Inter-industry trade and heterogeneous firms: Country size matters

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Inter-industry trade and heterogeneous firms: Country size matters*

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Abstract

This study investigates how industries with different patterns of firm heterogeneity are distributed across countries by developing a three-sector general-equilibrium model. There are two manufacturing industries in our setting: one in which firm productivity is homogeneous and the other in which it is heterogeneous. The higher degree of firm heterogeneity in the latter reflects the larger difference in firm heterogeneity between industries. We show that the larger country is more specialized in the industry with heterogeneous (homogeneous) firms when trade costs are low (high) and that an increase in the inter-industry difference in firm heterogeneity fosters the larger country's degree of specialization in the industry with heterogeneous firms. We also disclose the trade patterns across countries and show how they respond to trade liberalization. Moreover, wages are found to be higher in the larger country, with an increase in the inter-industry difference in firm heterogeneity enlarging the wage inequality across countries.

Keywords: Firm heterogeneity; Industrial specialization; Trade patterns
JEL Classification: F12 · F22 · R12

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1 Introduction

In the past decade or so, an extensive established literature has analyzed the role of firm heterogeneity in trade (Melitz 2003; Helpman et al. 2004; Bernard et al. 2007; Melitz and Ottaviano 2008), economic geography (Baldwin and Okubo 2006; Nocke 2006; Okubo et al. 2008), and production organization (Antràs and Helpman 2004), enriching our understanding of observed trade patterns, industrial agglomeration, spatial inequality, and firm integration.

Most of these studies are based on a single industry framework (usually with a homogeneous goods sector) to make the models more tractable. A few studies such as Bernard et al. (2007) have also considered the inter-industry trade, but they assume an identical pattern of firm heterogeneity across industries. To the best of our knowledge, little attention has thus far been paid to the fact that the degree of firm heterogeneity differs (sometimes enormously) across industries. For instance, using micro panel data for producers in seven two-digit manufacturing industries in South Korea and Taiwan, Aw et al. (2003) find that the within-industry productivity dispersion across producers, productivity differentials between surviving and failing producers, producer turnover, and so on, differ among manufacturing industries and countries. Exploiting Italian firm-level data, Gatto et al. (2008) also show that firm heterogeneity in productivity differs across industries and that more open industries feature less dispersion among firms' marginal costs.

In this regard, it is natural to examine how industries differing in their patterns of firm heterogeneity are distributed across countries during trade liberalization. Does country size matter? What are the roles of firm heterogeneity in industrial specialization and trade patterns? In this study, we provide preliminary answers to these questions by marrying the literature on inter-industry trade studies,[1] usually assuming homogeneous firm productivity, with that on the so-called 'new' New Economic Geography (NEG) literature that sheds light on how firm heterogeneity affects the existence and intensity of agglomeration economies (see Ottaviano (2011) for a review of this stream of the literature).

We investigate how industries differing in their patterns of firm heterogeneity are distributed across countries by developing a two-country three-sector general-equilibrium model. The countries differ in market size, and there are one homogeneous goods sector and two manufacturing sectors, which differ in firm heterogeneity. In particular, one of the manufacturing industries is modeled with heterogeneous firms *à la* Melitz (2003), while the other one, to keep the model tractable, is modeled with homogeneous firms. That is, the patterns of firm heterogeneity differ between the two manufacturing sectors.[2] Workers are immobile across countries but can move freely across the sectors within a country. Industrial agglomeration works via the entry and exit of firms stimulated by labor mobility across sectors.

We show that the larger country is more specialized in the industry with heterogeneous (homogeneous) firms when trade costs are low (high) and that an increase in the inter-industry difference in firm heterogeneity further fosters the larger country's specialization

[1] This body of the literature includes Amiti (1998), Laussel and Paul (2007), and Ricci (1999), to name a few.

[2] Alternatively, we could assume that the firms in both manufacturing industries are heterogeneous in productivity. However, this makes the model too heavy to provide any tractable results. By assuming homogeneous firms in one of the manufacturing industries, this simply reflects the inter-industry difference in firm heterogeneity and offers us more tractable results without losing many of the intuitive implications.

degree in the industry with heterogeneous firms. Furthermore, our results suggest that the larger country is a net exporter of both manufacturing industry goods when trade costs are high and is a net exporter (importer) of the industrial goods produced by heterogeneous (homogeneous) firms when trade costs are low. An increase in the inter-industry difference in firm heterogeneity increases (decreases) the larger country's net exports of the industrial goods produced by heterogeneous (homogeneous) firms.

Our findings on industrial specialization and trade patterns are related to the results presented by, for example, Amiti (1998) and Laussel and Paul (2007). Amiti (1998) finds that the larger country specializes more in the production of high elasticity goods and, hence, is a net exporter of high elasticity goods when trade costs are close to the levels of autarky or free trade. In a single-factor model, Laussel and Paul (2007) demonstrate that if the two countries have different market sizes and the demand elasticities differ across industries, the larger country specializes in the production of high elasticity goods and is always a net exporter of such high elasticity goods. However, their analyses rely on the assumption of homogeneous firm productivity across industries, which is somewhat unrealistic. Bernard et al. (2007) examine how comparative advantages, heterogeneous firm productivity and falling trade costs interact and affect reallocations of resources both within and across industries and countries. Although Bernard et al.'s model takes firm heterogeneity into account as well, our study differs from theirs, as they neglect the inter-industry difference in firm heterogeneity (in addition, they assume symmetric country sizes).

Owing to the interplay of the inter-industry difference in firm heterogeneity and other ingredients in traditional trade and NEG models (e.g., demand elasticity, country size, and transport costs), neglecting this point tends to cause inconsistency between the theoretical predictions and empirical findings. That is, the assumption of an identical pattern of firm productivity across industries is not innocuous in inter-industry trade studies.

The remainder of this paper is organized as follows. Section 2 further discusses the previous literature. Section 3 introduces the model setting. Section 4 solves the equilibrium and Section 5 examines the wages, industrial specialization, and trade patterns. The last section summarizes our main results and discusses some potential extensions.

2 Literature review

This section briefly describes the related literature on the approaches of inter-industry trade studies and on 'new' NEG models.

Amiti (1998) theoretically examines the relationship between the size of a country and characteristics of the goods it produces and trades. She builds a general-equilibrium model with two countries differing only in size and two imperfectly competitive industries that can differ in factor intensities, trade costs, and demand elasticities. Her results show that industrial specialization and trade patterns largely depend on the interplay between the market access effect and production cost effect. In contrast to the model of Amiti (1998) with two factors, Laussel and Paul (2007) build a one-factor two-sector general-equilibrium model and demonstrate that if the size of the two countries is different and demand elasticities differ across industries, the larger country is always a net exporter of the less differentiated goods. In a new trade theory framework, Ricci (1999) also investigates the relationship between agglomeration and industrial specialization by building a two-country three-sector model encompassing Ricardian comparative advantage, monopolistic competition, and trade costs. He shows that agglomeration in one country reduces its specialization within the manufacturing industry. Nonetheless, all these studies assume away firm heterogeneity in productivity and therefore fail to answer the aforementioned questions.

In the 'new' NEG literature, a growing number of studies examine the location decisions of heterogeneous firms or how firm heterogeneity alters existing results on agglomeration. For instance, Baldwin and Okubo (2006) introduce firm heterogeneity *à la* Melitz (2003) into the footloose capital model (Martin and Rogers 1995) and show that firm heterogeneity leads to the sorting of the most productive firms into larger regions. Based on the footloose capital model where the mobile factor repatriates all its earnings to its region of origin, their approach does not exhibit demand-linked or cost-linked circular causality as in the core-periphery model of Krugman (1991). Okubo (2009), by considering intermediate input linkages, further reveals that rather than catastrophic agglomeration, gradual trade liberalization causes gradual agglomeration. Ehrlich and Seidel (2013) succeed in introducing Melitz-type firm heterogeneity into the core-periphery model and shed light on the role of firm heterogeneity in agglomeration. They show that an increase in firm heterogeneity works in favor of agglomeration. By contrast, Zhou (2018) theoretically demonstrates that an increase in firm heterogeneity enlarges the range of trade costs in which dispersion is a stable equilibrium, by including Melitz-type firm heterogeneity into the model proposed by Murata and Thisse (2005) with urban costs.

Among others, in a linear model, Okubo et al. (2010) assume two types of firm productivities and investigate how heterogeneous firms respond to trade liberalization by choosing different locations. They uncover a bell-shaped relationship between trade liberalization and the international productivity gap. Specifically, they show that high productive firms are selected into the large market when trade costs fall; however, less productive firms also find it profitable to be located in the large market if trade costs fall further. By assuming two types of firm productivities, Saito et al. (2011) also disclose that low productivity firms relocate away from the region in which high productivity firms agglomerate during trade liberalization. Saito (2015) further examines the organization and location decisions of heterogeneous firms with multi-plant operations and the implications for regional productivity.

Indeed, the extensive literature on 'new' NEG has greatly enriched our understanding of the role of firm heterogeneity in economic agglomeration and regional development. However, by examining only a single manufacturing industry, existing studies in this strand of the literature are still inadequate for addressing the aforementioned questions.[3]

3 The model

We consider an economy involving two countries $j \in \{h, f\}$, two increasing returns to scale (IRS) sectors $v \in \{1, 2\}$, each producing differentiated varieties (of goods 1 and 2, respectively), and one constant returns to scale (CRS) sector producing a homogeneous commodity (A). The economy is endowed with a unit mass of skilled workers and L units of unskilled workers, each supplying one unit of labor inelastically.[4] Both types of workers

[3]While Forslid and Okubo (2014) consider the multi-industry case, however, their analysis focuses on regional firm sorting rather than industrial specialization and trade patterns. Moreover, they do not exploit the inter-industry difference in firm heterogeneity.

[4]Without loss of generality, the number of skilled workers is normalized to one for simplicity.

are mobile across sectors but immobile across countries. Let λ denote the proportion of skilled workers residing in country h, so that the mass of skilled workers in country f is given by $1-\lambda$. To rule out the Heckscher–Ohlin advantages, the share of unskilled workers in country h is given by λ as well. Without loss of generality, country h is assumed to be the larger one, namely $\lambda \in (1/2, 1)$. In the following analysis, we mainly describe the economy in country h for simplicity, as that in country f is almost symmetric.

3.1 Consumption

Preferences are identical across consumers and each consumer in country h maximizes the CES utility function given by

$$U_h = C_{1h}^{\alpha\beta} C_{2h}^{\alpha(1-\beta)} C_{Ah}^{1-\alpha}, \quad 0 < \beta < 1, \quad 0 \leq 2\alpha < 1, \tag{1}$$

with

$$C_{1h} = \left(\sum_{i=1}^{\Omega_1} c_{1hi}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)} \quad \text{and} \quad C_{2h} = \left(\sum_{i=1}^{\Omega_2} c_{2hi}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)},$$

where $\sigma > 1$ is the elasticity of substitution among the varieties of the same goods, α is the share of expenditure on the two differentiated goods,[5] of which β is allocated to good 1 and $1-\beta$ to good 2. Ω_1 and Ω_2 are the number of differentiated varieties for the two IRS industries available in country h, respectively. Maximizing the utilities, total demand for differentiated goods i in country h is derived as

[5]We set the expenditure share on IRS goods to be less than half to keep the A goods produced in both countries and maintain nominal wage equalization. See Baldwin and Krugman (2004, footnote 5) for more details. Further, as argued by Ricci (1999), such an assumption is clearly technical, but not too implausible: in most countries, the share of manufacturing in GDP does not exceed half.

$$d_{1h}(i) = \frac{p_{1h}(i)^{-\sigma}}{P_{1h}^{1-\sigma}} Y_h \alpha\beta, \quad d_{2h}(i) = \frac{p_{2h}(i)^{-\sigma}}{P_{2h}^{1-\sigma}} Y_h \alpha(1-\beta), \tag{2}$$

where $p_{1h}(i)$ and $p_{2h}(i)$ are the consumer prices for variety i, and $P_{1h} \equiv (\int_{i\in\Omega_1} p_{1h}(i)^{1-\sigma} di)^{\frac{1}{1-\sigma}}$ and $P_{2h} \equiv (\int_{i\in\Omega_2} p_{2h}(i)^{1-\sigma} di)^{\frac{1}{1-\sigma}}$ denotes the price indices. $Y_h = L\lambda\hat{w} + \lambda w_h$ is the national income of country h in which $\hat{w}$ and w_h are the wages of unskilled and skilled workers, respectively.

3.2 Production and technology

Following Helpman and Krugman (1985), the homogeneous goods sector is subject to CRS, perfect competition, and free trade. One unskilled worker is employed to produce one unit of the homogeneous goods. By choosing the homogeneous goods as the numeraire, the wages of unskilled labor in the two countries are pinned down to $\hat{w} = 1$. The two manufacturing industries are subject to Dixit–Stiglitz monopolistic competition and each variety of the differentiated goods is produced by a single firm under internal IRS. When a differentiated goods is shipped across countries, transport costs *à la* Samuelson (1954) occur: $\tau > 1$ units of the variety must be sent from the origin for one unit to arrive at the destination.

3.2.1 Industry 1 with heterogeneous firms

We follow Melitz (2003) by assuming that firms in Industry 1 differ in productivity φ, which is drawn from a commonly known distribution function. Firms do not know their productivity *ex ante*; to obtain this information, they have to incur an investment (e.g., R&D). We denote this entry cost in terms of skilled labor, namely $f_e w_h$. Based on this knowledge, a firm decides whether to start production or exit the industry if its productivity is too low to generate a profit. After that, for x_i units of output of variety i, each firm has a specific input requirement according to $x_i(\varphi) = h_i(\varphi)\varphi$, where $h_i(\varphi)$ denotes the marginal unskilled labor input subject to productivity φ. As in Melitz (2003), firms are heterogeneous w.r.t. their productivities whereas workers have the same skills. This can be rationalized by arguing that each firm possesses a specific technology, which in turn determines the labor productivity of its employees.

Moreover, to serve the local market, a firm is required to invest f units of skilled labor as a fixed input. This investment could take the form of, say, an equipment purchase or marketing activities that are independent of variable costs. A similar argument applies for the export market, as firms have to hire an additional f_x units of skilled labor to sell to overseas consumers.

Under Dixit–Stiglitz preferences, firms maximize their profits by choosing the optimal price. For domestic sales and exports, the consumer prices of variety i are derived as $p_{1hh}(i) = \sigma\hat{w}/(\sigma-1)\varphi$ and $p_{1hf}(i) = \sigma\tau\hat{w}/(\sigma-1)\varphi$, respectively. Together with the demand function (2), the revenue and profit of a representative firm in country h are

$$R_{1h}(\varphi) = \frac{p_{1hh}(\varphi)^{1-\sigma}}{P_{1h}^{1-\sigma}} Y_h \alpha\beta, \quad R_{1hx}(\varphi) = \frac{p_{1hf}(\varphi)^{1-\sigma}}{P_{1f}^{1-\sigma}} Y_f \alpha\beta,$$

$$\pi_{1h}(\varphi) = R_{1h}(\varphi)/\sigma - f w_h, \quad \pi_{1hx}(\varphi) = R_{1hx}(\varphi)/\sigma - f_x w_h,$$

where R_{1h} (π_{1h}) is the revenue (profit) from the domestic market and R_{1hx} (π_{1hx}) is that from the foreign market. Firms with higher productivity (higher φ) charge lower prices, sell more, and earn higher profits.

We follow the literature on heterogeneous firms in assuming Pareto-distributed productivity levels. Hence, the cumulative distribution function reads $G(\varphi) = 1 - \varphi^{-k}$, where $k > 0$ denotes the shape parameter. To simplify the notation, as in Ehrlich and Seidel (2013, 2015), we normalize the scale parameter to unity without loss of generality. This means that $\varphi = 1$ is the lowest productivity a firm will achieve. As noted by Ehrlich and Seidel (2013, 2015), the Pareto distribution offers the advantage that the shape parameter k is a straightforward measure of the heterogeneity of firms. The variance of the Pareto distribution $Var(\varphi) = k/[(k-1)^2(k-2)]$ is strictly decreasing in k for $k > 2$.[6] A high value of k implies that it becomes less likely to draw a high productivity level φ. In other words, only a few firms are highly productive and the number of low-productivity firms is high. In the extreme case of $k = \infty$, all firms are clustered at the lower bound (i.e., $\varphi = 1$). By contrast, a lower value of k implies a more heterogeneous distribution of productivity levels.

3.2.2 Industry 2 with homogeneous firms

In Industry 2, we assume all firms are homogeneous in productivity by setting $k = \infty$. In this way, a change in k in Industry 1 reflects not only a change in the degree of firm heterogeneity in this industry, but also the inter-industry difference in firm heterogeneity between the two manufacturing sectors. This simplified setting allows us to provide more tractable results without losing intuitive insights and implications.[7]

[6] Assuming $k > 2$ is necessary to ensure the Pareto distribution has finite variance. See also Helpman et al. (2004). Meanwhile, as in the literature, we impose $k > \sigma - 1$ to ensure that the integrals of the average productivity of the Pareto distribution converge.

[7] Alternatively, we could also assume a normal k in Industry 2; however, this makes the model too complicated to provide any tractable results. It also becomes difficult to capture how a change in the

By choosing units of the product, one skilled worker is employed for start-up and $(\sigma - 1)/\sigma$ units of unskilled labor are required to produce one unit of the product. The profit of a representative firm in Industry 2 is then given as

$$\pi_{2h}(i) = \left(p_{2h}(i) - \frac{\sigma - 1}{\sigma}\hat{w} \right) (d_{2hh}(i) + \tau d_{2hf}(i)) - w_h.$$

The F.O.C. gives the optimal price of variety i as $p_{2hh}(i) = \hat{w}$ and $p_{2hf}(i) = \tau\hat{w}$. Free entry and exit ensure zero profit of firms in the industry.

4 Equilibrium

4.1 Equilibrium of Industry 1 with heterogeneous firms

Firms in Industry 1 first decide whether to enter the industry until their expected profits can offset the entry costs. Based on their productivity draw, firms start producing as long as their profits are not negative; this applies to all firms with a level of productivity level φ that exceeds the cutoff level φ^*. Moreover, a subset of these domestically active firms with higher productivity may find it profitable to export to the foreign market.

To solve the equilibrium, we combine the *free-entry condition* with the *zero-cutoff-profit condition* to derive the domestic cutoff productivity level φ^*. Firms enter the industry as long as the expected profits (from both domestic sales and exports) are sufficient to cover

inter-industry difference in firm heterogeneity affects industrial specialization and trade patterns. By contrast, by assuming homogeneous firms in Industry 2, it serves as a benchmark case, which provides us more tractable results as well as insights into how the inter-industry difference in firm heterogeneity affects specialization and trade patterns across countries during trade liberalization.

the fixed market entry costs. Formally, this free-entry condition for country h is given by

$$(\varphi_h^*)^{-k}\tilde{\pi}_{1h} = f_e w_h, \tag{3}$$

where $\tilde{\pi}_{1h}$ denotes the average profits of surviving firms. Multiplied by the probability of surviving in the competitive market (i.e., $(\varphi_h^*)^{-k}$), we obtain the expected profits before firm-specific productivity levels have been realized.

Surviving firms expect to earn $\pi_{1h}(\tilde{\varphi}_h)$ domestically and $(\varphi_h^*/\varphi_{hx}^*)^k\pi_{1hx}(\tilde{\varphi}_{hx})$ from exports, where $\tilde{\varphi}_h$ and $\tilde{\varphi}_{hx}$ denote the average productivity levels of domestic and exporting firms, respectively. Here, $(\varphi_h^*/\varphi_{hx}^*)^k$ reflects the probability of becoming an exporter conditional on being active in the domestic market, with φ_{hx}^* denoting the cutoff productivity for exporting. Firms will only start producing for the domestic and export market as long as their revenue from the respective market covers the market-specific fixed costs. As a result, the marginal domestic and exporting firm will be formally given by $R_{1h}(\varphi_h^*) = \sigma f w_h$ and $R_{1hx}(\varphi_{hx}^*) = \sigma f_x w_h$. These two conditions can be used together with $R_{1fx}(\varphi_{fx}^*)$ to establish the link between the domestic cutoff in country h and exporter cutoff in country f:

$$\varphi_{fx}^* = \tau\left(\frac{f_x}{f}\frac{w_f}{w_h}\right)^{\frac{1}{\sigma-1}}\varphi_h^*. \tag{4}$$

As in the literature, we assume $f_x > f$, which reflects the reality that domestic sales are generally more profitable than exporting. This common assumption in the literature is used to avoid the case that a exporting firm does not serve local consumers.[8] Based

[8]If $w_h = w_f$, we have $\varphi_h^* = \varphi_f^*$ such that $\varphi_{fx}^* > \varphi_h^*$ implies $\varphi_{fx}^* > \varphi_f^*$, whereas $\varphi_{hx}^* > \varphi_f^*$ implies

on these insights, it is evident that the conditional export probability is limited to the range between zero and unity. Intuitively, a lower level of the shape parameter k (more heterogeneous in productivity) implies a higher export probability. Using Eq. (4), we can formulate the conditional export probability as

$$\left(\frac{\varphi_h^*}{\varphi_{hx}^*}\right)^k = \tau^{-k}\left(\frac{f_x}{f}\frac{w_h}{w_f}\right)^{\frac{k}{1-\sigma}}\left(\frac{\varphi_h^*}{\varphi_f^*}\right)^k. \tag{5}$$

Then, we can formulate average revenue in terms of the cutoff productivities, $\tilde{R_{1h}}(\tilde{\varphi}_h) = \left(\frac{\tilde{\varphi}_h}{\varphi_h^*}\right)^{\sigma-1} R_{1h}(\varphi_h^*)$. By combining the profits from domestic and export sales with the conditional export probability in Eq. (5), the zero-cutoff-profit condition can be derived as

$$\tilde{\pi}_{1h} = \left(\frac{\tilde{\varphi}_h}{\varphi_h^*}\right)^{\sigma-1} f w_h - f w_h + \left(\frac{\varphi_h^*}{\varphi_{hx}^*}\right)^k \left[\left(\frac{\tilde{\varphi}_{hx}}{\varphi_{hx}^*}\right)^{\sigma-1} f_x w_h - f_x w_h\right], \tag{6}$$

where the first two terms on the RHS are domestic profit, whereas the third one is profit from the export market. A Pareto distribution of productivity implies that average productivity results as a constant markup on the respective cutoff levels, that is $\tilde{\varphi}_h/\varphi_h^* = \tilde{\varphi}_{hx}/\varphi_{hx}^* = [k/(k-\sigma+1)]^{1/(\sigma-1)}$, which helps simplify the mathematical expressions.[9] Then, we can solve the domestic cutoff level of productivity in country h by combining Eqs. (3) and (6):

$$\varphi_h^* = \left[\frac{\sigma-1}{(f_e/f)(k-\sigma+1)}\frac{1-\mathcal{H}^2\tau^{-k}}{1-\mathcal{H}(w_f/w_h)^{\frac{k}{\sigma-1}}}\right]^{1/k}, \tag{7}$$

$\varphi_{hx}^* > \varphi_h^*$, which ensures that exporting firms also serve the domestic market. If $w_h \neq w_f$, we show $\varphi_{hx}^* > \varphi_f^* > \varphi_h^*$ and provide the necessary and sufficient conditions of $\varphi_{fx}^* > \varphi_f^*$ in the Online Appendix.

[9]Please see Ehrlich and Seidel (2013, p. 542; 2015, footnote 14).

where $\mathcal{H} \equiv (f_x/f)^{\frac{k-\sigma+1}{1-\sigma}} \in (0,1)$. This shows that the country with higher wages features lower cutoff productivity because higher wages reduce expected profits and result in less entry. This is consistent with the theoretical results proposed by Ehrlich and Seidel (2013) and Zhou (2018) as well as the empirical findings of Chen and Moore (2010).[10] Moreover, total expenditure in the domestic and foreign markets equals total revenue, which implies

$$Y_h \alpha \beta = n_{1h} \left(\frac{\tilde{\varphi_h}}{\varphi_h^*} \right)^{\sigma-1} \sigma f w_h + n_{1f} \left(\frac{\varphi_f^*}{\varphi_{fx}^*} \right)^k \left(\frac{\tilde{\varphi_{fx}}}{\varphi_{fx}^*} \right)^{\sigma-1} \sigma f_x w_f, \tag{8}$$

where n_{1j} is the number of active firms in country j. The LHS is the total expenditure in Industry 1 goods, while the RHS shows the revenue of domestic and foreign firms.

4.2 Equilibrium of Industry 2 with homogeneous firms

For a typical firm in Industry 2, free entry and exit ensures zero profit, and the payment to fixed input equals $1/\sigma$ share of the total revenue in equilibrium *à la* Dixit and Stiglitz (1977). Using Eq. (2), we thus have

$$\sigma w_h = \left(\frac{p_{2hh}^{1-\sigma} Y_h}{P_{2h}^{1-\sigma}} + \phi \frac{p_{2hh}^{1-\sigma} Y_f}{P_{2f}^{1-\sigma}} \right) \alpha(1-\beta), \tag{9}$$

where $\phi \equiv \tau^{1-\sigma} \in (0,1)$ denotes trade freeness. Finally, skilled workers are fully employed in the two differentiated goods sectors. The labor market-clearing condition of country h

[10] Based on a firm-level data for French multinational companies, Chen and Moore (2010) find that firms investing in less populous markets are on average more efficient. Both the cutoff and average total factor productivity are negatively correlated with the host country's market potential.

can be formulated as

$$\lambda = n_{1h} f + \left(\frac{\varphi_h^*}{\varphi_{hx}^*} \right)^k n_{1h} f_x + (\varphi_h^*)^k n_{1h} f_e + n_{2h}, \tag{10}$$

where the LHS is the skilled labor supply and the four terms on the RHS represent the amount of skilled labor employed in domestic production, export, fixed entry costs, and Industry 2, respectively. For Eqs. (8), (9), and (10), mirror expressions exist for country f, and we thus have six equations that endogenously determine the following six variables: w_h, w_f, n_{1h}, n_{2h}, n_{1f}, and n_{2f}.

5 Wages, industrial specialization, and trade patterns

This section analytically investigates wages, industrial specialization, and trade patterns in the equilibrium. Although closed-form solutions of the endogenous variables are not fully available, we are able to provide some tractable results and further confirm them by carrying out numerical experiments. We first exploit the impact of the inter-industry difference in firm heterogeneity as well as market size and trade liberalization on the wage inequality across countries. After establishing a theoretical foundation, we turn to analyze industrial specialization and trade patterns.

5.1 Home Market Effect (HME) in terms of wages

Proposition 1 *The wages in the larger country are higher than those in the smaller country in both the interior and the corner equilibria. An increase in the inter-industry difference in firm heterogeneity enlarges the wage inequality across countries when trade* *costs are close to the level of autarky.*

Proof: See Appendix A.

This shows that the wages in the larger country are higher, which was first addressed by Krugman (1980) with an intra-industry trade model and further confirmed by Takahashi et al. (2013), Mossay and Tabuchi (2015), and Zhou (2019). In particular, this was termed the "HME in terms of wages" by Takahashi et al. (2013), and was also confirmed in inter-industry trade studies (e.g., Amiti 1998; Laussel and Paul 2007).[11] Empirically, evidence on the role of market access in determining factor prices is found by Breinlich (2006) and Head and Mayer (2006, 2011) and summarized in Redding (2013).[12]

Intuitively, in countries with better market access, more value-added remains after deducting trade costs to remunerate factors of production, which results in higher nominal wages in the equilibrium.

Our analysis adds to these related results by generalizing the results to a multi-sector model with an inter-industry difference in firm heterogeneity. Moreover, in contrast to previous studies, we find that an increase in the inter-industry difference in firm heterogeneity enlarges the wage inequality across countries if trade costs are sufficiently high. This finding is confirmed by our numerical experiments as well. Fig. (1) shows the relationship between the wage differential, w_h/w_f, and trade freeness ϕ, given $\sigma = 3$ and $\lambda = 0.7$. It illustrates that an increase in the inter-industry difference in firm heterogeneity

[11] Amiti (1998) generalizes the wage advantages of the larger market to the two-industry case, which allows for industrial differences in factor intensity, the elasticity of substitution, and transport costs. Her findings on wage advantages are further confirmed by Laussel and Paul (2007) in a two-industry one-factor model. They extend the results by showing that the relative wage rate of the larger country is an overall increasing function of its market size.

[12] Breinlich (2006) and Head and Mayer (2006) find that wages increase with market access using EU data and exploiting both cross-sectional and time-series variation. Head and Mayer (2011) also confirm the strong correlation between changes in income and changes in market access by exploiting a country-level panel dataset.

(i.e., fall in k) enlarges w_h/w_f.

Since a growing stream of empirical studies has shown that firm heterogeneity in productivity exerts significant impacts on wage rates in addition to industrial locations and trade patterns (Aw et al. 2003; Aw and Lee 2008; Chen and Moore 2010), our analysis partly consolidates the theoretical foundation of the HME to a certain extent, making it more compatible with micro-level empirical analysis.

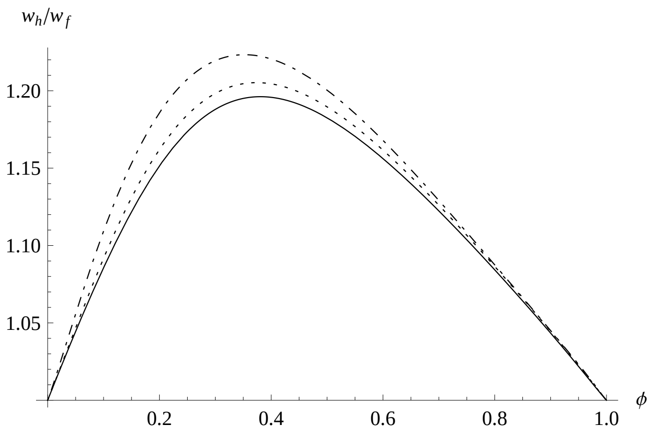

Figure 1: Wage differential w.r.t. ϕ. Solid: $k = 6$, Dotted: $k = 4$, DotDashed: k=3

The mechanisms under which firm heterogeneity in productivity affects the wage inequality across countries could be manifold, among which the following is particularly relevant. A smaller k implies that a firm in Industry 1 is more likely to draw a high productivity level, which leads to more efficient competitors. Accordingly, the least productive firms are forced to exit and a higher share of firms that survive find it profitable to enter the export market. Although the conditional export probability, $\left(\frac{\varphi_j^*}{\varphi_{jx}^*}\right)^k$, is higher in the smaller country,[13] the number of new exporters increases more in the larger country because of its market size advantage.

[13]Using Eq. (7), we have $\left(\frac{\varphi_h^*}{\varphi_{hx}^*}\right)^k < \left(\frac{\varphi_f^*}{\varphi_{fx}^*}\right)^k$, where the inequality stems from $w_h/w_f > 1$.

This is illustrated more intuitively by a numerical experiment, as shown in Fig. (2); the columns show the cases of a given ϕ and a changing σ, while the rows show the situations of a given σ and a changing ϕ. We find that the value of k falls, the number of exporters in the larger country increases relatively more than that in the smaller country. As a result, firms in the larger country sell more (than their counterparts) to the foreign market, which raises revenue and wages in Industry 1 in the larger country. Owing to the higher wage rate in Industry 1, workers in the larger country move from Industry 2 into Industry 1; this will be discussed in the next subsection.

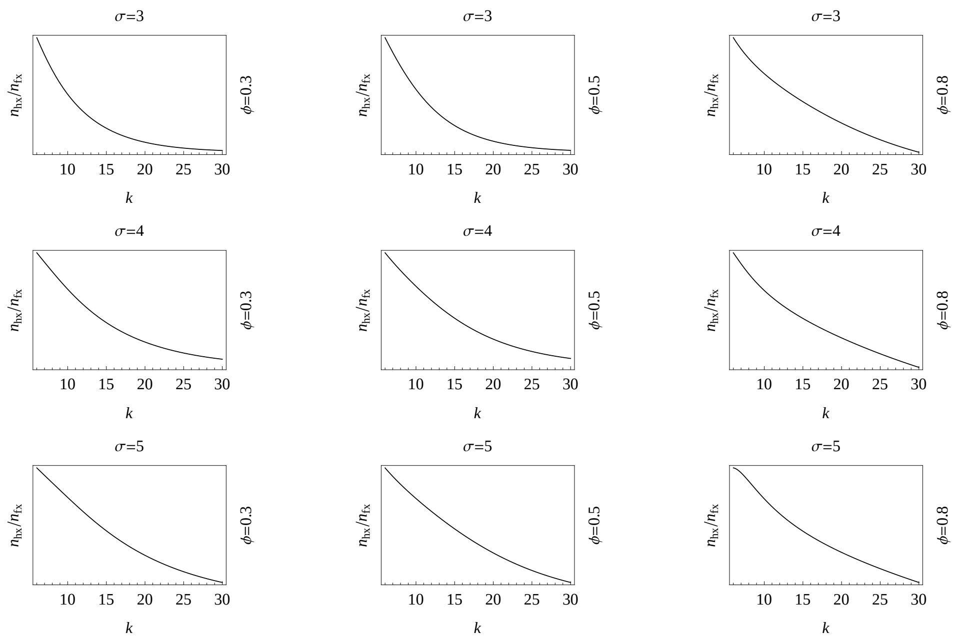

Figure 2: The relative number of exporters, n_{hx}/n_{fx}, w.r.t. k.

In addition, as shown in Fig. (1), the wage differential exerts bell-shaped pattern during trade liberalization.[14] Intuitively, owing to the market size advantage, firms in the larger country sell more and pay higher wages when trade costs are relatively high. As trade costs fall during economic integration, the market size advantage attenuates and

[14]Appendix A analytically derives $\frac{\partial(w_h/w_f)}{\partial\phi}\big|_{\phi=0} > 0$ and $\frac{\partial(w_h/w_f)}{\partial\phi}\big|_{\phi=1} < 0$. For intermediate values of trade freeness, the related results are shown by numerical experiments.

the disadvantage of higher labor costs begins to take the upper hand. As a result, firms in the larger market reduce the wage rate to sustain production, which results in the bell-shaped wage differential during trade liberalization. This result is consistent with that of inter-industry trade models (e.g., Amiti 1998) and single-industry studies (e.g., Takahashi et al. 2013; Zhou 2019).[15]

5.2 Sectoral agglomeration and industrial specialization

Following Ricci (1999), sectoral agglomeration is defined as

$$\eta_{vh} \equiv \frac{n_{vh}}{n_{vh} + n_{vf}}, \quad 0 \leq \eta_{vh} \leq 1, \quad \eta_{vh} + \eta_{vf} = 1, \quad \forall v = 1, 2.$$

Although the closed-form solutions of η_{vh} and η_{vf} are not available, we still obtain several tractable results.

Proposition 2 *In autarky, the degree of sectoral agglomeration η_{1h} is equal to the larger country's market size share λ. As trade costs fall, there exists a $\phi^* \in (0, 1)$ at which $\eta_{1h} = \lambda$. We have $\eta_{1h} < \lambda$ when ϕ is close to zero and $\eta_{1h} > \lambda$ when ϕ is close to one. Meanwhile, there exists a $\phi^\dagger \in (0, 1)$ at which $\eta_{2h} = \lambda$. We have $\eta_{2h} > \lambda$ when ϕ is close to zero and $\eta_{2h} < \lambda$ when ϕ is close to one. Moreover, a fall in k increases η_{1h} and decreases η_{2h} when ϕ is close to zero or one.*

Proof: See Appendix B.

[15] Theoretical support for the bell-shaped pattern of spatial inequality is mostly based on industrial location (e.g., Krugman and Venables 1995; Venables 1996; Puga and Venables 1997). Spatial inequality in wages is theoretically investigated by Amiti (1998), Takahashi et al. (2013), and Zhou (2019).

In autarky, two countries are isolated from each other, and the larger country is just a scale expansion of the smaller country. The inference on sectoral agglomeration is thus straightforward.

As trade costs decline from the level in autarky, firms in both industries begin to enter the export market. When trade costs are still relatively high, the firms in Industry 2 in the larger country enjoy the advantage of market size; they sell more in a larger domestic market that is free of trade cost (the market access effect). Consequently, the increase in factor demand bids up the wages in Industry 2. Meanwhile, the firms in Industry 1 in the larger country face high export barriers. Although they enjoy a larger local market as well, their average firm productivity is lower than their counterparts in the smaller country.[16] The domestic market in Industry 1 in the larger country is gradually invaded by the more competitive competitors from the smaller country. Hence, the revenue in Industry 1 falls and workers in Industry 1 flow out to Industry 2. The degree of sectoral agglomeration in Industry 1 (η_{1h}) thus decreases, while η_{2h} increases, when trade costs are still at relatively high levels, as illustrated in Fig. (3).

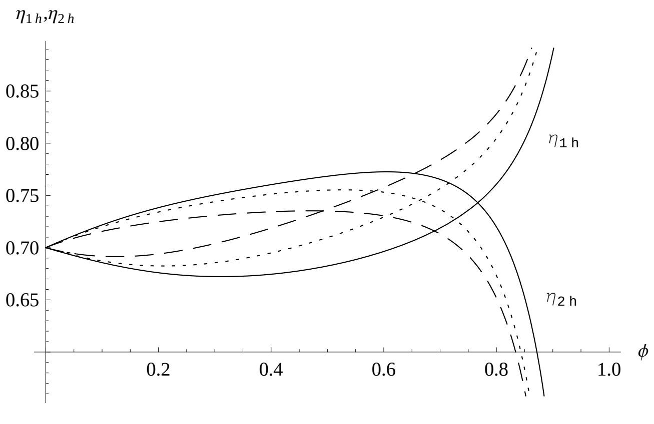

Figure 3: η_{1h} and η_{2h} w.r.t. ϕ. Solid: $k = 6$, Dotted: $k = 4$, Dashed: $k = 3$.

[16]This is because φ_h^* is lower than φ_f^*, as shown by Eq. (7) and $\tilde{\varphi}_h/\varphi_h^* = \tilde{\varphi}_f/\varphi_f^* = [k/(k-\sigma+1)]^{1/(\sigma-1)}$.

As trade liberalization advances further (i.e., trade costs reach a relatively low level), the market access effect attenuates; on the contrary, the disadvantage of higher labor costs begins to take the upper hand. The firms in Industry 2 in country h sell less than before, which, in turn, reduces the wages in this industry. Meanwhile, the firms in Industry 1 face much lower export barriers than before. Some firms that were not sufficiently productive to export begin to enter the export market. Although the conditional export probability is still lower in the larger country than in the smaller country, the number of new exporters increases more owing to the size advantage. Hence, revenue in Industry 1 gradually increases, bidding up the wages in the industry. Consequently, workers move from Industry 2 to Industry 1. As illustrated in Fig. (3), the degree of sectoral agglomeration η_{1h} increases and exceeds λ, while η_{2h} decreases, as trade liberalization proceeds further to a low level of trade costs.

Furthermore, our analytical results show how a change in the inter-industry difference in firm heterogeneity (i.e., change in k) affects industrial specialization across countries. A fall in k increases the agglomeration degree in Industry 1 and decreases that in Industry 2 in the larger country if the trade costs are close to the level of autarky or free trade. Our numerical experiments, as shown in Fig. (3), also confirm this trend. Intuitively, a smaller k implies that a firm in Industry 1 is more likely to draw a high productivity level, which leads to more efficient competitors. Accordingly, a higher share of firms that survive in the productivity draw find it profitable to enter the export market. As explained above, the number of new exporters then increases relatively more in the larger country because of its larger market size. Hence, firms in Industry 1 have higher revenue than before, which bids up wages in the industry. As a result, workers in the larger country move from

Industry 2 into Industry 1, which fosters the agglomeration of Industry 1 in the larger country.

Following Ricci (1999), we measure specialization using the level of industrial agglomeration. Specifically, the degree of national specialization in Industry v (S_{vj}) is measured as

$$S_{1h} \equiv \frac{\eta_{1h}}{\eta_{2h}}, \quad S_{2h} \equiv \frac{\eta_{2h}}{\eta_{1h}}, \quad S_{1f} \equiv \frac{\eta_{1f}}{\eta_{2f}}, \quad S_{2f} \equiv \frac{\eta_{2f}}{\eta_{1f}}.$$

Country j is more specialized in Industry v if $S_{vj} > 1$. According to the definition of η_{vh}, $S_{1h} > 1$ implies $S_{2f} > 1$ simultaneously. That is, if country h is more specialized in Industry 1, then country f is more specialized in Industry 2 and vice versa.

Proposition 3 *The larger country is more specialized in the industry with homogeneous firms when trade costs are close to the level of autarky and is more specialized in the industry with heterogeneous firms when trade costs are close to the level of free trade. An increase in the inter-industry difference in firm heterogeneity increases the larger country's degree of specialization in the industry with heterogeneous firms when trade costs are close to the level of autarky or free trade.*

Proof: From the definition of S_{vj} and Proposition 2, the results above are straightforward.
□

Our result indicates that the smaller country is more specialized in the industry with heterogeneous firms when trade costs are high. This supports the empirical findings of Chen and Moore (2010) using French multinational firm-level data. They find that in countries with below-average market potential, the productivity distribution of firms firstorder stochastically dominates those in countries with above-average market potential.[17] Moreover, we complement their results by predicting that if trade liberalization proceeds further, the export barriers for firms in the larger market decrease, and a larger proportion of firms in the larger market can penetrate the foreign market. This increases revenue, bids up wages, and encourages the larger country's degree of specialization in Industry 1.

Moreover, our results suggest that the assumption of homogeneous firm productivity across industries (e.g., Amiti 1998; Laussel and Paul 2007) may not be innocuous in inter-industry trade studies. Indeed, the inter-industry difference in firm heterogeneity does have significant impacts on wages, sectoral agglomeration, and industrial specialization.

5.3 Trade patterns

This section examines the trade patterns across countries during trade liberalization, and particularly how the inter-industry difference in firm heterogeneity affects those trade patterns. The net exports of the two types of industrial goods in country h are derived respectively as

$$EX_1(\phi) \equiv n_{1h}\left(\frac{\varphi_{1h}^*}{\varphi_{1hx}^*}\right)^k \left(\frac{\tilde{\varphi}_{1hx}}{\varphi_{1hx}^*}\right)^{\sigma-1} \sigma f_x w_h - n_{1f}\left(\frac{\varphi_{1f}^*}{\varphi_{1fx}^*}\right)^k \left(\frac{\tilde{\varphi}_{1fx}}{\varphi_{1fx}^*}\right)^{\sigma-1} \sigma f_x w_f, \tag{11}$$

$$EX_2(\phi) \equiv n_{2h}\phi\frac{p_{2hh}^{1-\sigma}}{P_{2f}^{1-\sigma}}Y_f\alpha(1-\beta) - n_{2f}\phi\frac{p_{2ff}^{1-\sigma}}{P_{2h}^{1-\sigma}}Y_h\alpha(1-\beta), \tag{12}$$

in which the LHSs are total exports, while the RHSs are total imports, both measured in numeraire units. Since there are only two countries, the larger country is a net exporter of one good if and only if the smaller country is a net importer of that good.

[17] In the extreme case of $k = \infty$, all the firms in Industry 2 cluster at the lower bound $\varphi = 1$, which is the lowest productivity a firm can draw.

Proposition 4 *The larger country is a net exporter of both industrial goods when trade costs are high and is a net exporter (importer) of the industrial goods produced by heterogeneous (homogeneous) firms when trade costs are low. An increase in the inter-industry difference in firm heterogeneity increases (decreases) the larger country's net exports of the goods produced by heterogeneous (homogeneous) firms when trade costs are close to the level of autarky or free trade.*

Proof: See Appendix C.

The larger country is relatively more specialized in the production of Industry 2 goods when trade costs are high, as shown in Proposition 3, and, naturally, it becomes a net exporter of Industry 2 goods. In addition, Proposition 3 shows that the degree of sectoral agglomeration (η_{1h}) is less than the demand share (λ) in the larger country when trade costs are high. However, the results here show that the larger country is also a net exporter of Industry 1 goods, as illustrated in Fig. (4). Intuitively, this pattern can be explained by the following points. First, firms in the larger country are relatively more shielded from their more competitive foreign counterparts when trade costs are high and imports are less. Second, although the conditional export probability in the larger country is lower, the absolute number of exporters could be higher because of its size advantage. Third, net exports here are measured in numeraire units and, therefore, the export values are higher for firms with higher wages and lower productivity. When the larger country is a net exporter of both industrial goods, trade is balanced by the net imports of the homogeneous goods.

As trade liberalization proceeds further (i.e., trade costs reach a relatively low level), the larger country becomes more specialized in Industry 1. In this scenario, it is straight-

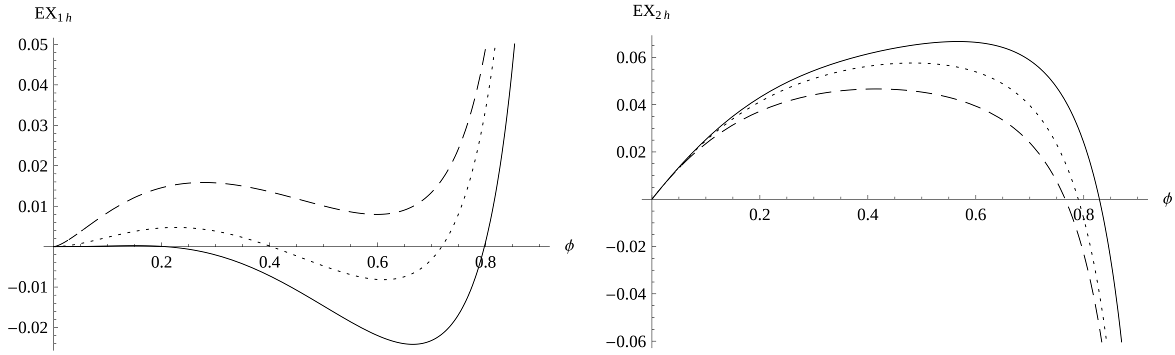

Figure 4: The net exports of country h, EX_{vh}, w.r.t. ϕ. Solid: k=6, Dotted: k=4, Dashed: k=3.

forward to find that the larger country is a net exporter (importer) of Industry 1 (2) goods. Moreover, as shown by Propositions 2 and 3, an increase in the inter-industry difference in firm heterogeneity (a fall in k) fosters the larger country's degree of specialization in Industry 1. As a result, the larger country produces more Industry 1 goods and thus exports more. This is also illustrated in Fig. (4): a fall in k increases (decreases) the larger country's net exports of the industrial goods produced by heterogeneous (homogeneous) firms. Our results on trade patterns are thus in contrast to those of Amiti (1998) and Laussel and Paul (2007), who assume homogeneous firm productivity across industries.[18]

To sum up, consistent with our analyses on wages and specialization, we find that the inter-industry difference in firm heterogeneity has significant impacts on trade patterns as well. The current study therefore implies that exploring the inter-industry difference in firm heterogeneity and its impacts on related issues enriches our understanding of the modern spatial economy with dynamics in firm productivity. Further, neglecting this point is likely to lead to a disagreement between the theoretical predictions and empirical

[18]Amiti (1998) finds that the larger country has positive net exports of high elasticity goods when trade costs are close to the level of free trade or autarky; it is a net importer of high elasticity goods at intermediate levels of trade costs. In a one-factor two-sector model, Laussel an Paul (2007) show that if the two countries are very different in size and demand elasticities differ across industries, the larger country is always a net exporter of the less differentiated goods.

findings.

6 Concluding remarks

Our main results may be summarized as follows. First, wages in the larger country are higher than those in the smaller country. The wage differential has a bell-shaped pattern during trade liberalization. An increase in the inter-industry difference in firm heterogeneity enlarges the wage inequality across countries. Second, the larger country is more specialized in the industry with homogeneous (heterogeneous) firms when trade costs are high (low). An increase in the inter-industry difference in firm heterogeneity increases (decreases) the larger country's specialization degree in the industry with heterogeneous (homogeneous) firms. Third, the larger country is a net exporter of both industrial goods when trade costs are relatively high and is a net exporter (importer) of the industrial goods produced by heterogeneous (homogeneous) firms when trade costs are low. An increase in the inter-industry difference in firm heterogeneity increases (decreases) the larger country's net exports of the industrial goods produced by heterogeneous (homogeneous) firms.

Our study contributes to the literature by disclosing how countries with different market sizes specialize in industries with different degrees of firm heterogeneity during trade liberalization. We also provide implications on how a change in the inter-industry difference in firm heterogeneity affects wage inequality, industrial specialization, and trade patterns. Our results indicate that the assumption of identical firm productivity across industries in related theoretical studies is likely to be not innocuous. Hence, the current study provides a theoretical foundation for future empirical research that aims to explore the dynamics of firms and industries in the context of globalization further.

Our framework suffers from some drawbacks. First, several results are tractable only when trade costs are close to the level of autarky or free trade. For intermediate levels of trade costs, our analyses still rely on numerical simulations. Extending our settings to a linear framework with firm heterogeneity (e.g., Melitz and Ottaviano 2008) may derive more tractable results and could allow for the robustness of our findings to be examined. This remains a task for future research. Second, our setting includes two manufacturing industries: one with heterogeneous firms and the other with homogeneous firms. Although this strategy helps simplify the mathematics and can still provide insights into the role of the inter-industry difference in firm heterogeneity in determining wages, industrial specialization, and trade patterns, it makes our assumptions less realistic. Extending our settings to two manufacturing industries that are both heterogeneous in productivity and are different in the degrees of firm heterogeneity may help us better understand the underlying mechanisms.

Appendices

Appendix A. Proof of Proposition 1

We first consider the interior equilibrium with $n_{1j} > 0$ and $n_{2j} > 0$. For Eqs. (8), (9), (10), mirror expressions exist for country f, which are respectively given as

$$Y_f \alpha\beta = n_{1f} \left(\frac{\tilde{\varphi_f}}{\varphi_f^*}\right)^{\sigma-1} \sigma f w_f + n_{1h} \left(\frac{\varphi_h^*}{\varphi_{fx}^*}\right)^k \left(\frac{\tilde{\varphi_{fx}}}{\varphi_{fx}^*}\right)^{\sigma-1} \sigma f_x w_h, \tag{A1}$$

$$\sigma w_f = \left(\frac{p_{2ff}^{1-\sigma} Y_f}{P_{2f}^{1-\sigma}} + \phi \frac{p_{2ff}^{1-\sigma} Y_h}{P_{2h}^{1-\sigma}}\right) \alpha(1-\beta), \tag{A2}$$

$$1 - \lambda = n_{1f} f + \left(\frac{\varphi_f^*}{\varphi_{fx}^*}\right)^k n_{1f} f_x + (\varphi_f^*)^k n_{1f} f_e + n_{2f}. \tag{A3}$$

By plugging Eqs. (4) - (6) into Eqs. (8) - (10) and (A1) - (A3), the six variables n_{1h}, n_{1f}, n_{2h}, n_{2f}, w_h and w_f are endogenously determined by the six equations. Suppose $w_h = w_f$ for $\phi \in (0, 1)$, Eqs. (8) - (10) and (A1) - (A3) solve

$$n_{2h} = -\left(\frac{\mathcal{H}\phi^{\frac{k-\sigma+1}{\sigma-1}}}{1 - \mathcal{H}\phi^{\frac{k-\sigma+1}{\sigma-1}}}\right)[\lambda - (1-\lambda)\phi] < 0,$$

which contradicts with $n_{2h} > 0$. Therefore, we have $w_h \neq w_f$ for $\phi \in (0, 1)$ in the interior equilibrium. On the other hand, at $\phi = 0$, Eqs. (8) - (10) and (A1) - (A3) uniquely solve

$$w_h = w_f = \frac{L\alpha}{\sigma - \alpha}, \quad n_{1h} = \frac{\beta\lambda(k - \sigma + 1)}{fk}, \quad n_{2h} = \lambda(1 - \beta), \tag{A4}$$

$$n_{1f} = \frac{\beta(1-\lambda)(k-\sigma+1)}{fk}, \quad n_{2f} = (1-\beta)(1-\lambda). \tag{A5}$$

By total differentiating Eqs. (8) - (10) and (A1) - (A3) w.r.t. ϕ, the derivatives of w_j w.r.t. ϕ, when ϕ is close to zero, are derived as

$$\left.\frac{\partial w_h}{\partial \phi}\right|_{\phi\to 0} = \frac{L\alpha\sigma(1-\beta)(2\lambda-1)}{\lambda(\sigma-\alpha)^2} > 0, \quad \left.\frac{\partial w_f}{\partial \phi}\right|_{\phi\to 0} = -\frac{L\alpha\sigma(1-\beta)(2\lambda-1)}{(1-\lambda)(\sigma-\alpha)^2} < 0. \quad \text{(A6)}$$

Using Eqs. (A4), (A6), we derive

$$\left.\frac{\partial (w_h/w_f)}{\partial \phi}\right|_{\phi\to 0} = \frac{\sigma(2\lambda-1)(1-\beta)}{\lambda(1-\lambda)(\sigma-\alpha)} > 0.$$

Therefore, at ϕ close to zero, we have $w_h > w_f$. Together with the result above, because of the continuity, we have $w_h > w_f$ for $\phi \in (0, 1)$. Furthermore, at ϕ close to zero, we derive

$$\left.\frac{\partial (w_h/w_f)}{\partial k}\right|_{\phi\to 0} = \frac{-\beta\sigma(2\lambda-1)\mathcal{H}\phi^{\frac{k}{\sigma-1}}}{k\lambda(1-\lambda)(\sigma-\alpha)} < 0,$$

which implies that a smaller k brings to a higher w_h/w_f when ϕ is close to zero. Meanwhile, at $\phi = 1$, total differentiating Eqs. (8) - (10) and (A1) - (A3) w.r.t. ϕ yields

$$\left.\frac{\partial (w_h/w_f)}{\partial \phi}\right|_{\phi=1} = -(2\lambda-1) < 0.$$

It implies that w_h/w_f increases with ϕ at ϕ close to 0, and decreases with ϕ at ϕ close to 1.

We then consider the case of corner equilibrium.[19] Denote the threshold value of trade freeness at which $n_{1f} = 0$ by $\phi^\sharp$. For $\phi \in (\phi^\sharp, 1)$, we have $n_{1f} = 0$, $n_{1h} > 0$ and $n_{2j} > 0$. Suppose $w_h = w_f \equiv w$ when $\phi \in (\phi^\sharp, 1)$. Plugging $n_{1f} = 0$ and Eq. (A3) into Eqs. (9)

[19]The possibilities of corner equilibria are examined in Online Appendix.

and (A2) solves

$$n_{2h} = \frac{(1-\lambda)[\lambda - (1-\lambda)\phi]}{1-\lambda-\lambda\phi} \quad \text{and} \quad w = \frac{L\alpha(1-\beta)(1-\lambda-\lambda\phi)}{\sigma[1-\lambda-(1-\lambda)\phi] - \alpha(1-\beta)(1-\lambda-\lambda\phi)}. \tag{A7}$$

Meanwhile, Eqs. (8), (10), (A1) together give

$$\mathcal{G} \equiv (L+w)\alpha\beta \left[1 + \frac{\lambda(\sigma-1)\left(1+\mathcal{H}\phi^{\frac{k}{\sigma-1}}\right)}{k-\sigma+1}\right] + (n_{2h}-\lambda)\left(\frac{k}{k-\sigma+1}\right)\sigma w = 0.$$

By plugging (A7) into $\mathcal{G}$, we have

$$\mathcal{G} > \frac{\alpha}{(1-\lambda)(1-\phi)} \left\{k\phi(2\lambda-1) - \beta\left[(1-\lambda)^2(\sigma-1)(1-\phi) - k(1-\lambda-\lambda\phi)\right]\right\} > 0,$$

where the second inequality comes from the monotonicity of β. Note that if $(1-\lambda)^2(\sigma-1)(1-\phi) - k(1-\lambda-\lambda\phi) > 0$, at $\beta = 1$, we have $\mathcal{G} > \alpha[k-(1-\lambda)(\sigma-1)] > 0$. It contradicts with $\mathcal{G} = 0$, and we therefore have $w_h \neq w_f$ for $\phi \in (\phi^\sharp, 1)$.

On the other hand, at $\phi = 1$, Eqs. (9) and (A2) together give $w_h = w_f$. By plugging $n_{1f} = 0$ and $w_h = w_f$ into Eqs. (8) - (10) and (A1) - (A3), we solve $n_{2f} = 1-\lambda$ and

$$w_h = w_f = \frac{L\alpha[k+\beta(\sigma-1)(\lambda\mathcal{H}-1+\lambda)]}{k\sigma - \alpha[k+\beta(\sigma-1)(\lambda\mathcal{H}-1+\lambda)]}, \tag{A8}$$

$$n_{2h} = \frac{k(1-\beta)}{k+\beta(\sigma-1)(\lambda\mathcal{H}-1+\lambda)} - (1-\lambda), \quad n_{1h} = \frac{\beta\lambda(k-\sigma+1)}{f[k+\beta(H\lambda-1+\lambda)(\sigma-1)]}. \tag{A9}$$

Total differentiating Eqs. (8) - (10) and (A2) - (A3) w.r.t. w_h, w_f and ϕ at $\phi = 1$ and plugging Eqs. (A8) into it, we derive

$$\left.\frac{\partial(w_h/w_f)}{\partial\phi}\right|_{\phi=1} = -(2\lambda-1) < 0,$$

which implies that $w_h > w_f$ when ϕ is close to 1. Together with the result that $w_h \neq w_f$ for $\phi \in (\phi^\sharp, 1)$, due to the continuity, we know $w_h > w_f$ when $\phi \in (\phi^\sharp, 1)$ in the corner equilibrium of $n_{1f} = 0$. □

Appendix B. Proof of Proposition 2

At $\phi = 0$, by using Eqs. (A4) and (A5), we solve $\eta_{1h} \equiv \frac{n_{1h}}{n_{1h}+n_{1f}} = \lambda$. Meanwhile, total differentiating Eqs. (8) - (10) and (A1) - (A3) w.r.t. ϕ and plugging Eqs. (A4), (A5) into it, using the definition of n_{1j}, we derive $\frac{\partial\eta_{1h}}{\partial\phi}\big|_{\phi\to 0} = -(1-\beta)(2\lambda-1) < 0$. It implies that at ϕ close to zero, we have $\eta_{1h} < \lambda$. At $\phi = 1$, in interior equilibrium, we solve

$$n_{1h} = \frac{\beta(k-\sigma+1)[\lambda-\mathcal{H}(1-\lambda)]}{fk(1-\mathcal{H}^2)} > 0, \quad n_{2h} = \frac{(1-\beta)\lambda(1-\mathcal{H})-\beta(2\lambda-1)\mathcal{H}}{1-\mathcal{H}}, \quad \text{(B1)}$$

$$n_{1f} = \frac{\beta(k-\sigma+1)(1-\lambda-\mathcal{H}\lambda)}{fk(1-\mathcal{H}^2)}, \quad n_{2f} = \frac{(1-\beta)(1-\lambda)(1-\mathcal{H})+\beta(2\lambda-1)\mathcal{H}}{1-\mathcal{H}} > 0, \quad \text{(B2)}$$

in which the positiveness of n_{2h} and n_{1f} is guaranteed by $\mathcal{H} < \min\{\frac{1-\lambda}{\lambda}, \frac{\lambda-\beta\lambda}{\lambda-\beta(1-\lambda)}\}$. By the definition of η_{1h}, we have $\eta_{1h} = \lambda + \frac{\mathcal{H}(2\lambda-1)}{1-\mathcal{H}} > \lambda$, at $\phi = 1$. On the other hand, in the case of corner equilibrium $n_{1f} = 0$, for $\phi \in [\phi^\sharp, 1)$, we have $\eta_{1h} = 1 > \lambda$ by the definition of η_{1h}. In both the interior and corner equilibria, because of the continuity of η_{1h}, there exists a $\phi^* \in (0,1)$ at which $\eta_{1h} = \lambda$. We have $\eta_{1h} < \lambda$ when ϕ is close to 0 and $\eta_{1h} > \lambda$

when ϕ is close to 1.

For the Industry 2, at $\phi = 0$, we solve $\eta_{2h} \equiv \frac{n_{2h}}{n_{2h}+n_{2f}} = \lambda$ and $\frac{\partial \eta_{2h}}{\partial \phi}\big|_{\phi \to 0} = \beta(2\lambda - 1) > 0$. Therefore, at ϕ close to zero, we have $\eta_{2h} > \lambda$. At $\phi = 1$, in interior equilibrium, η_{2h} is solved as

$$\eta_{2h} = \frac{(1-\mathcal{H})[\lambda - \beta(1-\lambda)] - \beta(2\lambda - 1)}{(1-\mathcal{H})(1-\beta)} < \lambda,$$

where the inequality is from $\lambda > 1/2$. Moreover, in the corner equilibrium, for $\phi \in [\phi^\sharp, 1)$, we have $n_{1f} = 0$ and $n_{2f} = 1 - \lambda$. Because $n_{1h} > 0$, we have $n_{2h} < \lambda$ by Eq. (10), which implies

$$\eta_{2h} \equiv \frac{n_{2h}}{n_{2h} + n_{2f}} < \frac{\lambda}{\lambda + (1-\lambda)} = \lambda.$$

In both the interior and corner equilibria, because of the continuity of η_{2h}, there exists a $\phi^\dagger \in (0, 1)$ at which $\eta_{2h} = \lambda$. We have $\eta_{2h} > \lambda$ when ϕ is close to 0 and $\eta_{2h} < \lambda$ when ϕ is close to 1.

Moreover, total differentiating Eqs. (8) - (10) and (A1) - (A3) w.r.t. k and plugging Eqs. (A4), (A5) into it, by the definition of n_{vh}, we derive

$$\left.\frac{\partial \eta_{1h}}{\partial k}\right|_{\phi \to 0} = -\frac{\mathcal{H}}{k}(1-\beta)(2\lambda - 1)\phi^{\frac{k}{\sigma-1}} < 0 \quad \text{and} \quad \left.\frac{\partial \eta_{2h}}{\partial k}\right|_{\phi \to 0} = \frac{\mathcal{H}}{k}\beta(2\lambda - 1)\phi^{\frac{k}{\sigma-1}} > 0.$$

It implies that a smaller k brings to a higher η_{1h} and a lower η_{2h} when ϕ is close to zero. On the other hand, at $\phi = 1$, in the interior equilibrium, differentiating η_{vh} w.r.t. k yields

$$\frac{\partial \eta_{1h}}{\partial k} = -\frac{(2\lambda - 1)\mathcal{H}\log\left(\frac{f_x}{f}\right)}{(\sigma - 1)(1-\mathcal{H})^2} < 0 \quad \text{and} \quad \frac{\partial \eta_{2h}}{\partial k} = \frac{(2\lambda - 1)\beta\mathcal{H}\log\left(\frac{f_x}{f}\right)}{(1-\beta)(\sigma - 1)(1-\mathcal{H})^2} > 0.$$

In the corner equilibrium of $n_{1f} = 0$, we have $\eta_{1h} = 1$. By using Eq. (A9), we derive

$$\frac{\partial \eta_{2h}}{\partial k} = \frac{\beta(1-\lambda)}{k^2(1-\beta)} \left[\mathcal{H}\lambda(\sigma - 1 + k\log(f_x/f)) - (1-\lambda)(\sigma-1)\right] > 0,$$

where the inequality comes from $k\log(f_x/f) > 0$ and $\mathcal{H} > \frac{1-\lambda}{\lambda}$ in corner equilibrium. □

Appendix C. Proof of Proposition 4

At $\phi = 0$, we solve $EX_1(0) = EX_2(0) = 0$. At ϕ close to zero, we derive

$$EX_1'(\phi)\big|_{\phi\to 0} = \frac{Lk\alpha\beta\sigma\mathcal{H}(2\lambda-1)}{(\sigma-\alpha)(\sigma-1)}\phi^{\frac{k-\sigma+1}{\sigma-1}} > 0, \quad EX_2'(\phi)\big|_{\phi\to 0} = \frac{L\alpha\sigma(1-\beta)(2\lambda-1)}{\sigma-\alpha} > 0,$$

which implies that the larger country is a net exporter of both industrial goods when trade costs are close to the level of autarky.

On the other hand, at $\phi = 1$, in the interior equilibrium, by plugging Eq. (B1), (B2) into (11) and (12), we solve

$$EX_1(1) = \frac{L\alpha\beta\sigma\mathcal{H}(2\lambda-1)}{(1-\mathcal{H})(\sigma-\alpha)} > 0 \quad \text{and} \quad EX_2(1) = -\frac{L\alpha\beta\sigma\mathcal{H}(2\lambda-1)}{(1-\mathcal{H})(\sigma-\alpha)} < 0,$$

which implies that the larger country is a net exporter (importer) of Industry 1 (2) goods when trade costs are close to the level of free trade. Furthermore, total differentiating $EX_1(1)$ and $EX_2(1)$ w.r.t. k derives

$$\frac{\partial EX_1(1)}{\partial k} = -\frac{L\alpha\beta\sigma\log(f_x/f)\mathcal{H}(2\lambda-1)}{(1-\mathcal{H})^2(\sigma-1)(\sigma-\alpha)} < 0, \frac{\partial EX_2(1)}{\partial k} = \frac{L\alpha\beta\sigma\log(f_x/f)\mathcal{H}(2\lambda-1)}{(1-\mathcal{H})^2(\sigma-1)(\sigma-\alpha)} > 0,$$

which means that, at $\phi = 1$, a smaller k increases (decreases) the larger country's net exports of Industry 1 (2) goods when trade costs are close to the level of free trade. On the other hand, in the corner equilibrium of $n_{1f} = 0$, by plugging Eqs. (A8), (A9) into (11) and (12), we solve

$$EX_1(1) = \frac{\mathcal{H}kL\alpha\beta\lambda\sigma}{k\sigma - \alpha k - \alpha\beta(\lambda + \mathcal{H}\lambda - 1)(\sigma - 1)} > 0 \quad \text{and}$$
$$EX_2(1) = -\frac{L\sigma\alpha\beta(1-\lambda)\left[k - (1 - \lambda - \mathcal{H}\lambda)(\sigma - 1)\right]}{k(\sigma - \alpha) + \alpha\beta(1 - \lambda - \mathcal{H}\lambda)(\sigma - 1)} < 0,$$

where the inequalities come from $k > 2$ and $\mathcal{H} > \frac{1-\lambda}{\lambda}$. Furthermore, we derive

$$\frac{\partial EX_1(1)}{\partial k} = -\frac{\mathcal{H}L\alpha^2\beta^2\ \sigma\lambda(\sigma - 1)(\mathcal{H}\lambda - 1 + \lambda)}{\left[k(\alpha - \sigma) - \alpha\beta(1 - \lambda - \mathcal{H}\lambda)(\sigma - 1)\right]^2} < 0 \quad \text{and}$$
$$\frac{\partial EX_2(1)}{\partial k} = \frac{L\alpha\beta\sigma(1-\lambda)(\sigma - 1)[\sigma - \alpha(1-\beta)](\mathcal{H}\lambda - 1 + \lambda)}{[k(\alpha - \sigma) - \alpha\beta(1 - \lambda - \mathcal{H}\lambda)(\sigma - 1)]^2} > 0,$$

where the inequalities come from $\mathcal{H} > \frac{1-\lambda}{\lambda}$ in the corner equilibrium. Therefore, the results are robust in the corner equilibrium of $n_{1f} = 0$. Meanwhile, at ϕ close to zero, we derive

$$\left.\frac{\partial EX_1(\phi)}{\partial k}\right|_{\phi\to 0} = -\frac{L\alpha\beta\sigma(2\lambda - 1)\mathcal{H}\phi^{\frac{k}{\sigma-1}}}{k(\sigma - \alpha)} < 0 \quad \text{and}$$
$$\left.\frac{\partial EX_2(\phi)}{\partial k}\right|_{\phi\to 0} = \frac{L\alpha\beta\sigma(1-\beta)(2\lambda - 1)[\alpha\lambda(2\lambda - 1) + \sigma - \alpha\lambda]\mathcal{H}\phi^{\frac{k+\sigma-1}{\sigma-1}}}{\lambda(1-\lambda)(\sigma - \alpha)^2(k + \sigma - 1)} > 0,$$

which imply that a smaller k increases (decreases) the larger country's net exports of Industry 1 (2) goods when trade costs are close to the level of autarky. □

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Online Appendix

1. Conditions of ensuring $\varphi_{jx}^* > \varphi_j^*$

In line with the empirical evidence that the exporting firms also serve the domestic market, we derive the conditions of ensuring $\varphi_{jx}^* > \varphi_j^*$. First of all, as shown by Appendix A, we have $w_h > w_f$ for $\phi \in (0,1)$ and, therefore, $\varphi_h^* > 0$. Using Eq. (7), we derive a necessary and sufficient condition $w_h/w_f < (f_x/f)^{\frac{k-\sigma+1}{k}}$ which ensures φ_f^* to be a positive real number. Second, we derive conditions that ensure $\varphi_{hx}^* > \varphi_h^*$ and $\varphi_{fx}^* > \varphi_f^*$. Note that $\varphi_h^* < \varphi_f^*$, as aforementioned, the countries with higher wages have a lower cutoff productivity level. From the mirror expression of Eq. (4), $w_h > w_f$ and $f_x > f$ imply $\varphi_{hx}^* > \varphi_f^* > \varphi_h^*$. By Eq. (4), we also have $\varphi_{hx}^* > \varphi_{fx}^*$ because of $\varphi_f^* > \varphi_h^*$ and $w_h > w_f$. To ensure $\varphi_{fx}^* > \varphi_f^*$, by using Eq. (7), the sufficient and necessary condition is derived as $\frac{w_h}{w_f} < \left(\frac{f_x}{f}\right)\left[\frac{\mathcal{H}(f_x/f)^{\frac{k}{1-\sigma}}\tau^{-k}+1}{\tau^{-k}+f_x/f}\right]^{\frac{\sigma-1}{k}} < \left(\frac{f_x}{f}\right)^{\frac{k-\sigma+1}{k}}$, where the second inequality comes from $1 > \mathcal{H} > 0$. Therefore, $\frac{w_h}{w_f} < \left(\frac{f_x}{f}\right)\left[\frac{\mathcal{H}(f_x/f)^{\frac{k}{1-\sigma}}\tau^{-k}+1}{\tau^{-k}+f_x/f}\right]^{\frac{\sigma-1}{k}}$ is the sufficient and necessary condition to ensure $\varphi_{hx}^* > \varphi_{fx}^* > \varphi_f^* > \varphi_h^*$.

2. Possibilities of corner equilibria

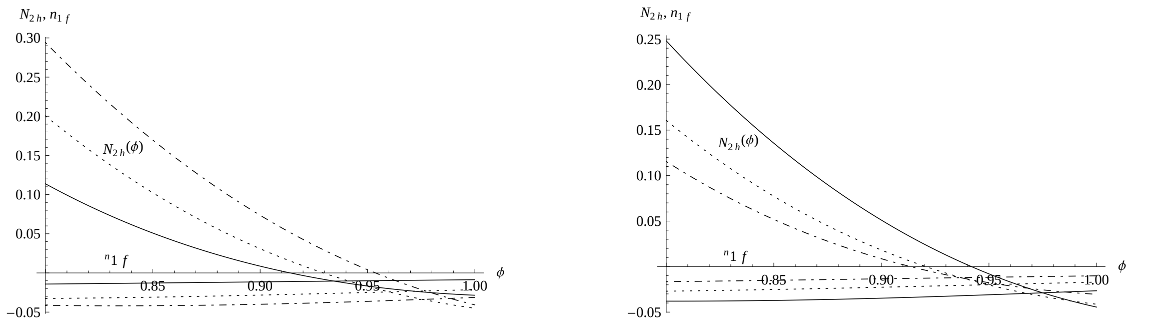
This section shows that $n_{2h} = 0$, $n_{2f} = 0$, and $n_{1h} = 0$ are not reasonable corner equilibria. Since closed-form solutions are not available here, we show it by numerical experiments.

2.1 $n_{2h} = 0$

Eq. (9) can be rearranged as

$$N_{2h} \equiv \left(\frac{p_{2hh}^{1-\sigma} Y_h}{P_{2h}^{1-\sigma}} + \phi \frac{p_{2hh}^{1-\sigma} Y_f}{P_{2f}^{1-\sigma}} \right) \alpha(1-\beta) - \sigma w_h.$$

Plugging Eqs. (4) - (6) and $n_{2h} = 0$ into (8), (10), (A1) - (A3), and N_{2h}, the threshold value of $\phi^\sharp$ is defined as the trade freeness at which $N_{2h} = 0$ holds. Although $\phi^\sharp$ is not tractable, numerical experiment shows that we always have $n_{1f} < 0$ at $\phi^\sharp$, as shown by Fig. 5. Therefore, $n_{2h} = 0$ is not a reasonable corner equilibrium.



(a) $\sigma = 4$, Solid: $k = 4$, Dotted: $k = 6$, DotDashed: $k = 8$ (b) $k = 7$, Solid: $\sigma = 4$, Dotted: $\sigma = 5$, DotDashed: $\sigma = 6$

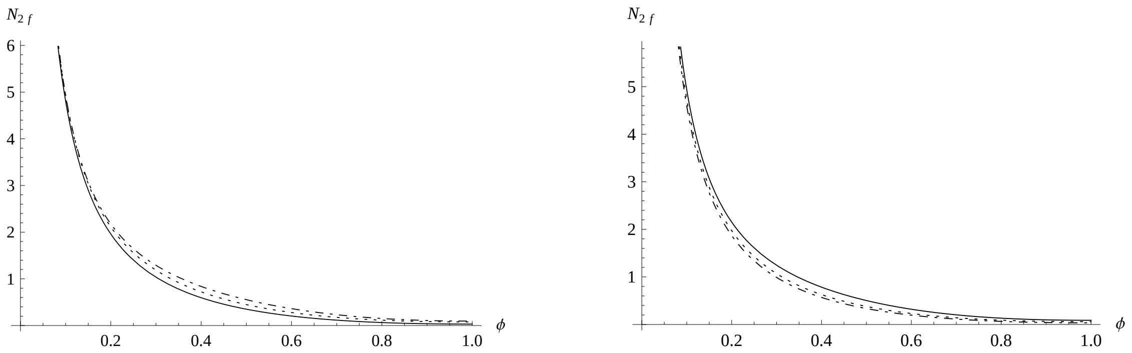
Figure 5: N_{2h} and n_{1f} w.r.t. ϕ.

2.2 $n_{2f} = 0$

Eq. (A2) can be rearranged as

$$N_{2f} \equiv \left(\frac{p_{2ff}^{1-\sigma} Y_f}{P_{2f}^{1-\sigma}} + \phi \frac{p_{2ff}^{1-\sigma} Y_h}{P_{2h}^{1-\sigma}} \right) \alpha(1-\beta) - \sigma w_f.$$

Plugging Eqs. (4) - (6) and $n_{2f} = 0$ into (8) - (10), (A1), (A3), and N_{2f}, the threshold value of $\phi^\sharp$ is defined as the trade freeness at which $N_{2f} = 0$ holds. Although $\phi^\sharp$ is not tractable, numerical experiment shows that such $\phi^\sharp \in (0,1)$ does not exist, as shown by Fig. 6. Therefore, $n_{2f} = 0$ is not a reasonable corner equilibrium.



(a) $\sigma = 4$, Solid: $k = 4$, Dotted: $k = 8$, DotDashed: $k = 12$

(b) $k = 10$, Solid: $\sigma = 4$, Dotted: $\sigma = 6$, DotDashed: $\sigma = 8$

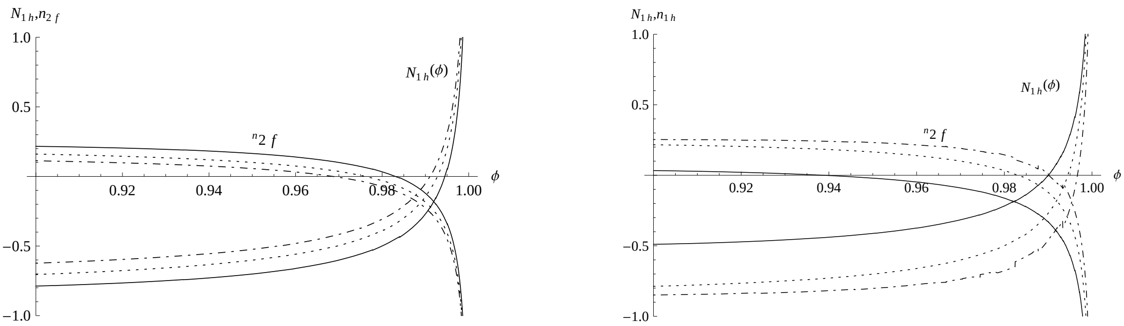
Figure 6: N_{2f} w.r.t. ϕ.

2.3 $n_{1h} = 0$

Eq. (8) can be rearranged as

$$N_{1h} \equiv n_{1h} \left(\frac{\tilde{\varphi_h}}{\varphi_h^*} \right)^{\sigma-1} \sigma f w_h + n_{1f} \left(\frac{\varphi_f^*}{\varphi_{fx}^*} \right)^k \left(\frac{\tilde{\varphi_{fx}}}{\varphi_{fx}^*} \right)^{\sigma-1} \sigma f_x w_f - Y_h \alpha \beta.$$

Plugging Eqs. (4) - (6) and $n_{1h} = 0$ into (9), (10), (A1) - (A3), and N_{1h}, the threshold value of $\phi^\sharp$ is defined as the trade freeness at which $N_{1h} = 0$ holds. Although $\phi^\sharp$ is not tractable, numerical experiment shows that we always have $n_{2f} < 0$ at $\phi^\sharp$, as shown by Fig. 7. Therefore, $n_{1h} = 0$ is not a reasonable corner equilibrium.



(a) $\sigma = 4$, Solid: $k = 6$, Dotted: $k = 7$, DotDashed: $k = 8$

(b) $k = 6$, Solid: $\sigma = 3$, Dotted: $\sigma = 4$, DotDashed: $\sigma = 4.5$

Figure 7: N_{1h} and n_{2f} w.r.t. ϕ.