New Essentials of Economic Theory I. Assumptions, Economic Space and Variables

Olkhov, Victor

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Victor Olkhov

TVEL, Moscow, Russia

victor.olkhov@gmail.com

Abstract

This paper develops economic theory framework free from general equilibrium assumptions. We describe macroeconomics as system of economic agents under action of *n* risks. Economic and financial variables of agents, their expectations and transactions between agents define macroeconomic variables. Agents variables depend on transactions between agents and transactions are performed under agents expectations. Agents expectations are formed by economic variables, transactions, expectations of other agents, other factors that impact macroeconomic evolution. We use risk ratings of agents as their coordinates on economic space and approximate description of economic and financial variables, transactions and expectations of numerous separate agents by description of variables, transactions and expectations as density functions on economic space. We describe evolution of macroeconomic density functions of variables, transactions and expectations and their flows induced by motion of separate agents on economic space due to change of agents risk rating. We apply our model to description of business cycles, present models of wave propagation for disturbances of economic variables and transactions, model asset price fluctuations and argue hidden complexities of classical Black-Scholes-Merton option pricing.

Keywords: economic theory, risk ratings, economic space, economic flows, density functions

JEL: C00, C02, C10, E00

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1. Introduction

Economic policy and regulation rely heavily on general equilibrium theory (GE) (Arrow and Debreu, 1954; Tobin, 1969; Arrow, 1974; Smale, 1976; Kydland and Prescott, 1990; Starr, 2011) and DSGE (Fernández-Villaverde, 2010; Komunjer and Ng, 2011; Negro, et al, 2013; Farmer, 2017) and define implementation of macroeconomic and financial management and policymaking. Existing flaws and weaknesses of GE and DSGE may bring economic authorities to unjustified decisions and add excess shocks into unsteady global economic and financial processes. Numerous papers study for pro and contra of GE (Hazlitt, 1959; Morgenstern, 1972; Ackerman, 1999; Stiglitz, 2017). A special issue of Oxford Review of Economic Policy on "Rebuilding macroeconomic theory" (Vines and Wills, Eds. 2018a) presents 14 papers of 18 authors those discuss: "What new ideas are needed? What needs to be thrown away? What might a new benchmark model look like? Will there be a 'paradigm shift'?" (Vines and Wills, 2018b).

We present economic model that entirely differs from mainstream GE. There is not much sense to argue pro and contra of our approach before we introduce main economic assumptions and formal frame of the model. Thus we avoid any general discussions and comparisons with GE and move forward to introduce the model.

The sketch of our approach is based on well-known economic statements. We treat macroeconomics as system of numerous economic agents. Agents have different economic and financial variables and are engaged into various economic and financial transactions with other agents. Agents perform transactions under different expectations. Agents form expectations on base of their forecasts of macroeconomic variables, transactions, expectations of other agents, policy, technology or regulatory changes, climate forecasts and so on. We describe relations between three core economic notions - variables, transactions and expectations.

This study has three Parts. In Part I we argue main economic assumptions and explain key concepts of our model (Sec.2). In Sec.3 we argue economic agents as simple units of macroeconomic processes and introduce economic space. In Sec.4 we discuss meaning of economic and financial variables and introduce notions of flows of economic variables on economic space. We derive equations that describe dynamics of economic and financial variables and their flows on economic space and argue their economic meaning. In Part II (Olkhov, 2019c) we study transactions and expectations on economic space and develop assets pricing model as result of equations on transactions and expectations. In Part III

(Olkhov, 2019d) we apply our model to description of several particular economic problems. We model business cycles, describe wave propagation for disturbances of different economic variables and transactions, describe asset pricing model and price fluctuations and argue hidden complexities of classical Black-Scholes-Merton option pricing model.
We number equations independently in each Part of the paper and refer (II.4) as equation (4) in Part II. We use bold letters to denote vectors and roman letters – scalars.

2. Main assumptions and economic model

Let's regard macroeconomics as a system of numerous economic agents. Under different expectations agents perform economic and financial transactions with other agents. Let's mention that our approach has almost nothing common with agent-based models (ABM) (Tesfatsion and Judd, 2005; Gaffard and Napoletano, 2012).
Agents expectations may reflect forecasts of economic growth, demand, expectations of other agents, assumptions on possible economic impact of policy, regulatory or technology changes and etc. Certain macroeconomic variables are determined as sum (without doubling) of corresponding variables of economic agents. For example, macroeconomic demand, supply, investment, credits are determined as sum of demand, supply, investment and credits of economic agents. Let's call such variables as additive. Other macroeconomic variables are determined as ratio of two additive variables and are non-additive. For example prices are determined as ratio of transactions trading values and trading volumes. Inflation, indexes are determined as ratio of prices in different moments of time and are non-additive also. We present these obvious considerations to make simple statement: agents additive variables those define additive macro variables describe all macroeconomic and financial variables.
Now let's argue variables those involved into transactions between agents. Any transaction imply that seller transfer certain volume of commodities, assets, service, investment and etc., to buyer. Let's call agents variables involved into transactions between agents as additive variables of type 1. Let's call other additive variables that are defined by additive variables type 1 as additive variables type 2. For example sum of agents value-added define macroeconomic additive variable – GDP (Fox, et al, 2014). As well agents value-added variables are not subject of any transaction and are determined as difference between agents aggregate sales and expenditures. Thus we call agents value-added as additive variables type 2. Sales and expenditures are result of transactions between agents and their linear functions define agents value-added. These easy examples result second simple statement: all agents variables are determined by additive variables of type 1 those involved into transactions

between agents. Hence description of transactions between agents permit model all agents variables and hence model all macroeconomic variables. This statement is well-known at least since Leontief's models (Leontief, 1941; 1955; Horowitz and Planting, 2006). Now let's present three issues that distinguish our approach from common economic treatment:

I. *We use risk ratings of economic agents as their coordinates on economic space*.

II. *We approximate description of economic and financial variables, transactions and expectations of numerous separate agents by description of variables, transactions and expectations as density functions on economic space*.

III. We take into account flows of economic variables, transactions and expectations induced by motion of separate agents on economic space due to change of agents risk ratings and describe macroeconomic impact of such economic flows.

Let's discuss these issues in details.

I. We use risk ratings of economic agents as their coordinates on economic space.

Our main issue concern assessments of agents risk ratings. International rating agencies as S&P, Moody's, Fitch (Metz and Cantor, 2007; S&P, 2014; Fitch, 2018) for decades provide risk assessments for major banks, corporations, securities and etc., and deliver distributions of biggest banks by their risk ratings (Moody's, 2018; South and Gurwitz, 2018). These assessments are basis for investment expectations of biggest hedge funds, investors, traders etc. According to current risk assessment methodologies (Altman, 2010; Moody's, 2010; S&P&, 2016; Fitch, 2018) risk ratings take values of risk grades like *AAA, AA, BB, C* etc. Different rating agencies use different risk assessment methodologies and risk grades notions differs slightly.

Let's outline that risk grades *AAA, AA, BB, C* can be treated as points $x_1,\ldots x_N$ of space that we call further as economic space. Risk assessment methodology use available economic statistics and determine number N of risk points. Let's propose that economic statistics and econometrics can provide sufficient data to assess risk ratings for all economic agents and for all risks that may hit macroeconomic evolution and growth. Let's assume that rating agencies may be able to estimate risk ratings for all economic agents: for large corporations and banks and for small companies, firms and even households. Now let's assume that risk assessment methodologies can define continuous spectrum of risk grades on space R. Risk methodology always can take continuous risk grades as $[0,1]$ with point 0 as most secure and 1 as most risky grades. A lot of different risks can disturb macroeconomic processes (McNeil, Frey and Embrechts, 2005;). Assessments of single risk, like credit risk, distributes agents over range $[0,1]$ of 1-dimensional economic space R. Assessments of two or three risks, like credit, exchange rate and liquidity for example, distribute economic agents over unit square or cube. For given configuration of n macroeconomic risks, assessments of agents risk rating distribute agents by their risk coordinates $\boldsymbol{x}=(x_1,\ldots x_n)$ over economic domain

$$0 \leq x_i \leq 1\,, i = 1, \ldots n \qquad (1.1)$$

of n-dimensional economic space R^n. Distribution of economic agents by their risk coordinates $\boldsymbol{x}=(x_1,\ldots x_n)$ over economic domain (1.1) mean that all economic and financial variables of agents are also distributed on economic domain (1.1). Aggregation of similar variables for agents with coordinates near point $\boldsymbol{x}=(x_1,\ldots x_n)$ of (1.1) define economic variables as functions of $\boldsymbol{x}$. Aggregations of similar transactions between agents with coordinates $\boldsymbol{x}$ and $\boldsymbol{y}$ determine transactions as functions of $\boldsymbol{x}$ and $\boldsymbol{y}$ on economic space. As we show below this helps describe dynamics of macroeconomic variables, transactions and expectations by partial differential equations on economic space.

Let's repeat our main assumptions:

1. We assume that economic statistics may provide sufficient data for risk assessment of almost all economic agents for wide range of macroeconomic risks. That permits distribute economic agents by their risk ratings as coordinates on economic space.
2. We propose that risk assessment methodologies may define continuous risk grades *[0,1]* on R for all macroeconomic risks. Ratings of n risks define risk coordinates $\boldsymbol{x}=(x_1,\ldots x_n)$ on economic domain (1.1) of n-dimensional economic space R^n.

II. *We approximate description of economic and financial variables, transactions and expectations of numerous separate agents by description of economic and financial variables, transactions and expectations as density functions on economic space*.

Transition from description of economic properties, like variables, transactions and expectations, of separate agents to same economic properties as density functions on economic space has clear economic meaning. Risk assessment distributes agents by their ratings as coordinates on economic domain (1.1). Description of variables and transactions of numerous separate agents requires a lot of econometric data. We propose approximation that gives more rough description but requires significantly less economic data. To establish such approximation let's aggregate variables, transactions or expectations of agents with risk coordinates inside small volume dV on economic domain (1.1) and then average them. To do that let's chose economic space scale d and time scale Δ. For n-dimensional economic space R^n let's take unit volume $dV=d^n$ near point $\boldsymbol{x}$ of (1.1) and assume that space scales $d \ll 1$ are small to compare with scales of economic domain (1.1) but many economic agents have risk coordinates inside this unit volume dV near point $\boldsymbol{x}$. The similar requirements concern time scale: *Δ* should be small to compare with time scale of the problem under consideration but many transactions should be performed during *Δ*. For example, the number of agents in economics with population around 10^8-10^9 can be estimated as 10^8-10^9. Thus space scale $d \sim 10^{-2}$ on 2-dimensional economic space defines unit volume $dV \sim 10^{-4}$ with estimate 10^4-10^5 agents inside it. Time scale *Δ=1 week* is small with time term one quarter or year. Assumption - 1 transaction between agents per second gives assessment of $6*10^5$ transactions per *Δ=1 week*. Thus scales $d \sim 10^{-2}$ and *Δ=1 week* may help approximate economic processes for time term one quarter or year. As example let's consider Credits provided by agents with coordinates inside *dV* near point ***x*** and average it during *Δ=1 week*. Let's take that $C(t,\boldsymbol{x})$ equals sum of credits over volume *dV* and averaged during time *Δ*. Function $C(t,\boldsymbol{x})$ has meaning of density of credits provided by agents from point ***x*** at moment *t*. Indeed, integral of $C(t,\boldsymbol{x})$ by $d\boldsymbol{x}$ over economic domain equals total credits provided by all economic agents in economics at moment *t*. Averaging over time *Δ* reduce high frequency fluctuations of the sum of credits and makes this variable smooth. Introduction of space scale *d* and time scale *Δ* reduce accuracy of the model approximation. If one chose space scale *d=1* then volume *dV* will be equal economic domain and aggregation of credits provided by agents inside economic domain equals all credits provided in macroeconomics. Thus introduction of scales $d<<1$ establishes economic approximation that is intermediate between precise description of variables of numerous separate economic agents and rough macroeconomic approximation based on aggregation of variables of all economic agents. Below we define density functions for economic and financial variables, transactions and expectations. Nevertheless expectations are not additive variables, we show in Part II (Olkhov, 2019c) how apply aggregation procedure to obtain correct form for density functions of expectations. Description of density functions of economic variables, transactions and expectations require significantly less economic data then same description with accuracy of each agent and hence simplifies the models. The same time descriptions of mutual relations between density functions of economic variables, transactions and expectations are much more informative then modeling relations between macroeconomic variables as functions of time only.

It is obvious that one may aggregate agents and their variables, transactions and expectations on economic domain (1.1) by various economic groups with section by different industry sectors, wealth, gender, age or other economic or financial conditions. Macroeconomic models based on aggregation of agents by various groups on economic domain may model relations between economic variables, transactions and expectations of different industry sectors or describe influence of any specifications those define grouping agents. For such models one may use different sets of risks and different risk measures for different groups of agents. For example risk assessment may differ for different industry sectors, for different wealthy level and etc. It is clear that any specific grouping and usage of different set of risks and risk measures induce additional complexity to the model. In current study we describe simplest framework that use aggregation of all economic agents without any additional specification and use one risk assessment measure for all agents.

The most important factor that impact evolution of density functions of variables, transactions and expectations is determined by aggregative flows of variables, transactions and expectations induced by motion of agents on economic space. Such economic flows are results of motion of agents on economic space due to change of their risk rating.

III. *We take into account flows of economic variables, transactions and expectations induced by motion of separate agents on economic space due to change of agents risk ratings. We describe macroeconomic impact of such economic flows.*

Change of agents risk ratings due to their economic activity, variation of economic environment, action of risk factors and other reasons cause change of agents risk coordinates on economic space. Such change means that agents move on economic space with certain speed $\boldsymbol{v}$. Motion of agent with speed $\boldsymbol{v}$ indicates that agents carry their economic and financial variables, expectations and transactions. For example if agent provides credits C and moves with speed $\boldsymbol{v}$ then it carries flow $\boldsymbol{P}_C$ of credits as $\boldsymbol{P}_C=C\boldsymbol{v}$. Flows of variables, expectations and transactions carried by agents due to change of their risk ratings have important impact on macroeconomic evolution. To describe action of these flows on macroeconomics let's develop approximation similar to one we use to describe densities functions of variables, expectations and transactions. As we show below, aggregations of flows of separate agents define densities of economic flows of variables, transactions and expectations. Motion of different flows of variables, expectations and transactions have certain parallels to flows of fluids but all properties of economic flows are completely different from hydrodynamics. Numerous flows of economic and financial variables, expectations and transactions induce on economic domain (1.1) a great variety of mutual interactions and economic effects.

Now let's argue derivation of equations that should govern density functions of variables, transactions and expectations and their flows. These equations have similar form and we explain their derivation for credit density function $C(t,\boldsymbol{x})$ as example. Credit density function $C(t,\boldsymbol{x})$ aggregates credits of agents with coordinates inside small volume dV at point $\boldsymbol{x}$. Each agent moves on economic space with some velocity $\boldsymbol{v}$ due to change of its risk ratings. This motion of agents induces aggregate credit flows $\boldsymbol{P}_C(t,\boldsymbol{x})=C(t,\boldsymbol{x})\boldsymbol{v}(t,\boldsymbol{x})$. Function $\boldsymbol{v}(t,\boldsymbol{x})$

describes velocity of flow of credit density $C(t,\boldsymbol{x})$. To describe change of credit density function $C(t,\boldsymbol{x})$ during time dt in a small volume dV on economic space let's take into account two factors of such change. The first factor describes change of $C(t,\boldsymbol{x})$ due to change of agents credits in time dt in a small volume dV. That can be presented as

$$\int dV \ \frac{\partial}{\partial t} C(t, \boldsymbol{x})$$

The second factor that impact change of credit density $C(t,\boldsymbol{x})$ is determined by credit flows $\boldsymbol{P}_C=C\boldsymbol{v}$ of agents that during time dt may flow in or flow out of small volume dV. Agents that flow in the volume dV with credit flow $\boldsymbol{P}_C=C\boldsymbol{v}$ increase credit density function $C(t,\boldsymbol{x})$ and agents that flow out of the volume dV with credit flow $\boldsymbol{P}_C=C\boldsymbol{v}$ decrease credit density function $C(t,\boldsymbol{x})$. Balance of aggregated $\boldsymbol{P}_C(t,\boldsymbol{x})=C(t,\boldsymbol{x})\boldsymbol{v}(t,\boldsymbol{x})$ credit flows in and credit flows out takes form of integral of credit flows $\boldsymbol{P}_C(t,\boldsymbol{x})=C(t,\boldsymbol{x})\boldsymbol{v}(t,\boldsymbol{x})$ over the surface of small volume dV:

$$\oint ds \, \boldsymbol{P}_{\boldsymbol{C}}(t, \boldsymbol{x}) = \oint ds \, C(t, \boldsymbol{x}) \, \boldsymbol{v}(t, \boldsymbol{x})$$

Due to well-known divergence theorem (Gauss' Theorem) (Strauss 2008, p.179), surface integral of the flows equals volume integral of the flows divergence. Thus balance of credit flows equals integral of the divergence of flow over small volume dV:

$$\oint ds \, C(t, \boldsymbol{x}) \, \boldsymbol{v}(t, \boldsymbol{x}) = \int dV \ \nabla \cdot \big(\, C(t, \boldsymbol{x}) \, \boldsymbol{v}(t, \boldsymbol{x}) \big) \tag{1.2}$$

Hence total change of credit density function during time dt in a small volume dV equals:

$$\int dV \left[\frac{\partial}{\partial t} C(t, \boldsymbol{x}) + \nabla \cdot \big(\, C(t, \boldsymbol{x}) \, \boldsymbol{v}(t, \boldsymbol{x}) \big) \right]$$

As small volume dV is arbitrary one can take equations on density functions as:

$$\frac{\partial}{\partial t} C(t, \boldsymbol{x}) + \nabla \cdot \big(\, C(t, \boldsymbol{x}) \, \boldsymbol{v}(t, \boldsymbol{x}) \big) = F_C(t, \boldsymbol{x}) \tag{1.3}$$

Function $F_C(t,x)$ in the right side (1.3) describes action of any factors defined by variables, transactions and expectations and their flows on credit density function $C(t,\boldsymbol{x})$. Equation (1.3) depends on flow $\boldsymbol{P}_C(t,\boldsymbol{x})=C(t,\boldsymbol{x})\boldsymbol{v}(t,\boldsymbol{x})$ and hence one should derive equation on this flow. Completely same considerations as above cause equations on flows $\boldsymbol{P}_C(t,\boldsymbol{x})=C(t,\boldsymbol{x})\boldsymbol{v}(t,\boldsymbol{x})$ as:

$$\frac{\partial}{\partial t} \boldsymbol{P}_C(t, \boldsymbol{x}) + \nabla \cdot \big(\, \boldsymbol{P}_C(t, \boldsymbol{x}) \, \boldsymbol{v}(t, \boldsymbol{x}) \big) = \boldsymbol{G}_C(t, \boldsymbol{x}) \tag{1.4}$$

Function $\boldsymbol{G}_C(t,\boldsymbol{x})$ describes action of any factors defined by variables, transactions and expectations and their flows on credit flows $\boldsymbol{P}_C(t,\boldsymbol{x})$. Let's underline that equations (1.3; 1.4) define "simple" relations for macroeconomic variables as functions of time only. Indeed, integral by $d\boldsymbol{x}$ of credit density $C(t,\boldsymbol{x})$ over economic domain (1.1) equals macroeconomic credits $C(t)$ issued by all agents:

$$C(t) = \int d\boldsymbol{x}\ C(t, \boldsymbol{x}) \tag{1.5}$$

Integral by $d\boldsymbol{x}$ for equations (1.3) over economic domain (1.1) equals

$$\frac{d}{dt}C(t) + \int d\boldsymbol{x} \nabla \cdot \big(C(t, \boldsymbol{x})\, \boldsymbol{v}(t, \boldsymbol{x}) \big) = \int d\boldsymbol{x}\ F_C(t, \boldsymbol{x}) = F_C(t) \tag{1.6}$$

Due to (1.2) integral in left side (1.6) equals zero as no in- or out- flows exist outside of economic domain (1.1) and no economic agents exist outside economic domain (1.1). Thus (1.6) takes simple form of ordinary differential equation:

$$\frac{d}{dt}C(t) = F_C(t) \tag{1.7}$$

The problems of (1.7) are hidden by function $F_C(t)$ determined by integral in the right side of (1.6). Function $F_C(t,\boldsymbol{x})$ may depend on other variables, transactions, expectations and their flows and integral in (1.6) may define $F_C(t)$ as very complicated function. Thus time evolution of macroeconomic variables like macro credits $C(t)$ may depend on properties of hidden dynamics of variables, transactions and expectations and their flows on economic space. Integral by $d\boldsymbol{x}$ for equations (1.4) over economic domain (1.1) define ordinary differential equation on new macroeconomic variables $\boldsymbol{P}_C(t)$:

$$\boldsymbol{P}_C(t) = \int d\boldsymbol{x}\ \boldsymbol{P}_C(t, \boldsymbol{x}) = C(t)\boldsymbol{v}(t) \tag{1.8}$$

$$\frac{d}{dt}\boldsymbol{P}_C(t) = \int d\boldsymbol{x}\ \boldsymbol{G}_C(t, \boldsymbol{x}) = \boldsymbol{G}_C(t) \tag{1.9}$$

Integral (1.8) define macroeconomic flows $\boldsymbol{P}_C(t)$ of credits $C(t)$ (1.5) with velocity $\boldsymbol{v}(t)$ and equation (1.9) describes evolution of macroeconomic credit flows $\boldsymbol{P}_C(t)$ determined by function $\boldsymbol{G}_C(t)$ in the right side of (1.9). Similar equations are valid to macroeconomic flows of other additive variables as demand and supply, investment and GDP and etc. Economic meaning of equations (1.9) is following. Velocity $\boldsymbol{v}(t)$ of macroeconomic flow $\boldsymbol{P}_C(t)$ of credits $C(t)$ describes motion on economic domain (1.1) that is bounded along each risk axes by most secure and most risky grades [0,1]. Thus for each axis motion from secure to risky direction with velocity $\boldsymbol{v}(t)$ should change by opposite motion from risky to secure area of (1.1). Thus velocity $\boldsymbol{v}(t)$ and macroeconomic flow $\boldsymbol{P}_C(t)$ of credits $C(t)$ should fluctuate in time and such fluctuations describe credit cycles of macroeconomics. Similar fluctuations of flows model business cycles of GDP, investment and etc. Description of correlations between cycles of different macro variables and particular models that define forms of functions $F_C(t)$ and $\boldsymbol{G}_C(t)$ should be studied for each economic case. In Part III (Olkhov, 2019d) we present one simple model of business cycles caused by interactions between transactions.

In Part II (Olkhov, 2019c) we show that equations on transactions have form similar to (1.3; 1.4) taking into account that transactions density functions depend on two coordinates $\boldsymbol{x}$ and

y. In Part II we argue that expectations of agents can't be treated as additive variables and derivation of equations on aggregated expectations requires further considerations. We propose that economic value or economic importance of agents expectations should be taken proportional to value of transactions approved by this particular expectation. In Part II we introduce additive factors that we call – expected transactions – that are proportional to product of transactions and expectations. Our approach permits define density functions of expected transactions and flows of expected transactions. Further we derive equations on expected transactions and their flows that have form similar to (1.3; 1.4). That permits derive definitions and equations for density functions of expectations and their flows. Further in Part II we show that considerations similar to those we use for description of expectations can be applied for description of prices as densities functions on economic space and we derive definitions and equations for price density functions and their flows. That allows model dynamics of asset pricing determined by corresponding transactions. It is well-known that asset pricing is one of the most important problems of economics and finance and papers by (Cochrane and Hansen, 1992; Cochrane and Culp, 2003; Hansen, 2013; Campbell, 2014; Fama, 2014; Cochrane, 2017) refer only few but important studies on asset pricing. These studies argue models that determine "correct" price of assets. In our paper we don't argue "correct" price and don't study why asset price should take certain value. We describe prices as results of transactions performed by agents in economy. In Part II we study different definitions of prices caused by different aggregations of transactions and show how economic equations on transactions, expectations and their flows determine equations on prices caused by transactions.

Let's argue some consequences of our macroeconomic approximations. As we mention above equations similar to (1.3; 1.4) describe density functions and flows of numerous economic and financial variables, transactions and expectations. Thus equations (1.3; 1.4) define macroeconomic approximations for each selected set of variables, transactions and expectations. Let's take a model that describes macroeconomics by set of k different transactions. As such transactions one can study for example credit transactions, investment, buy-sell transactions and etc. Each type k of transactions defines change of variables of sellers and buyers. For example credit transaction change value of credits provided by Creditor (seller) and amount of loans received by Borrowers (buyers). Hence each type of transactions can change only two additive variables of type 1 – one for seller and one for buyer and their prices. Thus k types of transactions can change $2k$ additive variables of type 1 and their prices. Transactions of each type can be performed under different expectations.

Let's assume that k types of selected transactions are performed under W expectations. To develop self-consistent macroeconomic model that describe $2k$ additive variables of type 1 determined by k types of selected transactions one should assume that all W expectations are formed by endogenous $2k$ additive variables, k selected transactions and their flows. If part of W expectations can depend on exogenous variables, transactions, expectations and their flows or exogenous shocks and etc., then one describes macroeconomic model in presence of exogenous factors. Expectations approve transactions and thus impact change of economic and financial variables. Hence expectations may transfer impact of exogenous variables, transactions or shocks on macroeconomic evolution, transactions and variables.

Importance of expectations is not reduced by their role as transfer of exogenous shocks on macroeconomic dynamics. As we argue above expectations can depend on economic flows of variables, transactions and other expectations. Dependence of expectations on economic flows makes them key factors that determine impact of economic flows on macroeconomics. Dynamics of economic flows like credit flows $\boldsymbol{P}_C(t,\boldsymbol{x})=C(t,\boldsymbol{x})\boldsymbol{v}(t,\boldsymbol{x})$, flows of variables, transactions and expectations and their mutual interactions on economic domain (1.1) establish very complex picture. For example economic flows on economic domain (1.1) generate business cycles that describe slow oscillations of macroeconomic variables. On the other hand perturbations of economic flows cause wave propagation of disturbances and shocks of economic variables, transactions and expectations those induce fast oscillations of economic parameters. Consistent macroeconomic model on base of economic equations (1.3; 1.4) that describe dynamics of $2k$ variables that depend on k transactions under action of W expectations establish a really tough problem. Reductions of complete system of equations permit study various approximations of macroeconomic evolution. In Part III (Olkhov, 2019d) we study approximations of equations (1.3; 1.4) that describe "simplified" model interactions between two variables, between two transactions, between transactions and expectations. Such approximations help describe model examples of business cycles and different examples of wave propagation of disturbances of economic variables and transactions inside economic domain and on surface of economic domain (1.1). Similar approximations permit develop model of price fluctuations induced by interactions between transactions and numerous expectations.

3. Economic space and economic agents

Notion of economic agents is a basic economic term (Giovannini, 2008): "One of the fundamental characteristics of activities defined as economic processes is that they involve relations between various agents. The definition of economic agent is therefore absolutely fundamental in determining the nature of the economic processes: economic agent refers to a person or legal entity that plays an active role in an economic process". There are a lot of studies of agent-based economic and financial models (Tesfatsion and Judd, 2005; Gaffard and Napoletano, 2012). Our approach has nearly nothing with them. We regard agent as economic unit that has a lot of economic or financial variables like asset and debts, investment and credits, supply and demand and etc. Economic and financial variables can be additive or non-additive. Additive variables are investment, credits, volume and cost of commodities and etc. Non-additive variables are prices, bank rates, inflation, indexes and etc. Non-additive variables can be presented as ratio of two additive variables or ratio of non-additive variables. For example price of commodity equals ratio of cost and volume of commodities purchased by particular transaction. Inflation index during time term *[0,T]* equals ratio of prices at moment *T* and at moment *0*. All additive macroeconomic or financial variables like GDP, investment, credits, supply and demand and etc., are composed as sum of agents variables. For example macroeconomic investment equals sum of investment (without doubling) of all agents of the entire economics. Non-additive macroeconomic variables like inflation, economic growth are determined as ratios of macroeconomic additive variables. Thus description of agents additive economic and financial variables determine evolution of all macroeconomic and financial variables. Let's introduce economic space notion and explain how macroeconomic additive variables can be described by additive variables of economic agents.

To define economic space let's use well-known economic tool – risk ratings. Risk management and risk assessment (Horcher, 2005; Skoglund and Chen, 2015) during at least 50 years establish well-developed sector of economics. Risk assessment is a core tool for banking and corporate management and is necessary issue for any investment and financial markets operations. Top international rating agencies provide risk assessments for major banks, financial securities and etc. Risk ratings of particular agent like bank or corporation or ratings of their securities impact on decisions of financial markets traders. There are many risks that affect macroeconomics and finance like credit, inflation, market risks and etc. We don't argue particular risks but treat any risks as factors that may affect economic and financial properties of agents and hence entire economics.

Let's treat assessments of risk ratings of agents as coordinates of agents alike to coordinates of physical particles. Let's note space that imbed agents by their risk coordinates as economic space (Olkhov, 2016a-b; 2017a-d). Current risk methodologies measure risk ratings by risk grades (Wilier, 1901; McNeil, Frey and Embrechts, 2005; Metz and Cantor, 2007; SEC, 2012; S&P, 2014) that have notations as *AAA, AA, BB, C* etc. Let's take current risk grades as points $x_1,...x_n$ of economic space. Such economic space imbed economic agents by their risk ratings $\boldsymbol{x}$. Risk grades of single risk establish *1*-dimensional economic space. Grades of two or three risks establish 2 or 3 dimensional economic space. Number of risk grades like *AAA, AA, BB, C* etc. depends on risk assessment methodology. Let's assume that one may extend risk methodology so that it adopts continuous risk grades. Then n-dimensional economic space that describe action of n risks can be treated as R^n. Let's propose that economic statistics provide sufficient data for risk assessments of all economic agents of the macroeconomics. Let's state that risk ratings take continuous values between most secure grade equals *0* and most risky grade equals *1*. Partition of agents by their risk ratings for n risks define economic domain (1.1) on economic space R^n. All agents have their risk coordinates inside economic domain (1.1). Partition of agents on economic domain (1.1) establishes distribution of agents economic and financial variables over economic domain. Change of agents risk ratings due to their economic activity, market dynamics, other endogenous or exogenous shocks induce evolution of agents variables and thus change macroeconomic variables. In the next section we show how usage of risk ratings as coordinates of economic agents describes evolution of macroeconomic variables.

4. Economic variables on economic space

Description of numerous separate agents and their economic and financial variables is too complex problem. Uncertainty of agents risk coordinates and of their economic and financial variables makes such description too ambiguity. To simplify macroeconomic model and develop more sustainable and reasonable model let's rougher our description. The main idea is simple: let's rougher description of separate agents and their variables and describe same variables as aggregates of variables of agents with coordinates at point $\boldsymbol{x}$ of economic space. Let's regard macroeconomics as system of numerous agents on n-dimensional economic domain (1.1). Let's state that agents at moment t have risk ratings coordinates $\boldsymbol{x}=(x_1,...x_n)$ and velocities $\boldsymbol{v}=(v_1,...v_n)$. Velocities $\boldsymbol{v}=(v_1,...v_n)$ describe change of agents risk coordinates. Let's assume that a unit volume $dV(\boldsymbol{x})$ at point $\boldsymbol{x}$ on economic space contains many agents but scales d (2) of a unit volume $dV(\boldsymbol{x})$ are small to compare with scales of domain (1.1)

$$d \ll 1 \quad ; \quad dV = d^n \tag{2}$$

Let's regard only additive variables of agents and assume that economic statistics able select "independent" agents. Let's call agents as "independent" if sum of their additive variables equals same variable of the entire group. For example sum of Credits of k agents equals Credits of the group of these k agents. Additive variables are Credits, Investment, Asset and etc. There are a lot of non-additive variables as bank rates, inflation, prices and etc. Non-additive variables are defined as ratio of additive variables or ratio of non-additive variables. For example non-additive variable - price of transaction equals the ratio of cost and volume of this deal. Hence all economic variables are determined by additive variables only. Let's show how description of additive variables models evolution of macroeconomic variables.

Let's define additive economic variable $A(t,\boldsymbol{x})$ at point $\boldsymbol{x}$ as sum of variables $A_i(t,\boldsymbol{x})$ of agents i with coordinates in a unit volume $dV(\boldsymbol{x})$ (2) and then average it during term Δ as:

$$A(t,\boldsymbol{x}) = \sum_{i\in dV(x);\,\Delta} A_i(t,\boldsymbol{x}) \tag{3}$$

$$\sum_{i\in dV(x);\,\Delta} A_i(t,\boldsymbol{x}) = \frac{1}{\Delta}\int_t^{t+\Delta} d\tau \ \sum_{i\in dV(x)} A_i(\tau,\boldsymbol{x}) \tag{4}$$

We use $i \in dV(\boldsymbol{x})$ to denote that risk coordinates $\boldsymbol{x}$ of agent i belong to unit volume $dV(\boldsymbol{x})$. For brevity we use left hand sum (4) to denote averaging during time Δ in a unit volume $dV(\boldsymbol{x})$. We repeat meaning of space scales d and time scale Δ given in Sec.2. Scales d<<1 of volume $dV(\boldsymbol{x})$ are small to compare with scales of economic domain (1.1) but volume $dV(\boldsymbol{x})$ contains a lot of economic agents. Scale Δ is small to compare with time scales of the problem under consideration but a lot of economic and financial transactions between agents are performed during time Δ. Time averaging smooth changes of variables under numerous transactions during time Δ. We aggregate values of variables of numerous agents with risk coordinates inside volume $dV(\boldsymbol{x})$, smooth their changes during time Δ and denote result as density function of variable at point $\boldsymbol{x}$. Density function $A(t,\boldsymbol{x})$ describes economic variable at point $\boldsymbol{x}$ on economic domain (1.1). For example let's take $A_i(t,\boldsymbol{x})$ as Credits of agent i. Then density of Credits $A(t,\boldsymbol{x})$ describe sum of Credits issued by all agents with coordinates $\boldsymbol{x}$ inside unit volume $dV(\boldsymbol{x})$ and averaged during time Δ. Total value of macroeconomic variable $A(t)$ is determined by integral (5) over economic domain (1.1):

$$A(t) = \int d\boldsymbol{x}\ A(t,\boldsymbol{x}) \tag{5}$$

Function $A(t)$ equals sum (without doubling) of variables $A_i(t,\boldsymbol{x})$ of all agents i of entire economics averaged during time Δ. For example Credits $A(t)$ issued in macroeconomics equal integral of Credits $A(t,\boldsymbol{x})$ by $d\boldsymbol{x}$ over economic domain (1.1). Thus function $A(t,\boldsymbol{x})$ (3) can be treated as economic density of variable $A(t)$ (5) on economic space. Now let's describe evolution of economic densities $A(t,\boldsymbol{x})$ defined by relations (3). Economic density $A(t,\boldsymbol{x})$ (3) is composed by variables $A_i(t,\boldsymbol{x})$ of agents i . Risk ratings of each agent can change during time Δ. Such time Δ can be equal a day, a week, a quarter etc. Let's describe change of agent's i risk coordinates on economic space during time Δ by velocity $\boldsymbol{v}_i=(v_1,\ldots v_n)$. Thus each agent i with economic variable $A_i(t,\boldsymbol{x})$ carries flow of this economic variable with velocity $\boldsymbol{v}_i=(v_1,\ldots v_n)$. Flow $\boldsymbol{p}_{iA}(t,\boldsymbol{x})$ of economic variable $A_i(t,\boldsymbol{x})$ of agent i with velocity $\boldsymbol{v}_i=(v_1,\ldots v_n)$ equals:

$$\boldsymbol{p}_{iA}(t,\boldsymbol{x}) = A_i(t,\boldsymbol{x})\boldsymbol{v}_i(t,\boldsymbol{x}) \tag{6}$$

Different agents induce different flows of economic variable A in different directions with different velocities. Let's aggregate flows of variable $A_i(t,\boldsymbol{x})$ in the direction of velocity $\boldsymbol{v}_i$ of agents i with coordinates in a unit volume $dV(\boldsymbol{x})$ (2) and then average this flow during time Δ similar to relations (3, 4). Let's define flow $\boldsymbol{P}_A(t,\boldsymbol{x})$ of variable $A(t,\boldsymbol{x})$ as:

$$\boldsymbol{P}_A(t,\boldsymbol{x}) = \sum_{i\in dV(\boldsymbol{x});\,\Delta} A_i(t,\boldsymbol{x})\boldsymbol{v}_i(t,\boldsymbol{x}) \tag{7}$$

Similar to (5) integral of (7) by $d\boldsymbol{x}$ over economic domain (1.1) define macro flow $\boldsymbol{P}_A(t)$ of variable $A(t)$ as:

$$\boldsymbol{P}_{\boldsymbol{A}}(t) = \int d\boldsymbol{x}\ \boldsymbol{P}_{\boldsymbol{A}}(t,\boldsymbol{x}) \tag{8}$$

Flow $\boldsymbol{P}_A(t,\boldsymbol{x})$ (7) of variable $A(t,\boldsymbol{x})$ (3) defines aggregated velocity $\boldsymbol{v}_A(t,\boldsymbol{x})$ of economic variable $A(t,\boldsymbol{x})$ that adjust the flow (7) as:

$$\boldsymbol{P}_A(t,\boldsymbol{x}) = A(t,\boldsymbol{x})\boldsymbol{v}_A(t,\boldsymbol{x}) \tag{9}$$

Thus (9) describes flow $\boldsymbol{P}_A(t,\boldsymbol{x})$ of economic variable $A(t,\boldsymbol{x})$ with velocity $\boldsymbol{v}_A(t,\boldsymbol{x})$. Relations (5) and (8) define macro velocity $\boldsymbol{v}_A(t)$ on domain (1.1) of macro variable $A(t)$ as:

$$\boldsymbol{P}_A(t) = A(t)\boldsymbol{v}_A(t) \tag{10}$$

Let's mention that due to (3; 5; 7-9 and 10) velocity $\boldsymbol{v}_A(t)$ is not equal to integral of velocity $\boldsymbol{v}_A(t,\boldsymbol{x})$ over economic domain (1.1). Aggregation of agents economic variables defines corresponding economic densities and velocities. Due to relations (3-10) different economic variables A define different economic flows $\boldsymbol{P}_A(t,\boldsymbol{x})$ and different velocities $\boldsymbol{v}_A(t,\boldsymbol{x})$. In other words – motion of different economic variables $A(t,\boldsymbol{x})$ on economic space has different velocities $\boldsymbol{v}_A(t,\boldsymbol{x})$. For example flow $\boldsymbol{P}_C(t,\boldsymbol{x})$ of Credits $C(t,\boldsymbol{x})$ has velocity $\boldsymbol{v}_C(t,\boldsymbol{x})$ different from velocity $\boldsymbol{v}_L(t,\boldsymbol{x})$ that describe flow $\boldsymbol{P}_L(t,\boldsymbol{x})$ of Loans $L(t,\boldsymbol{x})$ or velocity $\boldsymbol{v}_I(t,\boldsymbol{x})$ that describe flow $\boldsymbol{P}_I(t,\boldsymbol{x})$ of Investment $I(t,\boldsymbol{x})$ on economic space. Macroeconomic models should describe dynamics and mutual interactions between numerous economic and financial variables and their flows. Properties of economic flows are completely different from properties of any physical flows.

To model dynamics of economic variables $A(t,\boldsymbol{x})$ and flows $\boldsymbol{P}_A(t,\boldsymbol{x})$ let's describe their change in small unit volume dV. There are two factors that change $A(t,\boldsymbol{x})$ in a unit volume dV. The first factor describes change of $A(t,\boldsymbol{x})$ on a unit volume dV in time and can be presented by time derivative as:

$$\int d\boldsymbol{x}\ \frac{\partial}{\partial t}A(t,\boldsymbol{x}) \tag{11}$$

The second factor describes change of variable $A(t,\boldsymbol{x})$ due to flows $\boldsymbol{P}_A(t,\boldsymbol{x})$. Indeed, amount of economic density $A(t,\boldsymbol{x})$ in a unit volume dV during time dt can grow up or decrease due to in- or out- flows $\boldsymbol{P}_A(t,\boldsymbol{x})$. If there are more in-flows $\boldsymbol{P}_A(t,\boldsymbol{x})$ then out-flows then amount of $A(t,\boldsymbol{x})$ will increase in a volume dV. To calculate balance of in- and out-flows let's take integral of flow $\boldsymbol{P}_A(t,\boldsymbol{x})$ over the surface of a unit volume dV:

$$\oint ds\, \boldsymbol{P}_A(t,\boldsymbol{x}) = \oint ds\ \ A(t,\boldsymbol{x})\, \boldsymbol{v}_A(t,\boldsymbol{x}) \tag{12}$$

Due to divergence theorem (Strauss 2008, p.179) surface integral of flux $A(t,\boldsymbol{x})\boldsymbol{v}_A(t,\boldsymbol{x})$ through surface equals volume integral of divergence of flow $A(t,\boldsymbol{x})\boldsymbol{v}_A(t,\boldsymbol{x})$

$$\oint ds\, A(t,\boldsymbol{x})\, \boldsymbol{v}_A(t,\boldsymbol{x}) = \int d\boldsymbol{x}\ \nabla\cdot(A(t,\boldsymbol{x})\, \boldsymbol{v}_A(t,\boldsymbol{x})) \tag{13}$$

Relations (11,13) give total change of variable $A(t,\boldsymbol{x})$ in a unit volume dV :

$$\int dx\ \left[\frac{\partial}{\partial t}A(t,\boldsymbol{x}) + \nabla\cdot\big(A(t,\boldsymbol{x})\, \boldsymbol{v}_A(t,\boldsymbol{x})\big)\right]$$

As unit volume dV is arbitrary one can take equations on economic density $A(t,\boldsymbol{x})$ as

$$D_A A(t,\boldsymbol{x}) = \frac{\partial}{\partial t}A(t,\boldsymbol{x}) + \nabla\cdot\big(A(t,\boldsymbol{x})\, \boldsymbol{v}_A(t,\boldsymbol{x})\big) = F_A(t,\boldsymbol{x}) \tag{14}$$

Function $F_A(t,\boldsymbol{x})$ in right side (14) describe factors that impact change of economic density $A(t,\boldsymbol{x})$ as: other variables, transactions, expectations and etc. Equations like (14) are reproduced in any treatise on physics of fluids (Batchelor, 1967; Resibois and De Leener, 1977; Landau and Lifshitz, 1987) and are valid for any additive economic or financial variables defined as aggregates of agents variables on economic space similar to (3). Due to (13) integral of divergence of flow $\nabla\cdot\big(A(t,\boldsymbol{x})\, \boldsymbol{v}_A(t,\boldsymbol{x})\big)$ over economic domain (1.1) equals integral over surface of economic domain (1.1) and hence equals zero as no economic or financial flows exist outside of (1.1):

$$\int d\boldsymbol{x}\ \nabla\cdot\big(A(t,\boldsymbol{x})\, \boldsymbol{v}_A(t,\boldsymbol{x})\big) = \oint ds\, A(t,\boldsymbol{x})\, \boldsymbol{v}_A(t,\boldsymbol{x}) = 0$$

Hence integral over economic domain (1.1) for (14) due to (5) equals:

$$\int d\boldsymbol{x}\left[\frac{\partial}{\partial t}A(t,\boldsymbol{x}) + \nabla\cdot\big(A(t,\boldsymbol{x})\, \boldsymbol{v}_A(t,\boldsymbol{x})\big)\right] = \frac{d}{dt}\int d\boldsymbol{x}\ A(t,\boldsymbol{x}) = \frac{d}{dt}A(t) \tag{15}$$

Thus operator D_A (14) on economic space for economic or financial variable $A(t,\boldsymbol{x})$ (3) plays the same role as usual ordinary derivation by time d/dt for macro variable $A(t)$ (5). Let's underline that different variables $A(t,\boldsymbol{x})$ and $B(t,\boldsymbol{x})$ follow different operators (14) due to different velocities $\boldsymbol{v}_A(t,\boldsymbol{x})$ and $\boldsymbol{v}_B(t,\boldsymbol{x})$. So, economic variable $B(t,\boldsymbol{x})$ follows equations:

$$D_B B(t,\boldsymbol{x}) = \frac{\partial}{\partial t}B(t,\boldsymbol{x}) + \nabla\cdot\big(B(t,\boldsymbol{x})\, \boldsymbol{v}_B(t,\boldsymbol{x})\big) = F_B(t,\boldsymbol{x}) \tag{16}$$

Equations (14; 16) are valid for additive variables. Flow $\boldsymbol{P}_A(t,\boldsymbol{x})$ follows the same operator D_A (14) as variable $A(t,\boldsymbol{x})$ and

$$D_A\boldsymbol{P}_A(t,\boldsymbol{x}) \equiv \frac{\partial}{\partial t}\boldsymbol{P}_A(t,\boldsymbol{x}) + \nabla\cdot\big(\boldsymbol{P}_A(t,\boldsymbol{x})\,\boldsymbol{v}_A(t,\boldsymbol{x})\big) = \boldsymbol{G}_A(t,\boldsymbol{x}) \tag{17}$$

$$\nabla\cdot\big(\boldsymbol{P}_A(t,\boldsymbol{x})\,\boldsymbol{v}_A(t,\boldsymbol{x})\big) = \sum_{j=1,..n}\frac{\partial}{\partial x_j}\Big(\boldsymbol{P}_A(t,\boldsymbol{x})\,v_{Aj}(t,\boldsymbol{x})\Big)$$

Function $\boldsymbol{G}_A(t,\boldsymbol{x})$ in right side (17) describe factors that impact change of economic flow $\boldsymbol{P}_A(t,\boldsymbol{x})$ as: other variables, transactions, expectations and etc.

Equations (14, 17) describe evolution of $A(t,\boldsymbol{x})$ (3) and $\boldsymbol{P}_A(t,\boldsymbol{x})$ (9) under action of factors $F_A(t,\boldsymbol{x})$ and $\boldsymbol{G}_A(t,\boldsymbol{x})$. Integrals of (14; 17) by $d\boldsymbol{x}$ over domain (1.1) give ordinary differential equations as no economic or financial flows exist outside of (1.1) (Strauss 2008, p.179):

$$\int d\boldsymbol{x}\left[\frac{\partial}{\partial t}A(t,\boldsymbol{x}) + \nabla\cdot\big(A(t,\boldsymbol{x})\,\boldsymbol{v}_A(t,\boldsymbol{x})\big)\right] = \frac{d}{dt}A(t) = F_A(t) \tag{18.1}$$

$$\int d\boldsymbol{x}\left[\frac{\partial}{\partial t}\boldsymbol{P}_A(t,\boldsymbol{x}) + \nabla\cdot\big(\boldsymbol{P}_A(t,\boldsymbol{x})\,\boldsymbol{v}_A(t,\boldsymbol{x})\big)\right] = \frac{d}{dt}\boldsymbol{P}_A(t) = \boldsymbol{G}_A(t) \tag{18.2}$$

Ordinary differential equations (18.1, 18.2) describe time evolution of macroeconomic and financial variables of entire economics. It is clear that all complexity of economic dynamics is described by right hand side factors $F_A(t,\boldsymbol{x})$ and $\boldsymbol{G}_A(t,\boldsymbol{x})$ in (14, 17). Equations (14, 17) permit model self-consistent interactions between two macro variables. The simplest case of mutual dependence between two macro variables can be presented as

$$\frac{\partial}{\partial t}A(t,\boldsymbol{x}) + \nabla\cdot\big(A(t,\boldsymbol{x})\,\boldsymbol{v}_A(t,\boldsymbol{x})\big) = F_A(t,\boldsymbol{x}) \tag{19.1}$$

$$\frac{\partial}{\partial t}B(t,\boldsymbol{x}) + \nabla\cdot\big(B(t,\boldsymbol{x})\,\boldsymbol{u}_B(t,\boldsymbol{x})\big) = F_B(t,\boldsymbol{x}) \tag{19.2}$$

$$\frac{\partial}{\partial t}\boldsymbol{P}_A(t,\boldsymbol{x}) + \nabla\cdot\big(\boldsymbol{P}_A(t,\boldsymbol{x})\,\boldsymbol{v}_A(t,\boldsymbol{x})\big) = \boldsymbol{G}_A(t,\boldsymbol{x}) \tag{19.3}$$

$$\frac{\partial}{\partial t}\boldsymbol{P}_B(t,\boldsymbol{x}) + \nabla\cdot\big(\boldsymbol{P}_B(t,\boldsymbol{x})\,\boldsymbol{u}_B(t,\boldsymbol{x})\big) = \boldsymbol{G}_B(t,\boldsymbol{x}) \tag{19.4}$$

$$F_A(t,\boldsymbol{x}) = F_A(t,\boldsymbol{x}\,;B,\boldsymbol{P}_B) \;\;;\;\; F_B(t,\boldsymbol{x}) = F_B(t,\boldsymbol{x}\,;A,\boldsymbol{P}_A) \tag{19.5}$$

$$\boldsymbol{G}_A(t,\boldsymbol{x}) = \boldsymbol{G}_A(t,\boldsymbol{x}\,;B,\boldsymbol{P}_B) \;\;;\;\; \boldsymbol{G}_B(t,\boldsymbol{x}) = F_B(t,\boldsymbol{x}\,;A,\boldsymbol{P}_A) \tag{19.6}$$

Relations (19.5, 19.6) may describe dependence of $F_A(t,\boldsymbol{x})$ and $\boldsymbol{G}_A(t,\boldsymbol{x})$ on variable $B(t,\boldsymbol{x})$ and flow $\boldsymbol{P}_B(t,\boldsymbol{x})$ and $F_B(t,\boldsymbol{x})$ and $\boldsymbol{G}_B(t,\boldsymbol{x})$ on variable $A(t,\boldsymbol{x})$ and flow $\boldsymbol{P}_A(t,\boldsymbol{x})$. $F_A(t,\boldsymbol{x})$ and $\boldsymbol{G}_A(t,\boldsymbol{x})$ may depend on operators like divergence, gradient, rotor and etc. on functions $B(t,\boldsymbol{x})$ and $\boldsymbol{P}_B(t,\boldsymbol{x})$. It is obvious that in real economics macro variables depend on numerous economic and financial factors but (19.1-19.4) permit study simple approximations of mutual relations between two – three or four macroeconomic variables and their flows.

In Part II (Olkhov, 2019c) we describe economic transactions and expectations as density functions on economic space. We derive equations on transactions, expectations and their flows. We show how our approach helps describe asset pricing on economic space and derive equations on price evolution. In Part III (Olkhov, 2019d) of our paper we apply our model equations to description of particular economic problems.

5. Conclusions

The first part of our paper gives general economic treatment of economic model. We introduce notions of economic space, density functions of economic and financial variables and their flows. We derive economic equations on density functions and flows of economic variables and show that simple approximation permit study self-consistent relations between economic variables and their flows.

Our economic model uses no assumptions on market equilibrium, utility functions, rational expectations and etc., those ground general equilibrium (Arrow and Debreu, 1954; Tobin, 1969; Arrow, 1974; Smale, 1976; Kydland and Prescott, 1990; Starr, 2011). These assumptions are not necessary for economic modeling and economic theory can be based on economic statistics as source for agents risk assessments, alike to measuring the coordinates in physics. Hence excessive and unnecessary assumptions can be put aside of economic modeling or may be applied for description of some specific cases only.

Our approach uncovers a lot of economic problems that should be studied further to clarify elements of the economic model. Let's argue some those concern economic space. For example dimension of economic space is determined by choice of n risks those impact macroeconomic evolution. To develop reasonable economic model one should reduce number of risks and chose major two-three risks to define economic space of 2 or 3 dimensions. Hence one should develop methods to compare and forecast impact of risks on macroeconomic dynamics and procedure for selection most important risks. Choice of definite risks defines distribution of agents, form of density functions and economic dynamics on selected economic space. Different sets of risks cause different economic dynamics. Random nature of economic risks means that impact of some current risks may decline in time and influence of some new risk may unexpectedly grow up. Such collision underlines internal random properties of macroeconomic evolution and modeling. We state that economic development can occurs only under action of risks and different risks may set different directions for economic dynamics. Thus change of major risks results in change of dynamics determined by economic equations on density functions and flows of variables, transactions and expectations. In this paper we study economic evolution in the assumption that major risks and economic space don't change. The problems of random change of major risks should be studied further.

Risk assessments play central role for our model. It is clear that exact risks assessments of all agents in the entire economics are impossible. This is similar inability to measure coordinates of all physical particles of macro system. We propose the roughening procedure that transfers description of numerous separate particles to description of density functions on economic space. Such roughening procedure has parallels to transition from description of separate physical particles in kinetic approximation to description of continuous media or physics of fluids in hydrodynamic approximation. Such transition in physics significantly reduce amount of data required for model description. We seek the same effect in economic modeling. Roughening of risk ratings of separate agents and transition to description of density functions and flows of economic variables, transactions and expectations reduce amount of econometric statistics required for such approximation. Our approximation becomes intermediate between extra precise description based on modeling macroeconomics as system of numerous separate agents and description based on modeling macroeconomics as aggregated functions of time only. We propose that achievements of econometrics (Fox, et.al, 2014) and efforts in developing risks assessments methodologies will solve that complex problem for sure.

We assume that our approach to economic modeling may help improve description and forecasting of macroeconomic processes and impact development of economic policy making for sustainable economic growth.

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