\right)$, where $\pi^*(n)$ decreases with n and $n^M(f + \phi)$ decreases with ϕ; therefore, $\Phi^R(\phi)$ wealky increases with ϕ.

C.2 Upstream interaction

We now turn to the first stage and consider first the scenario where the two IP owners jointly determine a reciprocal upstream fee $\psi_1 = \psi_2 = \psi$. By setting

$$\psi = \frac{\pi^* \left(n^\Pi\right)}{2},$$

they can ensure that the second stage leads to $\Phi_1 = \Phi_2 = \pi^* \left(n^\Pi\right)$ and thus to the entry of n^Π downstream firms, and share equally the profit that an integrated IP owner could generate. To see this, given the above analysis of the second stage it suffices to show that $\pi^* \left(n^\Pi\right)/2$ is no higher than ϕ^D; but this derives directly from the fact that, due to double marginalization, a total fee of $2\phi^D$ for the two technologies generates less entry than is desirable for an integrated monopolist IP (that is, $n^D < n^\Pi$, and so $2\phi^D = \pi^* \left(n^D\right) > \pi^* \left(n^\Pi\right)$).

Finally, consider the alternative scenario where the two IP owners set their upstream fees simultaneously and independently. It is easy to check that, in the range $\psi_1, \psi_2 \leq \pi^m$:

- There is no equilibrium in which $\psi_i < \phi^R(\psi_j)$ for $i \neq j = 1, 2$: each U_i would obtain a profit $\Pi_i = n^*(\psi_1 + \psi_2)\psi_i$ and would thus deviate and increase its fee.

- There is no equilibrium in which $\psi_i \geq \phi^R(\psi_j)$ but $\psi_j < \phi^R(\psi_i)$: U_j would then obtain a profit $\Pi_j = n^R(\psi_j)\psi_j$, which increases with ψ_j, and would thus deviate and increase its fee.

- There is no equilibrium in which $\psi_i > \phi^R(\psi_j)$ for $i \neq j = 1, 2$, $\psi_j > \phi^D$ and U_i sells some licenses; this would require $\Phi_i < \Phi_j$ and $\Pi_i = n^R(\psi_j)\phi^R(\psi_j)$, but then U_i would profitably deviate by setting a fee ψ_i' just below $\phi^R(\psi_j)$, which would prompt U_j to sell $n^R(\psi_i') > n^R(\psi_j)$ ($\psi_j > \phi^D$ implies $\phi^R(\psi_j) < \phi^D < \psi_j$) and give U_i a greater profit $\Pi_i' = n^R(\psi_i')\psi_i' = n^R(\psi_i')\phi^R(\psi_j)$.

- There exist equilibria in which $\psi_i > \phi^R(\psi_j)$ for $i \neq j = 1, 2$, $\psi_j \leq \phi^D$ (which together imply $\psi_i > \phi^D \geq \psi_j$) and U_j sells complete licenses; in each such equilibrium the two IP owners obtain respectively

  $$\Pi_i = n^R(\psi_j)\phi^R(\psi_j), \Pi_j = n^R(\psi_j)\psi_j.$$

  In principle, U_j would want to deviate and increase its fee ψ_j, but such deviations can be deterred by "reverting" to a continuation equilibrium where U_j, rather than U_i sells the licenses for a fee $\Phi_j = \Phi^R(\psi_i)$, since in that case U_j

obtains $\Pi'_j = n^R(\psi_i)\,\phi^R(\psi_i)$, which is lower than Π_j since $\phi^R(\psi_i) < \psi_j$ and $\psi_i > \psi_j$ moreover implies $n^R(\psi_i) < n^R(\psi_j)$. This however requires that such continuation equilibrium exists, which in turn requires (see condition (15)):

$$n^R(\psi_i)\,\psi_i \geq \max_{\Phi \leq \Phi^R(\psi_i)} n^*(\Phi)(\Phi - \psi_j)\,.$$

The right-hand side decreases with ψ_j whereas the left-hand side increases with ψ_i, and they coincide for $\psi_i = \psi_j = \phi^D$. Therefore this condition determines a curve that goes through $\left(\phi^D, \phi^D\right)$ in the (ψ_1, ψ_2) plane and above which the two continuation equilibria coexist. The equilibrium that generates the greater joint profit is the one for which ψ_j is the lowest, and thus for which ψ_i is maximal: $\psi_i = \pi^m$ and ψ_j such that $n^R(\psi_j)\,\phi^R(\psi_j) = \underline{n}\pi^m$. This equilibrium gives both IP owners a larger total profit than the "double marginalization" outcome but only one IP owner benefits from it: $\psi_j < \phi^D$ and $\phi^R\left(\phi^D\right) = \phi^D$ indeed imply

$$\begin{aligned} \Pi_i &= n^R(\psi_j)\,\phi^R(\psi_j) > \Pi^D = n^R\left(\phi^D\right)\phi^R\left(\phi^D\right), \\ \Pi_j &= n^R(\psi_j)\,\psi_j < \Pi^D = n^R\left(\phi^D\right)\phi^D. \end{aligned}$$