Herd Behavior, Bank Runs and Information Disclosure

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Abstract

I develop a dynamic model of bank runs that allows me to study important phenomena such as the role of information externalities and herd behavior of depositors as a source of bank runs. I show that eliminating bank runs completely, even they can be generated by herd behavior of depositors, has costs. Furthermore, a deposit contract that allows for runs can achieve higher levels of depositor welfare than a contract that completely eliminates them. Since early liquidation of bank's assets is costly, a central bank that acts as a lender of last resort alleviates some of the costs associated with bank runs. Yet it cannot prevent runs on healthy banks in the absence of perfect information about the bank's asset quality. In those cases, a deposit contract, even with liquidity support from the central bank, cannot achieve the first-best efficient outcome. As a policy measure, any efforts to give market discipline a stronger role in achieving financial stability should be accompanied by transparency and disclosure of information on banks' soundness and management of the crisis.

1 Introduction

Throughout their history, banks have been subject to runs. It is a well-known fact that banks have illiquid assets and liquid liabilities. This matu-

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rity mismatch makes banks vulnerable to runs if depositors become worried about the bank's asset quality and demand the return of their savings.

Although financial crises including bank runs are no longer a matter of concern in some parts of the world, the reality is that banking crises are still a constant feature of economies for a majority of the countries. Lindgren, Garcia and Saal (1996) show that during the period 1980-96, of the 181 IMF member countries, 133 experienced significant banking problems. Such problems affected developed, as well as developing and transitional countries[1] and had significant costs as documented by many studies[2].

Since the emergence of banks, supervision and regulation of the industry evolved as government and private responses to banking panics. The studies on the issue date back to Thornton (1802) that works on optimal central bank policies for lending in times of distress. Over the two centuries since Thornton (1802), many policies have been developed to prevent and manage banking panics: suspension of convertibility, lender of last resort practices, deposit insurance, risk-based capital requirements, to cite a few. In light of the recent experiences, it is clear that the debate on these issues has not come to an end yet.

In his classic, *Lombard Street*, Bagehot (1873) argues that during times of panic, transparency on management of the crisis may reduce uncertainty and may have a calming effect on financial markets. In Bagehot's own words:

"If people could be really convinced that they could have money if they wait a day or two, and that utter ruin is not coming, most likely they would cease to run in such a mad way for money. Either shut the Bank (of England) at once and say it will not lend more than it commonly lends, or lend freely, boldly, and so that the public may feel you mean to go on lending. To lend a great deal, and yet not give the public confidence that you will lend sufficiently and effectually, is the worst of all policies."

In this paper, I concentrate on the role of information externalities and herd behavior of depositors as a source of bank runs and, consistent with Bagehot's advice, suggest that transparency and disclosure of information on banks' soundness and management of the crisis can alleviate and eliminate

[1] For example, over the past 25 years banking crises have hit such developed countries as Finland (1991-93), Norway (1988-92), Japan (1992-present), Spain (1977-85), and Sweden (1991) and have recently troubled such developing countries as Russia (1998), Brazil (1999), Turkey (2000) and (2001) and Argentina (2001), and various counties in Asia (1997-98).

[2] See Lindgren, Garcia and Saal (1996), Caprio and Klingebiel (1996), Lindgren et.al. (1999), Alexander et.al. (1997), Sundararajan and Baliño (1991), Hoggarth, Reis and Saporta (2001).

some of the problems related to bank runs.

In doing so, I search for answers to some important questions, that are closely related with the stability of the banking system, such as:

- Can information externalities and herd behavior of depositors trigger bank runs?
- What are the costs of preventing these types of bank runs?
- Does optimal deposit contract allow for bank runs even when they can be generated by herd behavior of depositors and can be on healthy banks?
- In the presence of noisy private information, can a deposit contract, even with liquidity support[3] from the central bank, achieve the first-best efficient outcome with public information?

When we look at the literature, we see that the bank run models have some common features. Most of them are static two-period equilibrium based models with a continuum of agents played in the simultaneous form. But Brunnermeier (2001) makes an important observation on the bank runs literature: "Although withdrawals by deposit holders occur sequentially in reality, the literature typically models bank runs as a simultaneous move game[4]". With static models, we cannot analyze the dynamics of bank runs and we cannot answer some of the questions I address. For these reasons, I diverge from these common features of bank run models.

In this paper, I build a model similar to the models of Diamond and Dybvig (1983) and Allen and Gale (1998) (DD and AG, respectively, from now on) but this is a dynamic model of bank runs where depositors move sequentially rather than simultaneously. This allows me to investigate the role of information externalities and herd behavior of depositors as a source of banking panics. The important feature of the model is that in an interim

[3]In this paper, liquidity support will be only against good collateral. Therefore, it is not a bailout of the bank. See Section 6 for a detailed discussion.

[4]A few exceptions where depositors move sequentially would be Chen (1999), Green and Lin (2003) and Schotter and Yorulmazer (2003). Chen (1999) extends the Diamond and Dybvig (1983) model to multiple banks and allows for interim revelation of information about some banks' performance. A sufficient number of bank failures results in pessimistic expectations about the general state of the economy and leads to runs on the remaining banks. This is a model with multiple banks that explains possible channels for contagion. In my model, there is only one bank. Therefore, my set up is not suitable to study contagion.

stage, depositors receive noisy private signals about the bank's asset quality and observe the actions of other depositors who move before them[5]. Since these signals are noisy, information revealed by the actions of depositors have value for other depositors. Depositors update their beliefs on the basis of these actions and their own private signals. A few number of withdrawals can lead depositors to withdraw even when they have a favorable signal about the prospects of the bank's investments. Hence, herd behavior of depositors can trigger a run on a healthy bank.

Early liquidation of bank's assets is costly, so the bank may choose a deposit contract that completely eliminates runs. To be able to do so, the bank has to promise a small return for those who want to withdraw early so that even a depositor who has observed a bad signal about the bank's prospects chooses not to withdraw. This will undermine the insurance provided by the deposit contract against a liquidity shock and is an important trade-off the bank needs to consider in choosing the deposit contract. I show that for a significant set of parameter values, it is optimal to allow for runs even if they can be generated by herd behavior of depositors and can be on healthy banks.

In the first-best efficient outcome with public information, when the bank cannot pay everybody the promised amount, the optimal arrangement is to distribute all the available funds equally among all depositors. But when bank runs are eliminated, early depositors will get their promised amount and if the return from the bank's investments is low, late depositors will get very little. This is another cost of eliminating runs completely the bank takes into account in choosing the optimal deposit contract (see AG for a detailed analysis).

In the bank runs literature, there are two main views about what triggers bank runs. One view states that bank runs are self-fulling phenomenon generated by "sunspots" as in the pioneering work of DD. They show that a deposit contract can give the optimal allocation by providing depositors insurance against uncertainty about their liquidity needs. If nobody thinks that there will be a run on the bank, only depositors with urgent liquidity needs withdraw early and there is no bank run. But, if depositors think that other depositors will withdraw their money, it is optimal for them to do so since there will not be anything left for them if they do not withdraw. This is the Pareto inferior bank run outcome. Sunspots determine which of these self-fulfilling outcomes will occur.

[5] To capture this, I use a very similar structure to the discrete signals model of Bikhchandani, Hirshleifer and Welch (1992).

Another view is that bank runs are natural consequences of the business cycle and they are information based. Gorton (1988) and Calomiris and Gorton (1991) empirically show that the banking crises in US in the late 19th and early 20th century are actual consequences of the business cycle and they are information driven.

In this paper there are both payoff and information externalities[6] allowing me to combine some features of these two main views of bank runs in one model[7]. In my model, bank runs are triggered by the withdrawal decisions of a few number of depositors, revealing that they have adverse news about the bank's performance. This makes bank runs information driven. Yet, because of payoff and information externalities, once a run is triggered, it is optimal for everyone else to join the run even if they had good news about the bank's asset quality initially. Since depositors have noisy private signals, runs can be on healthy banks. Saunders and Wilson (1996) provides historical evidence of runs on healthy banks, as I will later discuss in detail.

Here, I concentrate on some particular equilibria of the model which I find to be the most interesting[8]. There may be other equilibria, where "everybody runs" being a likely candidate. Since this is a dynamic game with information externalities, the public belief about the prospects of the bank evolves during the play of the game. When the public belief falls below a threshold, it is a dominant strategy to withdraw which makes a bank run the unique outcome from then on. When the public belief is above that threshold, "everybody runs" equilibrium can be eliminated by a constraint added to the deposit contract as long as the utility from receiving nothing from the bank is not extremely low[9].

There are models with unique equilibrium to the bank run game in the literature but their features differ from my model. Some of these share the limitations of the models mentioned before. Therefore they are not appropriate models for the purposes of this paper. Following is a discussion

[6] In many situations we observe both of these externalities in action but we have very few models studying this. Some would be Neeman and Orosel (1999), Dasgupta (2000) and Yorulmazer (2003).

[7] Though I have payoff externalities, the real reason that triggers a run is the information externalities. When the public belief about bank's prospects fall below a threshold, a run is triggered. This is a situation similar to the one sketched in Allen and Morris (2001). But with the help of payoff externalities, I show that once a run is triggered, everybody will join (see footnote 29).

[8] Postlewaite and Vives (1987) and Chari and Jagannathan (1988) build models of incomplete information with a unique Bayesian equilibrium that has a positive probability of bank runs.

[9] See footnote 29 for a similar discussion.

of these studies and my model's differences from them.

Green and Lin (2001), using a finite-player version of DD, develop a direct mechanism that implements the ex-ante efficient allocation as a unique outcome even when the bank uses the sequential-service constraint. In their model, aggregate uncertainty stems from the proportion of depositors that are subject to the liquidity shock. The return from the risky asset is constant. In my model, the return from the risky asset is random and this is the source of aggregate uncertainty. Their mechanism, under the sequential service constraint, pays different amounts to depositors who want to withdraw early, depending on the position they arrive the bank. This type of arrangement is not feasible in my contract space: depositors are treated equally in my model.

Goldstein and Pauzner (2002) use the equilibrium selection method[10] introduced by Carlsson and van Damme (1993) and find a unique equilibrium in which fundamentals determine whether a bank run will occur or not. But they have a continuum of agents and the bank run game is played in the simultaneous form. With their set up, it may not be possible to answer some of the questions my paper is interested in such as the role of information externalities and herd behavior of depositors as a source of bank runs. Also, Saunders and Wilson (1996) and Park (1991) empirically document runs on healthy banks which is in contrast with the predictions of their model.

Some studies show that deposit contracts that are subject to bank runs are optimal arrangements and provide efficient outcomes. But in those models, information about banks' asset quality is perfect, and is public observed (AG) or revealed through the actions of *insiders,* who are perfectly informed about the performance of their bank (Calomiris and Kahn (1991)).

Calomiris and Kahn (1991) show that the deposit contract is the optimal arrangement even though it might create opportunity for bank runs and costly liquidation. The rationale behind this argument is that depositors can monitor their banks, which can act against their interest, by liquidating their funds. The event that triggers a run is the withdrawal decisions of the insiders.

Allen and Gale (1998) builds a model consistent with the empirical evidence of Gorton (1988) and Calomiris and Gorton (1991) that banking panics are related with the business cycle and are not the result of sunspots. They show that bank runs allow efficient risk sharing between early and late

[10] Morris and Shin (2001) is an excellent article on the theory and the applications of global games including applications on bank runs. Morris and Shin (1998) uses the same technique to find unique equilibrium in a model of currency attacks.

withdrawing depositors and allow banks to hold efficient portfolios. When runs carry costs due to early liquidation of assets, the first-best efficient outcome can be achieved by liquidity support from the central bank. In their model, the asset quality of the bank is realized and publicly observed in an interim period. This is crucial for the deposit contract, with liquidity support of the central bank, to achieve the first-best efficient outcome.

In contrast with the models discussed above, some studies provide empirical evidence for runs on healthy banks. For example, Saunders and Wilson (1996) examine deposit flows in 163 failed and 229 surviving banks over the Depression era of 1929-1933 in US. In the years 1929 and 1933, they find evidence of "flight to quality" where withdrawals from failed banks were associated with deposit increases in surviving banks. However, they observe a decrease in deposits in both failed and surviving banks for the period of 1930-32. During this time, healthy banks experienced runs, too.

In another study, Park (1991)[11] looks at banking panics in US history and empirically shows that the government or banks stopped panics mainly by providing information on banks' solvency rather than liquidity[12]. He compares different arrangements used to manage and prevent banking panics according to their information provision and concludes that the arrangements that provided information on banks' solvency were successful while those that provided only liquidity failed. He examines three different arrangements: equalization of reserves, clearing house loan arrangements and suspension of payments.

Clearing House loan certificates were acquired by banks by depositing qualifying assets with the Clearing House Association and they were used in interbank settlements. They prevent costly liquidation of assets and improve bank's liquidity position. Since they were provided only when the Clearing House Association decided that the bank has enough assets to back them up, they also served the purpose of providing information that the bank was healthy.

Equalization of reserves was basically pooling all legal reserves of Clearing House Association member banks in an emergency and granting member banks equal access to the pooled resources. While equalization of reserves enhanced liquidity in the bank under attack, it did not provide any information on bank's asset quality. Because of this lack of information, equalization

[11] Gorton (1985), Gorton and Mullineaux (1987) and Gorton and Huang (2002) document similar experiences. Sprague (1910) is an excellent source for banking crises in US under the national banking system.

[12] For an excellent survey on the role of "lender of last resort" in crisis management, see Freixas, Giannini, Hoggarth and Soussa (1999).

of reserves was ineffective in preventing runs when used in the 1873 crisis. Instead, stability of the banking system was established by using Clearing House loan certificates.

Suspension of payments was another arrangement used to manage banking panics. Their major role was to provide information rather than liquidity. Suspensions were always followed by thorough examination of the banks' assets. While banks with solvency problems were liquidated, solvent banks were reopened. This selective reopening procedure assured the public that the reopened banks were healthy. As a result of this, runs on reopened banks did not recur in most cases.

The results of this paper are in parallel with the empirical evidence discussed. In this model, while liquidity support alleviates the costs of early liquidation, it cannot prevent runs on healthy banks. The policy measure that will prevent these type of runs is the disclosure of information on the bank's soundness and management of the crisis. A deposit contract can achieve the first-best efficient outcome only in the presence of perfect information about the banks' performance. In that sense, my results are complementary to the findings of the studies that suggest that a deposit contract, with liquidity support from the central bank, can achieve the first-best efficient outcome. In this paper, I argue that, while liquidity support is necessary, it is not sufficient in achieving the first-best efficient outcome. It has to be accompanied by transparency and disclosure of information on banks' soundness and management of crises.

This paper makes important suggestions on the policy side. In my model, deposit insurance can prevent runs and since early liquidation is costly, this can be welfare improving. But it is a well-known fact that in the presence of full insurance, depositors do not have any incentive to differentiate between sound and unsound banks. In such an environment, weak banks do not have any difficulty in attracting deposits. This creates an opportunity for moral hazard on banks' side that can cause adverse effects on the stability of the banking system.

Financial regulators and policy makers have been looking for alternatives for full deposit insurance (see Bhattacharya, Boot and Thakor (1998) for a comprehensive survey). Giving a stronger role on market discipline may lessen the moral hazard problem. But for market discipline to work effectively, depositors should have accurate information about the soundness of banks. This paper shows that in the absence of accurate information, information externalities and herd behavior of depositors can lead to runs on healthy banks. This undermines the role of market discipline in keeping an eye on bank activities. Therefore disclosure of information on banks' structure, performance and risk positions should be enforced[13]. This is "pillar three" of Basel II regime and this should be fully pursued to increase the effectiveness of market discipline, which can be an alternative for deposit insurance in achieving financial stability.

We can summarize the two main results of this paper as:

- In the presence of noisy private signals, for a significant set of parameter values, the optimal deposit contract allows for bank runs even if they can be generated by herd behavior of depositors and they can be on healthy banks.
- In the absence of perfect information on bank's performance, a deposit contract, with liquidity support from the central bank, cannot achieve the first-best efficient outcome.

The rest of the paper is organized as follows. The model is described in Section 2. In Section 3, I present the special case where the bank's return is publicly observed that serves as a benchmark model. Section 4 describes the information structure of the model with private noisy signals. I investigate different deposit contracts the bank can choose in Section 5. Section 6 analyzes the effect of liquidity support from the central bank and investigates whether it can prevent runs on healthy banks. Section 7 elaborates on policy implications of the model, in particular on transparency and disclosure of information on bank's performance and management of the crisis. Concluding remarks are in Section 8.

2 Model

The model is similar to the models of DD and AG. The main difference is that we have an infinite number of depositors rather than a continuum. This way, I can develop a model of bank runs, where depositors move sequentially, and therefore, incorporate information externalities into the model. As mentioned in Brunnermeier (2001), this is a more realistic representation of bank

[13] Rochet and Vives (2002) finds different results on the optimal disclosure policy of the Central Bank. They use the "global games" analysis of Carlsson and van Damme (1983) and find a unique equilibrium. In this unique equilibrium, there is an intermediate range of fundamentals where there is a potential for a coordination failure. Central Bank, by providing information that becomes common knowledge, can move the interbank market into a regime of multiple equilibria when the fundamentals are in that intermediate range. By adding noise to its announcements, Central Bank prevents switching to the regime of multiple equilibria. But they have a simultaneous move game while my model is a sequential move model of bank runs.

runs in the real world and will help me investigate the role of herding and information externalities as a source of bank runs.

There are three periods, $t = 0, 1, 2$ and a single consumption good available at each date.

There is a bank that offers depositors a deposit contract and makes investments on their behalf. I assume that the banking sector is competitive so that the bank chooses the deposit contract that maximizes the welfare of depositors.

There are two assets : a *safe* and a *risky asset*. The safe asset is a storage technology that pays one unit at date $t+1$ for each unit invested at date t. Both the depositors and the bank have access to the safe asset.

The risky asset that pays a random return of $\widetilde{R}$ at $t = 2$ for each unit invested at $t = 0$, where

$$\widetilde{R} = \begin{cases} R & \text{with probability} \quad 1/2 \\ r & \text{with probability} \quad 1/2 \end{cases} .$$

We have $r < 1 < R$ so that the risky asset does not dominate the safe asset. Only banks have access to the risky asset. Risky asset can be liquidated at $t = 1$ but only with some discount. When liquidated early, only a fraction τ of the return can be collected. Therefore the return at $t = 1$ is $\tau\widetilde{R}$ where $\tau \in (0, 1)$.

There is an infinite number of depositors. A depositor can be an *early* or a *late consumer*. Early consumers value consumption at $t = 1$ only, while late consumers value consumption at $t = 2$. I assume that the types are public information when realized but the bank cannot write type specific contracts[14]. Ex-ante, depositors do not know their type but they know the probability distribution over types. Probability of being an early consumer is λ and since there is an infinite number of depositors, λ is also the proportion of early consumers. As a result, there is no aggregate uncertainty resulting from the proportion of depositors that are subject to the liquidity shock.

Therefore consumers have the following preferences :

$$U(c_1, c_2) = \begin{cases} u(c_1) & \text{with probability} \quad \lambda \\ u(c_2) & \text{with probability} \quad 1 - \lambda \end{cases}$$

where c_t denotes consumption at date $t = 1, 2$. u is strictly increasing ($u' > 0$) and strictly concave ($u'' < 0$). Further, it satisfies the Inada conditions:

[14] See Section 3.2 for a discussion.

$u'(0) = -\infty$ and $u'(\infty) = 0$. Depositors have an initial endowment of 1 unit of the consumption good at $t = 0$ and nothing at $t = 1$ and $t = 2$.

3 Publicly Observed Return

In this section, I assume that the realization of $\widetilde{R}$ is publicly observed at $t = 1$. Since $\widetilde{R}$ can take only two values, the following is just a simplified version of AG. For this special case, I will find the first-best efficient outcome and show that a deposit contract with liquidity support from the central bank can achieve the first-best efficient outcome.

3.1 First-Best Efficient Outcome

For the moment, I assume that the bank can write contracts conditional on $\widetilde{R}$. Therefore, the resulting allocation will be the first-best allocation (for a detailed discussion see AG).

I assume that the risky asset cannot be liquidated at $t = 1$, for now.

The bank will choose a portfolio of the safe (L) and the risky asset (X) out of the initial endowment 1 at $t = 0$ and will also determine the consumption levels $c_t(\widetilde{R})$ conditional on the realization of $\widetilde{R}$ that maximize the ex-ante expected utility of the representative depositor. So the bank's problem can be written as:

$$(P1) \begin{cases} max & & E[\lambda u(c_1(\widetilde{R}) + (1-\lambda)u(c_2(\widetilde{R}))] \\ s.t. & (i) & L + X \leqslant 1 \\ & (ii) & \lambda c_1(\widetilde{R}) \leqslant L \\ & (iii) & \lambda c_1(\widetilde{R}) + (1-\lambda)c_2(\widetilde{R}) \leqslant L + \widetilde{R}X \\ & (iv) & c_1(\widetilde{R}) \leqslant c_2(\widetilde{R}) \end{cases}$$

The first constraint is the feasibility constraint for the portfolio chosen at $t = 0$. Since the risky asset cannot be liquidated, the second constraint insures that the bank has enough safe asset to pay early consumers. The third constraint insures that the consumption levels provided by the deposit contract can be achieved. The last constraint is an incentive compatibility constraint to separate early and late consumers.

Since $u(.)$ is strictly concave, the FOC uniquely characterizes the optimum. Depositors are risk averse so the best arrangement when everyone

state\outcome	c_1	c_2
R	$\frac{L}{\lambda}$	$\frac{XR}{1-\lambda}$
r	$L+rX$	$L+rX$

Table 1: Payoffs for early and late depositors in different states.

cannot be paid $\frac{L}{\lambda}$ is to distribute all funds equally among all depositors. This gives us the payoffs explained in Table 1 where the optimal level of L and X are given by the first-order conditions[15].

3.2 Optimal Deposit Contract

Now I will look at whether a bank that uses a deposit contract can achieve the first-best efficient outcome.

Though the bank observes the types of depositors when realized, I assume that it cannot write type specific contracts or suspend payments according to types. In my model, suspension of payments amounts to default on the deposit contract[16]. The bank has to use the sequential service constraint except for some cases, as I will explain now[17].

I assume that the bank uses the following payment rule: In making its payments, bank uses the first-come first-served rule. When there is a run, the bank abandons the first-come first-served rule. Rather, it pays everybody the promised if it can and if it cannot, it divides whatever it has equally among those who demand liquidation.

When the risky asset cannot be liquidated early, AG shows that the expected utility from the first-best efficient outcome can be achieved by a standard deposit contract that promises a fixed payment. Bank pays its promises using the liquid asset and if it does not have enough liquid asset to make the promised payment, it divides all the liquid asset equally among those who demand early liquidation.

Let $\overline{c}_1$ denote the fixed payment promised to the early consumers. First we simplify the bank's problem using the bank's payment rule, to get:

[15] See Allen and Gale (1998) for a detailed discussion.

[16] In a larger contract space, there can be other arrangements that achieve higher levels of depositor welfare. These types of arrangements are not feasible in my contract space.

[17] Though suspensions of payments occurred on some occasions in the nineteenth century in US, Commercial Bank Clearing House was responsible for deciding whether and when the suspension was appropriate. Marine National Bank was punished for suspending payments on its own in May 1884.

$$(P2)\begin{cases} Max & \frac{1}{2}\, u(c_1(r)) + \frac{1}{2}\left[\lambda\, u\left(\overline{c}_1\right) + (1-\lambda)\, u\left(c_2(R)\right)\right] \\ s.t. \quad (i) & L + X \leqslant 1 \end{cases}$$

If the bank chooses $\overline{c}_1 = L/\lambda$, the deposit contract can achieve the first-best efficient outcome given in Table 1.

Now we relax the assumption that the risky asset can be liquidated at $t = 1$. When liquidated at $t = 1$, the return is $\tau\widetilde{R}$ where $\tau \in (0, 1)$.

The bank has to liquidate the risky asset if it cannot pay $\overline{c}_1$ to depositors who demand early liquidation.

Under the possibility of costly liquidation, the deposit contract cannot achieve the first-best outcome from Table 1, since the bank has to go under costly liquidation when $\widetilde{R} = r$. But when the central bank provides liquidity, the bank does not have to go under costly liquidation. In that case, the deposit contract of $(P2)$ is feasible and can provide the expected utility from the first best allocation.

The following is a numerical example of the above discussion.

Example 1 *Assume that* $u(x) = ln(x)$ *and* $\lambda = 1/2$. *The optimal deposit contract can provide the first-best efficient expected utility. For low fundamentals* $(R = 4, r = 0.5)$, $E(U) = 0.2269$, *the optimal portfolio is* $(L, X) = (0.6404, 0.3596)$ *and* $\overline{c}_1 = 1.2808$. *For high fundamentals* $(R = 6, r = 0.75)$, $E(U) = 0.3861$, *the optimal portfolio is* $(L, X) = (0.5687, 0.4313)$ *and* $\overline{c}_1 = 1.1375$.

We saw that a deposit contract, with liquidity support from the central bank, can achieve the first-best efficient outcome when the return from the bank's investment is publicly observed. When the return is publicly observed, runs are correct runs. In other words, they occur only when they are needed to occur. But as Park (1991) empirically documents, in the absence of perfect information about the bank's asset quality, healthy banks have also experienced runs. In those cases, liquidity support did not prevent runs. The policies that prevented runs on sound banks revealed the asset quality of banks. In the following sections, consistent with the empirical evidence, I will show that liquidity support from the central bank cannot prevent runs on healthy banks in the absence of perfect information about the bank's assets. Under those conditions deposit contracts cannot achieve the first-best efficient outcome with public information.

4 Noisy Private Signals

In this section, I will explain the information structure of the model with noisy private signals. The structure is similar to the discrete signals social learning model of Bikhchandani, Hirshleifer and Welch (1992) (BHW from now on). Depositors will receive noisy signals about the quality of the bank's portfolio, namely about the return of the risky asset. These signals are informative but not perfect. Therefore the signal of each depositor have value for other depositors.

Initially, there is an exogenous order for depositors to make their decisions. At $t = 1$, depositors learn their type and if they happen to be a late depositor, their private signal about the bank's asset quality[18]. Basically, depositors receive a vector of signals with two dimensions: (Type, Signal). These vectors can be: $(e, \emptyset)$, (l, s_g), (l, s_b) where e stands for an early consumer, l stands for a late consumer, s_g stands for a *good signal* which predicts the high return R, while s_b stands for a *bad* signal which predicts the low return r.

As I assumed, the bank and the depositors can observe types[19]. The consequences of relaxing this assumption are discussed in the Appendix[20].

As explained, late depositors privately receive a noisy signal s about the likelihood of the high return. The signals of different depositors are assumed to be independent, conditional on the true return of the risky asset. Conditional probabilities for the signals, given the true state, are :

$$q = \Pr(s_g|\widetilde{R} = R) = \Pr(s_b|\widetilde{R} = r)$$

$$1 - q = \Pr(s_g|\widetilde{R} = r) = \Pr(s_b|\widetilde{R} = R).$$

Note that $q \in (1/2, 1)$ so that the signals are informative but not perfect. And as q increases, signals become more informative. Probabilities of signals in different states are summarized in Table 2.

[18] Early depositors can receive a signal about the bank's prospects, too. It would not effect my results.

[19] This assumption makes the analysis simple without sacrificing from the content of the model. Chari and Jagannathan (1988) builds a model where depositors are faced with a signal extraction problem. They see a queue in front of the bank but they cannot identify the exact reason, which can be a high realization of the liquidity shock or some bad news about the bank's performance.

[20] This will contribute noise to the model. Now a withdrawal decision will be noisy information about a bad signal. If a depositor decides to withdraw at $t = 1$ it can be for two reasons:

i) She may be an early consumer.

ii) She may be a late consumer and observed a bad signal.

state\signal	Good	Bad
$\widetilde{R} = R$	q	$1-q$
$\widetilde{R} = r$	$1-q$	q

Table 2: Probabilities of signals conditional on state.

4.1 Updating

Now I will show how depositors update their beliefs about the good state. I assume that they use Bayes' rule in updating their beliefs. Let $p_g(p)$ and $p_b(p)$ be the posterior probability of the good state attached by a depositor who observe the good and the bad signal respectively, starting with a prior of p. To save notation, I will use p_g and p_b instead of $p_g(1/2)$ and $p_b(1/2)$, respectively. Assuming that agents update their beliefs using Bayes' rule, we get:

$$p_g = \frac{(1/2)q}{(1/2)q + (1/2)(1-q)} = q.$$

In the same way, we get:

$$p_b = 1 - q.$$

I assume that depositors value signals equally. As a direct consequence of this, good news revealed by a good signal will be washed out by bad news from a bad signal. This result is formally stated below:

Claim 2 $\forall q \in \left(\frac{1}{2}, 1\right)$, $p_g(p_b(1/2)) = p_b(p_g(1/2)) = \frac{1}{2}$.

Proof. I will show that $p_g(p_b(1/2)) = 1/2$. The other case is similar. To find $p_g(p_b)$, I will substitute $p_b = (1-q)$ for p in $p_g(.)$ and get:

$$p_g(p_b) = \frac{(1-q)q}{(1-q)q + q(1-q)} = \frac{1}{2}.$$

■

Now we examine how depositors update their beliefs when they observe a history of signals.

I assume that the actions of depositors are observable. For the moment, suppose that the depositors can perfectly infer the signals of the other depositors from their actions. They will update their beliefs about the good state on the basis of their own private signal and the publicly observed actions of depositors who moved before them.

As I showed above, good news revealed by a good signal will be washed out by the bad news from a bad signal. For this reason, for depositor i, in updating her belief, the only thing that matters is the difference between the number of good signals (n_g^i) and the number of bad signals (n_b^i) revealed by the actions of depositors who moved before her, $(a^1, ..., a^{i-1})$, and her private signal (s^i). Therefore, all the information in $(a^1, ..., a^{i-1}, s^i)$ can be summarized as the difference of the number of good and bad signals. Formally speaking, $n^i = n_g^i - n_b^i$ is a sufficient statistic.

Note that the history is carried through the public belief about the good state. I will call the public belief of the high return, when it is player i's turn to move, as p^i, which is going to be the state variable.

I would like to explain the order in which depositors move, in more detail. This part of the analysis has differences from BHW and there will be an endogenous component in depositor's timing[21].

Initially, there is an exogenous order for players' decisions. Yet, while a withdrawal decision is irreversible, a non-withdrawal decision is not. A depositor who chose not to withdraw when it was her turn to move, is still in the game and can decide to withdraw later when it is her turn to move again[22]. In that sense the decisions are about the timing of withdrawals, which adds an endogenous component to the model. Figure 3 summarizes the sequence of events in this model.

5 Contracts with Noisy Private Information

In this section, I will investigate different deposit contracts the bank can choose. These contracts will have different probabilities of bank runs.

The first contract I will analyze, completely eliminates runs. I will call this the *run-proof contract*. Since runs carry early liquidation costs, this can be a desirable property of a deposit contract. But to be able to prevent runs, the bank will have to promise a small amount to early withdrawing depositors so that even a late depositor who observed the bad signal will get a higher expected payoff by not withdrawing at $t = 1$. Therefore the run-proof contract sacrifices from the insurance it provides against liquidity shocks.

[21] We do not have many social learning models where players' timing is endogenous. An excellent exception is Gale and Chamley (1994).

[22] For simplicity I assume that she goes to the back of the line.

Also, when we look at the first-best efficient outcome with public information, we see that when the bank cannot pay everybody the promised amount, the best arrangement is to distribute all the available funds equally among all depositors. But when bank runs are eliminated, early depositors will get their promised amount and if the return from the bank's investments is low, late depositors will get very little. This is another cost of eliminating runs completely. For these reasons, it is clear that the run-proof contract cannot achieve the first-best efficient outcome. Furthermore, it may be optimal to allow for bank runs.

Another contract, the bank can choose, is one that allows for runs. I will call this the *run contract*. This contract can provide higher levels of insurance against liquidity shocks since it is not bounded by the restrictions of the run-proof contract. But, because of the information structure of the model, withdrawal decisions can reveal information about the private signals of depositors. These signals possess valuable information for other depositors. As I will show, these information externalities can trigger a run on a sound bank.

When we compare the run contract with the first-best efficient outcome, we see that another shortcoming of this contract is that late depositors may choose to wait when they should run, as a result of information externalities. Though the run contract can perform better than the run-proof contract, for the explained reasons, it cannot achieve the first-best efficient outcome.

In this section I will formalize the deposit contracts discussed above.

5.1 Run-Proof Deposit Contract

In this section, I will analyze the deposit contract that completely eliminates runs. For this contract to work, the bank should promise a small enough return to the early withdrawing depositors, so that even a late consumer with the bad signal will prefer to wait until the last period.

When this deposit contract is used, no late depositor will withdraw early and the actions of late depositors will be independent of their private signals. No new information will be revealed during the course of the game. This creates an informational cascade as defined below.

Definition 3 *In an informational cascade, depositors choose the same action, regardless of their private signals*[23].

[23] In this discrete signals setup, herds are cascades are equivalent. For a distinction between the two, see Smith and Sorensen (2000).

Since no new information is revealed, the public belief p^i will always be equal to the prior belief of 1/2, for all depositors. All late depositors will be in the same situation as the first depositor and they will decide to wait until the last period. We will be in an informational cascade where no late depositor withdraws early. I will call this a *no-run cascade.*

There is another informational cascade in which late depositors choose to withdraw early regardless of their private signals. I will call this a *run-cascade.*

The run-proof contract can increase the welfare of depositors since runs carry early liquidation costs. But the liquidity insurance provided by these deposit contracts can be very low since the bank has to promise a small amount to early withdrawing depositors. And also, when the return from the risky asset is low, the optimal arrangement is to distribute the whole portfolio equally among all depositors, but the run-proof contract cannot achieve this.

Late depositors do not run with this type of contract and early depositors get the promised amount $\overline{c}_1$ in both states through the liquid asset. Since λ of the depositors will turn out to be early depositors, bank will invest λ of the funds in the safe asset. Late depositors, who will be $(1-\lambda)$ of the population, will get the funds invested in the risky asset at the end. Therefore, late depositors will get:

$$c_2(\widetilde{R}) = \begin{cases} \frac{(L-\lambda\overline{c}_1)+RX}{1-\lambda} & \text{if} \quad \widetilde{R} = R \\ \frac{(L-\lambda\overline{c}_1)+rX}{1-\lambda} & \text{if} \quad \widetilde{R} = r. \end{cases} \tag{1}$$

Now, let's look at the constraints needed for the run-proof contract to work. The first late depositor, upon observing the bad signal, should wait until the last period. If she withdraws, she gets $\overline{c}_1$. Now her posterior about the good state is $p_b = 1-q$ and if she does not withdraw, she will get an expected utility of $[(1-q)\ u(c_2(R)) + q\ u(c_2(r))]$. If the bank offers $\overline{c}_1$ such that

$$u(\overline{c}_1) \leqslant (1-q)\ u\left(c_2(R)\right) + q\ u\left(c_2(r)\right), \tag{2}$$

the first depositor, upon observing the bad signal, will not withdraw. The first depositor's decision did not reveal any information about her private signal. When it is the second late depositor's turn to move, the public belief about the good state is still 1/2. She is in the same situation as the first late depositor and because of inequality (2) she will not withdraw either. The analysis will continue in this fashion and no late depositor will withdraw early.

Under this contract, early depositors get $\overline{c}_1$ while the late depositors get $c_2(\widetilde{R})$.

We can write the bank's problem as:

$$(P3)\begin{cases} Max & \lambda\ u(\overline{c}_1) + (1-\lambda)\left[\frac{1}{2}\ u\left(c_2(R)\right) + \frac{1}{2}\ u\left(c_2(r)\right)\right] \\ s.t. \quad (i) & L + X \leqslant 1 \\ \qquad\ (ii) & u(\overline{c}_1) \leqslant (1-q)\ u\left(c_2(R)\right) + q\ u\left(c_2(r)\right) \end{cases}$$

where $c_2(R)$ and $c_2(r)$ are the same as in (1).

This contract will completely eliminate bank runs. Given that early liquidation is costly, preventing runs can increase welfare. But, note that the constraint (ii) becomes more binding as the informativeness of the private signals, q, increases. As q increases, the right-hand side of (ii) decreases. Therefore to satisfy this constraint, the bank has to offer a smaller amount to early depositors. This will undermine the insurance provided by the deposit contract against a liquidity shock. It is clear that the run-proof contract cannot achieve the first-best efficient outcome.

Also note that the constraint (ii) depends on the fundamentals. If the fundamentals are sound, formally high R and/or high r, then (ii) can be satisfied for a wider range of $\overline{c}_1$ values. Therefore when the fundamentals are not strong, preventing runs will have higher costs.

As $\overline{c}_1$ increases the left-hand side of (ii) increases while the right-hand side decreases. Therefore depending on the parameters of the model (q, R, r), there is a critical level of $\overline{c}_1$, denoted by $\underline{c}$, such that when $\overline{c}_1$ is above $\underline{c}$, (ii) is not satisfied. Note that $\underline{c}$ is decreasing in q, increasing in R and r (see Figure 1).

In the following example, I illustrate these ideas.

Example 4 *Assume that $u(x) = ln(x)$ and $\lambda = 1/2$. At $\overline{c}_1 = \underline{c}$, the left-hand side is equal to right-hand side in constraint (ii). We get*

$$\ln(\underline{c}) - \ln(2-\underline{c}) = (1-q)\ln(R) + q\ln(r).$$

When we take the exponential of both sides, we get

$$\frac{\underline{c}}{2-\underline{c}} = R^{(1-q)}r^q.$$

So that

$$\underline{c} = \frac{2R^{(1-q)}r^q}{1+R^{(1-q)}r^q}.$$

Note that $\frac{\partial \underline{c}}{\partial q} < 0$, $\frac{\partial \underline{c}}{\partial R} > 0$, $\frac{\partial \underline{c}}{\partial r} > 0$. *Figure 1 presents the relationship between* $\underline{c}$ *and* q *(dashed lines) for high* $(R = 6, r = 0.75)$ *and low fundamentals* $(R = 4, r = 0.5)$. *And Figure 2 shows the expected utility provided by the run-proof contract (dashed lines), as a function of* q, *for high* $(R = 6, r = 0.75)$ *and low fundamentals* $(R = 4, r = 0.5)$.

5.2 Deposit Contract with Bank Runs

In this section, I will analyze the deposit contract that allows for bank runs. This contract will not have the restrictions of the run-proof contract, therefore it can provide higher levels of insurance against liquidity shocks. Yet information externalities and herd behavior of depositors can trigger runs on healthy banks. This will be the counterfeit of the liquidity insurance provided by this contract.

I assume that actions of depositors are observable. The deposit contract I will analyze will have the following features[24]:

- A depositor with the posterior of $p_b = 1 - q$, will withdraw early.
- A depositor with the posterior of $1/2$, will not withdraw early.
- A depositor with the posterior of $p_g = q$, will not withdraw early.

If it is better to wait with a posterior of $1/2$, it should also be better to wait when the posterior is q. Therefore the third property will be automatically satisfied when the second one is satisfied.

In this deposit contract, whenever the public belief about the good state reaches a level of q, from then on no late depositor will withdraw. When the public belief reaches a level of q, even a depositor with the bad signal will have a posterior of $1/2$ and because of the second property, this depositor, say depositor i, will decide to wait until the last period. Note that her action does not depend on her private signal, therefore no new information has been revealed. So the public belief when it is the $(i+1)th$ depositor's turn to move is still q and she will be in the same position as the ith depositor. This starts a no-run cascade where no late depositor will withdraw early.

Also note that when the public belief drops to a level of $p(-2)$[25] even a depositor with the good signal will withdraw early, since her posterior,

[24] These features hold for a depositor for whom an informational cascade has not started yet.

[25] This notation has been explained in section 2.2. It corresponds to a sequence of signals where the number of bad signals is two more than the number of good signals, $n^i = n^i_g - n^i_b = -2$.

	No-Run Cascade	*No Cascade*	*Run Cascade*
$i = 2$	q	$(1-q)q$	$(1-q)^2$
$i = n$	$q\,[(1-q)q]^{(n/2)-1}$	$[(1-q)q]^{(n/2)}$	$(1-q)^2[(1-q)q]^{(n/2)-1}$

Table 3: Probabilities of cascades in the good state.

taking into account her private signal, will be equal to $(1-q)$ and by the first property, she should withdraw. Her action will not reveal any new information. All the depositors after her will be in the same situation and they will also withdraw early. Therefore, whenever the public belief drops to a level of $p(-2)$, a run on the bank will be triggered.

Now I can calculate the probabilities of each type of cascade (run and no-run cascades) in different states of nature using the discussed features of the deposit contract. The analysis will be very similar to that in BHW: I will start with the first late depositor and go forward sequentially. Since I assume that the depositors can observe each other's type, in updating their beliefs, late depositors will not take into account the withdrawal decisions of early depositors. Therefore, for the following discussion, I will concentrate on late depositors only.

I will start with the first late depositor. Her action, $a^1 \in \{W, N\}$ where W and N stand for withdraw and not-withdraw at $t = 1$ respectively, will depend on her private signal. If she had the good signal, she will not withdraw. Now the public belief is q and this starts a no-run cascade. If she had the bad signal, she would withdraw but a run-cascade will not start yet.

Let's look at the second late depositor. If the first late depositor did not withdraw, she is in a no-run cascade and will not withdraw, regardless of her signal. If the first late depositor withdrew early, then the public belief is $(1-q)$. If she observed the good signal, her posterior is $1/2$, therefore she will not withdraw. If she observed the bad signal, her posterior is $p(-2)$ and she will withdraw. This will trigger a run on the bank.

The only history where a cascade has not started by the third late depositor is $(a^1, a^2) = (W, N)$. For that history, the public belief is $1/2$ and the third depositor is in the same position as the first late depositor. If she observed the good signal, she will not withdraw and this will start a no-run cascade. If she observed the bad signal, she will withdraw early.

A cascade has not started by the fourth late depositor only when the history is $(a^1, a^2, a^3) = (W, N, W)$. In that case, the fourth late depositor is in the same situation as the second late depositor. Otherwise she is in a cascade and her action will not depend on her private signal.

state\outcome	No Run	Run
Good	$\frac{q}{1-q(1-q)}$	$\frac{(1-q)^2}{1-q(1-q)}$
Bad	$\frac{1-q}{1-q(1-q)}$	$\frac{q^2}{1-q(1-q)}$

Table 4: Probabilities of Run and No Run in Different States.

The analysis continues in this manner.

Next I will calculate the probabilities for run and no-run cascades in both states. Note that the only sequence of actions where a cascade does not start has repetitions of (W, N)s, possibly followed by a W. Otherwise, a cascade starts.

Now, suppose it is the good state. Table 3 shows the probabilities of these cascades for the second and the nth late depositor.

The probability of a no-run cascade **until** the nth late depositor is :

$$P_n(no - run|G) = \sum_{j=0}^{(n/2)-1} q[(1-q)q]^j = q\left(\frac{1-[(1-q)q]^{(n/2)}}{1-(1-q)q}\right)$$

When we look at the asymptotic properties of this probability, we get:

$$\lim_{n\to\infty} P_n(no - run|G) = \frac{q}{1-q(1-q)} = P(no - run|G).$$

Other probabilities can be calculated using the same analysis. Probabilities for run and no-run cascades in each state are given in Table 4.

Using these probabilities, I will formally write the run contract.

Now, I will write the constraints of the run contract. These constraints are valid for a depositor for whom a cascade has not started yet. The bank uses the payment rule explained in section 3.2. When a run starts, the bank will not use the first-come first-served rule but cumulate withdrawals[26]. The constraints for those cases will be discussed later.

The first property of this contract is that a depositor with a posterior of $(1-q)$[27] will prefer to withdraw. If she withdraws early, she guarantees herself $\overline{c}_1$. If she deviates and waits, others will think she observed the good signal. This will start a no-run cascade and no other late depositor will withdraw early. Note that this is the highest outcome a depositor can get

[26] Wallace (1988) and Wallace (1990) argue that the sequential service constraint is not the optimal arrangement under some conditions.

[27] The only case where a cascade has not started by this depositor is that the decisions before her were repetitions of (W, N)s and she observed the bad signal s_b.

state\outcome	No Run	Run
Good	$c_2(R) = \frac{(L-\lambda\overline{c}_1)+RX}{1-\lambda}$	$c_1^{Run}(R) = \begin{cases} \overline{c}_1 & if \quad \overline{c}_1 \leqslant \tau R \\ L+\tau RX & if \quad \overline{c}_1 > \tau R \end{cases}$
Bad	$c_2(r) = \frac{(L-\lambda\overline{c}_1)+rX}{1-\lambda}$	$c_1^{Run}(r) = L+\tau rX$

Table 5: Payoffs from not withdrawing when it is her turn to move.

from waiting, since liquidation of the risky asset is costly. To satisfy this, we need

$$u(\overline{c}_1) > q\ u\left(c_2(r)\right) + (1-q)\ u\left(c_2(R)\right). \tag{3}$$

According to the second property of this contract, a depositor with a posterior of 1/2[28] should wait. The consequences of this constraint is not as immediate as the previous one since this action will not start an informational cascade. The return from waiting will be random depending on the state as in other cases, but also on what the depositors after her will do.

We can have run and no-run cascades in both the good and the bad state. A player's payoff depends on the state as well as the occurrence of a run as summarized in Table 5.

If there is no run on the bank then late depositors get the return from the risky asset which is the same as in the run-proof contract, given in (1):

$$c_2(\widetilde{R}) = \begin{cases} \frac{(L-\lambda\overline{c}_1)+RX}{1-\lambda} & \text{if} \quad \widetilde{R} = R \\ \frac{(L-\lambda\overline{c}_1)+rX}{1-\lambda} & \text{if} \quad \widetilde{R} = r. \end{cases}$$

If a bank run starts at some point, the late depositor who chose not to withdraw when it was her turn can join the run. But now she cannot guarantee herself $\overline{c}_1$ since the bank cumulates withdrawals and pays everyone $\overline{c}_1$ if it can, but if it cannot, it equally divides whatever is available among those who demand early liquidation. If $\tau > \overline{c}_1/R$, even the risky asset is liquidated at a discount, everybody can get their promised of $\overline{c}_1$ in the good state. In the bad state and in the good state when $\tau < \overline{c}_1/R$, the bank cannot pay everybody the promised so it equally divides whatever it has among those who wants to withdraw.

[28] The only case where a cascade has not started by this depositor is that the decisions before her were repetitions of (W, N)s followed by a W, and she observed the good signal s_g.

Using these payoffs from Table 5 and the probabilities from Table 4, we get the constraint for a late depositor with a posterior of 1/2 not to withdraw, as:

$$\begin{aligned} u(\overline{c}_1) &< \frac{1}{2(1-q(1-q))} \left[\begin{array}{c} q\ u\left(c_2(R)\right) + (1-q)\ u\left(c_2(r)\right) + \\ (1-q)^2\ u\left(c_1^{Run}(R)\right) + q^2\ u(c_1^{Run}(r)) \end{array} \right] \\ &= E(U^{late}) \end{aligned}$$

When a run is triggered, the late depositors who chose not to withdraw when it was their turn to move can choose to join the run. This adds an endogenous component to the timing of withdrawal choices. We may need different constraints for them to choose to join[29].

Now we can state the bank's problem:

$$(P4) \left\{ \begin{array}{lll} Max & & E(U^{early}) + E(U^{late}) \\ s.t. & (i) & L + X \leqslant 1 \\ & (ii) & u(\overline{c}_1) > q\ u\left(c_2(r)\right) + (1-q)\ u\left(c_2(R)\right) \\ & (iii) & u(\overline{c}_1) < E(U^{late}) \end{array} \right.$$

where $c_2(R)$, $c_2(r)$, $c_1^{Run}(R)$ and $c_1^{Run}(r)$ are the same as in Table 5[30].

[29] A run is triggered when the public belief drops to $p(-2)$. The bank no longer uses the first-come first-served rule. Payoffs from not withdrawing when it is her turn to move but joining the run later are different from the payoffs in Table 5. If a late depositor joins the run and if it is the good state, she gets $\overline{c}_1$ when $\tau > \overline{c}_1/R$, she gets $L + \tau RX$ when $\tau < \overline{c}_1/R$. If she joins the run and if it is the bad state, she gets $L + \tau rX$. If she does not join the run, she gets $c_2^{Run}(\widetilde{R})$ which is $c_2(R)$ if it is the good state and $\tau > \overline{c}_1/R$, and 0 otherwise. That is:

$$c_2^{Run}(R) = \left\{ \begin{array}{ccc} c_2(R) & if & \overline{c}_1 \leqslant \tau R \\ 0 & if & \overline{c}_1 > \tau R \end{array} \right.$$

and $c_2^{Run}(r) = 0$.

For her to join the run we should have

$$(1-q)\ u(c_1^{Run}(R)) + q\ u(c_1^{Run}(r)) > (1-q)\ u(c_2^{Run}(R)) + q\ u(0).$$

Assuming that $u(0)$ is a small enough number, we can satisfy this constraint easily (for example $u(.) = ln(.)$).

[30] In this type of contract, if there is no run, all early depositors get the promised $\overline{c}_1$. If there is a run, they get $c_1^{Run}(R)$ in the good state and $c_1^{Run}(r)$ in the bad state. Ex-ante,

Now I would like to investigate how the parameters of the model (q, R, r, τ) will affect the constraints and the objective function.

First we observe that the critical value of $\overline{c}_1$ for constraint (ii) to be satisfied is $\underline{c}$, the critical value for the constraint in the run-proof contract. This constraint is satisfied for values of $\overline{c}_1 > \underline{c}$. We have seen before that $\underline{c}$ decreases in q and increases in R and r. Therefore keeping other parameters constant, as the informativeness of the signals increase, this constraint becomes less binding, which increases the welfare provided by the run-contract.

The LHS of (iii) is increasing while the RHS is decreasing in $\overline{c}_1$. Therefore there is a critical level of $\overline{c}_1$, denoted as $\overline{c}$, above which (iii) is not satisfied.

It is not clear whether the RHS of (iii) will increase or decrease as the signals become more informative. Therefore the effect of more informative signals is ambiguous on this constraint.

Now I would like to investigate how fundamentals (R, r) affect the constraint and the objective function. As R and r increases, RHS of (ii) increases so we need higher values of $\overline{c}_1$ to satisfy this constraint. It makes this constraint more binding. But this increases the RHS of (iii) and makes it less binding. So when the fundamentals are strong, it is not clear whether the choice set becomes larger or smaller. Clearly, strong fundamentals have a positive effect on the objective function.

As the liquidation cost goes down, constraint (ii) is not affected, while the RHS of constraint (iii) increases so that the bank's choice set becomes larger. This is because when the liquidation costs are low, the cost of bank runs is low. Note that as τ increases, the objective function also increases. Therefore a low liquidation cost makes the run contract more attractive.

Though the run contract can give better results than the run-proof contract, it is quite clear that it cannot achieve the first-best efficient outcome. In the next section I will investigate how the liquidity support can improve the run contract but even in that case, it cannot achieve the first-best efficient outcome.

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the probability of the good state is 1/2 and using the probabilities from Table 4, we find the expected utility of the early depositors as:

$$E(U^{early}) = \frac{1}{2(1-q(1-q))}\left[u(\overline{c}_1) + (1-q)^2\ u(c_1^{Run}(R)) + q^2\ u(c_1^{Run}(r))\right].$$

6 Liquidity Support

In this section I will introduce a central bank that will act as a *lender of last resort* and provide liquidity support to the bank. This liquidity support will improve the run contract since the bank does not have to go under costly liquidation of the risky asset when faced with a run. But as I will show, in the absence of public information about the bank's soundness, liquidity support will not be enough to achieve the first-best efficient outcome with public information.

Bagehot (1873) recommends that in a time of crisis, a central bank should lend freely at a very high rate of interest and these loans should be made on all good banking securities. In my model, the central bank will lend only against good collateral[31] as Bagehot (1873) recommends. Actually the best justification for this comes from Bagehot's own words: "Any aid to a present bad bank is the surest mode of preventing the establishment of a good bank[32]".

However, I will not use the second rule which recommends to lend at a very high rate. The central bank will lend at a zero interest rate in this model. Actually lending at a high rate can worsen the banking crisis[33] and can give managers the incentive to take very high, uneconomical risks. In practice, there have been episodes where Bagehot's rule of lending at a very high rate has been challenged, as Goodhart and Shoenmaker (1995) and Prati and Schinasi (1999) document.

With the liquidity support from the central bank, the bank does not have to go under costly liquidation of the risky asset. Simply it can lend the risky asset in its portfolio to the central bank which will provide liquidity at $t = 1$ in the amount that is equal to the value of the risky asset at $t = 2$. This contract is a special case of the run contract, $\tau = 1$ now.

The probabilities for run and no-run cascades in each state will be the

[31] I assume that the central bank knows the value of the bank's assets when providing liquidity. For a discussion, see Section 7.

[32] In practice, we see interventions when the bank is insolvent (Prati and Schinasi (1999) and Giannini (1999)). The justification for this is the possibility of contagion to the banking system (Diamond and Rajan (2002)). Since I have only one bank in my model, it is not a suitable model to study contagion. Another reason for interventions to insolvent banks is pointed out by Santomero and Hoffman (1998). They show that for US, between 1985 and 1991, access to the discount window was granted to banks that later failed. They argue that the reason for this was to keep institutions afloat so that the deposit insurance fund did not suffer further losses. This can be another destabilizing effect of deposit insurance.

[33] See Garcia and Plautz (1988).

state\outcome	No Run	Run
Good	$c_2(R) = \frac{(L-\lambda\overline{c}_1)+RX}{1-\lambda}$	$c_1^{Run}(R) = \overline{c}_1$
Bad	$c_2(r) = \frac{(L-\lambda\overline{c}_1)+rX}{1-\lambda}$	$c_1^{Run}(r) = L + rX$

Table 6: Payoffs from not withdrawing when it is her turn to move (with liquidity support).

same as in Table 4. The contract will be the same as the run contract in section 5.2. but the payoffs will come from Table 6 now.

Now we can state the run contract. The bank's problem is:

$$P(5)\begin{cases} Max & & E(U^{early}) + E(U^{late}) \\ s.t. & (i) & L + X \leqslant 1 \\ & (ii) & u(\overline{c}_1) > q\ u\left(c_2(r)\right) + (1-q)\ u\left(c_2(R)\right) \\ & (iii) & u(\overline{c}_1) < E(U^{late}) \end{cases}$$

where $c_2(R)$, $c_2(r)$, $c_1^{Run}(R)$ and $c_1^{Run}(r)$ are the same as in Table 6.

Since this is a special case of the run contract, the parameters of the model (q, R, r) will affect the constraints and the objective function in the same way as before.

The following example focuses on constraint (iii).

Example 5 *Suppose $u(x) = ln(x)$ and $\lambda = 1/2$. I would like to find the critical value $\overline{c}$. From constraint (ii) we get*

$$(1+q)^2 \ln(\overline{c}) - \ln(2-\overline{c}) - q^2 \ln\left(\frac{\overline{c} + (2-\overline{c})r}{2}\right) = q\ln(R) + (1-q)\ln(r).$$

Taking the exponential of both sides, we get

$$\frac{\overline{c}^{(1+q)^2}}{(2-\overline{c})\left(\frac{\overline{c}+(2-\overline{c})r}{2}\right)^{q^2}} = R^q\ r^{(1-q)}.$$

We can find the value of $\overline{c}$ below which (iii) is satisfied using the above equality. Figure 1 (solid lines) shows the values of $\overline{c}$ for low $(R = 4, r = 0.5)$ and high fundamentals $(R = 6, r = 0.75)$.

Because of the explained trade-off of preventing runs, for some parameter values, it may be optimal to allow for runs even if they can be generated by herd behavior of depositors and can be on healthy banks. The relation between the parameters and the utilities provided by the contracts were explained within the text when we analyze how the parameters affect the constraints and the objective functions for each contract. The example below illustrates these points.

Example 6 *The table below shows the expected utilities from different contracts the bank can choose for different values of q with low fundamentals* $(R = 4, r = 0.5)$

	Run Contract with Liquidity	*Run-Proof Contract*
$q = 0.60$	0.1319	0.1733
$q = 0.75$	0.1836	0.1695
$q = 0.90$	0.2147	0.1441

Figure 2 shows the expected utilities from run (solid lines) and run-proof (dashed lines) contracts with low $(R = 4, r = 0.5)$ *and high fundamentals* $(R = 6, r = 0.75)$. *For low values of q, the run-proof contract is better but as q increases, allowing for runs becomes optimal. From Figure 2, we see that, for low fundamentals, the range of q values for which the run contract performs better than the run-proof contract is larger than the same range with high fundamentals. This is because, preventing runs have higher welfare costs when fundamentals are not strong.*

Since the bank does not have to go under costly liquidation of the risky asset it is quite clear that the liquidity support improves the performance of the run contract. But as we saw, the liquidity support does not prevent runs on healthy banks, which is consistent with the historical evidence provided by Park (1991). For the run contract to achieve the first-best efficient outcome, it is crucial that runs are correct, that is, they occur only when they are needed to. For these reasons, in the absence of perfect information about banks' asset quality, a deposit contract, even with liquidity support from the central bank, cannot achieve the first-best efficient outcome with public information.

7 Policy Implications

This paper makes important suggestions on the policy side. In my model, deposit insurance can prevent runs and since early liquidation is costly, this can be welfare improving. Yet it is a well-known fact that with deposit insurance[34], depositors do not have any incentive to differentiate between sound and unsound banks. In such an environment, weak banks do not have any difficulty in attracting deposits. This creates an opportunity for moral hazard on banks' side. Actually, some studies argue that guarantees create distortions and make bank failures more likely. Demirguc-Kunt and Detragiache (2000) analyze panel data for 61 countries during 1980-97 and concludes: "Explicit deposit insurance tends to be detrimental to bank stability, the more so when institutional environment is weak, when the coverage is extensive and when the insurance is run by the government[35]."

Financial regulators and policy makers have been looking for alternatives for full deposit insurance. Giving a greater role on market discipline may lessen the moral hazard created by guarantees. For market discipline to work effectively, depositors should have accurate information about the soundness of banks. As this paper shows, in the absence of accurate information, information externalities and herd behavior of depositors can trigger runs on healthy banks. This in turn undermines the role of market discipline in keeping an eye on bank activities. Therefore disclosure of information on banks' structure, performance and risk positions should be enforced. This is pillar three of Basel II regime and this should be fully pursued to increase the effectiveness of market discipline, which can be an alternative for deposit insurance in achieving financial stability.

Another important point with the disclosure of information is the continuous review of banks' performance which is "pillar two" of Basel II regime. In my model, I assume that the central bank knows the value of the bank's assets when providing liquidity. In some situations, it can be difficult for a central bank to identify the exact source of a problem in a bank, i.e. whether it is a solvency or a liquidity problem[36]. Actually, Berger et.al (1998) test the hypothesis that supervisors have more accurate information than the market on the soundness of financial institutions for the US case. They show that shortly after supervisors have inspected a bank, supervisory assessment of the bank is more accurate than the market. But, for periods

[34] For an excellent survey on deposit insurance practices and current discussions, see Garcia (2000).

[35] In parallel with this finding, Schotter and Yorulmazer (2003) show that partial deposit insurance can diminish the severity of bank runs, therefore full deposit insurance can be inefficient.

[36] Diamond and Rajan (2001) build a model where liquidity and solvency problems interact. They show that bank failures can themselves cause liquidity shortages which may cause a total meltdown of the system.

where the supervisory information is not up-to-date, market has more accurate information than the supervisors. By continuously (or frequently) reviewing bank's performance, authorities can take preemptive action and act rapidly when faced with banking problems. This can minimize the costs generated by disruptions to the payments system. In that sense, pursuing pillar two of the Basel II regime is crucial.

8 Conclusion

In this paper, I built a dynamic model of bank runs that allowed me to study important phenomena such as the role of information externalities and herd behavior of depositors as a source of bank runs. I showed that, in the presence of noisy private information, information externalities and herd behavior of depositors can trigger runs on healthy banks. The bank can choose a deposit contract that completely eliminates runs but this has some costs. That type of contract sacrifices from the insurance provided against liquidity shocks. Furthermore, in cases where the bank cannot pay everybody the promised amount, it may be socially optimal to have a run, as shown in AG. These are some costs of completely eliminating runs.

The bank can also choose a deposit contract that allows for runs. For some parameter values, it can be optimal to allow for runs even they can be generated by herd behavior of depositors and they can be on healthy banks.

The results of the paper are in parallel with the empirical evidence on measures that prevent bank runs. While liquidity support alleviates the costs of early liquidation, it cannot prevent runs on healthy banks. The policy measure that will prevent wrong runs is the disclosure of information on banks' soundness and management of the crisis. A deposit contract, with liquidity support from the central bank, can achieve the first-best efficient outcome only in the presence of perfect information about the bank's performance.

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Appendix

A Unobserved Types

Now I will relax the assumption that types are observed by the depositors. As explained, this will contribute noise, but the previous results will continue to hold. For all the analysis below, I assume that an informational cascade has not started yet. The following analysis is for the case where the public belief is equal to 1/2 but it certainly holds for the general case.

If a depositor decides to withdraw at $t = 1$ it can be for two reasons:

i) She may be an early consumer (with probability λ) or

ii) She may be a late consumer and observed a bad signal (with probability $(1-\lambda)\left((1/2)(1-q) + (1/2)q\right) = 1/2(1-\lambda)$.[37]

So when a depositor decides to withdraw at $t = 1$, the conditional probability that she is a late consumer and observed a bad signal is :

$$\Pr(s_b|W) = \frac{(1/2)(1-\lambda)}{\lambda + (1/2)(1-\lambda)} = \frac{1-\lambda}{1+\lambda}.$$

Using the value $\lambda = \frac{1}{2}$, we get:

$$\Pr(s_b|W) = \frac{1/2}{3/2} = \frac{1}{3}.$$

If a depositor does not withdraw, it reveals that she has certainly observed a good signal.

$$\Pr(s_g|N) = 1.$$

In this case early consumers will add extra noise to the model and a withdrawal decision will now be noisy information for the realization of a bad signal.

Let p_N be the conditional probability that we are in the good state, given a non-withdrawal decision. Then we have:

$$p_N = \Pr(\widetilde{R} = R|N) = p_g = 1 - q.$$

[37] A depositor is a late consumer with probability $(1-\lambda)$. With probability 1/2 it is the good state and a late consumer observes a bad signal with probability $(1-q)$ in the good state. With probability 1/2 it is the bad state and a late consumer observes a bad signal with probability q in the bad state. Therefore the probability of this case is $(1-\lambda)\left((1/2)(1-q) + (1/2)q\right)$.

Let p_W be the conditional probability that we are in the good state given a withdrawal decision. With probability 1/2 we are in the good state and a depositor withdraws early in the good if she is an early consumer (with probability λ) or if she is a late consumer and observed the bad signal (with probability $(1-\lambda)(1-q)$). Therefore

$$\begin{aligned} p_W &= \frac{(1/2)\left(\lambda + (1-\lambda)(1-q)\right)}{\lambda + (1/2)(1-\lambda)} \\ &= \frac{1 - q(1-\lambda)}{(1+\lambda)}. \end{aligned}$$

Using the value $\lambda = \frac{1}{2}$, we get:

$$p_W = \frac{2-q}{3}.$$

Claim 7 *$\forall \lambda \in (0,1)$, $p_b = 1 - q < p_W < 1/2$.*

Proof. Note that $\frac{\partial p_W}{\partial \lambda} > 0$. When $\lambda = 0$, $p_W = 1 - q = p_b$. When $\lambda = 1$, $p_W = 1/2$. ■

The analysis is similar to that of the case without the early consumers. Since $p_W > p_b$, $p_W(p_g(p)) > p$ so that we will need more than one, (say, k^*) withdrawals to cancel out the effect of a non-withdrawal. We will now have to deal with the cases in which we observe one non-withdrawal and k withdrawals where $k < k^*$. So we will have more cases of interest for the purposes of writing the optimal deposit contract but the analysis will be similar.

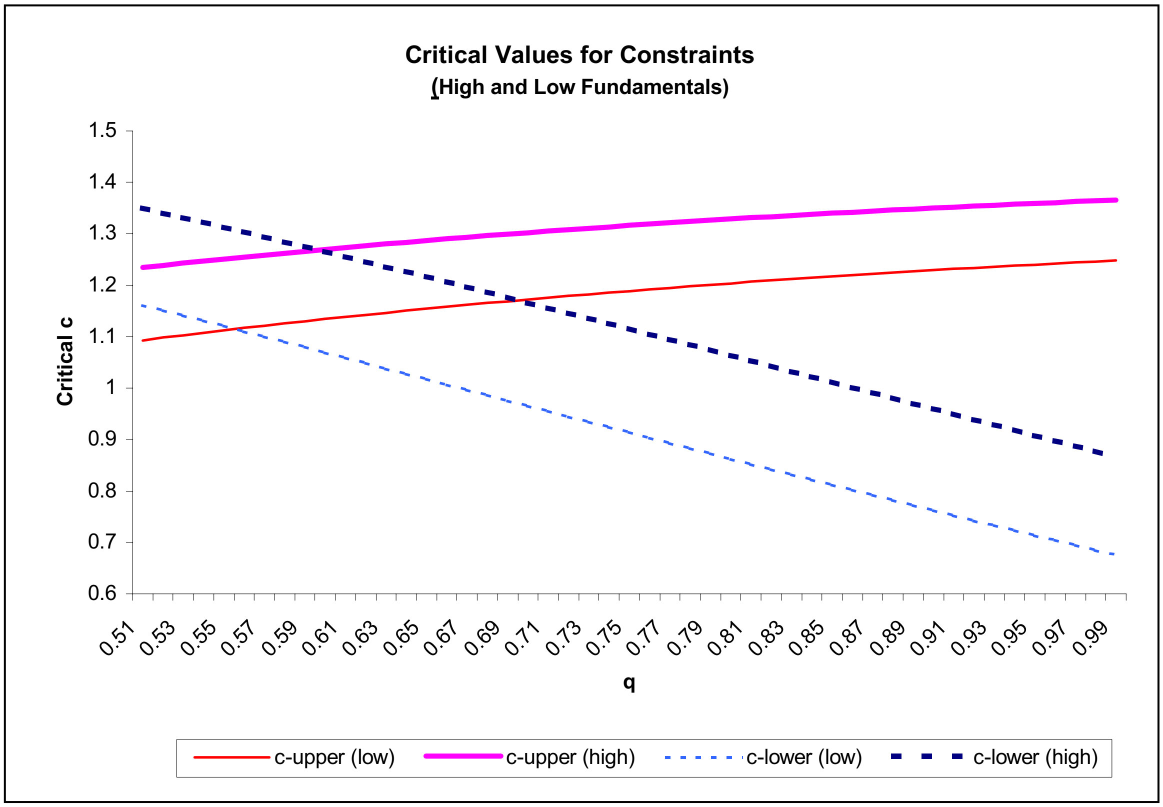

Figure 1: Critical values of c_1 for the constraints.

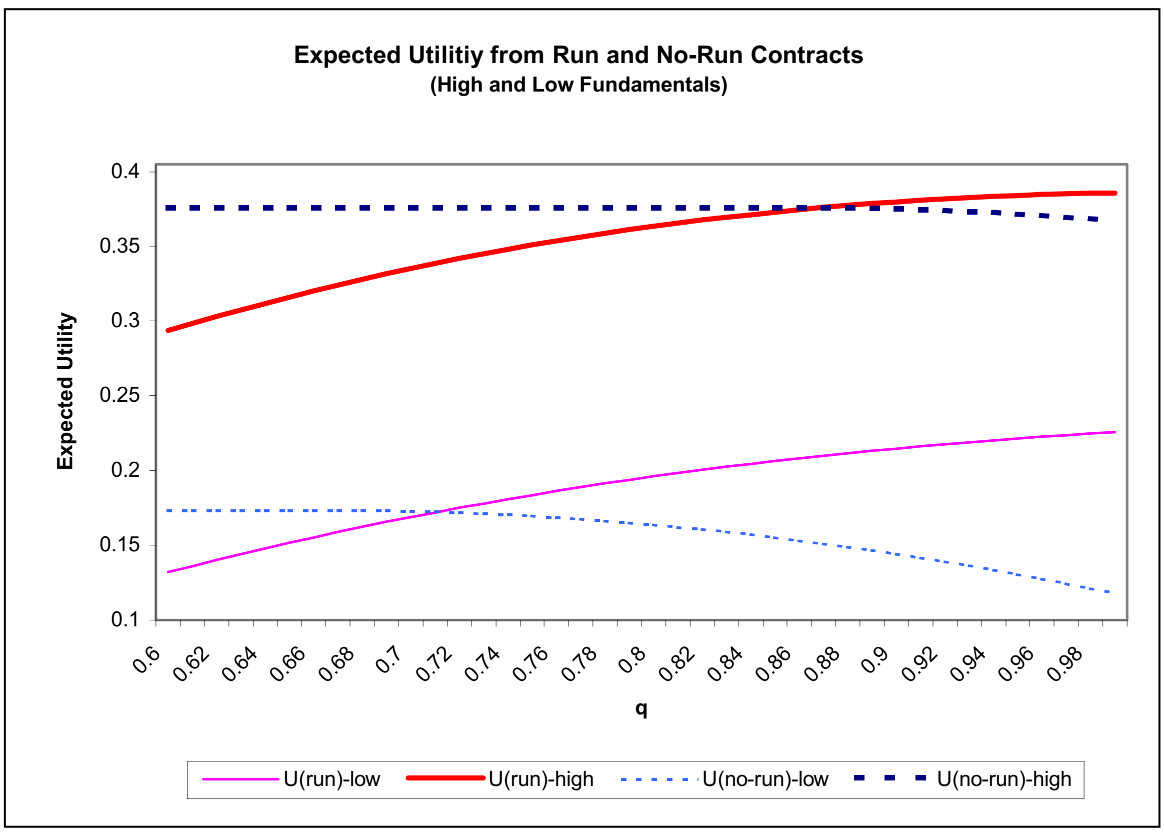

Figure 2: Expected utility for *Run* and *No-Run* contracts as a function of q for different fundamentals.

***t* = 0**	***t* = 1**	***t* = 2**
· Nature chooses the state.	· Depositors learn their type and receive private signals.	· Returns are realized.
· Banks choose the deposit contract to offer.	· Depositors make their withdrawal decisions.	· Late depositors who did not withdraw their return.
· Depositors make their investment choices.	· If there is no run, early depositors get their promised.	
· Bank chooses the portfolio of assets.	· If there is a run, bank cumulates withdrawals.	

Figure 3: Sequence of events.