Intersectorial contagion risk in Morocco

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Abstract
This paper proposes an evaluation of intersectorial contagion risk through the analysis of the network of the intersectorial expositions, on the one hand, and the implementation of the approach of the contingent claims analysis (CCA) on the other hand. From this point of view, the matrix of the intersectorial expositions was approximated and of the indicators of centrality, resulting from the network analysis, were estimated. Then, using the CCA, which rises from the pricing theory of the options, the indicators of risk were calculated for each sector, in particular, the distance to the default, probability of default and the leverage ratio. The results obtained confirm that the financial and non-financial companies are the most systemic institutional sectors being able to constitute a principal channel of contagion. Lastly, an estimate of the joint and the conditional probabilities of default of the economic sectors was carried out, while taking as a starting point the work by Goodhart and al (2009) and by using the Archimedean copulas.

Keywords: systemic risk, contagion, financial stability.
JEL Classification: G20, G33, G23.
1. Introduction

In period post-crisis, the contagion risk constituted one of the fundamental pillars of the evaluation of financial stability. Indeed, the financial crisis which was transmitted to the real economy put forward the importance to identify the transmission channels of the shocks between the various economic agents, in order to guard itself against the systemic risks likely to involve such crises. In addition, the increasing intensity of the interconnections between the economic agents because of the increase in the financial assets in circulation in the economy deteriorates the extension risk of the shocks, in particular between the various economic sectors.

The evaluation of financial stability is not interested solely in the vulnerabilities of the various components of the financial system, which are the financial institutions, the capital market and the payment and regulation system, but also wonders about their interconnections and the possible channels of propagation of the shocks. Within this framework, the macro-prudential policy aims at acting on two dimensions of the systemic risk namely: temporal dimension and transverse dimension (Borio, 2009). First aims to control the pro-cyclical character of the financial system, while the second is interested in the phenomenon of contagion and the interconnections in the financial system as a whole and with the real sphere.

The first theoretical and empirical work relating to contagion risk, first of all, was interested in the interconnections between the financial institutions and particularly in the level of the interbank market. This work evaluates the implications of the default of a financial institution on the continuity of the activities of the other institutions on the markets concerned. The risk of intersectorial contagion is interested, in addition, with the transmission channels of the shocks inside the economic system, in particular with a view identify the economic sectors at origin of the shock or systemic matter.

Although the interest carried to the risk of contagion is very recent, several empirical techniques were proposed in order to capture the transverse dimension of the systemic risk. These techniques aim at identifying the interdependences between the economic actors and in particular the financial institutions and to quantify the negative externalities of them. The empirical approaches which are often adopted divide into four families (GFRS April 2009, chapter 2):

- The approach of networks based on the bilateral expositions between financial institutions aims at simulating the negotiable instruments of the default of an actor within the system.
- The models Co-risk evaluate the simultaneous movements of some variables relative in particular to the credit risk of the financial institutions.
- The matrix of the dependences is based on the estimate of the multivariate distributions of the outputs of assets of the financial institutions. These last make it possible to measure the linear and nonlinear correlations as well as the joint probabilities of default of the financial institutions.
- The models of intensity of default are statistical reduced form models in particular making it possible to measure synchronization between the events of default. This type of models is conceived to measure the direct and indirect financial assets defaults and to evaluate the behavior of dependence between the rates of default.

Concerning the intersectorial contagion, empirical work on the question remains limited and is interested primarily in the developments and the interactions between the balance-sheets of the various economic sectors. The most recent work proposes a framework of analysis taking as a starting point the Merton model (1974) and by the approach of the contagion claims analysis (CCA). The Merton model, by integrating uncertainties and it not linearity, puts forward measures of risk by basing on the method of Black and Scholes (1973) for pricing options. In this direction,
Gray and al (2007) and Gary and Malone (2008) extended the Merton model to an economy sectors (five sectors), whose balance-sheets are interdependent by the means of the financial assets. Recently, Castrén and al (2009) implemented this methodology to the sectors of the Euro area and derived several indicators from the risk of contagion, such as the probability of default and the distance to the default of each economic sector. The test results confirm that the European banking sector plays a systemic role in the transmission of the shocks.

In addition, Silva (2010) adopts a similar step to analyze the intersectorial contagion in Portugal by integrating two major changes: the taking into account of the credit risk and the introduction of the real assets into the balance-sheet of the households.

The approach recommended by ECB (Castrén and all (2009)) was adopted on the level of this document. This one consists initially with the calculation of the matrix of the intersectorial bilateral exposition’s which will be used to describe the transmission channels of shocks of a sector to another. Then, of the indicators of risk exposure of the various economic sectors are calculated, while taking as a starting point the approach CCA. A prolongation of this approach was considered by proposing an evaluation of the matrix of the intersectorial dependences, through the use of the joint and conditional probabilities of default (Goodhart and Al (2009)).

The rest of this paper is structured as follows: the second part forwards financial flows in circulation in the economy and an approximation of the matrix of the bilateral intersectorial financial expositions. This section also forwards some indicators inspired of the graph theory aiming at describing the systemic character of the institutional sectors. Then, the third party proposes an analysis of the CCA of the institutional sectors in order to calculate the probabilities of default of these sectors. Lastly, the last party forwards a new approach of evaluation of the risk of intersectorial contagion aiming at measuring the joint and conditional probabilities of default which can constitute indicators of the risk of contagion.

2. Intersectorial expositions matrix

Following the work of the ECB (2009), the FED (2007), the Bank of Austria (2011) and the Bank of Portugal (2010), the relative data with financial flows between the various institutional sectors are used, in order to work out the matrix of the bilateral expositions making it possible to identify the institutional sectors in systemic matter. Within this framework, the first under-party describes financial flows between the various economic sectors. Second, as for it, the step of calculation of the matrix of the expositions crossed between the principal economic sectors forwards.

2.1. Financial flows

The table of the financial transactions, worked out by the HCP (High Commission of Planning), recalls the whole of financial flows which circulate within the economy. It describes the variations of the financial assets and liabilities in the whole of the economic sectors, namely: the households (M), not financial firms (SNF), finance companies (EC.), Central Bank (BAM), public administration (AP), insurance companies (SA), other financial institutions (AIF) and rest of the world (RM). These financial flows, emanating from the national accounting, are used to approximate the matrix of the bilateral expositions and to implement CCA method. Various financial flows are classified by types of financial instruments, i.e.:

- Cash and deposits
- Bonds and shares
Loans
• Mutual funds’ assets
• Technical reserves of insurances
• Other accounts

The analysis of the development of these financial flows, during the period 1999 and 2007, emphasizes a prevalence of the finance companies in the financing of the economic activities compared to the different sources of financing. Thus, in 2007 the finance companies drained the majority of flows relating to the financial assets, comparatively to the other economic agents, and contributed massively to the collection of the saving and the financing of the other economic sectors (figure 1).

Figure 1: financial flows per institutional sector

Figure 1 shows the composition of liabilities and assets of the various economic sectors in 2007. For the majority of the sectors, the assets are composed to have in currency (deposits) and of bonds in various proportions, except for the finance companies of which the assets are mainly made up of the loans to the various sectors of the economy.

On the other hand, the analysis of the liabilities of the various institutional sectors reveals characteristics more distinct from the composition of the funding sources. For the finance companies, the liabilities are composed primarily of the deposits. Not financial societies resort at the same time to the banking financing and the market capital. For the public administrations the liabilities are made up mainly of loans.

On the level of the individual financial accounts, liabilities and assets must coincide. However, this is not inevitably the case for financial flows on a sector scale insofar as certain sectors can have systemic deficits whereas others are characterized by permanent surpluses. No financial firms and the Government (Public administrations) are generally debtors Nets whereas the households belong to the principal creditor sectors. In addition, if the deficits posted by the sectors borrowers exceed the surpluses recorded by the sectors lenders, the difference is financed by loan near the rest of the world.
In order to describe the need or exceed it financing of each institutional sector, the clear financial wealth is calculated like the difference between the assets and the passive ones.

Graph 2 forwards the development of this clear financial wealth of the various economic sectors from 2001 to 2007. The households, the finance companies and the other financial institutions release a surplus. In addition, the public administrations start to release a surplus as from 2006. On the other hand, not-financial firms remain in insufficiency structural because of nature of their activity.

The difference between the assets and the liabilities ones of the various sectors forms one exceed or a deficit of financing. In the event of overall deficit, the financing is done near the rest of the world; in the event of surplus, this last can be the subject of placement near the Rest of the world.
The analysis of the sum of the financial net wealth of the sectors indicates that during the period 2001-2006, the various institutional sectors released from the financial surpluses which were placed on the level of the overseas markets to height of 15 billion Dirhams annually. However, as from 2007, the institutional sectors post a need of almost 600 million Dirhams which was filled by the recourse to the financing near the Rest of the world (Figure 3). Graph 3 illustrates well the change of the situation of surplus to that of deficit of financing of the Moroccan economy in particular reflected by the inversion of account balance running in 2007.

3. Matrix of bilateral expositions and centrality measures

The table of the financial transactions makes it possible to identify financial flows of each economic sector without providing detailed information on the counterparts of the held assets or the funding sources of each sector.

In order to identify the bilateral financial expositions between the various economic sectors technique of optimization of the cross entropy is used. This technique help to obtain the expositions between sectors $i$ and $j$ called $x_{ij}$ starting from the inputs $x_{ij}^0$ initial by supposing that the sum of the loans (liabilities) of the lines is equal to that of the sum of the columns (ready assets).

Then, the expositions are obtained by the minimization of a distance compared to $x_{ij}^0$ under constraints related to the structure of the table of the financial transactions. The disadvantage of this technique, is that it is based on an assumption of complete structure of the financial system, i.e. each economic sector with financial relationships to the whole of the other sectors.

Let us note the matrix of the exhibitions $X_k$ :

$$X_k = \begin{bmatrix} x_{i1} & \cdots & x_{ij} & \cdots & x_{iN} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i1} & \cdots & x_{ij} & \cdots & x_{iN} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{N1} & \cdots & x_{nj} & \cdots & x_{NN} \end{bmatrix}$$

The algorithm of optimization of the method of the cross entropy takes the following form:

$$MINx_{ij} = \sum_{j=1}^{n} \sum_{i=1}^{n} x_{ij} \ln \left( \frac{x_{ij}}{x_{ij}^0} \right)$$

Under the following constraints:

$$a_j = \sum_{i=1}^{n} x_{ij}$$

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1This method resulting from the information theory whose first work was initiated by Kullback and Al (1951), Jaynes (1957) and Shannon and Al (1975).
\[ p_i = \sum_{j=1}^{n} x_{ij} \quad (3) \]

with:
- \( x_{ij} \): The expositions of sector i compared to the sector j;
- \( a_j \): Summon assets of the sector j;
- \( p_i \): Summon passive sector i;

\[ x_{ij} \geq 0 \text{ et } x_{ij} = 0 \text{ while } x_{ij}^0 = 0, \ln(0/0) = 0 \text{ et } x_{ij}^0 = (p_i \times a_j)/\text{sum} \]

On the basis of series published by the HCP on the level of the table of the financial transactions and whose last official observations go back to 2007, the assets and the liabilities overall ones were used to obtain the initial distributions allowing to approximate the bilateral financial expositions. Following obtaining these bilateral expositions, the schematization of the sectors in networks (Figure 4) makes it possible to visualize the development of the financial expositions between the various sectors of the Moroccan economy, between 2000 and 2007, by taking account of the weights of the financial affairs.

The analysis of the development of the intersectorial network shows an increase in volumes of the transactions as well as a reinforcement of the interconnections between the finance companies, not-financial societies and the households between 2000 and 2007. The increase in the volume of the transactions is collected through the size in the nodes, whereas the intensity of the bilateral interconnections is apprehended by the thickness of the features linking the various sectors.

The network of the bilateral expositions thus reveals the important development of the volume of the financial assets of the finance companies, the households and not-financial society. In addition, the sector of the other financial institutions and the Rest of the world also know a considerable increase of the transactions because in particular to the increasingly important recourse to the capital market and the external financing. Central Bank also knows an increase in her assets as well as her interconnection with the other sectors and in particular the finance companies because of the increase of the external assets combined with that of her interventions on the interbank market (Figure 4).
Figures 4: networks of the bilateral expositions between the various economic sectors in Morocco (2000 (up) and 2007 (down))
The graph theory makes it possible to measure the systemic character of the economic sectors in particular through the indicators of centrality of each sector represented by a node. The degree of centrality measures the degree of systemicity of each sector. The systemic character is identified by the capacity of this component to deal with the whole of the other actors of the system. The degree of centrality can be broken up into two under-indicators namely: degrees entering (in degree) and outgoing (out degree). First is relating to financial flows coming from the other economic sectors, while second informs on those intended for the other components of the system. In addition, the clean vector informs about the most systemic components of the system by studying the structure and the position of each economic sector in the system starting from the factorial analysis of the network.
Figure 5 : Indicators of centrality between 1999 and 2007

Outdegree des secteurs économiques entre 1999 et 2007

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Vecteurs propres des secteurs économiques entre 1999 et 2007

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Figure 5 : Indicators of centrality between 1999 and 2007
The calculations carried out on the basis of matrix of the bilateral financial expositions confirm that in 2007 the financial companies are the actors the most systemic follow-ups of the households and the not-financial firms. The results of calculations relating to the degrees of centrality and the clean vectors confirm this conclusion (Figure 5).

4. Measures of risk exposure

The main objective of this party is to quantify the probability of default which can interrupt the normal functioning of the activities of each sector. The determination of the individual probabilities of default of each sector will then make it possible to measure the joint and conditional probabilities of default.

On the scale of the economy, the intersectorial contagion, represented by the transmission of shocks in the network of the intersectorial expositions, can take place via the channel of the balance-sheets. The basic assumptions subtending such a phenomenon of contagion suppose however the evaluation with the mark to market as well as an immediate adjustment of the assets value and passive of the balance-sheet.

In order to describe this phenomenon, let us suppose the existence of 3 sectors S1, S2 and S3, the S2 sectors and holding S3 of the shares of the S1 sector. Within this framework, the contagion of a shock on the balance-sheet of the S1 sector in S2 and S3 are carried out through the loss of clear financial wealth which takes place on the level of the shares of the S2 sectors and holding S3 of the shares of S1. Since the counterparts of S1 have a book-keeping with the mark-to-market, these last must also deduce the losses on their assets from the value from their own actions. The fall of the shares of S2 and S3 will be also reflected in the contagion of the assets of the sectors holding them and so on. The process of contagion of depreciations continues as much as certain sectors still record positive gains which cancel the initial losses, or stops when the shock reached a sector which is connected with no other sector or which is not prone to the mark-to-market. Consequently, it appears that the transmission system of the shocks is closely related to the method of valorization of the assets and accounting of more or depreciations [Castren (2011)].

From this point of view, Graya, Merton and al (2007) propose an approach based on the analysis of the contingent claims analysis (CCA), in order to measure the default risk of each economic sector. The approach suggested by the authors aims at evaluating the risk of intersectorial contagion while being based on a balance sheet analysis of the institutional sectors.

4.1. CCA approach

In the beginning, this approach was proposed by Merton (1974) to evaluate the risk of default of only one institution on the basis of its balance-sheet. Method CCA, based on the theory of the options of Black, Scholes and Merton (1973), compares a balance-sheet to an addition of options\(^2\). It consists thus of the generalization of the theory of the pricing of option initiated by Black & Scholes (1973) and Merton (1974) with liabilities and assets of a firm (or a given sector). Merton and al (2007) define Contingent Assets, as being a financial credit whose future value of the incomes depends on the value of another assets. The prototype of contingent assets is thus an option which gives the right to buy or to sell the assets subjacent at a price specific (strike price) to a given

\(^2\)The shareholders hold call options and the lenders have put options.
date. The value of a share $E$ issued by a firm (or a given sector) and whose value depends on the assets has this firm or (sector given) is also contingent assets.

The CCA, first of all, was implemented to the firms and banks before being wide by Merton et al. with the various sectors of the economy. The basic analytical tool of the model of Merton is thus the corporate balance sheet, of the bank or the sector, adjusted by dimension “risks” with a view evaluate the sensitivity of liabilities and assets to the external shocks. When the assets drop, the risky debt also drops and the spread credit increases. In this model, the default (distress) is represented by a fall of the assets on a level lower than the value of the passive ones. Also, starting from the distribution of the value of market of the assets, it is possible to calculate a certain number of indicators of systemic risk such as the probability of default (PoD) and the Distance-to-Distress

![Figure 6: distance to the default and probability of default of an optional balance-sheet](image)

The assets value ($A$) is regarded as a random variable whose evolution depends on its uncertainty and yield ($R, \sigma$). At one moment “$T$” given, this value can become lower (or higher) than the value of its debts ($B$). Thus, the probability that the firm is lacking is equal to the probability that “With” becomes lower than “$B$” and corresponds to the area grayed below the value of the debts. Approach CCA help to determine the indicators of key risk, in particular, the distance to the default, the probability of default, the rate of loss discounted and credit risk spread.

This analysis was generalized for the economic sectors. Indeed, using the balance-sheets of the latter, it is possible to evaluate the level of the risk incurred by each sector and to calculate the probability of default and its distance to the default. According to Graya and al (2007) and Castrén and al (2009), the value of a sector is equal the sum of its shares and its debt, less one put option which the lenders hold:

$$A_t = E_t + B_t - P_t \quad (4)$$

with:

- $A_t$ : Assets value of one sector at the moment $T$;
- $B_t$ : Overall debt at the moment $T$ (value with the balance-sheet);
$E_t$ : The stock's value (option to buy);
$P_t$ : Value of the put option;

In reference to the options theory, if the assets value is higher than the level of debt, the shareholders can profit from the difference. This implies that they hold an option to buy on the firm or the sector. However, in the contrary case, where the assets value is lower than the debt, the backers can ask for the liquidation of the firm by carrying out their put option ($P_t$).

Thus, the shareholders of the firm or the sector hold:

$$Max(0, A_t - B_t) \quad (5)$$

and the debtors profit from:

$$Min(0, B_t - A_t) \quad (6)$$

According to Black and Scholes (1973) and Merton (1974), the assets value of the economic sector follows a stochastic process modelled like a geometrical Brownian movement:

$$dA_t = rA_t dt + \sigma_A A_t dz \quad (7)$$

whit:

dz : Process of Wiener $\varepsilon\sqrt{t}$
$r$ : Risk-free interest rate
$\sigma_A$ : Volatility of the assets value

According to Black and Scholes (1973), the solution of the Brownian movement led to the following expression:

$$E_t = A_t N(d_1) - B_t e^{-r_t}N(d_2) \quad (8)$$

In addition the Lemma of Ito provides the relation between the volatility of the shares and the value and the volatility of the assets, making it possible to build a system of equation whose solution facilitates the calculation of the indicators of risk exposure:

$$\sigma_{fp} = \frac{N(d_1) * A_t * \sigma_A}{E_t} \quad (9)$$

with:

$N(\cdot)$ : Function of distribution of the normal law
$\sigma_{fp}$ : The volatility of the shares
$A_t/B_t$ : The inverse of leverage ratio

Let us note that:

$$d_1 = \frac{\ln(A_t/B_t) + (r + \sigma_A^2/2)t}{\sigma_A\sqrt{t}} \quad (10)$$
Then the distance to default (DD) is equal to:

\[ d_2 = d_1 - \sigma \sqrt{t} \]  

(11)

\[ DD = d_2 = \frac{\ln(A_t/B_t) + (r - \frac{\sigma^2}{2})t}{\sigma \sqrt{t}} \]  

(12)

Other indicators can be calculated, in particular, the leverage ratio and the implicit volatility of each sector. In order to determine the probabilities of default and the other indicators of risk, the relative informations with the assets value and its volatility are necessary. However this information is unobservable for the whole of the economic sectors. Within this framework, the standard technique of resolution suggested by Gray et al. (2007) and Moody' S (2002) consist in solving the system of equation 8 and 9 using a program of nonlinear optimization.

4.2. Data

The installation of this type of modeling requires to have the relative informations with the debt of each sector, the stock’s value and of the assets like their respective volatilities. The data used in this work result from the tables of the financial transactions and the accounts from the institutional sectors.

The practical application of Merton model in order to evaluate the risk exposure of the economic sectors in Morocco, raises certain relative questions with the data to use. First of all, certain economic sectors do not have value gone their shares, because they do not issue shares on the stock markets: it is in particular the case of the public administrations and the households. In this situation, Gray and al (2007) and Castrén and al (2009) propose to use for the case of the public administrations, the sum of the net financial wealth and the issued obligations and only the first for the case of the households. For the other sectors, namely: finance companies, not-financial firms, the insurances and the other financial institutions, the value of their shares can be approximated via the market capitalization on the Moroccan stock market.

Concerning the debt of the sectors, the solution by approach CCA adopted by Moody’ S (2002) was used in this work. The latter considers the point of distress (level of the debt beyond whose the firm is at default) as the sum of the debts of short term and half of the debts of long run. In this work, the financial instruments were classified according to their nature, assets of short and long run. Thus, the deposits, the fiduciary currency, the short-term loans, the debt on short term, the derivative instruments and the other accounts of amounts receivable were regarded as being debts of short term. In addition, the assets constituting the debts of long run are: debt of long run, long-term credits, assets of OPCVM, the nets assets in life assurances, pension funds and premium accounts of insurance (Graya and Al (2007) and Castrén and Al (2009)).

In addition, the relative data with the volatility of the shares are approximated in the following way: for the sectors present at the Casablanca Stock Exchange, volatility in 12 months of the sectorial indices is used. Concerning, the sectors which do not issue documents of title volatility in

\[ f(x) = 0 \]
12 months of interest rates of the Treasury bills with 10 years is considered, according to the step adapted by the ECB.
Lastly, for interest rate (R) to use in the model, the proposal of Graya and al (2007) and Castrén and al (2009), were considered and interest rate is equal to risk-free interest rate plus 45% of the volatility of the prices on the market\(^4\).

4.3. Results

Approach CCA implemented to the economic sectors makes it possible to calculate four indicators of risk namely: probability of default, the distance to default, the volatility of the assets and the leverage ratio.

With regard to the distance to the default\(^5\), the results of this work show a certain stagnation during the first six years (2000-2005) around a median value of 0.4. Thus, 0.4 times the volatility of the financial asset of the institutional sectors are necessary to arrive at a level of inheritance in lower part of the contracted debt. However and as from 2005, a fall of the distance to default was noted on the level of three sectors in particular the finance companies, not-financial society and the other financial institutions (see figure 8). It records at the end of 2007 level lower than 0.2 times the standard deviation of the financial asset. This implies an increase in the risks incurred by these sectors in particular.

![Figure 8: Distance to default of the institutional sector's in Morocco](image)

The probability of default of the institutional sectors recorded, as for it a stability during the period 2000-2005 with a significant decrease in 2003 for the sector of the insurances because of the reduction of their commitment compared to the stock's value. In addition, since the end of 2005, the probability of default increased to reach in the end of 2007 level of 45% for the three sectors, finance companies, not-financial society and other financial institutions (figure 9). Concerning the\(^4\)The level of volatility retained depends on each market.
\(^5\)More the distance to default (DD) decreases more the risk increases. A DD equal to 0 indicates a crisis situation.
other sectors, the probability of default stagnated in the neighborhoods of 30% over the two last years of the analysis.

Growth of the probability of default and the fall of the distance to the default of the finance companies, of not – financial enterprises and other financial institutions, rise from an increase in volatility in the assets in the sectors in question. Indeed, as from 2005 a significant rise of the volatility of the sectors is perceptible (Figure 10). However, for the other sectors volatility remains stationary around 2 during all the period of the study. In this case, the rise of the volatility of the sectors often represents the stock rise of the assets.

In addition, approach CCA makes it possible to provide an evaluation of the leverage ratio for each economic sector. Estimated by the relationship between the assets and the shares of each sector, the leverage ratio was marked by a specific development during the period 2000-2007. Indeed, for
the two sectors, insurance companies and finance companies, and a way less marked for not-financial enterprises and the public administrations, an increase in the ratio of leverage is to be stressed during the period 2003-2005. This can be explained, partly, by the increase in the funds at the disposal of these sectors and emanating of the other economic agents. However, as from 2005 a decrease tendency is observed because of the stock rise of market of the assets. For the households, the ratio of financial leverage was stabilized between the period 2001-2005 close relations of zero, before starting a perceptible rise as from 2006. This increase is explained primarily by the recourse more and more of the households to the mortgage and consumption loans, the increase in the transfers of the Moroccan’s residents foreign and raises its receipts of privatization (figure 11).

![Figure 11: leverage ratio of institutional sectors of Morocco](image)

The advantage of the Merton model is its capacity to evaluate the contingent financial assets prices and other indicators of default by adopting the options theory. However, to reach this model that point formulates some constraining assumptions in link with the dynamics of the assets prices, the nature of the risk and the choice of interest rates. Indeed, the model is developed in a world at the risk free (arbitrated prices), where, on the one hand, certain imperfections of market cannot exist (transaction and agency costs) and on the other hand, only involved interest rate is that risk-free. Moreover, in the model of Merton the contractual commitments can be regulated between two exact dates (except for the case of the American options), therefore the possibility of regulation by anticipation or rescheduling possible of the debt is excluded. A last fundamental limit of the model of Merton is that he considers that corporate stock follows a Wiener movement (geometrical movement Brownian) which supposes a normal distribution generally underestimating the probability of supervening of the extreme events such as the financial crises.

5. Joint and conditional probability of default

Work of Goodhart and al (2009) proposed an approach of measure of the united and conditional probabilities of default of the banks while being based on their individual probabilities of default. Within this framework, this section extends the methodology of Goodhart and al (2009) [Goodhart] with the economic sectors with a view determine their joint and conditional probabilities of default on the basis of individual probability obtained using approach CCA. The objective is to quantify
the probability that a sector is at default consequently default of another economic sector (conditional probability of default), or that two economic sectors are simultaneously at fault (joint probability of default).

According to Goodhart and al (2009), it is possible to forward the matrix of the dependences in the following form:

<table>
<thead>
<tr>
<th></th>
<th>Secteur 1</th>
<th>Secteur 2</th>
<th>Secteur 3</th>
<th>...</th>
<th>Secteur N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Secteur 1</td>
<td>1</td>
<td>P(S1/S2)</td>
<td>P(S1/S3)</td>
<td>...</td>
<td>P(S1/Sn)</td>
</tr>
<tr>
<td>Secteur 2</td>
<td>P(S2/S1)</td>
<td>1</td>
<td>P(S2/S3)</td>
<td>...</td>
<td>P(S2/Sn)</td>
</tr>
<tr>
<td>Secteur 3</td>
<td>P(S3/S1)</td>
<td>P(S3/S2)</td>
<td>1</td>
<td>...</td>
<td>P(S3/Sn)</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Secteur N</td>
<td>P(Sn/S1)</td>
<td>P(Sn/S2)</td>
<td>P(Sn/S3)</td>
<td>...</td>
<td>1</td>
</tr>
</tbody>
</table>

Source: Goodhart et al. (2009)

Probability that a sector X may be at default under condition of failure of the sector Y is the relationship between the joint probability of default of the two economic sectors X, Y and probability of default of Y:

\[ P(X / Y) = \frac{P(X \cap Y)}{P(Y)} \]  

(13)

For the determination of joint probabilities of default \( P(X, Y) \), of the economic sectors, the copulas are used with a view generate the structure of dependence of the sectors. The copulas make it possible to distinguish two types of information, on the random variables: (1) the relative information with the marginal distribution of each variable and (2) information on their structure of dependence (Sklar (1959)).

In order to illustrate this matter, are \( x \) and \( y \) two random variables with marginal probability distributions \( X \rightarrow F(X) \) and there \( Y \rightarrow H \) and is \( G(x, y) \) their joint distribution. With a view transform \( X \) and there of two random variables of uniform distribution \((0,1)\), let us pose \( F(x) = u \) and \( H(y) = v \), \( u \) and \( v \) having a function of joint density \( c(u, v) \). In this direction, the copula function \( C(u,v) \) allows to couple the functions of marginal distributions \( F(x) \) and \( H(y) \) to lead to a structure of single dependences. It is defined as follows:

\[ c[u, v] = \frac{g[x, y]}{f(x)h(y)} \]  

(14)

\(^6\text{When the distributions of the variables are not elliptic (of Levy type or with tendency) the standard correlation does not make it possible any more to generate structures of efficient dependences.}\)
The determination of the structure of dependence between the probabilities of default of the various institutional sectors will make it possible to generate the joint probabilities of default $P(X \cap Y)$, which will facilitate the conditional probability of default $P(X / Y)$. Concerning the choice of the parametric copula which will be used to identify the structure of dependence, a copula of Gumbel was retained in order to impose the asymmetrical one on the joint probability of default\(^7\).

The analysis of the structure of dependence, through the use of the nonparametric coefficient of correlation of Kendall, emphasized a strong correlation between the probabilities of default of 2007; the results obtained are forwarded to the level of the table below:

<table>
<thead>
<tr>
<th>Sectors</th>
<th>SNF</th>
<th>EC</th>
<th>AIF</th>
<th>SA</th>
<th>AP</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNF</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EC</td>
<td>0.884</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AIF</td>
<td>0.872</td>
<td>0.884</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SA</td>
<td>0.834</td>
<td>0.856</td>
<td>0.835</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AP</td>
<td>0.8079</td>
<td>0.872</td>
<td>0.842</td>
<td>0.886</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>0.8757</td>
<td>0.902</td>
<td>0.848</td>
<td>0.886</td>
<td>0.890</td>
<td>1</td>
</tr>
</tbody>
</table>

With regard to the distances to the default of the institutional sectors, the analysis of the structure of dependence gives the following results:

<table>
<thead>
<tr>
<th>Sectors</th>
<th>SNF</th>
<th>EC</th>
<th>AIF</th>
<th>SA</th>
<th>AP</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNF</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EC</td>
<td>0.67</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AIF</td>
<td>0.42</td>
<td>0.37</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SA</td>
<td>-0.19</td>
<td>-0.05</td>
<td>-0.39</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AP</td>
<td>0.02</td>
<td>0.27</td>
<td>0.13</td>
<td>0.18</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>0.21</td>
<td>0.36</td>
<td>0.11</td>
<td>0.19</td>
<td>0.31</td>
<td>1</td>
</tr>
</tbody>
</table>

The analysis of the structure of the correlations between the distances to default of the Moroccan economic sectors shows that:

- The evolution of the distance to default of the finance companies is positively correlated with those of not-finance enterprises, the households and other financial institutions. Thus, any development with the rise or the fall of the distance to default of these sectors coincides with an identical trend of the distance to default of the finance companies,
- The insurance companies are negatively correlated with the sectors of EC, SNF and AIF, which means that any development of the distance to the default of these sectors coincides with an opposite evolution of insurances sector, with a rather small proportion with regard to EC.

The matrix of the joint probability of default also makes it possible to measure the probability that two institutional sectors fall at fault simultaneously.

---

\(^7\)This copula help to take account of the positive dependences and does not impose symmetry against the other elliptic copulas.
### Matrix of joint probability of default (JPoD)

<table>
<thead>
<tr>
<th>Sectors</th>
<th>SNF</th>
<th>EC</th>
<th>AIF</th>
<th>SA</th>
<th>AP</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNF</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EC</td>
<td>0.005</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AIF</td>
<td>0.008</td>
<td>0.01</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SA</td>
<td>0.03</td>
<td>0.05</td>
<td>0.07</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AP</td>
<td>0.03</td>
<td>0.04</td>
<td>0.07</td>
<td>0.31</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>0.02</td>
<td>0.04</td>
<td>0.06</td>
<td>0.26</td>
<td>0.25</td>
<td>1</td>
</tr>
</tbody>
</table>

The results obtained help to deduce that the probability that two institutional sectors are lacking simultaneously is very weak, except for the case from the sectors from the households, the insurance companies and the public administrations. Thus, there is 26% of chance to record a default of the households and insurances at the same time and 25% of chance that the households and the public administrations are in difficulty.

From the results relating to the joint probabilities of default, it is possible to extract the conditional probabilities and to measure the impact of the default of a sector on the impact strength of another economic sector. The following table forwards the conditional probabilities of default for each sector.

### Matrix of conditional probability of default (CPoD)

<table>
<thead>
<tr>
<th>Sectors</th>
<th>SNF</th>
<th>EC</th>
<th>AIF</th>
<th>SA</th>
<th>AP</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNF</td>
<td>1</td>
<td>0.012</td>
<td>0.019</td>
<td>0.132</td>
<td>0.122</td>
<td>0.091</td>
</tr>
<tr>
<td>EC</td>
<td>0.012</td>
<td>1</td>
<td>0.027</td>
<td>0.184</td>
<td>0.170</td>
<td>0.127</td>
</tr>
<tr>
<td>AIF</td>
<td>0.018</td>
<td>0.026</td>
<td>1</td>
<td>0.277</td>
<td>0.257</td>
<td>0.191</td>
</tr>
<tr>
<td>SA</td>
<td>0.078</td>
<td>0.111</td>
<td>0.175</td>
<td>1</td>
<td>0.956</td>
<td>0.814</td>
</tr>
<tr>
<td>AP</td>
<td>0.075</td>
<td>0.106</td>
<td>0.167</td>
<td>0.99</td>
<td>1</td>
<td>0.778</td>
</tr>
<tr>
<td>M</td>
<td>0.062</td>
<td>0.088</td>
<td>0.139</td>
<td>0.93</td>
<td>0.866</td>
<td>1</td>
</tr>
</tbody>
</table>

The matrix of the conditional probabilities indicates that the finance companies have 13% of chance to be at default if the sector of the households is at default and a probability of default of 17% in the event of default of the public administrations. As regards the sector of the insurances, these last have 96% of chance to be at default if the public administrations and the households have difficulties. Concerning the other financial institutions, they have a probability of default bordering the 28% in the event of default of the insurance companies, the public administrations or the households.

The determination of the probability of conditional default facilitate to measure the extent of the transmission of the shocks between the institutional sectors. Thus, a high probability means that there is a strong interdependence between two sectors and the difficulties being able to force the activity of the one can affect the other. In addition, these conditional probabilities of default constitute important indicators of the systemic risk of contagion. They assist to follow the evolution of the interconnections between the institutional sectors and in the event of shock or of crisis in a specific sector, they can help to contain the transmission of the shocks by the implementation of the appropriate measures to the sectors at the origin of the shock or strong interdependence.
6. Conclusion

The international financial crisis put forward the importance to identify the transmission channels of the shocks in the financial system and between the various economic sectors. In particular, the reversal of the cycle and the recessions noted following the fall of Lehmann Brother (2008) revealed that the interconnections between the real sphere and the financial system could be a powerful channel of contagion.

Also, the installation of an operative paragraph of evaluation of the interactions between the economic sectors and the financial institutions of intermediation will help the financial authority to quantify the risk of contagion and to identify the transmission channels of the possible shocks. However, work on the intersectorial contagion remains very limited because of the complexity of the subject.

While taking as a starting point the recent work of the ECB (2009) and the IMF (2009), this paper proposed an evaluation of the intersectorial contagion while being based on two approaches. The first step uses the analysis in network and the CCA method and the second take as a starting point the work relating to the matrix of the dependences and the joint and conditional probabilities of default.

The results obtained confirm that the finance and no financial companies and the insurances constitute the most systemic sectors and most connected and thus the principal vectors of transmission of the shocks over the period 2000-2007.

In addition, in the event of advent of shock affecting a component of the economic system, the indicators of the risk of contagion proposed by this work can help with the formulation of a strategy of intervention aiming, initially, the sectors at the origin of the shock or those strongly interconnected. Lastly, the indicators of the risk of contagion must be confronted with the other analytical tools with a view lead to a complete evaluation of the situation of the economic agents.
Références


[16]. Haldane, A. (2009), Rethinking the financial network, Speech at Financial Student Association, Amsterdam, April.


