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Investment decisions and sensitivity analysis: NPV-consistency of rates of return

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Abstract

Investment decisions may be evaluated via several different metrics/criteria, which are functions of a vector of *value drivers*. The economic significance and the reliability of a metric depend on its compatibility with the Net Present Value (NPV). Traditionally, a metric is said to be NPV-consistent if it is coherent with NPV in signalling value creation. This paper makes use of Sensitivity Analysis (SA) for measuring coherence between rates of return and NPV. In particular, it introduces a new, stronger definition of NPV-consistency that takes into account the influence of value drivers on the metric output. A metric is strongly NPV-consistent if it signals value creation *and* the ranking of the value drivers in terms of impact on the output is the same as that provided by the NPV. The degree of (in)coherence is calculated with Spearman's (1904) correlation coefficient and Iman and Conover's (1987) top-down coefficient. We focus on the class of AIRRs (Magni 2010, 2013) and show that the average Return On Investment (ROI) enjoys strong NPV-consistency under several (possibly all) methods of Sensitivity Analysis.

Keywords. Finance, sensitivity analysis, investment decisions, NPV, Return On Investment, consistency, average, AIRR.

JEL codes. C4, C44, D81, M41, G31, G12, D92.

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1 Introduction

In capital budgeting many different criteria are used for evaluating a project, measuring economic efficiency, and making decisions. Net Present Value (NPV) is considered the most theoretically reliable tool, since it correctly measures shareholder value creation (Brealey and Myers 2000, Ross, Westerfield and Jordan 2011). However, in practice, many other metrics are used; in particular, relative measures of worth such as internal rate of return (IRR), profitability index (PI), modified internal rate of return (MIRR), Return On Investment (ROI), etc. Recently, a more general notion of rate of return, labeled AIRR (Average Internal Rate of Return) has been developed by Magni (2010, 2013), based on a capital-weighted mean of holding period rates. The AIRR approach consists in associating the capital amounts invested in each period with the corresponding period returns by means of a weighted arithmetic mean. Magni (2010, 2013) showed that any AIRR is NPV-consistent: decisions made by an investor who adopts NPV are the same as those made by an investor who adopts AIRR. Magni (2013) showed that many traditional metrics can be viewed as belonging to the class of AIRRs, including IRR, PI, MIRR. As a special case, this approach makes use of the Return On Investment (ROI) to get an *average ROI*, which is the ratio of the total project return to the total invested capital. Whatever the depreciation pattern, the average ROI exists and is unique, it has the unambiguous nature of investment rate, independent of the value drivers, and decomposes the economic value created into economic efficiency (the difference between average ROI and cost of capital) and investment scale (the sum of the committed amounts).

However, while traditional NPV-consistency is important, under uncertainty, an NPV or a rate of return are not the only factors that drive a decision. The investigation of the risk factors that mainly influence the value of the objective function is no less important.

Sensitivity analysis (SA) investigates the variation of an objective function under changes in the key inputs of a model, so aiming at identifying the most important risk factors affecting the output (and, therefore, the decision) and ranking them. There are many different SA techniques (see Pianosi et al. 2016, Borgonovo and Plischke 2016) and, given a technique, different objective functions may or may not lead to different results.

This paper positions itself in the interfaces of operational research (OR) and finance. The strict connections between operations management and finance were recognized long since (e.g., Small 1956, Weingartner 1963, Adelson 1965, Hespos and Strassman 1965, Teichroew et al. 1965a,b, Rivett 1974, Ignizio 1976) and scholarly contributions in the field have grown dramatically in the last decades (e.g., Rosenblatt and Sinuany-Stern 1989, Grubbström and Ashcroft 1991, Murthi et al. 1997, Meier et al. 2001, Gondzio and Kouwenberg 2001, Baesens et al. 2003, Steuer and Na 2003, Xu and Birge 2008, Koç et al. 2009, Fabozzi et al. 2010, Thomas 2010, Seifert et al. 2013).

The relation between OR and finance is bidirectional. On one side, finance provides a rich toolkit of theories, criteria, and methodologies which enable operational managers to better understand the impact of their decisions so as to maximize the shareholders' wealth: "In order to make decisions managers need criteria of goodness, decision tools, and an understanding of the environment in which they operate ... The main elements of

this are that the right criterion of goodness is the maximisation of shareholder wealth and that firms operate in something close to a perfect capital market." (Ashford et al˙ 1988, p. 144). On the other side, operational research sets the aims and scope of financial modeling for managerial purposes: As opposed to finance theory which uses financial modeling for describing the behavior of the "average" investor and deriving the pricing process of financial assets, operational managers use financial modeling from the point of view of an individual decision maker with specific needs, constraints and preferences (Spronk and Hallerbach 1997). Further, operations research itself provides techniques and tools that may be applied to several finance problems (Board et al. 2003).

This paper is in line with the bidirectional relation between operations and finance. Specifically, it recognizes the fundamental roles of economic and financial measures of worth such as the NPV and the ROI for decision-making and, at the same time, applies an OR technique (SA) to such financial measures in order to investigate their compatibility. As such, it falls within that strand of the OR literature which makes use of various economic efficiency measures for managerial purposes, including the NPV (e.g., Yang et al. 1993, Baroum and Patterson 1996, Herroelen et al. 1997, Cigola and Peccati 2005, Borgonovo and Peccati 2006a, Wiesemann et al. 2010, Leyman and Vanhoucke 2017), the IRR (Rapp 1980, Nauss 1988, Hazen 2003, 2009, Hartman and Shafrick 2004, Dhavale and Sarkis 2018), the ROI (e.g., Danaher and Rust 1996, Myung et al. 1997, Brimberg and ReVelle 2000, Brimberg et al. 2008, Li et al. 2008, Menezes et al. 2015, Magni 2016) and the return to outlay (Kumbhakar 2011). This work is strictly linked with some recent methodological papers within this field which evaluate rationality and robustness of various efficiency measures and/or their sensitivity to changes in the key parameters. Specifically, Magni (2015) showed that the average ROI (labeled average ROA) is reliable for measuring economic efficiency in industrial applications; Mørch et al. (2017) used the average ROI as the objective function in a problem of renewal of shippings, and compared the results with those obtained from the traditional NPV maximization. Borgonovo and Peccati (2004, 2006b) studied the impact of the key drivers of an industrial project on NPV, IRR, and value at any time. Borgonovo et al. (2010) applied SA in a project financing transaction to assess the degree of coherence between NPV and debt service coverage ratio. Talavera et al. (2010) applied SA to the IRR of photovoltaic grid-connected systems. Percoco and Borgonovo (2012) applied SA to IRR and NPV and studied the coherence between the two metrics in terms of importance of key drivers.

We investigate the coherence of average ROI and NPV and give a new, more stringent, definition of NPV-consistency (strong coherence), according to which a metric is strongly NPV-consistent under a given SA technique if it is NPV-consistent in the traditional sense and, in addition, the ranking of the project's value drivers (in terms of influence on the output) is the same. If a metric is not NPV-consistent, the degree of inconsistency may be measured by two alternative indices: Spearman's (1904) coefficient or Iman and Conover's (1987) top-down coefficient.

We find that the average ROI is strongly NPV-consistent under many techniques, even in a strict sense (the relevances of the parameters are the same). As a result, the average ROI is a reliable measure of worth which can coherently be associated with NPV

in investment evaluation, assessment of economic efficiency, and decision-making.

The remaining part of the paper is structured as follows. Section 2 presents the average ROI and the notion of NPV-consistency. Section 3 briefly describes some known SA methods and Section 4 introduces the notion of pairwise coherence according to which any two functions are strongly coherent if the ranking of the model parameters coincides. This section shows that, under many SA techniques, a function f and an affine transformation of it share the same (ranking and) relevances of parameters, so they are strongly coherent in a strict sense. Section 5 shows that the average ROI is strongly NPV-consistent in a strict sense under many SA techniques. Some numerical examples are illustrated in section 6. Some concluding remarks end the paper. (An Appendix is devoted to some other AIRRs, including non-strongly consistent ones such as IRR, MIRR and EAIRR.)

2 AIRR, average ROI, and NPV consistency

Let P be a project and let $\mathbf{F} = (F_0, F_1, \ldots, F_p) \neq \mathbf{0}$ its estimated stream of free cash flows (FCFs), where $F_0 < 0$ is the investment cost and p is the lifetime of the project. Let τ be the tax rate, R_t be the revenues, O_t be the operating costs, and let Dep_t denote depreciation, $t = 1, 2, \ldots, p$. Then,

$$\begin{aligned} F_t &= \overbrace{(R_t - O_t - \mathrm{Dep}_t)(1-\tau)}^{\text{operating profit}} + \mathrm{Dep}_t \\ &= (R_t - O_t)(1-\tau) + \tau \cdot \mathrm{Dep}_t. \end{aligned} \tag{1}$$

Revenues and costs are often estimated in terms of some key inputs such as prices, quantity produced and sold, unit costs, growth rates, etc. There may be several types of costs, such as energy, material, labor, selling, general, and administrative expenses, etc. For example,

$$F_t = \Big(q \cdot p_0(1+g_p)^t - \sum_{j=1}^{s} O_0^j (1+g_{O^j})^t\Big)(1-\tau) + \tau \cdot \mathrm{Dep}_t \tag{2}$$

where p_0 denotes the initial price, q denotes the annual quantity sold, O_0^j denotes the initial amount of the j-th item of cost, g_p and g_{O^j} are the growth rates, and s is the number of cost items involved in the project under consideration. Let k be the (assumed constant) cost of capital (COC). We assume that the COC is exogenously fixed by the decision-maker/analyst. It is well-known that net present value (NPV) measures the economic value created: $\mathrm{NPV} = \sum_{t=0}^{p} F_t(1+k)^{-t}$. Therefore, the NPV decision criterion may be stated as follows:

Definition 1. (NPV criterion) *A project creates value (i.e., it is worth undertaking) if and only if the project NPV, computed at the discount rate k, is positive: $NPV(k) > 0$.*

Let $\mathbf{C} = (C_0, C_1, \ldots, C_n)$ be any vector representing some notion of capital, such that $C_0 = -F_0$ and $C_n = 0$ and let $I_t = F_t + C_t - C_{t-1}$ be the associated return. An AIRR, denoted as $\bar{\imath}$, is defined as the ratio of the overall return $I = \sum_{t=1}^{p} I_t(1+k)^{-(t-1)}$ earned

by the investor to the overall capital committed $C = \sum_{t=1}^{p} C_{t-1}(1+k)^{-(t-1)}$:

$$\bar{\imath} = \frac{I}{C} \tag{3}$$

or, equivalently, as the weighted mean of period rates associated with the capital stream $\mathbf{C}$:

$$\bar{\imath} = \frac{\sum_{t=1}^{p} i_t C_{t-1} d_{t-1}}{\sum_{t=1}^{p} C_{t-1} d_{t-1}}$$

where $d_t = (1+k)^{-t}$ is the discounting factor and $i_t = I_t/C_{t-1}$ is the period rate of return, $t = 1, 2, \ldots, p$ (see Magni 2010, 2013).

Magni (2010, 2013) defined a project a net investment if $C > 0$ and a net financing if $C < 0$. In such a way, the financial nature of any project (and its associated rate of return) can be identified as an investment project or a financing project (respectively, an investment rate or a financing rate).

Traditionally, it is widely accepted that a metric/criterion φ is said to be NPV-consistent if and only if a decision maker adopting φ makes the same decision suggested by the NPV criterion. We can formalize this standard notion as follows.

Definition 2. (NPV-consistency) *A metric/criterion φ is NPV-consistent if, given a cutoff rate k, the following statements are true:*

(i) *an investment project creates value if and only if $\varphi > k$*

(ii) *a financing project creates value if and only if $\varphi < k$.*

Magni (2010, 2013) showed that, given a cash-flow stream $\mathbf{F}$, if $\varphi = \bar{\imath}$, then the metric is NPV-consistent, since, for any vector $\mathbf{C}$, the following product structure holds:

$$\mathrm{NPV}(1+k) = C(\bar{\imath} - k). \tag{4}$$

The above definition and eq. (4) are particularly interesting because they show that the AIRR approach enables reframing the NPV in terms of product of a capital base C and an excess return $\bar{\imath} - k$. This means that the economic value created is determined by two factors: The project scale (C) and the project's economic efficiency, $\bar{\imath} - k$. The same NPV can be created either by investing a large capital amount at a small rate or investing a small capital at a high rate. Furthermore, the general definition stated above enables the analyst to understand whether value is created because capital is invested at a rate of return which is higher than the COC or because capital is borrowed at a financing rate which is smaller than the COC (see also Magni 2015).

We now consider the special case of AIRR where $C_t = B_t$ is the capital which remains invested in the project at time t: $B_t = B_{t-1} - \mathrm{Dep}_t$ and $B_0 = -F_0$, so that I_t is the operating profit: $I_t = F_t + B_t - B_{t-1} = F_t - \mathrm{Dep}_t = (R_t - O_t - \mathrm{Dep}_t)(1-\tau)$. The associated period return rate is the Return on Investment (ROI):

$$\mathrm{ROI}_t = \frac{\text{Operating profit}}{\text{Invested capital}} = \frac{(R_t - O_t - \mathrm{Dep}_t)(1-\tau)}{B_{t-1}}. \tag{5}$$

Thus, the AIRR becomes

$$\bar{\imath}(B) = \frac{\text{Total Return}}{\text{Total Invested Capital}} = \frac{I}{B} \tag{6}$$

where $I = \sum_{t=1}^{p} \big((R_t - O_t - \text{Dep}_t)(1-\tau)\big) \cdot d_{t-1}$ is the overall operating profit generated by the project and $B = \sum_{t=1}^{p} B_{t-1} d_{t-1}$ is the overall invested capital, expressing the size of the investment. As seen above, $\bar{\imath}(B)$ may be viewed as a weighted average of ROIs:

$$\bar{\imath}(B) = \alpha_1 \text{ROI}_1 + \alpha_2 \text{ROI}_2 + \ldots + \alpha_p \text{ROI}_p \tag{7}$$

where $\alpha_t = B_{t-1} d_{t-1}/B$. We call $\bar{\imath}(B)$ average ROI.[1] As (4) holds for any $\mathbf{C}$ (and, therefore, for $\mathbf{B} = (B_0, B_1, \ldots, B_n)$ as well), $\text{NPV}(1+k) = B \cdot (\bar{\imath}(B) - k)$ so the average ROI is NPV-consistent (see also Magni 2015). It is also worth noting that the average ROI has the compelling property of existence and uniqueness for any project. Also, its financial nature does not depend on the value drivers nor the cost of capital: It is unambiguously determined as an investment rate, since $B_0 = -F_0 > 0$ and $\text{Dep}_t > 0$, which implies $B > 0$. This makes it a good candidate as a reliable measure of worth.

Owing to (1) and (2), the NPV is a function of several variables (the prospective revenues and costs). Practically, the analyst selects depreciation for every period, Dep_1, Dep_2, ... , Dep_p, then estimates the amount of sales, the initial price(s), the costs for labor, material, maintenance, energy, the growth rates, the tax rate, etc. These variables are risk factors, also known as *value drivers*, for they affect the FCFs. Hence, given the project COC, the project NPV is computed. For example, using (2),

$$\text{NPV} = F_0 + \sum_{t=1}^{p} \frac{\big(q \cdot p_0(1+g_p)^t - \sum_{j=1}^{s} O_0^j (1+g_{O^j})^t\big)(1-\tau) + \tau \cdot \text{Dep}_t}{(1+k)^t}. \tag{8}$$

It is evident that the average ROI depends on these same value drivers, given that ROI_t depends on them. From (5),

$$\text{ROI}_t = \frac{\big(q \cdot p_0(1+g_p)^t - \sum_{j=1}^{s} O_0^j (1+g_{O^j})^t - \text{Dep}_t\big)(1-\tau)}{B_{t-1}}.$$

Exploiting (4), one can describe the AIRR as a function of the overall capital C:

$$\bar{\imath} = \bar{\imath}(C) = k + \frac{\text{NPV}}{C}(1+k). \tag{9}$$

Figure 1 graphically describes the AIRR function $\bar{\imath}(C)$ for a value-creating project; each pair $(C, \bar{\imath}(C))$ represents an NPV-consistent rate of return; among the infinitely many AIRRs, we highlight the average ROI, which is the AIRR associated with the capital stream $\mathbf{B}$.

The project's aim is to check whether the coherence of average ROI and NPV, which is guaranteed in a traditional sense, remains valid if changes in value drivers are considered. The analysis of change in a model's inputs and the impact on the model output is the

[1] Magni (2015) used the expression average ROA for this measure.

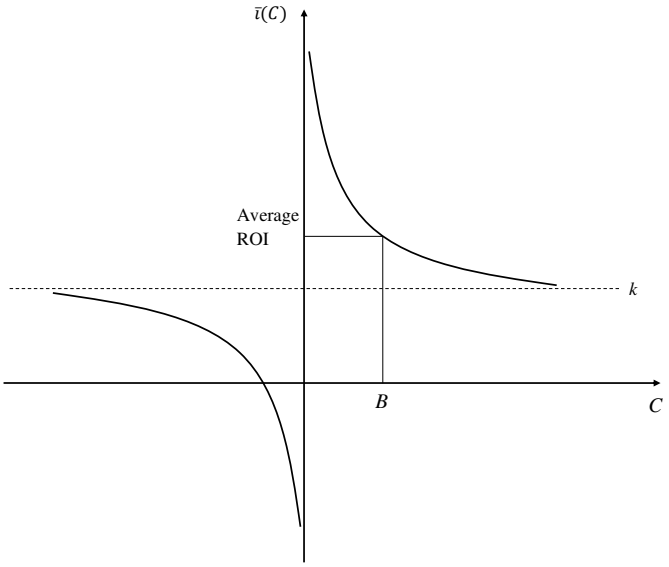

Figure 1: Graph of the AIRR function for a positive-NPV project

purpose of Sensitivity Analysis (SA).[2]

3 Sensitivity analysis

In the definition of Saltelli et al. (2004, p. 45), sensitivity analysis (SA) is the "study of how the uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input."

Given a model and a set of inputs (parameters), the SA investigates the relevance of parameters in terms of variability of the model output. In the literature there exist many SA techniques (see Borgonovo and Plischke 2016, Pianosi et al. 2016, for review of SA methods). A model can be described as consisting of an objective function f defined on the *parameter space* A, which maps vector of inputs onto a model output y:

$$f : A \subset \mathbb{R}^n \to \mathbb{R}, \quad y = f(\alpha), \quad \alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)\,. \tag{10}$$

The vector $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n) \in A \subset \mathbb{R}^n$ is the vector of inputs or parameters or key drivers and $y(\alpha)$ is the output of the model. Let $\alpha^0 = \left(\alpha_1^0, \alpha_2^0, \ldots, \alpha_n^0\right) \in A$ be the *base-case*, a representative value (e.g., mean value, most probable value, etc.). The relevance of a parameter α_i (also known as importance measure) quantifies the impact of α_i on the output variation. Let $R^f = \left(R_1^f, R_2^f, \ldots, R_n^f\right)$ be the vector of the relevances. The latter determines the ranking of the parameters in the following way. Input α_i is defined to be

[2]Ekern (1981) and Foster and Mitra (2003) provide conditions under which a project's NPV is greater than a second project's NPV irrespective of the COC. Assuming that the second project is the null alternative, those conditions identify those projects which are robust under changes in the COC. Those conditions hold for any AIRR as well, given that any AIRR is NPV-consistent in the traditional sense. In this paper, we measure the robustness of the project with respect to the estimates of revenues and costs and focus on their impact on NPV and rate of return.

more relevant than α_j if and only if $|R_i^f| > |R_j^f|$. The parameters are equally relevant for f if $|R_i^f| = |R_j^f|$. The rank of α_i, denoted as r_i^f, depends on the importance measure: α_i has a higher rank (it has a greater impact on the output) than α_j if it has greater relevance. Let $r^f = \left(r_1^f, r_2^f, \ldots, r_n^f\right)$ be the vector of ranks.

The average rank is $r_M^f = \frac{\sum_{i=1}^n i}{n} = \frac{\frac{n\cdot(n+1)}{2}}{n} = \frac{n+1}{2}$. The *high parameters* (or top parameters) are those whose rank is higher than the average rank r_M^f; the *low parameters* are those parameters whose rank is smaller than r_M^f.

Following we briefly describe some well-known (global and local) SA techniques.

(i) **Standardized regression coefficient** (global SA)

Let V denote variance and σ denote standard deviation. Consider the linear regression with dependent variable f and explanatory variables α_i, $\forall i = 1, \ldots, n$, estimated with OLS method: $f = \beta_0^f + \sum_{i=1}^n \beta_i^f \cdot \alpha_i + u$. The standardized regression coefficient SRC_i^f measures the importance of α_i (Saltelli and Marivoet 1990, Bring 1994, Saltelli et al. 2008):

$$SRC_i^f = \frac{\beta_i^f \cdot \sigma(\alpha_i)}{\sigma(f)}. \tag{11}$$

(ii) **Sensitivity indices in variance-based decomposition methods** (global SA)

In variance-based methods, the importance of a parameter is generally represented through the First Order Sensitivity Index (FOSI) and the Total Order Sensitivity Index (TOSI) (Saltelli et al. 2008). The FOSI, here denoted as $SI_i^{1,f}$, measures the individual effect of the parameter on the output variance:

$$SI_i^{1,f} = \frac{V(E(f|\alpha_i))}{V(f)}, \tag{12}$$

where $V(E(f|\alpha_i))$ is the variance of the expectation of f upon a fixed value of α_i.[3] The TOSI, here denoted as $SI_i^{T,f}$, measures the total contribution of α_i to the output variability, i.e., it is inclusive of the interaction effects with other parameters or groups of parameters. $SI_i^{T,f}$ can be calculated as (Saltelli et al. 2008)

$$SI_i^{T,f} = \frac{E(V(f|\alpha_{-i}))}{V(f)}, \tag{13}$$

where $f|\alpha_{-i} = f|\alpha_1, \alpha_2, \ldots, \alpha_{i-1}, \alpha_{i+1}, \ldots, \alpha_n$ (see also Sobol' 1993, Sobol' 2001, Saltelli et al. 2008).

(iii) **Finite Change Sensitivity Indices** (global SA)

The Finite Change Sensitivity Indices (FCSIs), introduced in Borgonovo (2010a, 2010b), focus on the output change due to a finite input change; there exist two versions of FCSIs: First Order FCSI and Total Order FCSI.

The First Order FCSI of a parameter measures the individual effect of the parameter's variation on f; the Total Order FCSI considers the total effect of a parameter's variation on f, including both the individual contribution and the interactions between a parameter and the other parameters.

[3] It can be shown that $V(E(f|\alpha_i)) = V(f) - E[V(f|\alpha_i)]$ (see Satelli et al. 2008).

Consider a change of parameters from α^0 to $\alpha^1 = \left(\alpha_1^1, \alpha_2^1, \ldots, \alpha_n^1\right) \in A$. The output variation is $\Delta f = f(\alpha^1) - f(\alpha^0)$. Let $(\alpha_i^1, \alpha_{(-i)}^0) = (\alpha_1^0, \alpha_2^0, \ldots, \alpha_{i-1}^0, \alpha_i^1, \alpha_{i+1}^0, \ldots, \alpha_n^0)$ be obtained by varying the parameter α_i to the new value α_i^1, while the remaining $n-1$ parameters are fixed at α^0. The individual effect of α_i on Δf is $\Delta_i f = f(\alpha_i^1, \alpha_{(-i)}^0) - f(\alpha^0)$ and the First Order FCSI of α_i, denoted as $\Phi_i^{1,f}$, is (Borgonovo 2010a):

$$\Phi_i^{1,f} = \frac{\Delta_i f}{\Delta f}. \tag{14}$$

Δf is equal to the sum of individual effects and interactions between parameters and groups of parameters. The total effect of the parameter α_i, denoted as $\Delta_i^T f$, is the sum of the individual effect of α_i and of the interactions that involve α_i. Borgonovo (2010a, Proposition 1) showed that $\Delta_i^T f$ can be obtained as $\Delta_i^T f = f(\alpha^1) - f(\alpha_i^0, \alpha_{(-i)}^1)$ for all $i = 1, 2, \ldots, n$, where $(\alpha_i^0, \alpha_{(-i)}^1)$ is the point with all the parameters equal to the new value α^1, except the parameter α_i, which is equal to α_i^0. The Total Order FCSI of the parameter α_i, denoted as $\Phi_i^{T,f}$, is (Borgonovo 2010a):

$$\Phi_i^{T,f} = \frac{\Delta_i^T f}{\Delta f} = \frac{f(\alpha^1) - f(\alpha_i^0, \alpha_{(-i)}^1)}{\Delta f}. \tag{15}$$

(iv) **Helton's index** (local SA)

Helton (1993) proposed a variance decomposition of f based on Taylor approximation. He assumed parameters are not correlated, so the variance of f can be approximated by

$$\hat{V}(f) = \sum_{i=1}^{n} \left[f'_{\alpha_i}(\alpha^0)\right]^2 \cdot V(\alpha_i). \tag{16}$$

The impact of input α_i on $V(f)$ can be measured by

$$H_i^f(\alpha^0) = \frac{\left[f'_{\alpha_i}(\alpha^0)\right]^2 \cdot V(\alpha_i)}{\hat{V}(f)}. \tag{17}$$

(v) **Normalized Partial Derivatives** (local SA)

Helton (1993) also proposed the adoption of normalized partial derivatives as sensitivity measures. He defined two versions of normalized partial derivatives (NPDs):

$$NPD1_i^f(\alpha^0) = f'_{\alpha_i}(\alpha^0) \cdot \frac{\alpha_i^0}{f(\alpha^0)}, \tag{18}$$

$$NPD2_i^f(\alpha^0) = f'_{\alpha_i}(\alpha^0) \cdot \frac{\sigma(\alpha_i)}{\hat{\sigma}(f)}, \tag{19}$$

where $\hat{\sigma}(f)$ is the square root of $\hat{V}(f)$ defined in (16). $NPD1_i^f(\alpha^0)$ measures the elasticity of f with respect to α_i in α^0 assuming that the relative change in α_i is fixed for $i = 1, 2, \ldots, n$ (Helton 1993, p. 329). $\left|NPD2_i^f(\alpha^0)\right|$ is the square root of (17).

(vi) **Differential Importance Measure** (local SA)

The total variation $f(\alpha^0 + \mathrm{d}\alpha) - f(\alpha^0)$ of a differentiable function f due to a local change $\mathrm{d}\alpha$ can be approximated by the total differential $\mathrm{d}f = \sum_{i=1}^{n} f'_{\alpha_i}(\alpha^0) \cdot \mathrm{d}\alpha_i$. The Differential

Importance Measure (DIM) of parameter α_i is the ratio of the partial differential of f with respect to α_i to the total differential of f (Borgonovo and Apostolakis 2001, Borgonovo and Peccati 2004):

$$DIM_i^f(\alpha^0, \mathrm{d}\alpha) = \frac{\mathrm{d}f_{a_i}}{\mathrm{d}f} = \frac{f'_{\alpha_i}(\alpha^0) \cdot \mathrm{d}\alpha_i}{\sum_{j=1}^n f'_{\alpha_j}(\alpha^0) \cdot \mathrm{d}\alpha_j}. \tag{20}$$

The DIM of a parameter represents the percentage of the function's variation due to the variation of that parameter (Borgonovo and Apostolakis 2001, Borgonovo and Peccati 2004).

4 Coherence between objective functions

Risk management problems are often characterized by the definition of more than one objective function (Borgonovo and Peccati 2006b, Borgonovo, Gatti and Peccati 2010). For a given technique, the analysis can be applied using different objective functions. A relevant aspect is the evaluation of the coherence (or compatibility) between the results of the sensitivity analysis for different functions.

We consider the objective functions $f, g : A \to \mathbb{R}$. The vector of importance measures are respectively $R^f = (R_1^f, R_2^f, \ldots, R_n^f)$ and $R^g = (R_1^g, R_2^g, \ldots, R_n^g)$; the ranking vectors are $r^f = (r_1^f, r_2^f, \ldots, r_n^f)$ and $r^g = (r_1^g, r_2^g, \ldots, r_n^g)$.

Definition 3. (Coherence) *Given a technique of SA and two objective functions f and g, they are coherent if the ranking vectors coincide: $r^f = r^g$. If, in addition, the vectors of the relevances coincide, $R^f = R^g$, they are strictly coherent.*

If two functions f and g are not coherent, the degree of incoherence can be alternatively measured through Spearman's rank correlation coefficient (Spearman 1904) or top-down correlation coefficient (Iman and Conover 1987).

Spearman's rank correlation coefficient (henceforth, Spearman's coefficient) between two stochastic variables is the correlation coefficient between the ranks of the stochastic variables (Spearman 1904). In SA, Spearman's coefficient between two objective functions f and g, denoted as $\rho_{f,g}$, is the correlation coefficient of the ranking vectors r^f and r^g:

$$\rho_{f,g} = \frac{\mathrm{Cov}(r^f, r^g)}{\sigma(r^f) \cdot \sigma(r^g)} = \frac{\sum_{i=1}^n (r_i^f - r_M^f) \cdot (r_i^g - r_M^g)}{\sqrt{\sum_{i=1}^n \left(r_i^f - r_M^f\right)^2} \cdot \sqrt{\sum_{i=1}^n \left(r_i^g - r_M^g\right)^2}}, \tag{21}$$

where, as seen, $r_M^f = r_M^g = \frac{n+1}{2}$. The coefficient $\rho_{f,g}$ attributes the same weight to top and low parameters and lies in the interval $[-1, 1]$. The coefficient $\rho_{f,g}$ is equal to 1 if and only if f and g are coherent according to Definition 3. Therefore, a value of $\rho_{f,g}$ smaller than 1 signals incoherence between f and g: The smaller the value of $\rho_{f,g}$, the higher the degree of incoherence. The difference $1 - \rho_{f,g}$ can be taken as representative of the degree of incoherence.

Iman and Conover (1987) introduced the top-down correlation coefficient, a compatibility measure that attributes a higher weight to top parameters than to low parameters.

This measure is based on *Savage Score* (Savage 1956). The Savage score of parameter α_i is $S_i^f = \sum_{h=r_i^f}^{n} \frac{1}{h}$. The vector of Savage scores is $S^f = \left(S_1^f, S_2^f, \ldots, S_n^f\right)$.[4] The average Savage score is $S_M^f = \frac{\sum_{i=1}^{n} S_i^f}{n} = 1$.

The top-down correlation coefficient between the objective functions f and g, denoted as ρ_{S^f,S^g}, is the correlation coefficient between the Savage scores' vectors S^f and S^g (Iman and Conover 1987):

$$\rho_{S^f,S^g} = \frac{\mathrm{Cov}(S^f, S^g)}{\sigma(S^f) \cdot \sigma(S^g)} = \frac{\sum_{i=1}^{n}(S_i^f - S_M^f) \cdot (S_i^g - S_M^g)}{\sqrt{\sum_{i=1}^{n}\left(S_i^f - S_M^f\right)^2} \cdot \sqrt{\sum_{i=1}^{n}\left(S_i^g - S_M^g\right)^2}}, \tag{22}$$

where $S_M^f = S_M^g = 1$. The coefficient ρ_{S^f,S^g} measures the compatibility between the parameters' ranking of f and g: The accordance between top parameters has a remarkable influence on ρ_{S^f,S^g}, while the discordance between low parameters has a weak influence on ρ_{S^f,S^g} (Iman and Conover 1987).

If the aim of the analysis is factor prioritization (i.e., identification of the most relevant parameters), the top-down coefficient should be preferred to Spearman's coefficient.

The maximum value of ρ_{S^f,S^g} is equal to 1. In case f and g have *no ties* (i.e., no relevance is equal), the minimum value is -1 for $n = 2$, it increases as n increases, and it tends to -0.645 as n tends to infinity (Iman and Conover 1987).

ρ_{S^f,S^g} is equal to 1 if and only if f and g are coherent. Therefore, a value of ρ_{S^f,S^g} smaller than 1 signals incompatibility between f and g. The smaller the value of ρ_{S^f,S^g}, the higher the incoherence level. The degree of incoherence of f and g can then be measured by $1 - \rho_{S^f,S^g}$.

Borgonovo et al. (2014) showed that an objective function f and a monotonic transformation g of it generate the same ranking of the parameters under several techniques. This means that they are coherent according to Definition 3.

We now show that, if g is an *affine* transformation of f, that is, $g(\alpha) = l \cdot f(\alpha) + q$ for all $\alpha \in A$, then f and g are strictly coherent under several techniques.

Proposition 1. *A function and an affine transformation of it are strictly coherent under the following techniques:*

(i) *Standardized regression coefficient*

(ii) *Sensitivity Indices in variance-based decomposition methods*

(iii) *Finite Change Sensitivity Indices*

(iv) *Helton's index*

(v) *Normalized Partial Derivative ($NPD2_i^f$)*

(vi) *Differential Importance Measure.*

Proof. By hypothesis, $g(\alpha) = l \cdot f(\alpha) + q$. Therefore,

[4]For example, if $n = 5$ and $r^f = (1, 2, 3, 4, 5)$, then $S^f = (2.28\bar{3}, 1.28\bar{3}, 0.78\bar{3}, 0.45, 0.2)$.

(i) $g = l \cdot (\beta_0^f + \sum_{i=1}^n \beta_i^f \cdot \alpha_i + u) + q = (l \cdot \beta_0^f + q) + \sum_{i=1}^n (l \cdot \beta_i^f) \cdot \alpha_i + l \cdot u$, whence

$$\begin{aligned}\beta_0^g &= l \cdot \beta_0^f + q,\\ \beta_i^g &= l \cdot \beta_i^f\end{aligned}$$

so that

$$SRC_i^g = \frac{\beta_i^g \cdot \sigma(\alpha_i)}{\sigma(g)} = \frac{l \cdot \beta_i^f \cdot \sigma(\alpha_i)}{l \cdot \sigma(f)} = SRC_i^f.$$

(ii) Denoting as $f|\alpha_i$ (and $g|\alpha_i$) the function f (and g) conditional to a specific value of α_i, $g|\alpha_i = (l \cdot f + q)|\alpha_i = (l \cdot f)|\alpha_i + q = l \cdot f|\alpha_i + q$. Therefore,

$$SI_i^{1,g} = \frac{V(E(g|\alpha_i))}{V(g)} = \frac{V(E(l \cdot f|\alpha_i + q))}{V(l \cdot f + q)} = \frac{l^2 \cdot V(E(f|\alpha_i))}{l^2 \cdot V(f)} = SI_i^{1,f}.$$

Analogously, $g|\alpha_{-i} = (l \cdot f + q)|\alpha_{-i} = l \cdot f|\alpha_{-i} + q$. Hence,

$$SI_i^{T,g} = \frac{E(V(g|\alpha_{-i}))}{V(g)} = \frac{E(V(l \cdot f|\alpha_{-i} + q))}{V(l \cdot f + q)} = \frac{l^2 \cdot E(V(f|\alpha_{-i}))}{l^2 \cdot V(f)} = SI_i^{T,f}.$$

(iii) Since $\Delta g = g(\alpha^1) - g(\alpha^0) = l \cdot f(\alpha^1) + q - l \cdot f(\alpha^0) - q = l \cdot \big(f(\alpha^1) - f(\alpha_0)\big) = l \cdot \Delta f$, and

$$\begin{aligned}\Delta_i g &= g(\alpha_i^1, \alpha_{(-i)}^0) - g(\alpha^0) = l \cdot f(\alpha_i^1, \alpha_{(-i)}^0) + q - l \cdot f(\alpha^0) - q\\ &= l \cdot \big(f(\alpha_i^1, \alpha_{(-i)}^0) - f(\alpha^0)\big) = l \cdot \Delta_i f,\end{aligned}$$

then

$$\Phi_i^{1,g} = \frac{\Delta_i g}{\Delta g} = \frac{l \cdot \Delta_i f}{l \cdot \Delta f} = \frac{\Delta_i f}{\Delta f} = \Phi_i^{1,f}.$$

As for the Total Indices,

$$\begin{aligned}\Delta_i^T g &= g(\alpha^1) - g(\alpha_i^0, \alpha_{(-i)}^1) = l \cdot f(\alpha^1) + q - l \cdot f(\alpha_i^0, \alpha_{(-i)}^1) - q\\ &= l \cdot \big(f(\alpha^1) - f(\alpha_i^0, \alpha_{(-i)}^1)\big) = l \cdot \Delta_i^T f\end{aligned}$$

so that

$$\Phi_i^{T,g} = \frac{\Delta_i^T g}{\Delta g} = \frac{l \cdot \Delta_i^T f}{l \cdot \Delta f} = \frac{\Delta_i^T f}{\Delta f} = \Phi_i^{T,f}.$$

(iv) From (16),

$$\begin{aligned}\hat{V}(g) &= \sum_{i=1}^n [g'_{\alpha_i}(\alpha^0)]^2 \cdot V(\alpha_i)\\ &= \sum_{i=1}^n [l \cdot f'_{\alpha_i}(\alpha^0)]^2 \cdot V(\alpha_i) = l^2 \cdot \sum_{i=1}^n [f'_{\alpha_i}(\alpha^0)]^2 \cdot V(\alpha_i) = l^2 \cdot \hat{V}(f).\end{aligned}$$

Hence,

$$H_i^g(\alpha^0) = \frac{[g'_{\alpha_i}(\alpha^0)]^2 \cdot V(\alpha_i)}{\hat{V}(g)} = \frac{l^2 \cdot [f'_{\alpha_i}(\alpha^0)]^2 \cdot V(\alpha_i)}{l^2 \cdot \hat{V}(f)} = H_i^f(\alpha^0).$$

(v) Straightforward, since $\left|NPD2_i^f\right|$ is the square root of $H_i^f(\alpha^0)$.[5]

(vi) From (20),

$$DIM_i^g(\alpha^0, \mathrm{d}\alpha) = \frac{g'_{\alpha_i}(\alpha^0) \cdot \mathrm{d}\alpha_i}{\sum_{j=1}^n g'_{\alpha_j}(\alpha^0) \cdot \mathrm{d}\alpha_j} = \frac{l \cdot f'_{\alpha_i}(\alpha^0) \cdot \mathrm{d}\alpha_i}{\sum_{j=1}^n l \cdot f'_{\alpha_j}(\alpha^0) \cdot \mathrm{d}\alpha_j} = \frac{f'_{\alpha_i}(\alpha^0) \cdot \mathrm{d}\alpha_i}{\sum_{j=1}^n f'_{\alpha_j}(\alpha^0) \cdot \mathrm{d}\alpha_j}$$
$$= DIM_i^f(\alpha^0, \mathrm{d}\alpha).$$

(This result is independent of the structure of $\mathrm{d}\alpha$.)

□

Remark 1. While we have proved that, for several SA techniques, a function and its affine transformation are coherent (even in a strict sense), it is intuitive to inductively believe that a function and its affine transformation share an *absolute* coherence, in that they are coherent for every existing SA technique. We leave the proof of this more general statement for future research.

5 Coherence between return rates and NPV

The investment risk can be defined as "the potential variability of financial outcomes" (White et al. 1997). The future outcomes of an investment are stochastic and the investor has limited information. Referring to NPV and IRR, Joy and Bradley (1973, p. 1255) wrote: "It has often been suggested that capital budgeting theory has over-emphasized the development of such techniques with little regard for the typically poor data used in project evaluation and the effect that errors in capital budgeting inputs have on project profitability." The practice of valuation criteria should be corroborated by a careful investment risk analysis.

Given an investment model based on a set of value drivers, SA allows the evaluator to identify the most relevant parameters in terms of variation of the value. The most relevant parameters are the risk factors that mainly influence the investment. After SA has been performed, the investment risk can be reduced through information insights on the main risk factors identified by the analysis; the collection of extra information on these parameters allows more precise estimates and a remarkable uncertainty reduction (Borgonovo and Peccati 2006b). Furthermore, the potential investor is able to appreciate the convenience of possible hedging strategies.

As the NPV is the main decision criterion in capital budgeting theory, the analysis of the parameters' relevance on NPV variability is fundamental. Any relative measure of worth should be consistent with NPV not only in terms of classical consistency but also in terms of output variability with respect to changes in the inputs.

[5] It is worth noting that f and g are coherent but not strictly coherent under $NPD1_i^f$ technique:

$$NPD1_i^g(\alpha^0) = g'_{\alpha_i}(\alpha^0) \cdot \frac{\alpha_i^0}{g(\alpha^0)} = l \cdot f'_{\alpha_i}(\alpha^0) \cdot \frac{\alpha_i^0}{g(\alpha^0)} \cdot \frac{f(\alpha^0)}{f(\alpha^0)} = l \cdot \frac{f(\alpha^0)}{g(\alpha^0)} \cdot NPD1_i^f(\alpha^0)$$

so that $|NPD1_i^f| > |NPD1_j^f|$ implies $|NPD1_i^g| > |NPD1_j^g|$. Therefore, the parameters' ranking in f and g is equal: $r^f = r^g$.

Definition 4. (Strong NPV-consistency) *Given an analysis technique T, a metric φ (and its associated decision criterion) is strongly NPV-consistent (or coherent with NPV) under T if it fulfills Definition 2 and NPV and φ are coherent functions. The metric φ is strictly NPV-consistent if the coherence is strict.*

If a metric/criterion possesses strong NPV-consistency, the investor can equivalently adopt NPV or such criterion for measuring value creation under uncertainty. In case a metric is not strongly NPV-consistent, the degree of incompatibility can be measured through Spearman's coefficient or through top-down coefficient, as seen in section 4.

We now show that the average ROI possesses strong NPV-consistency. To this end, we maintain the symbol $\alpha = (\alpha_1, \ldots, \alpha_n)$ as the vector of the project's value drivers and α^0 is the base value. We assume that the initial invested capital (or borrowed amount) is exogenously given, as well as the COC (and p). The economic profitability of P depends on the realization of the value drivers, which affect the FCFs, as seen in section 2: $F_t = F_t(\alpha)$, $t = 1, 2, \ldots p$. We now let $f(\alpha) = \text{NPV}(\alpha) = -B_0 + \sum_{t=1}^{p} F_t(\alpha)(1+k)^{-t}$ and $g(\alpha)$ denotes the average ROI.

Proposition 2. *For any fixed k, p and* $\mathbf{Dep} = (Dep_1, \ldots, Dep_p)$,[6] *average ROI and NPV are strongly consistent in a strict sense under the following techniques:*

(i) *Standardized regression coefficient*

(ii) *Sensitivity Indices in variance-based decomposition methods*

(iii) *Finite Change Sensitivity Indices*

(iv) *Helton's index*

(v) *Normalized Partial Derivative ($NPD2_i^f$)*

(vi) *Differential Importance Measure.*

Proof. The depreciation charge, Dep_t, does not depend on the value drivers; therefore, B does not depend on α and, using (9), one can write $g(\alpha) = q + l \cdot \text{NPV}(\alpha)$ where $q = k$ and $l = (1+k)/B$. The thesis follows from Proposition 1.[7] □

The above proposition guarantees that the value drivers' effect on the variability of $\bar{\imath}(B)$ and NPV is the same, not only in terms of ranks ($r^{\text{npv}} = r^{\bar{\imath}(B)}$) but also in terms of relevances ($R^{\text{npv}} = R^{\bar{\imath}(B)}$). Therefore, $\rho_{\bar{\imath}(B),\text{npv}} = \rho_{S^{\bar{\imath}(B)},S^{\text{npv}}} = 1$. This means that an investor can equivalently employ average ROI or NPV to analyze an investment under uncertainty.

6 Worked examples

In the previous sections we have shown that, given a depreciation plan, the average ROI is strongly consistent with NPV. The aim of this section is to discuss two models. The first analyzes an example with straight-line depreciation. The second one is a real-life application, illustrated in Hartman (2007, p. 344) and is based on declining balance depreciation. We will accomplish a SA by focusing on two techniques: FCSI and DIM.

[6] Obviously, to fix **Dep** is equivalent to fixing **B**.

[7] Evidently, $\bar{\imath}(B)$ is strongly NPV-consistent under $NPD1_i^f$ as well but not in a strict sense.

6.1 Straight-line depreciation

We discuss a simple model, consisting of a firm facing the opportunity of investing in a 4-period project whose estimated revenues and operating costs are R_t and O_t. We assume that the tax rate is zero, $\tau = 0$ (it is not a risk factor). This implies, from (1), $F_t = R_t - O_t$. The project's value drivers are then $\alpha_i = R_i$ for $i = 1, 2, 3, 4$ and $\alpha_i = O_{i-4}$ for $i = 5, 6, 7, 8$. Hence, the value driver's vector is $\alpha = \{R_1,\ R_2,\ R_3,\ R_4,\ O_1,\ O_2,\ O_3,\ O_4\}$. NPV is computed as:

$$\mathrm{NPV}(\alpha) = -B_0 + \frac{R_1 - O_1}{1+k} + \frac{R_2 - O_2}{(1+k)^2} + \frac{R_3 - O_3}{(1+k)^3} + \frac{R_4 - O_4}{(1+k)^4}.$$

We assume that straight-line (SL) depreciation is employed, which implies that the invested capital depreciates linearly with time: $\mathrm{Dep}_t = \gamma B_0$ where $\gamma = 1/p$. This means $B_t = B_0(1 - \gamma t)$ and, in turn, $B = B_0 \cdot \sum_{t=1}^{p}\big(1 - \frac{t-1}{p}\big)(1+k)^{-(t-1)}$. This implies $\mathrm{ROI}_t = (R_t - O_t - \gamma B_0)/(B_0(1 - \gamma(t-1)))$. The average ROI can be computed as a weighted average of the ROIs or as the ratio of overall profit to overall capital, B. Equivalently, using NPV, one can compute it as the value obtained by the AIRR function at $C = B$. Specifically, $\bar{\imath}(B) = k + \mathrm{NPV}(1+k)/B$.

Example 1. Assume $B_0 = 750$ and $k = 10\%$. Table 1 describes the base value

$$\alpha^0 = (R_1^0, R_2^0, R_3^0, R_4^0, O_1^0, O_2^0, O_3^0, O_4^0)$$

and reports the corresponding Free Cash Flows and valuation metrics. The NPV is $157.37 = -750 + 380/1.1 + 270/(1.1)^2 + 360/(1.1)^3 + 100/(1.1)^4$. Considering that depreciation charge is $750/4 = 187.5$, the vector of capitals associated with the average ROI is $\mathbf{B} = (750, 562.5, 375, 187.5, 0)$ and $B = 1712.15 = 750 + 562.5/1.1 + 375/(1.1)^2 + 187.5/(1.1)^3$. Therefore, the average ROI is equal to $\bar{\imath}(B) = 10\% + 157.37/1712.15 \cdot 1.1 = 20.11\%$.

Table 1: Investment evaluated in α^0

	0	1	2	3	4
R_t^0		580	570	560	400
O_t^0		200	300	200	300
F_t	−750	380	270	360	100
Valuation					
NPV	157.37				
$\bar{\imath}(B)$	20.11%				

Let α^1 be the vector of new values of revenues and costs (see Table 2), with the corresponding new values of F_t, NPV, and $\bar{\imath}(B)$. In α^1, NPV is 442.92, $\bar{\imath}(B)$ is 38.46%. The observed variations are: $\Delta\mathrm{NPV} = 285.55 = 442.92 - 157.37$; $\Delta\bar{\imath}(B) = 18.35\% = 38.46\% - 20.11\%$. Table 3 shows the First Order FCSIs ($\Phi_i^{1,f}$), the ranks (r_i^f), and the Savage scores of parameters (S_i^f) for NPV and $\bar{\imath}(B)$. The First Order FCSIs are equal: $\Phi_i^{1,\mathrm{npv}} = \Phi_i^{1,\bar{\imath}(B)}$. Hence, $\bar{\imath}(B)$ and NPV are strongly coherent in a strict sense and the degree of coherence is maximum: $\rho_{\bar{\imath}(B),\mathrm{npv}} = \rho_{S^{\bar{\imath}(B)},S^{\mathrm{npv}}} = 1$. (Note that, in this case, Total Order FCSIs and First Order FCSIs coincide, because the value drivers do not interact one another.)

Table 2: Investment evaluated in α^1

	0	1	2	3	4
R_t^1		800	810	780	630
O_t^1		350	250	380	600
F_t	−750	450	560	400	30
Valuation					
NPV	442.92				
$\bar{\imath}(B)$	38.46%				

Table 3: Finite Change Sensitivity Indices

	NPV			Average ROI		
Parameter	$\Phi_i^{T,\text{npv}} = \Phi_i^{1,\text{npv}}$	r_i^{npv}	S_i^{npv}	$\Phi_i^{T,\bar{\imath}(B)} = \Phi_i^{1,\bar{\imath}(B)}$	$r_i^{\bar{\imath}(B)}$	$S_i^{\bar{\imath}(B)}$
R_1	70.04%	2	1.718	70.04%	2	1.718
R_2	69.46%	3	1.218	69.46%	3	1.218
R_3	57.89%	4	0.885	57.89%	4	0.885
R_4	55.01%	5	0.635	55.01%	5	0.635
O_1	−47.76%	6	0.435	−47.76%	6	0.435
O_2	14.47%	8	0.125	14.47%	8	0.125
O_3	−47.36%	7	0.268	−47.36%	7	0.268
O_4	−71.76%	1	2.718	−71.76%	1	2.718
Correlations						
$\rho_{\bar{\imath}(B),\text{npv}}$	1					
$\rho_{S^{\bar{\imath}(B)},S^{\text{npv}}}$	1					

We now illustrate one numerical example where the DIM technique is used. It is a local SA technique, so it measures the value drivers' impact on the objective function in a neighbourhood of α^0. We assume that changes in the inputs are proportional to the base value ($\mathrm{d}\alpha_i = \xi \cdot \alpha_i^0$ for some $\xi \neq 0$) so the resulting DIM is

$$DIM_i^f(\alpha^0) = \frac{f'_{\alpha_i}(\alpha^0) \cdot \xi \cdot \alpha_i^0}{\sum_{j=1}^n f'_{\alpha_j}(\alpha^0) \cdot \xi \cdot \alpha_j^0} = \frac{f'_{\alpha_i}(\alpha^0) \cdot \alpha_i^0}{\sum_{j=1}^n f'_{\alpha_j}(\alpha^0) \cdot \alpha_j^0} \tag{23}$$

(Borgonovo and Apostolakis 2001, Borgonovo and Peccati 2004). In particular, the first partial derivatives of NPV(α), evaluated in α^0, are

$$\mathrm{NPV}'_{\alpha_i}(\alpha^0) = \begin{cases} (1+k)^{-i}, & i = 1,2,3,4; \\ -(1+k)^{-(i-4)}, & i = 5,6,7,8. \end{cases} \tag{24}$$

The first partial derivatives of $\bar{\imath}(B)$, evaluated in α^0, are

$$\bar{\imath}(B)'_{\alpha_i}(\alpha^0) = \mathrm{NPV}'_{\alpha_i}(\alpha^0) \cdot \frac{1+k}{B}. \tag{25}$$

Example 2. Consider a four-period investment P, with $B_0 = 900$ and $k = 8\%$. Hence, Dep_t=225 which implies $B = 2089.41$. The base value is

$$\alpha^0 = (900,\ 1000,\ 1100,\ 1200,\ 600,\ 700,\ 800,\ 900).$$

The corresponding cash-flow vector is $\mathbf{F} = (-900, 300, 300, 300, 300)$ and NPV $= 93.64$,

$\bar{\imath}(B) = 12.84\%$. Table 4 shows the DIMs, the ranks, and the Savage scores. As expected, the two metrics share the same rank and even the same DIMs. Therefore, they are strictly coherent.

Table 4: Coherence under DIM technique

		NPV			Average ROI		
Parameter	α^0	$DIM_i^{\text{npv}}(\alpha^0)$	r_i^{npv}	S_i^{npv}	$DIM_i^{\bar{\imath}(B)}(\alpha^0)$	$r_i^{\bar{\imath}(B)}$	$S_i^{\bar{\imath}(B)}$
R_1	900	83.87%	4	0.885	83.87%	4	0.885
R_2	1000	86.28%	3	1.218	86.28%	3	1.218
R_3	1100	87.88%	2	1.718	87.88%	2	1.718
R_4	1200	88.77%	1	2.718	88.77%	1	2.718
O_1	600	−55.91%	8	0.125	−55.91%	8	0.125
O_2	700	−60.40%	7	0.268	−60.40%	7	0.268
O_3	800	−63.91%	6	0.435	−63.91%	6	0.435
O_4	900	−66.58%	5	0.635	−66.58%	5	0.635
Correlations							
$\rho_{\bar{\imath}(B),\text{npv}}$	1						
$\rho_{S^{\bar{\imath}(B)},S^{\text{npv}}}$	1						

6.2 Declining-balance depreciation

We discuss a model based on (2). In particular, we borrow from Hartman (2007, p. 344) a real-life application. In 2003, Sunoco Inc. agreed to build a coke-making plant with an annual capacity of 550,000 tons per year in order to supply plants of International Steel Group (ISG) Inc. The cost of the plant was \$140 million and ISG agreed to purchase the coke (needed for producing steel) for the next 15 years.

Table 5 collects the (stochastic and non-stochastic) relevant data affecting the project's economic profitability. The 11 stochastic parameters are evaluated in the base case α^0. We assume that the facility is depreciated in 15 years with a double-declining balance switching to SL depreciation (DDB-SL), that is, $\text{Dep}_t = \max\big(2/p \cdot C_{t-1}; C_{t-1}/(p-t+1)\big)$. This implies that the depreciation schedule is

$$\mathbf{Dep} = (18.67, 16.18, 14.02, 12.15, 10.53, 9.13, 7.91, 6.86, 6.37, 6.37, 6.37, 6.37, 6.37, 6.37, 6.37).$$

From (2) the after-tax operating profit is obtained as

$$I_t = \big(q \cdot p_0(1+g_p)^t - M(1+g_m)^t - L(1+g_l)^t - E(1+g_e)^t - Ov - \text{Dep}_t\big)(1-\tau).$$

Table 6 describes the value drivers at α^0 and α^1 and the resulting value of NPV and average ROI. The individual and total contribution of the value drivers, as well as the ranking, are measured via the First Order FCSI and the Total Order FCSI respectively (Tables 7 and 8). Unlike the previous example, the two FCSIs are not equal, owing to nonzero interactions among the value drivers. As expected, the effect of each parameter on average ROI is the same as its effect on NPV, in terms of both magnitude and direction, which means that the average ROI and the NPV are strictly coherent.[8]

[8]It is interesting to note that, while the change in both NPV and average ROI is not so large, the effect of

Table 5: Sunoco project: Input data

Stochastic (value drivers)			
Annual production	q	0.55	million tons
Price	p_0	\$350	per ton
Price growth rate	g_p	2%	
Materials	M	\$27.5	million
Materials growth rate	g_m	2%	
Labor	L	\$75	million
Labor growth rate	g_l	5%	
Energy	E	\$20	million
Energy growth rate	g_e	3%	
Overhead	Ov	\$7	million
Tax rate	τ	35%	
Non-stochastic			
Investment		\$140	million
Salvage Value		\$0	million
COC		12%	
Periods		15	years
Dep Method		DDB-SL	

Table 6: Sunoco project evaluated in α^0 and α^1

Parameter	α^0	α^1
q	0.55	0.57
p_0	\$350	\$340
g_p	2.0%	2.5%
M	\$27.5	\$35.0
g_m	2.0%	3.6%
L	\$75	\$68
g_l	5.0%	4.0%
E	\$20	\$25
g_e	3.0%	2.0%
O	\$7	\$10
τ	35.0%	38.0%
Valuation	α^0	α^1
NPV	\$120.61	\$128.53
$\bar{\imath}(B)$	34.60%	36.08%

Table 7: Sunoco project: First Order FCSI

	NPV			Average ROI		
Parameter	$\Phi_i^{1,\text{npv}}$	r_i^{npv}	S_i^{npv}	$\Phi_i^{1,\bar{\imath}(B)}$	$r_i^{\bar{\imath}(B)}$	$S_i^{\bar{\imath}(B)}$
q	442.23%	3	1.520	442.23%	3	1.520
p_0	−347.47%	6	0.737	−347.47%	6	0.737
g_p	383.60%	4	1.187	383.60%	4	1.187
M	−473.82%	2	2.020	−473.82%	2	2.020
g_m	−183.24%	8	0.427	−183.24%	8	0.427
L	534.84%	1	3.020	534.84%	1	3.020
g_l	357.37%	5	0.937	357.37%	5	0.937
E	−336.21%	7	0.570	−336.21%	7	0.570
g_e	81.31%	11	0.091	81.31%	11	0.091
O	−167.82%	9	0.302	−167.82%	9	0.302
τ	−107.70%	10	0.191	−107.70%	10	0.191
Correlations						
$\rho_{\bar{\imath}(B),\text{npv}}$	1					
$\rho_{S^{\bar{\imath}(B)},S^{\text{npv}}}$	1					

Table 8: Sunoco project: Total Order FCSI

	NPV			Average ROI		
Parameter	$\Phi_i^{T,\text{npv}}$	r_i^{npv}	S_i^{npv}	$\Phi_i^{T,\bar{\imath}(B)}$	$r_i^{\bar{\imath}(B)}$	$S_i^{\bar{\imath}(B)}$
q	422.70%	3	1.520	422.70%	3	1.520
p_0	−354.32%	5	0.937	−354.32%	5	0.937
g_p	368.36%	4	1.187	368.36%	4	1.187
M	−499.62%	1	3.020	−499.62%	1	3.020
g_m	−222.45%	8	0.427	−222.45%	8	0.427
L	478.34%	2	2.020	478.34%	2	2.020
g_l	309.06%	6	0.737	309.06%	6	0.737
E	−301.30%	7	0.570	−301.30%	7	0.570
g_e	96.95%	11	0.091	96.95%	11	0.091
O	−160.07%	9	0.302	−160.07%	9	0.302
τ	−117.75%	10	0.191	−117.75%	10	0.191
Correlations						
$\rho_{\bar{\imath}(B),\text{npv}}$	1					
$\rho_{S^{\bar{\imath}(B)},S^{\text{npv}}}$	1					

We now use Sunoco's example to show the behavior of the two metrics with the DIM technique. The computation of DIMs is easy, given that the calculation of the partial derivatives of NPV with respect to each parameter is straightforward (see Appendix B for the list of derivatives) and the derivatives of the average ROI is obtained from (25).

The strict coherence obviously holds. It is interesting to note that, in this case, there are ties: The first rank is shared by two key drivers, the current price, p_0, and the quantity sold, q. An equal relative change of either parameter affects the average ROI (and the NPV) in the same way. The operating costs related to labor are top drivers (labor cost has rank 3 and its growth rate has rank 4). Much less impact have the growth rates in energy and materials (rank 10 and 11, respectively).

Table 9: Sunoco project: DIM technique

		NPV			Average ROI		
Parameter	α^0	$DIM_i^{\text{npv}}(\alpha^0)$	r_i^{npv}	S_i^{npv}	$DIM_i^{\bar{\imath}(B)}(\alpha^0)$	$r_i^{\bar{\imath}(B)}$	$S_i^{\bar{\imath}(B)}$
q	0.55	93.27%	1.5	2.520	93.27%	1.5	2.520
p_0	350	93.27%	1.5	2.520	93.27%	1.5	2.520
g_p	2%	11.54%	6	0.737	11.54%	6	0.737
M	27.5	−13.32%	5	0.937	−13.32%	5	0.937
g_m	2%	−1.65%	11	0.091	−1.65%	11	0.091
L	75	−43.95%	3	1.520	−43.95%	3	1.520
g_l	5%	−14.26%	4	1.187	−14.26%	4	1.187
E	20	−10.31%	7	0.570	−10.31%	7	0.570
g_e	3%	−1.95%	10	0.191	−1.95%	10	0.191
O	7	−3.00%	9	0.302	−3.00%	9	0.302
τ	35%	−9.64%	8	0.427	−9.64%	8	0.427
Correlations							
$\rho_{\bar{\imath}(B),\text{npv}}$	1						
$\rho_{S^{\bar{\imath}(B)},S^{\text{npv}}}$	1						

7 Concluding remarks

Many different investment criteria are available to managers, professionals and practitioners. NPV is considered a theoretically reliable measure of economic profitability. Industrial and financial investments are often evaluated through relative measures of worth as well. Recently, it has been introduced a new class of return rates named AIRR (Magni 2010, Magni 2013). This class includes the average ROI, which plays an important role in the appraisal of industrial investments (Magni 2015, Mørch et al. 2017). The average ROI exists and is unique, and is coherent with NPV in the sense that it correctly signals value creation or value destruction, just like the NPV (and, therefore, the decision made using either metric is the same).

This work provides a new definition of NPV-consistency making use of sensitivity analysis (SA). Given an SA technique, a metric is strongly consistent or coherent with NPV if it fulfills the classical definition of NPV-consistency and generates the same ranking of the

each parameter on the two metrics is extremely high. In this model, the NPV and the average ROI are highly sensitive to the contributions of each driver but, overall, the parameters' effects reciprocally compensate, in such a way that the resulting change is "smoothed".

value drivers as that generated by the NPV. If, in addition, the parameters' relevances are equal to the ones associated with NPV, then the metric and NPV are strongly consistent in a strict form.

We assume that the COC is exogenously fixed by the decision maker, as well as the initial investment and the lifetime of the project. After proving that an affine transformation of a function preserves the ranking, we show that the average ROI, being an affine transformation of NPV, is strongly NPV-consistent under several (possibly, all) different techniques of SA.

We have illustrated some simple numerical examples using FCSI (Borgonovo 2010a) and DIM (Borgonovo and Apostolakis 2001, Borgonovo and Peccati 2004), based on different depreciation plans (straight-line depreciation and accelerated depreciation). We have measured the degree of NPV-consistency via Spearman's (1904) coefficient and Iman and Conover's (1987) top-down coefficient. We have found that average ROI and NPV show perfect correlation and even strict consistency. However, we stress that not all AIRRs enjoy strong NPV-consistency, including the economic AIRR and the IRR, both showing degrees of incoherence that may be nonnegligible (see Appendix A).

The findings allow us to claim that the average ROI can be reliably associated with NPV, providing consistent pieces of information. Also, the average ROI is a good candidate for absolute NPV-consistency, to be intended as a strong coherence under any possible technique of SA (this should hold, given the affine relation between the average ROI and NPV). Future researches may be devoted to finding other relative measures of worth that enjoy strong NPV-consistency.

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Appendix A. (Non)strong consistency of other AIRRs

In principle, the class of AIRRs consists of infinitely many rates of return (albeit most of them non-economically significant), so it is no wonder that many of them are not strongly-

consistent. In this appendix we briefly focus on four special cases of AIRR, three of which are not strongly consistent with NPV.

Internal rate of return (IRR). Magni (2010, 2013) shows that the internal rate of return (IRR) is a special case of AIRR. Specifically, the IRR is a weighted mean of generally time-varying period rates, generated by any vector $\mathbf{C}$ fulfilling the following condition: $C = \sum_{t=1}^{p} \sum_{k=t}^{p} F_k(1+x)^{-(p-t+1)} \cdot (1+k)^{-(t-1)}$.[9] While the IRR is traditionally NPV-consistent (Hazen 2003), it suffers from some difficulties that have been extensively investigated in the literature. A part of it has been concerned with the necessary and sufficient condition for existence and uniqueness (e.g., Soper 1959, De Faro 1978, Bernhard 1980. See also Magni 2010 and references therein) or with project ranking (see Ekern 1981, Foster and Mitra 2003 for ranking of risk-free projects. See Ben-Horin and Kroll 2017 for ranking of nonequivalent-risk projects). In particular, Ekern (1981) and Foster and Mitra (2003) can be interpreted as supplying conditions of (non)existence of IRR in the interval $(0, +\infty)$, assuming that a project is ranked against the null alternative. Therefore, they provide a tool to measure the robustness of a value-creating project under changes in the COC and, at the same time, the conditions where IRR does not exist and cannot then be employed for ranking value drivers.[10] Percoco and Borgonovo (2012) show that, if the IRR exists and is unique, the ranking of value drivers provided by IRR is not equal to the ranking provided by the NPV, which means that the IRR is not strongly NPV-consistent. It is easy to see that its degree of NPV-inconsistency, as measured by Spearman's correlation coefficient or Iman and Conover's top-down coefficient, may be not negligible.[11] Also, the financial nature of the IRR is not unambiguously determined: An investment project may well turn to a financing project if value drivers change, which makes SA meaningless.

Economic AIRR (EAIRR). Another relevant AIRR is the economic AIRR, based on market values (Magni 2013, 2014, Barry and Robison 2014, Bosch-Badia et al. 2014). It is generated by picking $C_t = \sum_{k=t+1}^{p} F_k(1+k)^{t-k}$ for all $t = 1, \ldots, p-1$ (while $C_0 = -F_0$), which represents the economic value of the project at time t. The EAIRR is NPV-consistent in a traditional sense and, unlike the IRR, this AIRR always exists and is unique. However, just like the IRR, its financial nature may change under changes in the value drivers.

Strong consistency with NPV is not guaranteed because C_t (and, therefore, C) depends on $\mathbf{F}$ which, in turn, depends on the value drivers. Hence, it is not an affine transformation of NPV. The degree of inconsistency may be rather high.[12]

[9] This implies that the assumption $i_t = x$ for all t is sufficient but not necessary to generate a rate of return equal to IRR.

[10] It is usually believed that the case of no-IRR is very rare. However, in some engineering projects it is not infrequent that disposal and remedial costs occur at the terminal date, which is a necessary condition for inexistence of IRR. Most recently, Lima e Silva et al. (2017) focus on a very common transaction where the case of (multiple IRRs and) no IRR is the rule rather than the exception.

[11] For instance, in Example 1, the ranking generated by IRR with Total Order FCSIs is $(1, 2, 4, 6, 3, 8, 7, 5)$ and the top-down coefficient is $\rho_{S^{\text{npv}}, S^{\text{irr}}} = 0.409$. In Example 2, where the DIM technique is used, the parameters ranking supplied by IRR is $(1, 2, 3, 4, 8, 7, 5, 6)$ and the top-down coefficient is $\rho_{S^{\text{npv}}, S^{\text{irr}}} = 0.309$.

[12] In Example 1, the Total Order FCSIs for EAIRR generate the parameters' ranking $(1, 2, 4, 6, 3, 8, 5, 7)$ and $\rho_{S^{\text{npv}}, S^{\text{eairr}}} = 0.239$. In Example 2 the ranking is $(1, 2, 3, 4, 8, 7, 5, 6)$ (equal to the ranking of IRR) and, therefore, the top-down coefficient is equal as well: $\rho_{S^{\text{npv}}, S^{\text{eairr}}} = 0.309$.

Modified internal rate of return (MIRR). The MIRR approach, also known as the *external-rate-of-return* approach, consists of modifying project P by discounting and/or compounding some or all of its cash flows at an external rate so as to generate a modified project $P^{'}$ (with a modified cash-flow stream $\mathbf{F}^{'}$) bearing the same NPV as P but such that $\mathbf{F}^{'}$ has only one change in sign for the cash-flow stream. This guarantees that the IRR of $P^{'}$ (i.e., the MIRR of P) exists and is unique. The MIRR suffers from some ambiguities of definition: (i) it is not clear what the external rate should be, (ii) there are many ways to modify the project (resulting in different MIRRs), none of which seems to deserve a privileged status, and (iii) it does not actually measure P's rate of return (see Brealey and Myers 2000, Ross et al. 2011, Magni 2015).

Being an IRR of $P^{'}$, the MIRR is an AIRR of $P^{'}$ and is not strongly consistent. Further, the external rate from which it depends adds a source of uncertainty in the valuation process (it may be equal or different from the COC). This implies that the MIRR may not be NPV-consistent, not even in the traditional sense (see Magni 2015, Appendix).

Profitability Index (PI). PI is defined as $PI = \text{NPV}/B_0$. It is a well-known and widespread metric that measures the NPV per unit of initial investment. It is strongly consistent with NPV in a strict sense, as it is an affine transformation of NPV. It is also easily seen that the PI is strictly linked with AIRR; namely, if cash-flow accounting is used, that is, assets are expensed immediately (whence $B_t = 0$ for $t > 0$), then $B = B_0$ and, from (9), $\bar{\imath}(B_0) = k + \text{NPV}(1+k)/B_0$ whence $PI = (\bar{\imath}(B_0) - k)/(1+k)$. Therefore, PI is an affine transformation of the average ROI that is associated with a cash-flow-accounting depreciation system.

Appendix B. Partial derivatives

$$\mathrm{NPV}'_q(\alpha^0) = p_0 \cdot (1-\tau) \cdot \sum_{t=1}^{15} \left(\frac{1+g_p}{1+k}\right)^t ,$$

$$\mathrm{NPV}'_{p_0}(\alpha^0) = q \cdot (1-\tau) \cdot \sum_{t=1}^{15} \left(\frac{1+g_p}{1+k}\right)^t ,$$

$$\mathrm{NPV}'_{g_p}(\alpha^0) = p_0 \cdot q \cdot (1-\tau) \cdot \sum_{t=1}^{15} \frac{t \cdot (1+g_p)^{t-1}}{(1+k)^t} ,$$

$$\mathrm{NPV}'_M(\alpha^0) = -(1-\tau) \cdot \sum_{t=1}^{15} \left(\frac{1+g_m}{1+k}\right)^t ,$$

$$\mathrm{NPV}'_{g_m}(\alpha^0) = -M \cdot (1-\tau) \cdot \sum_{t=1}^{15} \frac{t \cdot (1+g_m)^{t-1}}{(1+k)^t} ,$$

$$\mathrm{NPV}'_L(\alpha^0) = -(1-\tau) \cdot \sum_{t=1}^{15} \left(\frac{1+g_l}{1+k}\right)^t ,$$

$$\mathrm{NPV}'_{g_l}(\alpha^0) = -L \cdot (1-\tau) \cdot \sum_{t=1}^{15} \frac{t \cdot (1+g_l)^{t-1}}{(1+k)^t} ,$$

$$\mathrm{NPV}'_E(\alpha^0) = -(1-\tau) \cdot \sum_{t=1}^{15} \left(\frac{1+g_e}{1+k}\right)^t ,$$

$$\mathrm{NPV}'_{g_e}(\alpha^0) = -E \cdot (1-\tau) \cdot \sum_{t=1}^{15} \frac{t \cdot (1+g_e)^{t-1}}{(1+k)^t} ,$$

$$\mathrm{NPV}'_O(\alpha^0) = -(1-\tau) \cdot \sum_{t=1}^{15} \frac{1}{(1+k)^t} ,$$

$$\mathrm{NPV}'_\tau(\alpha^0) = -\sum_{t=1}^{15} \frac{I_t}{(1-\tau) \cdot (1+k)^t} .$$