Heckscher-Ohlin Trade, Leontief Trade, and Factor Conversion Trade When Countries Have Different Technologies

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Abstract – Kurokawa (2011), Takahashi (2004), Simpson (2016), and Kozo and Yoshinori (2017) have demonstrated the evidence of the factor intensity reversals (FIRs) in their empirical studies. This is another big challenge for international economics after Leontief paradox. This paper demonstrates that there are three trade types in international trade: the Heckscher-Ohlin trade, the Leontief trade, and the conversion trade, by using the $2 \times 2 \times 2$ Trefler model. The conversion trade occurs when the model structure is with FIRs. The conversion trade is one that one country exports the commodity that uses its scarce factor intensively; another country exports the commodity that uses its abundant factor intensively. The conversion trade actually is the trade with factor content reversal[2], i.e. that if one country exports the services of capital and imports the services of labor, another country does the same. This study demonstrates that both the Leontief trade and the conversion trade are rooted in the Heckscher-Ohlin theories. The three trade types are under the generalized trade pattern that each country exports the commodity that uses its effective (virtual) [3] abundant factor intensively and imports the commodity that uses its effective (virtual) scarce factor intensively.

Keywords

Heckscher-Ohlin-Ricardo model, different technologies, Heckscher-Ohlin Trade, Leontief Trade, and Conversion Trade, factor content of trade, trade types, Leontief Paradox; virtual factor endowments;

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[2] The factor content reversal means that the directions (signs) of factor contents of trade of two countries are same, such as

$$\begin{bmatrix} F_K^H \\ F_L^H \end{bmatrix} = \begin{bmatrix} + \\ - \end{bmatrix}, \qquad \begin{bmatrix} F_K^F \\ F_L^F \end{bmatrix} = \begin{bmatrix} + \\ - \end{bmatrix}$$

where F_K^h is the capital content of trade in country h; F_L^h is the labor content of trade in country *h; h=H,F*.

[3] See Trefler (1993) and Feenstra and Taylor (2012, p102) for effective factor endowments. See Fisher (2011) for virtual factor endowment.

1. Introduction

The Leontief test (Leontief, 1953) showed that the US as a capital abundant country exported its labor-intensive commodities. It counters the common sense of international economics. Baldwin (1971) tested 1962 US trade data, the conclusion was the same as Leontief made. Leamer (1980) reformatted the Leontief data and showed that the capital/labor ratio embodied in production exceeded the capital/labor ratio in consumption. Other tests in the last century by using foreign countries data showed that the results are half-and-half being consistent with Heckscher-Ohlin theories.

The Leontief Paradox simulated the HOV studies to explore new approaches in international economics. Many studies, like Trefler (1993), are of opinion that the international factor price differences were the explanations for the paradox. Fisher and Marshall (2016) made the latest test and concluded that Leontief is not right. They used price analysis by the Trefler model.

Kwok and Yu (2005) re-investigated the 52 countries data by the approach of differentiated factor intensity techniques and concluded that Leontief paradox "is found to be either disappeared or eased".

Jones (1956) and Robinson (1956) argued that FIR could have been responsible for the Leontief Paradox in the US data. Wong (1995, pp128) thought that one possible reason for the Leontief paradox is the presence of the FIR. However, He believed that Leontief "computes and uses the wrong capital-labor ratio". When FIRs occur, Leamer(1995) referred it to "factor price insensitivities". Jones (1956) examined the possible trade results for the FIRs, he wrote "If the relatively labor abundant country exports its labor-intensive commodity, it must do so in exchange for the commodity that, in the relatively capital-abundant foreign country, is produced by labor-intensive techniques. Thus if one country satisfies the theorem, the other country cannot". This is the first study describing the detail trade features of the FIRs. The consequence of trade when countries have different technologies with the presence of FIRs still is mysterious, although it is curiosum in the studies of international economics.

Deardorff (1985) presented the diversification cones of the FIRs. He studied the double factor intensity reversal. He suggested a way to turn any model with FIRs into one without it, and vice versa, simply redefined goods.

Giri(2018) talked the reason of the FIRs as "Since firms are going to decide the units of labor and capital they employ in response to the prevailing wage rate and rental rate, it is possible that the factor intensity ranking in the equation above is reversed at a different factor-price ratio (w/r). This is termed as factor-intensity reversal

(FIRs).” He noticed that the different factor-price ratio is the result of equilibrium. He also implied that the FIRs is by the reversion of cones of diversification of factor endowments.

Feenstra (2004, p11) described the reality of the FIRs, “While FIRs might seem like a theoretical curiosum, they are actually quite realistic”. He thought that the FIRs is a typical case of factor technologies different across countries. This implied the FIRS is by the cone reversals.

Minhas (1962) first reported finding the evidence of FIRs. He also first provided a production function to form a case of FIRs. He investigated industry data for 19 countries. He found FIRs in 5 countries. Leontief (1964) revisited the Minhas test and showed fewer cases of FIRs.

Kurokawa (2011) showed “clear-cut evidence for the existence of the skill intensity reversal” in his empirical study of the USA-Mexico economy. Sampson (2016) interpreted his assignment reversals of skill workforce between North and South by factor intensity reversal. Takahashi (2004) studied the postwar Japan economy. He interpreted Japan economy growth by capital-intensity reversal. Reshef (2007) studied the model with factor intensity reversals in skill, which can explain the North-South skill premia increase well. Kozo and Yoshinori (2017) found evidence of factor intensity reversals in their study also. They wrote, “Using newly developed region-level data, however, we argue that the abandonment of factor intensity reversals in the empirical analysis has been premature. Specifically, we find that the degree of the factor intensity reversals is higher than that found in previous studies on average”. The factor intensity reversals they mentioned is the capital-labor factor intensity reversal. The FIRs are not just textbook interesting. The theory studies of FIRs are behind international trade observations. This study displayed that the FIRs always associated with the conversion trade. The FIRs implies the Leontief trade also. The reversion of factor content of trade is another big challenge for international economics. It is more paradox than the Leontief paradox.

Fisher and Marshall (2008) provided another insight approach to involve different technologies in the HOV model by using virtual factor endowments and the conversion matrix. Fisher (2011) proposed important terms of goods price diversification cone and the intersection of goods price diversification cones. Feenstra and Taylor (2012, p102) proposed another useful concept of effective factor endowments to measure the production efficiencies of factor endowments when countries are with different technologies across countries.

Guo (2015) proposed a solution to the general trade equilibrium for the $2 \times 2 \times 2$ Heckscher-Ohlin model. He demonstrated that equalized factor price and common world price at the equilibrium is the function of the world factor endowments (the rental-wage ratio equals to the world labor-capital ratio as $r/w = L^W/K^W$). Guo (2019) provided the price-trade equilibrium for the Trefler model, which is a typical expression for different productivities across countries. They are helpful to understand the Leontief trade and the conversion trade of this study.

This study showed that there are three trade types: the Heckscher-Ohlin trade, the Leontief trade, and conversion trade when countries have different productivities. The Heckscher-Ohlin trade and Leontief trades are widely known. The Heckscher-Ohlin trade is that each country exports the commodity that uses its abundant factor intensively. The Leontief trade, we defined here, is that each country exports the commodity that uses its scarce factor intensively. Most understanding of the cause of the Leontief is the presence of FIRs in the model structure. This study demonstrates that Leontief trade can occur without the presence of FIRs. For a $2 \times 2 \times 2$ Trefler model, the condition of the Leontief trade is $\frac{K^H L^F}{L^H K^F} < \frac{\pi_K}{\pi_L}$, if the home country is capital abundant, $\frac{K^H}{L^H} > \frac{K^F}{L^F}$, where k^h is factor k in country h, $k = K, L$; $h = H, F$; π_k is the factor productivity-argument parameter.

The conversion trade is a trade that one country does Heckscher-Ohlin trade, another does the Leontief trade, in which both countries export the same factor services and import the same factor services. The study also used a specified Trefler model to demonstrate that this is a result of trade equilibrium. When countries are under the FIRs model structure, international trade converts worldwide effective abundant factor into worldwide effective scarce factor. This is a new understanding of international trade. The conversion trade's behavior is very like the "black hole" [4] in astronomy. The trade has a mechanism that the effective abundant factor cannot "escape" from the market (or it is absorbed by the market). Meanwhile, the trade has also a "white hole"[5] function that the effective scarce factor can only leave the market to go toward to each country. It displays a different kind of gains from trade, the gains from the consumption side. Using the generalized trade pattern that a country exports the commodity that uses its virtual abundant factor intensively, we can explain the conversion trade and Leontief trade well.

Trefler (1993, pp965) have demonstrated that the equivalent-factor version (or same technology version) of his 1993 model satisfies both "factor price equalization hypothesis and HOV theorem". This implies that both Leontief trade and the conversion trade rooted in the Heckscher-Ohlin theory. This study normalizes both of them.

[4] Black hole in astronomy is defined as that a region of space having a gravitational field so intense that no matter or radiation can escape.

[5] In general relativity, a white hole is a hypothetical region of spacetime, which cannot be entered from the outside, although matter and light can escape from it. In this sense, it is the reverse of a black hole, which can only be entered from the outside and from which matter and light cannot escape.

The paper provides three ways to display trade types. The first way is by a multiscale generalized Integrated World Equilibrium (see Dixit and Norman, 1980). It presents the Heckscher-Ohlin trade, the Leontief trade, and conversion trade geometrically. This is a very useful visual tool to show complicated economics definitions. The second approach is by the trade equilibrium in the Trefler model. The third way is by using the intersection cone of commodity price (see Fisher, 2011) to present the trade direction based on the HOV analyses only[6]. This method can present numerical examples of three trade types very simply, without using any other new logic or new principles in international economics.

Is the issue of Leontief paradox over? Most scholars believe so. One reason is that Leontief trade has no theoretical background. Another reason is that some empirical studies declared that the conclusion of the Leontief test is not right. This study may help for changing the view on the Leontief trade. The normalization of Leontief trade may affect the usage of some popular sign prediction criteria in empirical studies. This paper reviewed fewer of the empirical studies in this century and found that the sign prediction criteria in their studies theoretically include all trade types, from the view of this paper now. The sign predictions are based on the HOV theorem with different technologies. The Leontief trade and the conversion trade are under the HOV theorem form this study. It implies that the sign predictions explained all of the trade types, including the Leontief trade and conversion trade. In addition, the evidence of factor intensity reversals just shows up that Leontief is right from the view of the conversion trade.

The paper is organized by six sections. Section 2 denotes the Heckscher-Ohlin-Ricardo model. It also reviews some related terms: the model technology patterns, the cone of commodity price, and intersection cone price. It demonstrates the generalized trade pattern as that each country exports the commodity that uses its virtual abundant factor intensively. Section 3 discusses the Heckscher-Ohlin trade and the Leontief trade by the model without the presence of FIRs. It illustrates that virtual (effective) factor abundance determines trade directions. Section 4 presents the conversion trade. It shows that the conversion trades occur when the model is with the presence of FIRs. It proposes a Trefler Hicks-Neutral FIRs model and shows that conversion trade is rooted in the Heckscher-Ohlin theories. Section 5 reviews HOV empirical studies and illustrates if the existing criteria of trade prediction in recent empirical studies favor all of the trade types of this paper. The final section is conclusion remarks.

[6] The trade equilibrium can display the three trade types directly. However, it may be somewhat sudden for some readers, since it is not a published paper. The another way, using the intersection cone of commodity price, is a straightforward logic in the HOV model with different technologies. It is available to play data examples of three trade types.

2. Preliminaries: Heckscher-Ohlin-Ricardo Model and Intersection Cone of Commodity Price

2.1 Heckscher-Ohlin-Ricardo Model

We refer to the Heckscher-Ohlin framework with different technologies across countries as the Heckscher-Ohlin-Ricardo model. Davis (1995) used this name first. Morrow (2010) named his study as Ricardian-Heckscher-Ohlin comparative advantage. Deardorff (2002)'s two cone analyses paved solid foundations for the source of technology difference in Heckscher-Ohlin framework. The Heckscher-Ohlin-Ricardo model is a general expression when countries have different technologies.

The Heckscher-Ohlin-Ricardo Model inherits all assumptions of the Heckscher-Ohlin model, except the assumptions of the same technologies and no factor intensity reversals. We take the basic assumptions for the Heckscher-Ohlin-Ricardo model as (1) different technologies across countries, (2) identical homothetic taste, (3) perfect competition in the commodities and factors markets, (4) no cost for international exchanges of commodities, (5) factors are completely immobile across countries but that can move costlessly between sectors within a country, (6) constant return of scale, (7) full employment of factor resources.

We denote the $2 \times 2 \times 2$ Heckscher-Ohlin-Ricardo model as

$$A^h X^h = V^h \qquad (h = H, F) \qquad \text{(2-2)}$$

$$(A^h)' W^{h*} = P^* \qquad (h = H, F) \qquad \text{(2-3)}$$

where A^h is the 2×2 matrix of factor input requirements with elements a_{ki} (r,w), $i = 1,2; k = K, L; and\ h = H, F$; V^h is the 2×1 vector of factor endowments with elements K as capital and L as labor; X^h is the 2×1 vector of output; W^{h*} is the 2×1 vector of factor prices with elements r as rent and w as wage; P^* is the 2×1 vector of commodity prices with elements $p_1^*\ and\ p_2^*$, when trade reached market equilibrium.

Trefler Model is a special case of the Heckscher-Ohlin-Ricardo model. We will use both of them to illustrate the trade types in this paper,

2.2 Model Structure Patterns

There are two model structure patterns for the Heckscher-Ohlin-Ricardo model. One is the nonexistence of factor intensity reversals (no presence of FIRs). Another is the existence of factor intensity reverse (the presence of FIRs) [7].

In the model of no presence of FIRS, if the home country is capital intensive in commodity 1, the foreign country is capital intensive in commodity 1 also. It can be characterized, for the $2 \times 2 \times 2$ model, by

$$\left|A^H\right|\left|A^F\right| > 0 \tag{2-4}$$

where $\left|A^h\right|$ is the determinant of technology matrix of country h, $h = H, F$. $\left|A^h\right| > 0$ means that it is capital-intensive in sector 1 for country h. $\left|A^h\right| < 0$ means labor-intensive in sector 1 for country h.

Another description of the model pattern is by the cost requirement ratio ranks. The following two ranks are typical for the no presence of FIRs (2-4),

$$\frac{a_{K1}^H}{a_{K2}^H} > \frac{a_{K1}^F}{a_{K2}^F} > \frac{a_{L1}^H}{a_{L2}^H} > \frac{a_{L1}^F}{a_{L2}^F} \tag{2-5}$$

or

$$\frac{a_{K1}^H}{a_{K2}^H} > \frac{a_{K1}^F}{a_{K2}^F} > \frac{a_{L1}^F}{a_{L2}^F} > \frac{a_{L1}^H}{a_{L2}^H} \tag{2-6}$$

In the model of the presence of FIRs, if the home country is capital intensive in sector 1 , the foreign country is capital intensive in sector 2. It is characterized by

$$\left|A^H\right|\left|A^F\right| < 0 \tag{2-7}$$

The following two cost requirement ratio ranks are typical for (2-7),

$$\frac{a_{K1}^H}{a_{K2}^H} \geq \frac{a_{L1}^F}{a_{L2}^F} > \frac{a_{L1}^H}{a_{L2}^H} \geq \frac{a_{K1}^F}{a_{K2}^F} \tag{2-8}$$

or

$$\frac{a_{K1}^H}{a_{K2}^H} \geq \frac{a_{L1}^F}{a_{L2}^F} > \frac{a_{K1}^F}{a_{K2}^F} > \frac{a_{L1}^H}{a_{L2}^H} \tag{2-9}$$

2.3 Cone of Commodity Price and Intersection Cone of Commodity Price

Fisher (2011) proposed the terms of "the output price diversification cone" and "the intersection of price cones". The output price diversification cone is the counterpart concept of the cone of diversification of factor endowments. We refer to it as the cone of commodity price briefly. The intersection of the output price cones

[7] Deardorff (1985) used this classification.

illustrates what makes sure that the rewards of two sets of local factor are positive when countries have different productivities.

The intersection cone of commodity prices for the case of inequity (2-5) can be expressed in algebra as

$$\frac{a_{K1}^F}{a_{K2}^F} > \frac{p_1^*}{p_2^*} > \frac{a_{H1}^H}{a_{H2}^H} \tag{2-10}$$

It identifies a full set of all possible commodity prices for equilibriums.

2.4 The generalized trade pattern when countries have different technologies

The HOV theorem originally says that a country will export the services of abundant factors and imports the services of scarce factors. When countries have different productivities, the effective factor abundance determines the trade direction of factor services (see Feestra and Taylor, 2012, p102-103). When countries have general technology differences, the virtual factor abundance (Fisher, 2011) determines the trade direction of factor services. Those rules in the HOV studies can be addressed as that a country exports the services of effective (virtual) abundant factors and imports the services of effective (virtual) scarce factors. This paper heavily depends on this principle. Appendix A is the proof for it.

The measurements of factor endowments by productivity equivalent unit (Trefler 1993) and by a giving country's technology (Fisher and Marshall, 2008) are insight concepts in HOV theories. The concepts showed that factor price equalization and HOV theorem hold when the factor endowments are adjusted by equivalent productivities or by a country's technology referred to. The concepts imply the trade pattern that each country exports the commodity that uses its effective (virtual) abundant factor intensively and imports the commodity that uses its effective (virtual) scarce factor intensively. We address it explicitly for the convenience to illustrate the trade types of this study. We may refer it to the generalized trade pattern of Heckscher-Ohlin-Ricardo model. This trade pattern can explain both the Leontief trade and the conversion trade of this paper well. For theoretical caution, we provided proof in Appendix A. It proved that the conversion trade by the factor intensity reversal has two features. One is factor content reversal. Another is factor abundance reversal that both countries are factor abundant at the same factor.

For simple, we use both effective factor abundance and virtual factor abundance this paper for different cases, with the same meaning, alternatively.

By HOV theorem, trade flows in the home country are

$$T^H = X^H - s^H X^W \tag{2-11}$$

A country's factor content is defined using the country's domestic technology matrix (see Bernhofen, 2011, pp104). The factor content of trade for the home country is

$$F^H = A^H T^H = V^H - s^H (V^H + A^H A^{F-1} V^F) \tag{2-12}$$

where s^H is the home country's shre of GNP, $s^H = P'X^H / P'X^W$. We denote the virtual factor endowments now. Using the home country's technology as a reference, the world virtual factor endowments (see Fisher 2011) is

$$V^{WH} = A^H (X^H + X^F) = V^H + V^{FH} = \begin{bmatrix} K^{WH} \\ L^{WH} \end{bmatrix} = \begin{bmatrix} K^H + K^{FH} \\ L^H + L^{FH} \end{bmatrix} \tag{2-13}$$

Where V^{Wh} is the vector of world virtual factor endowments that need to produce world commodities by referring to country h's technology matrix, $h = H, F$, and V^{fh} is the vector of virtual factor endowments of country f that needed to produce country f's commodities by referring to the technology matrix of country h.

Using the foreign country's technology as a reference, the world virtual factor endowment is

$$V^{WF} = A^F (X^H + X^F) = V^{HF} + V^F \tag{2-14}$$

We see that quantitatively, $V^{WH} \neq V^{WF}$.

We need to notice that effective or virtual factor abundance of a country, we mentioned, is measured by the country's domestic technology. This is important for this study.

2.5 Predicting Trade Direction by Using Intersection Cone of Commodity Price.

We introduce that the signs of trade, such as sign (T_1^H) and sign (F_K^H), remain the same for all commodity prices that lie in the intersection cone of commodity price. This implies that any commodity price lies in the intersection cone of commodity price can present trade direction by (2-11) and (2-12). We present the details of the proof in Appendix B.

Corresponding to the intersection cone of commodity price (2-10), there is a range of the share of GNP of the home country as

$$s_b\left(p_b\left(\frac{a_{K1}^F}{a_{K2}^F}, 1\right)\right) > \quad s^h \quad > \quad s_a\left(p_a\left(\frac{a_{L1}^H}{a_{L2}^H}, 1\right)\right) \tag{2-15}$$

where

$$s_a\left(p_a\left(\frac{a_{K1}^F}{a_{K2}^F}, 1\right)\right) = \frac{a_{K1}^F x_1^H + a_{K2}^F x_2^H}{a_{K1}^F x_1^W + a_{K2}^F x_2^W} = \frac{K^{HF}}{K^{WF}} \tag{2-16}$$

$$s_b\left(p_b\left(\frac{a_{L1}^H}{a_{L2}^H}, 1\right)\right) = \frac{a_{L1}^H x_1^H + a_{L2}^H x_2^H}{a_{L1}^H x_1^W + a_{L2}^H x_2^W} = \frac{L^H}{L^{WH}} \tag{2-17}$$

Giving a share of GNP in the range (2-15), it can predict trade direction and direction of factor content of trade by (2-11) and (2-12). The middle point of the share of GNP range (2-15) is a good candidate to use to display the trade direction numerically. For the Trefler model, it is just the equilibrium share of GNP of the home country.

3. Heckscher-Ohlin Trade and Leontief Trade

3.1 Heckscher-Ohlin Trade

We identify international trade types by evaluating commodity trade directions, excess factor supplies, factor intensity, factor abundance, and virtual factor abundance.

The Heckscher-Ohlin theorem and factor proportion concept only describe one type of factor content of trades. The Heckscher-Ohlin trade is of the symmetric factor content of trade, in which a country exports the commodity that uses its abundant factor intensively. We can use the Leamer theorem (Leamer 1980) to identify the Heckscher-Ohlin trade and the Leontief trade. The Leamer theorem says that if capital is abundant relatively for country h, its consumption capital/labor ratio is less than its capital/labor ratio as

$$\frac{K^h}{L^h} > \frac{K^h - F_K^h}{L^h - F_L^h} \qquad (h = H, F) \qquad \text{(3-1)}$$

If this is valid, it is the Heckscher-Ohlin trade. If it is not valid, it is the Leontief trade.

The Heckscher-Ohlin-Ricardo model also presents the Heckscher-Ohlin trade. It happens when its virtual factor abundance and its actual factor abundance are in the same direction when the model structure is the None-FIRs. A country's virtual capital abundance is specified as

$$\frac{K^h}{L^h} > \frac{K^{Wh}}{L^{Wh}} \qquad (h = H, F) \qquad \text{(3-2)}$$

The actual capital abundance of a country is specified as

$$\frac{K^h}{L^h} > \frac{K^W}{L^W} \qquad (h = H, F) \qquad \text{(3-3)}$$

3.2 Leontief Trade

The Leontief trade occurs when a country's virtual factor abundance and its actual factor abundance are at different directions when the model structure is the None-FIRs.

Leontief studied US trade from a unique angle. His view of factor abundance is the Heckscher-Ohlin view as (3-3), a professional view at that moment. When the virtual factor abundance of a country is in reversal to its actual factor abundance, the Leontief trade occurs.

We use the Trefler (1993) model to illustrate a simple Leontief trade. The major assumption in Trefler model is

$$A^H = \Pi A^F = \begin{bmatrix} \pi_K & 0 \\ 0 & \pi_L \end{bmatrix} A^F \tag{3-4}$$

where Π is a 2×2 diagonal matrix, its element π_k is factor productivity-argument parameter, $k = K, L$. This can be used to compose a typical Trefler model as

$$A^H X^H = V^H, \qquad (A^H)' W^H = P^H \tag{3-5}$$

$$\Pi^{-1} A^H X^F = V^F, \qquad (\Pi^{-1} A^H)' W^F = P^F \tag{3-6}$$

The world effective factor endowments by referring to the home technologies are

$$K^{WH} = K^H + \pi_K K^F \quad , \quad L^{WH} = L^H + \pi_L L^F \tag{3-7}$$

The world effective factor endowments by referring to foreign technologies are

$$K^{WF} = K^F + K^H/\pi_K \quad , \quad L^{WF} = L^F + L^H/\pi_L \tag{3-8}$$

We assume that the home country is actual capital abundance as

$$\frac{K^H}{L^H} > \frac{K^F}{L^F} \tag{3-9}$$

We also assume that the home country is virtual labor abundance as

$$\frac{K^H}{L^H} < \frac{K^{FH}}{L^{FH}} = \frac{\pi_K K^F}{\pi_L L^F} \tag{3-10}$$

Rewrite it as

$$\frac{K^H L^F}{L^H K^F} < \frac{\pi_K}{\pi_L} \tag{3-11}$$

If inequalities (3-9) and (3-11) holds, inequality (3-10) holds, Therefore (3-11) is the condition for the Leontief trade in the Trefler model.

We now see what happens in the foreign country. Rewrite (3-11) as

$$\frac{K^F}{L^F} > \frac{K^H/\pi_K}{L^H/\pi_L} = \frac{K^{HF}}{L^{HF}} \tag{3-12}$$

It means that the foreign country is virtual capital abundance. Inequalities (3-9) and (3-12) means that the foreign country does Leontief trade also. The numerical example 1 in Appendix C illustrates the Leontief trade by the Trefler model. Appendix D provides the analytical demonstration of the Leontief trade as that actual capital abundant country exports the labor-intensity commodity.

If the home country is actual labor abundant as $\frac{K^H}{L^H} < \frac{K^F}{L^F}$, the condition for the Leontief trade will be

$$\frac{K^H L^F}{L^H K^F} > \frac{\pi_K}{\pi_L} \tag{3-13}$$

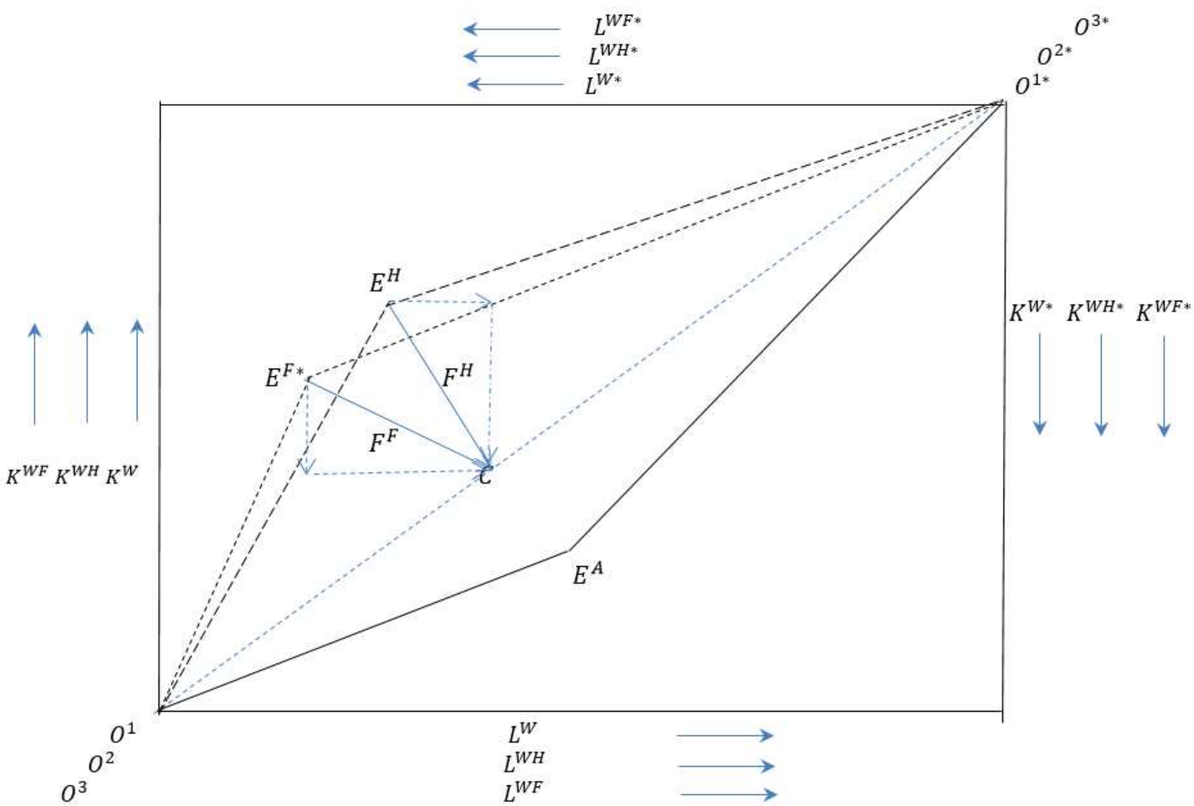

Figure 1 Multiscale IWE Diagram for Leontief Trade

Figure 1 draws a generalized IWE diagram with the Leontief trade. It is a multiscale diagram that merges the three diagrams together. The densities of each diagram's scales are different. The lower left corner is three origins for the home country. The right upper corner is three origins of the foreign country. Dimension O^1O^{1*} is for two countries' actual factor endowments. Dimension O^2O^{2*} is for two countries' virtual factor endowments measured by home technology. Dimension O^3O^{3*} is for two countries' virtual factor endowments measured by foreign technology. The diagram dimension just fits V^{WH}, V^{WF}, and V^W, althrough $V^{WH} \neq V^{WF} \neq V^W$. The goal is to make subtle changes to the feature density of each scale to avoid distortion of the factor content of trade and overall message.

We assume here that the ratios of capital to labor employed in the two countries lie in their diversification cone of factor endowments like

$$\frac{a_{K1}^{H}}{a_{L1}^{H}} \quad > \quad \frac{K^{H}}{L^{H}} \quad > \quad \frac{a_{K2}^{H}}{a_{L2}^{H}} \tag{3-14}$$

$$\frac{a_{K1}^{F}}{a_{L1}^{F}} \quad > \quad \frac{K^{F}}{L^{F}} \quad > \quad \frac{a_{K2}^{F}}{a_{L2}^{F}} \tag{3-15}$$

For a giving allocation of actual factor endowments of two countries at E^{A}, there are two respective allocations of virtual factor endowments E^{H} and E^{F*}. E^{A} is the vector from the home origin O^{1}. It is below the diagonal line. It indicates that the home country is actual labor abundance as

$$\frac{K^{H}}{L^{H}} < \frac{K^{W}}{L^{W}} \tag{3-16}$$

Point E^{H} indicates the allocation of virtual factor endowments of two countries, which are measured by the reference to the home country's technology. It is above the diagonal line. It signifies that the home country is virtual capital abundance as

$$\frac{K^{H}}{L^{H}} > \frac{K^{WH}}{L^{WH}} \tag{3-17}$$

Point E^{F*} indicates the allocation of virtual factor endowments of two countries, which are measured by the reference to the foreign country's technology. It is below the diagonal line from the view of the foreign origin. It signifies that the foreign country is virtual labor abundance as

$$\frac{K^{F}}{L^{F}} < \frac{K^{WF}}{L^{WF}} \tag{3-18}$$

Inequalities (3-16) and (3-17) implies that the home country is with the Leontief trade. In addition, Inequalities (3-16) and (3-18) implies that the foreign country is with Leontief trade too.

There are two vectors of factor content of trade, F^{H} and F^{F} in Figure 1. They point at C and reach the same share of GNP. Point C represents the trade equilibrium point. It indicates the sizes of the consumptions of the two countries. Vector F^{H} indicates that the home country as an actual labor abundant country exports capital services and imports labor services. Similarly, vector F^{F} indicates that the foreign country as an actual capital abundant country exports capital services and imports labor services.

When the factor endowments of E^{A} allocated above the diagonal line, it will be the Heckscher-Ohlin trade. For the Trefler model, by its single price cone property, E^{H} and E^{F*} will overlap together in the multiscale diagram.

We presented a numerical case to illustrate the details of the Leontief trade by example 2 in Appendix C.

The trade pattern of the Heckscher-Ohlin-Ricardo model explains the Leontief trade well.

4. Conversion Trade

4.1 The Model with the presence of FIRs

The first task to study the FIRs is to set up a simple FIRs model. Trefler (1993) empirical implementation of equivalent-productivity is useful to implement a model with the FIRs in theoretical analysis. Similarly, we now specify a "Hicks-Neutral" FIRs model by assuming technological matrices as

$$A^H = \psi A^F = \begin{bmatrix} 0 & \theta_K \\ \theta_L & 0 \end{bmatrix} A^F \tag{4-2}$$

where ψ is a 2×2 anti-diagonal matrix, its element θ_k is the factor productivity-across-factor-argument parameter, $k = K, L$. This composes a model with FIRs as

$$A^H X^H = V^H, \qquad (A^H)' W^H = P^H \tag{4-3}$$

$$\psi^{-1} A^H X^F = V^F, \qquad (\psi^{-1} A^H)' W^F = P^F \tag{4-4}$$

The world effective factor endowments by referring to the home technologies now are

$$K^{WH} = K^H + \theta_L L^F \quad , \quad L^{WH} = L^H + \theta_K K^F \tag{4-5}$$

The world effective factor endowments by referring to foreign technologies are

$$K^{WF} = K^F + L^F / \theta_L \quad , \quad L^{WF} = L^F + K^F / \theta_K \tag{4-6}$$

When $\theta_K = 1$ and $\theta_L = 1$, we have

$$A^F = \begin{bmatrix} a_{L1}^H & a_{L2}^H \\ a_{K1}^H & a_{K2}^H \end{bmatrix} \tag{4-7}$$

The foreign country' technology requirement coefficients for labor are as same as the home country's coefficients for capital. The requirements for factors in the foreign country are switched. It did turn the sector technologies across countries as the way Deardroff (1985) mentioned.

The cost requirement ratio ranks, which indicate the rays of the cone of commodity price by (4-3) and (4-4), are

$$\frac{a_{K1}^H}{a_{K2}^H} = \frac{a_{L1}^F}{a_{L2}^F} = \frac{a_{L1}^H/\theta_L}{a_{L2}^H/\theta_L} \quad > \quad \frac{a_{L1}^H}{a_{L2}^H} = \frac{a_{K1}^F}{a_{K2}^F} = \frac{a_{K1}^H/\theta_K}{a_{K2}^H/\theta_K} \tag{4-8}$$

This is a case of the single cone of commodity price, with different technologies across countries. Its equilibrium solution is comparatively simple.

Expression (4-8) implies that $|A^H||A^F| < 0$. The model by (4-3) and (4-4) is one of the existence of the FIRs. It results in the conversion trade.

The Hicks-Neutral FIRs model essentially is a Trefler (1993) model. By assuming $V^{FH} = \psi V^F$ and $W^{FH} = \psi^{-1} W^F$, we have the same-technology version of the Hicks-Neutral FIRs model as

$$A^H X^H = V^H, \qquad (A^H)' W^H = P^H \tag{4-9}$$

$$A^H X^F = V^{FH}, \qquad (A^H)' W^{FH} = P^F \tag{4-10}$$

For this version of the model, both factor price equalization hypothesis and the HOV theorem hold. Trefler (1993, pp965) had demonstrated this for his original model. Guo (2015) provided an equilibrium solution for the Heckscher-Ohlin model. It can be used directly on the model (4-9) and (4-10). Example 2 in Appendix C provides a numerical illustration for the conversion trade by the Hicks-Neutral FIRs model.

Statistically, for empirical study, the Hicks-Neutral FIRs model is not crazed any more than Hicks-Neutral Trefler model. In economics, Trefler's productivity-argument parameter π_i is much easier to accept than the productivity-across-factor-argument parameter θ_i. However, the Hicks-Neutral FIRs model is one possibility of international trade practice observed.

4.2 Conversion trade

Appendix D is the equilibrium solution for the $2 \times 2 \times 2$ Hicks-Neutral FIRs model, which explored the major features of the FIRs model as factor content reversal as

$$\text{Sign}\,(F_K^H) = \text{Sign}\,(F_K^F) \quad , \quad \text{Sign}\,(F_L^H) = \text{Sign}\,(F_L^F) \tag{4-11}$$

Appendix A also demonstrates this also. We call it the conversion trade. However, the trade volume balance definitely holds as

$$T^H = -T^F \tag{4-12}$$

Timing two sides of (4-12) by the home technology matrix A yields

$$F^H = -A^H T^F = -A^H {A^F}^{-1} F^F = -F^{FH} \tag{4-13}$$

where F^{FH} is the vector of the foreign factor content of trade measured by the home country's technology. This equation implies that the home country's factor content of trade equals negatively to foreign ocuntry's factor content of trade measured by the home country's technology. The conversion trade is always symmetrical and balanced under this meaning. The conversion trade is odd for "normal" understanding of international economics. Actually, it is normal and not with any paradox theoretically.

The FIRs model by the matrices (4-3) and (4-4) is a special case. It is with a single cone of commodity price. In general, under the model of the existence of FIRs, such as $\frac{a_{K1}^H}{a_{K2}^H} \geq \frac{a_{L1}^F}{a_{L2}^F} > \frac{a_{L1}^H}{a_{L2}^H} \geq \frac{a_{K1}^F}{a_{K2}^F}$, all trades are conversion trade.

4.3 Both countries are virtual factor abundance at the same factor

We now present another property of the FIRs model that both countries are virtual factor abundance at the same factor, i.e. that if the home country is effective capital abundance as $\frac{K^H}{L^H} > \frac{K^{WH}}{L^{WH}}$, the foreign country is effective capital abundance as $\frac{K^F}{L^F} > \frac{K^{WF}}{L^{WF}}$, also. It sourced conversion trade. Appendix A has demonstrated it logically. We use the Hicks-Neutral FIRs model to demonstrate it in details.

If the home country is effective capital abundance, it means that

$$\frac{K^H}{L^H} > \frac{K^{FH}}{L^{FH}} = \frac{\theta_L L^F}{\theta_K K^F}$$

It can be rewritten as

$$\frac{K^F}{L^F} > \frac{L^H/\theta_K}{K^H/\theta_L} = \frac{K^{WF}}{L^{WF}}$$

Therefore, both countries are effective capital abundance.

The factor, at which both countries are effective abundant, is a relatively plentiful factor in productions worldwide. Both countries export the service of that factor. Meanwhile, both countries import the services of another factor, effective scarce factor[8].

With factor content reversal, both countries will consume more on their scarce factor. International trade adjusts the consumption of factor content not only quantitatively but also in quality.

[8] Guo (2019) shows that the equilibrium solution of the Trefler model makes sure of gain from trades for the countries participating in trades. It is true for the conversion trade. In the coversion trade, both countries export commodities with comparative advantages in production. Those commodities use intensively their countries' effective abundant factors. Both countries import commodities without comparative advantages in production. This is just a meaningful part of the conversion trade. At this point, the conversion trade does the same as the Heckscher-Ohlin trade does.

Figure 2 shows a generalized multiscale IWE diagram with the conversion trade. E^A is from the home origin. It indicates the allocation of actual factor endowments of two countries. It shows that the home country is labor abundance as

$$\frac{K^H}{L^H} < \frac{K^W}{L^W} \tag{4-21}$$

E^H is the vector from home origin. It indicates the allocation of virtual factor endowments of two countries, which are measured by referring to the home country's technology. It is below the diagonal line. It signifies that the home country is virtual labor abundance as

$$\frac{K^H}{L^H} < \frac{K^{WH}}{L^{WH}} \tag{4-22}$$

E^{F*} is from the foreign origin, It indicates the allocation of the virtual factor endowments of two countries, which are measured by referring to the foreign country's technology. It is below the diagonal line from the view of foreign origin. It signifies that the foreign country is virtual labor abundance as

$$\frac{K^F}{L^F} < \frac{K^{WF}}{L^{WF}} \tag{4-23}$$

Inequalities (4-23) and (4-24) indicates that this is a conversion trade since both countries are virtual factor abundance at labor. Inequalities (4-21) and (4-22) implies that the home country is with the Heckscher-Ohlin trade since the home country are both actual labor abundance and virtual labor abundance. In addition, inequalities (4-21) and (4-23) implies that the foreign country is with Leontief trade since the foreign country is actual capital abundance and virtual labor abundance.

Vectors F^F and F^H indicates that both countries exports capital services and imports labor services. It illustrates how the conversion trade formed.

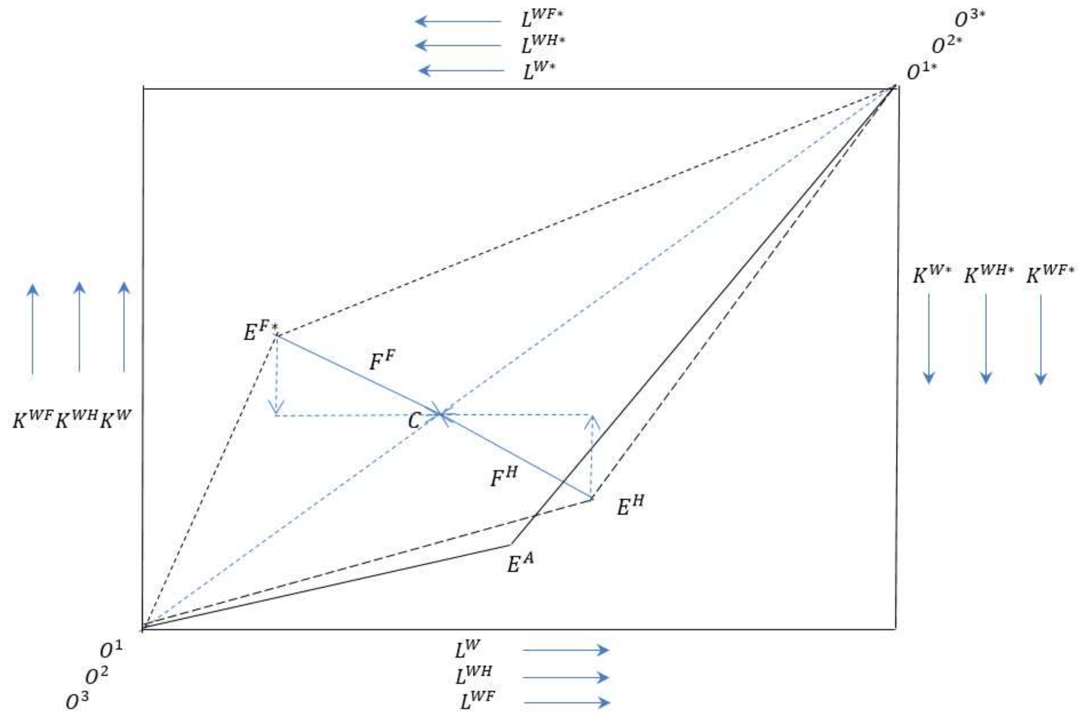

Figure 2 Multiscales IWE Diagram for Conversion Trade

Under the FIRs model structure, one country does the Heckscher-Ohlin trade; another does the Leontief trade. The virtual Heckscher-Ohlin theorem explains the conversion trade well.

4.4 Conversion Trade for Many Factors, Many Commodities, and Many Countries

In multiple-country trade analyses, a trade partner of a country is the rest of the world. So does the analyses of conversion trade and the Leontief trade. When the conversion trade occurs, a country and the rest world export same factor services and import the same factor services for at least a pair of factors.

The conversion trade occurs also in the context of the models with many commodities and many factors and many countries[9]. The numerical example 4 in Appendix C display a conversion trade for $4 \times 4 \times 2$ model.

A simple way to specify a FIRs model in high dimensions is by switching a pair of rows in its technology matrix. Row-switching matrix S_{ij}, as the following, switches all matrix elements on row i with their counterparts on row j.

[9] Prof. Furusawa, the editor of The Japanese Economic Review, suggested me to improve this study by adding the case of more commodities for the Leontief trade and the conversion trade, in 2015. Author appreciates what he did.

$$S_{ij} = \begin{bmatrix} 1 & & & & & & & & \\ & \ddots & & & & & & & \\ & & 1 & & & & & & \\ & & & 0 & 0 & 1 & & & \\ & & & 0 & \ddots & 0 & & & \\ & & & 1 & 0 & 0 & & & \\ & & & & & & 1 & & \\ & & & & & & & \ddots & \\ & & & & & & & & 1 \end{bmatrix}$$

The corresponding elementary matrix is obtained by swapping row *i* and row *j* of the identity matrix. Since the determinant of the identity matrix is unity, $\det[S_{ij}] = -1$. It follows that for any square matrix A (of the correct size), we have $\det[S_{ij}A] = -\det[A]$. Using a row-switching operation, we can implement a FIRs model. This is also available for non-square (not even) technology matrix. The conversion trade not only occurs for even model (factor number equals to commodity number) but also for the non-even model. To specify a non-even FIR model, just use Row-switching matrix S_{ij}. Numerical example 3 in Appendix C presents a case of conversion trade of two factors, three commodities, and three countries[10]. The factor content reversal in higher dimension cases will be “local” or “regional” phenomena in whole trade space.

5 Related Discussions

So far, we demonstrate that the Leontief trade and the conversion trade are normal theoretically. We say now that Leontief is more likely to be true than any before. We have three arguments as follows.

Many empirical HOV studies for trade pattern predictions predicted the trade direction successfully by the models incorporating different technologies across countries. The prediction accuracies were improved a lot. This paper based on their analyses and their results a lot. Due to the normalization of the Leontief trade and the conversion trade. The prediction criteria are not sufficient to deny the Leontief trade. Some popular sign prediction criteria are by the HOV theorem, from equations like (2-11) or (2-12). Theoretically, those predication signs designed include all of the three trade types of this paper. However, their explanations and presentations of the trade theories incorporating different technologies are right.

[10] To specify the trade direction for many factors and commodities model, we need to know how to identify the range of share of GNP in higher dimension. Guo (2018) demonstrated the higher-dimension Trefler model is the single price cone structure. Guo (2019) provide an equilibrium solution for higher dimension H-O model. Those two papers provide the theoretical background to show the conversion trade for many factors and commodities model.

The Trefler model (even without the FIRs) can generate cases of the Leontief trade. This is a new understanding of this paper. Empirical studies by Trefler model may have a chance to include the Leontief trade also for individual countries.

The factor intensity reversal always associated with conversion trade. The conversion trade is one kind of Leontief trade. Kurokawa (2011), Takahashi (2004), Simpson (2016), Kozo and Yoshinori (2017) and some other scholars have provided clear evidence of factor intensity reversals. Their studies imply that there exist both the Leontief trade and the conversion trade in international trade practice. More studies need to be done for further confirmation. We believe that all of the three trade types are true in the real life of international trade. Theories and empirical studies both pointed at it yet.

The factor content reversal displays a new kind of comparative advantage from the consumption side. Both countries consume more on virtual scarce factor. Trade converts the virtual scarce factor into virtual abundant factor, which are bundled in the trade flows.

Conclusion

We explored three trade types from the view of factor contents of trade by the Heckscher-Ohlin-Ricardo model, to reflect the international trade among countries with different technologies. The Leontief trade and the conversion trade counter common understanding of international economics somehow. Actually, they are rooted in the Heckscher-Ohlin theories. The generalized trade pattern, by which each country exports the commodity that uses its virtual abundant factor, explains the three trade types equally well. The factor prices and commodity price at equilibrium price make sure gains from both from conversion trade and from Leontief trade.

The new understanding for factor intensity reversal is that it causes factor price reversal, factor content reversals, and effective (virtual) factor abundance reversal. The new understanding for the Leontief trade is that it can occur both with the presence of FIRs and without the presence of FIRs.

This study answered the challenge of the Leontief paradox and the challenge of the factor intensity reversals.

There may be different formats of trade types when fulfilling a more complicted higher dimension’s analysis. International trades benefit countries by the diversifications of gains.

The factor price reversal and factor content reversal are the results of the general equilibrium of conversion trade. The general equilibrium of trade is the most important topic in international trade. The paper opened a new view of the real international trade.

Appendix A – A generalized trade pattern for 2 × 2 × 2 Heckscher-Ohlin-Ricardo model

We will prove two related rules.

The trade pattern of factor contents – Each country exports the service of virtual abundant factor and imports the services of virtual scarce factor.

The trade pattern of commodity - Each country exports the commodity that uses its virtual abundant factor intensively and imports the commodity that uses its virtual scarce factor intensively.

Leamer (1984, pp 8-9) provides a unique way to demonstrate the Heckscher-Ohlin theorem. He showed that for the Heckscher-Ohlin model, if the home country is capital abundant as

$$(K^H/K^W) > s > (L^H/L^W) \tag{A-1}$$

the excess factor supplies have signs

$$F^H = \begin{bmatrix} F_K^H \\ F_L^H \end{bmatrix} = \begin{bmatrix} + \\ - \end{bmatrix} \tag{A-2}$$

If the home country is capital intensive in commodity 1, the signs of trade flow will be

$$T^H = (A^H)^{-1}F^H = \begin{bmatrix} + & - \\ - & + \end{bmatrix} \begin{bmatrix} + \\ - \end{bmatrix} = \begin{bmatrix} + \\ - \end{bmatrix} \tag{A-3}$$

Therefore, the home country will export commodity 1 and import commodity 2.

We now generalize Leamer's analysis to the Heckscher-Ohlin-Ricardo model and demonstrate a generalized trade pattern.

With the Heckscher-Ohlin-Ricardo model (2-1) and (2-2), the vector of commodity exports in the home country is the difference between production and consumption:

$$T^H = X^H - C^H = A^{H-1}(V^H - s^H V^{WH}) \tag{A-4}$$

which is A^{H-1} times the vector of excess factor supplies:

$$F^H = V^H - s^H V^{WH} = \begin{bmatrix} K^H - s^H K^{WH} \\ L^H - s^H L^{WH} \end{bmatrix} = \begin{bmatrix} K^{WH}(K^H/K^{WH} - s^H) \\ L^{WH}(L^H/L^{WH} - s^H) \end{bmatrix} \tag{A-5}$$

The vector of commodity exports in the foreign country is

$$T^F = X^F - C^F = A^{F-1}(V^F - s^F V^{WF}) \tag{A-6}$$

which is A^{F-1} times the vector of excess factor supplies:

$$F^F = V^F - s^F V^{WF} = \begin{bmatrix} K^F - s^F K^{WF} \\ L^F - s^F L^{WF} \end{bmatrix} = \begin{bmatrix} K^{WF}(K^H/K^{WF} - s^F) \\ L^{WF}(L^H/L^{WF} - s^F) \end{bmatrix} \tag{A-7}$$

Corresponding to the four rays of the two cones of the commodity price of two countries, there are four boundaries of shares of GNP. The boundaries of the share of GNP by the commodity price cone of the home country are

$$s_b^h\left(p\left(\frac{a_{K1}^H}{a_{K2}^H}, 1\right)\right) = \frac{a_{K1}^H x_1^H + a_{K2}^H x_2^H}{a_{K1}^H x_1^w + a_{K2}^H x_2^w} = \frac{K^H}{K^{WH}} \tag{A-8}$$

$$s_a^h\left(p\left(\frac{a_{L1}^H}{a_{L2}^H}, 1\right)\right) = \frac{a_{L1}^H x_1^H + a_{L2}^H x_2^H}{a_{L1}^H x_1^w + a_{L2}^H x_2^w} = \frac{L^H}{L^{WH}} \tag{A-9}$$

The boundaries of the share of GNP by the commodity price cone of the foreign commodity price are

$$s_b^F\left(p\left(\frac{a_{K1}^F}{a_{K2}^F}, 1\right)\right) = \frac{a_{K1}^F x_1^F + a_{K2}^F x_2^F}{a_{K1}^H x_1^w + a_{K2}^H x_2^w} = \frac{K^F}{K^{WF}} \tag{A-10}$$

$$s_a^F\left(p\left(\frac{a_{L1}^F}{a_{L2}^F}, 1\right)\right) = \frac{a_{L1}^F x_1^F + a_{L2}^F x_2^F}{a_{L1}^F x_1^w + a_{L2}^F x_2^w} = \frac{L^F}{L^{WF}} \tag{A-11}$$

We first discuss the case that the mode is without the presence of FIRs, in which $|A^H| > 0$ and $|A^F| > 0$.
If the home country is virtual capital abundant, the home country's share of GNP must lie in the following range,

$$\frac{K^H}{K^{WH}} > s^H > \frac{L^H}{L^{WH}} \tag{A-12}$$

The home country will export the services of capital and import the service of labor by (A-5). Therefore, the vector of factor content of trade in the home country is with signs

$$F^H = \begin{bmatrix} + \\ - \end{bmatrix} \tag{A-13}$$

This implies that we proved the trade pattern of factor content, as that a country exports the services of virtual abundant factors and imports the services of virtual scarce factors.

The signs of trade flow from equation (A-13) will be

$$T^H = (A^H)^{-1} F^H = \begin{bmatrix} + & - \\ - & + \end{bmatrix} \begin{bmatrix} + \\ - \end{bmatrix} = \begin{bmatrix} + \\ - \end{bmatrix} \tag{A-14}$$

This is due to the home country is capital intensive in commodity 1.
By the international trade balance, the sign of trade flow in the foreign country is

$$T^F = -T^H = \begin{bmatrix} - \\ + \end{bmatrix} \tag{A-15}$$

For the factor content of trade in the foreign country, we discuss two cases, one is the model with the presence of FIRs, and another is the model without the presence of FIRs.

If it is without the presence of FIRs, $|A^H| > 0$, the vector of factor content of trade in the foreign country is with signs

$$F^H = A^F T^F = \begin{bmatrix} + & - \\ - & + \end{bmatrix} \begin{bmatrix} - \\ + \end{bmatrix} = \begin{bmatrix} - \\ + \end{bmatrix} \tag{A-16}$$

From (A-4), (A-15), and (A-16), we know that the foreign country is virtual labor abundant. This is just the result that each country exports the commodity that uses its virtual abundant factor intensively and imports the commodity that uses its virtual abundant factor intensively.

If it is with the presence of FIRs, $|A^F| < 0$, the vector of factor content of trade in the foreign country is with signs

$$F^H = A^F T^F = \begin{bmatrix} - & + \\ + & - \end{bmatrix} \begin{bmatrix} - \\ + \end{bmatrix} = \begin{bmatrix} + \\ - \end{bmatrix} \tag{A-17}$$

It implies

$$\frac{K^F}{K^{WF}} > s^F > \frac{L^F}{L^{WF}} \tag{A-18}$$

From (A-7), (A-17) and (A-1-18), we know that the foreign country is virtual capital abundant. This is just the result that each country exports the commodity that uses its virtual abundant factor intensively and imports the commodity that uses its virtual scarce factor intensively, also.

The commodity price must lie in the intersection cone of commodity price. The home country's share of GNP, corresponding the commodity price, should lie in the following range,

$$\frac{K^{HF}}{K^{WF}} > s^H > \frac{L^F}{L^{WF}} \tag{A-19}$$

The range by (A-19) is part of the range by (A-12). The relationships (A-14), (A-16), (A-17) that hold under (A-12) should hold also under (A-19) (see the trade boxes in Appendix B)

Appendix B

We demonstrate that trade directions remain same for any commodity price that lies within the intersection cone of commodity prices.

Figure 3 draws a generalized IWE diagram with the two trade boxes. It is a multiscale diagram that merges the two diagrams together. The densities of each diagram's scales are different. The lower left corner is two origins for the home country. The right upper corner is two origins of the foreign country. Dimension O^1O^{1*} is for two countries' virtual factor endowments measured by home technology. Dimension O^2O^{2*} is for two countries' virtual factor endowments measured by foreign technology. The diagram dimension just fits V^{WH} and V^{WF}, althrough $V^{WH} \neq V^{WF}$. The goal is to make subtle changes to the feature density of each scale to avoid distortion of the factor content of trade and overall message.

Giving factor endowments of two countries V^H and V^F, there are two respective allocations of virtual factor endowments E^H and E^{F*}. Allocation E^H is the vector from origin O^1. Allocation E^{F*} is the vector from origin O^{2*}. There are two factor trade vectors, $F^H$and F^F. Both of them point at C and reaches the same point of share of GNP. Point *C* represents trade equilibrium point. It indicates the sizes of the consumptions of the two countries.

Figure 2 also draws two trade boxes by the boundaries of shares of GNP (A-8) through (A-11). The solid-line box is for the home country; the dash-line box is for the foreign country. The intersection of the two trade boxes, indicated by the diagonal line C^2C^3, reflects the intersection cone of commodity prices in the IWE diagram. Point C will changes when giving different commodity price. However the signs of F_L^H, F_K^H, F_L^F and F_K^F will not change. Such as F_L^H is always negative, which means import the services of labor. F^Hcan end within C^2C^3, No matter which point it end at, the trade direction F_L^H and F_K^H remain the same.

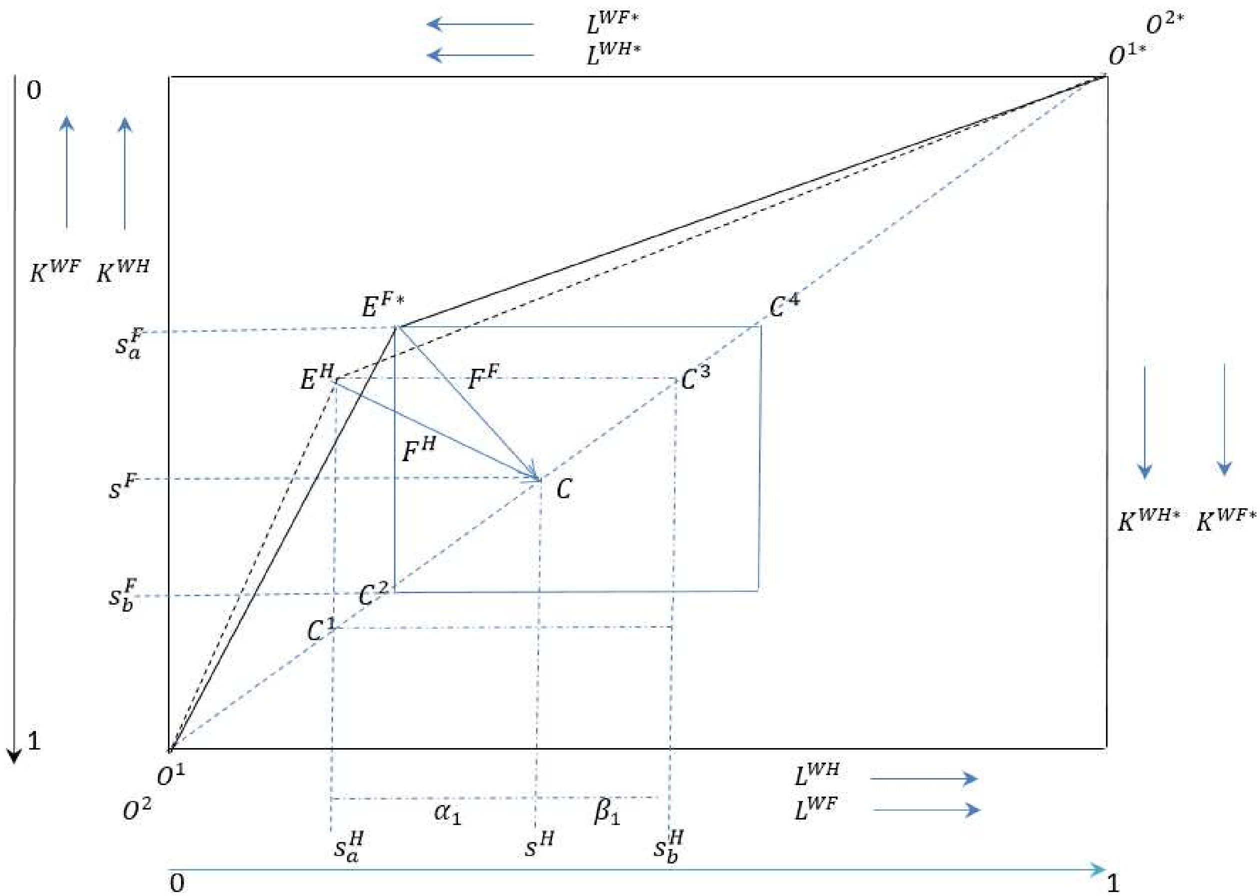

Figure 3 Multiscale IWE Diagram with the Two Trade Boxes

Appendix C

Numerical example 1 - Leontief trade

This is of a Trefler model. The technological matrix for the home country is

$$A^H = \begin{bmatrix} 3.0 & 1.0 \\ 1.5 & 2.0 \end{bmatrix}$$

The technological matrix for the foreign country is

$$A^F = \begin{bmatrix} 1/0.5 & 0 \\ 0 & 1/0.9 \end{bmatrix} \begin{bmatrix} 3.0 & 1.0 \\ 1.5 & 2.0 \end{bmatrix}$$

The factor intensities of the two countries are as

$$a_{K1}^H / a_{L1}^H = 2.0 > \; a_{K2}^H / a_{L2}^H = 0.5$$
$$a_{K1}^F / a_{L1}^F = \; 3.6 > a_{K2}^F / a_{L2}^F = 0.9$$

The home country is capital intensive in sector 1, and the foreign country is capital intensive in sector 1 too.

This is a model without FIRs. We take the factor endowments for the two countries as

$$\begin{bmatrix} K^H \\ L^H \end{bmatrix} = \begin{bmatrix} 4200 \\ 3000 \end{bmatrix}, \qquad \begin{bmatrix} K^F \\ L^F \end{bmatrix} = \begin{bmatrix} 4800.0 \\ 2833.3 \end{bmatrix}$$

The home country is actual labor abundant as

$$\frac{K^H}{L^H} = \frac{4200}{3000} = 1.4 < \quad \frac{K^F}{L^F} = \frac{4800}{2833.3} = 1.69$$

However, the home country is effective capital abundant as

$$\frac{K^H}{L^H} = \frac{4200}{3000} = 1.4 > \quad \frac{K^{FH}}{L^{FH}} = \frac{2400}{2550} = 0.94$$

Therefore, the home country exports commodity 1 and is with net excess of capital, since commodity 1 uses the capital intensively.

The foreign country is effective labor abundant as

$$\frac{K^F}{L^F} = \frac{4800}{2833.3} = 1.69 < \quad \frac{K^{HF}}{L^{HF}} = \frac{8400}{3333.3} = 2.52$$

Therefore, the foreign country exports commodity 2 and is with net excess of labor services since commodity 2 uses labor intensively. The home country is in the Leontief trade and the foreign country is in the Leontief trade too.

The Leontief trade criteria is

$$\frac{K^H L^F}{L^H K^F} = 0.82 < \frac{\pi_K}{\pi_L} = \frac{\pi_K}{\pi_L} = 0.555$$

The outputs of the two countries are

$$\begin{bmatrix} x_1^H \\ x_2^H \end{bmatrix} = \begin{bmatrix} 1200.0 \\ 600.0 \end{bmatrix}, \qquad \begin{bmatrix} x_1^F \\ x_2^F \end{bmatrix} = \begin{bmatrix} 500.0 \\ 900.0 \end{bmatrix}$$

The ranks of cost requirement ratios are

$$\frac{a_{K1}^H}{a_{K2}^H} = \frac{a_{L1}^F}{a_{L2}^F} = 3 \quad > \quad \frac{a_{L1}^H}{a_{L2}^H} = \frac{a_{K1}^F}{a_{K2}^F} = 0.75$$

The shares of GNP of the home country, corresponding to the rays of the intersection cone, are

$$s_b^H\left(p_a\left(\frac{a_{K1}^F}{a_{K2}^F}, 1\right)\right) = 0.636$$

$$s_a^H\left(p_b\left(\frac{a_{L1}^H}{a_{L2}^H}, 1\right)\right) = 0.540$$

The middle of the range of the share of GNP is $s_m^H = 0.5884$.

The exports and factor contents of trade by the share of GNP above are:

$$\begin{bmatrix} T_1^H \\ T_2^H \end{bmatrix} = \begin{bmatrix} 199.6 \\ -282.6 \end{bmatrix}, \qquad \begin{bmatrix} T_1^F \\ T_2^F \end{bmatrix} = \begin{bmatrix} -199.6 \\ 282.6 \end{bmatrix}$$

$$\begin{bmatrix} F_K^H \\ F_L^H \end{bmatrix} = \begin{bmatrix} 316.2 \\ -265.9 \end{bmatrix}, \qquad \begin{bmatrix} F_K^F \\ F_L^F \end{bmatrix} = \begin{bmatrix} -632.4 \\ 295.4 \end{bmatrix}$$

We see that the home country with actual labor abundance exports commodity 1 that uses intensively capital.

Numerical example 2- Conversion trade

This is of a Trefler style FIRs model. The technological matrix for the home country is

$$A^H = \begin{bmatrix} 3.0 & 1.0 \\ 1.5 & 2.0 \end{bmatrix}$$

The technological matrix for the foreign country is

$$A^F = \begin{bmatrix} 0.0 & 1/0.9 \\ 1/0.8 & 1.0 \end{bmatrix} \begin{bmatrix} 3.0 & 1.0 \\ 1.5 & 2.0 \end{bmatrix}$$

The factor intensities of the two countries are as

$$a_{K1}^H / a_{L1}^H = 2.0 > \ a_{K2}^H / a_{L2}^H = 0.5$$

$$a_{K1}^F / a_{L1}^F = \ 0.562 < a_{K2}^F / a_{L2}^F = 2.25$$

The home country is capital intensive in sector 1, and the foreign country is capital intensive in sector 2.

This is a FIRs structure. We take the factor endowments for the two countries as

$$\begin{bmatrix} K^H \\ L^H \end{bmatrix} = \begin{bmatrix} 4200 \\ 3000 \end{bmatrix}, \qquad \begin{bmatrix} K^F \\ L^F \end{bmatrix} = \begin{bmatrix} 3187.5 \\ 2666.6 \end{bmatrix}$$

The home country is actual labor abundant as

$$\frac{K^H}{L^H} = \frac{4200}{3000} = 1.4 < \ \frac{K^F}{L^F} = \frac{3187.5}{2666.6} = 1.19$$

However, the home country is effective capital abundant as

$$\frac{K^H}{L^H} = \frac{4200}{3000} = 1.4 > \ \frac{K^{FH}}{L^{FH}} = \frac{2400}{2550} = 0.94$$

Therefore, the home country exports commodity 1 and is with net excess of capital, since commodity 1 uses the capital intensively.

The foreign country is effective capital abundant as

$$\frac{K^F}{L^F} = \frac{318.75}{2666.6} = 1.19 > \ \frac{K^{HF}}{L^{HF}} = \frac{3750}{4666} = 0.80$$

Therefore, the foreign country exports commodity 2 and is with net excess of capital services since commodity 2 uses the capital intensively. The home country is in the Leontief trade and the foreign country is in the Heckscher-Ohlin trade too.

The outputs of the two countries are

$$\begin{bmatrix} x_1^H \\ x_2^H \end{bmatrix} = \begin{bmatrix} 1200.0 \\ 600.0 \end{bmatrix}, \qquad \begin{bmatrix} x_1^F \\ x_2^F \end{bmatrix} = \begin{bmatrix} 500.0 \\ 900.0 \end{bmatrix}$$

The ranks of cost requirement ratios are

$$\frac{a_{K1}^H}{a_{K2}^H} = \frac{a_{L1}^F}{a_{L2}^F} = 3 \quad > \quad \frac{a_{L1}^H}{a_{L2}^H} = \frac{a_{K1}^F}{a_{K2}^F} = 0.75$$

The shares of GNP of the home country, corresponding to the rays of the intersection cone, are

$$s_b^H \left(p_a \left(\frac{a_{K1}^F}{a_{K2}^F}, 1 \right) \right) = 0.636$$

$$s_a^H \left(p_b \left(\frac{a_{L1}^H}{a_{L2}^H}, 1 \right) \right) = 0.540$$

The middle of the range of the share of GNP is $s_m^H = 0.5884$.

The exports and factor contents of trade by the share of GNP above are:

$$\begin{bmatrix} T_1^H \\ T_2^H \end{bmatrix} = \begin{bmatrix} 199.6 \\ -282.6 \end{bmatrix}, \qquad \begin{bmatrix} T_1^F \\ T_2^F \end{bmatrix} = \begin{bmatrix} -199.6 \\ 282.6 \end{bmatrix}$$

$$\begin{bmatrix} F_K^H \\ F_L^H \end{bmatrix} = \begin{bmatrix} 316.2 \\ -265.9 \end{bmatrix}, \qquad \begin{bmatrix} F_K^F \\ F_L^F \end{bmatrix} = \begin{bmatrix} 332.8 \\ -351.3 \end{bmatrix}$$

We see that both countries export capital services and import labor services. The trade converts the global abundant factor into the global scarce factor. This is an interesting result.

Numerical Example 3 - Conversion trade for two factors, three commodities, and three countries model

We need to refer to Guo (2018) and (2019) for the equilibrium share of GNP for multiple countries.

The technological matrix for country 1 is

$$A^1 = \begin{bmatrix} 3 & 1.5 & 1 \\ 1.2 & 2 & 0.9 \end{bmatrix}$$

The technological matrix for the foreign country is

$$A^2 = \begin{bmatrix} 1/0.9 & 0 \\ 0 & 1/0.7 \end{bmatrix} \begin{bmatrix} 3 & 1.5 & 1 \\ 1.2 & 2 & 0.9 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0 & 1/0.8 \\ 1/0.6 & 0 \end{bmatrix} \begin{bmatrix} 3 & 1.5 & 1 \\ 1.2 & 2 & 0.9 \end{bmatrix}$$

For the non-even technology matrix, we need to have the outputs of the three countries first as

$$X^1 = \begin{bmatrix} 400 \\ 1300 \\ 410 \end{bmatrix}, \qquad X^2 = \begin{bmatrix} 250 \\ 700 \\ 900 \end{bmatrix}, \qquad X^3 = \begin{bmatrix} 800 \\ 900 \\ 1000 \end{bmatrix}$$

The factor endowments of the three countries are

$$V^1 = \begin{bmatrix} 3250 \\ 3320 \end{bmatrix}, \qquad V^2 = \begin{bmatrix} 2855.5 \\ 407.71 \end{bmatrix}, \qquad V^3 = \begin{bmatrix} 4537.5 \\ 8416.6 \end{bmatrix}$$

The share of GNP of each country is calculated by

$$s^h = \frac{1}{2}\left(\frac{v_1^h}{v_1^{Wh}} + \frac{v_2^h}{v_2^{Wh}}\right) \qquad\qquad h = (1,2,3)$$

where v_i^h is the factor endowment i in country h and v_i^{Wh} is the world equivalent factor endowment i by referring to country h's technology.

We obtain the shares of GNP of the three countries as

$$s^1 = 0.320 , \qquad s^2 = 0.262 , \qquad s^3 = 0.417$$

The commodity trade volumes are

$$T^1 = \begin{bmatrix} -212.18 \\ 371.68 \\ -240.22 \end{bmatrix}, \qquad T^2 = \begin{bmatrix} -120.45 \\ -62.07 \\ 337.44 \end{bmatrix}, \qquad T^3 = \begin{bmatrix} 332.63 \\ -309.62 \\ -134.22 \end{bmatrix}$$

The factor contents of trade are

$$F^1 = \begin{bmatrix} -319.22 \\ 272.55 \end{bmatrix}, \qquad F^2 = \begin{bmatrix} 47.82 \\ -72.01 \end{bmatrix}, \qquad F^3 = \begin{bmatrix} -272.69 \\ 239.54 \end{bmatrix}$$

We provide two ways to show the factor content reversal. The first one is to use a technology reference of one country of the rest of the world.

For country 1, the trade of the rest of the world is $T^2 + T^3$. The factor contents of $T^2 + T^3$ by the technology of country 2 and country 3 respectively are

$$A^2(T^2 + T^3) = \begin{bmatrix} -190.78 \\ 287.30 \end{bmatrix}$$

$$A^3(T^2 + T^3) = \begin{bmatrix} -218.04 \\ 191.53 \end{bmatrix}$$

They are in the same trade direction of F^1. The conversion trade occurs.

The second method is to use the monetary item to show the factor content reversal. We compare $r^1 F_K^1$ with $r^2 F_K^2 + r^3 F_K^3$, and comaprare $w^1 F_L^1$ with $w^2 F_L^2 + w^3 F_L^3$, to see if their signs are same.

Numerical Example 4 - Conversion trade for the 4 × 4 × 2 Model

We need to refer to Guo (2018) and Guo (2019) for the equilibrium share of GNP for multiple factors.

The technological matrix for the home country is

$$A^H = \begin{bmatrix} 3.0 & 1.2 & 1.3 & 0.9 \\ 1.1 & 2.0 & 0.9 & 1.4 \\ 0.7 & 1.5 & 2.1 & 1.0 \\ 1.6 & 1.7 & 0.8 & 1.5 \end{bmatrix}$$

For simple, the technological matrix for the foreign country is

$$A^F = \psi^{-1} A^H$$

where

$$\psi = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

The factor endowments of the two countries are

$$V^H = \begin{bmatrix} 4253 \\ 4189 \\ 3631 \\ 4098 \end{bmatrix}, \qquad V^F = \begin{bmatrix} 3690 \\ 4975 \\ 3865 \\ 4080 \end{bmatrix}$$

The outputs of the two countries are

$$X^H = \begin{bmatrix} 600.0 \\ 1300.0 \\ 410.0 \\ 400.0 \end{bmatrix}, \qquad X^F = \begin{bmatrix} 250.0 \\ 700.0 \\ 1500.0 \\ 600.0 \end{bmatrix}$$

The share of GNP of the home country is 0.4946 by

$$s^H = \frac{1}{4}\left(\frac{v_1^H}{v_1^{WH}} + \frac{v_2^H}{v_2^{WH}} + \frac{v_3^H}{v_3^{WH}} + \frac{v_4^H}{v_4^{WH}}\right)$$

where $v_i^h$is the factor endowment i in country h and v_i^{Wh} is the world equivalent factor endowment i by referring to country h's technology.

The trade volumes are

$$T^H = \begin{bmatrix} 179.55 \\ 310.70 \\ -534.78 \\ -94.65 \end{bmatrix}, \qquad T^F = \begin{bmatrix} -179.55 \\ -3100.70 \\ 534.78 \\ 94.65 \end{bmatrix}$$

The factor contents of trade are

$$F^H = \begin{bmatrix} 131.07 \\ 205.08 \\ -625.96 \\ 245.65 \end{bmatrix}, \qquad F^F = \begin{bmatrix} -245.65 \\ 625.95 \\ -205.08 \\ -131.07 \end{bmatrix}$$

We see that F_2^H and F_2^F are at same trade direction. In addition, F_3^H and F_3^F are at same trade direction. The conversion trade occurs at factor 2 and factor 3.

Appendix C

Assume

$$A^H = \Pi A^F = \begin{bmatrix} \pi_K & 0 \\ 0 & \pi_L \end{bmatrix} A^F \tag{C-1}$$

where Π is a 2×2 diagonal matrix, its element π_k is factor productivity-argument parameter, $k = K, L$.

The Trefler $2 \times 2 \times 2$ model can be denoted as

$$A^H X^H = V^H, \qquad (A^H)' W^H = P^H \tag{C-2}$$

$$\Pi^{-1} A^H X^F = V^F, \qquad (\Pi^{-1} A^H)' W^F = P^F \tag{C-3}$$

The world effective factor endowments by referring to the home technologies are

$$K^{WH} = K^H + \pi_K K^F \quad , \ L^{WH} = L^H + \pi_L L^F \tag{C-4}$$

The world effective factor endowments by referring to foreign technologies are

$$K^{WF} = K^F + K^H/\pi_K \quad , \ L^{WF} = L^F + L^H/\pi_L \tag{C-5}$$

Guo (2019) provides an equilibrium solution for the Trefler model. The equilibrium shares of GNP of the two countries are

$$s^H = \frac{1}{2}\left(\frac{K^H}{K^{WH}} + \frac{L^H}{L^{WH}}\right) \tag{C-6}$$

$$s^F = \frac{1}{2}\left(\frac{K^F}{K^{WF}} + \frac{L^F}{L^{WF}}\right) \tag{C-7}$$

Using them, we obtain the following trade-price equilibrium of the model. The prices are

$$W^{H*} = \begin{bmatrix} \frac{L^{WH}}{K^{WH}} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{L^H + \pi_L L^F}{K^H + \pi_K K^F} \\ 1 \end{bmatrix} \tag{C-8}$$

$$P^* = (A^H)' \, W^{H*} \tag{C-9}$$

$$W^{F*} = \Pi W^{H*} \tag{C-10}$$

Factor content of trade for the two countries are

$$F_K^h = K^h - s^h K^{Wh} = \frac{1}{2}\frac{K^h L^{Wh} - K^{Wh} L^h}{L^{Wh}} \qquad (h = H, F) \tag{C-11}$$

$$F_L^h = L^h - s^h L^{Wh} = -\frac{1}{2}\frac{K^h L^{Wh} - K^{Wh} L^h}{K^{Wh}} \qquad (h = H, F) \tag{C-12}$$

We demonstrate that the actual capital abundant country exports the labor-intensive commodity.

We assume that both countries are capital-intensity in commodity 1. We also assume that the home country is capital abundance.

Substituting (C-4) into (C-11) yields

$$F_K^H = \frac{1}{2}\frac{K^H \pi_L L^F - L^H \pi_K K^F}{L^H + \pi_L L^F} \tag{C-13}$$

If the numerator of F_K^H is less than zero, It means that its numerator is less than zero as

$$\frac{K^H L^F}{L^H K^F} < \frac{\pi_K}{\pi_L} \tag{C-14}$$

Rewrite it as

$$\frac{K^H}{L^H} < \frac{\pi_K K^F}{\pi_L L^F} = \frac{K^{FH}}{L^{FH}} \tag{C-15}$$

It means that the home country is virtual labor abundance. It implies that actual capital abundant country exports the services of labor under the condition (C-14).

Appendix D

Assume

$$A^H = \psi A^F = \begin{bmatrix} 0 & \theta_K \\ \theta_L & 0 \end{bmatrix} A^F \tag{D-1}$$

The Hicks-Neutral FIRs model can be denoted as

$$A^H X^H = V^H, \qquad (A^H)' W^H = P^H \tag{D-2}$$

$$\psi^{-1} A^H X^F = V^F, \qquad (\psi^{-1} A^H)' W^F = P^F \tag{D-3}$$

The world effective factor endowments by referring home technologies are

$$K^{WbH} = K^H + \theta_L L^F \quad , \ L^{WH} = L^H + \theta_K K^F \tag{D-4}$$

The world effective factor endowments by referring foreign technologies are

$$K^{WbF} = K^F + L^F / \theta_L \quad , \ L^{WF} = L^F + K^F / \theta_K \tag{D-5}$$

We demonstrate that F_K^H and F_K^F are at the same sign. Substituting (D-4) into (C-11) yields

$$F_K^H = \frac{1}{2}\frac{K^H K^F \theta_K - L^H L^F \theta_L}{L^H + \theta_K K^F} \tag{D-6}$$

If the numerator of F_K^H is greater than zero, It means

$$\frac{K^H K^F}{L^H L^F} > \frac{\theta_L}{\theta_K} \tag{D-7}$$

Similarly, substituting (D-5) into (C-11) yields

$$F_K^F = \frac{1}{2}\frac{K^H K^F / \theta_L - L^H L^F / \theta_K}{L^F + K^F / \theta_K} \tag{D-8}$$

If the numerator of F_K^H is greater than zero, it means,

$$\frac{K^H K^F}{L^H L^F} > \frac{\theta_L}{\theta_K} \tag{D-9}$$

Therefore, F_K^H and F_K^F are at the same sign always.

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