Project appraisal and the Intrinsic Rate of Return

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ABSTRACT

Building upon Magni (2011)'s approach, we propose a new rate of return measuring a project's economic profitability. It is called the intrinsic rate of return (IROR). It is defined as the ratio of project return to project's intrinsic value. The IROR approach decomposes the NPV into project scale and economic efficiency. In particular, NPV is found as the product of the project's total invested capital and the excess rate of return, obtained as the difference between the IROR and the minimum attractive rate of return (MARR). This approach provides correct project ranking and is capable of managing time-varying costs of capital. In case of levered projects, shareholder value creation is captured by the equity IROR, which we call *Intrinsic Return On Equity* (IROE) (net income divided by total equity capital invested). If the project is unlevered, the IROE and the IROR lead to the same decision; if the project is levered, and the nominal value of debt is not equal to the market value of debt, the IROE should be preferred to project IROR.

INTRODUCTION

As often reported in empirical studies, practitioners are interested in assessing economic profitability with a relative measure of worth no less than with an absolute measure of worth such as the Net Present Value (NPV). The use of a rate of return in place of or in conjunction with NPV is rather common (Remer and Nieto 1995a,b, Graham and Harvey 2001, Sandahl and Sjögren 2003). Furthermore, recent findings in the literature have revived the debate on relative measures of worth and their relations with NPV (Hazen 2003, Hartman and Schafrick 2004, Magni 2010, 2011, 2013, 2016, Lima e Silva et al. 2017, Ben-Horin and Kroll 2017). In particular, the ability of Chisini means of making sense of seemingly disparate measures of worth have been demonstrated (Magni et al. 2018) and a stronger definition of NPV-consistency has been recently advanced (Marchioni and Magni 2018). We present a new relative measure of worth for project evaluation, called the Intrinsic Rate of Return (IROR). Contrary to IRR, it does not require solving equations, it exists and is unique and is, literally, a *return on investment*, namely, the total profit generated by the project divided by the total invested capital, where the capital is expressed in terms of intrinsic or economic values. The IROR is a rational measure of worth, simple to use and intuitive, which may be used for project ranking as well as accept-reject decisions, for both levered and unlevered projects. It improves on the traditional NPV analysis for it decomposes NPV into two value drivers: The project's scale (total capital invested) and the project's economic efficiency (excess rate of return). A companion of IROR is the Intrinsic Return On Equity (IROE), which measures the equity rate of return. IROE is NPV-consistent as well, and it is preferable to IROR whenever the nominal value of debt differs from the market value of debt. Both IROR and IROE easily cope with time-varying costs of capital.

1. NPV and intrinsic value

Consider an n-period project and let Rev_t and OpC_t be the estimated incremental revenues and incremental operational costs associated with the project, respectively. The project's after-tax operating profit is

$$P_t = (\mathrm{Rev}_t - \mathrm{OpC}_t - \mathrm{Dep}_t)(1-\tau)$$

where Dep_t is the capital's depreciation charge and τ is the marginal corporate tax rate. The estimated free cash flow (FCF) stream is $\boldsymbol{F} = (F_0, F_1, \dots, F_n)$ and $C_0 = -F_0$ is the project cost, such that

$$F_t = P_t + \text{Dep}_t \tag{1}$$

for $t > 0$, assuming that working capital is equal to 0. Let r be the cost of capital, that is, the interest rate at which funds may be invested or borrowed in a normal, competitive financial market. If the project is levered, the cost of capital is often called weighted average cost of capital (WACC). The cost of capital expresses the minimum attractive rate of return (MARR). We assume, for the time being, that it is constant. The project's net present value is defined as

$$\text{NPV} = \frac{F_1}{1+r} + \cdots + \frac{F_n}{(1+r)^n} - C_0.$$

It measures the economic value created, that is, the investors' wealth increase. The project is worth undertaking if and only if $\text{NPV} > 0$. Consider now the following definition of IROR.

Definition (Intrinsic Rate of Return) *The IROR is equal to the ratio of total profit to total capital invested:*

$$i = \frac{\text{TP}}{\text{TC}} = \frac{\sum_{t=0}^{n} P_t}{\sum_{t=0}^{n} V_t} \tag{2}$$

where $V_t = \sum_{k=t+1}^{n} F_k (1+r)^{t-k}$ is the discounted sum of the prospective FCFs (with $P_0 = V_n = 0$). V_t expresses the *intrinsic* value of the project, that is, the value at which an equal-risk asset is traded in the market (or, equivalently, it is the price that the project would have if it were traded in the market). It is then an economic measure of the capital invested in the project at time $\boldsymbol{t}$. Note that, recursively,

$$V_t = V_{t-1}(1+r) - F_t$$

or, proceeding backward,

$$V_t = \frac{V_{t+1} + F_{t+1}}{1+r}.$$

Once profits are estimated, FCFs are derived from (1). Then, the intrinsic value is obtained from FCFs recursively as described above. In other words, V_t is the capital intrinsically invested at the beginning of period $[t, t+1]$, $t = 0, 1, \dots, n-1$. Summing the invested amounts, one gets the total capital, TC, invested in the span of n years.

The IROR in (2) is economically significant for it fulfills the literal definition of a rate of return: An amount of return per unit of invested capital.

The IROR may also be framed in a different-but-equivalent way, using cash flows instead of profits. Specifically, we first prove that the total profit coincides with the project's net cash flow:

$$\sum_{t=1}^{n} P_t = \sum_{t=0}^{n} F_t. \tag{3}$$

To this end, consider that, owing to (1),

$$F_0 + \sum_{t=1}^{n} F_t = -C_0 + \sum_{t=1}^{n} (P_t + \text{Dep}_t).$$

As $C_0 = \sum_{t=1}^{n} \text{Dep}_t$, then (3) is straightforward. As a result, the IROR may be alternatively viewed as a profit measure or as a cash-flow measure:

$$\frac{\overbrace{P_1 + P_2 + \cdots + P_n}^{\text{profit to capital}}}{V_0 + V_1 + \cdots + V_{n-1}} = \text{IROR} = \frac{\overbrace{F_0 + F_1 + \cdots + F_n}^{\text{cash flow to capital}}}{V_0 + V_1 + \cdots + V_{n-1}}.$$

It is a ratio of total profit to invested capital or a ratio of net cash flow to invested capital.

The following decision criterion is naturally derived from the IROR.

IROR decision criterion. An investment project is worth undertaking (i.e., it creates value) if and only if $i > r$. A financing project is worth undertaking (i.e., it creates value) if and only if $i < r$.

Whether the IROR criterion is economically rational or not depends on whether it is consistent with the NPV criterion. The NPV criterion recommends acceptance if and only if $\text{NPV} > 0$. We now show that such a consistency indeed holds.

2. NPV-consistency of IROR

Consider the following definition.

Investment project and financing project. If $\text{TC} > 0$, the project is an investment project and i is an investment rate; if $\text{TC} < 0$, the project is a financing (or borrowing) project and i is a financing rate.

(See also Magni 2010, 2013, 2016 on the difference between investment and financing). From section 1, we know that $V_t = V_{t-1}(1+r) - F_t$, whence

$$r = \frac{V_t + F_t - V_{t-1}}{V_{t-1}}$$

for every $t \geq 1$. The WACC, r, is the market return that would be earned by investors if they invested V_{t-1}

in the market instead of investing it in the project. More precisely, the project's cash-flow stream is $(-C_0, F_1, F_2, \dots, F_n)$ while the cash-flow stream of a portfolio replicating the project's prospective FCFs is $(-V_0, F_1, F_2, \dots, F_n)$. The return stream of the project is $(P_1, P_2, \dots, P_n)$ while the return stream of the replicating portfolio is $(rV_0, rV_1, \dots, rV_{n-1})$. Using (3), the difference between the total project return and the total market return is

$$\sum_{t=1}^{n} P_t - \sum_{t=1}^{n} rV_{t-1} = \sum_{t=0}^{n} F_t - \sum_{t=1}^{n} (F_t + V_t - V_{t-1}).$$

As $V_n = 0$, this means

$$\sum_{t=1}^{n} P_t - \sum_{t=1}^{n} rV_{t-1} = V_0 - C_0.$$

However, $V_0 = \sum_{t=1}^{n} F_t(1+r)^{-t}$ and

$$V_0 - C_0 = \sum_{t=0}^{n} F_t(1+r)^{-t} = \text{NPV}.$$

Therefore,

$$\text{NPV} = \sum_{t=1}^{n} P_t - \sum_{t=1}^{n} rV_{t-1}.$$

Dividing by $\text{TC} = \sum_{t=0}^{n} V_t$,

$$\text{NPV} = \overbrace{\text{TC}}^{\text{Project Scale}} \cdot \overbrace{(i - r)}^{\text{Economic Efficiency}} \qquad (4)$$

Equation (4) represents an economically significant decomposition of NPV. It says that the economic value created by the project is the result of two effects: The amount of capital that will be invested in the project (project scale) and the extent by which the project rate of return will exceed the MARR (economic efficiency). Note that this kind of information cannot be derived from a traditional NPV analysis. Equation (4) proves that the IROR is NPV-consistent.

Proposition 1. (NPV-consistency of IROR) *In an investment project,* $\text{NPV} > 0$ *if and only if* $i > r$. *In a borrowing project,* $\text{NPV} > 0$ *if and only if* $i < r$.

Note that, if the project is a financing project, then the IROR represents a financing rate, as well as r. Therefore, the project is worth undertaking if its financing cost is smaller than the borrowing cost prevailing in the market. (Financing projects may occur only if total assets are negative, which may occur whenever fixed assets are sufficiently small and the net working capital is negative and sufficiently high in absolute value. In these situations, cash is received from customers earlier than cash is paid out to suppliers.)

3. Time-varying WACCs

We now show how the MARR should be computed if the WACC is time-varying. Let $\boldsymbol{r} = (r_1, r_2, \dots, r_n)$ be the stream of WACCs holding in the various years, such that $r_t = (V_t + F_t - V_{t-1})/V_{t-1}$.

In this case, the equality $\text{NPV} = \sum_{t=1}^{n} P_t - \sum_{t=1}^{n} rV_{t-1}$ shown in the previous section generalizes to

$$\text{NPV} = \sum_{t=1}^{n} P_t - \sum_{t=1}^{n} r_t V_{t-1}.$$

Equation (4) still holds, with the understanding that r is redefined as a weighted mean of the WACCs:

$$r = \frac{\sum_{t=1}^{n} r_t V_{t-1}}{\sum_{t=1}^{n} V_{t-1}}. \qquad (5)$$

In other words, the MARR is the weighted average of the time-varying WACCs. An investment project is worth undertaking if and only if the IROR is greater than this MARR.

4. Equity perspective

Suppose that the project is levered and let Int_t be the interest expense associated with the debt. Let r_t^e be the required return to equity (equity cost of capital) in period t and let V_t^e be the intrinsic equity value:

$$V_t^e = \sum_{k=t+1}^{n} \frac{F_k^e}{(1 + r_{t+1}) \cdot (1 + r_{t+2}) \cdot \dots \cdot (1 + r_k)}$$

where F_k^e expresses the cash flow to equity (CFE) at time k. The latter is in turn obtained from the net income as follows. The net income is

$$\text{NI}_k = (\text{Rev}_k - \text{OpC}_k - \text{Dep}_k - \text{Int}_k)(1 - \tau)$$

or, equivalently, $\text{NI}_k = P_t - \text{Int}_k \cdot (1 - \tau)$, and the CFE is $F_k^e = \text{NI}_k + \text{Dep}_k + (D_k - D_{k-1})$, where $D_k - D_{k-1}$ is the change in the outstanding debt. We define the *equity IROR* (i^e) as the ratio of the project's overall net income to total equity (intrinsic) value:

$$i^e = \frac{\text{TNI}}{\text{TC}^\text{e}} = \frac{\sum_{t=1}^{n} (\text{Rev}_t - \text{OpC}_t - \text{Dep}_t - \text{Int}_t)(1 - \tau)}{\sum_{t=0}^{n} V_t^e} \qquad (6)$$

with $V_n^e = 0$. We will also call this ratio *Intrinsic Return On Equity* (IROE). The equity NPV is $\text{NPV}^\text{e} = V_0^e + F_0^e$ or, equivalently

$$\mathrm{NPV^e} = F_0^e + \sum_{k=1}^{n} \frac{F_k^e}{(1+r_1^e)\cdot(1+r_2^e)\cdot\ldots\cdot(1+r_k^e)}.$$

Applying (4) to the equity capitals and the net incomes, one may write

$$\mathrm{NPV^e} = \mathrm{TC^e}\cdot(i^e - r^e) \qquad (7)$$

where TC^e expresses the value of the equity invested in the project and

$$r^e = \frac{\sum_{t=1}^{n} r_t^e V_{t-1}^e}{\sum_{t=1}^{n} V_{t-1}^e}$$

is the weighted average of the costs of equity. This is the equity MARR.

Assuming the interest rate on debt is equal to the required return to debt (debt's cost of capital),[1] then the market value of debt coincides with the book value of debt, which implies that the equity NPV is equal to the project NPV. From (4) and (7),

$$\mathrm{TC}(i-r) = \mathrm{TC^e}(i^e - r^e)$$

This implies $i^e > r^e$ if and only if $i > r$ (assuming, as usual, that TC and TC^e have the same sign). The IROR and the IROE are reciprocally consistent.

If, instead, the interest rate on debt differs from the required rate of return to debt, then $\mathrm{NPV} \neq \mathrm{NPV^e}$. In this case, part of the value created by the project is captured (if $\mathrm{NPV} > \mathrm{NPV^e}$) or given up ($\mathrm{NPV} < \mathrm{NPV}^e$) by the debtholders and the project IROR will not be reliable as a measure of shareholder value creation any more; as shareholders' value creation is the goal of the firm, the IROE will be an appropriate intrinsic rate of return.

5. **Project ranking**

Choice between mutually exclusive projects and ranking of $m > 2$ projects may be accomplished by incremental analysis: If the incremental IROR of A−B is greater than the incremental MARR, then A is preferable to B. Specifically, let i^A and i^B the IRORs of project A and B and let r^A and r^B be the respective MARRs. Let also TC^A be the total intrinsic value of A and TC^B the total intrinsic value of B. Assuming, with no loss of generality, that $\mathrm{TC}^A > \mathrm{TC}^B$, then $\mathrm{NPV}^A > \mathrm{NPV}^B$ if and only if

$$\mathrm{TC}^A(i^A - r^A) > \mathrm{TC}^B(i^B - r^B)$$

which in turn holds if and only if $i^{A-B} > r^{A-B}$ where

$$i^{A-B} = \frac{\sum_{t=1}^{n}(P_t^A - P_t^B)}{\sum_{t=0}^{n}(V_t^A - V_t^B)} = \frac{\sum_{t=0}^{n}(F_t^A - F_t^B)}{\sum_{t=0}^{n}(V_t^A - V_t^B)}$$

is the incremental IROR and

$$r^{A-B} = \frac{\sum_{t=1}^{n}(r_t^A V_{t-1}^A - r_t^B V_{t-1}^B)}{\sum_{t=0}^{n}(V_t^A - V_t^B)}$$

is the incremental MARR. In other words, if investors undertake A instead of B, they earn money at an incremental rate of return equal to the incremental IROR, i^{A-B}, but, at the same time, they incur an incremental opportunity cost which is equal to the incremental MARR, r^{A-B}. If the incremental IROR exceeds the incremental MARR, then project A is preferable to project B.

6. **Numerical example**

Consider a 5-year project with input data as follows:

- Incremental revenues in first year: $350
- Growth rate for revenues: 6% annual
- Incremental operating costs: 30% of revenues
- Cost of the project: $800
- Dep_t: $160 (constant)
- Amount of debt: $300
- Type of debt: Bullet bond (4 years)
- Debt rate: 3%
- Required return to debt: 3%
- Required return to equity: 10% (constant)
- Tax rate: 30%

We use these data to compute the after-tax operating profit and the net income, as well as the equity capital invested, the outstanding debt, the CFE and the cash flow to debt (CFD) (see Table 1). Note that the relation among CFE, CFD and FCF is as follows: $F_t = F_t^e + F_t^d - \tau\cdot\mathrm{Int}_t$, where F_t^d denotes the CFD (see any corporate finance textbook for details) which shows the relation between tax shield and FCF.

The IROE is 19.1% and is greater than the equity MARR by $19.1\% - 10\% = 9.1\%$. The latter figure expresses the economic efficiency of the equity investment. Applied to a total equity value of $1,996, the equity NPV is found to be $\mathrm{NPV^e} = 182$. As we assume that interest rate on debt and cost of debt are equal, the nominal value of debt equates the intrinsic value of debt and the project NPV equates the equity

[1] The required return on debt is the interest rate required by the investors of a competitive, normal market who receive the same prospective cash flows as the debtholders. The interest rate on debt is the contractual rate at which the debt is actually granted by the debtholders. While the two rates are often assumed to be equal, there may be cases where they are not.

NPV, that is, $NPV^e = NPV = 182$. However, in the project perspective, a total \$3,196 is invested, obtained as

$$3{,}196 = 982 + 837 + 667.2 + 469.5 + 240.5$$

or, equivalently, as the sum of total equity value, \$1,996, and total debt value, \$1,200 (= $\$300 \cdot 4$). As the total afer-tax operating profit is \$406.8= $59.5 + 69.8 + 80.7 + 92.3 + 104.5$, dividing the latter by \$3,196 one gets the project IROR, which is equal to 12.73%. The WACC is computed as a weighted average of the cost of equity and the (after-tax) cost of debt, where the weights are the intrinsic value of equity and debt:

$$r_t = \frac{0.1 \cdot V_{t-1}^e + 0.03 \cdot V_{t-1}^d(1 - 0.3)}{V_{t-1}}$$

with $V_{t-1} = V_{t-1}^e + V_{t-1}^d$. It is time-varying because, while cost of equity and cost of debt are time-invariant, the intrinsic value of equity and debt changes over time.

In turn, the mean of the r_t's, weighted by the respective intrinsic values V_{t-1} (see eq. (5)) is equal to the project MARR, which is equal to $r = 7.03\%$, smaller than the IROR by 5.7%. This is the economic efficiency of the project. Applying this figure to the total intrinsic value, the NPV is found back.

7. CONCLUSIONS

The intrinsic rate of return (IROR) is a simple metric, since it is a mere ratio of total profit to total invested capital or, equivalently, the ratio of net cash flow to total invested capital. Therefore, it is, at the same time, an income-based as well as a cash-flow-based measure. It is ready-to-use and understandable by any practitioner. It may be applied to any engineering project as well as a financial investment, for both ex ante decision-making and ex post performance measurement. Multiplied by the total capital invested, it provides the shareholders' wealth increase. Contrary to IRR, it exists, is unique, no equation is required, and it is based on the economically meaningful measure of profit and intrinsic value. It is capable of coping with time-varying WACCs and of correctly ranking competing projects via incremental analysis.

REFERENCES

Ben-Horin M, Kroll Y 2017. A simple intuitive *NPV-IRR* consistent ranking. *The Quarterly Review of* Economics and Finance, 66 (November) 108-114.

Graham JR, Harvey CR 2001. The theory and practice of corporate finance: Evidence from the field. *Journal of Financial Economics* 60 (2-3) (May), 187–243.

Hartman JC, Schafrick IC 2004. The relevant internal rate of return. *The Engineering Economist*, 49, 139–158.

Hazen GB 2003. A new perspective on multiple internal rates of return. *The Engineering Economist*, 48(1), 31–51.

Lima e Silva, JL, Sobreiro VA & Kimura H 2017. Pre-purchase Financing Pool: Revealing the IRR Problem. *The Engineering Economist*, https://doi.org/10.1080/0013791X.2017.1333662, 1-14.

Magni CA 2010. Average Internal Rate of Return and investment decisions: A new perspective. *The Engineering Economist*, 55(2), 150–181.

Magni CA 2011. Aggregate Return On Investment and investment decisions: a cash-flow perspective. *The Engineering Economist*, 56(2), 181–182.

Magni CA 2013. The Internal-Rate-of-Return approach and the AIRR paradigm: A refutation and a corroboration. *The Engineering Economist*, 58(2), 73–111.

Magni CA 2016. Capital depreciation and the underdetermination of rate of return: A unifying perspective. *Journal of Mathematical Economics*, 67 (December), 54-79.

Marchioni A, Magni CA 2018. Investment decisions and sensitivity analysis: NPV-consistency of rates of return, *European Journal of Operational Research*. 268, 361-372.

Magni CA, Veronese P, Graziani R 2018. Chisini means and rational decision-making: Equivalence of investment criteria. *Mathematics and Financial Economics*, 12 193-217.

Remer, DS, Nieto AP 1995a. A compendium and comparison of 25 project evaluation techniques. Part1: Net present value and rate of return methods, *International Journal of Production Economics*, 42, 79–96.

Remer DS, Nieto AP 1995b. A compendium and comparison of 25 project evaluation techniques. Part2: Ratio, payback, and accounting methods, *International Journal of Production Economics* 42, 101–129.

Sandahl G, Sjögren S. 2003. Capital budgeting methods among Sweden's largest groups of companies. The state of the art and a comparison with earlier studies. *International Journal of Production Economics*. 84, 51–69.

Table 1

Year		0	1	2	3	4	5
Revenues	Rev_t		350.0	371.0	393.3	416.9	441.9
Operating Costs	OpC_t		105.0	111.3	118.0	125.1	132.6
Depreciation	Dep_t		160.0	160.0	160.0	160.0	160.0
Pre-tax operating profit	$\mathrm{Rev}_t - \mathrm{OpC}_t - \mathrm{Dep}_t$		85.0	99.7	115.3	131.8	149.3
Taxes on operating profit	$\tau \cdot (\mathrm{Rev}_t - \mathrm{OpC}_t - \mathrm{Dep}_t)$		25.5	29.9	34.6	39.5	44.8
After-tax operating profit	P_t		59.5	69.8	80.7	92.3	104.5

Pre-tax operating profit	$\mathrm{Rev}_t - \mathrm{OpC}_t - \mathrm{Dep}_t$		85.0	99.7	115.3	131.8	149.3
Interest	Int_t		9.0	9.0	9.0	9.0	0.0
Earnings before taxes (EBT)	$\mathrm{Rev}_t - \mathrm{OpC}_t - \mathrm{Dep}_t - \mathrm{Int}_t$		76.0	90.7	106.3	122.8	149.3
Taxes on EBT	$\tau \cdot (\mathrm{Rev}_t - \mathrm{OpC}_t - \mathrm{Dep}_t - \mathrm{Int}_t)$		22.8	27.2	31.9	36.8	44.8
Net income	NI_t		53.2	63.5	74.4	86.0	104.5

Equity capital		500	340	180	20	160	0
Debt capital		300	300	300	300	0	0

FCF	F_t	−800	219.5	229.8	240.7	252.3	264.5
CFE	F_t^e	−500	213.2	223.5	234.4	−54.0	264.5
CFD	F_t^d	−300	9.0	9.0	9.0	309.0	0.0

EQUITY perspective							
Intrinsic value	V_t^e	682.0	537.0	367.2	169.5	240.5	0.0
Total intrinsic value	TC^e	1,996.0					
Total net income	TNI	381.6					
IROE	i^e	19.1%					
MARR	r^e	10.0%					
equity NPV	NPV^e	182.0					

PROJECT perspective							
Intrinsic value	V_t	982.0	837.0	667.2	469.5	240.5	0.0
Total intrinsic value	TC	3,196.0					
Total operating profit	TP	406.8					
WACC	r_t		7.6%	7.2%	6.4%	5.0%	10.0%
IROR	i	12.73%					
MARR	r	7.03%					
Project NPV	NPV	182.0					