Innovation and FDI: Does the Target of Intellectual Property Rights Matter?

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ABSTRACT

This paper develops a North-South product-cycle model with innovation and foreign direct investment (FDI) to analyze the influences from strengthening intellectual property rights (IPR) protection. Innovation occurs in the North while imitation happens in the South. Southern firms can imitate either goods produced in the North or goods produced by multinationals in the South. We find that if the target of strengthening IPR protection is Northern-produced goods, then such a policy change reduces the innovation rate and raises the North-South relative wage in the long run. However, the effects on the long-run innovation rate and the North-South relative wage reverse if its target is Southern-produced goods by multinationals. As for the pattern of production, strengthening IPR protection raises the long-run extents of FDI and Southern production imitating goods produced by multinationals while reducing the long-run extents of Northern production and Southern production imitating goods produced in the North, regardless of the target of stronger IPR protection. In addition to examining the long-run effects of strengthening IPR protection, we also analyze its effects during the transitional dynamics. The quantitative analysis indicates that the two strengthening-IPR-protection policies cause welfare losses for both Northern and Southern consumers if we consider the accumulated effects during the transitional dynamics.

Keywords: FDI; Imitation; Innovation; IPR; R&D.

JEL Classification: E52; F23; O31.

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1. INTRODUCTION

The approval of the Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPs) under the Uruguay Round of negotiations based on the framework of the General Agreement on Tariffs and Trade (GATT) has caused considerable discussion on the effects from the strengthening of intellectual property rights (IPR) protection for both developed and developing countries. Since then, studies have increased concerning the effects of strengthening IPR protection for both developed and developing countries.

International production through foreign direct investment (FDI) is quite common nowadays due to technological progress, which improves transportation and telecommunications. The availability of FDI allows firms to choose to produce goods domestically or abroad, meaning strengthening IPR protection in one country causes cross-country influences due to the adjustment of the production pattern for firms in response to this policy change. It is commonly believed that such a policy can benefit both developed and developing countries. For developed countries, stronger IPR protection encourages innovation owing to the mitigation of imitation risk. Developing countries can benefit from stronger IPR protection by attracting firms to produce in those countries. Increases in FDI activities also have the added virtues of reducing the relative wage between the developed and the developing countries and bringing cutting-edge technologies to the developing countries. Firms, at the same time, can reduce production cost by shifting production from developed countries to developing countries. However, some people argue against stronger IPR protection, since it is doubtful whether it actually increases the rate of innovation and reduces the relative wage between developed and developing countries at the same time. Moreover, they doubt whether both Northern and Southern consumers can benefit from strengthening IPR protection.

Several studies have examined the effects of changes in IPR protection on innovation in developed countries and pattern of production based on an R&D model with FDI. The early study of Helpman (1993) develops a model where innovation occurs in developed countries and imitation happens in developing countries to examine the effects of IPR protection when firms can undertake production in developing countries through FDI. In his study, the innovation rate is assumed to be exogenous; as a result, this study does not analyze the impact of the strengthening of IPR protection on innovation. Due to dissatisfaction with the exogenous innovation rate, the Helpman (1993) model subsequently undergoes various modifications in several studies where the innovation rate is endogenously determined in order to examine the effects of IPR protection on innovation.

Based on a model where innovation raises the varieties of goods, Lai (1998) shows that both the innovation rate and FDI will increase with strengthening IPR protection. However, based on a quality-improvement (product-cycle) model with costly imitation, Glass and Saggi (2002) find the reverse effects of stronger IPR protection on the rate of innovation and the extent of FDI due to labor wastage and imitation tax effects if imitation is costly. Since R&D activities require high-skilled labor, Parello (2008) and Chen (2015b, 2018) introduce human capital into an R&D model and endogenize the skill choice of Northerners when examining the effects of strengthening IPR protection on innovation and wage inequality in developed countries. Changes in IPR protection affect not only the pattern of production, but also the demand of Northern skilled (unskilled) labor, which will in turn affect the wage inequality in developed countries.

In this study we develop a dynamic quality-improvement general-equilibrium model.[1] Our model is made up of a North country (a developed country) in which innovation occurs and a South country (a developing country) in which imitation happens. Innovation targets all types of goods and improves their quality. Northern workers can work either in the R&D sector or in the production sector. Northern firms choose either to carry out the entire production of the goods in the North or act as multinational firms and produce goods in the South through FDI. Southern firms could imitate goods produced in the North or goods produced by multinationals in the South at different (exogenous) rates. Once Southern firms succeed at imitation, they are able to use the state-of-the-art technologies to produce the highest quality products. We assume that the rate of imitation for goods produced by multinationals in the South is higher than the rate of imitation for goods produced in the North due to two reasons. First, developed countries usually have more comprehensive and complete IPR protection than developing countries; second, it is easier for Southern firms to imitate domestic goods (i.e. goods produced by multinationals in the South) than foreign goods (i.e. goods produced in the North). Therefore, multinational firms can make products in the South at lower costs by taking advantage of the lower Southern wage rate, but they face a higher risk of imitation by Southern firms.

We assume that the Northern standard of IPR protection determines the imitation rate of goods produced in the North while the Southern standard of IPR protection decides the imitation rate of goods produced by multinationals in the South.[2] Therefore, depending on the target of IPR protection, strengthening such protection is represented by a decrease in the rate of imitation for Northern-produced goods or Southern-produced goods by multinationals. Our assumption that Southern firms could imitate Northern-produced goods and Southern-produced goods by multinationals at different rates allows us to analyze the influences of strengthening IPR protection for different targets of IPR protection. Two scenarios of IPR protection policies are considered, with the first of these scenarios involving IPR protection targeting Northern-produced goods. We find that increasing the Northern

[1] The product-cycle model is originally considered by Vernon (1966) and subsequently developed by Segerstrom, Anant and Dinopoulos (1990), Grossman and Helpman (1991a, 1991b, 1991c, 1991d), and Aghion and Howitt (2009).

[2] The Northern and Southern standards of IPR protection are also considered in Lin (2019). However, in his study the Northern IPR standard determines the imitation rate of goods consumed exclusively in the North, whereas the Southern IPR standard determines the imitation rate of goods consumed exclusively in the South.

standard of IPR protection raises the North-South relative wage and reduces the rate of innovation in the long run. As regards the pattern of production, such a policy change will cause an increase in the long-run extents of FDI and Southern production imitating Southern-produced goods by multinationals, along with corresponding reductions in the long-run extents of Northern production and Southern production imitating Northern-produced goods.

We show that welfare for Northern (Southern) consumers is positively correlated with the rate of innovation and consumers' expenditure in the North (South) and negatively correlated with the price factor. Due to the model's complexity, we are not able to analytically determine the effects of stronger IPR protection on welfare for Northern (Southern) consumers. We thus conduct a numerical analysis to evaluate the effects of stronger IPR protection on welfare. Following Grossman and Helpman (1991), we first consider the effects of stronger IPR protection on the long-run (steady-state) welfare. Although such a policy change raises the steady-state Northern (Southern) expenditure, which is beneficial to Northern (Southern) welfare, it reduces the steady-state rate of innovation and raises the steady-state price factor, both harmful to Northern (Southern) welfare. We find overall that such a policy change generates welfare losses for both Northerners and Southerners.

Because our numerical analysis indicates that the speed of convergence is slow, changes in IPR protection would affect economic variables for sustained periods of time so that their accumulated effects during the transition from one equilibrium to another may therefore cause potentially large impacts on welfare during the transition. We then take changes during the transitional dynamics into consideration and re-examine welfare changes. We show that when considering welfare changes during the transitional dynamics, strengthening the Northern standard of IPR protection will also cause welfare losses for both Northerners and Southerners, but with much larger orders when comparing changes in steady-state welfare.[3]

In the second scenario we consider the effects caused by strengthening the Southern standard of IPR protection. In this scenario, strengthening the Southern standard of IPR protection is represented by a reduction in the imitation risk for Southern-produced goods by multinationals. Since such a policy motivates Northern firms to shift their production from the North to the South through FDI, the long-run extent of FDI will increase. Furthermore, the demand for Northern labor will decrease while the demand for Southern labor will increase, leading to a decrease in the North-South relative wage in the long run. Because more Northern workers are released from the production sector and become available for employment in the R&D sector, the long-run rate of innovation will increase. As regards

[3] Our quantitative analysis suggests that regarding steady-state welfare, a decrease in the imitation rate for Northern-produced goods from 5% to 4.75% (a 5% decrease) will respectively cause decreases in the steady-state welfare for Northerners and Southerners by 1.21% and 18.41% in consumption equivalence. If we take transitional dynamics into account, such a policy change will generate welfare losses in the order of 62.44% for Northerners and 62.49% for Southerners in consumption equivalence.

the pattern of production, the strengthening of IPR protection will cause reductions in the long-run extents of Northern production and Southern production imitating Northern-produced goods. However, the change in the long-run extent of Southern production imitating Southern-produced goods by multinationals is ambiguous.

Concerning steady-state welfare, our numerical analysis indicates that although strengthening IPR protection on goods produced in the South raises the steady-state innovation rate, which is beneficial to welfare, it also raises the steady-state price factor, which is harmful to welfare. The steady-state Northern consumers' expenditure will decrease while the steady-state Southern consumers' expenditure will increase. We find that there will be welfare gains for both Northern and Southern consumers. However, if we take the transitional dynamics into account, then the results of welfare reverse and such a policy change will generate welfare losses for both Northern and Southern consumers.[4] This finding highlights the tradeoffs involved for economic performance induced by strengthening IPR protection, especially innovation and welfare. Moreover, when examining the effects of stronger IPR protection on welfare, the results would be misleading if we only consider its effects at the steady state and do not take the transitional dynamics into consideration.

Our findings cast doubt on the common belief that strengthening IPR protection could raise the rate of innovation and FDI activities as well as benefit consumers both in developed and developing countries. We show that strengthening IPR protection may not necessarily raise the rate of innovation and that the target of stronger IPR protection plays an important role when determining the effects of IPR protection on the rate of innovation. While increasing the Northern standard of IPR protection reduces the innovation rate, increasing the Southern standard of IPR protection raises it. But both policies raise the extent of FDI. Our findings about the long-run effects of stronger IPR protection on innovation and FDI activities are different from those found by Lai (1998) and Glass and Saggi (2002). Regarding welfare, both policies generate welfare losses for Northern and Southern consumers if we consider the accumulated effects during the transitional dynamics.

Since previous studies of R&D have demonstrated that the stability of the steady-state equilibrium of an R&D model is sensitive to the model setting, we also examine the stability of the steady-state equilibrium under each scenario in this paper.[5] As summarized in Table 1, in addition to the target of imitation, which is the focus of this paper, the target of innovation is also the focus in the literature of IPR protection. Glass and Wu (2007) introduce costless imitation into a product-cycle model and find that the target of innovation matters when analyzing the effects of stronger IPR protection on innovation rate and FDI. They show that stronger IPR protection will reduce both

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4 Our quantitative analysis suggests that regarding steady-state welfare, a decrease in the imitation rate for Northern-produced goods by multinationals from 25% to 23.75% (a 5% decrease) will respectively cause increases in the steady-state welfare for Northerners and Southerners by 52.48% and 28.69% in consumption equivalence. If we take transitional dynamics into account, such a policy change will generate welfare losses in the order of 56.09% for Northerners and 55.19% for Southerners in consumption equivalence.

5 An early study that examines the stability of the steady-state equilibrium in an R&D model can be found in Tanaka, Iwaisako and Futagami (2007). In their study, they consider a product-cycle model with international transfer of technology through licensing.

innovation intensity and the extent of FDI in a model with inefficient followers where innovation targets only imitated products; however, stronger IPR protection will cause reverse effects on the innovation rate and the extent of FDI in a model with efficient followers where innovation targets all types of products.

Tanaka and Iwaisako (2014) find that the steady-state equilibrium is not attainable in a product-cycle model with inefficient followers and then introduce government subsidies for R&D and FDI and show that the steady state will become achievable if the subsidy rates are sufficiently high. Furthermore, if the interior steady state is stable, then stronger IPR protection promotes both innovation rate and FDI, which are opposite to the results found by Glass and Wu (2007), indicating that the effects of strengthening IPR protection are also sensitive to the model setting. By modifying our model to allow imitation targeting only Southern-produced goods by multinationals, our model can be returned to the model of Glass and Wu (2007) with efficient followers. We show that the economy will attain a stable steady-state equilibrium. Thus, our finding suggests that allowing innovation to target all types of goods (efficient followers) is another way to obtain a stable steady-state equilibrium. However, one should be careful when drawing results based on a product-cycle model with inefficient followers.

The remainder of this paper is organized as follows. In the next section, we develop a product-cycle model with innovation and FDI, derive the steady-state equilibrium, and analyze the stability condition of the steady-state equilibrium. Section 3 derives the steady state and also analyzes its stability property. Section 4 examines the effects of increasing the Northern (Southern) standard of IPR protection on key variables and the social welfare for Northern and Southern consumers under each type of IPR protection. A numerical analysis is also provided in this section. The final section concludes.

2. THE MODEL

We develop a product-cycle model composed of a developed Northern country (*N*) and a developing Southern country (*S*). There is no population growth in both countries and there are L_k ($k = \{N, S\}$) agents in country k. In every period, each agent in country k is endowed with one unit of time, and she spends all of the time at work to earn the wage $w_k(t)$. We normalize the Southern wage rate ($w_S(t)$) to 1. This implies that the North-South relative wage, which is measured by the ratio of the Northern wage rate to the Southern wage rate, is equal to $w_N(t)$.

2.1. Consumers

There is a continuum of products $z \in [0,1]$ available at different quality levels (j). Each quality level 'j' is better than quality level '$j-1$' by λ times, where the size of the quality increment λ is constant and $1 < \lambda < 2$.[6] All products begin at time $t = 0$ with a quality level $j = 0$ and a base quality $\lambda^0 = 1$. Therefore, each product of quality j provides quality $x_j(z) = \lambda^j$.

Let $q_{kj}(z,t)$ denote consumption in country k for quality level j of product z at time t. For the representative consumer in country k, the total expenditure ($E_k(t)$) for all products with different quality levels under price $p_{kj}(z,t)$ is:

$$E_k(t) = \int_0^1 \left[\sum_j p_{kj}(z,t) q_{kj}(z,t) \right] dz. \qquad (1)$$

Let $A_k(0)$ and $W_k(0)$ respectively denote the value of assets that the household holds and the sum of the discount wage income of the household at time $t = 0$ in country k. The cumulative interest rate, up to time t, is given by $R(t) = \int_0^t r(\tau)d\tau$, where $r(\tau)$ is the instantaneous real interest rate at time τ. The intertemporal budget constraint is:

$$\int_0^\infty E_k(t) e^{-R(t)} dt \leq A_k(0) + W_k(0). \qquad (2)$$

Consumers care about both the quantity and quality of goods. The instantaneous utility faced by a representative consumer in country k is:

$$log\, u_k\,(t) = \int_0^1 \log \left[\sum_j x_j(z) q_{kj}(z,t) \right] dz. \qquad (3)$$

Let ρ represent the subjective discount factor. The lifetime utility of the representative consumer in country k is:

$$U_k(0) = \int_0^\infty e^{-\rho t}\, log\, u_k(t)\, dt. \qquad (4)$$

The consumer's problem is solved by three steps. First, when considering the within-industry static optimization problem, consumers are willing to pay λ for a single quality-level improvement in a product; that is, consumers choose the quality that gives the lowest adjusted price, $\frac{p_j(z,t)}{x_j(z)}$. Second, consumers choose an expenditure across all products that is the same, because the elasticity of substitution between any two products is constant at unity. Let $E(t) = E_N(t)L_N + E_S(t)L_S$ represent global expenditure. This results in a demand function for product z of quality j at time t in country k equal to $q_{kj}(z,t) = E_k(t)/p_j(z,t)$. Finally, consumers maximize lifetime utility subject to the inter-temporal budget constraint to allocate lifetime wealth across time. This leads to the optimal expenditure path for the representative consumer in each country:

[6] The assumption of $1 < \lambda < 2$ is in line with empirical findings that a markup over the marginal/average cost is positive, but less than 100%.

$$\frac{\dot{E}_k(t)}{E_k(t)} = r(t) - \rho. \tag{5}$$

2.2. Producers

Innovation occurs only in the North. Northern firms hire Northern workers for R&D. Let $v_N(t)$ represent the expected discounted value of a Northern firm that has successfully improved a product by one higher quality level. Let $\phi_R(z,t)$ denote the rate of innovation. A Northern firm in industry z will achieve one level of quality improvement in the final product with a probability $\phi_R(z,t)dt$ for a time interval dt. In order to achieve this, $a_R\phi_R(z,t)dt$ units of labor are required at a cost of $w_N(t)a_R\phi_R(z,t)dt$. To generate a finite rate of innovation, the expected gains from innovation cannot exceed the costs:

$$v_N(t) \leq a_R w_N(t),$$

with equality being achieved when innovation occurs with positive intensity:

$$v_N(t) = a_R w_N(t) \quad \Leftrightarrow \quad \phi_R(z,t) > 0. \tag{6}$$

We assume that one unit of labor is required to produce one unit of the final product in the North or in the South. The cost of firms producing one unit of goods in the North is then w_N, and the cost of firms completing one unit of production in the South is $w_S = 1$. After succeeding at innovating a higher-level quality product, a Northern firm can hire Northern workers to produce goods in the North or undertake its production in the South through FDI, lowering its costs by hiring Southern workers. Let $v_F(z,t)$ represent capital gains from undertaking production in the South through FDI. Following Glass and Wu (2007) and Tanaka and Iwaisako (2014), we assume that FDI does not incur any cost. A Northern firm will feel then indifferent between producing in the North or in the South if:

$$v_F(t) = v_N(t). \tag{7}$$

Old technologies in which the designs have been improved are available internationally, so that Southern firms are able to produce final goods by using old technologies. Firms face Bertrand competition. Northern firms, which produce through the use of state-of-the-art technologies possessing one quality-level lead over their closest rivals will charge the price $p(t) = \lambda$ in order to just prevent their closest rivals from earning positive profits.

Let $\Pi_N(t)$ and $\Pi_F(t)$ denote the instantaneous profits for Northern production and FDI, respectively. The instantaneous profits for Northern production are:

$$\Pi_N(t) = \frac{E(t)}{\lambda}[\lambda - w_N(t)]. \tag{8}$$

The instantaneous profits for multinational firms are:

$$\Pi_F(t) = \frac{E(t)}{\lambda}(\lambda - 1). \tag{9}$$

Southern firms can imitate either Northern-produced goods at a rate of ϕ_{SN} or Southern-produced goods by multinationals at a rate of ϕ_{SF}. After succeeding at imitating a higher-level quality product, a Southern firm is able to capture the entire industry market by setting a price at 1 that is lower than λ. Because of the following two reasons, we assume that the imitation rate of goods produced in the North is smaller than that of goods produced in the South; that is, $\phi_{SN} < \phi_{SF}$. First, it is easier for Southern firms to imitate Southern-produced (domestic) goods than Northern-produced (foreign) goods. Second, developed countries usually have stricter IPR protection than developing countries, so that goods produced in the North are more protective. This implies that multinational firms can earn higher profits by charging the price $p(t) = \lambda$ and hiring Southern workers for production; however, they also face a higher rate of imitation. Depending on its target, strengthening IPR protection is represented by a decrease in ϕ_{SN} or ϕ_{SF}.

The no-arbitrage condition that determines $v_N(t)$ is:

$$r(t) = \frac{\dot{v}_N(t) + \Pi_N(t) - [\phi_R(t) + \phi_{SN}]v_N(t)}{v_N(t)}. \tag{10}$$

Equation (10) equates the real interest rate to the asset return per unit of asset for Northern production. The asset return includes (i) any potential capital gains ($\dot{v}_N(t)$); (ii) profits from successful innovation ($\Pi_N(t)$); (iii) the expected capital loss from creative destruction $(-\phi_R(t)v_N(t))$; and (iv) the expected capital loss from imitation ($-\phi_{SN}v_N(t)$).

The no-arbitrage condition that determines $v_F(t)$ is:

$$r(t) = \frac{\dot{v}_F(t) + \Pi_F(t) - [\phi_R(t) + \phi_{SF}]v_F(t)}{v_F(t)}. \tag{11}$$

Equation (11) equates the real interest rate to the asset return per unit of asset for multinational firms. The asset return is the sum of (i) any potential capital gains ($\dot{v}_F(t)$); (ii) profits from successful imitation ($\Pi_F(t)$); (iii) the expected capital loss from creative destruction $(-\phi_R(t)v_F(t))$; and (iv) the expected capital loss from imitation $(-\phi_{SF}v_F(t))$.

2.3. Type of industry

Let $n_N(t)$, $n_F(t)$, $n_{SN}(t)$, and $n_{SF}(t)$ respectively denote the extent of Northern production (the proportion of products produced completely in the North), the extent of FDI (the proportion of the goods for which production is carried out through FDI), the extent (proportion) of Southern production imitating Northern-produced goods, and the extent (proportion) of Southern-produced goods by multinationals. The extent of Southern production by imitation is thus defined as $n_S(t) = n_{SN}(t) + n_{SF}(t)$. The sum of these product measures equals one:

$$n_N(t) + n_F(t) + n_{SN}(t) + n_{SF}(t) = 1. \tag{12}$$

The law of motion of $n_{SN}(t)$ is governed by the following equation:

$$\dot{n}_{SN}(t)=\phi_{SN}n_N(t)-\phi_R(t)n_{SN}(t). \tag{13}$$

Equation (13) indicates that the change in the extent of Southern production imitating Northern-produced goods equals the flows going into Southern production imitating Northern-produced goods minus the flows coming out of it due to innovation.

Similarly, the change in the extent of Southern production imitating goods produced by multinationals equals the flows going into Southern production imitating goods produced by multinationals in the South minus the flows coming out of it due to innovation. This implies that the law of motion of $n_{SF}(t)$ is governed by the following equation:

$$\dot{n}_{SF}(t)=\phi_{SF}n_F(t)-\phi_R(t)n_{SF}(t). \tag{14}$$

Combining (13) and (14), we derive the law of motion for the extent of Southern production by imitation as:

$$\dot{n}_S(t)=\phi_{SN}n_N(t)+\phi_{SF}n_F(t)-\phi_R(t)n_S(t). \tag{15}$$

2.4. Labor market

Northern firms hire Northern workers for innovation and production. The labor-market-clearing condition for Northern labor is therefore:

$$a_R\phi_R(t)+n_N(t)\frac{E(t)}{\lambda}=L_N. \tag{16}$$

Multinational firms and Southern firms hire Southern workers for production. The labor-market-clearing condition for Southern labor is therefore:

$$[n_F(t)+\lambda(n_{SF}(t)+n_{SN}(t))]\frac{E(t)}{\lambda}=L_S. \tag{17}$$

2.5. Welfare

Using $q_{kj}(z,t)=E_k(t)/p_j(z,t)$, we express the instantaneous utility as:

$$\log u_k(t)=\log E_k(t)-\int_0^1 \log p_j(z,t)dz+\log(\lambda)\int_0^1 x_j(z)dz.$$

Because the expected number of innovations arriving in period *t* is $t\phi_R(t)$, the instantaneous utility becomes:[7]

$$\log u_k(t)=\log E_k(t)-\int_0^1 \log p_j(z,t)dj+t\phi_R(t)\log(\lambda).$$

[7] See Grossman and Helpman (1991d) for more details.

Since consumers pay the price of λ for goods produced by Northern firms in the North and by multinationals in the South and pay the price of 1 for goods produced by Southern firms, the price factor is:[8]

$$P_t = \int_0^1 \log p_j(z,t)dj$$
$$= [n_N(t) + n_F(t)]\log\lambda + n_S(t)\log 1 = [1 - n_S(t)]\log\lambda.$$

Note that the price factor positively depends on the extent of Northern production and the extent of FDI, but negatively depends on the extent of Southern production. Therefore, the instantaneous utility can be written as:

$$\log u_k(t) = \log E_k(t) - \big(1 - n_S(t)\big)\log\lambda + t\phi_R(t)\log\lambda. \qquad (18)$$

Substituting (18) into (4), we then derive the lifetime utility of consumers which represents welfare for consumers living in country k.

3. STEADY-STATE EQUILIBRUM AND SOCIAL WELFARE

We assume that the Northern standard of IPR protection determines the imitation rate of goods produced in the North, while the Southern standard of IPR protection decides the imitation rate of goods produced by multinationals in the South. This section first derives the steady-state equilibrium and examines its stability and then calculates the steady-state social welfare.

3.1. The steady-state equilibrium

We use variables with an upper bar to denote the steady-state values of the corresponding variables. Since $\frac{\dot{E}_k(t)}{E_k(t)} = r(t) - \rho$, we then have:

$$\frac{\dot{E}(t)}{E(t)} = r(t) - \rho. \qquad (19)$$

At the steady state, $\frac{\dot{E}(t)}{E(t)} = \dot{v}_N(t) = \dot{v}_F(t) = 0$ and $r(t) = \rho$. The no-arbitrage conditions of (10) and (11) can then be expressed as:

$$\bar{v}_N = \frac{\overline{\Pi}_N}{\rho + \overline{\phi}_R + \phi_{SN}}, \qquad (20)$$

$$\bar{v}_F = \frac{\overline{\Pi}_F}{\rho + \overline{\phi}_R + \phi_{SF}}. \qquad (21)$$

Substituting (6) and (8) into (20) gives us:

$$\frac{\bar{E}}{\lambda}(\lambda - \overline{w}_N) = \big(\rho + \overline{\phi}_R + \phi_{SN}\big)a_R\overline{w}_N. \qquad (22)$$

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8 See Tanaka and Iwaisako (2014).

Substituting (6), (7), and (9) into (21) yields:

$$\frac{\bar{E}}{\lambda}(\lambda-1)=(\rho+\overline{\phi}_R+\phi_{SF})a_R\overline{w}_N. \tag{23}$$

At the steady-state equilibrium, the flows going into Southern production equal the flows coming out of it - that is, $\dot{n}_S(t)=0$. Thus, (15) indicates:

$$\phi_{SN}\bar{n}_N+\phi_{SF}\bar{n}_F=\overline{\phi}_R\bar{n}_S. \tag{24}$$

The steady-state equilibrium is characterized by (12), (16), (17), and (22)-(24) with six variables $\{\overline{w}_N,\bar{E},\bar{n}_N,\bar{n}_F,\bar{n}_S,\overline{\phi}_R\}$.[9] We are now ready to calculate the steady state. Combining (22) and (23), we express the steady-state Northern wage ($\overline{w}_N$) as a function of the steady-state rate of innovation ($\overline{\phi}_R$):

$$\overline{w}_N=\overline{w}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})=\frac{\rho+\overline{\phi}_R+\lambda\phi_{SF}-(\lambda-1)\phi_{SN}}{\rho+\overline{\phi}_R+\phi_{SF}}. \tag{25}$$

Combining (22) and (25), we express the steady-state global expenditure ($\bar{E}$) as a function of $\overline{\phi}_R$:

$$\bar{E}(\overline{\phi}_R;\phi_{SN},\phi_{SF})=\frac{\lambda a_R\left[\lambda\phi_{SF}-(\lambda-1)\phi_{SN}+\rho+\overline{\phi}_R\right]}{\lambda-1}.$$

Appendix A shows that besides $\overline{w}_N$ and $\bar{E}$, variables $\{\bar{n}_N,\bar{n}_F,\bar{n}_S\}$ also can be expressed as functions of $\overline{\phi}_R$; that is, $\bar{n}_N=\bar{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})$, $\bar{n}_F=\bar{n}_F(\overline{\phi}_R;\phi_{SN},\phi_{SF})$, and $\bar{n}_S=\bar{n}_S(\overline{\phi}_R;\phi_{SN},\phi_{SF})$. Using 12), we replace $(\bar{n}_{SN}+\bar{n}_{SF})$ by $(1-\bar{n}_N-\bar{n}_F)$ in the market-clearing condition for Southern labor (equation(17)) and derive:

$$f(\overline{\phi}_R;\phi_{SN},\phi_{SF})=L_S, \tag{26}$$

where $f(\overline{\phi}_R;\phi_{SN},\phi_{SF})=\left[\lambda\left(1-\bar{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})\right)-(\lambda-1)\bar{n}_F(\overline{\phi}_R;\phi_{SN},\phi_{SF})\right]\frac{\bar{E}(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\lambda}$.

Equation (26) is used to implicitly solve for $\overline{\phi}_R$. Appendix A also shows that if $\frac{\rho}{\lambda-1}<\phi_{SN}$, then $\frac{\partial f(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\overline{\phi}_R}>0$, indicating that $f(\overline{\phi}_R;\phi_{SN},\phi_{SF})$ is an increasing function in $\overline{\phi}_R$. Since $\overline{\phi}_R\in(0,1)$, the solution of $\overline{\phi}_R$ will exist if $f(0;\phi_{SN},\phi_{SF})<L_S<f(1;\phi_{SN},\phi_{SF})$.[10] Therefore, there will exist a unique solution of $\overline{\phi}_R$ if the following two conditions hold.

Condition (P1) $\frac{\rho}{\lambda-1}<\phi_{SN}$.

Condition (P2) $f(0;\phi_{SN},\phi_{SF})<L_S<f(1;\phi_{SN},\phi_{SF})$.

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[9] See Appendix A for details of calculation.

[10] Note that $f(0;\phi_{SN},\phi_{SF})=\frac{[\lambda\phi_{SF}-(\lambda-1)\phi_{SN}][\lambda a_R\phi_{SF}-(\lambda-1)L_N]+\rho\lambda a_R\phi_{SF}}{(\lambda-1)\phi_{SF}}$ and $f(1;\phi_{SN},\phi_{SF})=\frac{[\lambda\phi_{SF}-(\lambda-1)\phi_{SN}+1][a_R(1+\lambda\phi_{SF})-(\lambda-1)(L_N-a_R)]+\rho a_R(1+\lambda\phi_{SF})}{(\lambda-1)(1+\phi_{SF})}$.

Once one derives the solution of $\overline{\phi}_R$, the remaining variables $\{\overline{w}_N, \overline{E}, \overline{n}_N, \overline{n}_F, \overline{n}_S\}$ can be solved accordingly. Note that at the steady-state equilibrium, the flows going into $n_{SN}(t)$ and $n_{SF}(t)$ equal the flows coming out of them - that is, $\dot{n}_{SN}(t) = \dot{n}_{SF}(t) = 0$. Thus, (13) and (14) indicate:

$$\phi_{SN}\overline{n}_N = \overline{\phi}_R \overline{n}_{SN}, \tag{27}$$

$$\phi_{SF}\overline{n}_F = \overline{\phi}_R \overline{n}_{SF}. \tag{28}$$

Using (27) and (28), we respectively derive $\overline{n}_{SN}$ and $\overline{n}_{SF}$ as $\overline{n}_{SN} = \frac{\phi_{SN}\overline{n}_N}{\overline{\phi}_R}$ and $\overline{n}_{SF} = \frac{\phi_{SF}\overline{n}_F}{\overline{\phi}_R}$.

3.2. Stability of the steady-state equilibrium

In the following analysis we derive the dynamical system, which characterizes the dynamic behavior of the economy, and then examine the stability of the steady-state equilibrium.[11] Appendix B shows that $E(t)$ can be expressed as a function of $w_N(t)$. Moreover, we present that $\phi_R(t)$, $n_N(t)$, and $n_F(t)$ can be expressed as functions of $w_N(t)$and $n_S(t)$; that is, $E(t) = E(w_N(t))$, $\phi_R(t) = \phi_R(w_N(t), n_S(t))$, $n_N(t) = n_N(w_N(t), n_S(t))$, and $n_F(t) = n_F(w_N(t), n_S(t))$. During the transitional dynamics, the real interest rate $r(t)$ changes over time, and its dynamic behavior can be characterized by the following function in $w_N(t)$and $n_S(t)$:

$$r(t) = r(w_N(t), n_S(t))$$
$$= \frac{1}{w_N(t)}\left[\frac{(\phi_{SF} - \phi_{SN})(\lambda - w_N(t))}{w_N(t) - 1} - \phi_R(w_N(t), n_{SN}(t), n_{SF}(t)) - \phi_{SN} + \rho(w_N(t) - 1)\right]. \tag{29}$$

From the no-arbitrage condition that determines $v_N(t)$ (equation (10)), we derive the relationship between $r(t)$ and $v_N(t)$ as:

$$\frac{\dot{v}_N(t)}{v_N(t)} = r(t) - \frac{(\phi_{SF} - \phi_{SN})(\lambda - w_N(t))}{w_N(t) - 1} + \phi_R(t) + \phi_{SN}.$$

Equation (6) indicates that $\frac{\dot{w}_N(t)}{w_N(t)} = \frac{\dot{v}_N(t)}{v_N(t)}$. Next we use (6) and (10) to derive the law of motion of $w_N(t)$ as:

$$\dot{w}_N(t) = (\rho + \phi_{SF})w_N(t) + (w_N(t) - 1)\phi_R(w_N(t), n_S(t)) - \lambda\phi_{SF} + \phi_{SN}(\lambda - 1) - \rho. \tag{30}$$

Equation (15) indicates that the law of motion of $n_S(t)$ is governed by the following equation:

$$\dot{n}_S(t) = \phi_{SN} n_N\big(w_N(t), n_S(t)\big) + \phi_{SF} n_F(w_N(t), n_S(t)) - \phi_R(w_N(t), n_S(t)) n_S(t). \tag{31}$$

The dynamical system of the economy is represented by (30) and (31) in $\{w_N(t), n_S(t)\}$. Linearizing (30) and (31) at the steady state yields:

[11] See Appendix B for details of calculation.

$$\begin{bmatrix} \dot{w_N}(t) \\ \dot{n_S}(t) \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} w_N(t) - \overline{w}_N \\ n_S(t) - \overline{n}_S \end{bmatrix},$$

where $J = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$ is the Jacobian matrix of this dynamical system evaluated at the steady-state values of $(\overline{w}_N, \overline{n}_S)$ with $d_{11} = \rho + \overline{\phi}_R + \phi_{SF} - \frac{(\phi_{SF}-\phi_{SN})[1+(\lambda-1)\overline{n}_S]}{\overline{w}_N-1}$, $d_{12} = (\phi_{SF} - \phi_{SN})\overline{w}_N(\lambda - 1)$, $d_{21} = \frac{L_S}{a_R\overline{w}_N{}^2} + \frac{(\phi_{SF}-\phi_{SN})[1+(\lambda-1)\overline{n}_S]\overline{n}_S}{(\overline{w}_N-1)^2}$, and $d_{22} = -\lambda(\phi_{SF} - \phi_{SN}) - \overline{\phi}_R - \phi_{SN} - \frac{(\phi_{SF}-\phi_{SN})\overline{w}_N(\lambda-1)\overline{n}_S}{\overline{w}_N-1}$.

Let η_1 and η_2 represent the two eigenvalues calculated from the Jacobian matrix (J). Because $n_S(t)$ is a state variable and $w_N(t)$ is a jump variable, then the steady-state equilibrium is stable if one eigenvalue is negative and one eigenvalue is positive, implying that the determinant of the Jacobian matrix is negative. This requires that $\eta_1\eta_2 = Det(J) = d_{11}d_{22} - d_{12}d_{21} < 0$, which generates the following condition.

Condition (P3) $(\lambda - 1)\left\{\rho\overline{n}_S + \frac{(\phi_{SF}-\phi_{SN})L_S}{a_R[\rho+\overline{\phi}_R+\lambda\phi_{SF}-(\lambda-1)\phi_{SN}]}\right\} > (2-\lambda)[\overline{\phi}_R + \lambda\phi_{SF} - (\lambda-1)\phi_{SN}]$.

We thus have the following proposition.

Proposition 1. There exists a unique non-trivial steady state that is a saddle point if conditions (P1)-(P3) hold.

We hereafter assume that conditions (P1)-(P3) are satisfied in the rest of the paper. Let η_1 and η_2 respectively represent the negative and positive eigenvalues. The paths for $w_N(t)$ and $n_S(t)$ are characterized by:

$$w_N(t) = \overline{w}_N + (w_N(0) - \overline{w}_N)e^{\eta_1 t}, \tag{32}$$

$$n_S(t) = \overline{n}_S + \frac{\eta_1 - a_{11}}{a_{12}}(n_S(0) - \overline{n}_S), \tag{33}$$

where $w_N(0)$ and $n_S(0)$ are the initial values of $w_N(t)$ and $n_S(t)$.

3.3. Steady-state social welfare

This section derives steady-state social welfare in the North and in the South. The steady-state price factor equals $\overline{P} = (1 - \overline{n}_S)\log\lambda$, indicating that the average price is determined by the steady-state value of the extent of Southern production ($\overline{n}_S$). Because at the steady state the innovation rate is constant, the expected number of innovations arriving in period *t* is $\overline{\phi}_R t$. Equation (18) indicates that the instantaneous utility at the steady state is:

$$log\, \bar{u}_k(t) = log\bar{E}_k - (1 - \bar{n}_S)\log\lambda + \bar{\phi}_R t log\lambda.$$

From (4), we derive the steady-state lifetime utility representing steady-state social welfare as:

$$\bar{U}_k(0) = \frac{1}{\rho}\left(log\bar{E}_k - (1 - \bar{n}_S)\log\lambda + \frac{\bar{\phi}_R}{\rho} log\lambda \right). \quad (34)$$

4. EFFECTS OF STRENGTHENING IPR PROTECTION

We are now ready to examine the long-run and short-run effects of strengthening IPR protection on key macroeconomic variables and social welfare for Northern and Southern consumers under two scenarios, depending on the target of IPR protection.

4.1. Strengthening the Northern standard of IPR protection

We first examine the long-run effects of strengthening the Northern standard of IPR protection.[12] Such a policy change is represented by a decrease in ϕ_{SN}. Note that (26) is used to derive the long-run rate of innovation. In Appendix C, we totally differentiate (26) with respect to $\bar{\phi}_R$ and ϕ_{SN} and find that $\frac{d\bar{\phi}_R}{d\phi_{SN}} > 0$, indicating that such a strengthening policy induces a decrease in the rate of innovation. Strengthening the Northern standard of IPR protection reduces imitation risk of Northern-produced goods, thereby motivating Northern firms to produce in the North. We find that there is an increase in the long-run sale of Northern-produced goods ($\bar{n}_N \frac{\bar{E}}{\lambda}$), implying that Northern labor employed by the production sector will increase, which crowds out labor employed in the R&D sector in the long run. As a result, the long-run rate of innovation will decrease.

Proposition 2. Strengthening the Northern standard of IPR protection reduces the rate of innovation in the long run.

Proof. See Appendix C.

Equation (25) indicates that this policy change affects the long-run North-South relative wage directly or indirectly through its effect on the long-run rate of innovation. Strengthening the Northern standard of IPR protection reduces imitation risk of Northern-produced goods, thereby motivating Northern firms to produce in the North. The no-arbitrage condition (20) implies that the market value of a Northern firm equals the sum of the present value of its instantaneous profit flow; that is, $\bar{v}_N =$

[12] Appendix C provides the details of calculation for the effects of strengthening Northern IPR standard on endogenous variables.

$\frac{\bar{\pi}_N}{\rho+\bar{\phi}_R+\phi_{SN}}$. Therefore, a lower ϕ_{SN} *ceteris paribus* raises $\bar{v}_N$, increasing the motivation of Northern production, raising the demand for Northern labor, and causing an increase in the North-South relative wage in the long run (the direct effect). Concerning the indirect effect, (25) implies that $\frac{\partial \bar{w}_N}{\partial \bar{\phi}_R} < 0$, meaning that a decrease in the long-run innovation rate will cause an increase in the long-run North-South relative wage (the indirect effect). Note that both the direct effect and the indirect effect show that the North-South relative wage will increase with such a strengthening policy.

We then decompose the effects of stronger IPR protection on the long-run sale of Northern-produced goods by separately analyzing its effects on the extent of Northern production and global expenditure. Although such strengthening reduces the imitation risk for Northern-produced goods and motivates firms to produce in the North, the increased North-South relative wage motivates firms to shift their production bases to the South. Appendix C shows that:

$$\frac{d\bar{n}_N}{d\phi_{SN}} = \frac{1}{\bar{\phi}_R + \phi_{SN} + \lambda(\phi_{SF} - \phi_{SN}) + \rho}\left[(\lambda-1)\bar{n}_N - (\lambda - 1 + \bar{n}_N)\left(\frac{d\bar{\phi}_R}{d\phi_{SN}}\right)\right].$$

This implies that strengthening the Northern standard of IPR protection will cause an overall decrease in the long-run extent of Northern production if such a strengthening policy causes a small change in innovation rate such that $\left(\frac{d\bar{\phi}_R}{d\phi_{SN}}\right) < \frac{(\lambda-1)\bar{n}_N}{\lambda-1+\bar{n}_N}$.

Regarding the effect of strengthening the Northern standard of IPR protection on global expenditure in the long run, Appendix C shows that:

$$\frac{d\bar{E}}{d\phi_{SN}} = \frac{\lambda a_R}{\lambda - 1}\left[\left(\frac{d\bar{\phi}_R}{d\phi_{SN}}\right) - (\lambda - 1)\right].$$

Note that if the change in the innovation rate induced by such a policy change is small enough, like $\left(\frac{d\bar{\phi}_R}{d\phi_{SN}}\right) < \frac{(\lambda-1)\bar{n}_N}{\lambda-1+\bar{n}_N}$, then $\left(\frac{d\bar{\phi}_R}{d\phi_{SN}}\right) < \lambda - 1$, implying that $\frac{d\bar{E}}{d\phi_{SN}} < 0$. This strengthening policy will thus raise global expenditure in the long run. We now have our next proposition.

Proposition 3. Strengthening the Northern standard of IPR protection leads to an increase in the long-run North-South relative wage. Moreover, if such a strengthening policy does not cause a large change in the long-run rate of innovation, then the long-run global expenditure will increase.

Proof. See Appendix C.

A decrease in the long-run extent of Northern production implies that Northern firms shift production to the South, indicating that there is an increase in the long-run extent of FDI. Since such a strengthening policy reduces the motivation for Southern firms to imitate Northern-produced goods, the long-run extent of Southern production imitating Northern-produced goods will decrease while the long-run extent of Southern production imitating Southern-produced goods by multinationals will increase. We summarize the results of the production pattern in the following proposition.

Proposition 4. Strengthening the Northern standard of IPR protection decreases the extent of Northern production and the extent of Southern production imitating Northern-produced goods while raising the extent of FDI and the extent of Southern production imitating Southern-produced goods by multinationals in the long run, provided that such a policy change does not cause a large change in the long-run rate of innovation.

Proof. See Appendix C.

Since a theoretical analysis may not be able to provide clear results of the effects on some endogenous variables and welfare, we next conduct a numerical analysis. For the benchmark model, we assign the discount factor $\rho = 0.01$ to generate a 1% real interest rate at the steady state. The one-stage quality improvement is set at $\lambda = 1.35$ to match the Northern markup of 35%.[13] We assign the Northern population to 0.9 and the Southern population to 1.35 so that the North-South relative wage is about 1.2.[14] Equation (16) indicates that the labor requirement for innovative R&D ($a_R \phi_R$) cannot be much larger than the Northern population in order to generate a positive value of $\overline{n}_N$. Thus, the labor intensity for innovative R&D (a_R) is set at 1.5 in order to generate a positive value of $\overline{n}_N$. We set ϕ_{SN} to 0.05 and ϕ_{SF} to 0.25, but we allow their values to vary in order to examine their impacts.[15]

Based on our parameterization, the North-South relative wage equals 1.20 and global expenditure equals 2.44 at the steady state. The resultant steady-state rate of innovation is 0.09. The respective extents of Northern production, FDI, Southern production imitating Northern-produced goods, and Sothern production imitating Southern-produced goods by multinationals are 42.23%, 9.29%, 23.08%, and 25.40% at the steady state. Table 2 presents the benchmark values and also summarizes the effects of strengthening IPR protection on the key macroeconomic variables and welfare. Under our parameterization, there are one positive eigenvalue and one negative eigenvalue ($\eta_1 < 0$ and $\eta_2 > 0$) in all numerical cases, indicating that the steady-state equilibrium is stable in all these cases.

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13 The parameter λ is set to 1.35 in Dinopoulos and Segerstrom (1999) and 1.3 in Cozzi and Impullitti (2016).

14 Cozzi and Impullitti (2016) mention that the data show that the ratio of skilled wage to unskilled wage equals 1.28. In our model, since innovation occurs only in the North, then Northern labor can be considered as skilled labor and Southern labor can be considered as unskilled labor.

15 Ideally, the data of $\overline{n}_N$, $\overline{n}_F$, $\overline{n}_{SN}$, and $\overline{n}_{SF}$ can be used to calibrate parameter values of L_N, L_S, ϕ_{SN}, and ϕ_{SF}. Unfortunately, such data are not available. In Appendix F, we summarize studies conducting numerical analysis based on a product-cycle model. Their calibration results are also given in Appendix F. Appendix F indicates that there is a wide variety of parameter values. Moreover, these parameter values generate very different results for the production pattern.

<Table 2 is inserted about here>

In order to analyze the effects of strengthening the Northern standard of IPR protection, we let ϕ_{SN} decrease by 5% from 5% to 4.75% (that is, $5\% * 0.95 = 4.75$). We find that a decrease in ϕ_{SN} by 5% raises the long-run North-South relative wage rate by 0.23% and long-run global expenditure by 0.10%. The long-run rate of innovation decreases by 0.48%. The extent of Northern production decreases by 0.02%, while the extent of FDI increases by 2.70% in the long run, resulting in a decrease in the extent of Southern production imitating Northern-produced goods by 4.56% and an increase in the extent of Southern production imitating Southern-produced goods by multinationals by 3.19% in the long run. As a result, the steady-state extent of Southern production decreases by 0.50%.

Ever since the study of Grossman and Helpman (1991), the literature has focused on examining the effects of IPR protection on the steady-state welfare.[16] Regarding changes to the steady-state welfare for Northerners and Southerners in response to such a policy change, we differentiate (34) with respect to ϕ_i, $i = SF, SN$, and obtain:[17]

$$\frac{d\bar{U}_k(0)}{d\phi_i} = \frac{1}{\rho}\left[\frac{1}{\bar{E}_k}\left(\frac{d\bar{E}_k}{d\phi_i}\right) + \left(\frac{d\bar{n}_S}{d\phi_i}\right)\log\lambda + \frac{log\lambda}{\rho}\left(\frac{d\overline{\phi}_R}{d\phi_i}\right)\right]. \quad (35)$$

Equation (35) indicates that strengthening the Northern standard of IPR protection affects the Northern (Southern) welfare through consumers' expenditure in the North (South), the average price, and innovation rate. An increase in Northern (Southern) consumers' expenditure means that Northern (Southern) consumers consume more goods, while an increase in the rate of innovation allows consumers to enjoy a better quality of products. These two effects are beneficial to Northern (Southern) consumers. However, a higher price factor reduces consumers' purchasing power and is harmful to the welfare of Northern (Southern) consumers.

A decrease in ϕ_{SN} by 5% raises the steady-state price factor by 0.47%. The decrease in the steady-state rate of innovation and the increase in the steady-state price factor hurt the steady-state welfare for both Northerners and Southerners. However, the increase in the steady-state Northern (Southern) consumers' expenditure by 0.17% (0.04%) benefits Northern (Southern) welfare.[18] We evaluate the welfare gains/losses as the percentage change in consumption necessary to equate the initial levels of welfare to what they would be following a policy change. Our numerical results show that under our parameterization, such a policy change will generate a 1.21% steady-state welfare loss

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[16] Grossman and Helpman (1991) consider a closed economy and show that it must jump immediately to the steady state in response to a policy change. Therefore, there is no need to consider welfare changes during the transitional dynamics.

[17] In this analysis, we consider welfare at the steady-state equilibrium and do not consider welfare changes during the transitional dynamics.

[18] We explain how strengthening the Northern standard of IIPR protection on the Northern and Southern consumers' expenditures in the next paragraph.

in consumption equivalence for Northerners and a 18.41% steady-state welfare loss in consumption equivalence for Southerners.

Tanaka and Iwaisako (2014) examine how welfare changes in response to strengthening IPR protection during the transitional dynamics. As shown in Table 2, the absolute value of the negative eigenvalue (η_1) is not large, indicating that the speed of convergence is slow (see (32) and (33)). Since such strengthening policy affects economic variables for sustained periods of time, we need to consider their accumulated effects on welfare during the transition from one equilibrium to another. Assuming that the economy is initially at the steady state and the strengthening policy is implemented at $t = 0$, Figure 1 presents the transitional dynamics of the North-South wage gap, the rate of innovation, global expenditure, and the pattern of production. It depicts both temporary (short-run) and permanent (long-run) effects of the strengthening policy on economic performance. It shows that the North-South relative wage jumps upward right after the policy change, then decreases, and converges to the new steady state. Conversely, the innovation rate jumps downward after the policy change, then increases and converges to the new steady state. Regarding the pattern of production, the extents of Northern and Southern production decrease over time while the extent of FDI increases over time.

<Figure 1 is inserted about here>

Equation (18) indicates that strengthening IPR protection affects the instantaneous utility through the consumers' expenditure, the extent of Southern production (or the price factor), and the innovation rate. Following Tanaka and Iwaisalo (2014), we assume that Southern consumers initially possess no assets. With this assumption and along with the normalization of the Southern wage rate to 1, the intertemporal budget constraint implies that the initial Southern consumer's expenditure is $E_S(0) = 1$. Equation (5) indicates the dynamic behavior of $E_S(t)$ follows $\frac{\dot{E}_S(t)}{E_S(t)} = r(t) - \rho$. Note that the transitional dynamics of the real interest rate are characterized by (29). From the definition of global expenditure, we derive the transitional dynamics of Northern consumers' expenditure as $E_N(t) = \frac{E(t) - L_S E_S(t)}{L_N}$. Figure 2 presents the transitional dynamics of the real interest rate, the price factor, the Northern consumers' expenditure, and the Southern consumers' expenditure. It indicates that the price factor, the Northern consumers' expenditure, and the Southern consumers' expenditure all increase over time.

<Figure 2 is inserted about here>

We find that there are welfare losses for both Northerners and Southerners if we consider the accumulated effects during transitional dynamics caused by strengthening the Northern standard of IPR protection. Our quantitative analysis suggests that such losses are in the order of 62.44% for Northerners and 62.49% for Southerners in consumption equivalence. These results demonstrate that if we do not consider the transitional dynamics, then we would underestimate welfare losses caused by an increase in the Northern standard of IPR protection.

4.2. Strengthening the Southern standard of IPR protection

We next turn to examine the long-run effects of strengthening the Southern standard of IPR protection.[19] Such a policy change is represented by a decrease in ϕ_{SF}. In Appendix D, we totally differentiate (26) with respect to $\overline{\phi}_R$ and ϕ_{SF} and find that $\frac{d\overline{\phi}_R}{d\phi_{SF}} < 0$, indicating that such a strengthening policy will cause an increase in the rate of innovation. Increasing the Southern standard of IPR protection reduces the imitation risk for Southern-produced goods by multinationals, thereby motivating Northern firms to shift production to the South. We find that there is a decrease in the long-run sale of Northern-produced goods ($\overline{n}_N \frac{\bar{E}}{\lambda}$), meaning that Northern labor employed by the production sector will decrease in the long run. With more labor released from the production sector in the North, employment in the R&D sector increases, resulting in a rise in the rate of innovation in the long run.

Proposition 5. Strengthening the Southern standard of IPR protection raises the rate of innovation in the long run.

Proof. See Appendix D.

We use (25) to examine the direct and indirect effects of such a strengthening policy on the long-run North-South relative wage. Increasing the Southern standard of IPR protection reduces imitation risk for Southern-produced goods by multinationals, thereby motivating Northern firms to shift production to the South. The no-arbitrage condition (21) indicates that the market value of multinationals equals the sum of the present value of the instantaneous profit flow; that is, $\overline{v}_F = \frac{\overline{\Pi}_F}{\rho+\overline{\phi}_R+\phi_{SF}}$. A lower ϕ_{SF} *ceteris paribus* raises $\overline{v}_F$, increasing the motivation of Northern firms to produce in the South. This will reduce the demand for Northern labor and increase the demand for Southern labor, causing a reduction in the North-South relative wage in the long run (the direct effect). Concerning the indirect effect, (25) implies that an increase in the long-run innovation rate will reduce the long-run North-South relative wage (the indirect effect). Therefore, the North-South relative wage will decrease in the long run.

[19] Appendix D provides the details of calculation for the effects of strengthening the Southern standard of IPR protection.

We also show in Appendix D that:

$$\frac{d\bar{E}}{d\phi_{SF}} = \frac{\lambda^2 a_R}{(\lambda - 1)\xi_1(\overline{\phi}_R + \phi_{SF})}\left\{\lambda[\overline{\phi}_R + \phi_{SN} + \lambda(\phi_{SF} - \phi_{SN})] - \frac{(\lambda - 1)L_S}{a_R}\right\}.$$

Therefore, if $L_S < \frac{\lambda a_R}{\lambda - 1}[\lambda(\phi_{SF} - \phi_{SN}) + \phi_{SF}]$, then $\frac{d\bar{E}}{d\phi_{SF}} > 0$. Therefore, such a strengthening policy will raise global expenditure in the long run if the Southern population is sufficiently small. We thus have the next proposition.

Proposition 6. Strengthening the Southern standard of IPR protection results in a decrease in the long-run North-South relative wage. Moreover, such a strengthening policy will reduce global expenditure in the long run, provided that Southern population is sufficiently small.

Proof. See Appendix D.

Concerning the pattern of production, we show that $\frac{d\bar{n}_N}{d\phi_{SF}} > 0$ in Appendix D, implying that such a policy change results in a reduction in the long-run extent of Northern production. Since Northern firms shift production to the South, the extent of FDI increases. We then use the steady-state condition for the extent of Sothern production imitating Northern-produced goods (equation (27)) to determine the change of extent of Southern production imitating Northern-produced goods. With a decrease in the long-run extent of Northern production, the flows going into Southern production imitating Northern-produced goods decrease. At the same time, an increase in the long-run rate of innovation spurs the flows going out of Southern production imitating Northern-produced goods. Therefore, the steady-state extent of Sothern production imitating Northern-produced goods must decrease in order to raise the flows going out of Southern production imitating Northern-produced goods so as to restore the steady-state condition.

Although an increase in the Southern standard of IPR protection reduces the motivation of Southern firms to imitate Southern-produced goods by multinationals, the released Southern labor from the sector of Sothern production imitating Northern-produced goods raises the extent of Southern production imitating Southern-produced goods by multinationals. As a result, there is an ambiguous change in the extent of Southern production imitating Southern-produced goods by multinationals. We summarize the results of this production pattern in the following proposition.

Proposition 7. Increasing the Southern standard of IPR protection causes the extent of Northern production and the extent of Southern production imitating Northern-produced goods to decrease and the extent of FDI to increase in the long run. However, the change in the long-run extent of Southern production imitating Southern-produced goods by multinationals is ambiguous.

Proof. See Appendix D.

In order to examine the effects of strengthening the Southern standard of IPR protection, we let ϕ_{SF} decrease by 5% from 25% to 23.75% (that is, $25\% * 0.95 = 23.75\%$) and examine its effects on macroeconomic performance and welfare. Table 2 presents the results.[20] A decrease in ϕ_{SF} by 5% in the Southern standard of IPR protection reduces the North-South relative wage by 1.14% and the global expenditure by 0.49% while raising the rate of innovation by 16.17% in the long run. The extent of Northern production decreases by 2.43% and the extent of FDI increases by 31.09% in the long run. Moreover, the extent of Southern production imitating Northern-produced goods decreases by 16.01% and the extent of Southern production imitating Southern-produced goods by multinationals increases by 7.21% in the long run. As a consequence, the extent of Southern production decrease by 3.85%.

Although the increase in the steady-state rate of innovation benefits the steady-state welfare for both Northerners and Southerners, the rise in the steady-state price factor by 3.62% reduces the steady-state welfare for both Northerners and Southerners. The steady-state Northern consumers' expenditure decreases by 1.63%, while the steady-state Southern consumers' expenditure increases by 0.34%. There are overall increases in the steady-state Northern and Southern welfare of 52.48% and 28.69% in consumption equivalence, respectively.

Figure 3 presents the transitional dynamics of the North-South wage gap, the rate of innovation, global expenditure, and the pattern of production after the strengthening policy of the Southern standard of IPR protection is implemented. It shows that the North-South relative wage jumps downward right after the policy change, then decreases, and converges to the new steady state. The innovation rate jumps upward after the policy change, then increases, and converges to the new steady state. Regarding the pattern of production, the extents of Northern and Southern production decrease over time while the extent of FDI increases over time.

<Figure 3 is inserted about here>

Concerning welfare changes during the transitional dynamics, we present the transitional dynamics of the real interest rate, the price factor, the Northern consumers' expenditure, and the Southern consumers' expenditure in response to an increase in the Southern standard of IPR protection in Figure 4. It indicates that the price factor and the Southern consumers' expenditure increase over time. The Northern consumers' expenditure jumps downward right after the policy change and then increases over time. In contrast to the changes of steady-state welfare, there are welfare losses for both Northerners

[20] Because ϕ_{SF} is larger than ϕ_{SN} under our parametrization, then a 5% decrease of ϕ_{SF} will cause larger impacts on endogenous variables than a 5% decrease of ϕ_{SN}.

and Southerners if we consider the accumulated effects caused by strengthening the Southern standard of IPR protection during the transitional dynamics. Our quantitative analysis suggests that such losses are in the order of 56.09% for Northerners and 55.19% for Southerners in consumption equivalence. Comparing results in our two scenarios under two IPR protection policies, we find that when examining the effects of stronger IPR protection on welfare, we should consider the accumulated effects caused by stronger IPR protection during the transitional dynamics. If we only consider changes in the steady state and ignore the accumulated effects during the transitional dynamics, the conclusions would be misleading or wrong.

<Figure 4 is inserted about here>

Comparing the effects caused by these two IPR protection policies, we find that the target of strengthening IPR protection matters when concerning its effects on the North-South relative wage and the rate of innovation. While increasing the Northern standard of IPR protection raises the North-South relative wage and reduces the rate of innovation, strengthening the Southern standard of IPR protection causes the opposite effects on these two variables.[21] Concerning the production pattern, our findings indicate that the strengthening of IPR protection leads to decreases in the extent of Northern production and the extent of Southern production imitating Northern-produced goods while raising the extent of FDI and the extent of Southern production imitating Southern-produced goods by multinationals, regardless of the targets of IPR protection. Table 3 summarizes the results found in Sections 4.1 and 4.2.

<Table 3 is inserted about here>

4.3. Discussion

We summarize the related studies that examine the influences of stronger IPR protection in a product-cycle model with FDI or outsourcing in Table 1. One important feature of this paper is that Southern firms can imitate either Northern-produced goods or Southern-produced goods by multinationals at different rates. The spirit of this feature is similar to the study of Glass and Saggi (2002). However, our model differs from their model in many ways. For example, the settings of imitation and the cost of FDI activities in this paper are all different from theirs.[22] More importantly, we assume that imitation rates are exogenous, so that we are able to separately analyze the effects of strengthening

[21] For studies finding that strengthening IPR protection raises the rate of innovation, see Glass and Wu (2007), Tanaka and Iwaisako (2014), and Chen (2015b). For studies finding that strengthening IPR protection reduces the rate of innovation, see Glass and Saggi (2001) and Chen (2015c).

[22] While we assume that imitation is costless and Northern firms can become multinational firms without any cost in this paper, they assume that imitation and FDI activities are costly. We also investigate the stability of the steady-state equilibrium and examine welfare changes in responses to changes of IPR protection whereas they do not conduct such an analysis.

IPR protection when its target is Northern-produced goods or Southern-produced goods by multinationals. However, they assume that the rate imitation is endogenous and that stronger IPR protection is represented by an increase in labor units used for imitation activities. As a result, stronger IPR protection raises *both* labor units required for imitation under their setting of the model and affects the rates of imitation for both Northern-produced goods and Southern-produced goods by multinationals at the same time. Therefore, they are not able to separate the effects caused by strengthening IPR protection on different targets. Because our model setting is different from theirs in many ways, the results herein are different from theirs. For example, they show that strengthening IPR protection impedes both innovation rate and FDI while the North-South relative wage is immune to such policy change. Depending on the target of stronger IPR protection, we show that such a policy change may increase or decrease innovation rate and the North-South relative wage. Furthermore, the extent of FDI will increase, regardless of the object of stronger IPR protection.

It is commonly known that the setting of an R&D model affects not only the impacts of strengthening IPR protection on macroeconomic performance qualitatively, but also the stability property of the steady-state equilibrium. Therefore, before analyzing the impacts of strengthening IPR protection on the steady-state equilibrium, we should make sure that the steady-state equilibrium is attainable. The analysis of Tanaka and Iwaisako (2014) introduces government subsidies on R&D and FDI into the case of inefficient followers in Glass and Wu (2007) and shows that these subsidies on R&D and FDI are important determinants for the stability of the steady-state equilibrium in a product-cycle model where innovation targets only imitated goods. If the government does not subsidize R&D and FDI, then the steady-state equilibrium is not attainable in Glass and Wu (2007); however, the steady-state equilibrium will be stable if these subsidies of R&D and FDI are sufficiently large.

In Appendix E we consider a model where Southern firms only imitate Southern-produced goods by multinationals by setting $\phi_{SN} = 0$ and $n_{SN} = 0$.[23] This corresponds to the case of efficient followers in Glass and Wu (2007).[24] We show that the steady state is determinant if $(\lambda - 1)\left[\rho\bar{n}_{SF} + \frac{(\lambda-1)\phi_{SF}L_S}{a_R(\rho+\bar{\phi}_R+\lambda\phi_{SF})}\right] > (2-\lambda)(\bar{\phi}_R + \lambda\phi_{SF})$.[25] The numerical analysis indicates that under our parameterization in Section 4.1, the two eigenvalues of the Jacobian matrix for the dynamical system represented in $\{w_N(t), n_{SF}(t)\}$ are -0.3163 and 1.5551, implying that the steady-state equilibrium is determinant since $n_{SF}(t)$ is a state variable and $w_N(t)$ is a jump variable. This

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[23] In this specific case, there is no need to consider the law of motion of n_{SN} (equation (13)).

[24] The Proposition 3 in Glass and Wu (1007) demonstrates that in this case, stronger IPR protection will increase innovation rate and extent of FDI while at the same time reducing global expenditure, the North-South relative wage, and the extent of Northern production. However, they do not analyze the stability property of the steady-state equilibrium. In Appendix E, we complete their analysis by examining the stability property of the steady-state equilibrium.

[25] See Appendix E for the details of calculation.

result demonstrates that allowing innovation to target all types of goods is another way to obtain a stable steady-state equilibrium. Moreover, one should be more cautious with the results based on a product-cycle model with inefficient followers.

5. CONCLUSION

This paper studies the effects of strengthening-IPR-protection policies on rate of innovation, the North-South relative wage, and the pattern of production based on a product-cycle model with FDI. We also examine the changes of social welfare for Northern (Southern) consumers in response to the strengthening of IPR protection.

We find that the target of strengthening IPR protection matters when concerning its effects on the rate of innovation and the North-South relative wage, but the target of strengthening IPR protection does not matter when concerning its effects on the production pattern. When the strengthening of IPR protection is applied to goods produced in the North, such a policy raises the North-South relative wage while reducing the rate of innovation. The extents of Northern production and Southern production imitating Northern-produced goods decrease while the extents of FDI and Southern production imitating Southern-produced goods by multinationals increase.

When the strengthening of IPR protection is applied to goods produced in the South by multinationals, this policy generates the reverse effects on the North-South relative wage and the rate of innovation, but it causes the same effects on the production pattern as when the strengthening of IPR protection is applied on Northern-produced goods. In Tanaka and Iwaisako (2014), strengthening IPR protection means that the risk of imitation of goods produced by multinationals decreases. In our second scenario, we find that if the policy of stronger IPR protection is applied to Southern-produced goods by multinationals, then such a policy will promote both innovation rate and FDI. These findings are consistent with theirs.[26]

A few notes are worth discussing. First, the imitation risk is exogenous in the model, and we use an exogenous reduction in imitation risk to represent the strengthening of IPR protection. However, Southern firms can devote labor input to the imitation sector to increase the rate of imitation. It would thus be interesting to endogenize the imitation risk and examine the effects of IPR protection. Second, our paper focuses on a theoretical analysis of the long-run effects of stronger IPR protection. The results shown herein provide a direction for empirical study, and it is important to know whether the data support these results.

[26] The major difference between our second scenario and what Tanaka and Iwaisako (2014) present is that in order to allow the economy to converge to a stable nontrivial steady-state equilibrium, we assume that innovation targets all types of goods, whereas they assume that the government subsidizes R&D and FDI.

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Table 1 Related studies

	Imitation targets only Southern-produced goods by multinationals	Imitation targets Northern-produced goods and Southern-produced goods by multinationals at different rates
Innovation targets only imitated goods	1. Glass and Wu (2007) (inefficient followers) 2. Tanaka and Iwaisako (2014)	Glass and Saggi (2002)
Innovation targets all types of goods	1. Glass (2004) 2. Glass and Wu (2007) (efficient followers) 3. This paper (ϕ_{SN}=0 and $n_{SN} = 0$)	This paper

Notes: Glass (2004) develops a model with outsourcing to study the influences of stronger IPR protection on the steady-state equilibrium. However, her study does now analyze the stability of the steady-state equilibrium and the transitional dynamics.

Table 2 Numerical results of the effects of strengthening IPR protection

Variables	Equilibrium values	ϕ_{SN} down by 5%	ϕ_{SF} down by 5%
Panel A: Effects on key variables			
$\overline{w}_N$	1.1992	0.2286	-1.1388
$\bar{E}$	2.4385	0.1038	-0.4947
$\overline{\phi}_R$	0.0915	-0.4782	16.1687
$\bar{n}_N$	0.4223	-0.0178	-2.4256
$\bar{n}_F$	0.0929	2.6953	31.0941
$\bar{n}_{SN}$	0.2308	-4.5605	-16.0063
$\bar{n}_{SF}$	0.2540	3.1887	7.2056
$\bar{n}_S$	0.4848	-0.5011	-3.8468
Panel B: Welfare gain/loss in consumption equivalence (steady state)			
$\overline{E}_N$	1.2095	0.1654	-1.6266
$\overline{E}_S$	1.0000	0.0447	0.3448
$\bar{P}$	0.1546	0.4715	3.6197
welfare(North)	278.0658	-1.2128	52.4789
welfare(South)	259.0484	-18.4121	28.6911
Panel C: Welfare gain/loss in consumption equivalence (transitional dynamics)			
welfare(North)	278.0658	-62.4414	-56.0889
welfare(South)	259.0484	-62.4905	-55.1872
Panel D: Stability property (eigenvalues)			
η_1	-0.3093	-0.3096	-0.3117
η_2	1.6282	1.6245	1.6272

Notes: All figures in the columns of ϕ_{SN} down by 5% and ϕ_{SF} down by 5% in Panels A, B, and C refer to the percentage changes in the key variables from their equilibrium values as a result of changes in IPR protection policies.

Table 3 The effects of strengthening IPR protection

	Strengthening IPR protection on Northern-produced goods	Strengthening IPR protection on Southern-produced goods by multinationals
	$\phi_{SN}\downarrow$	$\phi_{SF}\downarrow$
$\overline{w}_N$	$\uparrow$	$\downarrow$
$\bar{E}$	$\uparrow$	$\downarrow$
$\overline{\phi}_R$	$\downarrow$	$\uparrow$
$\bar{n}_N$	$\downarrow$	$\downarrow$
$\bar{n}_F$	$\uparrow$	$\uparrow$
$\bar{n}_{SN}$	$\downarrow$	$\downarrow$
$\bar{n}_{SF}$	$\uparrow$	N/A

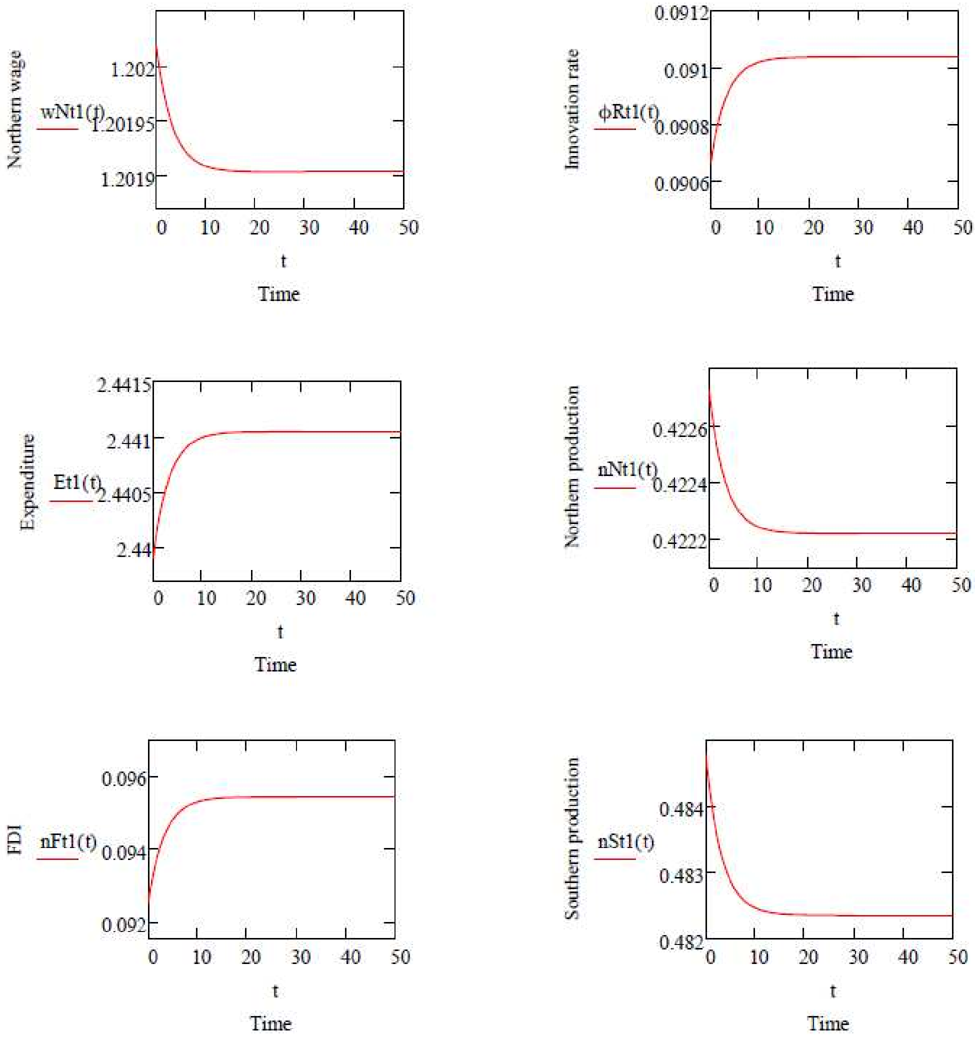

Figure 1. Transitional dynamics of macroeconomic variables
when ϕ_{SN} decreases by 5%

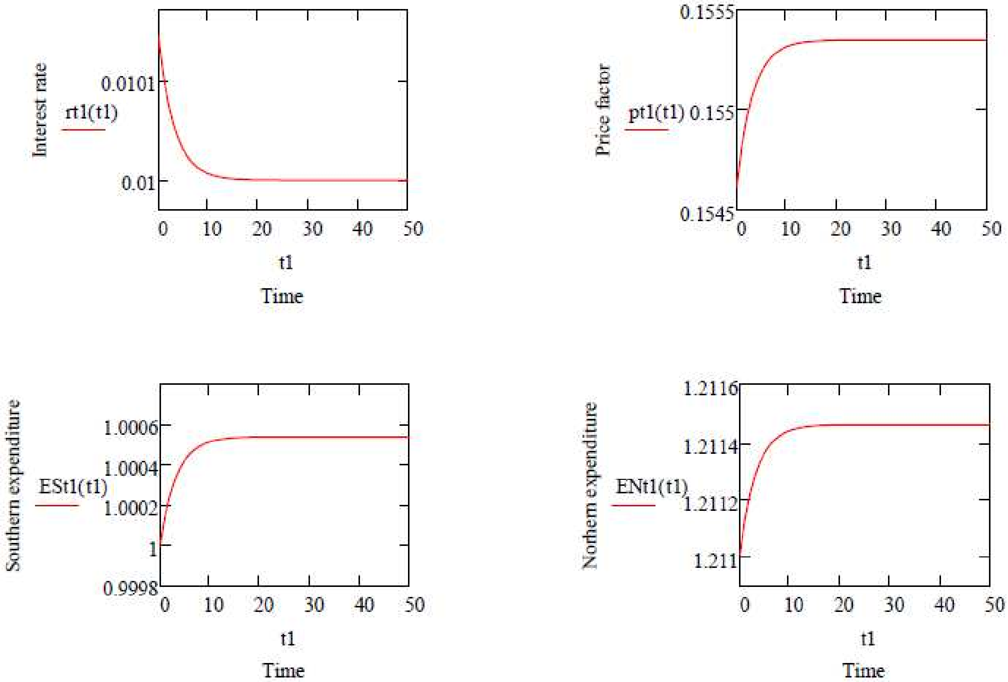

Figure 2. Transitional dynamics of macroeconomic variables relating to welfare when ϕ_{SN} decreases by 5%

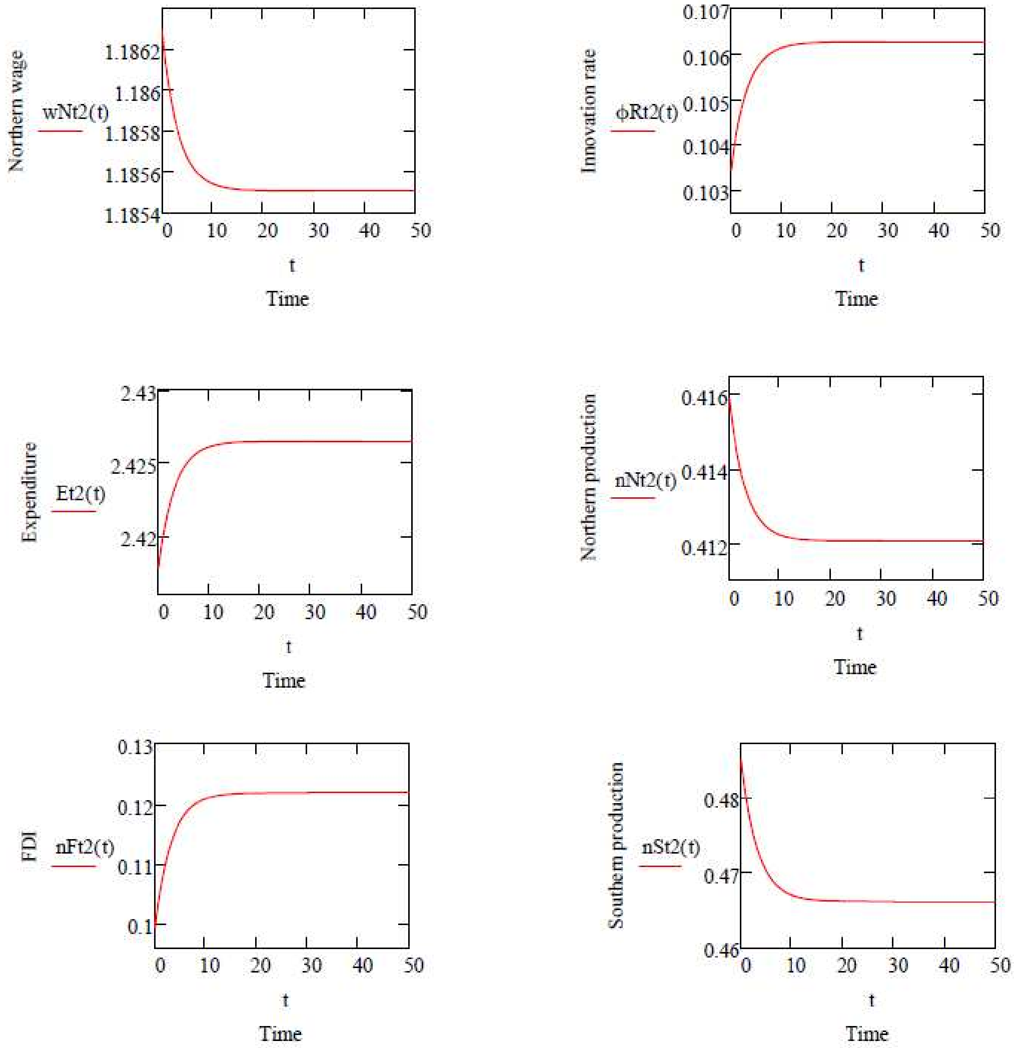

Figure 3. Transitional dynamics of macroeconomic variables
when ϕ_{SF} decreases by 5%

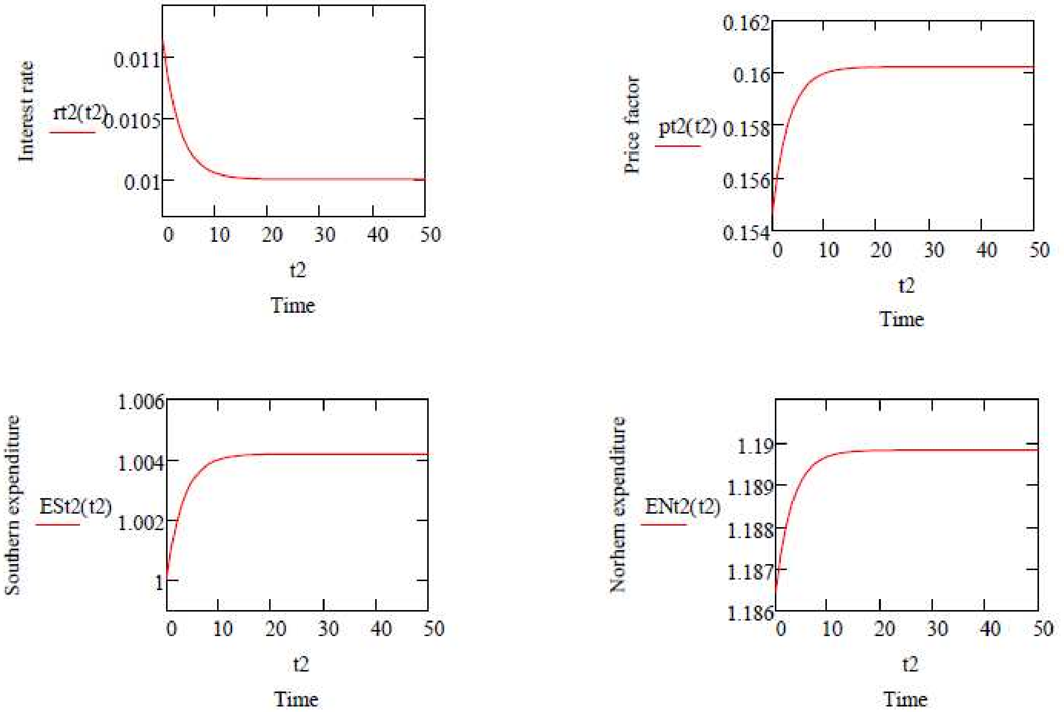

Figure 4. Transitional dynamics of macroeconomic variables relating to welfare when ϕ_{SF} decreases by 5%

APPENDIX A

The steady-state equilibrium

In this appendix, we derive the steady-state equilibrium. We use variables with an upper bar to denote the steady-state values of the corresponding variables. In Appendix B we show that $\frac{\dot{E}(t)}{E(t)} = \dot{v}_N(t) = \dot{v}_F(t) = 0$ and $r(t) = \rho$ in the long run. The no-arbitrage conditions of (10) and (11) can then be expressed as:

$$\bar{v}_N = \frac{\bar{\Pi}_N}{\rho + \bar{\phi}_R + \phi_{SN}}, \tag{A1}$$

$$\bar{v}_F = \frac{\bar{\Pi}_F}{\rho + \bar{\phi}_R + \phi_{SF}}. \tag{A2}$$

Substituting (6) and (8) into (A1) gives us:

$$\frac{\bar{E}}{\lambda}(\lambda - \bar{w}_N) = \left(\rho + \bar{\phi}_R + \phi_{SN}\right) a_R \bar{w}_N. \tag{A3}$$

Substituting (6), (7), and (9) into (A2) yields:

$$\frac{\bar{E}}{\lambda}(\lambda - 1) = \left(\rho + \bar{\phi}_R + \phi_{SF}\right) a_R \bar{w}_N. \tag{A4}$$

At the steady-state equilibrium, the flows going into $n_{SN}(t)$ and $n_{SF}(t)$ equal the flows coming out of them - that is, $\dot{n}_{SN}(t) = \dot{n}_{SF}(t) = 0$. Thus, (13) and (14) indicate:

$$\phi_{SN}\bar{n}_N = \bar{\phi}_R \bar{n}_{SN}, \tag{A5}$$

$$\phi_{SF}\bar{n}_F = \bar{\phi}_R \bar{n}_{SF}. \tag{A6}$$

The steady-state equilibrium is characterized by (12), (16), (17), and (A3)-(A6) with seven variables $\{\bar{w}_N, \bar{E}, \bar{n}_N, \bar{n}_F, \bar{n}_{SN}, \bar{n}_{SF}, \bar{\phi}_R\}$. Moreover, the extent of Southern production at the steady state can be derived as $\bar{n}_S = \bar{n}_{SN} + \bar{n}_{SF}$.

Combining (A3) and (A4) yields:

$$\frac{\lambda - \bar{w}_N}{\lambda - 1} = \frac{\rho + \bar{\phi}_R + \phi_{SN}}{\rho + \bar{\phi}_R + \phi_{SF}}. \tag{A7}$$

Using (A7), we can express $\bar{w}_N$ as a function of $\bar{\phi}_R$; that is:

$$\bar{w}_N = \bar{w}_N(\bar{\phi}_R; \phi_{SN}, \phi_{SF}) = \frac{\rho + \bar{\phi}_R + \lambda\phi_{SF} - (\lambda - 1)\phi_{SN}}{\rho + \bar{\phi}_R + \phi_{SF}}, \tag{A8}$$

with $\frac{\partial \bar{w}_N(\bar{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \bar{\phi}_R} = \frac{-(\lambda-1)(\phi_{SF} - \phi_{SN})}{(\rho + \bar{\phi}_R + \phi_{SF})^2} < 0$, $\frac{\partial \bar{w}_N(\bar{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \phi_{SN}} = \frac{-(\lambda-1)}{\rho + \bar{\phi}_R + \phi_{SF}} < 0$, and $\frac{\partial \bar{w}_N(\bar{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \phi_{SF}} = \frac{(\lambda-1)(\rho + \bar{\phi}_R + \phi_{SN})}{(\rho + \bar{\phi}_R + \phi_{SF})^2} > 0$. Note that (A8) indicates that $\bar{w}_N > 1$.

Combining (A4) and (A8), the steady-state global expenditure ($\bar{E}$) can be expressed as a function of $\overline{\phi}_R$:

$$\bar{E}(\overline{\phi}_R;\phi_{SN},\phi_{SF}) = \frac{\lambda a_R\left[\lambda\phi_{SF} - (\lambda-1)\phi_{SN} + \rho + \overline{\phi}_R\right]}{\lambda-1}, \tag{A9}$$

with $\frac{\partial \bar{E}(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial \overline{\phi}_R} = \frac{\lambda a_R}{\lambda-1} > 0$, $\frac{\partial \bar{E}(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial \phi_{SN}} = -\lambda a_R < 0$, and $\frac{\partial \bar{E}(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial \phi_{SF}} = \frac{\lambda^2 a_R}{\lambda-1} > 0$.

Substituting (A9) into (16), we derive the steady-state extent of Northern production ($\overline{n}_N$) as a function of $\overline{\phi}_R$:

$$\bar{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF}) = \frac{\lambda(L_N - a_R\overline{\phi}_R)}{\bar{E}(\overline{\phi}_R;\phi_{SN},\phi_{SF})} = \frac{(\lambda-1)(L_N - a_R\overline{\phi}_R)}{a_R[\lambda\phi_{SF} - (\lambda-1)\phi_{SN} + \rho + \overline{\phi}_R]}, \tag{A10}$$

with $\frac{\partial \overline{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial \overline{\phi}_R} = -\frac{\lambda}{\bar{E}^2}\left[a_R\bar{E}(\overline{\phi}_R;\phi_{SN},\phi_{SF}) + (L_N - a_R\overline{\phi}_R)\left(\frac{\partial \bar{E}(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial \overline{\phi}_R}\right)\right] < 0$, $\frac{\partial \overline{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial \phi_{SN}} = -\frac{\lambda(L_N - a_R\overline{\phi}_R)}{\bar{E}^2}\left(\frac{\partial E(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial \phi_{SN}}\right) > 0$, and $\frac{\partial \overline{n}_N(\overline{\phi}_R\phi_{SN},\phi_{SF})}{\partial \phi_{SF}} = -\frac{\lambda(L_N - a_R\overline{\phi}_R)}{\bar{E}^2}\left(\frac{\partial \bar{E}(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial \phi_{SF}}\right) < 0$.

Substituting (A10) into (A5), we derive the steady-state extent of Southern production imitating Northern-produced goods ($\overline{n}_{SN}$) as a function of $\overline{\phi}_R$:

$$\bar{n}_{SN}(\overline{\phi}_R;\phi_{SN},\phi_{SF}) = \frac{\phi_{SN}\overline{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\overline{\phi}_R}, \tag{A11}$$

with $\frac{\partial \overline{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial \overline{\phi}_R} = \frac{\phi_{SN}}{\overline{\phi}_R^{\,2}}\left[\overline{\phi}_R\left(\frac{\partial \overline{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial \overline{\phi}_R}\right) - \overline{n}_N\right] < 0$, $\frac{\partial \overline{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial \phi_{SN}} = \frac{\phi_{SN}}{\overline{\phi}_R}\left(\frac{\partial \overline{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial \phi_{SN}}\right) > 0$, and $\frac{\partial \overline{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial \phi_{SF}} = \frac{\phi_{SN}}{\overline{\phi}_R}\left(\frac{\partial \overline{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial \phi_{SF}}\right) < 0$.

Combining (12), (A6), and (A11) allows us to express the steady-state extent of FDI ($\overline{n}_F$) as a function of $\overline{\phi}_R$:

$$\bar{n}_F\left(\overline{\phi}_R;\phi_{SN},\phi_{SF}\right) = \frac{\overline{\phi}_R - \left(\overline{\phi}_R + \phi_{SN}\right)\overline{n}_N\left(\overline{\phi}_R;\phi_{SN},\phi_{SF}\right)}{\overline{\phi}_R + \phi_{SF}}, \tag{A12}$$

with $\frac{\partial \overline{n}_F(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial \overline{\phi}_R} = \frac{1}{\overline{\phi}_R + \phi_{SF}}\left[1 - \overline{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF}) - (\overline{\phi}_R + \phi_{SN})\left(\frac{\partial \overline{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial \overline{\phi}_R}\right) - \overline{n}_F(\overline{\phi}_R;\phi_{SN},\phi_{SF})\right] > 0$, $\frac{\partial \overline{n}_F(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial \phi_{SN}} = -\frac{1}{\overline{\phi}_R + \phi_{SF}}\left[\overline{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF}) + (\overline{\phi}_R + \phi_{SN})\left(\frac{\partial \overline{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial \phi_{SN}}\right)\right] < 0$, and $\frac{\partial \overline{n}_F(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial \phi_{SF}} = -\frac{1}{\overline{\phi}_R + \phi_{SF}}\left[\overline{n}_F(\overline{\phi}_R;\phi_{SN},\phi_{SF}) + (\overline{\phi}_R + \phi_{SN})\left(\frac{\partial \overline{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial \phi_{SF}}\right)\right]$.

Substituting (A10)-(A12) into (12), we derive the steady-state extent of Southern production imitating Southern-produced goods by multinationals ($\bar{n}_{SF}$) as a function of $\overline{\phi}_R$:

$$\bar{n}_{SF}(\overline{\phi}_R;\phi_{SN},\phi_{SF}) = 1 - \bar{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF}) - \bar{n}_F(\overline{\phi}_R;\phi_{SN},\phi_{SF}) - \bar{n}_{SN}(\overline{\phi}_R;\phi_{SN},\phi_{SF}). \tag{A13}$$

Using (12), we replace $(\bar{n}_{SN} + \bar{n}_{SF})$ by $(1 - \bar{n}_N - \bar{n}_F)$ and re-write (17) as:

$$f(\overline{\phi}_R;\phi_{SN},\phi_{SF}) = L_S, \tag{A14}$$

where $f(\overline{\phi}_R;\phi_{SN},\phi_{SF}) = \left[\lambda(1 - \bar{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})) - (\lambda-1)\bar{n}_F(\overline{\phi}_R;\phi_{SN},\phi_{SF})\right]\frac{\bar{E}(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\lambda}$.

The steady-state equilibrium is represented by (A14), which we use to implicitly solve for $\overline{\phi}_R$.

Note that $f(0;\phi_{SN},\phi_{SF}) = \frac{[\lambda\phi_{SF}-(\lambda-1)\phi_{SN}][\lambda a_R\phi_{SF}-(\lambda-1)L_N]+\rho\lambda a_R\phi_{SF}}{(\lambda-1)\phi_{SF}}$ and $f(1;\phi_{SN},\phi_{SF}) = \frac{[\lambda\phi_{SF}-(\lambda-1)\phi_{SN}+1][a_R\lambda(1+\phi_{SF})-(\lambda-1)L_N]+\rho a_R(1+\lambda\phi_{SF})}{(\lambda-1)(1+\phi_{SF})}$. Moreover, using (A14), we derive:

$$\begin{aligned}
\xi_1 &= \frac{\partial f(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\overline{\phi}_R} \\
&= -\left[\lambda\left(\frac{\partial\bar{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\overline{\phi}_R}\right) + (\lambda-1)\left(\frac{\partial\bar{n}_F(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\overline{\phi}_R}\right)\right]\frac{\bar{E}}{\lambda} \\
&\quad + \frac{[\lambda(1-\bar{n}_N) - (\lambda-1)\bar{n}_F]}{\lambda}\left(\frac{\partial\bar{E}(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\overline{\phi}_R}\right) \\
&= -\frac{\overline{\phi}_R + \phi_{SN} + \lambda(\phi_{SF}-\phi_{SN})}{\overline{\phi}_R + \phi_{SF}}\left(\frac{\partial\bar{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\overline{\phi}_R}\right)\frac{\bar{E}}{\lambda} + \frac{\bar{n}_F}{\lambda}\left(\frac{\partial\bar{E}(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\overline{\phi}_R}\right) \\
&\quad + \frac{a_R(1-\bar{n}_N-\bar{n}_F)}{(\lambda-1)(\overline{\phi}_R+\phi_{SF})}\left\{\lambda\phi_{SF}(2-\lambda) + \overline{\phi}_R + (\lambda-1)[(\lambda-1)\phi_{SN} - \rho]\right\}.
\end{aligned}$$

If $\frac{\rho}{\lambda-1} < \phi_{SN}$, then $\frac{\partial f(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\overline{\phi}_R} > 0$ and $f(\overline{\phi}_R;\phi_{SN},\phi_{SF})$ is an increasing function in $\overline{\phi}_R$. Since $\overline{\phi}_R \in (0,1)$, then the solution of $\overline{\phi}_R$ exists if $f(0;\phi_{SN},\phi_{SF}) < L_S < f(1;\phi_{SN},\phi_{SF})$. We consider the following conditions.

Condition (P1) $\frac{\rho}{\lambda-1} < \phi_{SN}$.

Condition (P2) $f(0;\phi_{SN},\phi_{SF}) < L_S < f(1;\phi_{SN},\phi_{SF})$.

Under conditions (P1) and (P2), there exists a unique solution of $\overline{\phi}_R$. Once one derives the solution of $\overline{\phi}_R$, the remaining endogenous variables can be solved accordingly by using (A8)-(A12).

APPENDIX B

The dynamical system and stability of the steady-state equilibrium

In this appendix we derive the dynamical system of the economy and examine the stability of the steady-state equilibrium. Variables with an upper bar are used to denote the steady-state values of the corresponding variables. Since $\frac{\dot{E}_k(t)}{E_k(t)} = r(t) - \rho$, we thus have:

$$\frac{\dot{E}(t)}{E(t)} = \frac{\dot{E}_k(t)}{E_k(t)} = r(t) - \rho. \tag{B1}$$

Equation (7) indicates that $\dot{v}_F(t) = \dot{v}_N(t)$. Using (10) and (11), we have:

$$\Pi_F(t) - \Pi_N(t) = (\phi_{SF} - \phi_{SN})v_N(t). \tag{B2}$$

Substituting (8) and (9) into (B2) gives:

$$E(t) = \frac{\lambda(\phi_{SF} - \phi_{SN})v_N(t)}{w_N(t) - 1}. \tag{B3}$$

Substituting (6) into (B3) allows us to express $E(t)$ as a function of $w_N(t)$:

$$E(t) = E(w_N(t)) = \frac{\lambda(\phi_{SF} - \phi_{SN})a_R w_N(t)}{w_N(t) - 1}. \tag{B4}$$

Using (17) and (B4), we derive $n_F(t)$ as a function of $w_N(t)$ and $n_S(t)$:

$$\begin{aligned} n_F(t) &= n_F(w_N(t), n_S(t)) \\ &= \lambda\left[\frac{L_S(w_N(t) - 1)}{\lambda(\phi_{SF} - \phi_{SN})a_R w_N(t)} - n_{SN}(t) - n_{SF}(t)\right] \\ &= \lambda\left[\frac{L_S(w_N(t) - 1)}{\lambda(\phi_{SF} - \phi_{SN})a_R w_N(t)} - n_S(t)\right]. \end{aligned} \tag{B5}$$

Combining (12) and (B5), we express $n_N(t)$ as a function of $w_N(t)$ and $n_S(t)$:

$$n_N(t) = n_N(w_N(t), n_S(t)) = 1 - n_F(w_N(t), n_S(t)) - n_S(t). \tag{B6}$$

Substituting (B4) and (B6) into (16) yields:

$$\begin{aligned} \phi_R(t) &= \phi_R(w_N(t), n_S(t)) \\ &= \frac{1}{a_R}\left[L_N - \frac{(\phi_{SF} - \phi_{SN})a_R w_N(t)}{w_N(t) - 1} n_N(w_N(t), n_S(t))\right]. \end{aligned} \tag{B7}$$

Using (B4), we derive:

$$\frac{\dot{E}(t)}{E(t)} = \frac{-1}{w_N(t) - 1}\frac{\dot{w}_N(t)}{w_N(t)}. \tag{B8}$$

Combining (8) and (10), we have:

$$r(t) = \frac{\dot{v}_N(t)}{v_N(t)} + \frac{E(t)(\lambda - w_N(t))}{\lambda v_N(t)} - (\phi_R(t) + \phi_{SN}). \tag{B9}$$

Using (B4) to replace $E(t)$ in (B9) gives:

$$\frac{\dot{v}_N(t)}{v_N(t)} = r(t) - \frac{(\phi_{SF} - \phi_{SN})(\lambda - w_N(t))}{w_N(t) - 1} + \phi_R(t) + \phi_{SN}. \tag{B10}$$

Equation (6) indicates that $\frac{\dot{v}_N(t)}{v_N(t)} = \frac{\dot{w_N}(t)}{w_N(t)}$. Next we re-write (B10) as:

$$\frac{\dot{w_N}(t)}{w_N(t)} = r(t) - \frac{(\phi_{SF} - \phi_{SN})(\lambda - w_N(t))}{w_N(t) - 1} + \phi_R(t) + \phi_{SN}. \tag{B11}$$

Substituting (B11) into (B8), we have:

$$\frac{\dot{E}(t)}{E(t)} = \frac{-1}{w_N(t) - 1}\left[r(t) - \frac{(\phi_{SF} - \phi_{SN})(\lambda - w_N(t))}{w_N(t) - 1} + \phi_R(t) + \phi_{SN}\right]. \tag{B12}$$

Combining (B1), (B7), and (B12), we express $r(t)$ as a function of $w_N(t)$ and $n_S(t)$:

$$r(t) = r(w_N(t), n_S(t))$$

$$= \frac{1}{w_N(t)}\left[\frac{(\phi_{SF} - \phi_{SN})(\lambda - w_N(t))}{w_N(t) - 1} - \phi_R(w_N(t), n_S(t)) - \phi_{SN} + \rho(w_N(t) - 1)\right]. \tag{B13}$$

Substituting (B7) and (B13) into (B11) gives:

$$\dot{w_N}(t) = (\rho + \phi_{SF})w_N(t) + (w_N(t) - 1)\phi_R(w_N(t), n_S(t)) - \lambda\phi_{SF} + \phi_{SN}(\lambda - 1) - \rho. \tag{B14}$$

Substituting (B5), (B6), and (B7) into (15) yields:

$$\dot{n}_S(t) = \phi_{SN} n_N(w_N(t), n_S(t)) + \phi_{SF} n_F(w_N(t), n_S(t)) - \phi_R(w_N(t), n_S(t)) n_S(t). \tag{B15}$$

The dynamical system is represented by (B14) and (B15) in $\{w_N(t), n_S(t)\}$. Linearizing (B14) and (B15) at the steady state yields:

$$\begin{bmatrix} \dot{w_N}(t) \\ \dot{n}_S(t) \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} w_N(t) - \overline{w}_N \\ n_S(t) - \bar{n}_S \end{bmatrix},$$

where $J = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$ is the Jacobian matrix of this dynamical system evaluated at the steady-state values of $(\overline{w}_N, \bar{n}_S)$ with $d_{11} = \rho + \overline{\phi}_R + \phi_{SF} - \frac{(\phi_{SF} - \phi_{SN})[1 + (\lambda - 1)\bar{n}_S]}{\overline{w}_N - 1}$, $d_{12} = (\phi_{SF} - \phi_{SN})\overline{w}_N(\lambda - 1)$, $d_{21} = \frac{L_S}{a_R \overline{w}_N{}^2} + \frac{(\phi_{SF} - \phi_{SN})[1 + (\lambda - 1)\bar{n}_S]\bar{n}_S}{(\overline{w}_N - 1)^2}$, and $d_{22} = -\lambda(\phi_{SF} - \phi_{SN}) - \overline{\phi}_R - \phi_{SN} - \frac{(\phi_{SF} - \phi_{SN})\overline{w}_N(\lambda - 1)\bar{n}_S}{\overline{w}_N - 1}$.

Let η_1 and η_2 represent the two eigenvalues calculated from the Jacobian matrix (J). The determinant of the Jacobian matrix is derived as:

$$Det(J) = d_{11}d_{22} - d_{12}d_{21} = \frac{-(\rho + \overline{\phi}_R + \lambda\phi_{SF})}{\lambda - 1}\Theta,$$

where $\Theta = (\lambda-1)\left\{\rho\bar{n}_S + \frac{(\lambda-1)(\phi_{SF}-\phi_{SN})L_S}{a_R[\rho+\overline{\phi}_R+\lambda\phi_{SF}-(\lambda-1)\phi_{SN}]}\right\} - (2-\lambda)[\overline{\phi}_R + \lambda\phi_{SF} - (\lambda-1)\phi_{SN}]$.

Because $n_S(t)$ is a state variable and $w_N(t)$ is a jump variable, then the steady-state equilibrium is stable if there are one negative eigenvalue and one positive eigenvalue of the Jacobian matrix (J). The steady-state equilibrium is stable if one eigenvalue is positive and the other eigenvalue is negative, implying that the determinant of the Jacobian matrix is negative. This requires that $\eta_1\eta_2 = Det(J) = d_{11}d_{22} - d_{12}d_{21} < 0$. Thus, the steady state determinacy requires the following condition.

Condition (P3) $(\lambda-1)\left\{\rho\bar{n}_S + \frac{(\phi_{SF}-\phi_{SN})L_S}{a_R[\rho+\overline{\phi}_R+\lambda\phi_{SF}-(\lambda-1)\phi_{SN}]}\right\} > (2-\lambda)[\overline{\phi}_R + \lambda\phi_{SF} - (\lambda-1)\phi_{SN}]$.

APPENDIX C

Proof of Propositions 2-4

To study how a decrease in ϕ_{SN} affects $\overline{\phi}_R$, we totally differentiate (A14) with respect to $\overline{\phi}_R$ and ϕ_{SN} and derive:

$$\frac{d\overline{\phi}_R}{d\phi_{SN}} = -\frac{\xi_2}{\xi_1}, \tag{C1}$$

where ξ_1 is defined in Appendix A and:

$$\begin{aligned}\xi_2 &= \frac{\partial f(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\phi_{SN}} \\ &= -\left[\lambda\left(\frac{\partial\bar{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\phi_{SN}}\right) + (\lambda-1)\left(\frac{\partial\bar{n}_F(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\phi_{SN}}\right)\right]\frac{\bar{E}}{\lambda} \\ &\quad + \frac{[\bar{n}_F + \lambda(1-\bar{n}_N-\bar{n}_F)]}{\lambda}\left(\frac{\partial\bar{E}(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\phi_{SN}}\right) \\ &= \frac{a_R}{\overline{\phi}_R+\phi_{SF}}\{\bar{n}_N[\rho-(\lambda-1)\phi_{SN}] - (\overline{\phi}_R+\lambda\phi_{SF})(1-\bar{n}_N)\}.\end{aligned}$$

If condition (P1) holds, then $\xi_1 > 0$ and $\xi_2 < 0$, indicating that $\frac{d\overline{\phi}_R}{d\phi_{SN}} > 0$.

From (A8), we derive:

$$\frac{d\bar{w}_N}{d\phi_{SN}} = \frac{\partial\bar{w}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\phi_{SN}} + \frac{\partial\bar{w}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\overline{\phi}_R}\frac{d\overline{\phi}_R}{d\phi_{SN}} < 0. \tag{C2}$$

From (A10), we derive:

$$\frac{d\bar{n}_N}{d\phi_{SN}} = \frac{\partial\bar{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\phi_{SN}} + \frac{\partial\bar{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\overline{\phi}_R}\frac{d\overline{\phi}_R}{d\phi_{SN}}. \tag{C3}$$

Substituting (C1) into (C3) yields:

$$\frac{d\bar{n}_N}{d\phi_{SN}} = \frac{1}{\xi_1}\left[\xi_1\left(\frac{\partial\bar{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\phi_{SN}}\right) - \xi_2\left(\frac{\partial\bar{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\overline{\phi}_R}\right)\right]$$

$$= \frac{1}{\overline{\phi}_R + \phi_{SN} + \lambda(\phi_{SF} - \phi_{SN}) + \rho}\left[(\lambda - 1)\bar{n}_N - (\lambda - 1 + \bar{n}_N)\left(\frac{d\overline{\phi}_R}{d\phi_{SN}}\right)\right]. \quad \text{(C4)}$$

Since $\frac{d\overline{\phi}_R}{d\phi_{SN}} > 0$, then (C4) indicates that $\frac{d\bar{n}_N}{d\phi_{SN}} > 0$ if and only if $\left(\frac{d\overline{\phi}_R}{d\phi_{SN}}\right) < \frac{(\lambda-1)\bar{n}_N}{\lambda-1+\bar{n}_N}$. This means that a decrease in ϕ_{SN} will raise $\bar{n}_N$ if it does not cause a large increase in $\overline{\phi}_R$ and vice versa.

Using (A9), we derive:

$$\frac{d\bar{E}}{d\phi_{SN}} = \frac{\partial \bar{E}(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \phi_{SN}} + \frac{\partial \bar{E}(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \overline{\phi}_R}\frac{d\overline{\phi}_R}{d\phi_{SN}}$$

$$= \frac{\lambda a_R}{\lambda - 1}\left[\left(\frac{d\overline{\phi}_R}{d\phi_{SN}}\right) - (\lambda - 1)\right]. \quad \text{(C5)}$$

Since $\frac{(\lambda-1)\bar{n}_N}{\lambda-1+\bar{n}_N} < (\lambda - 1)$, (C5) indicates that $\frac{d\bar{E}}{d\phi_{SN}} < 0$ if $\left(\frac{d\overline{\phi}_R}{d\phi_{SN}}\right) < \frac{(\lambda-1)\bar{n}_N}{\lambda-1+\bar{n}_N}$.

From (A12), we derive:

$$\frac{d\bar{n}_F}{d\phi_{SN}} = \frac{\partial \bar{n}_F(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \phi_{SN}} + \frac{\partial \bar{n}_F(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \overline{\phi}_R}\frac{d\overline{\phi}_R}{d\phi_{SN}}$$

$$= -\frac{1}{\overline{\phi}_R + \phi_{SF}}\left[\bar{n}_N - (1 - \bar{n}_N - \bar{n}_F)\left(\frac{d\overline{\phi}_R}{d\phi_{SN}}\right) + (\overline{\phi}_R + \phi_{SN})\left(\frac{d\bar{n}_N(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{d\phi_{SN}}\right)\right].$$

(C6)

If $\left(\frac{d\overline{\phi}_R}{d\phi_{SN}}\right) < \frac{(\lambda-1)\bar{n}_N}{\lambda-1+\bar{n}_N}$, then (C6) indicates that $\frac{d\bar{n}_F}{d\phi_{SN}} < 0$ since $\frac{(\lambda-1)\bar{n}_N}{\lambda-1+\bar{n}_N} < \frac{\bar{n}_N}{\lambda-1+\bar{n}_N}$.

Using (A11), we have:

$$\frac{d\bar{n}_{SN}}{d\phi_{SN}} = \frac{\partial \bar{n}_{SN}(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \phi_{SN}} + \frac{\partial \bar{n}_{SN}(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \overline{\phi}_R}\frac{d\overline{\phi}_R}{d\phi_{SN}}$$

$$= \frac{1}{\overline{\phi}_R^{\,2}}\left\{\bar{n}_N\left[\overline{\phi}_R - \phi_{SN}\left(\frac{d\overline{\phi}_R}{d\phi_{SN}}\right)\right] + (\overline{\phi}_R\phi_{SN})\left(\frac{d\bar{n}_N(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{d\phi_{SN}}\right)\right\}. \quad \text{(C7)}$$

Using (A5), we derive $\phi_{SN} = \frac{\bar{n}_{SN}\overline{\phi}_R}{\bar{n}_N}$. If $\left(\frac{d\overline{\phi}_R}{d\phi_{SN}}\right) < \frac{(\lambda-1)\bar{n}_N}{\lambda-1+\bar{n}_N}$, then we derive $\phi_{SN}\left(\frac{d\overline{\phi}_R}{d\phi_{SN}}\right) < \frac{(\lambda-1)\bar{n}_{SN}}{\lambda-1+\bar{n}_N}\overline{\phi}_R < \overline{\phi}_R$, implying that $\frac{d\bar{n}_{SN}}{d\phi_{SN}} > 0$.

From (A13), we have:

$$\frac{d\bar{n}_{SF}}{d\phi_{SN}} = -\left(\frac{d\bar{n}_N}{d\phi_{SN}}\right) - \left(\frac{d\bar{n}_F}{d\phi_{SN}}\right) - \left(\frac{d\bar{n}_{SN}}{d\phi_{SN}}\right). \quad \text{(C8)}$$

Using (C6) and (C7) to substitute $\left(\frac{d\bar{n}_F}{d\phi_{SN}}\right)$ and $\left(\frac{d\bar{n}_{SN}}{d\phi_{SN}}\right)$ into (C8) gives:

$$\frac{d\bar{n}_{SF}}{d\phi_{SN}} = -\frac{\phi_{SF}(\overline{\phi}_R + \phi_{SN})}{(\overline{\phi}_R + \phi_{SF})\overline{\phi}_R}\left(\frac{d\bar{n}_N}{d\phi_{SN}}\right)$$

$$-\frac{1}{(\overline{\phi}_R + \phi_{SF})\overline{\phi}_R^{\,2}}\left[\phi_{SF}\overline{\phi}_R\overline{n}_N + [\overline{\phi}_R^{\,2}(1-\overline{n}_N-\overline{n}_F) - (\overline{\phi}_R + \phi_{SF})\phi_{SN}\overline{n}_N]\left(\frac{d\overline{\phi}_R}{d\phi_{SN}}\right)\right].$$

(C9)

Using (A5) to replace $\phi_{SN}\overline{n}_N$ in (C9) by $\overline{n}_{SN}\overline{\phi}_R$ yields:

$$\frac{d\overline{n}_{SF}}{d\phi_{SN}} = -\frac{1}{(\overline{\phi}_R + \phi_{SF})\overline{\phi}_R}\left\{\phi_{SF}(\overline{\phi}_R + \phi_{SN})\left(\frac{d\overline{n}_N}{d\phi_{SN}}\right) + \phi_{SF}\overline{n}_N + [(1-\overline{n}_N-\overline{n}_F)\overline{\phi}_R - (\overline{\phi}_R + \phi_{SF})\overline{n}_{SN}]\left(\frac{d\overline{\phi}_R}{d\phi_{SN}}\right)\right\}. \quad \text{(C10)}$$

If $\left(\frac{d\overline{\phi}_R}{d\phi_{SN}}\right) < \frac{(\lambda-1)\overline{n}_N}{\lambda-1+\overline{n}_N}$, then we derive $\phi_{SF}\overline{n}_N > (\overline{\phi}_R + \phi_{SF})\overline{n}_{SN}\left(\frac{d\overline{\phi}_R}{d\phi_{SN}}\right)$, implying that $\frac{d\overline{n}_{SF}}{d\phi_{SN}} < 0$.

□

APPENDIX D

Proof of Propositions 5-7

To study how a decrease in ϕ_{SF} affects $\overline{\phi}_R$, we totally differentiate (A14) with respect to $\overline{\phi}_R$ and ϕ_{SF} and derive:

$$\frac{d\overline{\phi}_R}{d\phi_{SF}} = -\frac{\xi_3}{\xi_1}, \quad \text{(D1)}$$

where ξ_1 is given in Appendix A and:

$$\begin{aligned}\xi_3 &= \frac{\partial f(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\phi_{SF}} \\ &= -\left[\lambda\left(\frac{\partial\overline{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\phi_{SF}}\right) + (\lambda-1)\left(\frac{\partial\overline{n}_F(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\phi_{SF}}\right)\right]\frac{\overline{E}}{\lambda} \\ &\quad + \frac{[\lambda(1-\overline{n}_N)-(\lambda-1)\overline{n}_F]}{\lambda}\left(\frac{\partial\overline{E}(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\phi_{SF}}\right) \\ &= \left\{-[\overline{\phi}_R + \phi_{SN} + \lambda(\phi_{SF}-\phi_{SN})]\left(\frac{\partial\overline{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\phi_{SF}}\right) + (\lambda-1)\overline{n}_F\right\}\frac{\overline{E}}{\lambda(\overline{\phi}_R + \phi_{SF})} \\ &\quad + \frac{[\overline{n}_F + \lambda(1-\overline{n}_N-\overline{n}_F)]}{\lambda}\left(\frac{\partial\overline{E}(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\phi_{SF}}\right).\end{aligned}$$

Since $\frac{\partial\overline{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\phi_{SF}} < 0$ and $\frac{\partial\overline{E}(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial\phi_{SF}} > 0$, then $\xi_3 > 0$. If condition (P1) holds, then $\xi_1 > 0$. Thus, we have $\frac{d\overline{\phi}_R}{d\phi_{SF}} < 0$.

From (A8), we derive:

$$\frac{d\overline{w}_N}{d\phi_{SF}} = \frac{\partial \overline{w}_N(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \phi_{SF}} + \frac{\partial \overline{w}_N(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \overline{\phi}_R} \frac{d\overline{\phi}_R}{d\phi_{SF}} > 0. \quad \text{(D2)}$$

From (A9), we derive:

$$\frac{d\bar{E}}{d\phi_{SF}} = \frac{\partial \bar{E}(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \phi_{SF}} + \frac{\partial \bar{E}(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \overline{\phi}_R} \frac{d\overline{\phi}_R}{d\phi_{SF}}. \quad \text{(D3)}$$

Substituting (D1) into (D3) yields:

$$\frac{d\bar{E}}{d\phi_{SF}} = \frac{1}{\xi_1}\left[\xi_1\left(\frac{\partial \bar{E}(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \phi_{SF}}\right) - \xi_3\left(\frac{\partial \bar{E}(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \overline{\phi}_R}\right)\right]$$

$$= \frac{\lambda a_R^2\{\lambda[\overline{\phi}_R + \phi_{SN} + \lambda(\phi_{SF} - \phi_{SN})] - [\overline{\phi}_R + \phi_{SN} + \lambda(\phi_{SF} - \phi_{SN}) + \rho][\overline{n}_F + \lambda(1 - \overline{n}_N - \overline{n}_F)]\}}{(\lambda - 1)\xi_1(\overline{\phi}_R + \phi_{SF})}.$$

Using (A9) and (17), we respectively replace $[\overline{\phi}_R + \phi_{SN} + \lambda(\phi_{SF} - \phi_{SN}) + \rho]$ by $\frac{(\lambda-1)\bar{E}}{\lambda a_R}$ and $[\overline{n}_F + \lambda(1 - \overline{n}_N - \overline{n}_F)]$ by $\frac{L_S\lambda}{\bar{E}}$ and derive:

$$\frac{d\bar{E}}{d\phi_{SF}} = \frac{\lambda^2 a_R}{(\lambda - 1)\xi_1(\overline{\phi}_R + \phi_{SF})}\left\{\lambda[\overline{\phi}_R + \phi_{SN} + \lambda(\phi_{SF} - \phi_{SN})] - \frac{(\lambda - 1)L_S}{a_R}\right\}$$

$$> \frac{\lambda^2 a_R}{(\lambda - 1)\xi_1(\overline{\phi}_R + \phi_{SF})}\left\{\lambda[\lambda(\phi_{SF} - \phi_{SN}) + \phi_{SF}] - \frac{(\lambda - 1)L_S}{a_R}\right\}.$$

Therefore, $\frac{d\bar{E}}{d\phi_{SF}} > 0$ if $L_S < \frac{\lambda a_R}{\lambda-1}[\lambda(\phi_{SF} - \phi_{SN}) + \phi_{SF}]$.

From (A10), we derive:

$$\frac{d\overline{n}_N}{d\phi_{SF}} = \frac{\partial \overline{n}_N(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \phi_{SF}} + \frac{\partial \overline{n}_N(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \overline{\phi}_R} \frac{d\overline{\phi}_R}{d\phi_{SF}}. \quad \text{(D4)}$$

Substituting (D1) into (D4) yields:

$$\frac{d\overline{n}_N}{d\phi_{SF}} = \frac{1}{\xi_1}\left[\xi_1\left(\frac{\partial \overline{n}_N(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \phi_{SF}}\right) - \xi_3\left(\frac{\partial \overline{n}_N(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \overline{\phi}_R}\right)\right]$$

$$= \frac{-1}{\xi_1[\overline{\phi}_R + \phi_{SN} + \lambda(\phi_{SF} - \phi_{SN}) + \rho]}[\lambda\overline{n}_N\xi_1 - (\lambda - 1 + \overline{n}_N)\xi_3]$$

$$= \frac{a_R h_1(\overline{\phi}_R; \phi_{SF}, \phi_{SN})}{\xi_1(\overline{\phi}_R + \phi_{SF})[\overline{\phi}_R + \phi_{SN} + \lambda(\phi_{SF} - \phi_{SN}) + \rho]}, \quad \text{(D5)}$$

where $h_1(\overline{\phi}_R; \phi_{SN}, \phi_{SF}) = [\overline{\phi}_R + \phi_{SN} + \lambda(\phi_{SF} - \phi_{SN}) + \rho][\lambda\overline{n}_N + (\lambda - 1)\overline{n}_F](1 - \overline{n}_N) + [\lambda(1 - \overline{n}_N) - (\lambda - 1)\overline{n}_F)]\lambda(\overline{\phi}_R + \phi_{SF}) > 0$. Therefore, we have $\frac{d\overline{n}_N}{d\phi_{SF}} > 0$.

From (A12), we derive:

$$\frac{d\overline{n}_F}{d\phi_{SF}} = \frac{\partial \overline{n}_F(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \phi_{SF}} + \frac{\partial \overline{n}_F(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \overline{\phi}_R} \frac{d\overline{\phi}_R}{d\phi_{SF}}$$

$$= -\frac{1}{\overline{\phi}_R + \phi_{SF}}\left[(\overline{\phi}_R + \phi_{SN})\left(\frac{\partial \overline{n}_N(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \phi_{SF}}\right) + \overline{n}_F - (1 - \overline{n}_N - \overline{n}_F)\left(\frac{d\overline{\phi}_R}{d\phi_{SF}}\right)\right] < 0.$$

From (A11), we derive:

$$
\begin{aligned}
\frac{d\bar{n}_{SN}}{d\phi_{SF}} &= \frac{\partial \bar{n}_{SN}(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \phi_{SF}} + \frac{\partial \bar{n}_{SN}(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \overline{\phi}_R} \frac{d\overline{\phi}_R}{d\phi_{SF}} \\
&= \frac{1}{\xi_1}\left[\xi_1\left(\frac{\partial \bar{n}_{SN}(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \phi_{SF}}\right) - \xi_3\left(\frac{\partial \bar{n}_{SN}(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \overline{\phi}_R}\right)\right] \\
&= \frac{1}{\xi_1}\left\{\frac{\phi_{SN}}{\overline{\phi}_R}\xi_1\left(\frac{\partial \bar{n}_N(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \phi_{SF}}\right) - \frac{\phi_{SN}}{\overline{\phi}_R}\left[\left(\frac{\partial \bar{n}_N(\overline{\phi}_R; \phi_{SN}, \phi_{SF})}{\partial \overline{\phi}_R}\right) - \frac{\bar{n}_N}{\overline{\phi}_R}\right]\xi_3\right\}.
\end{aligned}
\tag{D6}
$$

Since $\frac{d\bar{n}_N}{d\phi_{SF}} > 0$, (D5) indicates that $\xi_1\left(\frac{\partial \bar{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial \phi_{SF}}\right) > \xi_3\left(\frac{\partial \bar{n}_N(\overline{\phi}_R;\phi_{SN},\phi_{SF})}{\partial \overline{\phi}_R}\right)$. Thus, we have $\frac{d\bar{n}_{SN}}{d\phi_{SF}} > 0$. Since an increase in ϕ_{SF} raises $\bar{n}_N$ and $\bar{n}_{SN}$ while reducing $\bar{n}_F$ and $\bar{n}_{SF} = 1 - \bar{n}_N - \bar{n}_F - \bar{n}_{SN}$, the change of $\bar{n}_{SF}$ is ambiguous.

□

APPENDIX E

A model where imitation targets only Southern-produced goods by multinationals

In this appendix, we consider a model where imitation targets only goods produced through FDI. Since Southern firms only imitate goods produced in the South through FDI, it implies that $\phi_{SN} = 0$ and $n_{SN} = 0$. Moreover, there is no need to consider (13). We use variables with an upper bar to represent the steady-state values of the corresponding variables. Since $\frac{\dot{E}_k(t)}{E_k(t)} = r(t) - \rho$, we thus have:

$$\frac{\dot{E}(t)}{E(t)} = \frac{\dot{E}_k(t)}{E_k(t)} = r(t) - \rho. \tag{E1}$$

Equation (7) indicates that $\dot{v}_F(t) = \dot{v}_N(t)$. Using (10) and (11), we have:

$$\Pi_F(t) - \Pi_N(t) = \phi_{SF} v_N(t). \tag{E2}$$

Substituting (8) and (9) into (E2) gives:

$$E(t) = \frac{\lambda \phi_{SF} v_N(t)}{w_N(t) - 1}. \tag{E3}$$

Substituting (6) into (E3) allows us to express $E(t)$ as a function of $w_N(t)$:

$$E(t) = E(w_N(t)) = \frac{\lambda \phi_{SF} a_R w_N(t)}{w_N(t) - 1}. \tag{E4}$$

Using (17) and (E4), we derive $n_F(t)$ as a function of $w_N(t)$ and $n_{SF}(t)$:

$$n_F(t) = n_F\big(w_N(t), n_{SF}(t)\big) = \lambda\left[\frac{L_S(w_N(t) - 1)}{\lambda \phi_{SF} a_R w_N(t)} - n_{SF}(t)\right]. \tag{E5}$$

Combining (12) and (E5), we express $n_N(t)$ as a function of $w_N(t)$ and $n_{SF}(t)$:

$$n_N(t) = n_N(w_N(t), n_{SF}(t)) = 1 - n_F(w_N(t), n_{SF}(t)) - n_{SF}(t). \tag{E6}$$

Substituting (E4) and (E6) into (16) yields:

$$\begin{aligned}\phi_R(t) &= \phi_R(w_N(t), n_{SF}(t)) \\ &= \frac{1}{a_R}\left[L_N - \frac{\phi_{SF} a_R w_N(t)}{w_N(t) - 1} n_N(w_N(t), n_{SF}(t))\right].\end{aligned} \tag{E7}$$

Using (E4), we derive:

$$\frac{\dot{E}(t)}{E(t)} = \frac{-1}{w_N(t) - 1}\frac{\dot{w_N}(t)}{w_N(t)}. \tag{E8}$$

Combining (8) and (10), we have:

$$r(t) = \frac{\dot{v}_N(t)}{v_N(t)} + \frac{E(t)(\lambda - w_N(t))}{\lambda v_N(t)} - \phi_R(t). \tag{E9}$$

Using (E4) to replace $E(t)$ in (E9) gives:

$$\frac{\dot{v}_N(t)}{v_N(t)} = r(t) - \frac{\phi_R(t)(\lambda - w_N(t))}{w_N(t) - 1} + \phi_R(t). \tag{E10}$$

Equation (6) indicates that $\frac{\dot{v}_N(t)}{v_N(t)} = \frac{\dot{w_N}(t)}{w_N(t)}$. Thus, we re-write (E10) as:

$$\frac{\dot{w_N}(t)}{w_N(t)} = r(t) - \frac{\phi_R(t)(\lambda - w_N(t))}{w_N(t) - 1} + \phi_R(t). \tag{E11}$$

Substituting (E11) into (E8), we have:

$$\frac{\dot{E}(t)}{E(t)} = \frac{-1}{w_N(t) - 1}\left[r(t) - \frac{\phi_{SF}(\lambda - w_N(t))}{w_N(t) - 1} + \phi_R(t)\right]. \tag{E12}$$

Combining (E1), (E7), and (E12), we express $r(t)$ as a function of $w_N(t)$ and $n_{SF}(t)$:

$$\begin{aligned}r(t) &= r(w_N(t), n_{SF}(t)) \\ &= \frac{1}{w_N(t)}\left[\frac{\phi_{SF}(\lambda - w_N(t))}{w_N(t) - 1} - \phi_R(w_N(t), n_{SF}(t)) + \rho(w_N(t) - 1)\right].\end{aligned} \tag{E13}$$

Substituting (E7) and (E13) into (E11) gives:

$$\dot{w_N}(t) = (\rho + \phi_{SF})w_N(t) + (w_N(t) - 1)\phi_R\big(w_N(t), n_{SF}(t)\big) - \lambda\phi_{SF} - \rho. \tag{E14}$$

Substituting (E5), (E6), and (E7) into (15) yields:

$$\dot{n}_{SF}(t) = \phi_{SF} n_F(w_N(t), n_{SF}(t)) - \phi_R(w_N(t), n_{SF}(t)) n_{SF}(t). \tag{E15}$$

The dynamical system is represented by (E14) and (E15) in $\{w_N(t), n_{SF}(t)\}$. Linearizing (E14) and (E15) at the steady state yields the following Jacobian matrix (J):

$$\begin{bmatrix}\dot{w_N}(t) \\ \dot{n}_{SF}(t)\end{bmatrix} = \begin{bmatrix}u_{11} & u_{12} \\ u_{21} & u_{22}\end{bmatrix}\begin{bmatrix}w_N(t) - \overline{w}_N \\ n_{SF}(t) - \overline{n}_{SF}\end{bmatrix},$$

where $u_{11} = \rho + \overline{\phi}_R + \phi_{SF} - \frac{\phi_{SF}[1+(\lambda-1)\bar{n}_{SF}]}{\overline{w}_N - 1}$, $u_{12} = \phi_{SF}\overline{w}_N(\lambda - 1)$, $u_{21} = \frac{L_S}{a_R \overline{w}_N{}^2} + \frac{\phi_{SF}[1+(\lambda-1)\bar{n}_{SF}]\bar{n}_{SF}}{(\overline{w}_N - 1)^2}$, and $u_{22} = -\lambda\phi_{SF} - \overline{\phi}_R - \frac{\phi_{SF}\overline{w}_N(\lambda-1)\bar{n}_S}{\overline{w}_N - 1}$.

Let η_3 and η_4 represent the two eigenvalues calculated from the Jacobian matrix of this dynamical system evaluated at the steady-state values of $(\overline{w}_N, \bar{n}_{SF})$. Because $n_{SF}(t)$ is a state variable and $w_N(t)$ is a jump variable, the steady-state equilibrium is therefore stable if there are one negative eigenvalue and one positive eigenvalue. The steady-state equilibrium is then stable if the determinant of the Jacobian matrix ($Det(J)$) is negative; that is, $\eta_3\eta_4 = Det(J) < 0$. The determinant of the Jacobian matrix is derived as:

$$Det(J) = u_{11}u_{22} - u_{12}u_{21}$$
$$= \frac{-(\rho + \overline{\phi}_R + \lambda\phi_{SF})}{\lambda - 1}\left\{(\lambda - 1)\left[\rho\bar{n}_{SF} + \frac{(\lambda - 1)\phi_{SF}L_S}{a_R(\rho + \overline{\phi}_R + \lambda\phi_{SF})}\right] - (2 - \lambda)(\overline{\phi}_R + \lambda\phi_{SF})\right\}.$$

Therefore, the steady-state determinacy requires that $(\lambda - 1)\left[\rho\bar{n}_{SF} + \frac{(\lambda-1)\phi_{SF}L_S}{a_R(\rho+\overline{\phi}_R+\lambda\phi_{SF})}\right] > (2 - \lambda)(\overline{\phi}_R + \lambda\phi_{SF})$.

APPENDIX F (Not Intended for Publication)

Literature for calibration

For the benchmark model, we assign the discount factor $\rho = 0.01$ to generate a 1% real interest rate at the steady state. The one-stage quality improvement is set at $\lambda = 1.35$ to match the Northern markup of 35%. The Northern population is assigned to 0.9 and the Southern population is assigned to 1.35 so that the North-South relative wage is about 1.2. We set the labor intensity for R&D (a_R) to 1.5. Ideally, the data of $\bar{n}_N$, $\bar{n}_F$, $\bar{n}_{SN}$, and $\bar{n}_{SF}$ can be used to set the parameter values of L_N, L_S, ϕ_{SN}, and ϕ_{SF}. Unfortunately, such data are not available.

We survey the literature and look for studies that conduct numerical analysis based on a product-cycle model. These studies include Glass (1999), Glass and Saggi (2001, 2002), Glass (2004), Tanaka, Iwaisako, and Futagami (2007), and Tanaka and Iwaisako (2014). Table F1 summarizes the parameter values used in these studies. As shown in Table F1, there is a wide variety of parameter values. Unfortunately, all these studies do not explain why these parameter values are assigned.

Table F2 also indicates that these parameter values generate very different results for $\bar{n}_N$, $\bar{n}_F$, and $\bar{n}_S$. As shown in Table F2 in this report, the numerical results of $\bar{n}_N$, $\bar{n}_F$, and $\bar{n}_S$ vary a lot in previous studies. It seems that there is no consensus on what the reasonable values for $\bar{n}_N$, $\bar{n}_F$, and $\bar{n}_S$ should be.

Table F1 Summary of parameter values used in the related literature

	ρ	λ	a_R	a_F/a_O	ϕ_S	L_S	L_N
Glass (1999)	4%	3	2	N/A	endogenized	2	4
Glass and Saggi (2001)	$\frac{1}{6}$	2	2	1	N/A	1	$\frac{17}{4}$
Glass and Saggi (2002)	0.05	4	3	0.2	endogenized	6	3
Glass (2004)	$\frac{1}{12}$	3	1	0	0.5	1	2
Tanaka , Iwaisako, and Futagami (2007)	0.05	1.5	7	3.5	N/A	2	1
Tanaka and Iwaisako (2014)	0.05	4	123.5	0	0.037	6	3

Notes: 1. The parameters a_O and a_F respectively represent the labor resources required for outsourcing and FDI. 2. The parameter (variable) ϕ_S represents the rate of imitation. Imitation risk is not a concern in Glass and Saggi (2001) and Tanaka et al. (2007). The imitation rate ϕ_S is exogenously given at 0.5 in Glass (2004) and at 0.037 in Tanaka and Iwaisako (2014) while it is endogenously determined in Glass (1999) and Glass and Saggi (2002).

Table F2 Summary of results in the related literature

	$\overline{w}_N$	$\overline{n}_N$	$\overline{n}_F$	$\overline{n}_S$
Glass (1999)	1.3	72%	20%	8%
Glass and Saggi (2001)	1.5	not reported	not reported	not reported
Glass and Saggi (2002)	1.5	22.1%	32.3%	45.6%
Glass (2004)	2.17	66.8%	18.1%	15.1%
Tanaka , Iwaisako, and Futagami (2007)	not reported	not reported	not reported	not reported
Tanaka and Iwaisako (2014)	not reported	not reported	not reported	not reported

Note: The focus of Tanaka et al. (2007) and Tanaka and Iwaisako (2014) is on welfare, and they do not report the values of $\overline{w}_N$, $\overline{n}_N$, $\overline{n}_F$, and $\overline{n}_S$.

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