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Bara, Aman Appolinus and Chakraborty, Bidisha

Department of Economics, Jadavpur University, Department of Economics, Jadavpur University

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Bara Aman Appolinus*
Jadavpur University
Bidisha Chakraborty**
Jadavpur University

Abstract: This paper develops an endogenous growth model with the non-rival but excludable public good. We seek to answer the question that whether this kind of infrastructure should be provided by pure private firm or by state or by Public-Private Partnership (PPP). And, if the government invests in this type of infrastructure, how should it finance the manufacturing cost-through accumulating debt or imposing a tax or by charging user-fees? In this paper, PPP in infrastructure is defined as a profit-making private firm-producing infrastructure with the partial cost borne by the government. The authors make a comparison of the macro-economic performances under the purely private provision, purely public provision and PPP provision of infrastructure in an economy. In the purely public provision of infrastructure, if the government runs a balanced budget or has constant debt, our model suggests that government should finance the infrastructure solely by charging user fees instead of imposing the tax. The model finds the condition under which the PPP provision of infrastructure is justified. The present paper finds the user fees and growth rate under the private provision and also user fees and growth-maximizing tax rate in 3 budgetary regimes: (a) when the government has constant debt, (b) when public debt is zero and (c) when there is accumulating debt, under the pure public provision of infrastructure and PPP provision of infrastructure. We find that there exists a unique, equilibrium steady state balanced growth rate in all the regimes. We compare the user fees and growth rates across different regimes.

Keywords: *Infrastructure, Public-Private Partnership, Endogenous growth, Public debt*

***JEL Classification*: E62, H44, O40**

Corresponding authors:

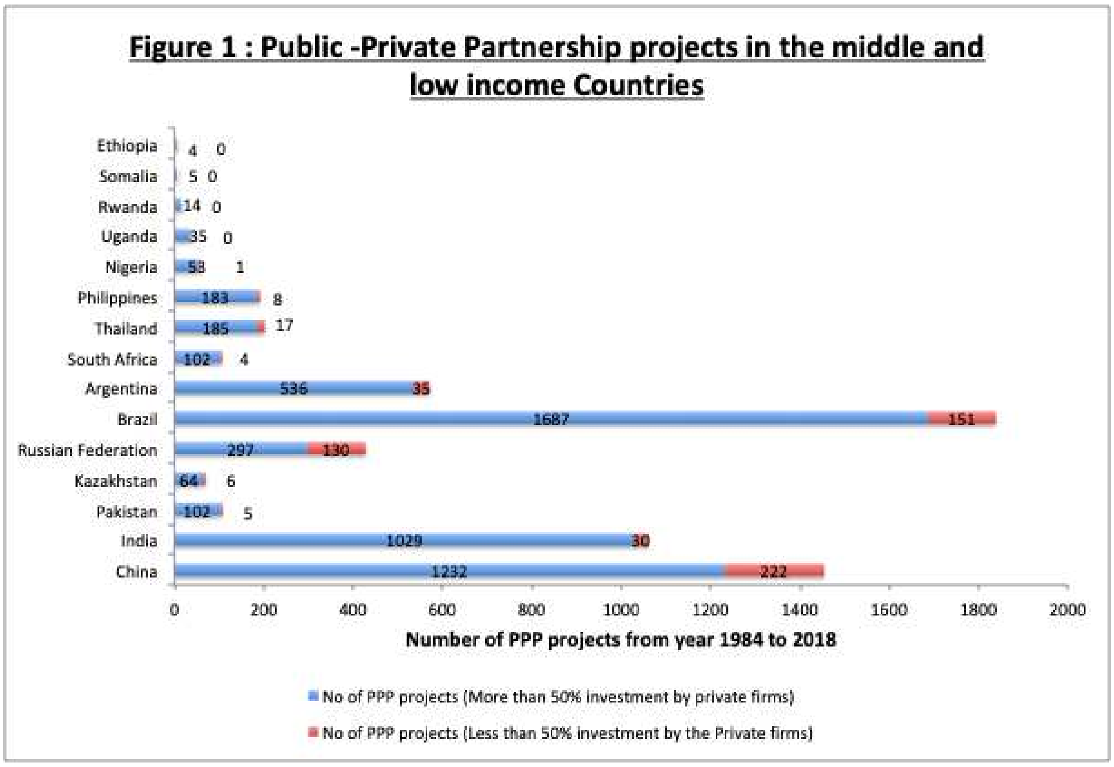
*,** Address:188 Raja S.C. MullickRoad,Department of Economics, Jadavpur University, Kolkata-700032, India. * Email:amanbara@gmail.com, ** Email: bidisha.chakraborty@gmail.com .

Why should the government provide the infrastructure through the Public-Private Partnership mode?

1. Introduction:

The government has been the unique provider of public goods and services traditionally in most of the developing nations. Much of the literature on infrastructure and endogenous growth theory concentrates on the non-rival and non-excludable pure public good. To name a few, Barro (1990), Futagami et al. (1993), Dasgupta (1999), Turnovsky and Pintea (2006) and Bhattacharya (2014) study the non-rival and non-excludable publicly provided infrastructure and find the optimal fiscal policy in a balanced budget framework. Infrastructure service is an important development tool that catalyzes growth in the long run due to its effect on the reduction of production cost, thereby increasing the productivity of private capital and rate of return on capital. But, it is difficult for the governments of the low-income developing countries to bear the enormous fund required for the cost of construction of infrastructure. Therefore, governments are looking at the Public-Private Partnership (PPP) in infrastructure provision as a solution to their problem of infrastructure crunch. The government wants to reduce the fiscal deficit and therefore the governments in the financing of infrastructure provision seek private sector participation. In PPP, the government makes payment for a part of the total up-front cost and the rest of the construction phase's cost is taken care by the private firm and therefore government pays little or nothing throughout the infrastructure project. The government gets the political credit for delivering the project in the current period and has the advantage of improving the current budgetary position and minimizing the government deficit. But the question draws our attention is that, "Is PPP mode of infrastructure provision a better option compared to the private and public mode of provision?" The complete privatization is different from PPP, the former has no direct government role in the ongoing operations of the projects and the private firm has the monopoly status with little or no regulation, whereas in case of PPP the government retains the share of responsibility for investment as well as for the operational function when it is handed over the ownership by the manufacturing firm after it has made its profit over the years. Therefore, PPP provision reduces the need for high current taxation, reduces the financial cost on part of the government and therefore unbinds the public spending to other sectors. In recent years many developed and developing nations have adopted the Public-Private Partnership (PPP) in the provisioning of infrastructure services. We investigate the possibility that why the government cannot produce the impure public goods, which are non-rival but excludable and charge the user fees itself. Why does it need the help of the private firm for the manufacturing of infrastructure? Kateja (2012), suggests that private partnership in infrastructure along with public investment offers significant advantages in terms of enhancing efficiency through competition in the provision of services to users. In real life, there are number of instances where PPP is being successfully implemented, for example metro rail system of New Delhi, India; roads in Chile, Argentina, United States of America, Hong Kong, Hungary and Italy; water system of Singapore, Airports of New Delhi and Mumbai of India; rural electrification of Guatemala; port expansion in Colombo, Sri Lanka, etc; are some examples of successful PPP projects among many PPP projects investment taking place around the world. Figure 1 depicts the PPP projects undertaken in different middle and low-income countries from year 1984-2018.

Most of the middle-income countries show the up-rise in the investment of PPP projects after 1991. However, after 1998 the lower-income groups of African countries have only few PPP projects.



Source: Author's own compilation from Private participation in Infrastructure Database, World Bank.

In Figure 1 we observe that number of PPP projects undertaken by Brazil, China, India, Russia are quite high. There is a contract between private firm and the government stating that in the contract period the revenue from the project will be earned by the private firm and the management responsibility will also lie with them. Usually, the contract period is in the range of 10-50 or more number of years after which the ownership is transferred from the private firm to the government. In the figure above, the number of PPP projects under the category of more than 50% investment by private firms is quite high and numbers of PPP projects under the category of less than 50% investment by the private firms are few. The World Bank in its PPI (Private Participation in Infrastructure) Annual Report 2017 reported China (73 projects worth US dollar 17.5 billion), Brazil (24 projects worth US dollar 7.3

billion) and Pakistan (4 projects worth US dollar 5.9 billion) among the top 5 PPI investment countries. The World Bank's PPI Annual Report 2018 again reported China to be the leader in PPP projects with 37 projects worth US dollar 11.6 billion. Also, India (24 projects worth US dollar 3.8 billion) and Brazil (11 projects worth US dollar 3 billion) were featured among the top 5 PPI investment countries in 2018. The World Bank PPI Report noted that SAR (India, Pakistan) and ECA (Kazakhstan, Russian Federation, Turkey) countries have a sizable portion of public debt financing with 27% and 26% respectively.

There exists small literature dealing with a comparative evaluation of different mode of provisioning infrastructure. Chatterjee and Morshed (2011) compare the impact of private and government provision of infrastructure on an economy's aggregate performance. Barro and Sala-i-Martin (1992), Futagami et. al. (1993), Fisher and Turnovsky (1998), Devarajan et. al. (1998) study the interaction between public and private capital in an endogenous growth context, where the public good is non-excludable. However, excludability feature of the public good is ignored in the above studies. A study by Ott and Tunovsky (2006) make a comparative study of the non-excludable and an excludable public good. Government is the unique supplier of public input and sets monopoly pricing of the user fees. According to them, tax plus user fees alone could be sufficient to finance for the provision of the entire infrastructure. However, this kind of financing is possible only in the case of developed nations like the United Kingdom, the United States and EU nations but for developing nations like India, only user fee cannot alone suffice the financing of the excludable public good. Privately provided roads, power, water, transportation, communication and irrigation etc; are quite common in the developed nations. However, in the developing nations, there is a dearth of private investors and the government fails to attract private investments. So, the governments of these countries must use the policy instruments such as subsidy, tax holiday and sharing of partial cost of manufacturing public good which is called Viability Gap Funding (VGF); for example in India, the government under the VGF bears 20% - 40% of the manufacturing cost of the infrastructure investment. Therefore, VGF could be an important policy tool for infrastructure provision in developing nations.

In this paper, we attempt to address primarily two questions: why should the government go for PPP for infrastructure provision? And how should the government finance the cost of infrastructure production - through imposition of tax, through bond financing or though charging only user-fees? We build a closed economy model of infrastructure provision to answer these questions. In this model, infrastructure may be provided by the pure private firm, pure public entity or through the partnership of private firm and public entity (PPP). We are considering different possibilities: the government may run balanced budget or may have budget deficit. In case of budget deficit there are two possibilities: government may have constant debtor or may accumulate debt over time. In the real world, the government borrows from the capital market, banks and issues public bonds to finance the subsidy and public investments. Following, Greiner (2008, 2012) and Kamaiguchi and Tamai (2012), we assume that the government may run a deficit but it must set the primary surplus, such that it is a positive linear function of public debt, which guarantees that the public debt is sustainable. Greiner (2008), studies the public investment in three different situations: the first being, the balanced budget situation, second being the situation when public debt grows at the lesser rate than the public capital and consumption and at last the situation when public debt grows at the same rate as capital and consumption. However, the investment through PPP mode and its comparative study with other mode of provision like the public and private provision has not been studied. Bara and Chakraborty (2019), study the optimality of PPP in an endogenous growth model, where public capital and private capital are treated as substitute and complementary goods. However, in their paper only balanced budget case is considered and debt-financing situation is not studied. Present paper attempts to find whether the PPP mode under the varying budgetary regime for infrastructure provision is growth maximizing or not, as compared to the public provision and private provision. We find the impact of the fiscal policy in different budgetary regime: (a) when public debt is constant, (b) When public debt is zero, (c) When public debt is accumulating. In present paper, we find that there exists a unique steady state balanced growth rate under all the different kinds of provision and regimes. We also find that the user fees and growth rates are the same for the constant public debt and zero public debt under both pure public provision and PPP provision. However, the growth rate is found to be different for the permanent deficit under both the provision. Also, the user fee is less under the PPP provision as compared to private provision and public provision under certain conditions.

The structure of the paper is organized in the following manner. Section 2 describes the competitive economy model of pure private infrastructure provision where the behavior of firms and the representative household are studied. Section 3 describes the model of pure public provision. Followed by section 4, which describes the public-private partnership provision of infrastructure where both the private firm and the government play the role in the provision of infrastructure. Section 3 and 4 also study the steady state balanced growth rates for the constant debt case and the permanent deficit case. Section 5 deals with the explanation of valid reasons supporting the PPP investment by the government. Section 6 explains the justification of debt financing or accumulating debt regime for the provision of infrastructure. Lastly, the concluding remarks are made in section 7.

2. Pure private provision of infrastructure:

In a competitive economy closed model, we have three agents namely, the representative household, two aggregative firms. In the pure private provision of infrastructure the private firm sponsors the manufacturing of infrastructure for the commercial use. The government does not play any role in the provision of infrastructure; therefore the cost of production to the infrastructure-manufacturing firm is unity.

2.1 Firms:

There are 2 profit-making firms in the economy. Firm 1 produces final goods for consumption and capital accumulation. Firm 2 produces the infrastructure services. Following Barro (1990), we assume that infrastructure services are flow in nature. Both the firms are run by profit-maximizing private entities. Infrastructure is used for final good production.

The household supplies physical capital (K) required for the production of both final goods (Y) and infrastructure (G). The production function of final goods is given as,

$$Y = A(uK)^{1-\alpha} G^{\alpha} \quad , \qquad 0 < \alpha < 1 \qquad (1)$$

Y is used for consumption as well as capital accumulation. uK denotes part of private capital used to produce Y. Flow of infrastructure good at time t is denoted by G. A is the technology parameter for production of Y. $1-\alpha$ and α are output elasticities with respect to uK and G respectively. Final good is assumed to be a numeraire commodity. Hence, the price of Y is considered to be unity.

The production function of infrastructure service is given as,

$$G = \delta(1-u)K \tag{2}$$

In equation (2), $(1-u)K$ denotes the remaining part of private physical capital used to produce infrastructure. δ is the technology parameter of infrastructure services production, which is a constant. The infrastructure services are productive investment and are flow in nature. We obtain the ratio of infrastructure to private physical capital from equation (2),

$$\frac{G}{K} = \delta(1-u) \tag{3}$$

The profit function of firm 1 is given by,

$$\Pi^1 = A(uK)^{1-\alpha}G^{\alpha} - ruK - \mu G \tag{4}$$

μ is the user fees or price paid by firm 1 for using infrastructure services.

The profit function of firm 2 producing flow of infrastructure service is given by,

$$\Pi^2 = \mu G - r(1-u)K \tag{5}$$

In equation (5), μ is the total price charged by firm 2 for providing infrastructure services (e.g. revenue earned by selling tickets of metro rail, user fees for the usage of roads, etc).

Substituting the value of G in equation (5), we rewrite the equation as,

$$\Pi^2 = \delta\,(1-u)K - r(1-u)K \tag{6}$$

Both firms take input prices as given and choose input quantities so as to maximize their profit. Differentiating Π^1 with respect to uK and G, the first order condition gives,

$$r = \frac{A(1-\alpha)}{u^{\alpha}}\left(\frac{G}{K}\right)^{\alpha} \tag{7}$$

$$\mu = Au^{1-\alpha}\alpha\left(\frac{G}{K}\right)^{\alpha-1} \tag{8}$$

Substituting the value of $\left(\frac{G}{K}\right)$ in the above equation,

$$r = A(1-\alpha)\delta^{\alpha}\left(\frac{1-u}{u}\right)^{\alpha} \tag{9}$$

$$\mu = A\alpha\delta^{\alpha-1}\left(\frac{1-u}{u}\right)^{\alpha-1} \tag{10}$$

Differentiating Π^2 with respect to $(1-u)K$, the first order condition gives,

$$r = \mu\delta \tag{11}$$

Equating the rate of interest of firm 1 and firm 2, we find the value of u,

$$u = 1-\alpha \tag{12}$$

Now again substituting the value of u in equation (10), the user fees under the private provision is given as,

$$\mu = A\delta^{\alpha-1}\alpha^{\alpha}(1-\alpha)^{1-\alpha} \tag{13}$$

2.2 The Households:

For the manufacturing of infrastructure the households provides the private physical capital. The rate of accumulation of private physical capital is given by,

$$\dot{K} = rK - C \tag{14}$$

The rate of growth of private physical capital is,

$$\frac{\dot{K}}{K} = r - \frac{C}{K} \tag{15}$$

It is assumed that the household derives utility from direct consumption of final good. The utility function of the representative household is given by,

$$U = \int_0^{\infty}(\gamma \ln C)\, e^{-\rho t} dt \tag{16}$$

ρ is constant and denotes positive discount rate at which future utility is discounted. It is assumed that the disposable income over expenditure is accumulated as wealth. The total wealth/ asset (W) of the household is equal to the total private physical capital (K) in the economy. Therefore, $W = K$.

In competitive economy, the representative household maximizes the current-value Hamiltonian, subject to equation (14),

$$H_c = \gamma \ln C + \eta[rK - C] \quad (17)$$

The control variable is C. The first – order maximization conditions are given as follows:

$$\frac{\gamma}{C} = \eta \quad (18)$$

Time derivative of the co-state variable is given by the following equation,

$$\frac{\dot{\eta}}{\eta} = \rho - r \quad (19)$$

Taking the log and derivative of equation (18), we get,

$$-\frac{\dot{C}}{C} = \frac{\dot{\eta}}{\eta} = \rho - r \quad (20)$$

The growth rate equation is,

$$\frac{\dot{C}}{C} = r - \rho = g \quad (21)$$

2.3 Steady State Balanced growth:

At steady state balanced growth rate under the private provision, $\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = g$. If $\frac{\dot{K}}{K}$ is constant, $\frac{C}{K}$ is also constant. The steady state growth rate, g is constant and positive. Now setting $\frac{\dot{C}}{C} = \frac{\dot{K}}{K}$, we get,

$$\frac{C}{K} = \rho \quad (22)$$

Therefore, the growth rate equation after substituting the value of r from equation (11) is,

$$g = \mu\delta - \rho \quad (23)$$

Now we substitute the value of user fees from equation (13), the growth rate under the private provision of infrastructure is,

$$g = A\alpha^{\alpha}(1-\alpha)^{1-\alpha}\delta^{\alpha} - \rho \quad (24)$$

Proposition 1: *There exists an unique steady state balanced growth rate and an user fee to be charged for using infrastructure under the private provision.*

3. Pure public provision of infrastructure:

In the pure public provision of infrastructure the government provides the infrastructure services without the help of private firm. Therefore in a closed economy model, we have three agents namely; the representative household, firm producing finished goods and the government. Since, the infrastructure is fully sponsored by the government, therefore the share of the manufacturing cost of production of infrastructure to the government is unity. We also assume that government charges user fees for the usage of infrastructure to the firms. Also, the government charges capital income tax for the financing of infrastructure.

3.1 Firm:

Firm 1 is a profit-making firm, which produces the final goods for consumption and capital accumulation. The benevolent government provides the infrastructure services. Following Barro (1990), we assume that infrastructure services are flow in nature. Infrastructure is used for final good production.

The household supplies physical capital (K) required for the production of both final goods (Y) and infrastructure (G). The production function of final goods is given as,

$$Y = A(uK)^{1-\alpha}G^{\alpha} \quad , \qquad 0 < \alpha < 1 \tag{25}$$

$Y$is used for consumption as well as capital accumulation. uK denotes part of private capital used to produce Y. Flow of infrastructure good at time t is denoted by G. A is the technology parameter for the production of Y. $1-\alpha$ and α are output elasticities with respect to uK and Grespectively. Final good is assumed to be a numeraire commodity. Hence, the price of Y is considered to be unity.

The production function of the infrastructure provided by the benevolent government is given by,

$$G = \delta(1-u)K \tag{26}$$

The ratio of infrastructure to private physical capital is given by,

$$\frac{G}{K} = \delta(1-u) \tag{27}$$

The profit function of firm 1 is given by,

$$\Pi^1 = A(uK)^{1-\alpha}G^{\alpha} - ruK - \mu G \tag{28}$$

μ is the user fees or price paid by firm 1 for using infrastructure services.

Differentiating the profit function Π^1 with respect to uK and G, the first-order condition gives,

$$r = \frac{A(1-\alpha)}{u^{\alpha}}\left(\frac{G}{K}\right)^{\alpha} \tag{29}$$

$$\mu = Au^{1-\alpha}\alpha\left(\frac{G}{K}\right)^{\alpha-1} \tag{30}$$

Substituting the value of $\left(\frac{G}{K}\right)$ in the above equations,

$$r = A(1-\alpha)\delta^{\alpha}\left(\frac{1-u}{u}\right)^{\alpha} \tag{31}$$

$$\mu = A\alpha\delta^{\alpha-1}\left(\frac{1-u}{u}\right)^{\alpha-1} \tag{32}$$

Infrastructure demand of firm 1 is obtained from equation (30),

$$G = \left[\frac{\mu}{Au^{1-\alpha}\alpha}\right]^{\frac{1}{\alpha-1}}K \tag{33}$$

Now using equations (26) and (33), the supply of infrastructure is equated with the demand for infrastructure for finding the equilibrium quantity of private physical capital.

$$u = \frac{(A\alpha)^{\frac{1}{\alpha-1}}\delta}{\mu^{\frac{1}{\alpha-1}}+(A\alpha)^{\frac{1}{\alpha-1}}\delta} \tag{34}$$

Substituting the value of u in the equation (31), the rate of interest is given as,

$$r = A^{\frac{1}{1-\alpha}}(1-\alpha)\left(\frac{\mu}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} \tag{35}$$

3.2 The Government:

In a closed economy under the pure public provision of infrastructure the government is mainly engaged in 3 activities: (1) It provides the infrastructure and charges user fees for the usage of infrastructure. (2) It imposes capital income tax in order to finance its cost. (3) It also issues government bonds. Therefore, interest on the bond adds to the debt burden of the government while tax revenue reduces the government debt.

The bond accumulation function is given by,

$$\dot{B} \ = \ (1-\tau)rB - (T-E) \tag{36}$$

T is the tax revenue at time t and E is the public expenditure at time t. $T - E$ shows the public investment to tax revenue. It indicates how much of it is used for the debt service.

$$T = \ \tau \, r \, uK + \mu G \tag{37}$$

$$E = r \, (1-u)K \tag{38}$$

Substituting the value of T and E in equation (36), the bond accumulation function is given as,

$$\dot{B} = (1-\tau)rB - [\tau ruK + \mu G - r(1-u)K] \tag{39}$$

Substituting the value of G from equation (26) in the above equation,

$$\dot{B} = (1-\tau)rB - [\tau ruK + \mu\delta(1-u) - r(1-u)K] \tag{40}$$

The rate of growth of bond is given as,

$$\frac{\dot{B}}{B} = (1-\tau)r - \frac{K}{B}[\tau ru + \mu\delta(1-u) - r(1-u)] \tag{41}$$

3.3 The Households:

It is assumed that the household derives utility from direct consumption of final good. The utility function of the representative household is given by,

$$U = \int_0^\infty \gamma \ \ln C \, e^{-\rho t} \, dt \tag{42}$$

ρ is constant and denotes positive discount rate at which future utility is discounted. It is assumed that the disposable income over expenditure is accumulated as wealth. The wealth/asset of the household denoted by W is defined as the sum of bond holding (B) and capital holding (K). Therefore,

$$W = B + K \tag{43}$$

Total disposable wealth of a household over consumption expenditure and payment for using infrastructure services is accumulated as wealth. The rate of accumulation of wealth is given by,

$$\dot{W} = (1-\tau)rW - C \tag{44}$$

τ is the tax on capital income , r is the interest rate, C is consumption at time t.

In the economy, the representative household maximizes the current-value Hamiltonian, subject to equation (44),

$$H_c = \gamma \ln C + \eta[(1-\tau)rW - C] \tag{45}$$

The control variable is C. The first – order maximization conditions are given as follows:

$$\eta = \frac{\gamma}{C} \tag{46}$$

Time derivative of the co-state variable is given by the following,

$$\frac{\dot{\eta}}{\eta} = \rho - (1-\tau)r \tag{47}$$

Taking the log and derivative of equation (46), we get,

$$-\frac{\dot{C}}{C} = \frac{\dot{\eta}}{\eta} = \rho - (1-\tau)r \tag{48}$$

Also using equation (43), we have,

$$\frac{\dot{W}}{W} = \frac{\dot{B}/B + \dot{K}/K \cdot K/B}{1 + K/B} \tag{49}$$

Since $\frac{\dot{B}}{B} = \frac{\dot{K}}{K} = g$, now substituting these values in the above equation, we get,

$$\frac{\dot{W}}{W} = g \tag{50}$$

Therefore, combining equations (48) and (50), the growth rate equation is obtained as,

$$\frac{\dot{C}}{C} = \frac{\dot{G}}{G} = \frac{\dot{\eta}}{\eta} = \frac{\dot{W}}{W} = (1-\tau)r - \rho = g \tag{51}$$

3.4 Steady State Balanced growth:

The steady state balanced growth equilibrium is defined as a situation when consumption, private physical capital and infrastructure capital grow at the same strictly positive constant growth rate, i.e;$\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{G}}{G} = \frac{\dot{B}}{B} = \frac{\dot{Y}}{Y} = g$. Where g is positive and constant. If $\frac{\dot{B}}{B}$ is constant, then $\frac{B}{K}$ is also constant. Setting $\frac{\dot{C}}{C} = \frac{\dot{B}}{B}$, we get the value of $\frac{B}{K}$ as,

$$\frac{B}{K} = \frac{\tau r u + \mu\delta(1-u) - r(1-u)}{\rho} \tag{52}$$

3.5 Steady State Balanced growth rate under the pure public provision at Zero Debt regime:

At steady state balanced growth rate under the pure public provision, the government doesn't have any debt such that the government's tax revenue is equal to the total expenditure of the government. In other words, the government observes the balanced budget. Setting $T = E$, we have,

$$\tau r u K + \mu G = r(1-u)K \tag{53}$$

Substituting the value of G in the above equation we have,

$$r[1 - u(1+\tau)] = \mu\delta(1-u) \tag{54}$$

Now again substituting the value of r and u in the above equation, we get the value of μ under the pure public provision zero-debt regime,

$$\mu = \left[\frac{\delta}{A^{\frac{1}{1-\alpha}}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}} + (A\alpha)^{\frac{1}{\alpha-1}}\delta\tau\right]^{\alpha-1} \tag{55}$$

3.5.1Growth rate under the pure public provision at zero debt regime:

The growth rate is given as,

$$g = (1 - \tau)r - \rho \tag{56}$$

Substituting the value of r in the above equation, we have,

$$g = (1 - \tau)A^{\frac{1}{1-\alpha}}(1 - \alpha)\left(\frac{\mu}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \rho \tag{57}$$

Now substituting the value of user fees under the pure public provision zero-debt case,

$$g = (1 - \tau)A^{\frac{1}{1-\alpha}}(1 - \alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left[\frac{\delta}{A^{\frac{1}{1-\alpha}}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}} + (A\alpha)^{\frac{1}{\alpha-1}}\delta\tau\right]^{\alpha} - \rho \tag{58}$$

3.5.2 Growth maximizing tax rate under the pure public provision at zero debt or balanced budget regime:

Differentiating the growth rate equation (58) with respect to τ, the first-order condition is,

$$\frac{\delta g}{\delta\tau} = -A^{\frac{1}{\alpha-1}}\ \alpha^{\frac{2\alpha-1}{\alpha-1}}\frac{\delta}{(1-\alpha)} - A^{\frac{1}{\alpha-1}}\alpha^{\frac{\alpha}{\alpha-1}}\ \delta\,\tau\ \frac{(\alpha+1)}{\alpha} < 0 \tag{59}$$

Therefore, equation (59) implies the growth maximizing tax rate is zero.

3.5.3 Maximum growth rate:

Substituting the growth maximizing tax rate in the growth rate equation (58), we get the maximum growth rate under the balanced budget for the pure public provision,

$$g_{\hat{\tau}} = A\alpha^{\alpha}(1 - \alpha)^{1-\alpha}\delta^{\alpha} - \rho \tag{60}$$

Proposition 2: *The growth maximizing tax rate under pure public provision of infrastructure is zero. It is optimal for the government to charge user fees instead of imposing tax for financing infrastructure. And, the maximum growth rate under the balanced budget for the pure public provision of infrastructure is equal to the maximum growth rate under the pure private provision of infrastructure.*

3.6 Steady State Balanced growth rate under the pure public provision at constant Debt regime:

At steady state balanced growth rate, the government experiences the constant debt, such that $\dot{B} = 0$ and also $\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{G}}{G} = g$ is constant and positive. However, when $\dot{B} = 0$, it does not necessarily imply that public debt equals zero. If the level of initial debt is positive, the debt to capital ratio and also the debt to GDP ratio are positive but decline over time and converge to zero in the long run.

Therefore, setting $\dot{B} = 0$, we find that the user fees under the pure public provision at constant debt regime is same as under the pure public provision at zero debt regime. Therefore the growth rate and growth maximizing tax rate are also same. Equation (61)-(64) of Appendix A1 show that the user fees, growth rate, growth maximizing tax rate, and the maximum growth rate respectively, under the balanced budget for the pure public provision at constant debt regime, which are exactly same as the balanced budget regime.

Proposition 3: *For the public provision of infrastructure, the user fee charged for infrastructure services and growth rates under the constant debt regime and the balanced budget (zero debt) regime are the same.*

3.7 Steady state balanced growth rate under the pure public provision at permanent deficit or accumulating debt regime:

At steady state balanced growth rate, the government experiences the case when debt is accumulating; this is the case of a permanent deficit. Steady state balanced growth rate g is positive and constant and therefore, all the variables grow at the same strictly positive constant rate, such that $\frac{\dot{C}}{C} = \frac{\dot{G}}{G} = \frac{\dot{K}}{K} = \frac{\dot{W}}{W} = \frac{\dot{B}}{B} = g$.The permanent deficit case is characterized by the public deficits, where the government debt grows at the same rate as all other endogenous variables in the long run.

For the sustainability of public debt in our model, we apply the primary surplus rule, which states that the primary surplus relative to GDP is a function that positively depends on the debt to GDP ratio. According to Greiner (2013), 'the economic rationale behind the rule is to make the debt ratio a mean-reverting process when the reaction of the primary surplus is sufficiently large, preventing the debt to GDP ratio from exploding'. There are also empirical evidences revealing that the governments follow such a rule of primary surplus. For example, Bohn (1998) and Greiner et al. (2007) have shown that this rule holds for the USA and for selected European countries, respectively, using OLS estimations. Also Fincke and Greiner (2012) find that the reaction coefficient determining the response of the primary surplus to public debt is not a constant but time varying with the average of that coefficient being strictly positive for some euro area countries. The model by Barro (1990), has been extended by Kamaiguchi and Tamai (2012) by integrating public deficit and public debt into the model to analyze the conditions for simultaneous growth and sustainability of public debt.

Therefore, following Greiner (2008) and Kamaiguchi and Tamai (2012), we assume that the ratio of the primary surplus to gross domestic income ratio is a positive linear function of the debt to gross domestic income ratio with an intercept. Hence, the primary surplus ratio can be written as,

$$\frac{T-E}{Y} = \xi + \beta \frac{B}{Y} \tag{65}$$

'Where ξ, β are the real numbers and are constant. β determines how strongly the primary surplus reacts to changes in public debt. ξ determines whether the level of the primary surplus rises or falls with an increase in gross domestic income. If $\xi <$

0 implies that primary surplus declines as GDP rises and the government increases it's spending with higher GDP. In this case of negative ξ, β must be sufficiently large. If β is sufficiently low, then the government must be a creditor for the economy to achieve sustained growth. If $\xi > 0$ implies that primary surplus rises as GDP increases. In this case, β must not be too large. A high β implies that government does not invest sufficiently and he must be a creditor in order to finance its investment, in order to achieve sustained growth.' (Greiner, 2008)

From equation (65),

$$T - E = \xi\, Y + \beta\, B \tag{66}$$

Substituting the values of T, E and Y in the above equation, we get another value of $\frac{B}{K}$ under the permanent deficit case, which is different from the balanced budget $\frac{B}{K}$, therefore,

$$\frac{B}{K} = \frac{\tau\, r\, u - \xi\, A\, u^{1-\alpha} \delta^{\alpha} (1-u)^{\alpha} - (1-u)(r-\mu\delta)}{\beta} \tag{67}$$

The bond accumulation function for the sustainability of public debt is,

$$\dot{B} = (1-\tau) r B - (\xi\, Y + \beta\, B) \tag{68}$$

After substituting the value of Y in the above equation, the rate of growth of bond is,

$$\frac{\dot{B}}{B} = (1-\tau) r - \xi\, A\, u^{1-\alpha} \frac{K}{B} \left(\frac{G}{K}\right)^{\alpha} - \beta \tag{69}$$

Now substituting the value of $\frac{K}{B}$ and $\frac{G}{K}$ in the above equation, the rate of growth of bond under the pure public provision at accumulating debt regime is given as,

$$\frac{\dot{B}}{B} = (1-\tau) r - \frac{\xi\, A u^{1-\alpha} [\delta(1-u)]^{\alpha} \beta}{\tau\, r\, u - \xi\, A\, u^{1-\alpha} \delta^{\alpha} (1-u)^{\alpha} - (1-u)(r-\mu\delta)} - \beta \tag{70}$$

Resorting to equation (51) and (70), we set $\frac{\dot{C}}{C} = \frac{\dot{B}}{B}$, we obtain the value of user fees under the pure public provision at accumulating debt regime.

$$\mu = \frac{1}{A\alpha} \left[\tau\, \delta - \frac{\delta\, (\rho\, \xi + (\rho - \beta) \alpha^{\alpha})}{(\rho - \beta)(1 - \alpha)}\right]^{\alpha - 1} \tag{71}$$

3.7.1 Growth rate under the pure public provision at accumulating debt or permanent deficit regime:

Substituting the value of user fees under the pure public provision at accumulating debt regime, we obtain the growth rate equation as,

$$g = \frac{(1-\tau)}{A}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}\left[\tau\,\delta - \frac{\delta\,(\rho\,\xi+(\rho-\beta)\alpha^{\alpha})}{(\rho-\beta)(1-\alpha)}\right]^{\alpha} - \rho \tag{72}$$

3.7.2 Growth maximizing tax rate under the pure public provision at accumulating debt regime:

We differentiate the growth rate equation (72) with respect to τ. Therefore, the first order condition gives the value of optimal tax rate under the pure public provision at accumulating debt regime,

$$\tau = \frac{1}{(1-\alpha)}\left[\frac{\rho\,\xi+(\rho-\beta)\alpha^{\alpha}}{(\rho-\beta)(1-\alpha)} + \alpha\right] \tag{73}$$

From the second order condition, we have,

$$\frac{(1-\alpha)}{A}\alpha^{\frac{2\alpha}{1-\alpha}}\left[\tau\,\delta - \frac{\delta(\rho\xi+(\rho-\beta)\alpha^{\alpha})}{(\rho-\beta)(1-\alpha)}\right]^{\alpha-1}\delta\left[-2 + \frac{(1-\tau)(\alpha-1)\,\delta}{\left[\tau\,\delta-\frac{\delta(\rho\,\xi+(\rho-\beta)\alpha^{\alpha})}{(\rho-\beta)(1-\alpha)}\right]}\right] < 0 \tag{74}$$

Therefore, $\frac{(1-\tau)(\alpha-1)\,\delta}{\left[\tau\,\delta-\frac{\delta(\rho\,\xi+(\rho-\beta)\alpha^{\alpha})}{(\rho-\beta)(1-\alpha)}\right]} > 2$ is a sufficient condition for $\frac{d^2 g}{d\,\tau^2} < 0$.

3.7.3 Maximum growth rate under the pure public provision at accumulating debt regime:

Substituting the value of growth maximizing tax rate under the pure public provision at accumulating debt regime in the growth rate equation (72), we obtain the maximum growth rate as,

$$g_{\hat{\tau}} = \left[1-\left(\frac{\rho\,\xi+(\rho-\beta)\alpha^{\alpha}\delta+\alpha(\rho-\beta)(1-\alpha)}{(\rho-\beta)(1-\alpha)}\right)\right]\frac{\alpha^{\frac{2\alpha}{1-\alpha}}}{A}\left[\frac{\delta}{(1-\alpha)}\left(\frac{\rho\,\xi+(\rho-\beta)\alpha^{\alpha}\delta+\alpha(\rho-\beta)(1-\alpha)}{(\rho-\beta)(1-\alpha)}\right) -\right.$$

ξ $\delta\rho+\rho-\beta\alpha\alpha\delta\rho-\beta1-\alpha\alpha-\rho$

(75)

Proposition 4: *The user fees and maximum growth rates under the pure public provision at accumulating debt regime are different from the user fees and maximum growth rates under the public provision at constant debt and balanced budget regime.*

4. Public-Private Partnership provision of infrastructure:

In the Public-Private Partnership provision of infrastructure the government provides the infrastructure services with the help of private firm. However, in the partnership venture the ownership lies with the private firm and the government makes a small partial investment for manufacturing. In real life there are number of PPP contracts such as Build-Operate-Transfer (BOT), Build-Own-Operate-Transfer (BOOT), Build-Own-Operate-Transfer (BOOT), Design-Build-Finance-Operate (DBFO), Design-Construct-maintain-Finance (DCMF) etc. The private firm transfers the ownership to the government in 10-50 years or more time period after making its profit revenues from the user fees. In countries, where PPP projects are implemented, there is VGF (Viability Gap Funding) through which government makes a direct investment up to a certain percentage (say, 20% in case of India) of the total cost to the infrastructure-producing firm. The government, if it so decides may provide additional grants out of its budget up to further 20% of the total project cost. VGF is in the form of a capital grant to the infrastructure-manufacturing firm for the construction of the infrastructure project.

In our model, we construct a model where the government makes the partial investment in the private firm. Therefore, following the real life examples of viability gap funding, it is assumed that the government bears $(1-\phi)$ fraction of total cost of manufacturing infrastructure$(1-u)K$. The model is a competitive economy closed model, we have four agents namely; the representative household, two aggregative firms and the government. Firm 1 produces the finished goods and firm 2 manufactures the infrastructure. The government invests in firm 2 not only because of the positive externality derived from the infrastructure but also because infrastructure is essential for final good production and welfare enhancement. First we look at the manufacturing sector, followed by the government sector and then the household sector.

4.1 Firms:

Firm 1 produces final goods for consumption and capital accumulation. Firm 2 provides the infrastructure services and charges user fees for it. Following Barro (1990), we assume that infrastructure services are flow in nature. Both the firms are run by profit maximizing private entities. Infrastructure is used for final good production.

The household supplies physical capital (K) required for the production of both final goods (Y) and infrastructure (G). The production function of final goods is given as,

$$Y = A(uK)^{1-\alpha}G^{\alpha} \quad , \qquad 0 < \alpha < 1 \qquad (76)$$

Y is used for consumption as well as capital accumulation. uK denotes part of private capital used to produce Y. Flow of infrastructure good at time t is denoted by G.A is the technology parameter for production of Y. $1-\alpha$ and α are output elasticities with respect to uK and Grespectively. Final good is assumed to be a numeraire commodity. Hence, the price of Y is considered to be unity.

The production function of infrastructure service is given as,

$$G = \delta(1-u)K \qquad (77)$$

In equation (77), $(1-u)K$ denotes the remaining part of private physical capital used to produce infrastructure. δ is the technology parameter of infrastructure services production, which is a constant. The infrastructure services are productive investment and are flow in nature. We obtain the ratio of infrastructure to private physical capital from equation (77),

$$\frac{G}{K} = \delta(1-u) \tag{78}$$

The profit function of firm 1 is given by,

$$\Pi^1 = A(uK)^{1-\alpha}G^{\alpha} - ruK - \mu G \tag{79}$$

μ is the user fees or price paid by firm 1 for using infrastructure services.

The profit function of firm 2 producing flow of infrastructure service is given by,

$$\Pi^2 = \mu G - \phi\, r\,(1-u)K \tag{80}$$

In equation (80), μ is the total price charged by firm 2 for providing infrastructure services (e.g. revenue earned by selling tickets of metro rail, user fees for the usage of roads, etc) and ϕ is the share of cost borne by firm 2 for manufacturing infrastructure. Substituting the value of G from equation (77) in the above equation, we have,

$$\Pi^2 = \mu\delta(1-u)K - \phi\, r\,(1-u)K \tag{81}$$

Both firms take input prices as given and choose input quantities so as to maximize their profit. Differentiating Π^1 with respect to uK and G, the first order condition gives,

$$r = \frac{A(1-\alpha)}{u^{\alpha}}\left(\frac{G}{K}\right)^{\alpha} \tag{82}$$

$$\mu = Au^{1-\alpha}\alpha\left(\frac{G}{K}\right)^{\alpha-1} \tag{83}$$

Substituting the value of $\left(\frac{G}{K}\right)$ in the above equations,

$$r = A(1-\alpha)\delta^{\alpha}\left(\frac{1-u}{u}\right)^{\alpha} \tag{84}$$

$$\mu = A\alpha\delta^{\alpha-1}\left(\frac{1-u}{u}\right)^{\alpha-1} \tag{85}$$

Infrastructure demand of firm 1 is obtained from equation (83),

$$G = \left[\frac{\mu}{Au^{1-\alpha}\alpha}\right]^{\frac{1}{\alpha-1}}K \tag{86}$$

Now using equations (77) and (86), the supply of infrastructure is equated with the demand for infrastructure for finding the equilibrium quantity of private physical capital.

$$u = \frac{(A\alpha)^{\frac{1}{\alpha-1}}\delta}{\mu^{\frac{1}{\alpha-1}} + (A\alpha)^{\frac{1}{\alpha-1}}\delta} \tag{87}$$

Substituting the value of u in the equation (84), the rate of interest under the public provision is given as,

$$r = A^{\frac{1}{1-\alpha}}(1-\alpha)\left(\frac{\mu}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} \tag{88}$$

4.2 The Government:

In the decentralized economy under the public-private partnership provision of infrastructure the government is mainly engaged in 3 activities: (1) It bears the partial cost burden of manufacturing infrastructure. (2) It imposes capital income tax in order to finance its cost. (3) It also issues government bonds.

The government in this economy receives fund by imposing income tax and by issuing government bonds. The interest on the bond and expenditure incurred for sharing of cost of producing infrastructure service and subsidizing its production add to the debt burden of the government while tax revenue reduces the government debt. Hence the bond accumulation function is given by,

$$\dot{B} = (1-\tau)rB - (T - E) \tag{89}$$

T is the tax revenue at time t and E is the public expenditure at time t. $T - E$ shows the public investment to tax revenue. It indicates how much of it is used for the debt service.

$$T = \tau\, r\, uK \tag{90}$$

$$E = (1-\phi)r\,(1-u)K \tag{91}$$

$(1-\phi)$ in the above equation represents the share of cost of manufacturing infrastructure, borne by the government.

Substituting the value of T and E in equation (89), the bond accumulation function is given as,

$$\dot{B} = (1-\tau)rB - \tau ruK + (1-\phi)r(1-u)K \quad (92)$$

The rate of growth of bond is given as,

$$\frac{\dot{B}}{B} = (1-\tau)r - \frac{K}{B}[\tau ruK - (1-\phi)r(1-u)K] \quad (93)$$

4.3 The Households:

It is assumed that the household derives utility from direct consumption of final good. The utility function of the representative household is given by,

$$U = \int_0^\infty \gamma \ \ln C \ e^{-\rho t} \, dt \quad (94)$$

ρ is constant and denotes positive discount rate at which future utility is discounted. It is assumed that the disposable income over expenditure is accumulated as wealth. The wealth/asset of the household denoted by W is defined as the sum of bond holding (B) and capital holding (K). Therefore,

$$W = B + K \quad (95)$$

Total disposable wealth of a household over consumption expenditure and payment for using infrastructure services is accumulated as wealth. The rate of accumulation of wealth is given by,

$$\dot{W} = (1-\tau)rW - C \quad (96)$$

τ is the tax on capital income , r is the interest rate, C is consumption at time t.

In competitive economy, the representative household maximizes the current-value Hamiltonian, subject to equation (96),

$$H_c = \gamma \ \ln C + \eta[(1-\tau)rW - C] \quad (97)$$

The control variable is C. The first – order maximization conditions are given as follows:

$$\eta = \frac{\gamma}{C} \tag{98}$$

Time derivative of the co-state variable is given by the following,

$$\frac{\dot{\eta}}{\eta} = \rho - (1 - \tau)r \tag{99}$$

Taking the log and derivative of equation (98), we get,

$$-\frac{\dot{C}}{C} = \frac{\dot{\eta}}{\eta} = \rho - (1 - \tau)r \tag{100}$$

Also using equation (95), we have,

$$\frac{\dot{W}}{W} = \frac{\dot{B}/B + \dot{K}/K \cdot K/B}{1 + K/B} \tag{101}$$

Since $\frac{\dot{B}}{B} = \frac{\dot{K}}{K} = g$, now substituting these values in the above equation, we get,

$$\frac{\dot{W}}{W} = g \tag{102}$$

Therefore, combining equations (100) and (102), the growth rate equation is obtained as,

$$\frac{\dot{C}}{C} = \frac{\dot{G}}{G} = \frac{\dot{\eta}}{\eta} = \frac{\dot{W}}{W} = (1 - \tau)r - \rho = g \tag{103}$$

4.4 Steady State Balanced growth:

The steady state balanced growth equilibrium is defined as a situation when consumption, private physical capital and infrastructure capital grow at the same strictly positive constant growth rate, i.e; $\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{G}}{G} = \frac{\dot{B}}{B} = \frac{\dot{Y}}{Y} = g$. Where g is positive and constant. If $\frac{\dot{B}}{B}$ is constant, then $\frac{B}{K}$ is also constant. Setting $\frac{\dot{C}}{C} = \frac{\dot{B}}{B}$, we get the value of $\frac{B}{K}$ as,

$$\frac{B}{K} = \frac{\tau r u - (1-\phi) r (1-u)}{\rho} \tag{104}$$

4.5 Steady State Balanced growth rate under the PPP provision at Zero Debt regime:

At steady state balanced growth rate under the PPP provision, the government doesn't have any debt such that the government's tax revenue is equal to the total expenditure of the government. In other words, the government observes the balanced budget. Setting $T = E$, we have,

$$\tau ruK = (1-\phi)r(1-u)K \tag{105}$$

Substituting the value of u in the above equation, we get the value of μ under the PPP provision at zero-debt regime,

$$\mu = \frac{\tau^{\alpha-1}(A\,\delta^{\alpha-1}\alpha)}{(1-\phi)^{\alpha-1}} \tag{106}$$

4.5.1 Growth rate under the PPP provision at zero debt regime:

The growth rate is given as,

$$g = (1-\tau)A^{\frac{1}{1-\alpha}}(1-\alpha)\left(\frac{\mu}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \rho \tag{107}$$

Now substituting the value of user fees of the PPP at zero-debt regime the growth rate equation is,

$$g = (1-\tau)A(1-\alpha)\frac{\tau^{\alpha}\delta^{\alpha}}{(1-\phi)^{\alpha}} - \rho \tag{108}$$

4.5.2 Growth maximizing tax rate under the PPP provision at zero debt regime:

To find the growth maximizing tax rate, we do the logarithmic transformation of growth rate equation (108),

$$\ln(g+\rho) = \ln(1-\tau) + \ln A + \ln(1-\alpha) + \alpha\,\ln\tau + \alpha\,\ln\delta - \alpha\,\ln(1-\phi) \tag{109}$$

Differentiating the log-transformed growth rate equation with respect to τ, the first order condition gives,

$$\frac{-\tau+\alpha(1-\tau)}{\tau(1-\tau)} = 0 \tag{110}$$

The optimal tax rate under the PPP provision at zero debt regime is,

$$\tau = \frac{\alpha}{(1+\alpha)} \tag{111}$$

The second order condition gives,

$$\frac{(\alpha-1)-\frac{\alpha}{\tau}+\frac{1-2\tau}{1-\tau}}{\tau(1-\tau)} < 0 \tag{112}$$

$\tau < 0.5$ is a sufficient condition for the growth maximizing tax rate under the PPP provision at zero debt regime.

Proposition 5: *At balanced budget and constant debt regime, for the public provision the growth maximizing tax rate is zero, but for the PPP provision the growth maximizing tax should be less than 50%.*

4.6 Steady State Balanced growth rate under the PPP provision at constant Debt regime:

At steady state balanced growth rate, the government experiences the constant debt, such that $\dot{B} = 0$ and also $\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{G}}{G} = g$ is constant and positive. However, when $\dot{B} = 0$, it does not necessarily imply that public debt equals zero. If the level of initial debt is positive, the debt to capital ratio and also the debt to GDP ratio are positive but decline over time and converge to zero in the long run.

Therefore, setting $\dot{B} = 0$, we find that the user fees under the PPP provision at constant debt regime is same as under the PPP provision at zero debt regime. Therefore the growth rate and growth maximizing tax rate are also same.

Equation (113)-(116) of Appendix A2 shows the user fees, growth rate, growth maximizing tax rate, and maximum growth rate respectively for the PPP provision at the constant debt regime, which are exactly same as the balanced budget regime.

Proposition 6: *For the PPP provision, the user fee charged for infrastructure services and growth rates under the constant debt regime and the balanced budget regime are the same.*

4.7 Steady state balanced growth rate under the PPP provision at permanent deficit regime:

At steady state balanced growth rate, the government experiences the case when debt is accumulating; this is the case of a permanent deficit. Steady state balanced growth rate g is positive and constant and therefore, all the variables grow at the same strictly positive constant rate, such that $\frac{\dot{C}}{C} = \frac{\dot{G}}{G} = \frac{\dot{K}}{K} = \frac{\dot{W}}{W} = \frac{\dot{B}}{B} = g$.The permanent deficit case is characterized by the public deficits, where the government debt grows at the same rate as all other endogenous variables in the long run.

Applying the primary surplus rule, we get

$$T - E = \xi\, Y + \beta\, B \tag{117}$$

Substituting the values of T, E and Y in the above equation, we get another value of $\frac{B}{K}$ under the permanent deficit case, which is different from the balanced budget $\frac{B}{K}$, therefore,

$$\frac{B}{K} = \frac{\tau\, r\, u - \xi\, A\, u^{1-\alpha} \delta^{\alpha} (1-u)^{\alpha} - (1-\phi)\, r(1-u)}{\beta} \tag{118}$$

The bond accumulation function for the sustainability of public debt is,

$$\dot{B} = (1-\tau)rB - (\xi\, Y + \beta\, B) \tag{119}$$

After substituting the value of Y in the above equation, the rate of growth of bond is,

$$\frac{\dot{B}}{B} = (1-\tau)r - \xi\, A\, u^{1-\alpha} \frac{K}{B} \left(\frac{G}{K}\right)^{\alpha} - \beta \tag{120}$$

Now substituting the value of $\frac{K}{B}$ and $\frac{G}{K}$ in the above equation, the rate of growth of bond under the PPP provision at permanent deficit regime is given as,

$$\frac{\dot{B}}{B} = (1-\tau)r - \frac{\xi\, A u^{1-\alpha} [\delta(1-u)]^{\alpha} \beta}{\tau\, r\, u - \xi\, A\, u^{1-\alpha} \delta^{\alpha} (1-u)^{\alpha} - (1-\phi)\, r(1-u)} - \beta \tag{121}$$

Resorting to equation (103) and (121), we set $\frac{\dot{C}}{C} = \frac{\dot{B}}{B}$, we obtain the value of user fees under the PPP provision at permanent deficit regime.

$$\mu = \frac{\alpha}{A}\left[\left(\tau - \frac{\xi\,\rho\,\delta}{(\rho-\beta)(1-\alpha)}\right)\frac{\delta}{(1-\phi)}\right]^{\alpha-1} \tag{122}$$

4.7.1 Growth rate under the PPP provision at constant debt regime:

Substituting the value of user fees under the PPP provision at permanent deficit regime, we obtain the growth rate equation as,

$$g = \frac{(1-\tau)}{A}(1-\alpha)\left[\left(\tau - \frac{\xi\,\rho\,\delta}{(\rho-\beta)(1-\alpha)}\right)\frac{\delta}{(1-\phi)}\right]^{\alpha} - \rho \tag{123}$$

4.7.2 Growth maximizing tax rate under the PPP provision at permanent deficit regime:

We differentiate the growth rate equation with respect to τ. Therefore, the first order condition gives the value of optimal tax rate under the PPP provision at permanent deficit regime,

$$\tau = \frac{1}{(\alpha+1)}\left[\frac{\xi\,\rho\,\delta}{(\rho-\beta)(1-\alpha)} + \alpha\right] \tag{124}$$

From the second order condition, we have,

$$\frac{(1-\alpha)}{A}\,\alpha\left[\left(\tau - \frac{\xi\,\rho\,\delta}{(\rho-\beta)(1-\alpha)}\right)\frac{\delta}{(1-\phi)}\right]^{\alpha-1}\left(\frac{\delta}{1-\phi}\right)\left[-2 + \frac{(1-\tau)(\alpha-1)}{\left(\tau - \frac{\xi\,\rho\,\delta}{(\rho-\beta)(1-\alpha)}\right)}\right] < 0 \tag{125}$$

Therefore, $\frac{(1-\tau)(\alpha-1)}{\left(\tau - \frac{\xi\,\rho\,\delta}{(\rho-\beta)(1-\alpha)}\right)} > 2$ is a sufficient condition for $\frac{d^2 g}{d\,\tau^2} < 0$.

4.7.3 Maximum growth rate under the PPP provision at accumulating debt regime:

Substituting the value of optimal tax rate for the PPP provision under the accumulating debt regime, we get the maximum growth rate equation as,

$$g_{\hat{\tau}} = \frac{\alpha}{\alpha+1}\left[\frac{\xi\,\rho\,\delta + \alpha\,(\rho-\beta)(1-\alpha)}{(\rho-\beta)(1-\alpha)}\right]\frac{1-\alpha}{A}\left[\frac{\alpha\delta}{(\alpha+1)(1-\phi)}\right]^{\alpha} - \rho \tag{126}$$

Proposition 7: *The user fees and maximum growth rates under the PPP provision at accumulating debt regime are different from the user fees and maximum growth rates under the PPP provision at constant debt and balanced budget regime.*

5. Justification of Public-private partnership provision of infrastructure:

We need to know whether the growth rates or rather the maximum growth rates for which provision is better and could be suggested for policy prescription. For that matter we make a comparison of the PPP mode with the private / public mode. Therefore, comparing the maximum growth rates under the PPP provision at balanced budget or constant debt and the maximum growth rates under the public / private provision at balanced budget or constant debt, we find that,

$$g_{\hat{\tau}}(PPP) - g_{\hat{\tau}}(public\ or\ private)$$
$$= A\delta^{\alpha}\alpha^{\alpha}(1-\alpha)\left[\frac{1}{(1-\phi)^{\alpha}(1+\alpha)^{\alpha+1}} - \frac{1}{(1-\alpha)^{\alpha}}\right] \tag{127}$$

Therefore the Public-Private Partnership is justified under the condition, if $\frac{(1-\alpha)}{(1+\alpha)^{\frac{\alpha+1}{\alpha}}} > (1-\phi)$. In other words, as the share of manufacturing cost of infrastructure borne by the government is low, PPP provision of infrastructure yields higher growth rate.

Suppose, $L = \frac{(1-\alpha)}{(1+\alpha)^{\frac{\alpha+1}{\alpha}}}$. Taking the logarithmic transformation of L. We obtain the equation,

$$\ln L = \ln(1-\alpha) - \left(\frac{\alpha+1}{\alpha}\right)\ln(1+\alpha) \tag{128}$$

Differentiating above equation with respect to α,

$$\frac{\partial \ln L}{\partial \alpha} = \frac{2\alpha-1}{\alpha(1-\alpha)} + \frac{1}{\alpha^2}\ln(1+\alpha) \tag{129}$$

From equation (129), $\alpha > 0.5$ is a sufficient condition for $\frac{\partial \ln L}{\partial \alpha}$ to be positive. It follows that when output elasticity from the infrastructure capital is more than 0.5, then with increase in α, there is higher chance that the condition (A) is satisfied justifying the adoption of the Public-Private Partnership (PPP) provision of infrastructure. Hence, the countries whose output elasticity from the infrastructure capital is sufficiently high should go for the PPP mode for the provision of infrastructure.

Proposition 8: *In zero debt case, if the output elasticity from the infrastructure capital should be sufficiently high and share of manufacturing cost of infrastructure borne by the government is low, PPP provision of infrastructure yield higher growth rate compared to pure public or pure private provision of infrastructure.*

Also comparing the user fees under the private provision and public provision (at both accumulating debt and balanced budget/constant debt case) with the PPP provision (at both accumulating debt and balanced budget / constant debt case), we have the following results:

$$\mu_{private} - \mu_{PPP}^{\dot{B}=0} = A\delta^{\alpha-1}\left[\frac{\alpha^{\alpha}}{(1-\alpha)^{\alpha-1}} - \frac{\tau^{\alpha-1}\alpha}{(1-\phi)^{\alpha-1}}\right] \quad (130)$$

From equation (130) it follows that, when $\frac{\alpha^{\alpha}}{(1-\alpha)^{\alpha-1}} > \frac{\tau^{\alpha-1}}{(1-\phi)^{\alpha-1}}$, user fee charged under pure private provision of infrastructure is higher than that under PPP. In PPP, if growth maximizing tax rate is imposed, the above condition reduces to $\phi < \frac{2\alpha}{1+\alpha}$. Thus, if the share of manufacturing cost borne by the private firm is low in PPP, user-fee charged under private provision is higher than that under PPP in zero debt case, because VGF brings down the user fees charged under the PPP mode.

This justifies PPP from the consumer's utility perspective as well and therefore, the government could opt for the PPP provision if the output elasticity for the infrastructure capital increases due to VGF financing by the government.

Proposition 9: *If the share of cost borne by the government for infrastructure provision* $(1-\phi)$ *is high, then user fee charged under PPP would be lower, which may give another reason to justify PPP.*

6. Justification for debt financing:

Another question is that why should government go for the debt financing and not the balanced budget or constant debt. In order to show the necessity of debt financing we make a comparison between the maximum growth under the PPP mode at accumulating debt regime with that of the maximum growth rate under the PPP mode at constant debt or zero debt regimes. We find that,

$$g_{\hat{\tau}}^{\dot{B}>0}(PPP) - g_{\hat{\tau}}^{\dot{B}=0}(PPP) = \frac{(1-\alpha)(\alpha\delta)^{\alpha}}{(1+\alpha)^{1+\alpha}(1-\phi)^{\alpha}}\left[\frac{\alpha}{A}\left(\frac{\xi\rho\delta+\alpha(\rho-\beta)(1-\alpha)}{(\rho-\beta)}\right) - A\right] \qquad (131)$$

From equation (131), it follows that when $\frac{\xi\rho\delta+\alpha(\rho-\beta)(1-\alpha)}{(\rho-\beta)} > \frac{A^2}{\alpha}$ then debt financing under the PPP mode of provision is feasible.

Proposition 10: *For technologically poor countries or less developed countries, debt financing is desirable.*

7. Conclusion:

This paper explains when the PPP mode of infrastructure provision is desirable in a model with non-rival yet excludable infrastructure in a closed economy model. The model shows that if output elasticity of infrastructure is sufficiently high and share of manufacturing cost borne by the government is low, PPP provision of infrastructure yields higher growth rate. There are situations when PPP provision may require charging lower user fees as well. PPP provision may be justified on that ground too. The model also shows that for the technologically poor countries debt financing may be desirable.

There is a vast literature that deals with financing problem of public good through pure public provision of infrastructure. But, there is not a single paper on endogenous growth theory, which deals with the financing problem of public good through PPP. This paper tries to answer the two questions: (1) why should the government go for the PPP for the infrastructure provision? (2) How should the government finance the cost of infrastructure production – (a) through imposition of tax (b) through bond financing and (c) through charging user fees? We find the conditions under which the maximum growth rate under the PPP mode of provision is greater than the private and public provision. Also, the user fees and growth maximizing tax rates under different regimes have been found out. User-fee under the PPP mode is less as compared to the pure private user fees and pure public user fees, due to VGF investment by the government, which keeps the PPP user fees lower. We also find that in zero debt case, for pure public provision of infrastructure, charging user fees is better than the imposition of tax. This paper also explains why in some nations there are huge number of PPP investments and why in some nations we find only few PPP projects. The nations where, the output elasticity from the infrastructure is very high say, more than 50%, there only the PPP projects yield higher growth rates.

However, like any other theoretical model, this model also has few limitations. This model does not contain many aspects of the reality - like the imperfect competition in the production of infrastructure, infrastructure as a stock variable etc. In our future research we aspire to find optimal fiscal policy by taking these aspects into the consideration. However, this paper contributes to the literature being the first one to incorporate the public private partnership in infrastructure provision in endogenous growth model with constant, zero and accumulating government debt.

APPENDIX

Appendix A1. For the pure public provision under the constant debt, $\dot{B} = 0$. Therefore, from equation (36), we have,

$$(1-\tau)rB - (T-E) = 0$$

Hence, $T - E = (1-\tau)rB$

Substituting the value of T and E in the above equation, we have,

$$\tau\, r\, u\, K + \mu\, \delta(1-u)K - r(1-u)K = (1-\tau)\, rB$$

Substituting the value of r, u we get,

$$A^{\frac{1}{1-\alpha}}(1-\alpha)\left(\frac{\mu}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}\left[\frac{\tau(A\alpha)^{\frac{1}{\alpha-1}}\delta}{\mu^{\frac{1}{\alpha-1}} + (A\alpha)^{\frac{1}{\alpha-1}}\delta} - \left(\frac{\mu^{\frac{1}{\alpha-1}}}{\mu^{\frac{1}{\alpha-1}} + (A\alpha)^{\frac{1}{\alpha-1}}\delta}\right) - (1-\tau)\frac{B}{K}\right]$$

$$= -\mu\delta\left(\frac{\mu^{\frac{1}{\alpha-1}}}{\mu^{\frac{1}{\alpha-1}} + (A\alpha)^{\frac{1}{\alpha-1}}\delta}\right)$$

Since $\frac{B}{K}$ converges to zero in the long run, therefore from the above equation we have,

$$A^{\frac{1}{1-\alpha}}(1-\alpha)\left(\frac{\mu}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}\left[\frac{\tau(A\alpha)^{\frac{1}{\alpha-1}}\delta}{\mu^{\frac{1}{\alpha-1}} + (A\alpha)^{\frac{1}{\alpha-1}}\delta} - \left(\frac{\mu^{\frac{1}{\alpha-1}}}{\mu^{\frac{1}{\alpha-1}} + (A\alpha)^{\frac{1}{\alpha-1}}\delta}\right)\right]$$

$$= -\mu\delta\left(\frac{\mu^{\frac{1}{\alpha-1}}}{\mu^{\frac{1}{\alpha-1}} + (A\alpha)^{\frac{1}{\alpha-1}}\delta}\right)$$

By simplifying the above equation we obtain the value of μ for the constant debt regime.

$$\mu = \left[\tau\,(A\alpha)^{\frac{1}{\alpha-1}}\delta + \frac{\delta}{A^{\frac{1}{1-\alpha}}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}}\right]^{\alpha-1} \tag{61}$$

Now substituting the value of user fees under the pure public provision constant-debt regime,

$$g = (1-\tau)A^{\frac{1}{1-\alpha}}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left[\frac{\delta}{A^{\frac{1}{1-\alpha}}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}} + (A\alpha)^{\frac{1}{\alpha-1}}\delta\tau\right]^{\alpha} - \rho \tag{62}$$

Differentiating equation (62) with respect to τ, the first order condition is,

$$-A^{\frac{1}{\alpha-1}}\ \alpha^{\frac{2\alpha-1}{\alpha-1}}\frac{\delta}{(1-\alpha)} - A^{\frac{1}{\alpha-1}}\alpha^{\frac{\alpha}{\alpha-1}}\ \delta\ \tau\ \frac{(\alpha+1)}{\alpha} < 0 \tag{63}$$

Since the first-order condition is negative for the pure public provision under constant debt, the growth maximizing tax rate is zero.

Now the maximum growth for the pure public provision under the constant debt regime is equal to the private provision growth rate.

$$g = A\alpha^{\alpha}(1-\alpha)^{1-\alpha}\delta^{\alpha} - \rho \tag{64}$$

Appendix A2. For the PPP provision under the constant debt, $\dot{B} = 0$. Therefore, from equation (88), we have,

$$(1-\tau)rB - (T-E) = 0$$

Hence, $T - E = (1-\tau)rB$

Substituting the value of T and E in the above equation, we have,

$$\tau\, r\, u\, K - (1-\phi)r(1-u)K = (1-\tau)\, rB$$

From above equation, $\tau\, u - (1-\phi)(1-u) = (1-\tau)\frac{B}{K}$

Since $\frac{B}{K}$ converges to zero in the long run, therefore from the above equation we have,

$$\frac{\tau}{(1-\phi)} = \frac{(1-u)}{u}$$

Substituting the value of u in the above equation, we obtain the value of user fees for the infrastructure services under the PPP provision for the constant debt regime.

$$\mu = \frac{\tau^{\alpha-1}A\, \delta^{\alpha-1}\alpha}{(1-\phi)^{\alpha-1}} \tag{113}$$

Now substituting the value of user fees under the PPP provision constant-debt regime,

$$g = (1-\tau)A(1-\alpha)\frac{\tau^{\alpha}\delta^{\alpha}}{(1-\phi)^{\alpha}} - \rho \tag{114}$$

To find the growth maximizing tax rate, we do the logarithmic transformation of equation (113),

$$\ln(g+\rho) = \ln(1-\tau) + \ln A + \ln(1-\alpha) + \alpha\ \ln\tau + \alpha\ \ln\delta - \alpha\ \ln(1-\phi)$$

Differentiating the log-transformed growth rate equation with respect to τ, the first-order condition gives,

$$\frac{-\tau+\alpha(1-\tau)}{\tau(1-\tau)} = 0$$

The optimal tax rate under the PPP provision at constant debt regime is,

$$\tau = \frac{\alpha}{(1+\alpha)} \tag{115}$$

The second order condition gives,

$$\frac{(\alpha-1)-\frac{\alpha}{\tau}+\frac{1-2\tau}{1-\tau}}{\tau(1-\tau)} < 0$$

$\tau < 0.5$ is a sufficient condition for the growth maximizing tax rate under the PPP provision at constant debt regime.

Substituting the value of optimal tax rate under the PPP provision at constant debt regime, we get the maximum growth rate,

$$g_{\hat{\tau}} = \frac{A\,(1-\alpha)\alpha^{\alpha}\delta^{\alpha}}{(1+\alpha)^{1+\alpha}(1-\phi)^{\alpha}} - \rho \quad (116)$$

Now the maximum growth for the PPP provision under the constant debt regime is equal to the PPP provision under the balanced budget regime.

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