General Trade Equilibrium of Integrated World Economy

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Abstract – The price-trade equilibrium as a central task of international economics is not fulfilled yet, even for the simplest 2x2x2 Heckscher-Ohlin model, due to its complexities. This paper explored the equilibrium by a simple geometrical derivation within the IWE. The study demonstrated that the endogenous factor price equalized is the function of world factor endowments and it has four important features: (i) it makes sure that countries participating in free trade gain from trade. (ii) it is the Dixit-Norman price that remains the same when the allocation of factor endowments changes within the IWE box. The FPE is unique for a giving IWE problem. (iii) the price-trade equilibrium displays the Heckscher-Ohlin theorem directly. (iv) the relative factor price (wage/rental ratio) is not related to technologies. The study also demonstrated the equilibrium relationship for the model with the context of two factors, two commodities, and multiple countries. The new economic logic from this the equilibrium is that world factor endowments determine world price (common commodity price and factor price).

Keywords:
Factor content of trade; factor price equalization; General equilibrium of trade; Integrated World Equilibrium; IWE

1. Introduction

Essentially the Heckscher-Ohlin theorem and the factor-price equalization (FPE) theorem paved the road toward general equilibrium. The general equilibrium of trade and the FPE are the same issues by different angles. McKenzie (1955)'s cone of diversification of factor endowments is an insight concept to understand FPE and trade from production supply constraints. He provided a mathematical demonstration of the existence of the FPE for many factors and many goods.

Vanek (1968)'s HOV model extends the usability of Heckscher-Ohlin theories on empirical trade analyses. The share of GNP in the HOV model engaged prices with trade and consumption. It also resulted in the application issue on how to convert the assumption of homothetic taste into consumption balance.

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The one focus of studies on the general equilibrium for constant returns and perfect competition is by the social utility function and direct and indirect trade utility function (offer curve). It is not easy neither for this approach to get complete price-trade equilibrium.

The Integrated World Equilibrium (Dixit and Norman, 1980) is remarkable to illustrate the FPE by trade. It displayed a mobile property of the FPE. Helpman and Krugman (1985) normalize the assumption of integrated equilibrium, which presented equilibrium analyses in a simple way. Deardorff (1994) derived the conditions of the FPE for many goods, many factors, and many countries by using the IWE approach. He discussed the FPE for all possible allocations of factor endowments within lenses identified.

The factor price equalization theorem is with a fundamental influence, with long and involved discussion in the literature. The FPE could imply the trade equilibrium and the Heckscher-Ohlin theorem. The FPE in the IWE does provide some hints on the price-trade equilibrium. This study found that behind the mobile factor price equalization (PFE) by the IWE, there is a clear relationship of the general equilibrium of trade that embedded just in the IWE. This is why the IWE is so correct and accepted widespread.

Woodland (2013, pp39) described the importance of the general equilibrium, “General equilibrium has not only been important for a whole range of economics analyses, but especially so for the study of international trade” Deardorff (1984, pp685) said, “A trade equilibrium is somewhat more complicated”. The Heckscher-Ohlin theories still do not achieve this important goal, even for the simplest 2x2x2 model. This study derived a price-trade equilibrium hiding in the IWE and demonstrated that the equalized factor price and common commodity price at the equilibrium is the function of the world factor endowments. The result is consistent with the insight inference that Dixit and Norman made four decades ago that prices remain same when the allocation of factor endowments changes.

This paper is divided into five sections. Section 2 introduces the solution of price–trade equilibrium by a geometric method within the IWE diagram. Section 3 provides a way to estimate the autarky price. The logic is that the autarky factor endowment determines the autarky price. It demonstrates that the FPE at its equilibrium ensures gains from trade for countries participating in the trade. Section 4 presents the equilibrium for cases of two factors, two commodities, and multiple countries. Section 5 is the discussions of equilibria and autarky price.

2. The Price-Trade Equilibrium by Geometric Analyses within The IWE

We take the following normal assumptions of the Heckscher-Ohlin model in this study: (1) identical technology across countries, (2) identical homothetic taste, (3) perfect competition in the commodities and factors markets, (4) no cost for international exchanges of commodities, (5) factors are completely immobile across countries but that can move costlessly between sectors within a country, (6) constant return of scale and no factor intensity reversals (7) full employment of factor resources.

We denote the Heckscher-Ohlin model as follows. The production constraint of full employment of factor resources is

$$AX^h = V^h \qquad (h = H, F) \qquad (2\text{-}1)$$

where A is the 2×2 technology matrix (the matrix of direct factor inputs), X^h is the 2×1 vector of commodities of country h, V^h is the 2×1 vector of factor endowments of country h. The elements of matrix A is $a_{ki}(w/r), k = K, L, i = 1,2$. We assume that A is not singular.

The zero-profit unit cost condition is

$$A'W^h = P^h \qquad (h = H, F) \qquad (2\text{-}2)$$

where $W^h$is the 2×1 vector of factor prices, its elements are r rental for capital and w wage for labor , P^h is the 2×1 vector of commodity prices.

Figure 1 is a regular IWE diagram. The dimensions of the box represent world factor endowments. The origin for the home country is the lower left corner, for the foreign country is the right upper corner. ON and OM are the rays of the cone of diversifications. Any point within the parallelogram formed by ONO^*M is an available allocation of factor endowments of two countries. Suppose that an allocation of the factor endowments is at point *E*, where the home country is capital abundant. Point *C* represents the trade equilibrium point. It indicates the sizes of the consumptions of the two countries.

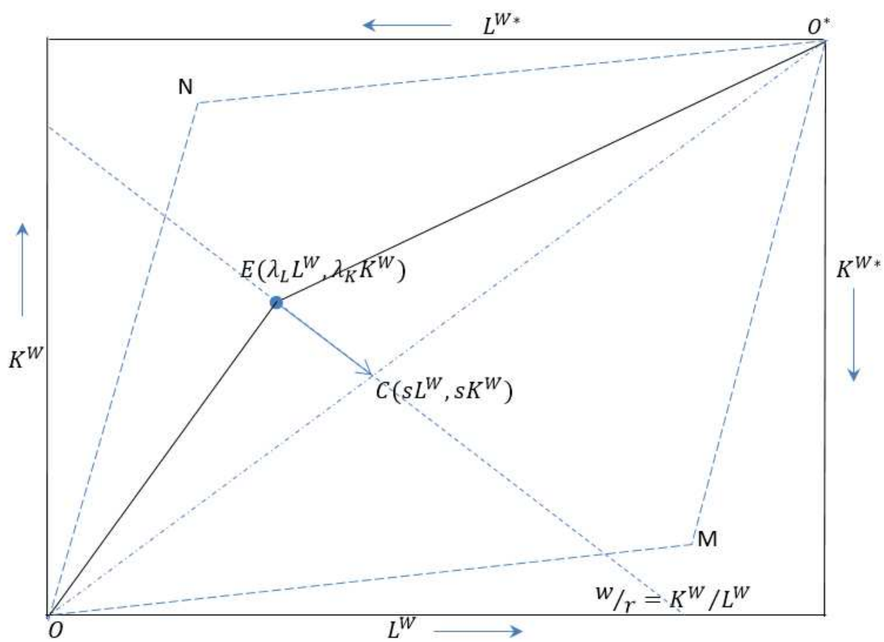

Figure 1 IWE Diagram

Dixit and Norman (1980) have shown that the same equalized factor price (FPE) occurs when the allocation of factor endowments of two countries changes within the parallelogram formed by ONO^*M . It implied price-trade equilibrium implicitly. The mobile property of the FPE provided a hint for what trade equilibrium is. We explore the price-trade equilibrium behind the FPE in this section.

We introduce two parameters, which are the shares of the home country's factor endowment to their world factor endowments respectively,

$$0 \le \lambda_L \le 1 \tag{2-3}$$

$$0 \le \lambda_K \le 1 \tag{2-4}$$

We denote the factor endowments of the home country as

$$L^H = \lambda_L L^W \tag{2-5}$$

$$K^H = \lambda_K K^W \tag{2-6}$$

where K^W is the world capital endowment, and L^W is the world labor endowment. When λ_L and λ_K changes, they can present any allocation of factor endowments in the IWE diagram. The allocation of point E in Figure 1 is $E(\lambda_L L^W, \lambda_K K^W)$.

The factor contents of trade are

$$F_K^H = K^H - sK^W = (\lambda_K - s)K^W \tag{2-7}$$

$$F_L^H = L^H - sL^W = (\lambda_L - s)L^W \tag{2-8}$$

Using trade balance of factor contents yields

$$\frac{r^*}{w^*} = \frac{(s-\lambda_L)L^W}{(\lambda_K - s)K^W} \tag{2-9}$$

where r^* is the equalized rental, w^* is the equalized wage.

Introduce a constant q as

$$q = \frac{(s-\lambda_L)}{(\lambda_K - s)} \tag{2-10}$$

Substituting it into (2-9) yields

$$\frac{r^*}{w^*} = q\frac{L^W}{K^W} \tag{2-11}$$

The factor price ratio (r^*/w^*) and factor price are unchanged within the parallelogram by ONO^*M on the IWE diagram. That was proofed by Dixit and Norman (1980) and other studies. Therefore, q should be a constant. Equation (2-11) illustrates that the rental/wage ratio is the function of the world factor endowments. This is why the FPE holds within the parallelogram formed by ONO^*M in the IWE diagram.

We have interesting to know what value q takes. At point $C(sL^W, sK^W)$, We see that $\lambda_L = s$ and $\lambda_K = s$, where s is the home country's share of GNP. There is no trade at this point.

We now suppose that allocation E is nearby to C or imagine point E moves to close to its equilibrium point C. If the allocation E is above the diagonal line OO', there are always $s-\lambda_L>0$ and $\lambda_K-s>0$.

Taking $\lambda_L \to s$ and $\lambda_K \to s$ yields

$$\lim_{\substack{\lambda_L\to s\\ \lambda_k\to s}} \frac{(s-\lambda_L)}{(\lambda_K-s)} = 1 = q \tag{2-12}$$

We see that constant q equals to 1. Subsituting q=1 into equation (2-10), we have the share of GNP at equilibrium as

$$s = \frac{1}{2}(\lambda_L+\lambda_K) = \frac{1}{2}\left(\frac{K^H}{K^W}+\frac{L^H}{L^W}\right) \tag{2-13}$$

In addition, equation (2-11) is reduced as

$$\frac{r^*}{w^*} = \frac{L^W}{K^W} \tag{2-14}$$

This is true for any allocation of factor endowments within parallelogram ONO^*M.

Is it properly to use point $C(sL^W, sK^W)$ to illustrate equals to 1? Helpmand and Krugman (1985, pp16) thought that the point, like C, was a right point for the FPE, they write, "the FPE is not empty because it always contains the diagonal OO'." The FPE implies trade balance. At point C, there is no trade but price.

Dixit and Norman (1980, p112) used a numerical example as $\lambda_L=1/3$, and $\lambda_K=1/2$ in their original study to illustrate how the IWE works. The share of GNP for their example is 5/12 by equation (2-13). Let convince that this result is true. The rest of factor endowments should generate the rest of the share of GNP. The rest of factor endowments are $\lambda_L=2/3$ and $\lambda_K=1/2$. The rest share of GNP is 7/12 by equation (2-13). The sum of 5/12 and 7/12 is 1. This just demonstrates that the derivation for (2-13) is right.

With the equilibrium share of GNP (2-13) and the rental/wage ratio (2-14), we now obtain the whole equilibrium solution of the Heckscher-Ohlin model as

$$r^* = \frac{L^W}{K^W} \tag{2-15}$$

$$w^* = 1 \tag{2-16}$$

$$p_1^* = a_{k1}\frac{L^W}{K^W} + a_{L1} \tag{2-17}$$

$$p_2^* = a_{k2}\frac{L^W}{K^W} + a_{L2} \tag{2-18}$$

$$F_K^h = \frac{1}{2}\frac{K^hL^W-K^WL^h}{L^W}, \qquad F_L^h = -\frac{1}{2}\frac{K^hL^W-K^WL^h}{K^W}, \quad (h=H,F) \tag{2-19}$$

$$T_1^h = x_1^h - \frac{1}{2}\frac{K^hL^W+K^WL^h}{K^WL^W}x_1^W, \qquad T_2^h = x_2^h - \frac{1}{2}\frac{K^hL^W+K^WL^h}{K^WL^W}x_2^W, \quad (h=H,F) \tag{2-20}$$

where p_i^* is world price for commodity i; T_i^h is the trade volume of commodity i in country h. We assumed $w^* = 1$ by using Walras' equilibrium condition to drop one market clear condition.

We now view the equilibrium above from the angle of trade competition by a trade box in the IWE diagram.

We suppose here that the home country is capital-abundant as

$$\frac{K^H}{L^H} > \frac{K^F}{L^F} \tag{2-21}$$

Trades redistribute national welfares, which are measured by GNP. This is a major trade consequence.

The commodity price is under the following constraint,

$$\frac{a_{K1}}{a_{K2}} > \frac{p_1^*}{p_2^*} > \frac{a_{L1}}{a_{L2}} \tag{2-22}$$

This condition will make sure that the factor rewards from unit cost equation (2-2) are positive. Fisher (2011) proposed this insight concept and called it "goods price diversification cone".

Figure 2 is an IWE diagram added with a trade box. The dimensions of the box represent world factor endowments.

The boundaries of the share of GNP corresponding the goods price diversification cone (2-22) can be calculated as

$$s_b^H(p) = s\left(p\left(\frac{a_{K1}}{a_{K2}}, 1\right)\right) = \frac{a_{K1}x_1 + a_{K2}x_2}{a_{K1}x_1^W + a_{K2}x_2^W} = \frac{K^H}{K^F + K^H} = \lambda_K \tag{2-23}$$

$$s_a^H(p) = s\left(p\left(\frac{a_{L1}}{a_{L2}}, 1\right)\right) = \frac{a_{L1}x_1 + a_{L2}x_2}{a_{L1}x_1^W + a_{L2}^H x_2^W} = \frac{L^H}{L^F + L^H} = \lambda_L \tag{2-24}$$

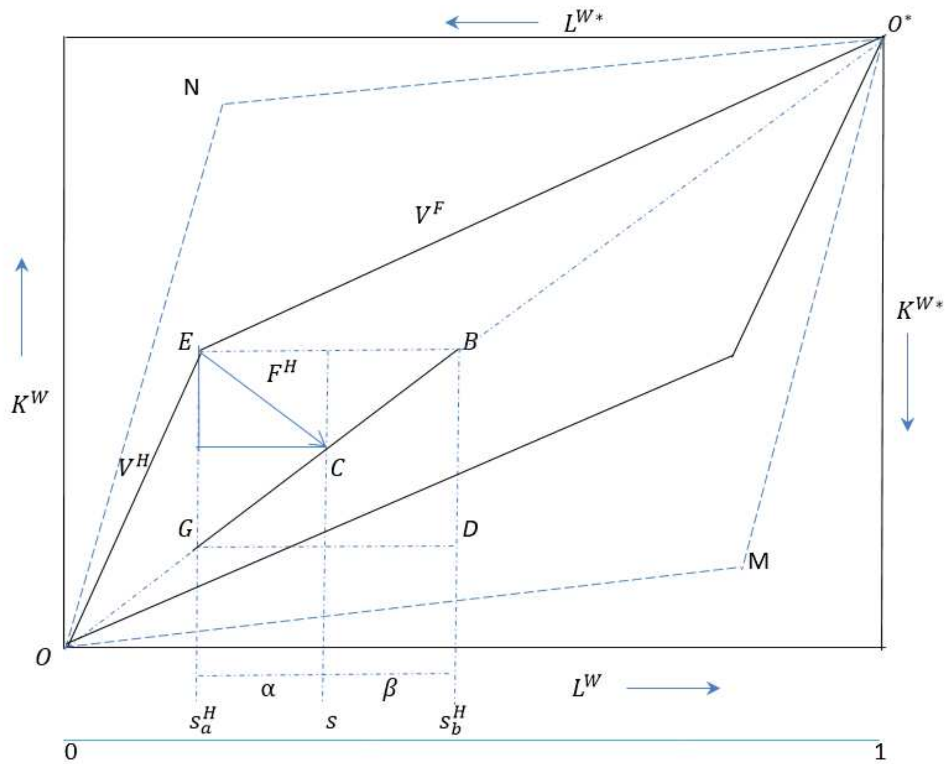

Figure 2 IWE with Trade Box

They identify the trade box $EBDG$ in Figure 2. If a commodity price lies in the commodity price cone, the share of GNP will lie in the trade box.

Trade Competition

The the home country's share of GNP s divides the trade box into two parts in Figure 2. Their lengths are α and β respectively as

$$\alpha = (\lambda_K - s), \qquad \beta = (s - \lambda_l) \tag{2-25}$$

When α increases, the the home country's share of GNP increases and the foriegn country's share of GNP decreases, and vice versa. In trade competitions, the both countries want to reach their maximum GNP share in free trade.

We notice that the trade box not only is the trade area but also is the redistributable area of the share of GNP for the two countries. Outside the box, they are not redistributable by trade (the trade outside of the trade box will course a negative factor reward). Therefore, α is redistributable part of the home country's share of GNP; β is redistributable part of the foreign country's share of GNP.

We see that the equilibrium occurs at

$$s = \frac{1}{2}(\lambda_k + \lambda_L) = \frac{1}{2}\left(\frac{K^H}{K^W} + \frac{L^H}{L^W}\right) \tag{2-26}$$

It implies that the two countries equally share the redistributable shares of GNP identified by trade box.

The volume of Factor Content of Trade

Helpman and Krugman (1985, pp23) introduced the term "volume of trade" as

$$VT = 2p_1(x_1^h - sx_1^W) = 2p_2(x_2^h - sx_2^W) \qquad (h = H, F) \tag{2-27}$$

Based on their concept, we introduce the volume of factor content of trade as[2]

$$VF = 2r^H(K^H - sK^W) = 2w^H(sL^W - L^H) \tag{2-28}$$

We now demonstrate that α is the size of VF.

The home country exports EG as capital service and imports GC as labor service. The GC indicates the share of GNP of capital service EG plus labor service GC. GC is the share of GNP measured at the diagonal OO' direction. Its size equals to α numerically.

We see $\alpha = \beta$ when trade reaches its equilibrium. They both are the share of GNP of F^H as

$$\alpha = \beta = \frac{VF}{World\ GNP} = \frac{2w^{H*}(sL^W - L^H)}{w^{H*}L^W + r^*K^W} \tag{2-29}$$

The share of world trade volume of factor content is

$$WVF = 2 \times VF = \alpha + \beta = (\lambda_k - \lambda_K) = \left(\frac{K^H}{K^{EW}} - \frac{L^H}{L^W}\right) \tag{2-30}$$

It implies that the size of world factor content of trade equals to the size of the trade box identified by the cone of commodity price.

From the consumption view, α is the size of the consumption built by trade for the home country. β is the size of the consumption built by trade for the foreign country. They should be the same in size.

The price solution above illustrates that Dixit-Norman price more stable. The technology matrix A keeps unchanging no matter $A = A(w/r)$ or $A = A(w/p)$ in the IWE diagram.

From the factor content of trade (2-19), we see that when $\frac{K^H}{L^H} > \frac{K^W}{L^W}$, then $F_K^H > 0$. This just states the Heckscher-Ohlin theorem.

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2 We assume that $\frac{K^H}{L^H} > \frac{K^W}{L^W}$.

The relative factor price, rental/wage ratio, is proportional in reversely to their world factor endowments. It does not relate to technologies. Moreover, it does not relate to commodity prices.

Dixit and Norman (1980) illustrated that when the allocation of the factor endowments changes, the factor price and the commodity price will remain the same. Their major argument is that the new allocation of factor endowments of the two countries leaves the same world supply of goods and, hence incomes unchanged and so supplies will still match the unchanged world demand. The price solution (2-15) through (2-18) proved it analytically in a more strict condition. It explained why the same FPE holds within the parallelogram by $ONO'M$ on the IWE diagram

The changes of allocations of factor endowments within parallelogram $ONO'M$ in the IWE box do cause the changes of shares of GNP of two countries and the changes of trade volumes of two countries. This still does not affect world commodity price and equalized factor price.

3. Autarky Price and Comparative Advantage

It is difficult to know autarky prices before free trade for countries. Therefore, it is not easy to show comparative advantages and gains from trade for the Heckscher-Ohlin model. We now propose an approach to estimate autarky prices.

By the logic that world factor resource determines world price in the last section, we imagine a country with an isolated market. Its "autarky" price can be determined by its "autarky" factor endowments.

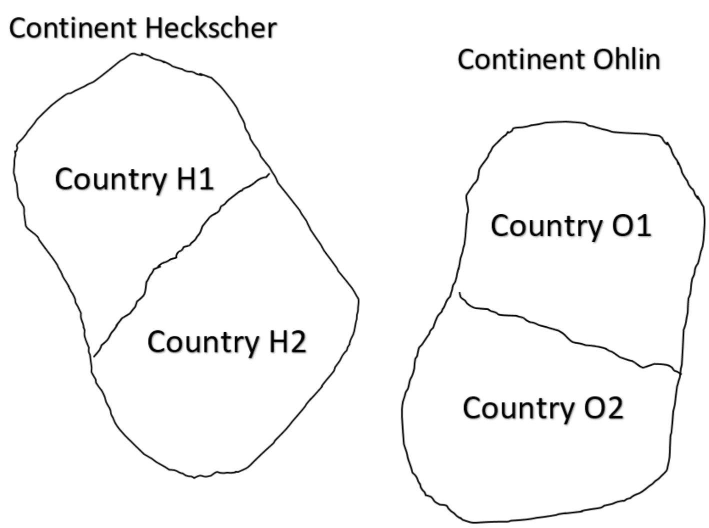

Figure 3 Autarky Price Formation

A good case to explain the estimation of autarky price is by Figure 3. There are two geographic continents, Heckscher and Ohlin, separated by an ocean. Continent Heckscher is with two free trade countries, H1 and H2. In addition, Continent Ohlin is with two free trade countries, O1 and O2. Two continents start to free trade by no-cost shipping. Knowing the total factor endowments of each continent, we can estimate the prices of each continent by the expression of world price (2-16) through (2-19). The prices of each continent can serve as autarky price for the trade of two-continent economy.

The IWE diagram itself supports the logic that autarky factor resources determine autarky price analytically. Assuming that one country shrinks to very small, another country's autarky price is then the world price of the current trade. Mathematically, when $V^H \to 0$, inside the IWE box, then $V^F \to V^W$ and the relative factor price r^* after trade will close to the relative autarky factor price of the foreign country,

$$r^* = \frac{L^W}{K^W} = \frac{L^H + L^F}{K^H + K^F} \to r^{Fa} = \frac{L^F}{K^F} \quad (3\text{-}1)$$

Moreover, the common commodity price will close to the foreign country's autarky commodity price. Therefore, we proved the autarky price formation mathematically.

Based on the above discussion, we present the autarky prices of countries that participate in free trade as

$$r^{ha} = \frac{L^h}{K^h} \qquad (h = H, F) \quad (3\text{-}2)$$

$$w^{ha} = 1 \qquad (h = H, F) \quad (3\text{-}3)$$

$$p_1^{ha} = a_{k1}\frac{L^h}{K^h} + a_{L1} \qquad (h = H, F) \quad (3\text{-}4)$$

$$p_2^{ha} = a_{k2}\frac{L^h}{K^h} + a_{L2} \qquad (h = H, F) \quad (3\text{-}5)$$

where superscript ha indicates the autarky price of country h.

When the home country is capital abundance, the condition for the comparative advantage is

$$\frac{a_{K1}}{a_{K2}} > \frac{p_1^{Fa}}{p_2^{Fa}} > \frac{p_1^*}{p_2^*} > \frac{p_1^{Ha}}{p_2^{Ha}} > \frac{a_{L1}}{a_{L2}} \quad (3\text{-}6)$$

The gains from trade are measured by

$$-W^{ha\prime}F^h > 0 \qquad (h = H, F) \quad (3\text{-}7)$$

$$-P^{ha\prime}T^h > 0 \qquad (h = H, F) \quad (3\text{-}8)$$

We add a negative sign in inequalities above since we expressed trade by net export, T^h . In most other literatures, they express trade by net import. Appendix B is the proof of the gain from trade by inequality (3-7). It implies that the world prices at the equilibrium will ensure the gains from trade for both countries, by the autarky prices inference.

The result of gains from trade is another good side effect of the trade equilibrium. It is an important property of the equilibrium and the FPE.

The equilibrium price should have some optimal properties. Guo (2019) demonstrates that the relative commodity price p_1^*/p_2^* reached its maximum value respective to world capital endowment (or to either country's capital endowments) if we assume that $K^H/K^F > L^H/L^F$ and $a_{K1}/a_{K1} > a_{L1}/a_{L2}$. In addition, it reached its minimum value respective to world labor endowment (it implies that p_2^*/p_1^* reached its maximum respective to labor factor endowments). This result means that both countries export their products with comparative advantage at the maximum price simultaneously. It implies that both countries get their maximum benefits through trade.

Theorem – The comparative advantage theorem

At the equilibrium, each country exports the good that has a comparative advantage. The ratio of world commodity prices at the equilibrium is between the ratios of autarky prices of the countries. The world factor endowments, fully employed, determine world prices, which assure the gains from trade for countries participating in trade. The equilibrium displays the Heckscher-Ohlin theorem. The factor price equalized when equilibrium reached.

Proof

The solution (2-15) through (2-18) shows how the world prices are determined and why it remains the same with mobile factor endowments in the IWE box. The relative factor price w/r is an angle in Figure 1. The angle is unique for a giving IWE. Therefore, the solution is unique. The FPE is true for an IWE. If the solution is unique and it satisfies the Dixit-Norman price inference, it is true.

Appendix B proved the gains from trade as inequality (3-7).

End Proof

The equilibrium shows the unification of the Heckscher-Ohlin theorem, The FPE theorem, gains from trade, and Dixit-Norman price. Each of them means each other of them.

4. General equilibrium of trade for the case of two factors, two commodities, and multiple countries

We generalize the equilibrium result in the last section to the model of two factors, two commodities, and multiple countries in this section.

In a two-country system, home and foreign, they are trade partners with each other. In a 3-country system, who is the trade partner for whom? We specify that trades are one that a country trade with the rest of the world. The trade relations are very simple now. It just likes the scenario of the two-country system from the analyses view.

Figure 4 draws an IWE diagram for two factors, two commodities, and three countries. The dimensions of the box represent world factor endowments. The vector $V^h(L^h, K^h)$ represents the vector of factor endowments of country h, h=1, 2, and 3. The factor endowment vector V^1 of country 1 is arranged to start at origin point O. The rest of the world factor endowment is $V^2 + V^3$. It starts at the origin point O'.

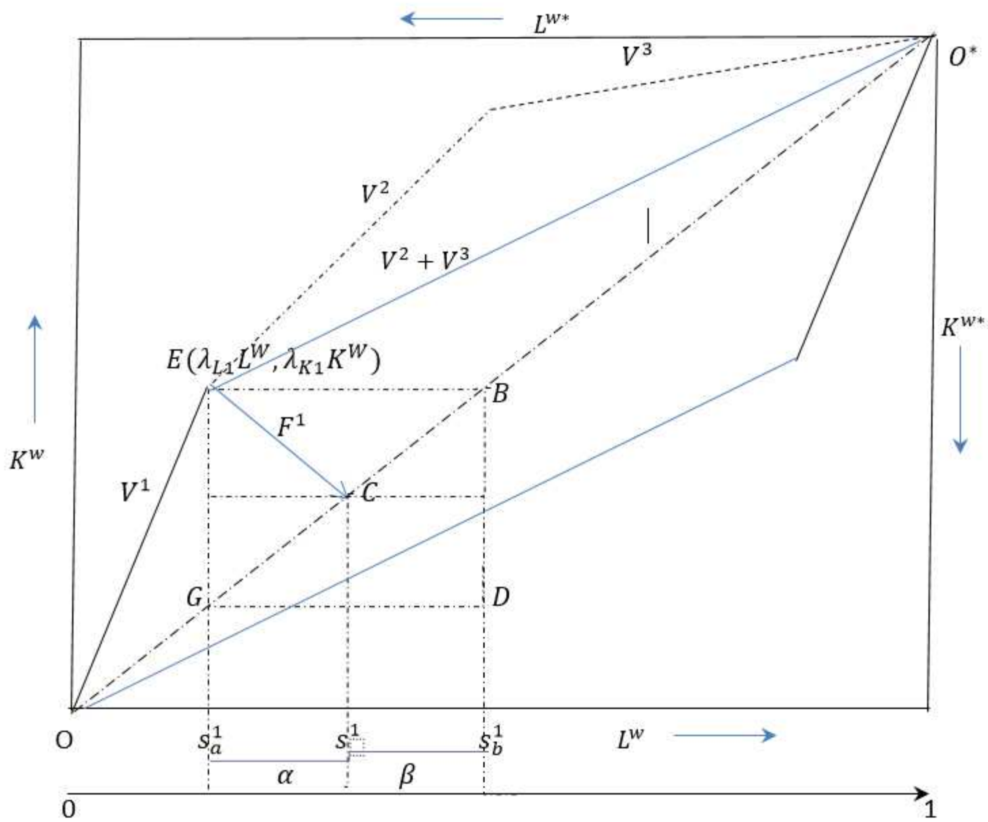

Figure 4 IWE diagram with GNP Share Box for 2 × 2 × 3 model

The algebra expression for the 2 x 2 x M model is as same as equation (2-1) and (2-2); the only difference is the country number. The country number now goes from 1 to M (In Figure 4, we only present 3 countries for illustration).

We now introduce two sets of parameters, which are the shares of factor endowments of country h to their world factor endowments respectively as

$$0 \le \lambda_{Lh} \le 1 \;, \quad 0 \le \lambda_{Kh} \le 1 \qquad (h = 1,2,\dots,M) \tag{4-1}$$

$$\sum_{h=1}^{M} \lambda_{Lh} = 1 \quad , \qquad \sum_{h=1}^{M} \lambda_{Kh} = 1 \tag{4-2}$$

We denote the factor endowments of country h as

$$L^h = \lambda_{Lh} L^W \qquad (h = 1,2,\dots,M) \tag{4-3}$$

$$K^h = \lambda_{Kh} K^W \qquad (h = 1,2,\dots,M) \tag{4-4}$$

The allocation of factor endowments of country 1 in Figure 3 is $E(\lambda_{L1}L^W, \lambda_{K1}K^W)$. It shows how a country trades with the rest of the world.

The factor contents of trade of country h are

$$F_K^h = K^h - s^h K^W = (\lambda_{Kh} - s^h)K^W \qquad (h = 1,2,\dots,M) \tag{4-5}$$

$$F_L^h = L^h - s^h L^W = (\lambda_{Lh} - s^h)L^W \qquad (h = 1,2,\dots,M) \tag{4-6}$$

Using trade balance of factor contents yields

$$\frac{r^{*h}}{w^{*h}} = \frac{(s^h - \lambda_{Lh})L^W}{(\lambda_{Kh} - s^h)K^W} \qquad (h = 1,2,\dots,M) \tag{4-7}$$

where r^{*h} is the equalized rental in country h, w^{*h} is the equalized wage in country h. It displays the trade balance between country h and the rest world. Extending the result (2-12) in the last section to the equation above, we have

$$\frac{(s^h - \lambda_{Lh})}{(\lambda_{Kh} - s^h)} = 1 \qquad (h = 1,2,\dots,M) \tag{4-8}$$

$$\frac{r^{*h}}{w^{*h}} = \frac{L^W}{K^W} \qquad (h = 1,2,\dots,M) \tag{4-9}$$

This means that the relative factor price is the same for all countries.

$$\frac{r^{*h}}{w^{*h}} = \frac{L^W}{K^W} = \frac{r^*}{w^*} \tag{4-10}$$

By assuming $w^* = 1$ to drop one market-clearing condition by Walras's equilibrium, we obtain

$$s^h = \frac{1}{2}\frac{K^h L^W + K^W L^h}{K^W L^W} \qquad (h = 1,2,\dots,M) \tag{4-11}$$

$$\frac{r^*}{w^*} = \frac{L^W}{K^W} \tag{4-12}$$

$$w^* = 1 \tag{4-13}$$

$$p_1^* = a_{k1}\frac{L^W}{K^W} + a_{L1} \tag{4-14}$$

$$p_2^* = a_{k2}\frac{L^W}{K^W} + a_{L2} \tag{4-15}$$

$$F_K^h = \frac{1}{2}\frac{K^h L^W - K^W L^h}{L^W} \qquad (h = 1,2,\dots,M) \tag{4-16}$$

$$F_L^h = -\frac{1}{2}\frac{K^h L^W - K^W L^h}{K^W} \qquad (h = 1,2,\dots,M) \tag{4-17}$$

$$x_1^h = x_1^h - \frac{1}{2}\frac{K^h L^W + K^W L^h}{K^W L^W} x_1^W \qquad (h = 1,2,\dots,M) \tag{4-18}$$

$$x_2^h = x_1^h - \frac{1}{2}\frac{K^h L^W + K^W L^h}{K^W L^W} x_2^W \qquad (h = 1,2,\dots,M) \tag{4-19}$$

We see that

$$\sum_{h=1}^{H} s^h = \sum_{h=1}^{H} \frac{1}{2} \frac{K^h L^W + K^W L^h}{K^W L^W} = 1 \tag{4-20}$$

Those are the equilibrium solution for the $2 \times 2 \times M$ model. We can demonstrate that all countries participating in trade gain from trade. It confirmed that world factor endowments determine world price in the multi-country economy.

5. Discussions of the equilibrium

The price-trade equilibrium displayed the root of the FPE in the IWE. It used the assumption of FPE in the IWE and demonstrated it in return. The trade box illustrates how the redistributable shares of GNP are distributed to each country. It is a Pareto Optimal since the trade box shows how social trade-off played. It is a balanced trade that the share of a country in world spending equals to its share in world income.

Dixit (2010) mentioned, "The Stolper-Samuelson and factor price equalization papers did not actually produce the Heckscher-Ohlin theorem, namely the prediction that the pattern of trade will correspond to relative factor abundance, although the idea was implicit there. As Jones (1983, 89) says, 'it was left to the next generation to explore this 2×2 model in more detail for the effect of differences in factor endowments and growth in endowments on trade and production patterns.' That, plus the Rybczynski theorem which arose independently, completed the famous four theorems." The equalized factor price at the equilibrium of this study presented the Heckscher-Ohlin theorem. Guo (2019) provide a trade effect analyses based on the equilibrium of this paper, it displayed that the trade effect of changes of factor endowments is a chain effect of the Rybczynski' trade effect triggering the Stolper-Samuelson' trade effect. The equilibrium solution put all of the four-core theorems together.

The multiple-country equilibrium is more intricate in economic logic. The equation (4-21) shows that the sum of the shares of GNP of all countries equals to 1. It indicates that both the solution and the approach are right. This is the first analytical result for multiple countries price-trade equilibrium.

At the equilibrium, the ratio of factor content of trade equals to consumption ratio. It reflects Leamer theorem (Leamer, 1980). We provide a chain of inequalities that includes the Heckscher-Ohlin theorem, the Leamer theorem, the Factor Price Equalization theorem, and the Dixit and Norman IWE price, as the follows,

$$\frac{a_{K1}}{a_{L1}} > \frac{K^H}{L^H} > \frac{K^H - F_K^H}{L^H - F_L^H} = \frac{K^W}{L^W} = \frac{w^*}{r^*} = \left| \frac{F_K^H}{F_L^H} \right| = \frac{K^F - F_K^F}{L^F - F_L^F} > \frac{K^F}{L^F} > \frac{a_{K2}}{a_{L2}} \tag{5-1}$$

It is a mathematical brief statement for the Heckscher-Ohlin theorem, the Leamer theorem, the Factor Price Equalization theorem, and the Dixit and Norman IWE price principle, which united at equilibrium.

Conclusion

The paper attained the general equilibrium of trade in the 2 x 2x M Heckscher-Ohlin model. The equilibrium addresses the Heckscher-Ohlin theorem with trade volume, the factor-price equalization theorem with price structure, and comparative advantage with gains from trade.

The study illustrates the economic logic that world factor resources determine world prices. Its first application is to identify autarky prices.

The price-trade equilibrium result matched and presented the Heckscher-Ohlin core theories. It is ascertained by Dixit and Norman price inference that the price remains the same when the allocation of factor endowments changes.

Appendix A

We express the gains from trade for the home country as

$$-W^{Ha\prime}F^{H} > 0 \qquad \text{(A-1)}$$

Adding trade balance condition $W^{*\prime}F^{H} = 0$ on (A-1) yields

$$-(W^{Ha\prime} - W^{*\prime})F^{H} > 0 \qquad \text{(A-2)}$$

We see

$$W^{Ha} = \begin{bmatrix} \frac{L^H}{K^H} \\ 1 \end{bmatrix} \quad , \qquad W^{*} = \begin{bmatrix} \frac{L^W}{K^W} \\ 1 \end{bmatrix} \qquad \text{(A-3)}$$

Substituting them into (A-2) yields,

$$-\begin{bmatrix} \frac{L^H}{K^H} - \frac{L^W}{K^W} & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2}\frac{K^H L^W - K^W L^H}{L^W} \\ -\frac{1}{2}\frac{K^H L^W - K^W L^H}{L^W} \end{bmatrix} > 0 \qquad \text{(A-4)}$$

It can be reduced to

$$-(\frac{L^H}{K^H} - \frac{L^W}{K^W}) \times \frac{1}{2}\frac{K^H L^W - K^W L^H}{L^W} > 0 \qquad \text{(A-5)}$$

It means

$$-\left(\frac{L^H}{K^H} - \frac{L^W}{K^W}\right) \times \frac{1}{2}\frac{\frac{L^W}{K^W} - \frac{L^H}{K^H}}{L^W} K^W K^H = \left(\frac{L^H}{K^H} - \frac{L^W}{K^W}\right)^2 \times \frac{1}{2L^W} K^W K^H > 0 \qquad \text{(A-6)}$$

It is true. So that (A-1) holds.

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