Modeling Deterrence by Denial and by Punishment

Nakao, Keisuke

University of Hawaii at Hilo

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Keisuke Nakao†

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Abstract

We explore a defender's prewar allocation of military resources between denial and punishment strategies for deterrence. While denial disproportionately raises the probability to countervail aggression by disrupting military forces ("guns"), punishment proportionately raises costs on the aggressor by damaging civilian values ("butter"). Because these countervailing and deterrence effects are so divergent, the deployment that minimizes the risk of war can vary, depending on the defender's military capacity relative to the aggressor's. Namely, inferior parties resort only to punishment (e.g., post-Cold War North Korea), competitive parties concentrate solely on denial (e.g., Germany, Italy, and Japan), and superior parties develop both denial and punishment capabilities (e.g., Permanent Five). JEL: D30, D74, F51, F52.

Keywords: denial vs. punishment, countervailing vs. deterrence effects, guns vs. butter, military strategy.

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†College of Business and Economics. University of Hawaii at Hilo. 200 W. Kawili St., Hilo, HI 96720 USA. E-mail: keisuken@hawaii.edu.

Deterrence involves problems of choice among weapons, vehicles, and targets.

Bernard Brodie (1959: viii)

1 Introduction

Among a variety of tradeoffs associated with its own security, a sovereign state faces two kinds of tradeoffs concerning "guns" and "butter."[1] One is of how to allocate productive resources between national security and economic prosperity. Especially in the context of arms races, this *production* tradeoff has long been studied by theorists in Economics and International Relations (Acemoglu et al. 2012; Baliga and Sjöström 2004; Brito and Intriligator 1985; Downs 1991; Downs and Rocke 1990; Fearon 2010; Hirshleifer 1995; Intriligator and Brito 1984; Jackson and Morelli 2009; Kadera and Morey 2008; Kydd 1997, 2000; Powell 1993; Richardson 1919; Skaperdas 1992; Slantchev 2005; Snyder 1971).

The other is of how to distribute military budget between the capabilities of disrupting military forces ("guns") and of damaging civilian values ("butter"). A successful foreign policy may need to address the balance between these two capabilities (Kissinger 1957), but this *destruction* tradeoff has been largely overlooked by formal theorists. The rarity of studies on this issue contrasts sharply with the maturing literature on the production tradeoff as shown above. This article thus explores the problem as to the allocation military resources between denial and punishment capabilities.[2]

The military strategy that intends to disrupt enemy forces is called *denial* (Snyder 1961). By affecting the balance of military strength against an opponent, denial aims to produce strategic advantage in war (with the countervailing effect). To promote its denial capabilities, a state mainly develops conventional forces such as army tanks, navy destroyers, and air fighters. On the other hand, the military strategy that targets civilian values is known as *punishment* (Schelling 1966). By inflicting unbearable

[1]Other tradeoffs include: the allocation of armed forces between offense and defense (Brown et al. 2004; Quester 1988); the distribution of offensive measures across battlefields (Borel 1921; Golman and Page 2009; Roberson 2006); the investment of defensive resources to harden targeted assets (Bier et al. 2007; Hausken and Levitin 2011; Powell 2007a, 2007b); the spending between hard and soft power to exert foreign influence either by threatening or through persuasion (Nye 2005; Wilson 2008).

[2]Denial and punishment in our words correspond to the counterforce and coutervalue strategies in the literature of nuclear war (Intriligator and Brito 1984).

costs on the opponent, punishment aims to influence enemy behavior through its psychological impacts (with the deterrence effect).[3] Not necessarily constrained to conventional forces, punishment may resort even to the use of unconventional forces such as chemical, biological, and nuclear weapons (Brodie 1946; Kahn 1960). In illuminating the destruction tradeoff, we also address how a defender adopts and combines these two strategies to deter a potential aggressor.

For this end, we develop a game-theoretic model, where Defender allocates her military resources between denial and punishment, and Aggressor then decides whether or not to fight Defender. Aggressor is deterred if his expected payoff from fighting is short of the payoff from the status quo, or if

$$\begin{bmatrix} \text{Benefit} \\ \text{from winning} \end{bmatrix} \times \begin{bmatrix} \text{Probability} \\ \text{of winning} \end{bmatrix} - \begin{bmatrix} \text{Cost of} \\ \text{fighting} \end{bmatrix} < \begin{bmatrix} \text{Payoff from} \\ \text{status quo} \end{bmatrix},$$

for which the probability of winning the war is decreased by denial, while the cost of fighting is increased by punishment. By deriving the game's equilibrium, we seek the deployment of denial and punishment capabilities that minimizes the risk of war.

There exist plenty of empirical studies on military strategies (Arreguin-Toft 2011; Bennett and Stam 1996, 1998; Goemans 2000; Mearsheimer 1983; Reiter 1999; Reiter and Meek 1999; Reiter and Stam 1998, 2002; Stam 1996; Wallace 2008). Among them, Pape (1996) and Toft and Zhukov (2012) examined the effectiveness of denial and punishment, concluding denial to be more effective than punishment. In contrast to the empirical studies, theoretical studies remain sparse and limited (Baliga & Sjöström 2008; Lindsey 2015; Meirowitz & Sartori 2008; Nakao 2019; Powell 1988, 1989; Sandler and Siqueira 2006; Slantchev 2010; Tarar 2016). Notably, two theoretical studies closest to ours are Snyder (1961) and Intriligator and Brito (1984). However, unlike Snyder (1961), whose numerical model presumes binary choice of between denial and punishment strategies, our analytical model allows continuity in allocating resources for these strategies. Also unlike Intriligator and Brito (1984), whose model of nuclear arms races regards weapons as versatile to strike either couterforce or countervalue targets in war, ours treats forces as strategy-specific and is concerned about the

[3] Historically, cost-inflicting strategies were adopted by both the Allies and Axis during World War II, by People's Liberation Army during the Chinese Civil War, by both the United States and North Vietnam during the Vietnam War, and by al-Qaeda against the United States and other liberal democracies markedly on and after 9/11.

armament of two qualitatively distinct forces before a war's outbreak.[4] To the best of our knowledge, ours is the first theoretical study on the prewar allocation of military resources between denial and punishment for deterrence with an analytical model.

The rest of the article proceeds as follows. Section 2 presents a model, which will be analyzed and solved in Section 3. Section 4 concludes.

[4]While some weapons are target-specific (e.g., MD system for denial, ICBM for punishment), others can be used for denial and punishment interchaneably (e.g., multi-role fighters, cruise missiles).

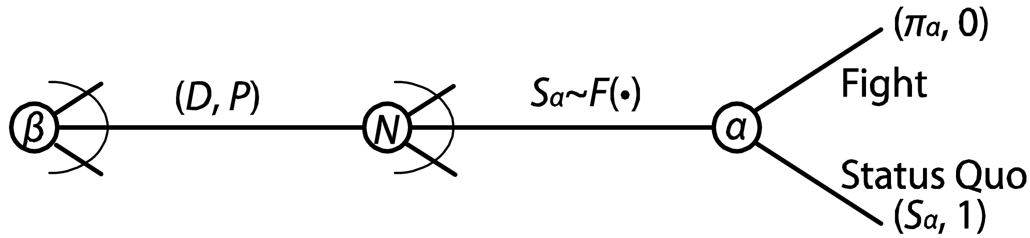


Figure 1: The game in extensive form.

2 The Model of Deterrence by Denial and by Punishment

To explore the prewar allocation of military resources between denial and punishment, we develop a game-theoretic model, which depicts the interplay between Defender and Aggressor. While denial raises the likelihood of winning a war upon its outbreak, punishment inflicts the cost of fighting on an opponent.

2.1 Basic Setup

In the game, there are two players: Aggressor α and Defender β.[5] At the game's onset, Defender β determines the levels of denial and punishment (D, P) within her capacity constraint $\overline{Q} > 0$ such that $D + P \leq \overline{Q}$ with $D \geq 0$ and $P \geq 0$. In contrast, Aggressor α's denial and punishment capabilities are exogenously given as $\left(\overline{D}, \overline{P}\right)$ with $\overline{D} > 0$ and $\overline{P} \geq 0$.[6] After observing β's decision (D, P), α decides to fight a war (F) or to honor the status quo (SQ).

Aggressor α's payoffs from fighting and from the status quo are denoted as π_α (specified later) and $S_\alpha \geq 0$, respectively. When β makes her decision, she is uncertain about the true value of S_α but knows its cumulative distribution $F(\cdot)$ and probability density $f(\cdot)$. This randomness generates the possibility of the war's outbreak. On the other hand, Defender β's sole purpose for military deployment is the preservation of peace, so that her payoff is set to be one from the status quo and zero from fighting.

[5] For the use of the model, we assign the feminine pronoun ("she") to Defender and the masculine pronoun ("he") to Aggressor.

[6] If α's capabilities $\left(\overline{D}, \overline{P}\right)$ are endogenized, he would spend all his resources for denial, because he is interested in waging and winning war, and as specified later, punishment has no impact on the probability of winning it.

The game's extensive form appears in Figure 1.

2.2 Denial and Punishment

As the war evolves with clashes of forces on battlefields, its outcomes is determined by the relative size of denial capabilities between the belligerents.[7] The probabilities of β's winning and losing the war are thus assumed as:

$$\begin{aligned} \Pr(win) &\equiv \frac{D^A}{D^A + \overline{D}^A} \\ \Pr(loss) &\equiv 1 - \Pr(win), \end{aligned} \tag{1}$$

where A is Lanchester's (1916) power, which determines the relative advantage to the stronger side. It is naturally assumed that $A > 1$ on the ground that substantially weaker parties (e.g., Iraq) can have very little chances to defeat their overpowering adversaries (e.g., the U.S.) by conventional forces, or that the stronger side is given *disproportionate* advantage in waging war. Put formally, it is ensured by $A > 1$ that $\frac{d\Pr(win)}{dD}$ is positive but negligible when D is near zero:

$$\frac{d\Pr(win)}{dD} = \frac{AD^{A-1}}{\left(D^A + \overline{D}^A\right)^2}\overline{D}^A, \tag{2}$$

which approaches zero with a decreasing D.

Aggressor's expected payoff from fighting is set to be:

$$\pi_\alpha \equiv \Pr(loss)\, W_\alpha - \Pr(win)\, L_\alpha - cP,$$

where $W_\alpha > 0$ is α's payoff from winning, $L_\alpha > 0$ his payoff from losing, and $c > 0$ the cost of fighting per unit of punishment (P). These payoffs imply that the cost of fighting increases *proportionately* to punishment. This setting could be justified on the ground that even very weak parties (e.g., individual terrorists) can inflict substantial damages on their opponent's values (e.g., unarmed civilians in the U.S.). The difference in effectiveness between denial and punishment, as shown above, can

[7]Empirical studies suggest that punishment has limited effects on winning wars (Belkin et al. 2000; Biddle 2002; Carr 2003: 190-191, 248-251; Horowitz and Reiter 2001; Kocher et al. 2011; Lambeth 2000; Pape 1996; Tooze 2006). In Brodie's (1959: viii) words, "[d]eterrence capability must be distinguished from war-winning capability in certain important respects."

influence Defender's allocation of military resources.

Given Aggressor α's sequentially rational decision, Defender β aims to maximize her *ex ante* payoff, or equivalently to minimize the risk of war:

$$(D^*, P^*) \equiv \arg\max_{(D,P)} \Pr\left(SQ|\left(D,P\right)\right),$$

where $\Pr\left(SQ|\left(D,P\right)\right)$ (and $\Pr\left(F|\left(D,P\right)\right)$) are the probabilities of the status quo (and of fighting) conditional on (D, P), respectively.[8]

[8] In Appendix, we consider the extension that instead of minimizing the risk of war, Defender maximizes her *ex ante* payoff comprising *ex post* payoffs W_β from winning the war, $-L_\beta$ from losing it, and S_β from the status quo. Even with this extension, the model generates similar results.

3 Military Deployment for Deterrence

By adopting subgame perfect Nash equilibrium as the game's solution, we will derive the equilibrium by backward induction. We first determine Aggressor α's rational decision. Because α decides to fight if his payoff from fighting exceeds his payoff from the status quo ($\pi_\alpha > S_\alpha$) and not to fight otherwise, the probabilities of fight and of the status quo are shown as:

$$\begin{aligned} \Pr(F|(D,P)) &= \Pr(S_\alpha < \pi_\alpha) \\ &= F(\pi_\alpha) \\ \Pr(SQ|(D,P)) &= 1 - F(\pi_\alpha). \end{aligned}$$

3.1 Countervailing Effect and Deterrence Effect

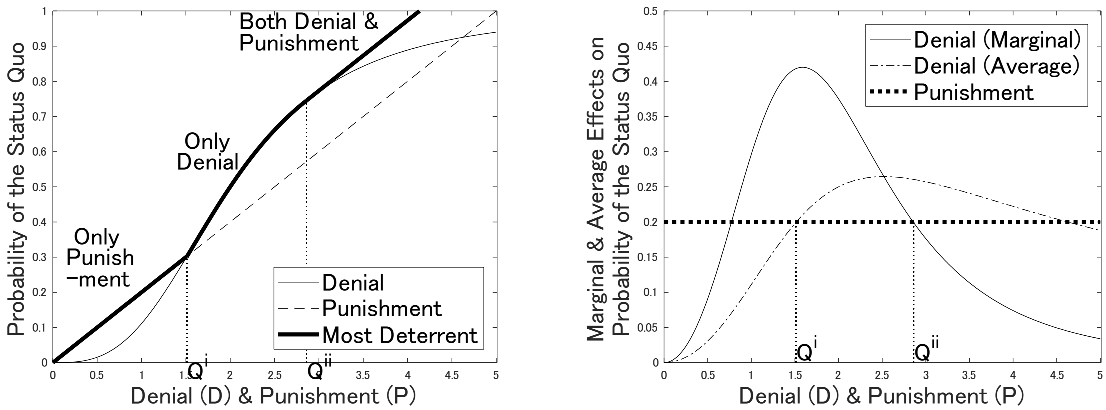
Both denial and punishment can reduce the risk of war through generating the disincentive on α from fighting; i.e., they both make the status quo more likely to be maintained:

$$\begin{aligned} \frac{d\Pr(SQ|(D,P))}{dD} &= -\frac{dF(\pi_\alpha)}{d\pi_\alpha}\frac{d\pi_\alpha}{dD} \\ &= (W_\alpha + L_\alpha)\frac{d\Pr(win)}{dD} f(\pi_\alpha) > 0 \\ \frac{d\Pr(SQ|(D,P))}{dP} &= -\frac{dF(\pi_\alpha)}{d\pi_\alpha}\frac{d\pi_\alpha}{dP} \\ &= cf(\pi_\alpha) > 0. \end{aligned}$$

While punishment directly influence Aggressor's behavior by inflicting the cost (c) on him upon the deterrence failure, denial indirectly deter the aggression through contervailing aggression ($\frac{d\Pr(win)}{dD}$). The two effects of denial and punishment are summrized in Table 1.

Table 1: Two effects of denial and punishment.

Strategy	*Denial*	*Punishment*
Countervailing effect (on $\Pr(win)$)	Disproportionate $\left(\frac{d\Pr(win)}{dD}\right)$	None
Deterrence effect (on $\Pr(F)$)	Indirect through countervailing	Proportionate & direct (c)



(a) (b)

Figure 2: Risk-minimizing deployment.

3.2 Minimization of the Risk of War

The relative effectiveness of denial and punishment determines the risk-minimizing deployment:

Proposition 1 *The risk of war* $\Pr(F|(D,P))$ *is minimized with the following allocation:*

$$(D^*, P^*) = \begin{cases} (0, \overline{Q}) & \text{if } \overline{Q} \in (0, Q^i) \\ (\overline{Q}, 0) & \text{if } \overline{Q} \in (Q^i, Q^{ii}) \\ (Q^{ii}, \overline{Q} - Q^{ii}) & \text{if } \overline{Q} \in (Q^{ii}, \infty), \end{cases}$$

where

$$Q^i \equiv \min\{Q \mid \Pr(F|(Q,0)) = \Pr(F|(0,Q))\} \tag{3}$$

$$Q^{ii} \equiv \max\left\{Q \mid \frac{d\Pr(F|(Q,0))}{dQ} = \frac{d\Pr(F|(0,Q))}{dQ}\right\}. \tag{4}$$

If c *is so large that there exists no* Q^i*,* $(D^*, P^*) = (0, \overline{Q})$*.*

Proof. The proof appears in Appendix. ■

The most deterrent military deployment (D^*, P^*) and its thresholds (Q^i, Q^{ii}) are graphically illustrated in Figure 2.[9] In (a), the relationships between the probability of the status quo $(\Pr(SQ|(D,P)))$ and each of denial (with the solid line), punishment

[9] For Figure 2, the following parameter values and function are adopted: $W_\alpha = 5$; $L_\alpha = 5$; $c = 2$; $A = 3$; $\overline{D} = 2$; $\overline{P} = 1$; $F(S_\alpha) = \frac{S_\alpha + L_\alpha}{W_\alpha + L_\alpha}$. The thresholds are: $Q^i = 1.51$; $Q^{ii} = 2.86$.

(with the dotted line) and the most deterrent deployment (with the bold line) are shown. In (b), the marginal and average effects of denial and the (constant) effect of punishment are shown.[10]

Proposition 1 implies that the most deterrent deployment hinges on the size of Defender's military capacity $\overline{Q}$: (i) for small $\overline{Q} \in (0, Q^i)$, because denial has little chances to defeat Aggressor, deterrence resorts only to punishment; (ii) for medium $\overline{Q} \in (Q^i, Q^{ii})$, denial can produce substantial chances to defeat Aggressor, so that deterrence is most likely to succeed solely by denial; (iii) for large $\overline{Q} \in (Q^{ii}, \infty)$, denial's marginal contribution to deterrence falls below punishment's;[11] thus deterrence is most effective with the denial level of Q^{ii} and remaining $\overline{Q} - Q^{ii}$ spent for punishment. Put formally, the following relationships hold:

$$
\begin{aligned}
&\text{(i) for } \overline{Q} \in (0, Q^i), && \Pr\left(SQ|\left(\overline{Q}, 0\right)\right) < \Pr\left(SQ|\left(0, \overline{Q}\right)\right); \\
&\text{(ii) for } \overline{Q} \in (Q^i, Q^{ii}), && \Pr\left(SQ|\left(\overline{Q}, 0\right)\right) > \Pr\left(SQ|\left(0, \overline{Q}\right)\right) \text{ and} \\
& && \frac{d\Pr\left(SQ|\left(\overline{Q}, 0\right)\right)}{d\overline{Q}} > \frac{d\Pr\left(SQ|\left(0, \overline{Q}\right)\right)}{d\overline{Q}}; \\
&\text{(iii) for } \overline{Q} \in (Q^{ii}, \infty), && \frac{d\Pr\left(SQ|\left(\overline{Q}, 0\right)\right)}{d\overline{Q}} < \frac{d\Pr\left(SQ|\left(0, \overline{Q}\right)\right)}{d\overline{Q}}.
\end{aligned}
$$

Proposition 1 might be understood with its analogy to the economic theory of production. Defender has two inputs to produce "security." While denial has a changing marginal product that once rises and then falls, punishment has a constant marginal product. Because the marginal product of denial is infinitesimal at small input levels (constituting a fixed cost), denial is adopted only when Defender has sufficient resources ($\overline{Q} > Q^i$) that both the marginal and average products of denial surpass those of punishment. Moreover, because the marginal product of denial falls below that of punishment with sufficiently large levels of investment ($\overline{Q} > Q^{ii}$), any additional resources above this threshold are spent for punishment. Therefore, denial is adopted only in the intermediate range of $\overline{Q} \in (Q^i, Q^{ii})$.

[10] Of denial, the marginal effect is defined as $\frac{d\Pr(SQ|(D,P))}{dD}$, and the average effect as $\frac{\Pr(SQ|(D,P))}{D}$. Of punishment, the marginal and average effects are constant and thus coincide.

[11] $\frac{d\Pr(win)}{dD}$ diminishes with a sufficiently large D (Equation (2); Equation (8) and Inequality (10) in Appendix).

3.3 Implications toward Contemporary World Politics

As we apply the results to the contemporary world politics, (i) those with a small $\overline{Q} \in (0, Q^i)$ correspond to parties significantly inferior to their rivals. Because they lack military resources to directly counter their rivals, they avoid confrontation of armed forces on battlefields and instead develop punitive measures to deter their opponents from aggression. An exemplary state might be North Korea, which has focused its very scarce resources on nuclear programs to deter the U.S. military interventions—while leaving its conventional forces more and more obsolete—since it lost the Soviet military supports at the end of the Cold War. Terrorist organizations that challenge interventions by liberal democracies may also fall into this category. (ii) Those with a medium $\overline{Q} \in (Q^i, Q^{ii})$ might be states who maintain competitive positions in their security environments. They do not afford to invest on punishment forces such as of weapons of mass destruction. Instead, they would develop their denial capabilities for deterrence. States with conventional forces such as Germany, Italy, and Japan might constitute this category. (iii) Those with a large $\overline{Q} \in (Q^{ii}, \infty)$ are states predominating over their neighbors. This group might be represented by the Permanent Five, which have deployed sizable amounts of both conventional and nuclear forces.

A caveat is that our model depicts only a dyadic situation and has limitations in its application to multi-lateral world orders. To analyze interplay across three or more parties, it might be essential to incorporate more players, who can choose to be allied aggressors, deterrers, protégés, or bystanders.

Table 2: Deployment by Defender relative military capacity.

Defender capacity	*Inferior*	*Competitive*	*Superior*
Deployment	Only punishment	Only denial	Both denial & punishment
Examples	North Korea & terrorist organizations	Germany, Italy, & Japan	Permanent Five

4 Conclusion

Despite the development of the literature on deterrence over the past decades, there have been very few theoretical studies on the allocation of military resources between denial and punishment to counter a potential aggressor. The scarcity of studies on the problem as to the *destruction* tradeoff between "guns" and "butter" is in sharp contrast to the accumulation of theoretical studies on the *production* tradeoff between them. We have taken the first step toward the formal theorization of the destruction tradeoff associated with the prewar armament for deterrence.

In exploring the interplay between Defender and Aggressor, we found that the deployment of denial and punishment that minimizes the risk of war depends on Defender's military capacity relative to Aggressor's. Namely, if Defender is no match for Aggressor in conventional fights, she should invest all her resources for punishment (e.g., post-Cold War North Korea and terrorist organizations).[12] In contrast, if Defender is more or less in balance with Aggressor, she should focus on denial (e.g., Germany, Italy, and Japan). Only if Defender has considerable resource advantage, she should develop both denial and punishment (e.g., Permanent Five). The risk-minimizing deployment can vary, because denial's marginal effect on deterrence appears inverted U-shaped, while punishment's marginal effect more stable (Figure 2-b). These results are largely consistent with the contemporary global security environments (Table 2).

Our theory suggests that denial and punishment have distinct rationales. According to Snyder (1961), one of the drawbacks of punishment lies in its difficulty in producing credible commitment to retaliation upon deterrence failure. On a very different ground, our theory upholds denial rather than punishment in competitive

[12] Chemical and biological weapons are often referred to as "the poor man's atomic bomb."

security environments, not because punitive strikes on civilians are so devastating (Brodie 1946), but because denial could outperform punishment in affecting Aggressor's decision calculus to wage war by significantly undermining his prospect of successful aggression. On the other hand, if there exists significant imbalance in military capacity between Defender and Aggressor, punishment might be worth preparing, in part because even very limited spending for punishment (by inferior Defender) can generate tremendous psychological impacts, but also because affluent spending for denial (by superior Defender) may suffer its diminishing marginal returns—predominant Defender is very likely to defeat Aggressor regardless of additional denial capabilities. This reasoning is also novel to the existent proposition that the invulnerability of unconventional forces favors punishment (Coldfelter 1989; Pape 1996).

To recap our work's innovation, we took the first step toward the theoretical research on the resource allocation for military strategies. Further fruitful questions might be garnered by delving into other military strategies (such as attrition, fait accompli, guerrilla, and maneuver), by modeling strategies in more detailed manners (such as combination, dynamics, determinants, and effects), or by incorporating relevant factors (such as alliance, bargaining, geography, and intelligence). Military strategy will remain a promising research agenda.

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APPENDIX

Proof of Proposition 1. The proposition seeks the allocation (D^*, P^*) that minimizes $\Pr(F|(D,P))$, or $F(\pi_\alpha)$. Because $F(\pi_\alpha)$ monotonically increases with π_α, $\Pr(F|(D,P))$ is minimized when π_α is minimized. Below we consider the minimization of π_α, which decreases with both D and P:

$$\frac{d\pi_\alpha}{dD} = -(W_\alpha + L_\alpha)\frac{d\Pr(win)}{dD} < 0 \tag{5}$$

$$\frac{d\pi_\alpha}{dP} = -c < 0, \tag{6}$$

for which $\frac{d\Pr(win)}{dD} > 0$ (Equation (2)). In addition, $\Pr(win)$ holds the following properties in its relation to D:

$$\lim_{D\to 0}\frac{d\Pr(win)}{dD} = 0 \tag{7}$$

$$\lim_{D\to \infty}\frac{d\Pr(win)}{dD} = 0 \tag{8}$$

$$\frac{d^2\Pr(win)}{dD^2} = \frac{AD^{A-2}\overline{D}^A}{\left(D^A+\overline{D}^A\right)^3}\left(-(A+1)D^A + (A-1)\overline{D}^A\right)$$

$$> 0 \text{ if } D < \left(\frac{A-1}{A+1}\right)^{1/A}\overline{D} \tag{9}$$

$$< 0 \text{ if } D > \left(\frac{A-1}{A+1}\right)^{1/A}\overline{D}. \tag{10}$$

Those say, $\Pr(win)$ is monotonically increasing with D (Equation (2)), convex for $D < \left(\frac{A-1}{A+1}\right)^{1/A}\overline{D}$ (Inequality (9)) and concave for $D > \left(\frac{A-1}{A+1}\right)^{1/A}\overline{D}$ (Inequality (10)).

Because punishment has a constant effect on π_α (Equation (6)) and no effect on $\Pr(win)$ (Equation (1)), the convexity of $\Pr(win)$ for $D < \left(\frac{A-1}{A+1}\right)^{1/A}\overline{D}$ implies that the solution is at corner (with either $D^* = 0$ or $P^* = 0$) when $\overline{Q}$ is sufficiently small, while the concavity of $\Pr(win)$ for $D > \left(\frac{A-1}{A+1}\right)^{1/A}\overline{D}$ implies that the solution is interior (with both $D^* > 0$ and $P^* > 0$) when $\overline{Q}$ is sufficiently large.

If c is not so large, there exist Q^i and Q^{ii} that satisfy the following properties:

(i) For $Q \in (0, Q^i)$, because the marginal effect of denial is so low with a small D (Equations (5) and (7)), punishment is more effective than denial (Equation (6)); i.e., $\pi_\alpha(Q,0) > \pi_\alpha(0,Q)$, or $\Pr(F|(Q,0)) > \Pr(F|(0,Q))$. (Let $\pi_\alpha(D,P)$ denote

π_α with Defender's allocation (D, P).) Thus, $(D^*, P^*) = \left(0, \overline{Q}\right)$.

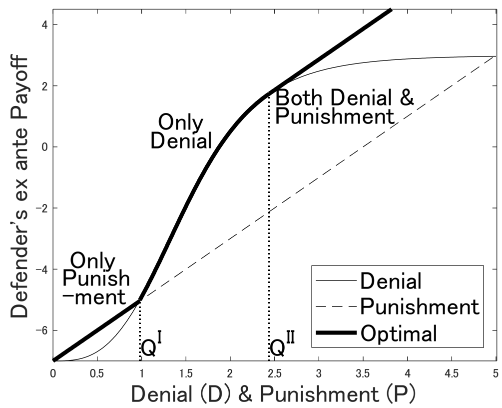
(ii) For $Q \in (Q^i, Q^{ii})$, both the marginal and average effects of denial surpass the effect of punishment, or $\frac{d\pi_\alpha(Q,0)}{dQ} < \frac{d\pi_\alpha(0,Q)}{dQ}$ and $\pi_\alpha(Q, 0) < \pi_\alpha(0, Q)$ (Equations (5), (6) and Inequality (9)). Thus, $(D^*, P^*) = \left(\overline{Q}, 0\right)$.

(iii) For $Q \in (Q^{ii}, \infty)$, because the marginal effect of denial decreases and converges to be zero with a sufficiently large Q (Equations (5), (8) and Inequality (10)), $\frac{d\pi_\alpha(Q,0)}{dQ} > \frac{d\pi_\alpha(0,Q)}{dQ}$. Thus additional resources after $Q = Q^{ii}$ are spent for punishment: $(D^*, P^*) = \left(Q^{ii}, \overline{Q} - Q^{ii}\right)$.

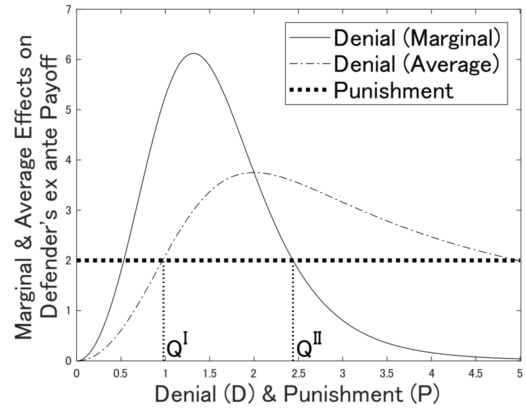
Moreover, because of the convex-then-concave shape of $\Pr(win)$ with respect to D (Equations (2, 7, 8); Inequalities (9, 10)), Q^i is the smaller of the two values of Q such that $\Pr(F|\,(Q, 0)) = \Pr(F|\,(0, Q))$, while Q^{ii} the larger of the two values of Q such that $\frac{d\Pr(F|(Q,0))}{dQ} = \frac{d\Pr(F|(0,Q)|)}{dQ}$. Thus Identities (3, 4) hold.

If c is sufficiently large, punishment is more effective than denial; i.e., $\pi_\alpha(Q, 0) > \pi_\alpha(0, Q)$ regardless of Q. Thus, there exists no Q^i, and $(D^*, P^*) = \left(0, \overline{Q}\right)$. This occurs with a sufficiently large c, because both $\Pr(win)$ is bounded from above (Equation (1)).

If c takes a special value, there exists only one value of Q such that $\Pr(F|\,(Q, 0)) = \Pr(F|\,(0, Q))$. (In this case, $Q^i = Q^{ii}$.) Even in this special case, the proposition still holds: (i) for $\overline{Q} \in (0, Q^i)$, $(D^*, P^*) = \left(0, \overline{Q}\right)$; (ii) for $Q = Q^i$, $(D^*, P^*) \in \left\{\left(\overline{Q}, 0\right), \left(0, \overline{Q}\right)\right\}$ (i.e., D and P have the same effect on $\Pr(F)$); (iii) for $\overline{Q} \in (Q^{ii}, \infty)$, $(D^*, P^*) = \left(Q^{ii}, \overline{Q} - Q^{ii}\right)$. ■



(a)



(b)

Figure 3: optimal deployment in light of *ex post* payoffs upon deterrence failure.

Extension of the Model. Below we extend the model by incorporating *ex post* payoffs that accrue from war outcomes upon deterrence failure. With this extension, Defender takes into account the prospects of winning and losing the war upon its outbreak so as to maximize her *ex ante* payoff:

$$\max_{(D,P)} \Pr\left(F|\left(D,P\right)\right)\left(\Pr\left(win\right)W_{\beta}-\Pr\left(loss\right)L_{\beta}-c\overline{P}\right)+\Pr\left(SQ|\left(D,P\right)\right)S_{\beta},$$

where W_{β}, L_{β}, and S_{β} are her *ex post* payoffs from the victory, defeat, and status quo, respectively.

Even with this extension, the results are similar. Figure 3 shows the optimal deployment with the same parameter values and function as in the baseline model ($W_i = 5$, $L_i = 5$, for $i \in \{\alpha, \beta\}$; $c = 2$; $A = 3$; $\overline{D} = 2.\overline{P} = 1$; $F\left(S_{\alpha}\right) = \frac{S_{\alpha}+L_{\alpha}}{W_{\alpha}+L_{\alpha}}$; $S_{\beta} = 3$). The optimal deployment still depends on the military capacity $\overline{Q}$ but with different thresholds ($Q^{I} = 0.98$; $Q^{II} = 2.44$): only punishment for $\overline{Q} \in \left(0, Q^{I}\right)$; only denial for $\overline{Q} \in \left(Q^{I}, Q^{II}\right)$; both denial and punishment for $\overline{Q} \in \left(Q^{II}, \infty\right)$.