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Abstract

This study presents voting on policies including public education, taxes, and public debt in an overlapping-generations model with physical and human capital accumulation and analyzes the effects of a debt ceiling on the government's policy formation and its impact on growth and welfare. The debt ceiling induces the government to shift the tax burdens from the older to younger generations and increase public education spending, resulting in a higher growth rate. However, it creates a trade-off between generations in terms of welfare. Alternatively, the debt ceiling is measured from the viewpoint of a benevolent planner; lowering the debt ceiling makes it possible for the government to approach the planner's allocation in an aging society.

- Keywords: Debt ceiling; Probabilistic voting, Public debt, Economic growth, Overlapping generations
- JEL Classification: D70, E24, H63,

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1 Introduction

Many developed countries have experienced large budget deficits and growing public debt in the past two decades. In 1998, the average general government debt as a percentage of GDP was 73.58% in Organisation for Economic Co-operation and Development (OECD) member countries, while it was 93.70% in 2017. In particular, the ratio increased more than 40 points in France, Greece, Japan, Portugal, Spain, the United Kingdom, and the United States.[1] The increased public debt is also a feature of developing countries (World Bank, 2019). Given this background, various types of fiscal rules have been introduced to control deficits, spending, and debt (e.g., Budina et al., 2012). In 2013, rules were in place in 97 countries (Halac and Yared, 2018).

The present study focuses on debt ceilings that control public debt and deficits. Imposing debt ceilings constrains fiscal policy choice (Heinemann, Moessinger, and Yeter, 2018) and thus may have the potential to improve welfare since political frictions in fiscal policymaking mean that the equilibrium public debt level is too high relative to the efficient one (Battaglini and Coate, 2008). Specifically, imposing debt ceilings may create long-run benefits of a lower debt burden at the cost of potentially short-run increased tax burdens and thus a net benefit in terms of welfare. The possibility of such welfare improvement is shown to be achieved by introducing a balanced budget rule (Azzimonti, Battaglini, and Coate, 2016) and an austerity program with a target level for debt and a time horizon (Barseghyan and Battaglini, 2016).

A welfare improvement is possible in the frameworks of Azzimonti, Battaglini, and Coate (2016) and Barseghyan and Battaglini (2016) because they assume infinitely lived households that can make up the short-run increased tax burdens with the benefits of reduced debt burdens accruing in the future. Such an improvement is not seen when we alternatively assume overlapping generations of finitely lived households (Arai, Naito, and Ono, 2018). In particular, agents who owe the costs of increased tax burdens today would not be alive in the future to enjoy the benefits of reduced debt burdens, suggesting a limitation of debt ceilings.

The present study reexamines debt ceilings from the viewpoint of the political economy. To pursue our analysis, we present an overlapping-generations model with physical and human capital accumulation (e.g., Lambrecht, Michel, and Vidal, 2005; Kunze, 2014; Ono and Uchida, 2016). Each generation comprises many identical individuals who live over three periods: young, middle, and old ages. Public education spending and parental human capital are inputs in the human capital formation process, thereby contributing to children's human capital formation and economic growth. Governments, as elected

[1] Source: https://data.oecd.org/gga/general-government-debt.htm (Accessed on June 14, 2019).

representatives, finance public education spending through taxes on capital and labor income and public debt issues.

Under this framework, we consider the politics of fiscal policy formation. In particular, following Song, Storesletten, and Zilibotti (2012) and the studies that have followed, we assume probabilistic voting à la Lindbeck and Weibull (1987) to demonstrate the extent to which generations face conflict over such policies. In each period, middle-aged and old individuals vote.[2] The government in power maximizes the political objective function of the weighted sum of the utilities of the middle-aged and old populations. In this voting environment, the current policy choice affects the decision on future policy via physical and human capital accumulation. To demonstrate this intertemporal effect, we employ the concept of a Markov-perfect equilibrium under which fiscal policy today depends on the current payoff-relevant state variables, namely physical and human capital and public debt. Given this process, we characterize the political equilibrium in the absence or presence of the debt ceiling, which yields the following findings.

Firstly, we provide a characterization of the political equilibrium in the absence of the debt ceiling and then compare it with an alternative scenario, called tax financing, in which the government is prohibited from issuing public debt; hence, its expenditure is financed solely through taxation. This scenario, while an extreme one, enables us to investigate the effect of controlling debt issues in a tractable way. We show that the labor tax rate increases, while the capital tax rate decreases if the government changes its instrument from debt financing to tax financing. Consequently, changes in tax rates could produce two opposing effects. In addition, tax financing removes the crowding-out effect of public bonds on physical capital and thus enhances human capital accumulation. Distant future generations benefit from this positive accumulation effect that outweighs the effects through taxes, whereas the initial generation does not. Therefore, the change in financing produces a trade-off in terms of utility across generations.

Secondly, we consider a more realistic fiscal rule that sets an explicit ceiling for public debt as a percentage of GDP, which has been widely introduced in developed countries (Schaechter et al., 2012). We investigate the effect of such a fiscal tightening rule (i.e., lowering the ceiling) based on a numerical analysis and show that this fiscal tightening rule is growth-enhancing, but not Pareto-improving. We also find that even if the rule is imposed only for limited periods, it has a long-lasting effect on utility across generations since increased human capital is bequeathed from generation to generation. Our result suggests the importance of connecting seemingly unrelated subjects, namely public

[2]The young may also have an incentive to vote since they benefit from public education in the future. However, for the tractability of the analysis, we assume that politicians do not care about the young's preferences following Saint-Paul and Verdier (1993), Bernasconi and Profeta (2012), and Lancia and Russo (2016). This assumption is supported in part by the fact that a large number of the young are below the voting age.

education and public debt, to consider the impact of debt ceilings over time and across generations.

Thirdly, we evaluate the optimality of the political equilibrium from the viewpoint of the planner's allocation. The long-lived planner has an incentive to invest more in education than short-sighted politicians. This implies that the share of the resources devoted to education (consumption) is higher (lower) in the planner's allocation than at the political equilibrium in the absence of a debt ceiling, showing the sub-optimality of the political equilibrium. To resolve this, we control the debt ceiling to approach the planner's allocation and show that the realization of the approach depends on the political power of the old, represented by their weight in the political objective function. In particular, under certain conditions, lowering the debt ceiling enables politicians to approach the planner's allocation when the political weight of the old is high, whereas it does not when the political weight is low, suggesting a rationale for strengthening fiscal discipline in an aging society.

In the main analysis, we assume that public spending is limited to education and that the old do not benefit from any public expenditure. When the old directly benefit from public expenditure such as public good provision, they may induce politicians to raise expenditure on them as well as place the fiscal burden onto future generations by issuing more public debt. We examine this possibility and show that such a case does not arise because the middle-aged, who also benefit from public good provision, find it optimal to reduce debt issues and increase public good provision in their old age. This disciplined effect (as also found by Song, Storesletten, and Zilibotti, 2012) works to reduce rather than increase public debt issues.

Relation to the Literature The present study follows Cukierman and Meltzer (1989), Song, Storesletten, and Zilibotti (2012, 2016), Röhrs (2016), and Ono and Uchida (2018) by employing the overlapping-generations model with public debt and introduces debt ceilings into the model. The study departs from previous ones and contributes to the literature in three ways. First, it assumes two different taxes, the labor income tax on the middle-aged and the capital income tax on the retired old, rather than a single tax instrument, a tax on labor income, as assumed in previous studies. This assumption enables us to demonstrate how the costs of the debt ceiling are distributed between generations through tax burdens and how this distribution in turn affects growth and welfare over time and across generations.

Second, the present study focuses on public education as a source of economic growth through human capital accumulation.[3] This enables us to demonstrate the endogenous

[3] Ono and Uchida (2018) also present a model with physical and human capital accumulation, but it assumes that only one generation participates in voting, thus lacking generational conflict over taxes and

determination of an interest rate and its impact on policy formation. This general equilibrium effect is absent in Arai, Naito, and Ono (2018), who employ AK technology, as well as Battaglini and Coate (2008, 2016), Barseghyan, Battaglini, and Coate (2013), Azzimonti, Battaglini, and Coate (2016), and Cunha and Ornelas (2018), who assume constant interest rates. Exceptions are Song, Storesletten, and Zilibotti (2012), Röhrs (2016), Arawatari and Ono (2017), Katagiri, Konishi, and Ueda (2019), and Barseghyan and Battaglini (2016); however, the first four consider economies without economic growth. Barseghyan and Battaglini (2016) demonstrate the general equilibrium effect by considering public investment as a source of productivity growth. In particular, they show that a temporary austerity program induces only a temporary effect on policies and economic growth. By contrast, the present study shows that the temporary program has a long-lasting effect through human capital accumulation that benefits future generations. This result suggests the potential importance of public education and human capital when we evaluate the effect of debt rules in the short and long run.

Third, the present study introduces an imaginary social planner who cares about all generations and aims to maximize the weighted sum of utilities across generations. Then, it investigates under what conditions the debt ceiling induces politicians to choose a fiscal policy that approximates the planner's allocation. Song, Storesletten, and Zilibotti (2012) use the planner's allocation to evaluate the optimality of the political equilibrium in the absence of a debt ceiling, whereas Arai, Naito, and Ono (2018) rely on the Pareto criterion to evaluate the effects of a debt ceiling on welfare across generations. The present study is thus an attempt to bridge the gap between these two studies. The study is also related to Cunha and Ornelas (2018), who point out that a tight debt ceiling can exacerbate political economy distortions, focusing on the degree of political turnover. We instead focus on the political power of the old and offer an alternative insight into the effect of debt ceilings.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 describes the political equilibrium. Section 4 compares the debt- and tax-financing political equilibria; it also investigates the effects of the debt ceiling rule. Section 5 provides a characterization of the planner's allocation and compares it with the political equilibrium. Section 6 investigates the case including public good provision. Section 7 presents concluding remarks.

2 Model

The discrete time economy starts in period 0 and consists of overlapping generations. Individuals are identical within a generation and live for three periods: youth, middle,

spending.

and old ages. Each middle-aged individual gives birth to $1+n$ children. The middle-aged population for period t is N_t and the population grows at a constant rate of $n(>-1)$: $N_{t+1}=(1+n)N_t$. Individuals who are middle-aged in period t are called generation t.

2.1 Individuals

Individuals display the following economic behavior over their lifecycles. During youth, they make no economic decisions and receive public education financed by the government. In middle age, individuals work, receive market wages, and make tax payments. They use after-tax income for consumption and savings. Individuals retire in their old age and receive and consume returns from savings.

Consider an individual born in period $t-1$. In period t, the individual is middle-aged and endowed with h_t units of human capital inherited from his or her parents. The individual supplies them inelastically in the labor market and obtains labor income $w_t h_t$, where w_t is the wage rate per efficient unit of labor in period t. After paying tax $\tau_t w_t h_t$, where $\tau_t \in (0,1)$ is the period t labor income tax rate, the individual distributes the after-tax income between consumption c_t and savings invested in physical capital s_t. Therefore, the period t budget constraint for the middle-aged becomes

$$c_t + s_t \leq (1-\tau_t) w_t h_t.$$

The period $t+1$ budget constraint in elderly age is

$$d_{t+1} \leq \left(1-\tau^k_{t+1}\right) R_{t+1} s_t,$$

where d_{t+1} is consumption, τ^k_{t+1} is the period $t+1$ capital income tax rate, $R_{t+1}(>0)$ is the gross return from investment in physical capital, and $R_{t+1}s_t$ is the return from savings. The results are qualitatively unchanged if capital income tax is on the net return from saving rather than the gross return.

Children's human capital in period $t+1$, h_{t+1}, is a function of government spending on public education, x_t, and parents' human capital, h_t. In particular, h_{t+1} is formulated using the following equation:

$$h_{t+1} = D\left(x_t\right)^{\eta}\left(h_t\right)^{1-\eta}, \tag{1}$$

where $D(>0)$ is a scale factor and $\eta \in (0,1)$ denotes the elasticity of education technology with respect to education spending.[4]

[4]Private investment in education may also contribute to human capital formation. For example, the parents' time (Glomm and Ravikumar, 1995, 2001, 2003; Glomm and Kaganovich, 2008) or spending (Glomm, 2004; Lambrecht, Michel, and Vidal, 2005; Kunze, 2014) devoted to education may complement public education. In the present study, we abstract private education from the main analysis to simplify the presentation of the model.

The preferences of an individual born in period $t-1$ are specified by the following expected utility function in the logarithmic form:

$$U_t = \ln c_t + \beta \ln d_{t+1},$$

where $\beta \in (0,1)$ is a discount factor. We substitute the budget constraints into the utility function to form the following unconstrained maximization problem:

$$\max_{\{s_t\}} \ln\left[(1-\tau_t)w_t h_t - s_t\right] + \beta \ln\left(1-\tau^k_{t+1}\right) R_{t+1} s_t.$$

By solving this problem, we obtain the following savings and consumption functions:

$$\begin{aligned} s_t &= \frac{\beta}{1+\beta} \cdot (1-\tau_t)w_t h_t, \qquad (2) \\ c_t &= \frac{1}{1+\beta} \cdot (1-\tau_t)w_t h_t, \\ d_{t+1} &= \left(1-\tau^k_{t+1}\right) R_{t+1} \cdot \frac{\beta}{1+\beta} \cdot (1-\tau_t)w_t h_t. \end{aligned}$$

2.2 Firms

Each period contains a continuum of identical firms that are perfectly competitive profit maximizers. According to Cobb–Douglas technology, they produce a final good Y_t using two inputs: aggregate physical capital K_t and aggregate human capital $H_t \equiv N_t h_t$. Aggregate output is given by

$$Y_t = A\left(K_t\right)^{\alpha}\left(H_t\right)^{1-\alpha},$$

where $A(>0)$ is a scale parameter and $\alpha \in (0,1)$ denotes the capital share.

Hereafter, we denote by $\hat{x}_t$ the ratio of X_t to aggregate human capital, H_t, and by x_t per capita X_t : $\hat{x}_t \equiv X_t/H_t$ and $x_t \equiv X_t/N_t$. Thus, $\hat{k}_t \equiv K_t/H_t$ denotes the ratio of physical to human capital. The first-order conditions for profit maximization with respect to H_t and K_t are

$$w_t = (1-\alpha)A\left(\hat{k}_t\right)^{\alpha}, \qquad (3)$$

$$\rho_t = \alpha A\left(\hat{k}_t\right)^{\alpha-1}, \qquad (4)$$

where w_t and ρ_t are labor wages and the rental price of capital, respectively. These conditions state that firms hire human and physical capital until the marginal products are equal to the factor prices. Capital is assumed to depreciate fully within each period.

2.3 Government Budget Constraint

Public education expenditure is financed by both taxes on capital and labor income and public bond issues. Let B_t denote aggregate inherited debt. The government budget constraint in period t is

$$B_{t+1} + \tau_t^k R_t s_{t-1} N_{t-1} + \tau_t w_t h_t N_t = N_{t+1} x_t + R_t B_t,$$

where B_{t+1} is newly issued public bonds, $\tau_t^k R_t s_{t-1} N_{t-1}$ is aggregate capital tax revenue, $\tau_t w_t h_t N_t$ is aggregate labor tax revenue, $N_{t+1} x_t$ is aggregate expenditure on public education, and $R_t B_t$ is debt repayment. We assume a one-period debt structure to derive analytical solutions from the model. We also assume that the government in each period is committed to not repudiating the debt.

By dividing both sides of the above expression by N_t, we obtain a per capita form of the constraint:

$$(1+n) b_{t+1} + \tau_t^k R_t \frac{s_{t-1}}{1+n} + \tau_t w_t h_t = (1+n) x_t + R_t b_t, \tag{5}$$

where $b_t \equiv B_t / N_t$ is per capita public debt.

2.4 Economic Equilibrium

Public bonds are traded in the domestic capital market. The market-clearing condition for capital is $B_{t+1} + K_{t+1} = N_t s_t$, which expresses the equality of total savings by the middle-aged population in period t, $N_t s_t$, to the sum of the stocks of aggregate public debt and aggregate physical capital at the beginning of period $t+1$, $B_{t+1} + K_{t+1}$. By using $\hat{k}_{t+1} \equiv K_{t+1}/H_{t+1}$, $h_{t+1} = H_{t+1}/N_{t+1}$, and the savings function in (2), we can rewrite the condition as

$$(1+n)\left(\hat{k}_{t+1} h_{t+1} + b_{t+1}\right) = \frac{\beta}{1+\beta}(1-\tau_t) w_t h_t. \tag{6}$$

The following defines the economic equilibrium in the present model.

Definition 1. Given a sequence of policies, $\left\{\tau_t^k, \tau_t, x_t\right\}_{t=0}^{\infty}$, an *economic equilibrium* is a sequence of allocations $\left\{c_t, d_t, s_t, \hat{k}_{t+1}, b_{t+1}, h_{t+1}\right\}_{t=0}^{\infty}$ and prices $\{\rho_t, w_t, R_t\}_{t=0}^{\infty}$ with the initial conditions $\hat{k}_0 (> 0)$, $b_0 (\geq 0)$ and $h_0 (> 0)$, such that (i) given $\left(w_t, R_{t+1}, \tau_t^k, \tau_t, x_t\right)$, $\left(c_t^y, c_{t+1}^o, s_t\right)$ solves the utility maximization problem; (ii) given (w_t, ρ_t), $\hat{k}_t$ solves a firm's profit maximization problem; (iii) given (w_t, h_t, R_t, b_t), $\left(\tau_t^k, \tau_t, x_t, b_{t+1}\right)$ satisfies the government budget constraint; (iv) an arbitrage condition $\rho_t = R_t$ holds; and (v) the capital market clears: $(1+n) \cdot \left(\hat{k}_{t+1} h_{t+1} + b_{t+1}\right) = s_t$.

In the economic equilibrium, the indirect utility of the middle-aged population in period t, V_t^M, and that of the old population in period t, V_t^o, can be expressed as functions of fiscal policy, physical and human capital, and public debt as follows:

$$\begin{aligned} V_t^M &= (1+\beta)\ln(1-\alpha)A\left(\hat{k}_t\right)^{\alpha} h_t\left(1-\tau_t\right) + \beta\ln R\left(\hat{k}_{t+1}\right) \\ &\quad + \beta\ln\left(1-\tau_{t+1}^k\right) + \phi^M, \end{aligned} \tag{7}$$

$$V_t^o = \ln\left(1-\tau_t^k\right) + \phi^O\left(\hat{k}_t, h_t, b_t\right), \tag{8}$$

where $R\left(\hat{k}_{t+1}\right) \equiv \alpha A\left(\hat{k}_{t+1}\right)^{\alpha-1}$ is the gross return from investment in physical capital, and ϕ^M and $\phi^O\left(k_t, \hat{h}_t, \hat{b}_t\right)$, including policy-irrelevant and constant terms, are defined by

$$\begin{aligned} \phi^M &\equiv \left(\ln\frac{1}{1+\beta} + \beta\ln\frac{\beta}{1+\beta}\right) + \beta\ln\alpha A, \\ \phi^O\left(\hat{k}_t, h_t, b_t\right) &\equiv \ln\alpha A\left(\hat{k}_t\right)^{\alpha-1}(1+n)\left(\hat{k}_t h_t + b_t\right), \end{aligned}$$

respectively.

3 Political Equilibrium

In this section, we consider voting on fiscal policy. In particular, we employ probabilistic voting à la Lindbeck and Weibull (1987). In this voting scheme, there is electoral competition between two office-seeking candidates. Each candidate announces a set of fiscal policies subject to the government budget constraint. As demonstrated by Persson and Tabellini (2000), the two candidates' platforms converge in the equilibrium to the same fiscal policy that maximizes the weighted average utility of voters.

In the present framework, the young, middle-aged, and elderly have an incentive to vote. While the young may benefit from current public education expenditure through human capital accumulation, we assume that their preferences are not taken into account by politicians. We impose this assumption, which is often used in the literature (e.g., Saint-Paul and Verdier, 1993; Bernasconi and Profeta, 2012; Lancia and Russo, 2016), for tractability reasons. However, the assumption could be supported in part by the fact that a large number of the young are below the voting age.

Thus, the political objective is defined as the weighted sum of the utility of the middle-aged and old, given by $\tilde{\Omega}_t \equiv \omega V_t^o + (1+n)(1-\omega)V_t^M$, where $\omega \in [0,1]$ and $1-\omega$ are the political weights placed on the old and middle-aged in period t, respectively. The weight of the middle-aged is adjusted by the gross population growth rate, $(1+n)$, to reflect their share of the population. To gain the intuition, we divide $\tilde{\Omega}_t$ by $(1+n)(1-\omega)$ and redefine the objective function as follows:

$$\Omega_t = \frac{\omega}{(1+n)(1-\omega)}V_t^o + V_t^M,$$

where the coefficient $\omega/(1+n)(1-\omega)$ of V_t^o represents the relative political weight of the old.

We substitute V_t^M in (7) and V_t^o in (8) into Ω_t and obtain

$$\begin{aligned}\Omega_t \simeq & \frac{\omega}{(1+n)(1-\omega)} \ln\left(1-\tau_t^k\right) + (1+\beta)\ln(1-\alpha) A\left(\hat{k}_t\right)^{\alpha} h_t (1-\tau_t) \\ & + \beta \ln R\left(\hat{k}_{t+1}\right) + \beta \ln\left(1-\tau_{t+1}^k\right). \end{aligned} \tag{9}$$

We use the notation $\simeq$ because irrelevant terms are omitted from the expression of Ω_t. With the use of (3)–(6), we can reformulate the expression in (9) as follows:

$$\begin{aligned}\Omega_t \simeq & \frac{\omega}{(1+n)(1-\omega)} \ln\left(1-\tau_t^k\right) + (1+\beta)\ln Z\left(\tau_t^k, x_t, b_{t+1}, \hat{k}_t, b_t, h_t\right) \\ & +\beta \ln R\left(\hat{K}\left(b_{t+1}, x_t, Z\left(\tau_t^k, x_t, b_{t+1}, \hat{k}_t, b_t, h_t\right)\right)\right) + \beta \ln\left(1-\tau_{t+1}^k\right), \end{aligned} \tag{10}$$

where $Z(\cdot)$, $\hat{K}(\cdot)$, and $R(\cdot)$ represent the after-tax income of the middle-aged, the next-period ratio of physical to human capital, and the gross return from investment in physical capital, respectively. With (3)–(6), they are defined as follows:

$$\begin{aligned} & Z(\tau_t^k, x_t, b_{t+1}, \hat{k}_t, b_t, h_t) \equiv A\left(\hat{k}_t\right)^{\alpha} h_t - \left(1-\tau_t^k\right)\alpha A\left(\hat{k}_t\right)^{\alpha-1}\left(\hat{k}_t h_t + b_t\right) - (1+n)x_t + (1+n)b_{t+1}, \\ & \hat{K}\left(b_{t+1}, x_t, Z\left(\tau_t^k, x_t, b_{t+1}, \hat{k}_t, b_t, h_t\right)\right) \equiv \frac{\frac{\beta}{1+\beta}Z(\cdot) - (1+n)b_{t+1}}{(1+n)D(x_t)^{\eta}}, \\ & R\left(\hat{K}(\cdot)\right) \equiv \alpha A\left(\hat{K}(\cdot)\right)^{\alpha-1}. \end{aligned}$$

The political objective function in (10) suggests that the current policy choice affects the decision on future policy via physical and human capital accumulation. In particular, the period t choices of τ_t^k, x_t, and b_{t+1} affect the formation of physical and human capital in period $t+1$. This in turn influences the decision making on period-$t+1$ fiscal policy. To demonstrate such an intertemporal effect, we employ the concept of a Markov-perfect equilibrium under which fiscal policy today depends on the current payoff-relevant state variables.

In the present framework, the payoff-relevant state variables are the ratio of physical to human capital, $\hat{k}_t$, public debt, b_t, and human capital, h_t. Thus, the expected rate of capital income tax for the next period, τ_{t+1}^k, is given by the function of the period-$t+1$ state variables, $\tau_{t+1}^k = T^k\left(\hat{k}_{t+1}, b_{t+1}, h_{t+1}\right)$. We denote by $-\underline{\tau}(<0)$ and $-\underline{\tau}^k(<0)$ the arbitrary lower limits of τ and τ^k, respectively. By using recursive notation with z' denoting the next period z, we can now define a Markov-perfect political equilibrium in the present framework as follows.

Definition 2. A *Markov-perfect political equilibrium* is a set of functions, $\langle T, T^k, X, B\rangle$, where $T : \Re^3_+ \to (-\underline{\tau}, 1)$ is a labor income tax rule, $\tau = T(\hat{k}, b, h)$, $T^k : \Re^3_+ \to \left(-\underline{\tau}^k, 1\right)$ is a capital income tax rule, $\tau^k = T^k(\hat{k}, b, h)$, $X : \Re^3_+ \to \Re_+$ is a public education expenditure rule, $x = X(\hat{k}, b, h)$, and $B : \Re^3_+ \to \Re_+$ is a public debt rule, $b' = B(\hat{k}, b, h)$, such that the following conditions are satisfied:

(i) The capital market clears,

$$(1+n)\left(\hat{k}'h' + b'\right) = \frac{\beta}{1+\beta}(1-\tau)(1-\alpha)A\left(\hat{k}\right)^{\alpha} h; \tag{11}$$

(ii) Given $\hat{k}$, b, and h, $\left\langle T(\hat{k}, b, h), T^k(\hat{k}, b, h), X(\hat{k}, b, h), B(\hat{k}, b, h)\right\rangle = \arg\max \Omega$ subject to $\tau^{k\prime} = T^k\left(\hat{k}', b', h'\right)$, the capital market-clearing condition in (11), the government budget constraint,

$$(1+n)b' + \tau(1-\alpha)A\left(\hat{k}\right)^{\alpha} h + \tau^k \alpha A\left(\hat{k}\right)^{\alpha-1}\left(\hat{k}h + b\right) = (1+n)x + \alpha A\left(\hat{k}\right)^{\alpha-1} b, \tag{12}$$

and the human capital formation function, $h' = D(h)^{1-\eta}\left(X(\hat{k}, b, h)\right)^{\eta}$, where Ω is defined by (10).

Two remarks are in order. First, Definition 2 allows the tax rates to be negative. However, the following analysis shows that the capital income tax rate from period 1 onward is positive. It also establishes the conditions for the capital income tax rate in period 0 and the labor income tax rate in the long run to be positive. Second, the state variables do not align in compact sets because they grow across periods. To define the equilibrium more precisely, we need to redefine the equilibrium as a mapping from a compact set to a compact set. We can do this by introducing the following notations: $\tilde{x}_t \equiv x_t / A\left(\hat{k}_t\right)^{\alpha} h_t$ and $\tilde{b}_{t+1} \equiv b_{t+1} / A\left(\hat{k}_t\right)^{\alpha} h_t$. However, for the simplicity of the exposition, we define the equilibrium as in Definition 2.

3.1 Characterization of the Political Equilibrium

To obtain the set of policy functions in Definition 2, we conjecture the following capital tax rate in the next period:

$$\tau^{k\prime} = 1 - T^k_{un} \frac{1}{\alpha\left(1 + \frac{b'}{\hat{k}'h'}\right)}, \tag{13}$$

where $T^k_{un} (> 0)$ is constant. The subscript "un" means that public debt issuance is "unconstrained." In the next section, we consider a case where public debt issuance is "constrained" by a constitutional rule and compare it with the unconstrained case.

Given the conjecture in (13), we consider the maximization of Ω in (10). The first-order conditions with respect to b', τ^k, and x are as follows:

$$b' : \underbrace{\frac{1+\beta}{Z}\frac{\partial Z}{\partial b'}}_{\text{(b.1)}} + \frac{\beta}{R}\frac{\partial R}{\partial \hat{K}}\left(\underbrace{\frac{\partial \hat{K}}{\partial Z}\frac{\partial Z}{\partial b'}}_{\text{(b.2)}} + \underbrace{\frac{\partial \hat{K}}{\partial b'}}_{\text{(b.3)}}\right) + \frac{\beta}{1-\tau^k}\left\{\frac{\partial\left(1-\tau^{k\prime}\right)}{\partial \hat{K}}\left[\underbrace{\frac{\partial \hat{K}}{\partial Z}\frac{\partial Z}{\partial b'}}_{\text{(b.4)}} + \underbrace{\frac{\partial \hat{K}}{\partial b'}}_{\text{(b.5)}}\right] + \underbrace{\frac{\partial\left(1-\tau^{k\prime}\right)}{\partial b'}}_{\text{(b.6)}}\right\} \leq 0, \tag{14}$$

$$\tau^k : \underbrace{-\frac{\omega}{(1+n)(1-\omega)}\frac{1}{1-\tau^k}}_{\text{(k.1)}} + \underbrace{\frac{1+\beta}{Z}\frac{\partial Z}{\partial \tau^k}}_{\text{(k.2)}} + \underbrace{\frac{\beta}{R}\frac{\partial R}{\partial \hat{K}}\frac{\partial \hat{K}}{\partial Z}\frac{\partial Z}{\partial \tau^k}}_{\text{(k.3)}} + \underbrace{\frac{\beta}{1-\tau^{k\prime}}\frac{\partial\left(1-\tau^{k\prime}\right)}{\partial \hat{K}}\frac{\partial \hat{K}}{\partial Z}\frac{\partial Z}{\partial \tau^k}}_{\text{(k.4)}} \leq 0, \tag{15}$$

$$x : \underbrace{\frac{1+\beta}{Z}\frac{\partial Z}{\partial x}}_{\text{(x.1)}} + \frac{\beta}{R}\frac{\partial R}{\partial \hat{K}}\left(\underbrace{\frac{\partial \hat{K}}{\partial Z}\frac{\partial Z}{\partial x}}_{\text{(x.2)}} + \underbrace{\frac{\partial \hat{K}}{\partial x}}_{\text{(x.3)}}\right) + \frac{\beta}{1-\tau^{k\prime}}\left\{\frac{\partial\left(1-\tau^{k\prime}\right)}{\partial \hat{K}}\left[\underbrace{\frac{\partial \hat{K}}{\partial Z}\frac{\partial Z}{\partial x}}_{\text{(x.4)}} + \underbrace{\frac{\partial \hat{K}}{\partial x}}_{\text{(x.5)}}\right] + \underbrace{\frac{\partial\left(1-\tau^{k\prime}\right)}{\partial h'}\frac{\partial h'}{\partial x}}_{\text{(x.6)}}\right\} = 0, \tag{16}$$

where a strict inequality holds in (14) if $b' = 0$ and in (15) if $\tau^k = 0$. By using these conditions, we can verify the conjecture in (13) and obtain the following result.

Proposition 1. *There is a Markov-perfect political equilibrium characterized by $b' > 0$. The corresponding policy functions of b', τ^k, x, and τ are as follows:*

$$(1+n)b' = B_{un}A\left(\hat{k}\right)^{\alpha}h,$$
$$\tau^k = 1 - T^k_{un}\frac{1}{\alpha\left(1+\frac{b}{\hat{k}h}\right)},$$
$$(1+n)x = X_{un}A\left(\hat{k}\right)^{\alpha}h,$$
$$\tau = 1 - T_{un},$$

where B_{un}, T^k_{un}, X_{un}, and T_{un} are defined by

$$B_{un} \equiv \frac{\beta(1-\alpha)}{\frac{\omega}{(1+n)(1-\omega)} + 1 + \beta\left[\alpha + \eta(1-\alpha)\right]},$$
$$T^k_{un} \equiv \frac{\frac{\omega}{(1+n)(1-\omega)}}{\frac{\omega}{(1+n)(1-\omega)} + 1 + \beta\left[\alpha + \eta(1-\alpha)\right]},$$
$$X_{un} \equiv \frac{\beta\eta(1-\alpha)}{\frac{\omega}{(1+n)(1-\omega)} + 1 + \beta\left[\alpha + \eta(1-\alpha)\right]},$$
$$T_{un} \equiv \frac{1+\beta}{1-\alpha}\cdot\frac{1}{\frac{\omega}{(1+n)(1-\omega)} + 1 + \beta\left[\alpha + \eta(1-\alpha)\right]}.$$

Proof. See Appendix A.1.

The result in Proposition 1 states that the government always borrows in the capital market and thus passes a part of the tax burden onto future generations by issuing public debt. To understand the mechanism behind this result, recall the first-order condition with respect to b' in (14). The condition indicates that the issue of public debt creates a crowding-out effect on physical capital and thus raises the return from savings as presented by the term (b.3). However, under the conjecture in (13), it also raises the next period capital income tax burden as presented by the terms (b.5) and (b.6).

The issue of public debt enables the government to cut labor income tax on the middle-aged. This lowers their tax burden and thus increases their lifetime consumption, as represented by the term (b.1). At the same time, it increases the saving of the middle-aged, the ratio of physical to human capital, and thus lowers the next period capital income tax burden as represented by the term (b.4). However, an increase in the physical to human capital ratio lowers the return from saving, R, as presented by the term (b.2). To summarize, three marginal benefits, represented by the terms (b.1), (b.3), and (b.4), and three marginal costs, represented by the terms (b.2), (b.5), and (b.6), arise from public debt issues. The sum of the former ones outweigh the sum of the latter ones in the present framework. Therefore, the government finds it optimal to issue public debt.

Next, recall the first-order condition with respect to τ^k in (15) to consider the formation of the policy function of the capital income tax rate. The term (k.1) shows the marginal cost of taxation for the old; raising the tax rate increases their tax burden and thus lowers their consumption. The terms (k.2)–(k.4) present the marginal cost or benefit for the middle-aged. The government can cut the labor income tax rate and thus lower the tax burden on the middle-aged by raising the capital income tax rate. This creates a positive income effect on the consumption of the middle-aged, as presented by the term (k.2). In addition, it creates a positive income effect on saving and physical capital formation, and thus lowers the capital income tax burden in the next period, as presented by the term (k.4). However, at the same time, it lowers the return from saving, as presented by the term (k.3). Therefore, there are two marginal benefits, presented by the terms (b.2) and (b.4), and two marginal costs, presented by the terms (b.1) and (b.3). The net effect of these forces is summarized in the following corollary.

Corollary 1. *For $\omega \in [0,1)$, the capital income tax rate is* (i) *positive for $t \geq 1$; and* (ii) *positive in period* 0 *if and only if the following condition holds:*

$$\frac{b_0}{\hat{k}_0 h_0} > \frac{\frac{\omega}{(1+n)(1-\omega)}}{\alpha\left[\frac{\omega}{(1+n)(1-\omega)} + 1 + \beta\left(\alpha + \eta\left(1-\alpha\right)\right)\right]} - 1.$$

Proof. See Appendix A.2.

The result in Corollary 1 suggests that the political weight of the old, denoted by ω, is crucial to determining the capital income tax rate in the political equilibrium. For $t \geq 1$, the capital income tax rate is given by

$$\tau_t^k = 1 - \frac{1}{1 + \frac{(1+n)(1-\omega)}{\omega}\left\{1 + \beta\left[\alpha + \eta\left(1-\alpha\right)\right]\right\}}.$$

The old want to reduce their tax burdens because they obtain no benefit from public education expenditure. This implies that the tax rate decreases as the political weight of the old rises. In particular, the tax rate becomes zero as the weight of the old approaches 100%. In other words, the capital income tax rate remains positive as long as the government attaches some weight to the middle-aged. Thus, the result in Corollary 1 implies that the presence of generational conflict allows the government to levy a capital income tax rate on the old.

Finally, recall the first-order conditions with respect to x in (16) to consider the formation of the policy function of public education expenditure. By comparing (14) with (16), we find that the effects of a decrease in public debt issues are qualitatively equivalent to the effects of an increase in public education expenditure. Thus, the government chooses the expenditure to balance the sum of the marginal benefits, represented by the terms (x.2), (x.5), and (x.6), and the sum of the marginal costs, represented by the terms (x.1), (x.3), and (x.4). Given the policy functions of b', x, and τ^k, the labor income tax rate τ is determined to satisfy the government budget constraint.

3.2 Steady State

Having established the policy functions, we are now ready to demonstrate the accumulation of physical and human capital. We substitute the policy functions in Proposition 1 into the capital market-clearing condition in (11) and human capital formation function in (1), and obtain

$$\hat{k}' = \Psi_K \left[A\left(\hat{k}\right)^{\alpha}\right]^{1-\eta}, \tag{17}$$

$$\frac{h'}{h} = D\Psi_H \left[A\left(\hat{k}\right)^{\alpha}\right]^{\eta}, \tag{18}$$

where Ψ_K and Ψ_H are defined by

$$\Psi_K \equiv \frac{\alpha\beta}{\frac{\omega}{(1+n)(1-\omega)} + 1 + \beta\left[\alpha + \eta\left(1-\alpha\right)\right]}\left\{(1+n)D\left[\frac{X_{un}}{1+n}\right]^{\eta}\right\}^{-1}, \tag{19}$$

and

$$\Psi_H \equiv \left[\frac{X_{un}}{1+n}\right]^{\eta}, \tag{20}$$

respectively. Appendix A.3 shows the derivation of (17) and (18).

Given $\{\hat{k}_0, h_0\}$, the sequence $\{\hat{k}_t, h_t\}$ is distinguished by the above two equations in (17) and (18). A *steady state* is defined as a political equilibrium with $\hat{k}_t = \hat{k}_{t+1}$. In other words, the ratio of physical to human capital is constant in a steady state. Equation (17) indicates that there is a unique, stable steady state. Along the steady-state path, the capital income tax rate remains within $[0, 1)$, as shown in Corollary 1, and human capital increases, as suggested in (18). The following proposition summarizes the argument thus far and identifies the conditions under which the labor income tax rate is set within the range $[0, 1)$ in the steady state.

Proposition 2. *There is a unique, stable steady-state equilibrium with $b' > 0$, $\tau^k \in [0, 1)$, and $\tau \in [0, 1)$ if*

$$\frac{1+\beta}{1-\alpha} - \{1 + \beta [\alpha + \eta (1-\alpha)]\} \leq \frac{\omega}{(1+n)(1-\omega)}. \tag{21}$$

Proof. See Appendix A.3.

The result in Proposition 3 suggests that the relative political weight of the old, represented by $\omega/(1 + n)(1 - \omega)$, plays a crucial role in shaping the labor income tax rate. When $\omega/(1 + n)(1 - \omega)$ is below the lower bound in (21), the relative political weight of the old is too low to incentivize the government to tax the young. The government would rather subsidize the young by choosing $\tau < 0$. We rule out this possibility by imposing the upper bound of $\omega/(1 + n)(1 - \omega)$.

4 Fiscal Rules

In the previous section, we considered fiscal policy and economic growth in the absence of constraints on public bond issues except for the flow budget constraint. In other words, we assumed no rule on public bond issuance. However, in practice, many countries have introduced fiscal rules that control public debt (Schaechter et al., 2012; Budina et al., 2012). In addition, in the present framework, public bond issuance creates a crowding-out effect on physical capital formation and economic growth, which in turn triggers the welfare loss for future generations. This observation motivates us to consider the question of how fiscal rules shape the choice of fiscal policy and affect economic growth and welfare across time and generations.

To answer this question, in Section 4.1 we first consider the following alternative scenario in which the government is prohibited from issuing public bonds and thus its expenditure is financed solely through taxation. As shown in Proposition 1, in the absence of the tax-financing rule, the government borrows in the capital market and issues public bonds. In other words, the government wants to issue public bonds to finance its expenditure, but their issuance is prohibited when the tax-financing rule is introduced. We then compare the tax rates, expenditure, and economic growth in the debt-financing case in the previous section with those in the tax-financing case. We also investigate the welfare consequences of shifting from debt financing to tax financing.

The requirement for tax financing is somewhat extreme because in reality the government is allowed to issue public bonds as long as their issuance is below some debt ceiling. Hence, in Section 4.2, we overcome this shortcoming by considering an alternative fiscal rule for managing the debt issuance-to-GDP ratio widely introduced in developed countries.

4.1 Tax Financing versus Debt Financing

The policy functions in the tax-financing case are obtained by assuming $b' = 0$ in the first-order conditions with respect to τ^k, b', and x in (15)–(16) (see Appendix A.4). To investigate the differences between the tax-financing and debt-financing cases, we compare their tax rates, τ^k and τ, public education expenditure-to-GDP ratio, $(1+n)x/y$, where $y \equiv Y/N$ is per capita GDP, and economic growth, h'/h. The variables in the tax-financing and debt-financing cases are denoted by the subscripts "tax" and "debt,"respectively.

Proposition 3. *Given the initial conditions k_0 and b_0, tax financing and debt financing are compared as follows:*

$$\tau_0^k\big|_{tax} = \tau_0^k\big|_{debt}; \;\; \tau_t^k\big|_{tax} < \tau_t^k\big|_{debt} \text{ for } t \geq 1; \;\; \tau|_{tax} > \tau|_{debt};$$
$$(1+n)x/y|_{tax} = (1+n)x/y|_{debt}; \text{ and } \; h'/h|_{tax} > h'/h|_{debt}.$$

Proof. See Appendix A.4.

In the initial period, the government needs to finance the repayment of outstanding public debt, b_0, regardless of the financing method. Thus, the capital tax rates are equal in the two financing cases in the initial period. However, from period 1 onward, the government incurs no repayment costs in the tax-financing case, while it still incurs such costs in the debt-financing case. Because of this difference, the capital tax rate is lower in the tax-financing case than in the debt-financing case from period 1.

By contrast, the labor tax rate is higher in the tax-financing case than in the debt-financing case. When the tax-financing rule is introduced, the government needs to compensate for the loss of revenue from bond issues by raising the labor income tax rate. An increase in revenue from the labor tax is offset by a decrease in revenue from the capital tax and public bond issues. Thus, the education expenditure-to-GDP ratio remains unchanged. However, the introduction of tax financing removes the crowding-out effect of public bonds. This positive effect on physical capital enhances human capital accumulation and economic growth.

The result in Proposition 3 suggests that the shift from debt financing to tax financing increases the growth rate and benefits future generations, but may worsen the current middle-aged population because of the increased labor tax burden. We investigate this welfare implication and obtain the following result.

Proposition 4. *Individuals who are middle-aged in period t are called generation t.* (i) *The welfare of the initial old population is unaffected; generation 0 is made worse off by shifting from debt financing to tax financing.* (ii) *There is a critical period, denoted by $\hat{t}(>1)$, such that generation $t \leq \hat{t}$ is made worse off, whereas generation $t > \hat{t}$ is made better off by shifting from debt financing to tax financing.*

Proof. See Appendix A.5.

The welfare of the initial old population is unaffected by shifting to tax financing since their tax burden is unchanged. However, the choice of tax financing has two opposing effects on current and future generations. Tax financing raises the tax burden of the middle-aged population as demonstrated in Proposition 3. This lowers the lifetime income of the middle-aged and thus lowers their lifetime utility of consumption. This is the negative effect of tax financing. However, tax financing removes the crowding-out effect of public bonds on capital and thus enhances human capital accumulation. This positive effect appears from generation 1 onward and accumulates over time, but generation 0 cannot enjoy this benefit. Therefore, generation 0 suffers from a negative effect, whereas distant future generations benefit from a positive effect that outweighs the negative one.

4.2 Debt Ceiling

This section extends the analysis of the previous section by considering the following debt rule:

$$\frac{B_{t+1}}{Y_t} \leq \bar{u}.$$

This rule resembles the debt rule that sets an explicit ceiling for public debt as a percentage of GDP (Schaechter et al., 2012). This is reformulated as

$$(1+n)b_{t+1} \leq \bar{u}A\left(\hat{k}_t\right)^{\alpha} h_t, \tag{22}$$

where $\bar{u}$ is defined by

$$\bar{u} \equiv \varepsilon B_{un}, \quad \varepsilon \in [0,1),$$

and the definition of B_{un} is provided in Proposition 1. The rule resembles the tax-financing case in Section 4.1 when $\varepsilon \to 0$ and the unconstrained debt-financing case in Section 3 when $\varepsilon \to 1$.

Debt issuance in the absence of the rule in (22) is given by $(1+n)b' = B_{un}A\left(\hat{k}\right)^{\alpha}\hat{h}$ as demonstrated in Proposition 1. When the debt rule in (22) is introduced, it is always binding since $\varepsilon < 1$. Thus, the issue of public bonds in the presence of the rule in (22) is

$$(1+n)b' = \varepsilon B_{un}A\left(\hat{k}\right)^{\alpha} h.$$

With the use of $(1+n)b' = \varepsilon B_{un} A \left(\hat{k}\right)^{\alpha} h$, the political objective function in (10) is reformulated as follows:

$$\Omega \simeq \frac{\omega}{(1+n)(1-\omega)} \ln\left(1-\tau^k\right) + (1+\beta)\ln Z\left(\tau^k, x, \frac{\varepsilon B_{un} A \left(\hat{k}\right)^{\alpha} h}{1+n}, \hat{k}, b, h\right)$$
$$+ \beta \ln R\left(\hat{K}\left(\frac{\varepsilon B_{un} A \left(\hat{k}\right)^{\alpha} h}{1+n}, x_t, Z\left(\tau^k, x, \frac{\varepsilon B_{un} A \left(\hat{k}\right)^{\alpha} h}{1+n}, \hat{k}, b, h\right)\right)\right) + \beta \ln\left(1-\tau^{k\prime}\right). \tag{23}$$

Following the procedure described in Section 3, we consider the maximization of Ω with respect to τ^k and x, and obtain the following result.

Proposition 5. *In the presence of the debt rule in (22), a Markov-perfect political equilibrium is characterized by the following policy functions:*

$$\tau^k = 1 - T^k_{con} \frac{1}{\alpha\left(1 + b/\hat{k}h\right)},$$
$$(1+n)x = X_{con} A \left(\hat{k}\right)^{\alpha} h,$$
$$\tau = 1 - \frac{1}{1-\alpha}\left[(1+\varepsilon B_{un}) - \left(1 + \frac{\omega}{(1+n)(1-\omega)} \cdot \frac{1}{\beta\eta(1-\alpha)}\right) X_{con}\right],$$

where B_{un} is defined in Proposition 1, and T^k_{con}, X_{con}, and the associated variables are defined as

$$T^k_{con} \equiv \frac{1}{\alpha} \cdot \frac{\omega}{(1+n)(1-\omega)} \cdot \frac{1}{\beta\eta(1-\alpha)} \cdot \frac{H - \sqrt{(H)^2 - 4GI}}{2G},$$
$$X_{con} \equiv \frac{H - \sqrt{(H)^2 - 4GI}}{2G},$$
$$G \equiv \left[1 + \frac{\omega}{(1+n)(1-\omega)} \cdot \frac{1}{\beta\eta(1-\alpha)}\right]\left[\frac{\omega}{(1+n)(1-\omega)} + 1 + \beta\left[\alpha + \eta(1-\alpha)\right]\right] > 0,$$
$$H \equiv \beta\eta(1-\alpha)\left[1 + \frac{\omega}{(1+n)(1-\omega)} \cdot \frac{1}{\beta\eta(1-\alpha)}\right]\left[(1+\varepsilon B_{un}) + \left(1 - \frac{\varepsilon B_{un}}{\beta}\right)\right]$$
$$+ \alpha\beta(1+\varepsilon B_{un}) + \left(1 - \frac{\varepsilon B_{un}}{\beta}\right)$$
$$> 0,$$
$$I \equiv \beta\eta(1-\alpha)(1+\varepsilon B_{un})\left(1 - \frac{\varepsilon B_{un}}{\beta}\right) > 0.$$

Proof. See Appendix A.6.

Following the procedure described in Section 3, we show the existence and uniqueness of steady-state capital. Recall the capital market-clearing condition in (11), which is rewritten as follows:

$$\begin{aligned}\hat{k}' &= \frac{\frac{\beta}{1+\beta}(1-\tau)(1-\alpha)-\varepsilon B_{un}}{(1+n)D(h)^{1-\eta}(x)^{\eta}} A(\hat{k})^{\alpha} h \\ &= \frac{\frac{\beta}{1+\beta}(1-\tau)(1-\alpha)-\varepsilon B_{un}}{(1+n)D\left(\frac{X_{con}}{1+n}\right)^{\eta}} \left[A(\hat{k})^{\alpha}\right]^{1-\eta}, \end{aligned} \tag{24}$$

where the first equality comes from the human capital formation function given by $h' = D(h)^{1-\eta}(x)^{\eta}$ and the second equality comes from the policy function of x presented in Proposition 5. Equation (24) indicates that there is unique, stable steady-state capital. In the next section, we focus on steady states and compare cases in the presence and absence of the debt rule in (22) in terms of the education expenditure-to-GDP ratio, capital and labor taxes, and growth rates. We also compare the cases in terms of utility across generations.

4.3 Numerical Analysis

Our task here is to compare cases in the absence and presence of the debt rule in (22) based on numerical methods. Our strategy is to calibrate the model economy such that the steady-state equilibrium with $b > 0$ matches some key statistics of average OECD countries during 1995–2014.[5] We fix the share of capital at $\alpha = 1/3$ following Song, Storesletten, and Zilibotti (2012) and Lancia and Russo (2016). Each period lasts 30 years; this assumption is standard in quantitative analyses of the two- or three-period overlapping-generations model (e.g., Gonzalez-Eiras and Niepelt, 2008; Lancia and Russo, 2016). Our selection of β is 0.99 per quarter, which is also standard in the literature (e.g., Kydland and Prescott, 1982; de la Croix and Doepke, 2002). Since agents in the present model plan over generations that span 30 years, we discount the future by $(0.99)^{120}$.

We assume an annual population growth rate of 1.0059, which was the OECD average during 1995–2014. This assumption implies that the net population growth rate for 30 years is $(1.0059)^{30} - 1$. Following Song, Storesletten, and Zilibotti (2016) and Lancia and Russo (2016), we set ω to 0.48.

For η, we focus on the education expenditure-to-GDP ratio in the steady state:

$$\frac{N_{t+1}x_t}{Y_t} = X_{un} = \frac{\beta\eta(1-\alpha)}{\frac{\omega}{(1+n)(1-\omega)} + 1 + \beta[\alpha + \eta(1-\alpha)]}.$$

Given $\alpha = 1/3$, $\beta = (0.99)^{120}$, $1 + n = (1.0059)^{30}$, and $\omega = 0.48$, we can solve this

[5]Source: World Development Indicators, https://data.worldbank.org/products/wdi (Accessed on March 3, 2018).

expression for η by using the average ratio observed in OECD countries of 0.051 and obtain $\eta = 0.504$.

To determine the two productivity parameters, A and D, we normalize the steady-state wage, w, to unity. Thus, we have $w = (1-\alpha)A(k)^{\alpha} = 1$, or

$$(1-\alpha)\left[\frac{\frac{\beta}{1+\beta}(1-\tau)(1-\alpha) - B_{un}}{(1+n)\left(\frac{X_{un}}{1+n}\right)^{\eta}}\right]^{\alpha/(1-\alpha(1-\eta))} (D)^{-\alpha/(1-\alpha(1-\eta))}(A)^{1/(1-\alpha(1-\eta))} = 1. \quad (25)$$

We also use the data on the per capita GDP gross growth rate of 1.02, which was the OECD average during 1995–2014. We substitute these data and the values of α, β, n, η, and ω into the following equation expressing the per capita GDP gross growth rate:

$$\frac{h'}{h} = \left(\frac{X_{un}}{1+n}\right)^{\eta}\left[\frac{\frac{\beta}{1+\beta}(1-\tau)(1-\alpha) - B_{un}}{(1+n)\left(\frac{X_{un}}{1+n}\right)^{\eta}}\right]^{\alpha\eta/(1-\alpha(1-\eta))} (D)^{(1-\alpha)/(1-\alpha(1-\eta))}(A)^{\eta/(1-\alpha(1-\eta))}$$
$$= (1.02)^{30}. \quad (26)$$

We solve the two equations, (25) and (26), for A and D, and obtain $A = 4.58$ and $D = 7.24$.

The economy is assumed to be in a steady state in period 0. The initial capital $\hat{k}_0$ is computed by solving equation (24) for $\hat{k}$. The initial value of human capital, h_0, is normalized at $h_0 = 1$. From the result in Section 3, in the absence of any fiscal rule, the ratio $b/\hat{k}h$ in the steady state is given by $b/\hat{k}h = (1-\alpha)/\alpha$. Thus, we set b_0 at $b_0 = [(1-\alpha)/\alpha]\hat{k}_0 h_0$ and compare the cases with and without a debt rule for the same initial conditions.

4.3.1 Comparative Statics

We study how the steady-state equilibrium responds to changes in the debt rule in (22). In particular, we focus on ε. When $\varepsilon = 1$, the equilibrium policy functions and corresponding economic growth rate coincide with those in the absence of the debt rule as in Section 3. When $\varepsilon = 0$, they coincide with those in the tax-financing case as in Section 4.1. In the following, we consider a decrease in ε that aims to tighten fiscal discipline and investigate its impact on fiscal policy, economic growth, and welfare across generations. Figure 1 plots the education expenditure-to-GDP ratio, labor and capital tax rates, ratio of physical to human capital, and per capita growth rate in the steady state, taking ε on the horizontal axis from 0 to 1.[6]

[Figure 1 here.]

[6] In Figure 1, each panel illustrates two cases, $\omega = 0.48$ and 0.6. The analysis here is restricted to the case of $\omega = 0.48$. The case of $\omega = 0.6$ is discussed in Section 5.

Fiscal tightening (i.e., a decrease in ε) has the following effects on the education expenditure-to-GDP ratio. Firstly, the government raises the labor tax rate to compensate for the loss of revenue from public bond issues, as depicted in Panel (a). This lowers the disposable income of the middle-aged population, which in turn raises the marginal cost of public education expenditure in terms of utility. This is a negative effect of fiscal discipline on public education expenditure. Secondly, a decrease in the disposable income leads to less saving and thus a lower ratio of physical to human capital. At the same time, fiscal tightening reduces the crowding-out effect of public debt. The net effect on the ratio of physical to human capital is positive as shown in Panel (b), implying a negative effect on the interest rate. This general equilibrium effect through the interest rate in turn leads the government to increase public education expenditure, which reduces savings and thus works to raise the interest rate. In sum, the two opposing effects on the choice of public education expenditure produces an initial increase in the education expenditure-to-GDP ratio followed by a decrease, as depicted in Panel (c).

Next, consider the effect of fiscal tightening on the choice of the capital tax rate. Firstly, this raises the marginal benefit of capital taxation. This positive effect parallels the effect on the choice of public education expenditure as described above. However, an additional effect on the marginal benefit arises through the term $\hat{b}/kh$ in the policy function of τ^k. Fiscal tightening lowers the ratio $\hat{b}/kh$ and thus produces a negative effect on the marginal benefit. This negative effect outweighs the positive effect. Therefore, the government chooses a lower capital tax rate to balance the marginal cost and benefit as ε decreases, as depicted in Panel (d). In other words, fiscal tightening shifts the tax burden from the old to the middle-aged. Finally, fiscal tightening raises the per capita growth rate as shown in Panel (e) because the positive effects through the increased ratio of physical to human capital and decreased capital tax rate outweigh the negative effect through the increased labor tax and non-monotone effect through public education expenditure.

4.3.2 Comparative Dynamics

The comparative static analysis shows that the physical-to-human capital ratio and steady-state growth rate increase as ε decreases. This finding suggests that future generations benefit from increased physical and human capital. However, are all generations made better off by fiscal tightening? To answer this question, Figure 2 plots the evolution of the key indicators from the initial old population for three scenarios, $\varepsilon = 0.2$, 0.5, and 0.8. In Panels (a)–(e), we take the ratio of a variable in the presence of the debt ceiling to that in its absence for each period. The lines in each panel imply the ratio of the relevant variable in the presence of the debt ceiling to that in its absence. Each ratio implies that the presence of the debt ceiling outweighs its absence when the ratio is above unity.

In Panel (f), we plot the difference in utility between the presence and absence of the debt ceiling from generation -1 to generation 4. Generation t attains higher (lower) utility in the presence of the debt ceiling than in its absence when the difference is positive (negative). The figure indicates that the initial old population as well as generation 1 onward are made better off by the introduction of the debt rule in (22), whereas generation 0 is made worse off. Thus, fiscal tightening is not Pareto-improving.

[Figure 2 here.]

The mechanism behind the result is straightforward. Under the present assumption, the government's optimal choice of the debt-to-GDP ratio is B_{un}; however, its choice is limited up to $\varepsilon B_{un}(< B_{un})$ by the rule in (22). Because of this constraint, the government in period 0 is unable to attain an "interior optimum." In particular, the constraint hits the middle-aged population in generation 0. Governments from period 1 are also constrained by the rule, but they benefit from the higher levels of physical and human capital bequeathed from past generations. This benefit outweighs the cost of the constraint in (22). Therefore, the introduction of the debt rule creates a trade-off between generations in terms of utility.

The effects of decreased ε on utility is monotone from generation 0 onward, as shown in Figure 2. However, the effect is non-monotone for the initial old population. In particular, a decrease in ε from 1.0 to 0.5 improves their utility, but a further decrease worsens it. This non-monotone effect stems from the initial decrease followed by an increase in the period 0 capital tax rate, as depicted in Figure 3. The U-shaped pattern of the period 0 capital tax rate parallels the hump-shaped pattern of public education expenditure described above.

[Figure 3 here.]

Finally, we consider the case where the debt rule is imposed only for limited periods. In particular, the debt rule in (22) is introduced in period 2, but terminated at the end of period 2, 3, or 4, meaning that successive governments from the termination period onward are free to choose policies with no rule. Figure 4 illustrates the effect of this temporary implementation of the fiscal rule on fiscal policies, the ratio of physical to human capital, per capita human capital, per capita GDP, and consumption and utility across generations when $\varepsilon = 0.5$. As expected, the rule produces a temporary effect on fiscal policies, the physical-to-human capital ratio, and per capita GDP. However, it has a long-lasting effect on utility across generations owing to the increased human capital bequeathed from generation to generation. This long-lasting effect of the temporary rule, which was not shown by Barseghyan and Battaglini (2016), is caused by human capital accumulation stimulated by debt-financed public education expenditure. Therefore, this result suggests the importance of connecting seemingly unrelated subjects, public education and public debt, to consider the impact of a debt rule over time and across generations.

[Figure 4 here.]

5 Planner's Allocation

In the previous section, we use the Pareto criterion to evaluate the welfare consequence of the debt rule. Here, we take an alternative approach by deriving an optimal allocation that maximizes an infinite discounted sum of generational utilities for an arbitrary social discount factor. In particular, we consider a benevolent planner who can commit to all his or her choices at the beginning of a period, subject to the human capital formation function and the resource constraint. Assuming such a planner, we evaluate the political equilibrium in the presence and absence of the debt ceiling by comparing it with the planner's allocation in terms of consumption, per capita GDP, and welfare over time and across generations.

The planner is assumed to value the welfare of all generations. In particular, the objective of the planner is to maximize a discounted sum of the lifecycle utility of all current and future generations:

$$SW = \sum_{t=-1}^{\infty} \gamma^t U_t, \ 0 < \gamma < 1,$$

under the human capital formation function in (1) and the resource constraint:

$$N_t c_t + N_{t-1} d_t + K_{t+1} + N_{t+1} x_t = A \left(K_t\right)^{\alpha} \left(H_t\right)^{1-\alpha},$$

or

$$c_t + \frac{d_t}{1+n} + (1+n)\hat{k}_{t+1} h_{t+1} + (1+n) x_t = A \left(\hat{k}_t\right)^{\alpha} h_t,$$

where $\hat{k}_0$ and h_0 are given. The parameter $\gamma \in (0,1)$ is the planner's discount factor. Reverse discounting, $1/\gamma (> 1)$, must be applied to U_{-1} (i.e., the utility of the old generation in the initial period) to preserve dynamic consistency.

Solving the problem leads to the following characterization of the planner's allocation.

Proposition 6. *Given $\hat{k}_0$ and h_0, a sequence of the planner's allocation,* $\left\{c_t, d_t, x_t, \hat{k}_{t+1}, h_{t+1}\right\}_{t=0}^{\infty}$, *satisfies the human capital formation function in* (1) *and the following:*

$$\begin{aligned} c_t &= \frac{\gamma(1-\gamma)[1-\alpha\gamma(1-\eta)]}{(\gamma+\beta)[1-\gamma(1-\eta)]} A\left(\hat{k}_t\right)^{\alpha} h_t, \\ \frac{d_t}{1+n} &= \frac{\beta(1-\gamma)[1-\alpha\gamma(1-\eta)]}{(\gamma+\beta)[1-\gamma(1-\eta)]} A\left(\hat{k}_t\right)^{\alpha} h_t, \\ (1+n)\hat{k}_{t+1} &= \frac{\alpha\gamma}{D\left[\frac{\gamma\eta(1-\alpha)}{(1+n)[1-\gamma(1-\eta)]}\right]^{\eta}} \left[A\left(\hat{k}_t\right)^{\alpha}\right]^{1-\eta}, \\ (1+n)x_t &= \frac{(1-\alpha)\gamma\eta}{1-\gamma(1-\eta)} A\left(\hat{k}_t\right)^{\alpha} h_t. \end{aligned}$$

Proof. See Appendix A.7.

In the following, we compare the planner's allocation with the political equilibrium based on numerical methods. The parameter values for the analysis are the same as those in Section 4.3 if not otherwise specified.

5.1 Comparison of the Planner's Allocation with the Political Equilibrium in the Absence of a Debt Ceiling

We first compare the planner's allocation with the political equilibrium in the absence of a debt ceiling. In Figure 5, we assume $\gamma = (0.99)^{120}$ in the planner's allocation and plot the evolution of physical capital (Panel (a)), human capital (Panel (b)), per capita GDP (Panel (c)), consumption in middle age (Panel (d)), and consumption in old age (Panel (e)) from periods $t = 0$ to 5. We take the ratio of a variable at the political equilibrium to that in the planner's allocation for each period. The lines in each figure imply the ratio of the relevant variable at the political equilibrium to that in the planner's allocation. Each ratio implies that the political equilibrium outweighs the planner's allocation when the ratio is above unity. In Panel (f), we plot the difference in utility between the political equilibrium and the planner's allocation from generation -1 to generation 4. Generation t attains higher (lower) utility at the political equilibrium than in the planner's allocation when the difference is positive (negative).

[Figure 5 here.]

The planner's allocation attains higher physical and human capital and higher per capita GDP than at the political equilibrium, as depicted in Panels (a), (b), and (c) of Figure 5. The share of the resources devoted to education is higher in the planner's allocation than at the political equilibrium because the long-lived planner has an incentive to invest more in education than short-lived politicians. Investment in education stimulates human capital formation and increases the output and resources devoted to physical capital formation in the planner's allocation. Therefore, the planner's allocation attains higher physical and human capital and per capita GDP than at the political equilibrium from period 1 onward.

The argument above implies that the share of the resources devoted to consumption is lower in the planner's allocation than at the political equilibrium. Because of this property, the middle- and old-age consumption levels in period 0 are lower in the planner's allocation than at the political equilibrium (see Panels (d) and (e)). However, the available resources in the planner's allocation are larger than at the political equilibrium from period 1 onward. Because of this positive income effect, the planner's allocation attains higher levels of consumption for the middle-aged and old from period 1 onward than at the political equilibrium (see Panels (d) and (e)). These results imply that generations -1 and 0 are made better off at the political equilibrium than in the planner's allocation, whereas agents from generation 1 onward are made worse off. In other words, short-sighted politicians produce a trade-off between generations in terms of welfare.

5.2 Comparison of the Planner's Allocation with the Political Equilibrium in the Presence of a Debt Ceiling

We next compare the planner's allocation with the political equilibrium in the presence of a debt ceiling and examine the way of approximating the planner's allocation through the control of the parameter ε representing the debt ceiling. As demonstrated above, generations from $t = 1$ onward are worse off at the political equilibrium in the absence of a debt ceiling than in the planner's allocation. In addition, a change in the debt ceiling, ε, creates an intergenerational trade-off in terms of utility, as shown in Section 4. These results imply that all generations from $t = 1$ onward may benefit from strengthening fiscal discipline (i.e., lowering ε) at the expense of generation $t = 0$. However, the sum of the benefits of all future generations may outweigh the cost incurred by generation $t = 0$, suggesting a rationale for focusing on generations from $t = 1$ onward. Therefore, we hereafter consider the effect of the debt ceiling on generations from $t = 1$ onward.

Three cases are considered for the planner's discount factor: high, moderate, and low (i.e., $\gamma = (0.99)^{120}$, $(0.985)^{120}$, and $(0.98)^{120}$). Each case is compared with the two cases of the political equilibrium: the case of the low political weight of the old, $\omega = 0.48$, called a young society, and the case of the high political weight of the old, $\omega = 0.6$, called an aging society. The debt ceiling in the political equilibrium has three scenarios: tight discipline, $\varepsilon = 0.2$, moderate discipline, $\varepsilon = 0.5$, and lax discipline, $\varepsilon = 0.8$. We consider these three cases of the debt ceiling as well as the absence of a debt ceiling, $\varepsilon = 1$, and compare them with the planner's allocation. In particular, we explore the conditions for the adjustment of the debt ceiling to approach the planner's allocation.

Figures 6, 7, and 8 depict the numerical results for the $\gamma = (0.99)^{120}$, $(0.985)^{120}$, and $(0.98)^{120}$ cases, respectively. Each figure contains two subfigures: the upper (lower) subfigure compares the planner's allocation with the political equilibrium with $\omega = 0.48$ (0.6). Following Figure 5, we plot the evolution of the physical-to-human capital ratio (Panel (a)), human capital (Panel (b)), per capita GDP (Panel (c)), middle-age consumption (Panel (d)), old-age consumption (Panel (e)), and the distribution of utility across generations (Panel (f)).

First, suppose that the planner's discount factor is high such that $\gamma = (0.99)^{120}$ holds (see Figure 6). The planner attaches a large weight to future generations, which incentivizes him or her to invest considerably in human capital formation. This strong incentive implies that generations from $t = 1$ onward experience higher levels of human capital, per capita GDP, and middle- and old-age consumption in the planner's allocation than at the political equilibrium regardless of the political weight of the old, ω, and the debt ceiling, ε. In other words, the planner's allocation outweighs the political equilibrium in terms of human capital, per capita GDP, and consumption. However, physical and human capital and per capita GDP increase at the political equilibrium as the debt ceiling, ε, lowers (see Section 4.3). Thus, the political equilibrium can close the gap to the planner's allocation in both young and aging societies by strengthening fiscal discipline.

[Figure 6 is here.]

Second, suppose that the planner's discount factor is moderate such that $\gamma = (0.985)^{120}$ holds (see Figure 7). The planner in this case has less incentive to invest in physical and human capital than in the first case. In other words, in the present case, the political equilibrium may outweigh the planner's allocation in terms of physical and human capital. In particular, this outweighing effect at the political equilibrium increases as the political weight of the old decreases, as illustrated in Panel (d) of Figure 1. This property implies that in the young society with $\omega = 0.48$, the political equilibrium outweighs the planner's allocation in terms of the utility of generations from $t = 1$ onward. In addition, the gap between the political equilibrium and the planner's allocation widens as the debt ceiling, ε, falls. This result suggests that strengthening fiscal discipline is not beneficial from the planner's viewpoint.

[Figure 7 is here.]

In the aging society with $\omega = 0.6$, the large political weight of the old has a negative effect on physical and human capital formation. This negative effect cancels out the abovementioned outweighing effect when ε is above 0.5. Thus, generations from $t = 1$ onward attain lower utility at the political equilibrium than in the planner's allocation when initial fiscal discipline is weak such that $\varepsilon > 0.5$ holds. Strengthening fiscal discipline improves utility and closes the gap to the planner's allocation. However, when initial fiscal discipline is strict such that $\varepsilon < 0.5$ holds, the implication is reversed. Generations from $t = 1$ onward attain higher utility at the political equilibrium than in the planner's allocation and the gap to the planner's allocation widens as the debt ceiling, ε, lowers. The results described thus far suggest that when the planner's discount factor is $\gamma = (0.985)^{120}$, the welfare implications of the debt ceiling depends critically on the political weight of the old, ω, as well as the initial condition of the debt ceiling, ε.

Third, suppose that the planner's discount factor is low such that $\gamma = (0.98)^{120}$ holds (see Figure 8). Compared with the second case, the present case gives the planner less incentive to invest in physical and human capital. Because of this property, the planner's allocation is outweighed by the political equilibrium in terms of physical and human capital, per capita GDP, and middle- and old-age consumption, regardless of the political weight of the old, ω, or the debt ceiling, ε. This implies that lowering the debt ceiling, ε, in the political equilibrium improves utility but widens the gap to the planner's allocation. Thus, strengthening fiscal discipline is not beneficial from the planner's viewpoint.

[Figure 8 is here.]

The results described thus far indicate that the political equilibrium may approximate the planner's allocation by controlling the debt ceiling, ε, but that its realization depends on the political weight of the old, ω, as well as the initial condition of the debt ceiling, ε. When the planner's discount factor γ is high (low) such that $\gamma = (0.99)^{120}$ ($(0.98)^{120}$), lowering (raising) the debt ceiling enables us to close the gap to the planner's allocation. However, in the moderate case where the planner's discount factor is $\gamma = (0.985)^{120}$, lowering the debt ceiling is beneficial from the planner's viewpoint in an aging society with $\omega = 0.6$ and $\varepsilon > 0.5$, whereas it is detrimental in a young society with $\omega = 0.48$. Therefore, the political power of the old, ω, as well as the initial condition of the debt ceiling, ε, matter when we evaluate the effect of strengthening fiscal discipline from the planner's viewpoint.

6 Public Good Provision

Thus far, we have assumed that the old do not benefit from any public expenditure. However, when the old directly benefit from public expenditure such as public good provision, they may induce politicians to raise public expenditure on them as well as place the fiscal burden on future generations by issuing more public debt. This section examines this possibility.

For the analysis, consider the following utility function:

$$U_t = \ln c_t + \theta \ln g_t + \beta \left[\ln d_{t+1} + \theta \ln g_{t+1}\right], \tag{27}$$

where g is per capita public good provision and $\theta(>0)$ is the weight of the utility of public good provision. Aggregate public good expenditure in period t is $(N_t + N_{t-1})\, g_t$, meaning that the government budget constraint becomes

$$(1+n)b_{t+1} + \tau_t^k R_t \frac{s_{t-1}}{1+n} + \tau_t w_t h_t = (1+n)x_t + \frac{2+n}{1+n} g_t + R_t b_t. \tag{28}$$

We assume the no public debt rule.

In this setting, the political objective function in (10) is reformulated as follows:

$$\begin{aligned}\Omega_g \simeq\; & \frac{\omega}{(1+n)(1-\omega)} \left[\ln\left(1-\tau^k\right) + \theta \ln g\right] + (1+\beta)\ln\left[Z\left(\tau^k, x, b', \hat{k}, b, h\right) - \frac{2+n}{1+n} g\right] \\ & + \beta \ln R\left(\hat{K}\left(b', x, Z\left(\tau^k, x, b', \hat{k}, b, h\right)\right)\right) + \beta \ln\left(1-\tau^{k\prime}\right) + \theta \ln g + \beta\theta \ln g',\end{aligned}$$

where Ω_g denotes the political objective function in the presence of public good provision. The function Ω_g differs from the function in the baseline model, Ω, in that (i) after-tax income $Z(\cdot)$ is replaced by $Z(\cdot) - (2+n)g/(1+n)$ and (ii) the utility of public good provision enters additively into the function. Keeping this difference in mind, we solve the problem of the government and obtain the following policy functions.

Proposition 7. *In the presence of public good provision, a Markov-perfect political equilibrium is characterized by the following policy functions:*

$$\tau^k = 1 - \frac{1}{1+\theta} T_{un}^k \frac{1}{\alpha\left(1 + \frac{b}{\hat{k}h}\right)},$$

$$(1+n)x = X_{un} A \left(\hat{k}\right)^\alpha h,$$

$$\tau = \begin{cases} 1 - \frac{1}{1+\theta} T_{un} & \text{if } 1 > \alpha(1+\theta), \\ 1 - \frac{1}{1+\theta} \cdot \frac{1+\alpha\beta(1+\theta)}{1+\beta} T_{un} & \text{if } 1 \leq \alpha(1+\theta), \end{cases}$$

$$(1+n)b' = \begin{cases} \frac{1}{1+\theta} \cdot \frac{1-\alpha(1+\theta)}{1-\alpha} B_{un} A \left(\hat{k}\right)^\alpha h & \text{if } 1 > \alpha(1+\theta), \\ 0 & \text{if } 1 \leq \alpha(1+\theta), \end{cases}$$

$$\frac{2+n}{1+n} g = \frac{\theta\left[\frac{\omega}{(1+n)(1-\omega)} + 1\right]}{(1+\theta)\left[\frac{\omega}{(1+n)(1-\omega)} + 1 + \beta\left[\alpha + \eta(1-\alpha)\right]\right]} A \left(\hat{k}\right)^\alpha h.$$

Proof. See Appendix A.8.

Comparing the results in the absence and presence of public good provision (i.e., Propositions 1 and 8, respectively), we find that the debt-to-GDP ratio in the presence of public good provision is lower than that in its absence. The old may thus have an incentive to pass the burden of public good provision by issuing public debt. However, the middle-aged, who also benefit from public good provision, find it optimal to reduce debt issues and increase public good provision in their old age from the viewpoint of their utility. Because of this disciplined effect, as also found by Song, Storesletten, and Zilibotti (2012), the government representing both the middle-aged and the old finds it optimal to reduce rather than increase public debt issues.

To understand the disciplined effect more precisely, consider the first-order condition with respect to b', which is given as follows:

$$\frac{\partial \Omega}{\partial b'} + \frac{\beta\theta}{g'} \cdot \frac{\partial g'}{\partial \hat{K}} \cdot \left(\underbrace{\frac{\partial \hat{K}}{\partial \left[Z(\cdot) - \frac{2+n}{1+n}g\right]} \cdot \frac{\partial \left[Z(\cdot) - \frac{2+n}{1+n}g\right]}{\partial b'}}_{\text{(d.1)}} + \underbrace{\frac{\partial \hat{K}}{\partial b'}}_{\text{(d.2)}} \right) \leq 0,$$

where the first term on the left-hand side, denoted by $\partial\Omega/\partial b'$, includes all the terms observed in the absence of public good provision and the second term shows the disciplined effect that is specific to the present case. The disciplined effect is composed of two parts: (i) increased after-tax income, which in turn increases saving and physical capital and thus raises the provision of public goods in the next period, as presented by the term (d.1), and (ii) the crowding out of physical capital, which in turn decreases public good provision in the next period, as presented by the term (d.2). Thus, there are two opposing effects of public debt issues on the provision of public goods and the net effect is negative in the present framework. Because of this negative effect, the government reduces public debt issues as the weight of the public good, denoted by θ, increases. In particular, when θ is high such that $\alpha(1+\theta) \geq 1$ holds, there is no public debt issue; spending is solely financed by taxation.

7 Conclusion

This study developed an overlapping-generations model with physical and human capital accumulation and analyzed voting on fiscal policy. In particular, it considered the effect of debt ceilings on fiscal policy formation and its impact on growth and welfare over time and across generations. The efficiency of the debt ceiling was measured based on the Pareto criterion. It was shown that the introduction of the debt ceiling is not Pareto-improving; it increases economic growth, but creates a trade-off between generations in terms of welfare.

The study further evaluated the debt ceiling from an alternative viewpoint, that is, an imaginary benevolent planner who can allocate resources across generations. It was found that the planner's allocation can be approached by controlling the debt ceiling. In particular, under certain conditions, lowering the debt ceiling enables us to approach the planner's allocation when the political power of the old is strong, suggesting a rationale for strengthening fiscal discipline in an aging society.

While the present study shed light on the evaluation of the debt ceiling from the political economy viewpoint, the analysis could be extended in several directions. For instance, the main analysis assumed away public good provision that benefits the retired old. In Section 6, the case including public good provision was briefly analyzed, whereas the evaluation of the debt ceilings in that case was left untouched. In addition, the analysis focused on the debt ceiling and alternative fiscal rules such as balanced budget rules, expenditure rules, and revenue rules were left untouched, which are common in many countries. The exploration of these extensions is left to future work.

A Proofs and Supplementary Explanations

A.1 Proof of Proposition 1

The conjecture in (13) is reformulated by using (11) and (12) as follows:

$$
\begin{aligned}
\tau^{k\prime} &= 1 - T_{un}^{k} \frac{\hat{k}'h'}{\alpha\left(\hat{k}'h' + b'\right)} \\
&= 1 - T_{un}^{k} \frac{A\left(\hat{k}\right)^{\alpha} h - \left(1-\tau^{k}\right)\alpha A\left(\hat{k}\right)^{\alpha-1}\left(\hat{k}h + b\right) - (1+n)x + (1+n)b' - \frac{1+\beta}{\beta}(1+n)b'}{\alpha\left[A\left(\hat{k}\right)^{\alpha} h - \left(1-\tau^{k}\right)\alpha A\left(\hat{k}\right)^{\alpha-1}\left(\hat{k}h + b\right) - (1+n)x + (1+n)b'\right]} \\
&= 1 - T_{un}^{k} \frac{Z(\cdot) - \frac{1+\beta}{\beta}(1+n)b'}{\alpha Z(\cdot)}. \qquad (29)
\end{aligned}
$$

By using (29) and rearranging the terms, the first-order conditions in (15)–(16) are reformulated as

$$
\tau^{k} : -\frac{\omega}{(1+n)(1-\omega)}\frac{1}{1-\tau^{k}} + \frac{\alpha A\left(\hat{k}\right)^{\alpha-1}\left(\hat{k}h+b\right)}{Z(\cdot)} + \frac{\alpha\beta\alpha A\left(\hat{k}\right)^{\alpha-1}\left(\hat{k}h+b\right)}{Z(\cdot) - \frac{1+\beta}{\beta}(1+n)b'} \le 0, \qquad (30)
$$

$$
b' : \frac{1+n}{Z(\cdot)} - \frac{\alpha(1+n)}{Z(\cdot) - \frac{1+\beta}{\beta}(1+n)b'} \le 0, \qquad (31)
$$

$$
x : \frac{\beta\eta\left(1-\alpha\right)}{x} - \frac{1+n}{Z(\cdot)} - \frac{\alpha\beta(1+n)}{Z(\cdot) - \frac{1+\beta}{\beta}(1+n)b'} = 0. \qquad (32)
$$

Suppose that $b' = 0$ holds. Eq. (31) implies that $b' = 0$ if $(1+n) - \alpha(1+n) \le 0$, that is, if $\left(1-\alpha\right)/\alpha \le 0$, which never holds $\forall\alpha \in (0,1)$. Thus, we obtain $b' > 0$.

Given that $b' > 0$ holds, the first-order condition with respect to b' in (31) holds with an equality. From (31) and (32), we obtain

$$
\frac{\left(1+\beta\right)\left(1+n\right)}{Z} = \frac{\beta\eta\left(1-\alpha\right)}{x}. \qquad (33)
$$

From (30) and (32), we have

$$
\alpha A\left(\hat{k}\right)^{\alpha-1}\left(\hat{k}h+b\right)\left(1-\tau^{k}\right) = \frac{\frac{\omega}{(1+n)(1-\omega)}}{\beta\eta\left(1-\alpha\right)}(1+n)x. \qquad (34)
$$

We substitute (34) into the first-order condition with respect to b' in (31). After rearranging the terms, we obtain the following relation between x and b' :

$$
\left(1-\alpha\right)\left[A\left(\hat{k}\right)^{\alpha} h - \left(\frac{\frac{\omega}{(1+n)(1-\omega)}}{\beta\eta\left(1-\alpha\right)} + 1\right)(1+n)x\right] = \left(\alpha + \frac{1}{\beta}\right)(1+n)b'. \qquad (35)
$$

We can obtain another relation between x and b' by substituting (34) into (33):

$$(1+n)b' = \left[\frac{1+\beta}{\beta\eta(1-\alpha)} + 1 + \frac{\frac{\omega}{(1+n)(1-\omega)}}{\beta\eta(1-\alpha)}\right](1+n)x - A\left(\hat{k}\right)^{\alpha} h. \tag{36}$$

By solving (35) and (36) for x and b', we obtain the policy functions of x and b' as in Proposition 1.

We substitute the obtained policy function, X, into (34) to derive

$$1-\tau^k = \frac{\frac{\omega}{(1+n)(1-\omega)}}{\frac{\omega}{(1+n)(1-\omega)} + 1 + \beta\left[\alpha + \eta(1-\alpha)\right]} \cdot \frac{1}{\alpha\left(1 + \frac{b}{\hat{k}h}\right)}.$$

Finally, we can compute the labor tax rate by substituting the obtained τ^k, x, and b' into the government budget constraint in (12).

■

A.2 Proof of Corollary 1

Recall the policy function of τ^k in Proposition 1. The second part of the corollary is immediately obtained by setting $\tau_0^k > 0$. For the proof of the first part, consider the ratio $b_t/\hat{k}_t h_t$, in period $t \geq 1$. We use the capital market-clearing condition in (11) and the policy functions in Proposition 1 to reformulate the ratio $b_t/\hat{k}_t h_t$ for $t \geq 1$ as follows:

$$\begin{aligned}
\frac{b_t}{\hat{k}_t h_t} &= \frac{\frac{1}{1+n} B_{un} A\left(\hat{k}_{t-1}\right)^{\alpha} h_{t-1}}{\frac{1}{1+n}\frac{\beta}{1+\beta}\left(1-\tau_{t-1}\right)(1-\alpha) A\left(\hat{k}_{t-1}\right)^{\alpha} h_{t-1} - b_t} \\
&= \frac{\frac{1}{1+n} B_{un} A\left(\hat{k}_{t-1}\right)^{\alpha} h_{t-1}}{\frac{1}{1+n}\frac{\beta}{1+\beta}\frac{1+\beta}{1-\alpha}\frac{1}{\frac{\omega}{(1+n)(1-\omega)}+1+\beta[\alpha+\eta(1-\alpha)]}(1-\alpha) A\left(\hat{k}_{t-1}\right)^{\alpha} h_{t-1} - \frac{1}{1+n} B_{un} A\left(\hat{k}_{t-1}\right)^{\alpha} h_{t-1}} \\
&= \frac{1-\alpha}{\alpha}.
\end{aligned}$$

By using this result, we can reformulate τ_t^k, $t \geq 1$, as

$$\begin{aligned}
\tau_t^k &= 1 - T_{un}^k \frac{1}{\alpha\left(1 + \frac{1-\alpha}{\alpha}\right)} \\
&= 1 - \frac{1}{1 + \frac{(1+n)(1-\omega)}{\omega}\left\{1 + \beta\left[\alpha + \eta(1-\alpha)\right]\right\}}.
\end{aligned}$$

The expression shows that τ_t^k is decreasing in ω with $\lim_{\omega\to 1}\tau_t^k = 0$ for $t \geq 1$.

■

A.3 Derivation of (17) and (18) and Proof of Proposition 2

Recall the human capital formation function, $h' = D(x)^{\eta}(h)^{1-\eta}$. With the use of the policy function of x in Proposition 1, this function is rewritten as

$$\frac{h'}{h} = D\left[\frac{1}{1+n}\frac{\beta\eta(1-\alpha)}{\frac{\omega}{(1+n)(1-\omega)}+1+\beta[\alpha+\eta(1-\alpha)]}\right]^{\eta}\left[A\left(\hat{k}\right)^{\alpha}\right]^{\eta}, \tag{37}$$

or,

$$\frac{h'}{h} = D\Psi_H\left[A\left(\hat{k}\right)^{\alpha}\right]^{\eta},$$

where Ψ_H is defined as in (20).

Next, consider the capital market-clearing condition in (11). With the use of the policy function of τ in Proposition 1, (11) is rewritten as

$$(1+n)\hat{k}'h' = \frac{\alpha\beta}{\frac{\omega}{(1+n)(1-\omega)}+1+\beta[\alpha+\eta(1-\alpha)]}A\left(\hat{k}\right)^{\alpha}h. \tag{38}$$

By substituting (37) into (38) and rearranging the terms, we obtain

$$\hat{k}' = \frac{\alpha\beta}{\frac{\omega}{(1+n)(1-\omega)}+1+\beta[\alpha+\eta(1-\alpha)]}\left\{(1+n)D\left[\frac{X_{un}}{1+n}\right]^{\eta}\right\}^{-1}\left[A\left(\hat{k}\right)^{\alpha}\right]^{1-\eta}, \tag{39}$$

which is reduced as in (17).

In the steady state, $b/\hat{k}h = b'/\hat{k}'h'$ holds, so

$$\frac{b}{\hat{k}h} = \frac{\frac{\beta(1-\alpha)}{\frac{\omega}{(1+n)(1-\omega)}+1+\beta[\alpha+\eta(1-\alpha)]}A\left(\hat{k}\right)^{\alpha}h}{\frac{\alpha\beta}{\frac{\omega}{(1+n)(1-\omega)}+1+\beta[\alpha+\eta(1-\alpha)]}A\left(\hat{k}\right)^{\alpha}h} = \frac{1-\alpha}{\alpha}, \tag{40}$$

where the first equality comes from the policy function of b' in Proposition 1 and the capital market-clearing condition in (38).

From (40), the tax rate on capital, τ^k, becomes

$$\tau^k = 1 - \frac{\frac{\omega}{(1+n)(1-\omega)}}{\frac{\omega}{(1+n)(1-\omega)}+1+\beta[\alpha+\eta(1-\alpha)]}\cdot\frac{1}{\alpha\left(1+\frac{1-\alpha}{\alpha}\right)},$$

or

$$\tau^k = 1 - \frac{\frac{\omega}{(1+n)(1-\omega)}}{\frac{\omega}{(1+n)(1-\omega)}+1+\beta[\alpha+\eta(1-\alpha)]} \in (0,1).$$

The tax rate on labor, τ, is given by

$$\tau = 1 - \frac{1+\beta}{1-\alpha}\frac{1}{\frac{\omega}{(1+n)(1-\omega)}+1+\beta[\alpha+\eta(1-\alpha)]} < 1.$$

We obtain

$$\tau \geq 0 \Leftrightarrow \frac{1+\beta}{1-\alpha} - \{1+\beta[\alpha+\eta(1-\alpha)]\} \leq \frac{\omega}{(1+n)(1-\omega)}, \tag{41}$$

as expressed in Proposition 2.

■

A.4 Proof of Proposition 3

When $b' = 0$, the first-order conditions with respect to τ^k in (52) and x in (53) are rewritten as

$$\tau^k : \frac{\omega}{(1+n)(1-\omega)}\frac{1}{1-\tau^k} = \frac{\alpha A\left(\hat{k}\right)^{\alpha-1}(1+\alpha)\left(\hat{k}h+b\right)}{Z(\cdot)}, \tag{42}$$

$$x : \frac{\beta\eta\,(1-\alpha)}{(1+n)x} = \frac{(1+n)(1+\alpha\beta)}{Z(\cdot)}, \tag{43}$$

respectively. From (42) and (43), we obtain

$$(1+n)x = \frac{(1+n)(1-\omega)}{\omega}\beta\eta\,(1-\alpha)\,\alpha A\left(\hat{k}\right)^{\alpha-1}\left(\hat{k}h+b\right)\left(1-\tau^k\right). \tag{44}$$

We substitute (44) into (42) and rearrange the terms to obtain

$$\tau^k = 1 - T_{un}^K\frac{1}{\alpha\left(1+\frac{b}{\hat{k}h}\right)}. \tag{45}$$

This is identical to the corresponding policy function in the debt-financing case. By plugging (45) into (42), we obtain the policy function of X as

$$(1+n)x = X_{un}A\left(\hat{k}\right)^{\alpha}h.$$

This is also identical to the corresponding policy function in the debt-financing case.

We can obtain the labor income tax rate by substituting the policy functions of τ^k, x, and $b' = 0$ into the government budget constraint in (12) as follows:

$$\tau = 1 - \frac{1}{1-\alpha}\frac{1+\alpha\beta}{\frac{\omega}{(1+n)(1-\omega)}+1+\beta\left[\alpha+\eta\,(1-\alpha)\right]}.$$

This differs from the corresponding policy function in the debt-financing case.

Finally, the accumulation of physical and human capital is computed by substituting the policy functions into the capital market-clearing condition in (1) and the human capital formation function in (11):

$$\hat{k}' = \Psi_{K,b'=0}\left[A\left(\hat{k}\right)^{\alpha}\right]^{1-\eta},$$

$$\frac{h'}{h} = D\Psi_H\left[A\left(\hat{k}\right)^{\alpha}\right]^{\eta},$$

where Ψ_H is defined in (20) and $\Psi_{K,b'=0}$ is

$$\Psi_{K,b'=0} \equiv \frac{\beta}{1+\beta}\frac{1+\alpha\beta}{\frac{\omega}{(1+n)(1-\omega)}+1+\beta\left[\alpha+\eta\,(1-\alpha)\right]}\left\{(1+n)D\left[\frac{X_{un}}{1+n}\right]^{\eta}\right\}^{-1}.$$

To compare the two cases, consider first the capital tax rates. Given $\hat{k}_0$ and b_0, we immediately find that $\tau_0^k\big|_{tax} = \tau_0^k\big|_{debt}$ holds. For $t \geq 1$, $\tau_t^k\big|_{tax} < \tau_t^k\big|_{debt}$ holds because $b/\hat{k}h = 0(>0)$ holds in the tax-financing (debt-financing) case.

Next, compare the labor tax rates and growth rates. Direct comparison leads to

$$\tau|_{tax} > \tau|_{debt} \Leftrightarrow 0 < \frac{1-\alpha}{\alpha},$$

and

$$h'/h|_{tax} > h'/h|_{debt} \Leftrightarrow 0 < \frac{1-\alpha}{\alpha}.$$

Finally, we immediately obtain $(1+n)x/y|_{tax} = (1+n)x/y|_{debt}$ from Proposition 1.

■

A.5 Proof of Proposition 4

(i) Recall the indirect utility function of the old given by (8). Because $\tau_0^k\big|_{tax} = \tau_0^k\big|_{debt}$ holds, as demonstrated in Proposition 5, we immediately obtain $V_0^o|_{tax} = V_0^o|_{debt}$.

To consider the effect on the utility of generation 0, V_0^M, recall the political objective function in period 0:

$$\Omega_0 = \frac{\omega}{(1+n)(1-\omega)} V_0^o + V_0^M.$$

The objective function, Ω_0 is maximized by choosing $b' > 0$, as shown in Proposition 1. However, the government choice is constrained when it is forced to finance its expenditure solely by taxes. That is, the government attains a lower value of its objective under tax financing than under debt financing: $\Omega_0|_{tax} < \Omega_0|_{debt}$. This implies $V_0^M\big|_{tax} < V_0^M\big|_{debt}$ since $V_0^o|_{tax} = V_0^o|_{debt}$ holds.

(ii) Recall the indirect utility function of the middle-aged in (7). Suppose that from some period $t_0(\geq 1)$ onward, the economy is in a steady state regardless of the government's financing method. The indirect utility function in (7) becomes

$$\begin{aligned}
V_t^M\big|_j &= (1+\beta)\ln(1-\alpha)A\left(\hat{k}\Big|_j\right)^{\alpha}\left(h_t|_j\right)\left(1-\tau|_j\right) + \beta\left[(\alpha-1)+\gamma\alpha\right]\ln \hat{k}\Big|_j \\
&+ \beta\ln\left(1-\tau_{t+1}^k\big|_j\right) + \phi^M, \\
&\simeq (1+\alpha\beta)\ln \hat{k}\Big|_j + (1+\beta)\ln\left(1-\tau|_j\right) + \beta\ln\left(1-\tau_{t+1}^k\big|_j\right) \\
&+ (1+\beta)\ln h_t|_j\,,\ j = tax, debt \qquad (46)
\end{aligned}$$

where the constant terms are omitted from the expression.

With the use of the policy functions presented in Proposition 1 and Appendix A.4, (46) is rewritten as follows:

$$V_t^M\big|_j \simeq (1+\alpha\beta)\ln \hat{k}\Big|_j + (1+\beta)\ln\left(1-\tau|_j\right) + \beta \ln \frac{1}{1+\left(b_{t+1}/\hat{k}_{t+1}h_{t+1}\right)\Big|_j}$$
$$+ (1+\beta)\ln h_t|_j\,,$$

or

$$V_t^M\big|_j \simeq (1+\alpha\beta)\ln \hat{k}\Big|_j + (1+\beta)\ln\left(1-\tau|_j\right)$$
$$+ (1+\beta)\ln\left(h'/h|_j\right)^{t-t_0} h_{t_0}|_j\,, \quad j = tax, debt.$$

The direct comparison of $V_1^M\big|_{tax}$ and $V_1^M\big|_{debt}$ leads to

$$\begin{aligned} V_1^M\big|_{tax} \gtrless V_1^M\big|_{debt} \Leftrightarrow (t-t_0)(1+\beta)\ln\frac{h'/h|_{tax}}{h'/h|_{debt}} \\ \gtrless (1+\alpha\beta)\ln\frac{\hat{k}\Big|_{debt}}{\hat{k}\Big|_{tax}} + (1+\beta)\ln\frac{1-\tau|_{debt}}{1-\tau|_{tax}} \\ + (1+\beta)\ln\frac{h_{t_0}|_{debt}}{h_{t_0}|_{tax}}, \end{aligned} \tag{47}$$

where the left-hand and right-hand sides of (47) are denoted by LHS and RHS, respectively. These satisfy the following properties:

$$\partial LHS/\partial t > 0,\ LHS|_{t=t_0} = 0,\ \lim_{t\to\infty} LHS = \infty,\ \text{and}\ \partial RHS/\partial t = 0.$$

Therefore, there is a positive integer, denoted by $\hat{t}$, such that $LHS \gtrless RHS \Leftrightarrow V_t^M\big|_{tax} \gtrless V_t^M\big|_{debt}$ for $t \gtrless \hat{t}$.

■

A.6 Proof of Proposition 5

With the use of $(1+n)b' = \varepsilon B_{un} A\left(\hat{k}\right)^{\alpha} h$, the government budget constraint in (12) is reformulated as

$$\varepsilon B_{un} A\left(\hat{k}\right)^{\alpha} h + \tau(1-\alpha)A\left(\hat{k}\right)^{\alpha} h + \tau^k \alpha A\left(\hat{k}\right)^{\alpha-1}\left(\hat{k}h + b\right) = (1+n)x + \alpha A\left(\hat{k}\right)^{\alpha-1} b,$$

and the capital market-clearing condition in (11) is rewritten as

$$\varepsilon B_{un} A\left(\hat{k}\right)^{\alpha} h + (1+n)\hat{k}'h' = \frac{\beta}{1+\beta}(1-\tau)(1-\alpha)A\left(\hat{k}\right)^{\alpha} h.$$

We conjecture the following capital tax rate in the next period:

$$\tau^{k\prime} = 1 - T^k_{con} \frac{1}{\alpha\left(1 + \frac{b'}{\hat{k}'h'}\right)},$$

where the subscript "con" of T^k_{con} implies that public debt issuance is "constrained" by the rule in (22). By using the capital market-clearing condition and government budget constraint, we can rewrite the conjecture as follows:

$$\tau^{k\prime} = 1 - T^k_{con} \frac{(1-\tau)(1-\alpha) - \frac{1+\beta}{\beta}\varepsilon B_{un}}{\alpha(1-\tau)(1-\alpha)}. \tag{48}$$

We substitute (48) into the political objective function in (23) and rearrange the terms to obtain

$$\Omega \simeq \frac{\omega}{(1+n)(1-\omega)} \ln\left(1-\tau^k\right) + \ln \tilde{Z}^1 + \alpha\beta \ln \tilde{Z}^2 + \beta\eta(1-\alpha)\ln x, \tag{49}$$

where

$$\tilde{Z}^1 \equiv A\left(\hat{k}\right)^{\alpha} h - \left(1-\tau^k\right)\alpha A\left(\hat{k}\right)^{\alpha-1}\left(\hat{k}h + b\right) - (1+n)x + \varepsilon B_{un} A\left(\hat{k}\right)^{\alpha} h, \tag{50}$$

$$\tilde{Z}^2 \equiv A\left(\hat{k}\right)^{\alpha} h - \left(1-\tau^k\right)\alpha A\left(\hat{k}\right)^{\alpha-1}\left(\hat{k}h + b\right) - (1+n)x - \frac{1}{\beta}\varepsilon B_{un}. \tag{51}$$

The first-order conditions with respect to τ^k and $\tilde{x}$ are

$$\tau^k : -\frac{\omega}{(1+n)(1-\omega)}\frac{1}{1-\tau^k} + \alpha A\left(\hat{k}\right)^{\alpha-1}\left(\hat{k}h + b\right)\left[\frac{1}{\tilde{Z}^1} + \frac{\alpha\beta}{\tilde{Z}^2}\right] = 0, \tag{52}$$

$$x : \frac{\beta\eta(1-\alpha)}{(1+n)x} - \left[\frac{1}{\tilde{Z}^1} + \frac{\alpha\beta}{\tilde{Z}^2}\right] = 0. \tag{53}$$

These conditions are summarized as

$$1 - \tau^k = \frac{\omega}{(1+n)(1-\omega)} \cdot \frac{(1+n)x}{\beta\eta(1-\alpha)} \cdot \frac{1}{\alpha\left(1 + b/\hat{k}h\right)}. \tag{54}$$

With the use of (54), $\tilde{Z}^1$ and $\tilde{Z}^2$ in (50) and (51) are rewritten as follows:

$$\begin{cases} \tilde{Z}^1 = (1+\varepsilon B_{un}) A\left(\hat{k}\right)^{\alpha} h - \left[1 + \frac{\omega}{(1+n)(1-\omega)} \cdot \frac{1}{\beta\eta(1-\alpha)}\right](1+n)x, \\ \tilde{Z}^2 = (1+\varepsilon B_{un}) A\left(\hat{k}\right)^{\alpha} h - \left[1 + \frac{\omega}{(1+n)(1-\omega)}\frac{1}{\beta\eta(1-\alpha)}\right](1+n)x - \frac{1+\beta}{\beta}\varepsilon B_{un} A\left(\hat{k}\right)^{\alpha} h. \end{cases} \tag{55}$$

The substitution of (55) into the first-order condition with respect to x in (53) leads to

$$\frac{\beta\eta(1-\alpha)}{(1+n)x} = \frac{1}{\tilde{Z}^1} + \frac{\alpha\beta}{\tilde{Z}^1 - \frac{1+\beta}{\beta}\varepsilon B_{un}}. \tag{56}$$

Figure A.1 illustrates the graph of (56), taking $(1+n)\tilde{x}$ on the horizontal axis. The figure indicates that there are two candidates for a solution to (56). However, the larger one is not feasible since $\tilde{Z}^1 < 0$ holds. Therefore, the smaller one is the solution to (56).

[Figure A.1 here.]

To derive the solution to (56), we reformulate (56) as follows:

$$\frac{\beta\eta(1-\alpha)}{(1+n)x}=\frac{\tilde{Z}^1-\frac{1+\beta}{\beta}\varepsilon B_{un}+\alpha\beta\tilde{Z}^1}{\tilde{Z}^1\left(\tilde{Z}^1-\frac{1+\beta}{\beta}\varepsilon B_{un}\right)},$$

or

$$f\left((1+n)\tilde{x}\right)\equiv G\left((1+n)x\right)^2-H(1+n)x+I=0, \tag{57}$$

where G, H, and I are defined in Proposition 5. Note that $H>0$ and $I>0$ hold because $\varepsilon B_{un}<\beta$ holds.

By solving (57) for $(1+n)x$ and taking the smaller solution, we obtain

$$(1+n)x=X_{con}\equiv\frac{H-\sqrt{(H)^2-4GI}}{2G}. \tag{58}$$

The substitution of (58) into (54) yields

$$1-\tau^k=\frac{\omega}{(1+n)(1-\omega)}\cdot\frac{\frac{H-\sqrt{(H)^2-4GI}}{2G}}{\beta\eta(1-\alpha)}\cdot\frac{1}{\alpha\left(1+b/\hat{k}h\right)}, \tag{59}$$

which verifies the initial guess. Finally, the labor tax rate is derived by substituting (58) and (59) into the government budget constraint.

■

A.7 Proof of Proposition 6

In the present framework, the state variable h_t does not line in a compact set because it continues to grow along an optimal path. To reformulate the planner's problem into one in which the state variable lies in a compact set, we undertake the following normalization:

$$\tilde{c}_t\equiv c_t/h_t, \tilde{d}_t\equiv d_t/h_t, \tilde{x}_t\equiv x_t/h_t.$$

Then, the resource constraint, $c_t+d_t/(1+n)+(1+n)\hat{k}_{t+1}h_{t+1}+(1+n)x_t=A\left(\hat{k}_t\right)^\alpha h_t$, is rewritten as

$$\tilde{c}_t+\frac{\tilde{d}_t}{1+n}+(1+n)\hat{k}_{t+1}\frac{h_{t+1}}{h_t}+(1+n)\tilde{x}_t=A\left(\hat{k}_t\right)^\alpha.$$

With the use of $h_{t+1}=D\left(h_t\right)^{1-\eta}\left(x_t\right)^\eta$, this is further reformulated as

$$\tilde{c}_t+\frac{\tilde{d}_t}{1+n}+(1+n)\hat{k}_{t+1}D\left(\tilde{x}_t\right)^\eta+(1+n)\tilde{x}_t=A\left(\hat{k}_t\right)^\alpha. \tag{60}$$

The utility functions are rewritten as follows:

$$
\begin{aligned}
U_{-1} &= \beta \ln \tilde{d}_0 + \beta \ln h_0, \\
U_0 &= \ln \tilde{c}_0 + \ln h_0 + \beta \ln \tilde{d}_1 + \beta \ln D \left(\tilde{x}_0\right)^{\eta} h_0, \\
U_1 &= \ln \tilde{c}_1 + \ln D \left(\tilde{x}_0\right)^{\eta} h_0 + \beta \ln \tilde{d}_2 + \beta \ln DD \left(\tilde{x}_0\right)^{\eta} \left(\tilde{x}_1\right)^{\eta} h_0, \\
&\vdots \\
U_t &= \ln \tilde{c}_t + \ln \left(D\right)^{t} \left(\tilde{x}_{t-1}\right)^{\eta} \left(\tilde{x}_{t-2}\right)^{\eta} \cdot \cdots \cdot \left(\tilde{x}_0\right)^{\eta} h_0 \\
&\quad + \beta \ln \tilde{d}_{t+1} + \beta \ln \left(D\right)^{t+1} \left(\tilde{x}_t\right)^{\eta} \left(\tilde{x}_{t-1}\right)^{\eta} \cdot \cdots \cdot \left(\tilde{x}_0\right)^{\eta} h_0, \\
&\vdots
\end{aligned}
$$

In particular, generation-t utility is rewritten as

$$
U_t = \ln \tilde{c}_t + \beta \ln \tilde{d}_{t+1} + \eta \left(1+\beta\right) \sum_{j=0}^{t-1} \ln \tilde{x}_j + \eta \beta \ln \tilde{x}_t + \left(1+\beta\right) \ln h_0 + \left[t + \beta \left(t+1\right)\right] \ln D.
$$

Thus, omitting the politically unrelated terms, the social welfare function becomes

$$
\begin{aligned}
SW \simeq\ & \frac{\beta}{\gamma} \ln \tilde{d}_0 \\
& + \ln \tilde{c}_0 + \beta \ln \tilde{d}_1 + \beta \eta \ln \tilde{x}_0 \\
& + \gamma \cdot \left[\ln \tilde{c}_1 + \beta \ln \tilde{d}_2 + \left(1+\beta\right) \eta \ln \tilde{x}_0 + \eta \beta \ln \tilde{x}_1 \right] \\
& + \gamma^2 \cdot \left[\ln \tilde{c}_2 + \beta \ln \tilde{d}_3 + \left(1+\beta\right) \eta \ln \tilde{x}_0 + \eta \left(1+\beta\right) \ln \tilde{x}_1 + \eta \beta \ln \tilde{x}_2 \right] \\
& + \cdots,
\end{aligned}
$$

that is,

$$
SW \simeq \sum_{t=0}^{\infty} \gamma^t \cdot \left\{ \ln \tilde{c}_t + \frac{\beta}{\gamma} \ln \tilde{d}_t + \eta \left[\beta + \frac{\gamma}{1-\gamma} \left(1+\beta\right) \right] \ln \tilde{x}_t \right\}. \tag{61}
$$

Plugging (60) into (61), the planner's problem becomes

$$
\begin{aligned}
& \max \sum_{t=0}^{\infty} \gamma^t \cdot \left\{ \ln \left[A \left(\hat{k}_t\right)^{\alpha} - \frac{\tilde{d}_t}{1+n} - (1+n) \hat{k}_{t+1} D \left(\tilde{x}_t\right)^{\eta} - (1+n) \tilde{x}_t \right] \right. \\
& \left. + \frac{\beta}{\gamma} \ln \tilde{d}_t + \eta \left[\beta + \frac{\gamma}{1-\gamma} \left(1+\beta\right) \right] \ln \tilde{x}_t \right\} \\
& \text{given } \hat{k}_0.
\end{aligned}
$$

We can express the Bellman equation for the problem as follows:

$$
\begin{aligned}
V(k) = \max_{\left\{\tilde{d}, \tilde{x}, \hat{k}'\right\}} & \left\{ \ln \left[A \left(\hat{k}\right)^{\alpha} - \frac{\tilde{d}}{1+n} - (1+n) \hat{k}' D \left(\tilde{x}\right)^{\eta} - (1+n) \tilde{x} \right] \right. \\
& \left. + \frac{\beta}{\gamma} \ln \tilde{d} + \eta \left[\beta + \frac{\gamma}{1-\gamma} \left(1+\beta\right) \right] \ln \tilde{x} + \gamma V(\hat{k}') \right\}.
\end{aligned} \tag{62}
$$

We make the guess $V(k') = z_0 + z_1 \ln k'$, where z_0 and z_1 are undetermined coefficients. For this guess, (62) becomes

$$V(k) = \max_{\{\tilde{d},\tilde{x},\hat{k}'\}} \left\{ \ln \left[A\left(\hat{k}\right)^{\alpha} - \frac{\tilde{d}}{1+n} - (1+n)\hat{k}' D\left(\tilde{x}\right)^{\eta} - (1+n)\tilde{x} \right] \right.$$
$$\left. + \frac{\beta}{\gamma} \ln \tilde{d} + \eta \left[\beta + \frac{\gamma}{1-\gamma}(1+\beta) \right] \ln \tilde{x} + \gamma \left[z_0 + z_1 \ln \hat{k}' \right] \right\}. \tag{63}$$

The first-order conditions with respect to $\tilde{d}$, $\tilde{x}$, and $\hat{k}'$ are

$$\tilde{d} : \frac{-1/(1+n)}{A\left(\hat{k}\right)^{\alpha} - \tilde{d}/(1+n) - (1+n)\hat{k}' D\left(\tilde{x}\right)^{\eta} - (1+n)\tilde{x}} + \frac{\beta}{\gamma} \cdot \frac{1}{\tilde{d}} = 0, \tag{64}$$

$$\tilde{x} : \frac{-(1+n)\left[\eta \hat{k}' D\left(\tilde{x}\right)^{\eta-1} + 1\right]}{A\left(\hat{k}\right)^{\alpha} - \tilde{d}/(1+n) - (1+n)\hat{k}' D\left(\tilde{x}\right)^{\eta} - (1+n)\tilde{x}} + \frac{\eta\left[\beta + \frac{\gamma}{1-\gamma}(1+\beta)\right]}{\tilde{x}} = 0, \tag{65}$$

$$\hat{k}' : \frac{-(1+n) D\left(\tilde{x}\right)^{\eta}}{A\left(\hat{k}\right)^{\alpha} - \tilde{d}/(1+n) - (1+n)\hat{k}' D\left(\tilde{x}\right)^{\eta} - (1+n)\tilde{x}} + \frac{\gamma z_1}{\hat{k}'} = 0. \tag{66}$$

Eqs. (64) and (65) lead to

$$\frac{\tilde{d}}{1+n} = \frac{\beta}{\gamma\eta} \cdot \frac{(1+n)\tilde{x}\left[\eta \hat{k}' D\left(\tilde{x}\right)^{\eta-1} + 1\right]}{\beta + \frac{\gamma}{1-\gamma}(1+\beta)}, \tag{67}$$

and Eqs. (64) and (66) lead to

$$\frac{\tilde{d}}{1+n} = \frac{\beta}{\gamma} \cdot \frac{\hat{k}'}{\gamma z_1}(1+n) D\left(\tilde{x}\right)^{\eta}. \tag{68}$$

By plugging (67) into (68) and rearranging the terms, we obtain

$$\eta D \hat{k}' = \frac{\left(\tilde{x}\right)^{1-\eta}}{\frac{1}{\gamma z_1}\left[\beta + \frac{\gamma}{1-\gamma}(1+\beta)\right] - 1}. \tag{69}$$

In addition, by plugging (69) into (67) and rearranging the terms, we obtain

$$\frac{\tilde{d}}{1+n} = \frac{\beta}{\gamma\eta} \cdot \frac{\frac{1}{\gamma z_1}}{\frac{1}{\gamma z_1}\left[\beta + \frac{\gamma}{1-\gamma}(1+\beta)\right] - 1}(1+n)\tilde{x}. \tag{70}$$

We substitute (69) and (70) into the first-order condition with respect to $\tilde{d}$ in (64) to obtain

$$(1+n)\tilde{x} = \frac{1}{\phi}\left\{ \frac{1}{\gamma z_1}\left[\beta + \frac{\gamma}{1-\gamma}(1+\beta)\right] - 1 \right\} A\left(\hat{k}\right)^{\alpha}, \tag{71}$$

where ϕ is defined by

$$\phi \equiv \frac{\gamma+\beta}{\gamma\eta} \cdot \frac{1}{\gamma z_1} + \frac{1}{\eta} + \frac{1}{\gamma z_1}\left[\beta + \frac{\gamma}{1-\gamma}(1+\beta)\right] - 1.$$

The substitution of (71) into (70) leads to the following policy function of $\tilde{d}$:

$$\frac{\tilde{d}}{1+n}=\frac{1}{\phi}\cdot\frac{\beta}{\gamma\eta}\cdot\frac{1}{\gamma z_1}A\left(\hat{k}\right)^{\alpha}. \tag{72}$$

With (69) and (71), we have

$$(1+n)\hat{k}'D\left(\tilde{x}\right)^{\eta}=\frac{1}{\phi\eta}A\left(\hat{k}\right)^{\alpha}. \tag{73}$$

We substitute (71), (72), and (73) into the resource constraint in (60) to obtain the following policy function of $\tilde{c}$:

$$\tilde{c}=\frac{1}{\phi\gamma\eta z_1}A\left(\hat{k}\right)^{\alpha}. \tag{74}$$

We also obtain from (71) and (73) the law of motion of physical capital:

$$\hat{k}'=\frac{\left[A\left(\hat{k}\right)^{\alpha}\right]^{1-\eta}}{\phi\eta(1+n)D\left\{\frac{1}{\phi(1+n)}\left[\frac{1}{\gamma z_1}\left(\beta+\frac{\gamma}{1-\gamma}\left(1+\beta\right)\right)-1\right]\right\}^{\eta}}. \tag{75}$$

Substituting (71), (72), (74), and (75) into the Bellman equation gives

$$V(k)=\alpha\left\{1+\frac{\beta}{\gamma}+\eta\left[\beta+\frac{\gamma}{1-\gamma}\left(1+\beta\right)\right]+\gamma z_1\left(1-\eta\right)\right\}\ln\hat{k}+C\left(z_0,z_1\right),$$

where $C\left(z_0,z_1\right)$ includes constant terms. The guess is verified if $z_0=C\left(z_0,z_1\right)$ and

$$z_1=\alpha\left\{1+\frac{\beta}{\gamma}+\eta\left[\beta+\frac{\gamma}{1-\gamma}\left(1+\beta\right)\right]+\gamma z_1\left(1-\eta\right)\right\}.$$

Therefore, z_1 is given by

$$z_1=\frac{\alpha\left\{1+\frac{\beta}{\gamma}+\eta\left[\beta+\frac{\gamma}{1-\gamma}\left(1+\beta\right)\right]\right\}}{1-\alpha\gamma\left(1-\eta\right)},$$

and the corresponding policy functions are obtained as expressed in Proposition 6.

■

A.8 Proof of Proposition 7

The difference of the model in Section 7 from the baseline model is the presence of public good provision, appeared as $\theta\ln g_t+\beta\theta\ln g_{t+1}$ in the utility function of (27) and as $(2+n)g/(1+n)$ in the government budget constraint of (28). Thus, in the presence of public good provision, the after-tax income of the middle-aged is $Z(\cdot)-(2+n)g/(1+n)$ and the next-period ratio of physical to human capital is

$$\hat{k}'=\hat{K}\left(b',x,Z(\cdot)-\frac{2+n}{1+n}g\right)\equiv\frac{\frac{\beta}{1+\beta}\left[Z(\cdot)-\frac{2+n}{1+n}g\right]-(1+n)b'}{(1+n)D\left(x\right)^{\eta}}.$$

The political objective function in (10) is reformulated as follows:

$$\Omega \simeq \frac{\omega}{(1+n)(1-\omega)}\left[\ln\left(1-\tau^k\right)+\theta\ln g\right]+(1+\beta)\ln\left[Z\left(\tau^k,x,b',\hat{k},b,h\right)-\frac{2+n}{1+n}g\right]$$
$$+\beta\ln R\left(\hat{K}\left(b',x,Z\left(\tau^k,x,b',\hat{k},b,h\right)-\frac{2+n}{1+n}g\right)\right)+\beta\ln\left(1-\tau^{k\prime}\right)+\theta\ln g+\beta\theta\ln g', \tag{76}$$

and the conjecture of $\tau^{k\prime}$ in (29) is reformulated as follows:

$$\tau^{k\prime}=1-T_g^k\frac{Z(\cdot)-\frac{2+n}{1+n}g-\frac{1+\beta}{\beta}(1+n)b'}{\alpha\left[Z(\cdot)-\frac{2+n}{1+n}g\right]}, \tag{77}$$

where $T_g^k(>0)$ is constant. We also conjecture the public good provision in the next period as

$$g'=G_g\cdot A\left(\hat{k}'\right)^{\alpha}h'.$$

This leads to

$$\beta\theta\ln g'\simeq\beta\theta\alpha\ln\left[Z(\cdot)-\frac{1+\beta}{\beta}(1+n)b'-\frac{2+n}{1+n}g\right]+(1-\alpha)\eta\ln x. \tag{78}$$

By substituting (77) and (78) into (76) and rearranging the terms, we have

$$\Omega\simeq\frac{\omega}{(1+n)(1-\omega)}\ln\left(1-\tau^k\right)+\ln\left[Z(\cdot)-\frac{2+n}{1+n}g\right]$$
$$+\beta\alpha\left(1+\theta\right)\ln\left[Z(\cdot)-\frac{1+\beta}{\beta}(1+n)b'-\frac{2+n}{1+n}g\right]+\left[\frac{\omega}{(1+n)(1-\omega)}+1\right]\theta\ln g$$
$$+\beta\eta\left(1+\theta\right)\left(1-\alpha\right)\ln x. \tag{79}$$

The first-order conditions with respect to τ^k, b', x, and g are as follows:

$$\tau^k:-\frac{\omega}{(1+n)(1-\omega)}\cdot\frac{1}{1-\tau^k}+\frac{\alpha A\left(\hat{k}\right)^{\alpha-1}\left(\hat{k}h+b\right)}{Z(\cdot)-\frac{2+n}{1+n}g}+\frac{\beta\alpha\left(1+\theta\right)\alpha A\left(\hat{k}\right)^{\alpha-1}\left(\hat{k}h+b\right)}{Z(\cdot)-\frac{1+\beta}{\beta}(1+n)b'-\frac{2+n}{1+n}g}=0, \tag{80}$$

$$b':\frac{1+n}{Z(\cdot)-\frac{2+n}{1+n}g}-\frac{\alpha\left(1+\theta\right)\left(1+n\right)}{Z(\cdot)-\frac{1+\beta}{\beta}(1+n)b'-\frac{2+n}{1+n}g}\leq 0, \tag{81}$$

$$x:\frac{\beta\eta\left(1+\theta\right)\left(1-\alpha\right)}{x}-\frac{1+n}{Z(\cdot)-\frac{2+n}{1+n}g}-\frac{\alpha\beta\left(1+\theta\right)\left(1+n\right)}{Z(\cdot)-\frac{1+\beta}{\beta}(1+n)b'-\frac{2+n}{1+n}g}=0, \tag{82}$$

$$g:\frac{\left[\frac{\omega}{(1+n)(1-\omega)}+1\right]\theta}{g}-\frac{\frac{2+n}{1+n}}{Z(\cdot)-\frac{2+n}{1+n}g}-\frac{\alpha\beta\left(1+\theta\right)\frac{2+n}{1+n}}{Z(\cdot)-\frac{1+\beta}{\beta}(1+n)b'-\frac{2+n}{1+n}g}=0, \tag{83}$$

where a strict inequality holds in (81) if $b'=0$.

$b' = 0$ Case Suppose that $b' = 0$ holds. Eq. (81) implies that

$$b' = 0 \text{ if } 1 \leq \alpha (1+\theta). \tag{84}$$

When $b' = 0$ holds, Eqs. (80) and (82) are rewritten as follows:

$$\frac{\omega}{(1+n)(1-\omega)} \cdot \frac{1}{1-\tau^k} = \frac{(1+\alpha\beta(1+\theta))\alpha A\left(\hat{k}\right)^{\alpha-1}\left(\hat{k}h+b\right)}{Z(\cdot) - \frac{2+n}{1+n}g}, \tag{85}$$

$$\frac{\beta\eta(1+\theta)(1-\alpha)}{(1+n)x} = \frac{1+\alpha\beta(1+\theta)}{Z(\cdot) - \frac{2+n}{1+n}g}. \tag{86}$$

Eqs. (85) and (86) lead to the optimal relation between x and τ^k:

$$(1+n)x = \frac{\beta\eta(1+\theta)(1-\alpha)}{\frac{\omega}{(1+n)(1-\omega)}}\left(1-\tau^k\right)\alpha A\left(\hat{k}\right)^{\alpha-1}\left(\hat{k}h+b\right). \tag{87}$$

Next, recall Eq. (83). When $b' = 0$, this is reduced to

$$\frac{\left[\frac{\omega}{(1+n)(1-\omega)}+1\right]\theta}{\frac{2+n}{1+n}g} = \frac{1+\alpha\beta(1+\theta)}{Z(\cdot) - \frac{2+n}{1+n}g}. \tag{88}$$

Eqs. (86) and (88) lead to the optimal relation between g and x :

$$\frac{2+n}{1+n}g = \frac{\left[\frac{\omega}{(1+n)(1-\omega)}+1\right]\theta}{\beta\eta(1+\theta)(1-\alpha)}(1+n)x. \tag{89}$$

In addition, (85) and (88) lead to the optimal relation between g and τ^k :

$$\frac{2+n}{1+n}g = \frac{\left[\frac{\omega}{(1+n)(1-\omega)}+1\right]\theta}{\frac{\omega}{(1+n)(1-\omega)}}\left(1-\tau^k\right)\alpha A\left(\hat{k}\right)^{\alpha-1}\left(\hat{k}h+b\right). \tag{90}$$

The substitution of (87) and (90) into (85) leads to the policy function of τ^k :

$$1-\tau^k = \frac{\frac{\omega}{(1+n)(1-\omega)}}{(1+\theta)\left\{\frac{\omega}{(1+n)(1-\omega)}+1+\beta\left[\alpha+\eta(1-\alpha)\right]\right\}} \cdot \frac{1}{\alpha\left(1+b/\hat{k}h\right)}. \tag{91}$$

The substitution of (91) into (87) leads to the policy function of x :

$$(1+n)x = \frac{\beta\eta(1-\alpha)}{\left\{\frac{\omega}{(1+n)(1-\omega)}+1+\beta\left[\alpha+\eta(1-\alpha)\right]\right\}} \cdot A\left(\hat{k}\right)^{\alpha}h, \tag{92}$$

and the substitution of (92) into (89) leads to the policy function of g :

$$\frac{2+n}{1+n}g = \frac{\left[\frac{\omega}{(1+n)(1-\omega)}+1\right]\theta}{(1+\theta)\left\{\frac{\omega}{(1+n)(1-\omega)}+1+\beta\left[\alpha+\eta(1-\alpha)\right]\right\}} \cdot A\left(\hat{k}\right)^{\alpha}h. \tag{93}$$

Finally, plugging (91)–(93) with $b' = 0$ into the government budget constraint leads to the policy function of τ :

$$\tau = 1 - \frac{1}{1-\alpha} \cdot \frac{1+\alpha\beta(1+\theta)}{(1+\theta)\left\{\frac{\omega}{(1+n)(1-\omega)}+1+\beta\left[\alpha+\eta(1-\alpha)\right]\right\}}.$$

A.8.1 $b' > 0$ Case

Alternatively, suppose that $b' > 0$ holds. The first-order condition with respect to b' in (81), holding with an equality, is rewritten as follows:

$$(1+n)b' = \frac{\beta\left[1-\alpha\left(1+\theta\right)\right]}{1+\alpha\beta\left(1+\theta\right)} \cdot \left[A\left(\hat{k}\right)^{\alpha} h - \left(1-\tau^{k}\right)\alpha A\left(\hat{k}\right)^{\alpha-1}\left(\hat{k}h+b\right) - (1+n)x - \frac{2+n}{1+n}g\right]. \tag{94}$$

Eqs. (80) and (83) lead to the optimal relation between g and τ^k :

$$\left(1-\tau^{k}\right)\alpha A\left(\hat{k}\right)^{\alpha-1}\left(\hat{k}h+b\right) = \frac{\frac{\omega}{(1+n)(1-\omega)}}{\left[\frac{\omega}{(1+n)(1-\omega)}+1\right]\theta} \cdot \frac{2+n}{1+n}g, \tag{95}$$

and Eqs. (82) and (83) lead to the optimal relation between x and g :

$$(1+n)x = \frac{\beta\eta\left(1+\theta\right)\left(1-\alpha\right)}{\left[\frac{\omega}{(1+n)(1-\omega)}+1\right]\theta} \cdot \frac{2+n}{1+n}g. \tag{96}$$

In addition, Eqs. (81) and (83) lead to

$$\begin{aligned}
&\left\{(1+\beta) + \left[\frac{\omega}{(1+n)(1-\omega)}+1\right]\theta\right\} \cdot \frac{2+n}{1+n}g \\
&= \left[\frac{\omega}{(1+n)(1-\omega)}+1\right]\theta \\
&\times \left[A\left(\hat{k}\right)^{\alpha} h - \left(1-\tau^{k}\right)\alpha A\left(\hat{k}\right)^{\alpha-1}\left(\hat{k}h+b\right) - (1+n)x + (1+n)b'\right].
\end{aligned} \tag{97}$$

The substitution of (95) and (96) into (94) leads to

$$\begin{aligned}
&\left[\frac{\omega}{(1+n)(1-\omega)}+1\right]\theta(1+n)b' \\
&= \frac{\beta\left[1-\alpha\left(1+\theta\right)\right]}{1+\alpha\beta\left(1+\theta\right)} \cdot \left\{\left[\frac{\omega}{(1+n)(1-\omega)}+1\right]\theta \cdot A\left(\hat{k}\right)^{\alpha} h\right. \\
&\left. - \left[\frac{\omega}{(1+n)(1-\omega)} + \beta\eta\left(1+\theta\right)\left(1-\alpha\right) + \left[\frac{\omega}{(1+n)(1-\omega)}+1\right]\theta\right] \cdot \frac{2+n}{1+n}g\right\},
\end{aligned} \tag{98}$$

and the substitution of (95) and (96) into (97) leads to

$$\begin{aligned}
&\left[\frac{\omega}{(1+n)(1-\omega)}+1\right]\theta(1+n)b' \\
&= \left\{(1+\beta) + \left[\frac{\omega}{(1+n)(1-\omega)}+1\right]\theta + \frac{\omega}{(1+n)(1-\omega)} + \beta\eta\left(1+\theta\right)\left(1-\alpha\right)\right\} \\
&\times \frac{2+n}{1+n}g - \left[\frac{\omega}{(1+n)(1-\omega)}+1\right]\theta \cdot A\left(\hat{k}\right)^{\alpha} h.
\end{aligned} \tag{99}$$

With the use of (98) and (99), we can obtain the policy functions of g and b' as follows:

$$\frac{2+n}{1+n}g = \frac{\left[\frac{\omega}{(1+n)(1-\omega)}+1\right]\theta}{(1+\theta)\left\{\frac{\omega}{(1+n)(1-\omega)}+1+\beta\left[\alpha+\eta\left(1-\alpha\right)\right]\right\}}A\left(\hat{k}\right)^{\alpha}h, \tag{100}$$

$$(1+n)b' = \frac{\beta\left[1-\alpha\left(1+\theta\right)\right]}{(1+\theta)\left\{\frac{\omega}{(1+n)(1-\omega)}+1+\beta\left[\alpha+\eta\left(1-\alpha\right)\right]\right\}}A\left(\hat{k}\right)^{\alpha}h. \tag{101}$$

Eq. (101) indicates that $b' > 0$ holds if and only if $1 > \alpha\left(1+\theta\right)$.

The substitution of (100) into (95) leads to the policy function of τ^k :

$$1-\tau^k = \frac{\frac{\omega}{(1+n)(1-\omega)}}{(1+\theta)\left\{\frac{\omega}{(1+n)(1-\omega)}+1+\beta\left[\alpha+\eta\left(1-\alpha\right)\right]\right\}}\cdot\frac{1}{\alpha\left(1+b/\hat{k}h\right)}.$$

The substitution of (100) into (96) leads to the policy function of x :

$$(1+n)x = \frac{\beta\eta(1-\alpha)}{\left\{\frac{\omega}{(1+n)(1-\omega)}+1+\beta\left[\alpha+\eta\left(1-\alpha\right)\right]\right\}}\cdot A\left(\hat{k}\right)^{\alpha}h.$$

Finally, we substitute the policy functions derived above into the government budget constraint and rearrange the terms to obtain the policy function of τ :

$$\tau = 1-\frac{1+\beta}{1-\alpha}\cdot\frac{1}{(1+\theta)\left\{\frac{\omega}{(1+n)(1-\omega)}+1+\beta\left[\alpha+\eta\left(1-\alpha\right)\right]\right\}}.$$

■

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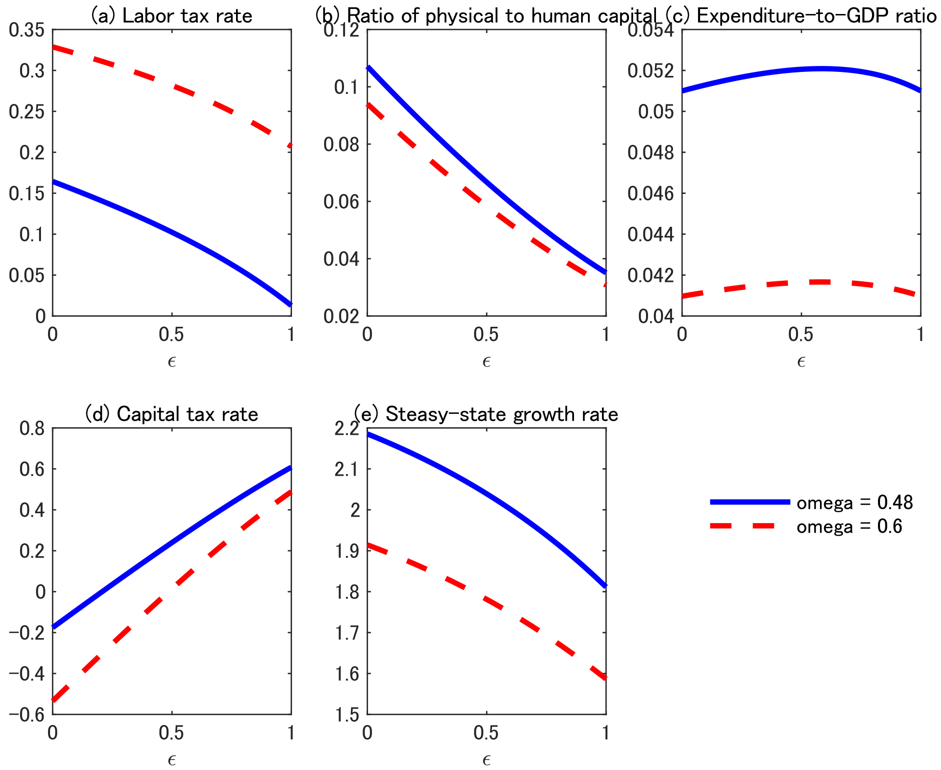

Figure 1: Effects of decreased ε on the education expenditure-to-GDP ratio (Panel (a)), labor tax rate (Panel (b)), capital tax rate (Panel (c)), ratio of physical to human capital (Panel (d)), and steady-state growth rate (Panel (e)). The solid and dashed curves correspond to the cases of $\omega = 0.48$ and 0.6, respectively.

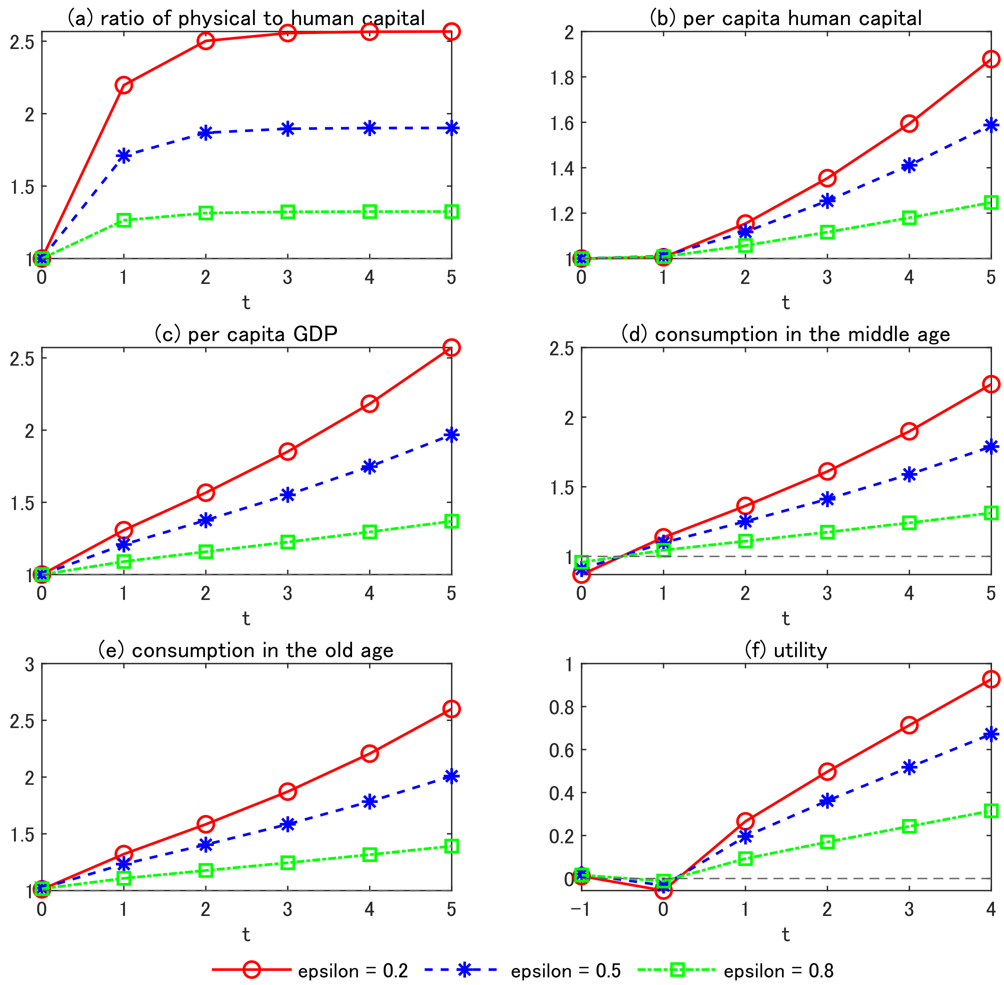

Figure 2: Evolution of the ratio of physical to human capital (Panel (a)), human capital (Panel (b)), per capita GDP (Panel (c)), middle-age consumption (Panel (d)), old-age consumption (Panel (e)), and utility (Panel (f)). The solid, dashed, and dot-dashed curves correspond to the cases of $\varepsilon = 0.2$, 0.5, and 0.8, respectively.

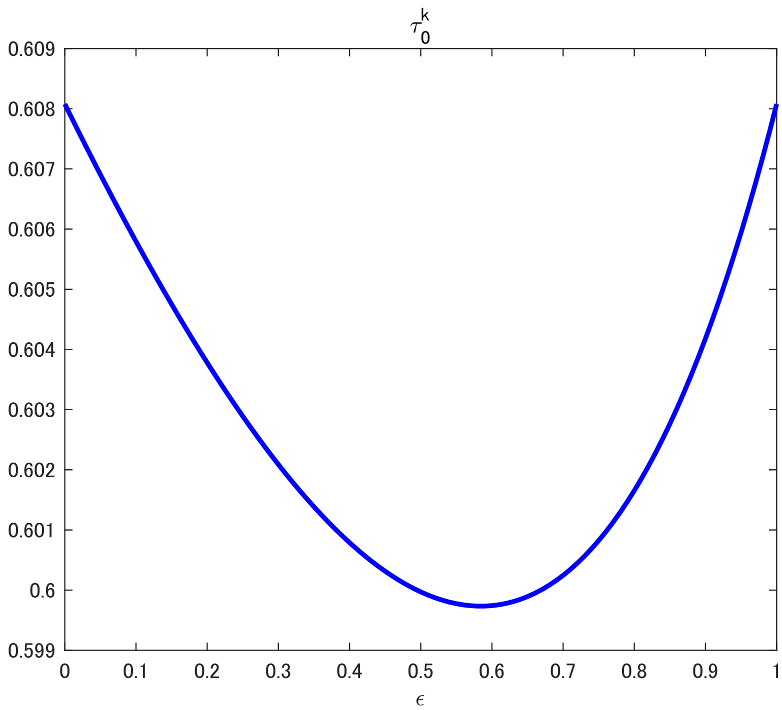

Figure 3: Period 0 capital tax rate.

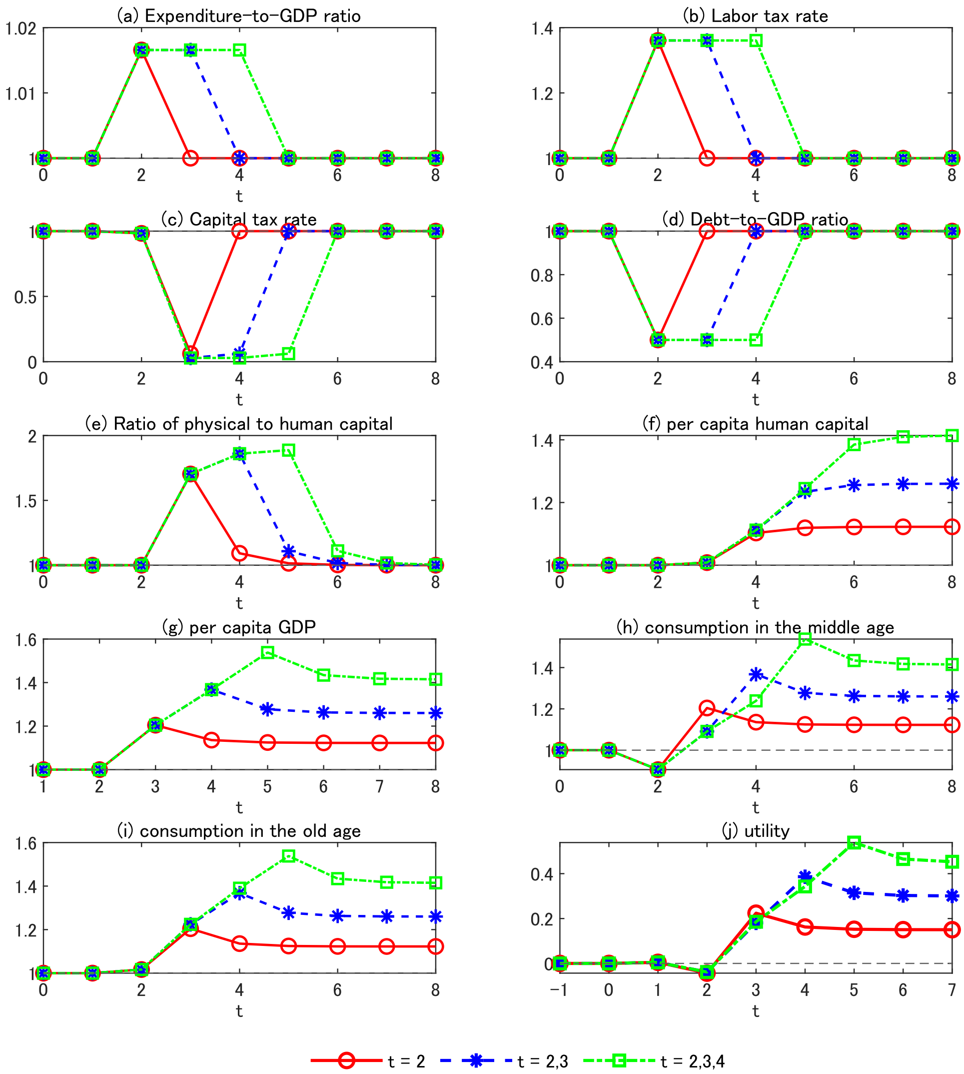

Figure 4: Effects of the temporary implementation of the fiscal rule in period 2. The solid, dashed, and chain lines depict the cases where the rule is terminated at the end of periods 2, 3, and 4, respectively.

Note: Panels (a)–(i) show the response by focusing on the ratio of a variable in the presence of the temporary implementation to that in its absence. Panel (j) plots the difference in utility between the presence and absence of the temporary implementation.

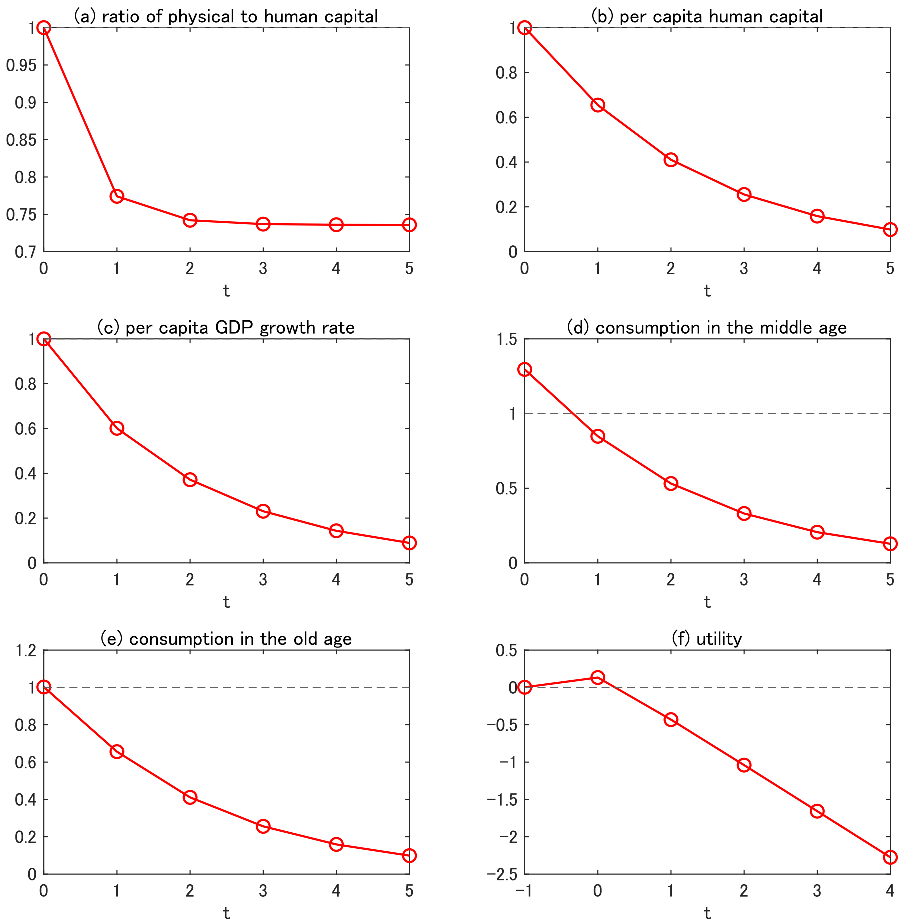

Figure 5: Evolution of the ratio of physical to human capital (Panel (a)), human capital (Panel (b)), per capita GDP (Panel (c)), middle-age consumption (Panel (d)), old-age consumption (Panel (e)), and utility (Panel (f)).

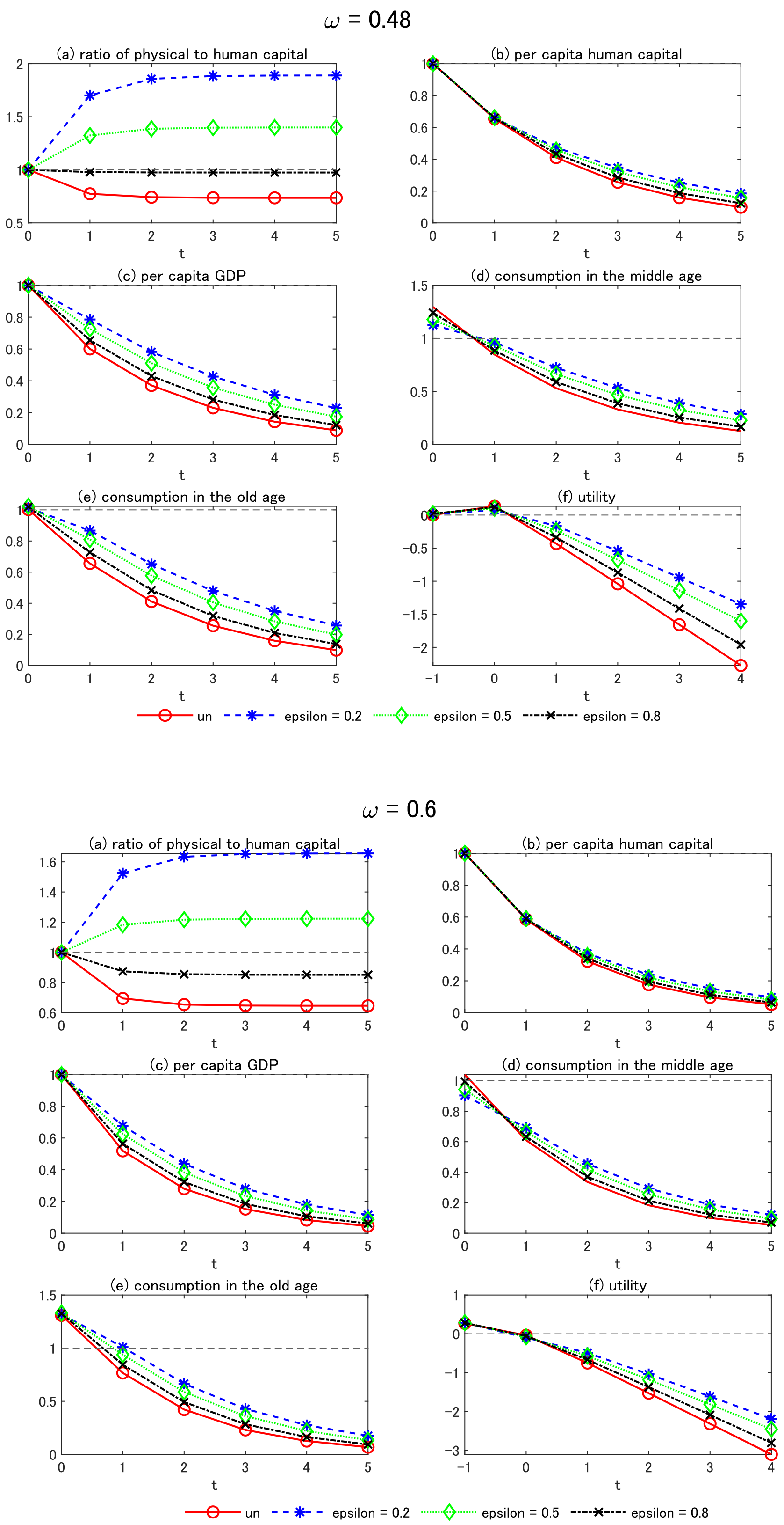

Figure 6: Evolution of the ratio of physical to human capital (Panel (a)), human capital (Panel (b)), per capita GDP (Panel (c)), middle-age consumption (Panel (d)), old-age consumption (Panel (e)), and utility (Panel (f)) when $\gamma = (0.99)^{120}$.

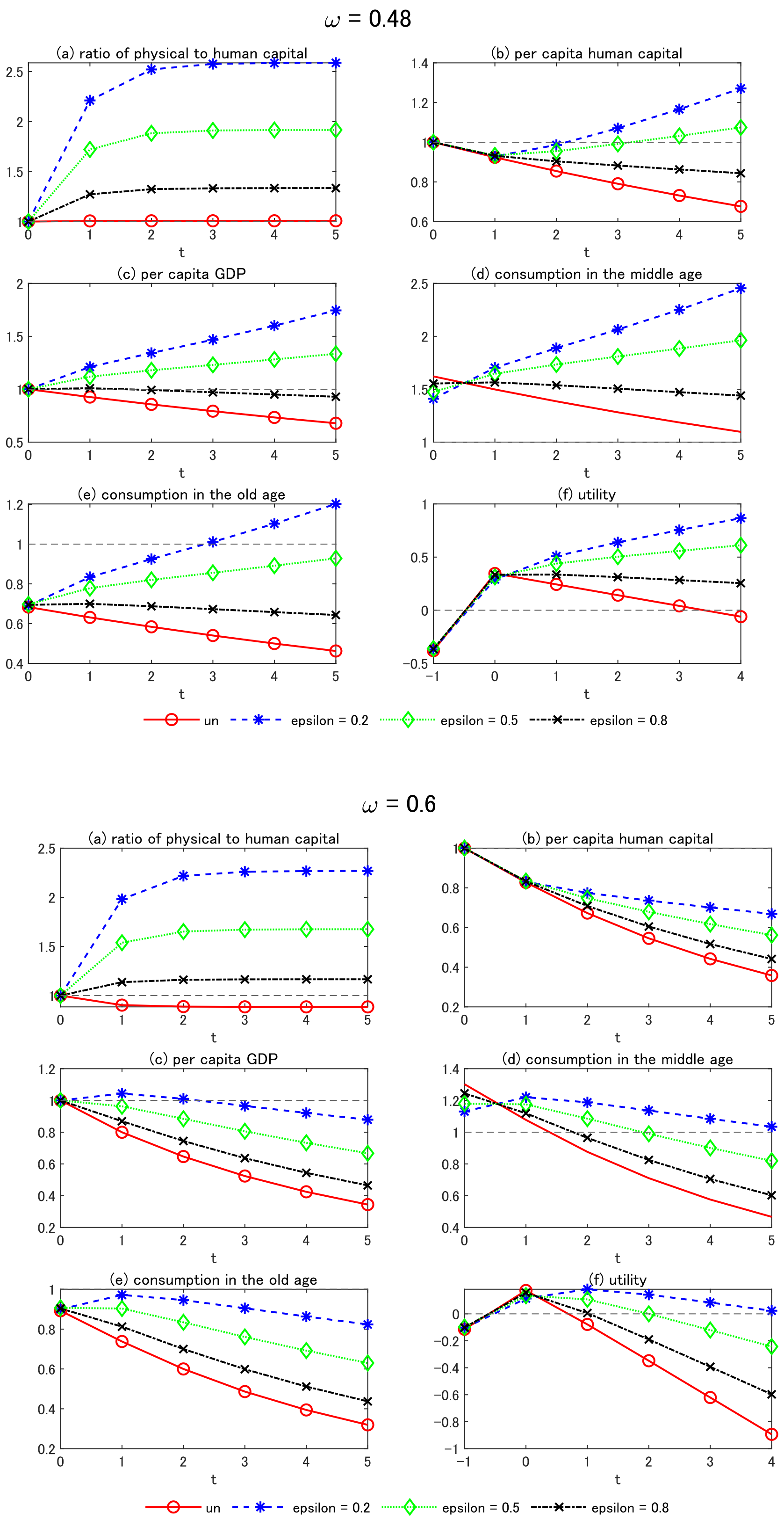

Figure 7: Evolution of the ratio of physical to human capital (Panel (a)), human capital (Panel (b)), per capita GDP (Panel (c)), middle-age consumption (Panel (d)), old-age consumption (Panel (e)), and utility (Panel (f)) when $\gamma = (0.985)^{120}$.

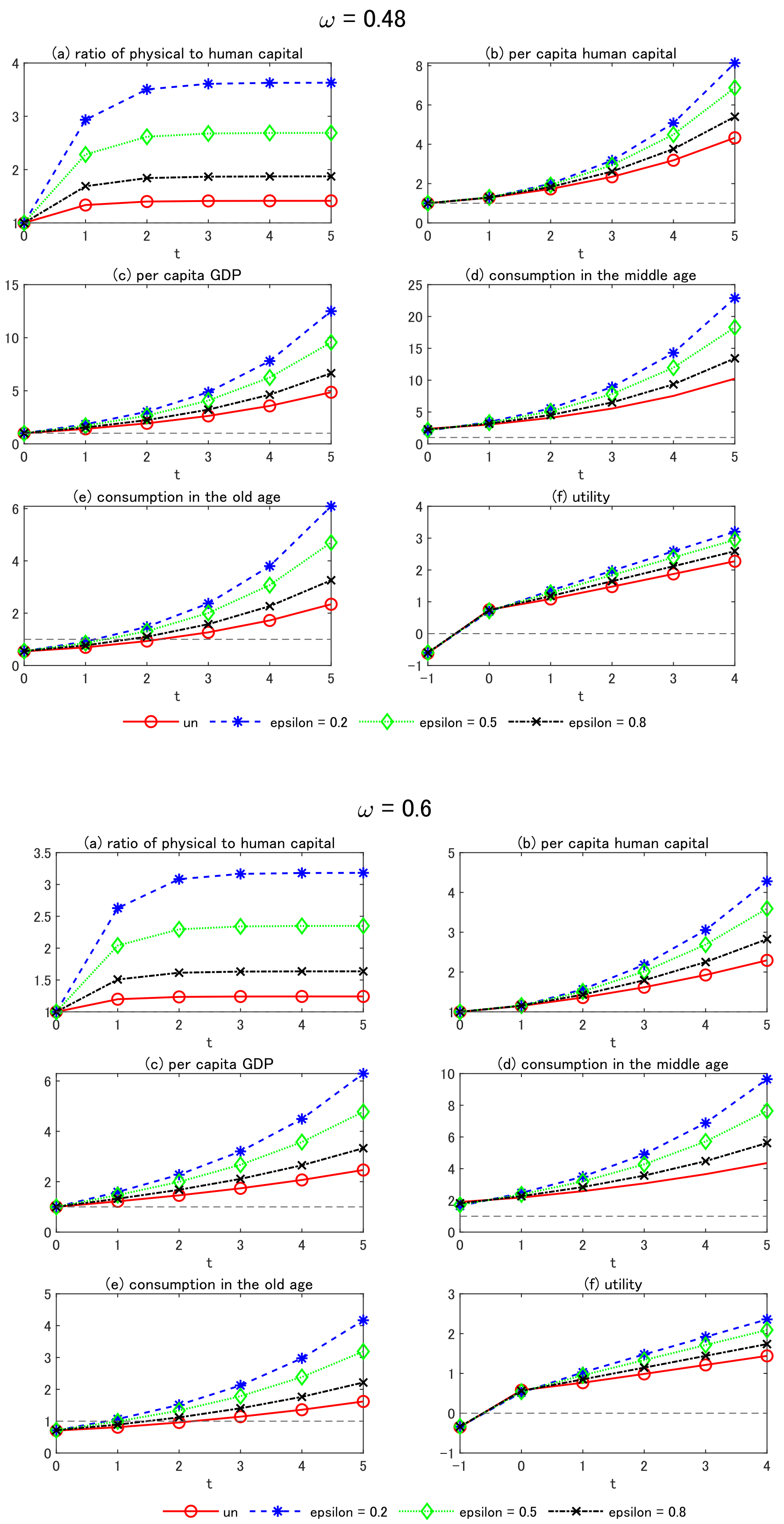

Figure 8: Evolution of the ratio of physical to human capital (Panel (a)), human capital (Panel (b)), per capita GDP (Panel (c)), middle-age consumption (Panel (d)), old-age consumption (Panel (e)), and utility (Panel (f)) when $\gamma = (0.98)^{120}$.

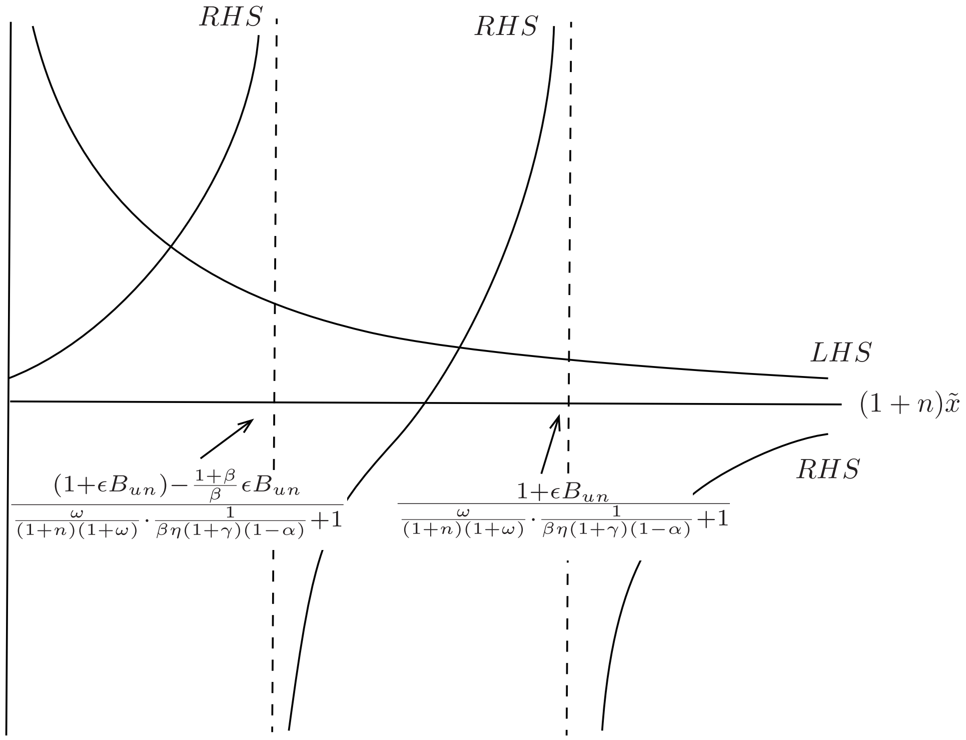

Figure A.1: Illustration of the left-hand side and right-hand side of Eq. (56).